

The Structural Design of Maillart's Chiasso Shed (1924): A Graphic Procedure

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Summary

This paper shows how the geometry of one of Robert Maillart's most intriguing projects, the Chiasso Shed, was created using graphics. Originating in the sixteenth century, graphic vectorial equilibrium has been used for studying a wide variety of structural problems. Among other things, it allows a form to be adjusted so that it only encounters axial loads while guaranteeing equilibrium. Here, we see that the general appearance of the structure was determined with reference to a uniformly distributed load, and then concrete was placed along force trajectories. Maillart allowed for some geometric inaccuracies remaining with regard to the way he designed asymmetric loadings to be supported by stiffening members. In so doing, emphasis is placed on efficient axial forces, similar to our modern strut-and-tie approach, in the design of concrete structures.

Keywords: structural design; graphic statics; concrete; strut-and-tie; Maillart; Chiasso; Magazzini Generali.

Introduction

Over a career spanning 40 years, Robert Maillart (1872–1940) designed and collaborated nearly 300 different projects. These include around 50 well-known railway and road bridges; others are mainly structures for buildings. Among these, the Chiasso Shed is perhaps the most interesting and intriguing. Maillart designed the five-storey warehouse building (Fig. 1) with architect Brenni in early 1924, but he found a solution for the shed only 6 months later.¹ Shortly after completion, the form of the shed was criticised for being forced and arbitrary.²

Many authors have given their own justification to explain the shed form: an analogy with natural forms,³ stylistic references,⁴ an analogy of the flexural behaviour of a simply supported

beam with cantilevers,⁵ reference to the Vierendeel truss⁶ etc. Despite Max Bill's observation that "the form follows the flow of forces",⁴ it seems that

few would have considered the form as resulting from a simple vectorial equilibrium obtained by graphic statics in accordance with the contextual constraints imposed by the warehouse building. Vectorial equilibrium is particularly evident in the scheme of forces published by Bill (Fig. 2). Furthermore, studying the Maillart archives leaves little doubt that he used graphic statics for designing the structures. However, the working drawings of the Chiasso Shed are lost.

Graphic Statics

The first use of graphic vectorial representation to analyse a structure can be encountered as early as 1586 in the writings of Simon Stevin⁷ who established a kind of parallelogram law of forces.



Fig. 1: Magazzini Generali in Chiasso, Robert Maillart, 1924 (ETH-Bibliothek Zurich, image archive/Robert Maillart archive)

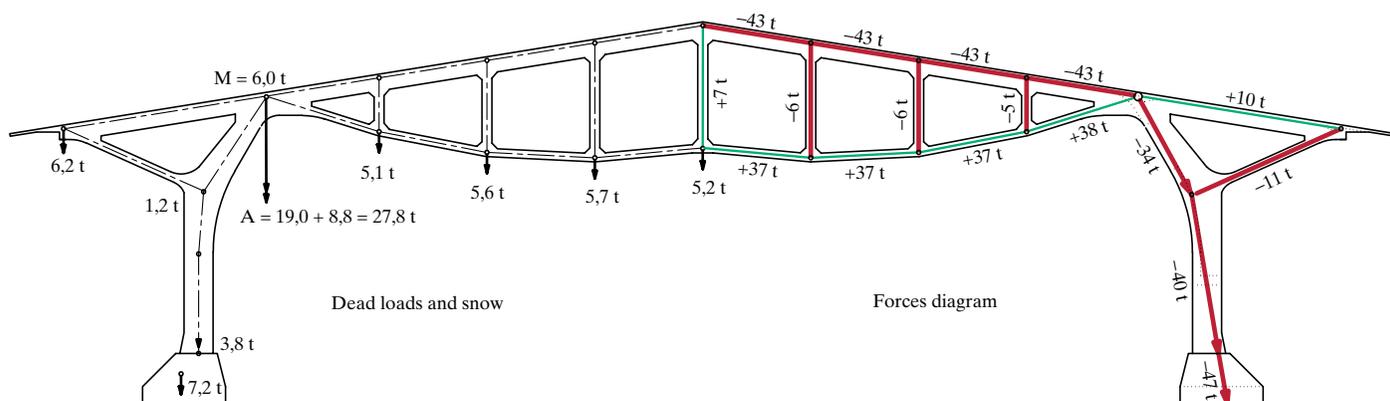


Fig. 2: Loads (left) and forces (right) in Robert Maillart's Magazzini Generali Shed in Chiasso, after Bill (1955)⁴

Pierre Varignon^{8,9} demonstrates the law of force polygon and introduces the use of funicular polygons. Graphic means were used or investigated by Poleni (1748), Lamé and Clapeyron (1826), Poncelet (1840), Rankine (1858) and Maxwell (1864).¹⁰⁻¹⁴ However, it is Karl Culmann who is considered the founder of the science of graphic statics with the publication of *Die graphische Statik* in 1866.¹⁴ After him, Cremona, Bow, Mohr, Ritter and Levy, all contributed to developing and broadening the use of graphic statics in Europe. Culmann also belongs to the first generation of professors at the ETH in Zürich, whose students included Maurice Koechlin (co-designer of the Eiffel Tower) and Robert Maillart (who attended classes given by Wilhelm Ritter, Culmann's successor at ETH). Graphic statics use at least two figures simultaneously: a force polygon that guarantees the equilibrium of translation forces, and either a truss or a funicular polygon that guarantees rotational equilibrium. A reciprocal relationship exists between the lines of the two polygons, as shown by Maxwell in 1864.

Design of the Chiasso Shed

When superimposing sections of the warehouse and the shed, we immediately see that the entire height of the central part of the shed structure is

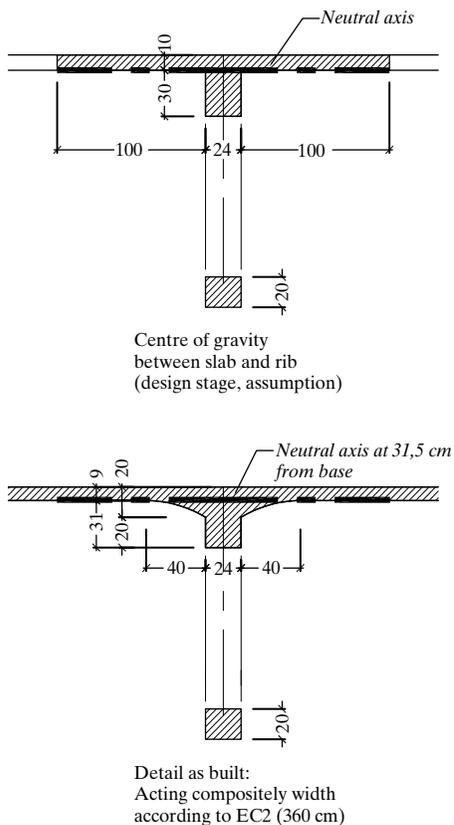


Fig. 3: Neutral axis position, (units: cm)

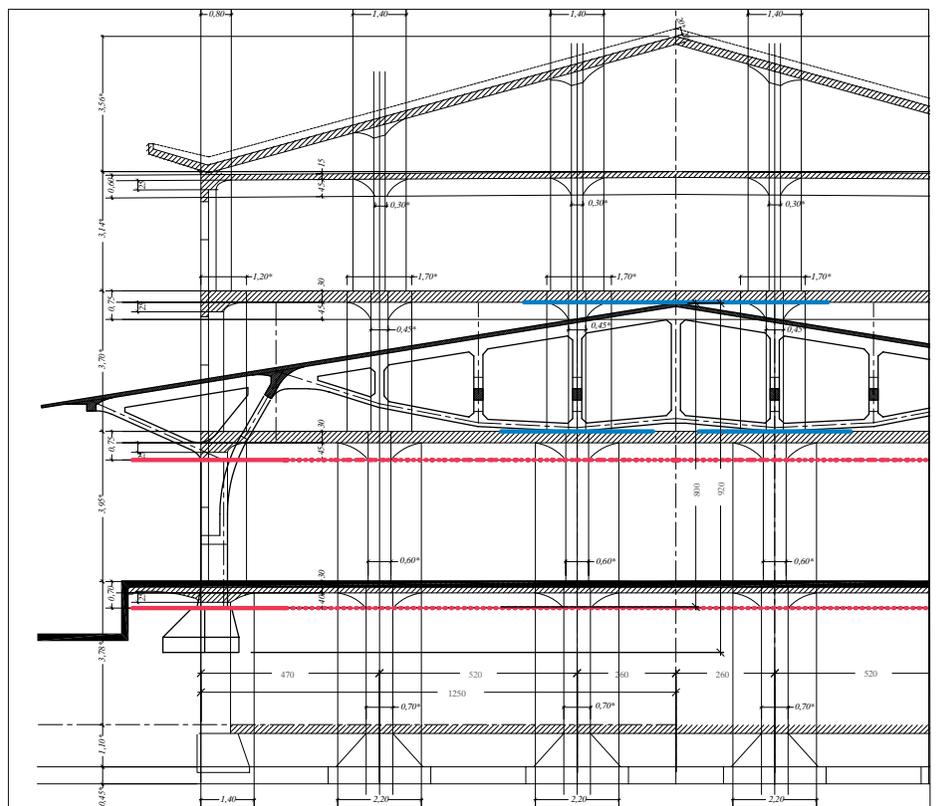


Fig. 4: Shed section superimposed on the warehouse construction lines (units: cm)

held between the surfaces of the first and second floor (in blue in Fig. 4). In the same way, new structural components emerge from others at the same height, as the capitals emerge from the columns (red in Fig. 4).

The span combines a compressed concrete slab and a (funicular) lower chord in tension. In so doing, through graphical construction we can determine the right form to encounter only axial forces under one reference loading case. But, since structures encounter a wide variety of loadings, another structural component is useful in guaranteeing that the structure behaves correctly. Maillart achieved this by giving the slab a rib, making a kind of T-beam. While the upper chord of the structure is maintained in axial compression (i.e. the ideal case), dimensions are given to place the centre of gravity just between the slab and the rib. Thus, during the design phase the compression chord is modelled as a 10-cm slab with 1-m flanges on each side of a 30-cm deep rib. At a later stage, the neutral axis will be maintained at the same level with a 9 cm slab and a curved junction with the slab (see Fig. 3). It will be demonstrated (Fig. 5) that, due to an inclination of the slab, the lower chord must rise in the central part of the structure beyond the axis *FG* (according to Robert Bow's interval notation: see below).

With interval notation, the numbers in the main diagram represent areas restricted by structural elements and the letters represent spaces between the lines of action of the applied forces, so that the bars of the structure are labelled by two numbers or by a number and a letter (Fig. 5). Thus, in the main diagram 4E represents the segment of upper chord between areas labelled 4 and E and the force in this segment is represented by the ray 4e in a force polygon. The corresponding small letters are used on the force polygon to label the two marks defining the length of the vectorial force associated with a bar or an axis in the main diagram. The lengths on the force polygon define the forces' magnitude at scale. On the main figure, forces are labelled with capital letters placed directly on either side of their lines of action, which are the trajectories of the forces up to the point of contact with the structure. Zalewski and Allen¹² provide a clear overview of graphical design procedures using Bow's notation.

Here, the vertical axes pattern is used as a support for the force trajectories applied on the roof's structure. The axes of the shed are spaced at half the distance between the axes of the warehouse column (Fig. 4). So, doubling axes allows the roof ridge to be supported and the span of the upper chord to be limited.

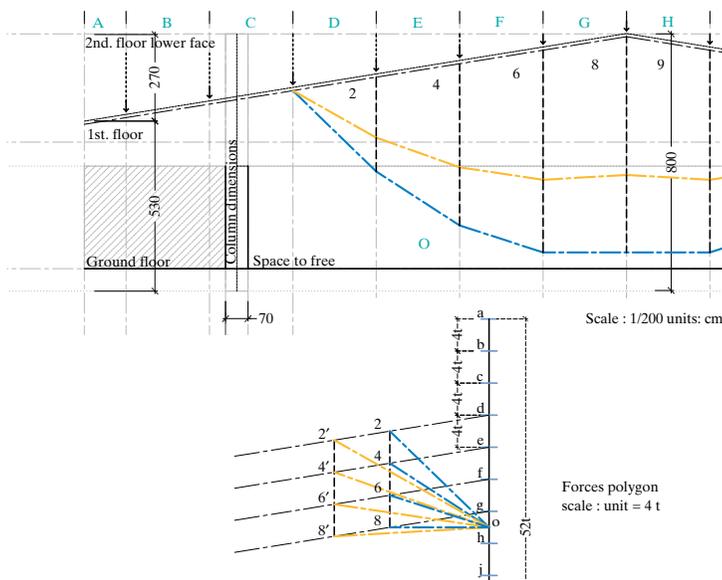


Fig. 5: Two trial funiculars: the blue one is horizontal at the centre

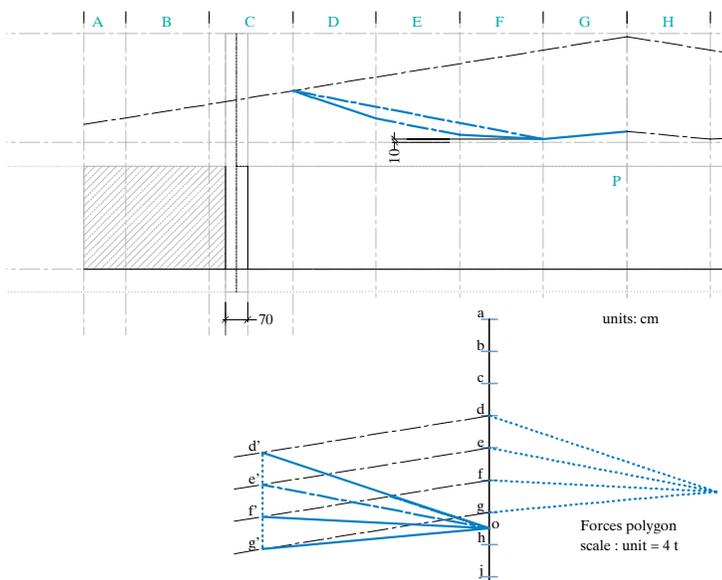


Fig. 6: Point o is placed at the mid-point of gh , orientation of $e'o$ is given by the main figure (between intersection of the CD axis with the roof, and a point on the FG axis 10 cm above the first floor slab); d' , f' and g' are vertically aligned on e'

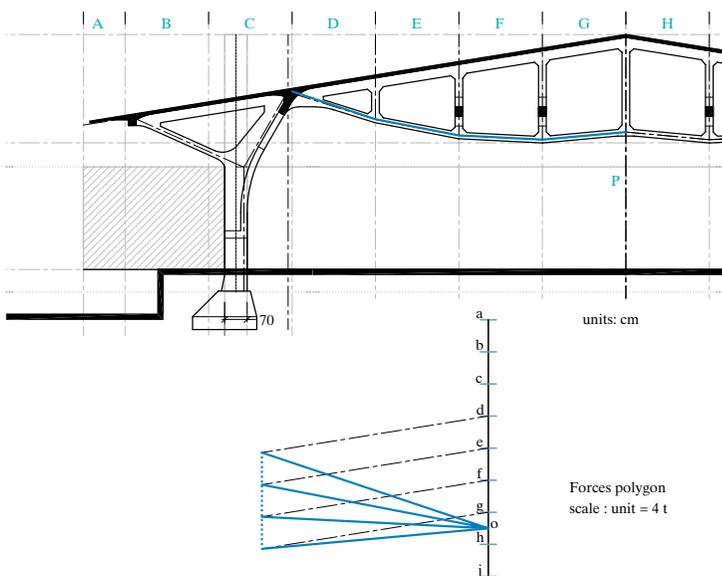


Fig. 7: Funicular polygon corresponds to the lower chord of the structure's geometry for segments E, F, G ; segment D follows other rules

Values of the applied forces are a simplification of those obtained with the weight information in Fig. 2, i.e. self weight of $2,5 \text{ t/m}^3$ ($\sim 25 \text{ kN/m}^3$) and a snow load of 100 kg/m^2 ($\sim 1 \text{ kN/m}^2$). The loads shown in Fig. 2 include all the geometric details (thus a verification stage), while we will use simplified ones for the design. In so doing, we use only the weight of a 10 cm deep slab + snow, obtaining 4,11 t on each axis, rounded to 4 t (tonnes and centimetres were Robert Maillart's preferred working units) for a longitudinal span of 4,5 m. Referring to Fig. 5, the purpose is to equilibrate the pull of the funicular lower chord at its anchorage with a compressed upper chord (the roof slab, acting as a strut). The segments of a funicular polygon are drawn parallel to the rays joining the corresponding loads to a pole o on a force polygon. Force vectors are drawn first (vertical loads line: a, b, \dots, j, \dots). Due to the symmetry of both the structure and the loading, the pole must remain on a horizontal symmetry axis passing through the mid-length of segment gh (corresponding vector of the load of the roof's ridge). In order to avoid a horizontal pull at the extremities of lower chord, the horizontal distance between o and the mid-length of gh must be zero.

If the blue central segment $o\delta$ is horizontal, so as to define an intersection δ with $g\delta$ (parallel to the roof's axis), the vertical line $2-4-6-8$ can be constructed and all the rays can be drawn. But the corresponding funicular on the main figure (shown in blue) appears too low to be effective. Therefore, all shallower funiculars need to have a central segment inclined towards the exterior (see $o\delta'$ for example). Since the width of the shed's columns and their position unit: cms are determined by those of the warehouse, no axis corresponds to the axis of the column. So the support for the central part of the structure is logically better placed on the CD axis.

We can now undertake a new geometric construction with a central part of the structure situated above the first floor (the lower chord is 20 cm high) and passing through the intersection between the upper chord and the CD axis (Fig. 6). Superimposing the resulting funicular on the structure (Fig. 7), we verify that the geometry is the same except for segment D of the lower chord, which varies only slightly. (The structure is drawn according to dimensions given in Maillart's plan no. 4020/41/2.) Thus, we only need to use

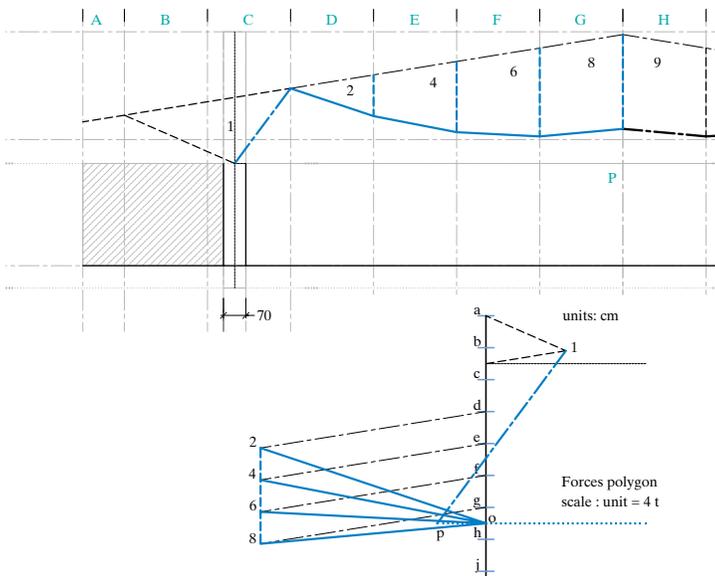


Fig. 8: Drawing the vertical struts on both figures and constructing the connection with the column; on force polygon, p is defined as the intersection between the blue axis line and the horizontal line at the mid-point of vector gh . However, o and p should correspond

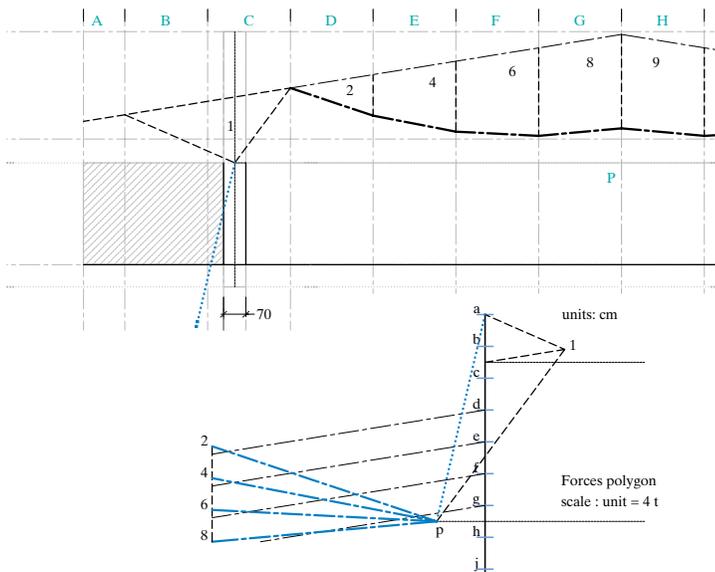


Fig. 9: Corrected force polygon (pole of the funicular rays is translated on point p); resultant force in the column is obtained by drawing dotted line ap

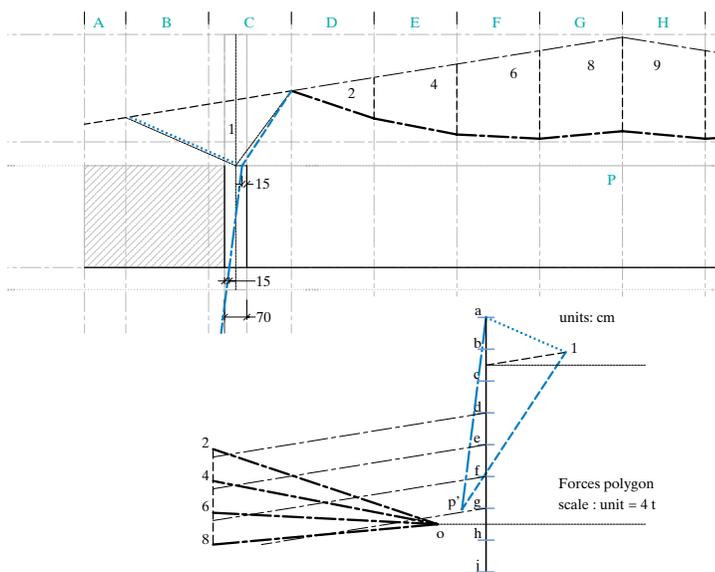


Fig. 10: Adjusting the resultant force inside the column, and thus, correcting the struts' axes. Force polygon is not yet entirely correct since o must correspond with p'

vertical members between the upper and lower chords. In fact, to Maillart, for economical and practical reasons it is preferable to avoid diagonal members, since such members complicate nodes, particularly in a concrete structure. So Maillart opted for a Vierendeel-like structure. For the same reason, Maillart avoided using a third member between IA and IP .

We can now examine the structural part of the work associated with columns and ground connection. In Fig. 2, we see that all vectorial resultants are situated within the thickness of the members. The reason is that in so doing, all forces can theoretically be borne by any material that can carry only compression forces, such as unreinforced concrete. Such a design procedure can be seen today as an application of the lower bound theorem.¹⁵ Therefore, Maillart did not need large amounts of reinforcing steel or members of a significant size. Subsequently, the bearing area of the foundation was evenly distributed around the resulting normal force and he did not need fixed-end conditions in the ground, which were difficult (and costly) to obtain for a simple industrial building in 1924.

The force polygons are completed by adding the vertical members (Fig. 8: 2-4, 4-6, 6-8, 8-9). On the main figure, we draw the axes of struts starting from the column (blue axis line IP and dashed lines AI and BCI).

Thereafter, we construct the corresponding parallel forces on the force polygon ($a1$, $bc1$ and $1p$). Point p is obtained by intersecting the horizontal symmetry axis passing through the mid-point of gh parallel to IP . But for the equilibrium, pole o of the funicular rays and point p must correspond. Thus, we need to correct the force polygon (on Fig. 9). On the same figure, we draw line ap (with the dotted blue line) on the force polygon, and immediately after on the main figure with a parallel line passing through the intersection of struts AI and IP . The resulting vector goes beyond the column envelop; therefore, the bending moment in the column is not negligible.

We can graphically correct the direction of the resultant force on the main figure. To do this, we impose the direction of the resulting force inside the column, for example, at a maximum distance of 15 cm from the edges (see the blue axis line on the main figure of Fig. 10). However, we also need to correct the orientation of strut AI and

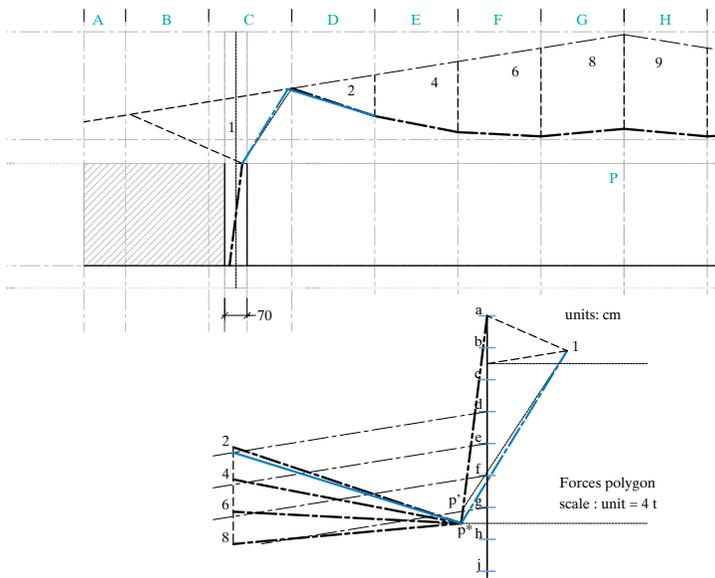


Fig. 11: Strut vector (Ip^*) orientation must be corrected in force polygon to intersect symmetry axis line; extreme segments on funicular polygon must therefore be re-oriented

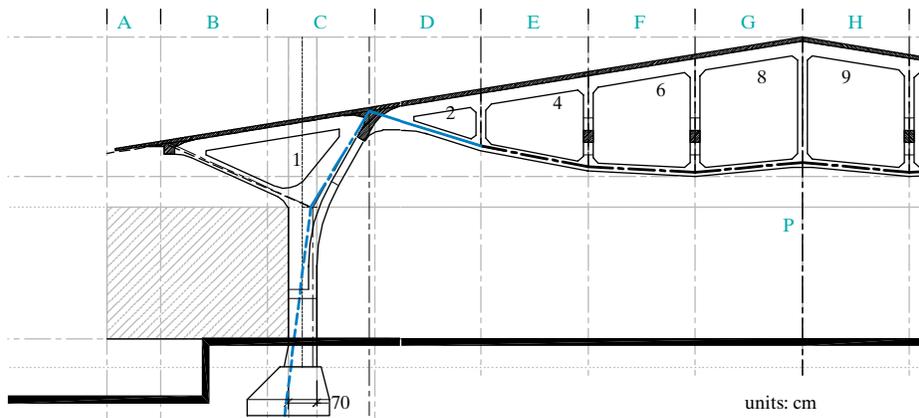


Fig. 12: Equilibrium geometry scheme fits execution plan; tolerance is 1 cm

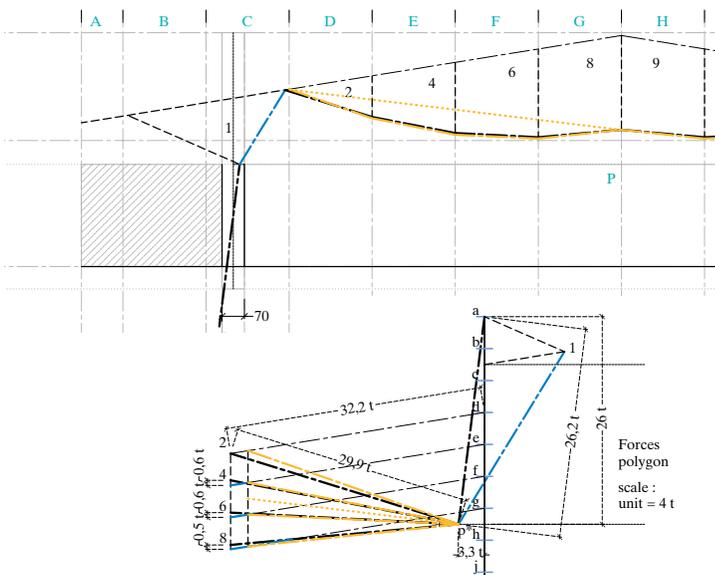


Fig. 13: Correct funicular shape and unbalanced loads in force polygon

IP by moving the former and inclining the latter. We can see that the main figure is no longer entirely correct: force AB is now applied just on the side of the top of strut IA , causing bending.

Further correction of the force polygon is now needed: segments ap' and Ip' are drawn, but their intersection, p' , no longer lies on the horizontal symmetry axis.

This must be corrected by inclining Ip' to Ip^* on the force polygon (blue axis line on Fig. 11). The rays of the funicular's segments must again be placed so that their pole corresponds with p^* . On the main figure, IP must be drawn parallel to Ip^* ; IP and $2P$ no longer intersect on the axis of the upper chord. Therefore, a new $2P$ segment is drawn to intersect with IP (blue continuous line). Again, $2p^*$ must be corrected on the force polygon.

We can compare the newly obtained geometry with that of the real structure from the construction drawings: all lines now match (Fig. 12), with a tolerance of less than 1 cm (all drawings are made using a CAD system at full size). Consequently, we now understand how Maillart determined the details of the form: just below the ground, the column widens to maintain the resulting forces in the material; the foundation had been shaped for equilibrium. The column has taken a T-shape in the upper zone while loaded on one side (and later rounded for sustaining detailed loading as in Fig. 2) but remains massive in the lower part.

We can also see that the main figure includes some errors when lines of action (axes) no longer correspond to the intersections of members. Similarly, we can see that intersections between forces of the upper and the lower chords do not converge on the force polygon if transition members have to remain vertical. Maillart could have corrected this by inclining the segments of the lower chord, but he did not.

Would Maillart disregard an appropriately worked out geometry? If we construct a new correct solution (in yellow on Fig 13) at a median position on the main figure, we can observe a positioning error of between 3,7 and 6 cm. If Maillart drew his figure on an A1 document at a scale of 1:50, where a real 5 cm is represented by 1 mm on the drawing, this correction would not be practical. But to aim for an optimum would require detailing the self weight of the structure; so it makes no sense to correct the geometry for our simplified loading.

Discussion

Our examination of the values of forces in Fig. 2 would lead us to suppose that geometry was determined with reference to the given (detailed) loads: forces in the lower and upper chords are said to be constant, as in a well-designed

constant-stress truss. Values are said to be the same: approximately 40 t minus a shared 6 t horizontal force for column equilibrium. However, they are not. We can show that, under this loading, funicular geometrical error is even greater. The author's recent doctoral dissertation¹⁶ demonstrates that because of the way asymmetric loads are managed by the T-shape beam (made of the slab with its rib), such a correction would be meaningless. We must conclude from this that Maillart first imagined the geometry of the Chiasso Shed with reference to a uniformly distributed loading (regardless of whether snow is included or not since, this is uniformly distributed). Following this, he designed a stiffening system on the same principles he used during this period with his stiffened arch bridges.^{1,6} And this T-beam will effectively take all flexural moments.

Conclusion

Maillart often earned commissions by providing the cheapest possible solution. He achieved this by using concrete with structural efficiency and sound consideration of building methods. Graphic statics permitted him to guarantee that vectorial equilibrium was

satisfied, so that he achieved mainly efficient axial forces, just as when we are tempted today to design structures with struts and ties according to plastic theory. By using graphical methods to determine the form, Maillart created an original and elegant structure for the Chiasso Shed roof.

Acknowledgements

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