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Econometric Analysis of Alternative Assets with Applications to the Art Market

by

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Abstract

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This dissertation aims at improving econometric methodologies used in the field of economics of alternative assets, with a specific focus on heterogeneous goods, more particularly goods exchanged in the fine art market and fine wines. This thesis covers new methods to track market prices through time, as well as volatility of heterogeneous assets. Statistically, this requires a methodology able to extract a common trend in prices from prices of goods that exhibit different characteristics. A challenge of this problem is to design an estimator that is relevant and coherent with applications of economists and business practitioners. Such applications include the use of estimated price indices to compute returns and volatility of an investment in art and comparing it with other financial indices. Two estimators are suggested: first, a generalized Nadaraya-Watson estimator allows estimating prices as a continuous function of time. Second, a Kalman-filter based estimator provides a consistent estimator of marginal impact of time on prices as well as an unbiased estimator of volatility of the underlying market. It is shown that these estimators have better statistical properties than estimators currently used in the applied literature, while presenting less constraining assumptions. Finally, the methodology is applied to tackle a regulatory requirement in the alternative funds industry.

Seller now in terrible straits and needs cash. Are you interested in making a cruel and offensive offer? Come on, want to try?

Gagosian Gallery, email to art collector, 2009

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Chapter 1

Introduction

Adapted from Bocart, Bastiaensen, and Cauwels (2011)

1.1 Alternative assets

After a crisis in financial markets and falling property prices in the USA at the end of the noughties, some investors turned to alternative investments to diversify their portfolio. A new class of assets, nicknamed *SWAG* (for Silver, Art, Wine and Gold) is popularly seen as a way to fight both interest rates stuck below inflation and a depressing economic environment in Western economies. In Europe, though the economic slowdown strucked art acquisition policy of large investment and commercial banks, services devoted to art and wine (Bocart and Hafner, 2013b) as a pure investment are now commonplace at private banks[1]. Hedge funds active in alternative investments such as art and wine are readily available to investors who can easily buy shares in art collections, libraries of precious books and wine caves like never before. Financial products related to art are also more sophisticated. Since the 1970ies, auction houses had been guaranteeing prices to sellers of expensive artworks before the auction, in order to gain market share (Greenleaf, Rao, and Sinha, 1993). Such guarantees are equivalent to shorting a put option: the buyer of the option has the right, but not the obligation to sell his or her artwork to the auction house at a certain strike price. Following heavy losses during the financial

[1] for instance: Berenberg Capital, ABN AMRO Private Banking, Schroders, Citi Private Bank, Société Générale Private Banking, HSBC Private Bank, Intesa Sanpaolo Private Banking, UBS Private Bank, Emirates NBD, Unicredit, etc

crisis (Sothebys lost \$60.2 million on guaranteed property in 2008), auction houses changed their strategy and now exchange part or all of this risk with third parties, effectively creating an Over The Counter (OTC) market for art derivatives.

OVERVIEW | LOT NOTES | FEATURES

Lot Description
Mark Rothko (1903-1970)
Black Stripe (Orange, Gold and Black)
signed and dated 'MARK ROTHKO 1957' (on the reverse)
oil on canvas
68 1/8 x 38 3/8 in. (173.1 x 97.5 cm.)
Painted in 1957.

Special Notice
On occasion, Christie's has a direct financial interest in lots consigned for sale, which may include guaranteeing a minimum price or making an advance to the consignor that is secured solely by consigned property. Christie's may choose to assume this financial risk on its own or may contract with a third party for such third party to assume all or part of this financial risk. When a third party agrees to finance all or part of Christie's interest in a lot, it takes on all or part of the risk of the lot not being sold, and will be remunerated in exchange for accepting this risk. The third party may also bid for the lot. Where it does so, and is the successful bidder, the remuneration may be netted against the final purchase price. If the lot is not sold, the third party may incur a loss. Christie's guarantee of a minimum price for this lot has been fully financed through third parties.

Figure 1.1: In this example of November 2012, Christie's New York discloses the use of a financial derivative on art price on its internet website.

Despite the sophistication and financialization of alternative assets like SWAGs, metrics used to control features such as risks and returns of these investments still rely on methodologies whose statistical properties can be improved. This is the goal of this thesis: improving statistical methodologies aimed at better grasping price dynamics of alternative assets with a particular focus on precious assets (such as art and wines) traded at auction.

1.2 Price indices as tools to track risks in the art market

The need to track art market swings and volatility can be illustrated by the price bubble on Impressionist art in the late 1980ies: this market experienced some of the most extreme conditions in its history. In March 1987, Van Gogh' *Sunflowers* were lifted by a Japanese company for £25 millions at Christie's London. A few months later, Van Gogh's *Irises* were hammered for £30 millions at Sotheby's

London. In May 1990, Pierre-Auguste Renoir's *Bal du moulin de la Galette* was acquired by a private Japanese collector for USD 78.1 millions at Christie's New York. By contrast, in December 1992, *The Jardin à Auvers*, one of the very last landscapes Vincent Van Gogh painted before his suicide, fetched only £6.5 millions. The Nikkei 225, on the other hand, followed a very similar path. In December 1989, this equity index was at its all-time high after a massive 106% rise in three years. In August 1992, five months prior to the *The Jardin à Auvers*' sale, the same index had declined 63%.
Could there be a link between the rise and fall of painting prices, especially on French painters (Barre, Docclo, and Ginsburgh, 1994) and the development of bubble-like behaviour in Japanese stocks? Singer (1997) writes about *Japanese speculators in the Impressionist market*. Roehner and Sornette (1999) also show evidences of speculation in the stamp market in the 1980s, with a strong outperformance of XIXth century stamps and Van Gogh stamps.

1.3 Measuring returns of artworks

To compare performance of artworks against performance of stocks at the end of the 1980ies, one must first estimate returns of artworks that exhibit very different characteristics, such as their size, medium, and so on. Literature devoted to price indices of heterogeneous assets often distinguishes methods based on "Repeated Sales Regression" from methods based on "Hedonic Regression". The former consists of averaging returns of the exact same goods sold through time while the latter consists of eliminating heterogeneity by regressing a function of price (generally the logarithm) on common predictors in order to extract a single trend in prices of different goods. The debate about which methodology is most appropriate to estimate returns has been particularly fierce in the field of art econometrics and economics of housing. Shiller (1991) observes a tight relationship between the two methods, in particular, the fact that the RSR is a nested case of hedonic regression.
Chanel, Gérard-Varet, and Ginsburgh (1996) construct an impressionist art price index based on 1972 observations and, using bootstrapping techniques, find that the hedonic method yields *"much smaller"* standard deviation of estimated returns than RSR. For real estate, Wallace and Meese (1997) equally favour hedonic regression over RSR or hybrid procedures. Using Australian housing data, Hansen (2009)

finds that hedonic and repeat-sales methods provide similar results when the sample is large.
Methodology-wise, Shiller (1991) highlights that *"the [Geometric Repeat Sales] estimator can be derived as a sort of special case of a hedonic estimator where hedonic variables consist only of house dummy variables"*. This observation constitutes also the basis of Case and Quigley (1991)'s *"hybrid estimator"*.
For the art market, Ginsburgh, Mei, and Moses (2006) formally introduce the bridge between the two methods, by starting from a hedonic regression equation:

$$p_{i,t} = \sum_{k=1}^{K} \alpha_k v_{i,k} + \sum_{\tau=1}^{T} \beta_\tau d_{i,\tau} + \epsilon_{i,t}, \tag{1.1}$$

where $p_{i,t}$ is the logged price of good i at time t with K time-invariant characteristics $v_{i,k}$. $d_{i,\tau}$ is a dummy variable taking the value 0 if good i is sold at time t and 0 otherwise. The β_t can be seen as time series whose values stand for returns. In the particular case of all goods sharing the exact same characteristics, their difference can be expressed as:

$$p_{i,t_2} - p_{i,t_1} = \beta_{t_2} - \beta_{t_1} + (\epsilon_{i,t_2} - \epsilon_{i,t_1}) = \beta_{t_2} - \beta_{t_1} + \eta_i. \tag{1.2}$$

OLS can be used to estimate β parameters. The authors proceed with a comparison between the two methods using a Monte Carlo simulation and reach the conclusion that *"hedonic regression performs much better than RSR"* and that *"RSR methods should not be used for time frames that include less than 20 years, unless the number of pairs is large"*.
Since this important conclusion, practitioners seem to have endorsed the hedonic approach for measuring returns in the art market. Table 1.3 presents recently published articles exploiting art price indices to estimate returns of investment in artworks. Though not exhaustive, the list tends to indicate that in recent years, art economists have overwhelmingly favoured hedonic-based art price indices.

RSR is avoided mainly because of sample selection issues (Collins, Scorcu, and Zanola, 2009) and because of the considerable waste of data since only artworks selling at least twice are taken into account in the regression. On the other hand, a traditional critique of hedonic regression is that the choice of the functional form and the choice of characteristics is important – Ginsburgh, Mei, and Moses (2006) or Oosterlinck (2010) – nevertheless, it has been shown in Ginsburgh, Mei, and Moses (2006) that hedonic regression still performs well, even in case of model misspecification. Furthermore, Bauwens and Ginsburgh (2000) show that even art experts

Table 1.1: Methodologies used by authors in the field of art economics. 2009 - 2012

Authors	Market	Journal	Hedonic Reg.	RSR	Not specified
Agnello (2010)	African American art	Journal of Black Studies	×		
Atukeren and Seçkin (2009)	Turkish art	Applied Financial Economics	×		
Campos and Barbosa (2009)	Latin American art	Oxford economic papers	×		
Collins, Scorcu, and Zanola (2009)	Symbolism	Economics letters	×		
Erdős and Ormos (2010)	International art	Journal of Banking & Finance		×	×
Goetzmann, Renneboog, and Spaenjers (2011)	International art	The American Economic Review		×	
Hiraki et al. (2009)	International art	Journal of Financial and Quantitative Analysis			×
Hodgson (2011)	French Canadian art	Applied Economics	×		
Kraeussl and Logher (2010)	Emerging art	Emerging Markets Review	×		
Mandel (2009)	International art	The American Economic Review		×	
Marinelli and Palomba (2011)	Italian Contemporary Art	The Quarterly Review of Economics and Finance	×		
Nahm (2010)	Korean art	Journal of Cultural Economics	×		
Renneboog and Spaenjers (2011)	Russian art	Journal of Alternative Investments	×		
Scorcu and Zanola (2011)	Picasso	Journal of Alternative Investments	×		
Taylor and Coleman (2011)	Australian Aboriginal art	Journal of Banking & Finance	×	×	

do not seem to take fully into account the information that is contained in sales catalogues. Generally, predictors used in the regression correspond to data available in auction catalogues. Table 1.3 presents regressors used by authors mentioned in Table 1.3.

The most common variables are those related to size and auction houses. Auction house variables are often dummies taking the value 1 if the artwork was sold in a given auction house, and 0 otherwise. Auction houses variables are independent of the underlying artwork. This is also true for other variables, such as the amount of citations, the presence in catalogue raisonné, the type of attribution, etc.

RSR being a particular case of hedonic regression has implications for the statistical estimation of parameters. Let us first consider the situation where the practitioner has only access to a set of data exclusively made of repeated sales. As stated in Shiller (1991), one can rephrase the RSR in its equivalent hedonic form:

$$p_{i,t} = \sum_{k=1}^{K} \alpha_k v_{i,k} + \sum_{\tau=1}^{T} \beta_\tau d_{i,\tau} + \epsilon_{i,t}, \tag{1.3}$$

In the RSR case, $v_{i,k}$ correspond to dummy variables taking the value 1 in case artwork i belongs to pair k. There are K pairs, or groups. To avoid colinearity issues, it is suggested to put β_1 and α_1 to 0.
An important remark is that, at least for OLS estimation, the quality of estimation of the parameters of interest (β) as measured by their standard deviation does not depend on the selected form. In this regard, it is trivial that equations (1.2) and (1.3) are perfectly mirroring each other and the estimated β's share the exact same properties.

Table 1.2: Variables used in hedonic regression

Authors	**Variables**
Agnello (2010)	Area, area squared, auction house, presence of an illustration in the catalogue
Atukeren and Seçkin (2009)	Area, area squared, auction house, technique, support, presence of title
Campos and Barbosa (2009)	log(area), auction house, age of artist, artist alive or not, technique, subject, presence of signature, existence of past exhibitions, presence in catalogue raisonnes, order of sale
Collins, Scorcu, and Zanola (2009)	Area, auction house, age of artist, technique, subject, number of past exhibitions, number of experts assessments, number of citations, nationality of artist
Hodgson (2011)	Height, width, artist, auction house, technique, support, presence of date, signature, title
Kraeussl and Logher (2010)	log(area), auction house,technique, support, presence of signature, Presence of pre-sale estimate, logarithm of reputation score
Marinelli and Palomba (2011)	Area, artist, auction house, artist alive or not, technique, support, presence of date, signature, title, dummy for authenticity, presence in a catalogue raisonné, recognition by experts, citations in literature, number of exhibitions, number of previous experts
Nahm (2010)	Area, area squared, auction house, age of artist, artist alive or not, technique, support
Renneboog and Spaenjers (2011)	Height, width, height squared, width squared, artist, artist alive or not, auction house, subject, presence of date, signature, markings, monthly seasonal factor, type of attribution
Scorcu and Zanola (2011)	Area, auction house, technique, support
Taylor and Coleman (2011)	Height, area, artist, auction house, artist alive or not, technique, support, monthly seasonal factor, artist of the year or not, logarithm of mean of historical prices

A first conclusion is that, apart from its more compact form, there is no special reason to favour a RSR form over an equivalent hedonic form. To the contrary, exploiting the hedonic form would allow practitioners to implement all existing developments designed to improve estimation of β parameters. Many improvements further detailed in this thesis, including heckit correction (Collins, Scorcu, and Zanola, 2009) that corrects sample selection bias due to the exclusion of bought-ins, smearing factor (Jones and Zanola, 2011) that corrects a retransformation bias, semi-parametric estimation (Hodgson and Vorkink, 2004) that improves stability of OLS estimators, etc. Most are expressed for the traditional hedonic form and can be directly applied to any regression aimed at computing art price indices, even when repeated sales data are used.

A second important remark is related to well known comments made by Case and Quigley (1991) for housing: RSR *"[...] is inappropriate when any of the characteristics of the properties have been changed between sales dates"*. This is also true for the art market. The authors suggest augmenting equation (1.3) with additional explanatory variables. In Ginsburgh, Mei, and Moses (2006), it is stipulated that *"the suggestion is hard to apply to paintings"* but the authors confirm that *"results [...] provide estimates with smaller standard deviations"*.

In the case of OLS estimation, dimensionality issues arise when the amount of lines in the regression is smaller than the amount of columns. For RSR, it happens when repeated sales do not "crossover" each other enough through time, this is, when $K \leq T$ (in fact, as $K < T$ is impossible in practice, the problem is practically constrained to the situation where $K = T$). Any situation where $K > (T + 1)$ paves the way to include at least one extra variable in the linear equation.

Not using available variables to correct estimation bias due to time-dependent characteristics of the transaction is a choice that necessarily leads to misspecification bias. Because RSR is in fact a nested case of a hedonic regression, it happens that it is equally concerned by the choice of its functional form. Actually, repeated-sales tackle only issues related to time-invariant characteristics, but clearly fail to cope with the time-dependent variables related to the market's microstructure, such as the proven influence of auction houses on prices.

For all these reasons, this thesis focuses on the generalized hedonic approach, since all conclusions drawn from hedonic methodology can be trivially translated to the RSR setting.

1.4 Example of a hedonic price index

To illustrate the practical construction of a hedonic art index, let us focus on the 1980ies bubble on impressionist art. The following study is based on three datasets. The first one consists of sales of works on paper from artists belonging to Impressionism and Post-Impressionism art movements. The analysis is confined to 3862 artworks from thirteen artists, born between 1820 and 1880, and cited in Galenson and Weinberg (2001): Pissarro, Manet, Degas, Cezanne, Monet, Redon, Renoir, Gauguin, Van Gogh, Seurat, Toulouse-Lautrec, Bonnard, and Vlaminck. The second dataset is made of 2563 sales from 39 Old Masters of Dutch, French and Italian origin cited in Ginsburgh and Schwed (1992). The third dataset consists of 2650 artworks made by 9 different artists who can be seen as painters of Modern Art (André Derain, Fernand Léger, Georges Braque, Jean Arp, Joan Miro, Juan Gris, Marc Chagall, Pablo Picasso and Robert Delaunay).

Only sales of drawings, studies and sketches are included, as supply in this segment of the art market is larger than in the market for paintings. By contrast, the market for works on canvas is reputed much more expensive and concerns mainly high net worth individuals (Goetzmann, Renneboog, and Spaenjers, 2011).

The database has been built using sales catalogues and results files published by auction houses. All sales were observed between January 1975 and December 1994. Prices are expressed in USD and are deflated using the 1995 OECD price index. We grossly estimate buyer's transaction fees using Christie's and Sotheby's fees' policy: +25% for works below 50,000 USD and +20% otherwise.

In order to focus on the broad art market, and not on very expensive art that may bias the index, the 5% most expensive artworks are eliminated, on semi-annual basis[2]. After eliminating further sales lacking information (size of the work, title, etc.) the first, second and third dataset eventually consists of 3463, 2410 and 2382 sales respectively.

Financial data are imported from Datastream and consist of 5218 daily observations of closing prices of the Nikkei 225 and the daily JPY/USD exchange rate, between the first of January, 1975 and the 30th of December, 1994. Equation (1.1) is used, where the K variables $v_{i,k}$ reflect specific characteristics of the piece of art i. For instance, for the Impressionist/Post Impressionist art dataset, these include: the *height* and *surface*, the *lot* number, twelve dummies related to the artists (Pis-

[2] Indices are less robust and more volatile when no trimming is made, but conclusions stay the same.

sarro as a benchmark), six dummies whose values depend on the *auction houses* (Sotheby's, Christie's, Koller, Blache, Ader Picard and Tajan, Phillips, Bonhams), one dummy taking the value 1 if the sales' session is devoted to a specific *collection* or not, six dummies for the *weekday* the sale occurs (as some (un)important sales may be more likely to happen certain days), three dummies for the *city* where the sale occurs (New York, London and Zurich), one dummy taking the value 1 if the drawing is a *study*, 0 otherwise, and fourteen dummies corresponding to different *subjects* that are not mutually exclusive: landscape, peasants, animals, portrait, people, still life, urban scenes, family of the artist, self-portrait, dancers, bath scenes, women, nude, religious scenes. The number of objects in the analysis (n) totals 3463.

After eliminating variables that are not significant at a 5% level in a standard ordinary least squares regression, 31 variables (in addition to the 39 semiannual dummies) are restrained. This linear model results in an R^2 of nearly 60% for Impressionism and Post-Impressionism. The same methodology yields an R^2 of 40% for Old Masters and more than 67% for Modern Art.

As an improvement to the estimation, it is chosen to follow Hodgson and Vorkink (2004) and implement the modified Bickel's adaptive estimator (Bickel, 1982) to gain more efficiency. Note that a valid application of this method requires symmetry of the residuals, which is empirically verified in the current case.
The semiparametric estimator is built as follows:
Let us consider $X_i' = (d_{i1}, ..., d_{i,t}, v_{i,1}, ..., v_{i,K})$ and $\beta = (\gamma_1, ..., \gamma_T, \alpha_1, ..., \alpha_K)$
Let $\hat{\beta}$ be the estimator of β, based on ordinary least squares:

$$\hat{\beta} = (X'X)^{-1}(X'p). \tag{1.4}$$

Let $\hat{\epsilon} = p - X\hat{\beta}$ be the vector of residuals. Let us define K(.), a gaussian kernel: $(K(\lambda) = \frac{e^{-0.5\lambda^2}}{\sqrt{2\pi}})$. Then,

$$\widehat{f(\epsilon_i)} = \frac{1}{2h(n-1)} \sum_{i \neq j}^{n} (K(\frac{\hat{\epsilon}_i + \hat{\epsilon}_j}{h}) + K(\frac{\hat{\epsilon}_i - \hat{\epsilon}_j}{h})). \tag{1.5}$$

$$\widehat{f'(\epsilon_i)} = \frac{1}{2h(n-1)} \sum_{i \neq j}^{n} (K'(\frac{\hat{\epsilon}_i + \hat{\epsilon}_j}{h}) + K'(\frac{\hat{\epsilon}_i - \hat{\epsilon}_j}{h})), \tag{1.6}$$

where h is a bandwidth obtained using Silverman's rule of thumb (Silverman, 1986) . The score function ψ is computed using a trimming parameter ($t_1 = 2.5, t_2 =$

$e^{2.5^2/2}, t3 = 2.5$) following Hsieh and Manski (1987).

$$\hat{\psi}_i(\hat{\epsilon}_i) = \frac{\hat{f'(\epsilon_i)}}{\hat{f(\epsilon_i)}} \text{ if } \mid \epsilon_i \mid < t_1 \text{ and } f(\epsilon_i) > t_2 \text{ and } f'(\epsilon_i) < t_3. \tag{1.7}$$

$$\hat{\psi}_i(\hat{\epsilon}_i) = 0 \text{ otherwise.} \tag{1.8}$$

The sample score vector is then estimated as follows:

$$\hat{S} = \frac{\sum_{i=1}^{n} X_i \hat{\psi}_i(\hat{\epsilon}_i)}{n}. \tag{1.9}$$

Similarly to Hodgson and Vorkink (2004), the information matrix is approximated:

$$\hat{I} = \frac{\sum_{i=1}^{n} (\hat{\psi}_i(\hat{\epsilon}_i))^2}{n^2} \sum_{i=1}^{n} X_i X_i'. \tag{1.10}$$

According to Bickel (1982):

$$\tilde{\beta} = \hat{\beta} + \hat{I}^{-1}\hat{S}. \tag{1.11}$$

$$\sqrt{n}(\tilde{\beta} - \beta) \rightarrow N(0, I^{-1}). \tag{1.12}$$

White (1980) consistent estimator of variance is used to obtain robust estimators of $\tilde{\beta}$'s variance.
Finally, the index is built[3]. The semi-annual Impressionist Works On Paper index is further referred to as $IWOP$. The Modern Art Works on Paper index is referred to as $MAWOP$ and the Old Master Drawings index is referred to as OMD. The first semester of 1975 is considered as the base of the index, whose value is arbitrarily put to 100.

$$IWOP_t = 100e^{\tilde{\gamma}_t - \tilde{\gamma}_1}, \tag{1.13}$$

where $t = 1, ..., T$.

The OMD (Old Master Drawings) index and $MAWOP$ (Modern Art Work On Paper) index are built in a similar fashion.
To conclude this example, the usefulness of constructing art price indices is highlighted by the Figure 1.2 that summarizes results from these hedonic regressions and compare them with the Japanese stock index.

[3]Details of regressions and computations are available in the working paper Bocart, Bastiaensen, and Cauwels (2011)

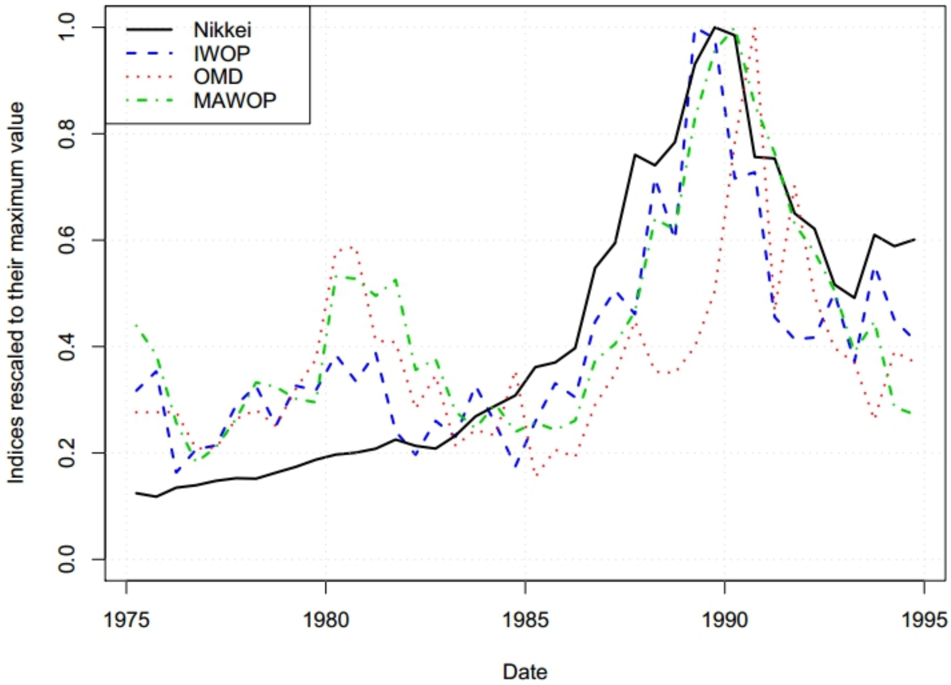

Figure 1.2: IWOP (USD), MAWOP (USD), OMD (USD) and Nikkei (JPY), rescaled to 1 in their maximum value.

1.5 Possible improvements

Unfortunately, a direct use of these indices *as if* they were directly observed requires caution. Indeed, time dummies-based estimators of price indices exhibit undesirable features, as compared to traditional financial indices, in particular, the low frequency of data points and unclear properties of volatility estimators directly derived from these indices.

To tackle these issues, we try three possible methods: first, in the next chapter, we investigate whether stocks of art companies could be in any way related to prices of physical art and whether or not they exhibit the diversification benefits expected from artworks. A relationship between the two markets could help estimate behaviour of physical art by observing stock prices of art companies, in a fashion similar to gold companies and gold prices.

As we observe no relationship between physical art and stock prices, we turn to a continuous setting of the existing method, exploiting a kernel-based regression. We use total variation to measure the variability of art prices. This is presented in

Chapter 3.

In Chapter 4, we exploit the fact that auction data actually are panel data. The model can be estimated using maximum likelihood in combination with the Kalman filter. We derive theoretical properties of the volatility estimator and show that it outperforms the standard estimator. This methodology is finally exploited in Chapter 5 to tackle a regulatory challenge faced by alternative funds. The case of fine wines funds is discussed. The last chapter concludes the thesis.

Chapter 2

Investment in art companies

Adapted from Bocart and Noh (2013)

2.1 Abstract

An equally-weighted basket of six quoted companies related to art is computed and compared with three hedonic art indices built using more than 75000 auction sales for 99 artists. We find that returns of art-related equities are not directly linked to returns of median and top prices of physical art, but are positively correlated to returns of the 5% cheapest artworks. They are also positively correlated to changes in volumes at auction, once a 6 months lag is taken into account. The basket of art stocks is then included when calculating the optimal portfolios for a rolling window of 500 trading days using a new approach based on GARCH filtering and R-Vine copulae. Such optimal portfolio can be seen as an ideal investment from an ex-post point of view. We find that art-related companies brought no diversification benefits during the banking crisis of 2008-2009, but were of interest in 2009-2012, during the European debt crisis. These patterns closely follow evolution of exchanged volumes of art at auction. We conclude that investment in art companies offers exposure to volumes in the art market, but not to the price levels of physical art.

2.2 Introduction

The question of "art as an investment" has frequently been debated. Attempts to know whether or not art is something worth investing in has led to disparate results. Baumol (1986) initially concluded that *"art is a floating crap game"*, Goetzmann (1993) reaffirmed that risks taken by an art investment are not compensated enough by financial returns, while Worthington and Higgs (2004) found no diversification benefit of art in a Markowitz mean-variance efficient portfolio. Renneboog and Spaenjers (2013) also concluded that art is not an investment that can possibly beat the expected return of equities. On the contrary, Buelens and Ginsburgh (1993) highlighted the fact that returns depend on artistic movements and that some of them can significantly outperform equity and bond markets. Mei and Moses (2002) stated that art as a whole is attractive for portfolio diversification. Hodgson and Vorkink (2004) found that Canadian art brings diversification benefits to a Canadian equity portfolio. Oosterlinck (2010) demonstrated that art played the important role as a safe haven during Nazi occupation in France. He showed that art was in high demand and outperformed most asset classes (except gold) during the war. In particular, smaller artworks outperformed larger ones that were harder to hide or carry, revealing the pragmatic approach of investors to art as a physical asset.
The notion of "investment in art" is a synonym for speculation: since a painting or a sculpture does not yield a significant financial dividend or revenue of some sort, but only an aesthetic dividend (Mandel, 2009), the hopeful investor can only buy an artwork with the view of selling it at a higher price. Investing in art is not straightforward: in addition to authenticity risks (Bocart and Oosterlinck, 2011) transaction fees can be as much as 25% of the value of an object sold at auction (Ashenfelter and Graddy, 2003). This is rather significant, especially when considering unattractive expected long term returns of an investment in art – +3.97% per year according to Renneboog and Spaenjers (2013). Nevertheless, diversified financial vehicles exposed to art exist, such as art funds, probably the closest form of a diversified art portfolio an investor can purchase. One of the main advantages of art funds is that they can possibly benefit from valuable insider information, in a fashion similar to art dealers. Unfortunately, their diversification strategies are rarely disclosed, let alone the artworks held by these vehicles. Liquidity of art funds can also be an issue, since they are not always quoted on stock exchanges. Another method for the rational investor to catch some diversification benefits related to the art market could be to invest in listed art-related companies, in a fashion similar to exposure to gold price

by investing in gold-related companies. Naturally, be definition, shares in companies are different of art as a physical asset since the value of shares incorporate all public information and expectations on future revenues of companies, whereas prices of art are spot prices of a physical asset. These securities are readily accessible, they are liquid and transaction costs are truly negligible, at least compared to the standards of the art market. However, little is known as to the diversification benefits of investing in a basket of art-related companies. Our goal in this paper is to further investigate the diversification benefits of liquid, publicly traded corporations whose activities are closely related to the art market: do such investments in stocks offer the same safe-haven characteristics as what is expected from physical art as an asset class ?

In the next sections, we investigate how art-related shares are linked to price of physical art, and how they behaved in a period ranging from October 2001 to October 2012. In particular, we want to know if these companies were of any interest from a diversification perspective during two consecutive financial crises. We describe a strategy genuinely available to an investor who has the possibility to allocate funds between equity and bond indices, energy commodities, agricultural commodities and an art-composite index easily constructable using art-related equities.

Our method to investigate the appropriateness of shares of art companies as a diversification tool during the last decade is comparable to the one that Goetzmann and Ukhov (2006) used for foreign British investments: by construction, an "optimal" portfolio can be seen as an ideal investment from an ex-post point of view. Focusing on the financial crisis, we interpret that an increase in the weight of art stocks in an "optimal" asset allocation during the crisis would reveal that art-related companies were a good diversification tool against distress. On the other hand, a drop in the weight of art companies in an optimal asset allocation as compared to other financial assets during the crisis would reveal that art-related stocks were not effective diversification tools in case of financial distress and did not share the expected benefits of physical art. Section 2 presents data we use in this study and introduces the methodology. Section 3 presents the empirical results we have obtained with the method. The last section presents our conclusions.

2.3 Data and methodology

In order to study investment in art-related companies, we construct a composite index with equal weights of public companies that openly disclose their financial exposure to visual arts: artnet, artprice.com, Sotheby's, Weng Fine Art, Art Vivant and Seoul Auction (all converted in U.S. dollars). They explicitly mention their exposure to the art market in their annual reports. The rationale behind constructing an index rather than considering each stock individually is that we want to appreciate the general performance of "art stocks" in a diversified portfolio, and not a given stock in particular.

These six companies are quoted daily on national exchanges. Other peers active in the market for collectibles, such as Noble Investment PLC, Collectors Universe or Stanley Gibbons Group Ltd were discarded because of their low exposure to visual arts. Out of the six companies used to build the Art Composite Index (ACI), only four were listed in October 2001: artprice.com, artnet, Sotheby's and Art Vivant. Seoul Auction was first listed on the Korea Exchange in August 2008 while Weng Fine Art was listed on the Deutsche Borse in January 2012. Artprice.com and artnet are two European companies providing databases of art auction prices. In 2011, both companies also started online services of art auction. Sotheby's is an auction house active internationally. It sold its real estate unit, "Sotheby's International Realty", in 2004. Art Vivant is a Japanese company that manages a network of gallery and sells prints, oil and watercolor paintings, carvings, and glass works. Seoul Auction is an auction house specialized in Korean art. Weng Fine Art operates as an art dealer. Its website states that it purchases, with an annual budget of about 10 million euros, more than 1,000 works of art every year. Its focus of trade includes around 500 well-known artists of the 20th Century, such as Picasso, Matisse, Warhol, Lichtenstein and Hirst. Table 2.1 presents key financial figures for each company constituting the index.

Table 2.1: Financial information on the companies constituting the basket

	artprice.com	artnet	Sotheby's	Art Vivant	Seoul Auction	Weng
Turnover 2011 (mUSD)	6.75	17.27	831.00	44.37	13.24	8.44
Net Income 2011 (mUSD)	0.12	0.04	171.00	10.25	- 0.03	0.99
Market Cap. 29-10-2012 (mUSD)	304.91	31.01	2,086.00	30.96	48.37	54.91

A market capitalization weighted index is impracticable because Sotheby's would considerably outweigh all other stocks. As a consequence, the index is equally weighted on the 29th of October, 2001 and rebalanced every 30 trading days:

$$p_{k,t} = \begin{cases} 1 & when\ t = 1 \\ \frac{1}{K}\sum_{k=1}^{K} p_{k,t-1}e^{r_{k,t}} & when\ t = 30, 60, 90, 120, \ldots \\ p_{k,t-1}e^{r_{k,t}} & otherwise \end{cases}$$

and

$$ACI_t = \frac{1}{K}\sum_{k=1}^{K} p_{k,t},$$

where ACI_t is the Art Composite Index at time t, K are the number of companies in the index, $p_{k,t}$ is the price of individual component k at time t, and $r_{k,t}$ is the stock return for company k between $t-1$ and t.

Rebalancing is necessary to avoid a situation in which a single stock would constitute too large a share of the index. Blume and Stambaugh (1983) discussed the long term bias related to portfolio rebalancing. Since in our case the index is rebalanced only every 30 days and we consider limited periods of 500 trading days, this effect is largely negligible and does not significantly impact on our results[1].

This "art composite index" (ACI) is compared with world government bonds, world credit bonds, world equities, agricultural commodities, energy commodities, companies active in the real estate business and gold, all denominated in U.S. dollars.

The study covers a period of eleven years ranging from the 29th of October, 2001 to the 29th of October, 2012. We have selected this combination of dates because it is the one that maximized the amount of available data for a sample of 11 years at the time of data collection. Financial data are provided by Thomson Reuters Datastream. We use the Citigroup World Government Bond Index (converted in U.S. dollar), the Citigroup World Big Corporation Bond Index, the MSCI World Equity Index, the S&P GSCI Agricultural Index Spot (a basket of agricultural commodities), the S&P GSCI Energy Spot (a basket of energy commodities), the Gold spot price per fine ounce from the Bank of England. For real estate, we follow the same rationale as for art, and use the MSCI World Real Estate, a basket of real-estate related stocks. An important goal of this research paper is also to describe relationships between art-related companies and the underlying physical art market. To achieve this purpose, we build a semi-annual physical art index: we use 75,763 sold lots at auction, coming from Tutela Capital S.A.'s[2] database. The sales concern 99 artists arbitrarily selected to cover the main segments of the art market: old

[1] the following robustness checks have been performed: computing ACI with and without rebalancing, with and without Weng Fine Art, and using Sotheby's only. In all cases, the conclusions are unchanged

[2] a company specialized in art as an investment

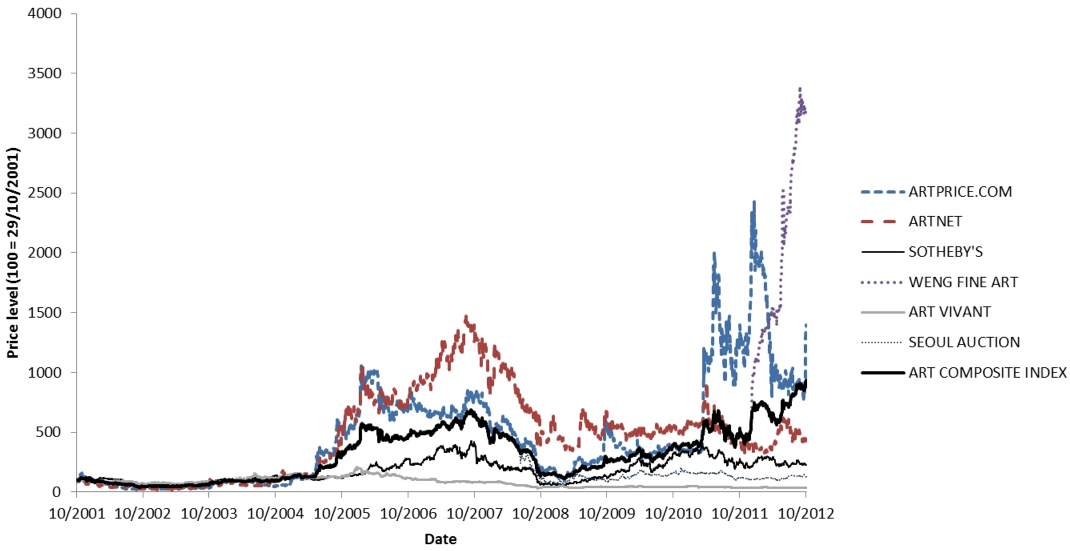

Figure 2.1: Price level of the six quoted companies constituting the Art Composite Index

masters, impressionism, post-impressionism, modern art and contemporary art, all gathered in a single art index. Different methods are available to construct art indices based on data collected at auction. Buelens and Ginsburgh (1993) derived an art index using a hedonic regression methodology. The goal of hedonic regression is to regress the logarithm of the price of each artwork on its characteristics in order to extract the marginal impact of time on returns. Mei and Moses (2002) built an art index using a particular case of hedonic regression in which artworks are organized by pairs of resold artworks. This nested case is generally called the Repeat Sales Regression (RSR).

Ordinary Least Squares (OLS) is frequently used to estimate art indices. However, the estimation based on it is not immune to criticisms: Collins, Scorcu, and Zanola (2009) introduced a method to correct for sample selection bias linked to reserve prices and time instability of parameters. Scorcu and Zanola (2011) presented a quantile hedonic regression to take into account the segmentation of the market according to price level. Jones and Zanola (2011) showed that in case of heteroskedasticity, a retransformation bias must be corrected when using OLS. Hodgson and Vorkink (2004) used Bickel's semi-parametric approach to correct for violation of normality assumption. Nahm (2010) modified the linear form of the hedonic regression by including a component that should be estimated non-parametrically.

Bocart and Hafner (2012a) presented a semi-parametric estimator of functional art returns based on a generalized Nadaraya-Watson estimator. Bocart and Hafner (2013c) showed that volatility as estimated from OLS-based hedonic indices is biased. They offer a Kalman-filtering approach to estimate returns and volatility of an investment in heterogeneous goods.

Since we desire to summarize relationships between art companies and price segments in the art market, we select Scorcu and Zanola (2011)'s quantile approach to compute the art index. We compute three hedonic art indices, for quantiles 5%, 50% and 95%. In our dataset, quantiles 5%, 50% and 95% for the transaction prices stand at 2571 USD, 17000 USD and 910299 USD respectively. Table 2.2 presents the variables used in the model and their estimated coefficients from the quantile regressions and the OLS regression. Table 2.3 shows the estimates of the time dummy variables from the regressions.

In a second stage, we want to assess the appropriateness of holding a composite index of stocks of art companies in a financial portfolio. When constructing an "optimal" portfolio, a typical framework for it is the Mean-Variance model proposed by Markowitz (1952), which characterizes portfolio return with expected return and portfolio risk with variance. However, since employing variance as a proxy for risk is valid only for normally distributed returns, it was necessary that variance should be replaced with an appropriate risk measure considering the non-normal and fat-tailed nature of financial random variables such as bonds and equities. After the pioneering work of Rockafellar and Uryasev (2000), Conditional Value at Risk (CVaR) has been spotlighted as a useful risk measure due to its attractive features: it inherits most advantages of VaR and is a coherent measure of risk. Hence, we apply the Mean-CVaR model for portfolio optimization, which finds the tradeoff between the expected return and the risk characterized by CVaR.
To find the optimal asset allocation in the Mean-CVaR model, we should generate a random sample of moderate size drawn from the distribution of a multivariate time series which is assumed to represent the distinctive feature of the assets of interest. For such purpose, we consider the copula-GARCH model, which models each marginal time series using GARCH models and combines each time series by considering a certain copula, which links the innovations of each marginal time series together. This model has been popularly used nowadays in financial applications because it is able to capture not only asymmetric co-movements thanks to the copula modeling part, but also marginal univariate features such as excess kurtosis and volatility clustering due to the GARCH modeling part. The details of the copula-

GARCH model are given in the following section.

2.4 Copula-GARCH model

We now formally introduce the copula-GARCH model. For simplicity, we consider a GARCH(1,1) model. Suppose that the observations $\{\mathbf{R}_t = (R_{1,t}, \dots, R_{p,t})^\top\}_{t=1}^n$ follow the model

$$R_{j,t} = \mu + h_{j,t}\,\epsilon_{j,t}, \quad h_{j,t}^2 = \alpha_{j,0} + \alpha_{j,1} R_{j,t-1}^2 + \beta_{j,1} h_{j,t-1}^2, \; j = 1, \dots, p, \tag{2.1}$$

where $\{\boldsymbol{\epsilon}_t = (\epsilon_{1,t}, \dots, \epsilon_{p,t})^\top\}_{t=1}^n$ is a sequence of *i.i.d.* random vectors with $E[\epsilon_{j,t}] = 0$ and $E[\epsilon_{j,t}^2] = 1$, and the joint distribution function $F_{\boldsymbol{\epsilon}}$ of $\boldsymbol{\epsilon}_t$ is assumed to take the semiparametric form:

$$F_{\boldsymbol{\epsilon}}(\epsilon_1, \dots, \epsilon_p) = C(F_{\epsilon,1}(\epsilon_1), \dots, F_{\epsilon,p}(\epsilon_p); \theta_0), \tag{2.2}$$

where $C(u_1, \dots, u_p; \theta)$ is a parametric copula function up to unknown $\theta \in \Theta \subset \mathbb{R}^l$, for $j = 1, \dots, p$, $F_{\epsilon,j}$ is the marginal distribution function of $\epsilon_{j,t}$, which is assumed to be continuous. By Sklar's Theorem, any multivariate distribution with continuous marginals can be uniquely represented by its copula function evaluated at its marginals. Let $C_{\boldsymbol{\epsilon}}$ denote the unique copula corresponding to the true joint distribution $F_{\boldsymbol{\epsilon}}$ of the GARCH residual vector $\boldsymbol{\epsilon}_t$. We call $C_{\boldsymbol{\epsilon}}$ the residual copula, which is defined as $C_{\boldsymbol{\epsilon}}(u_1, \dots, u_p) = F_{\boldsymbol{\epsilon}}(F_{\epsilon,1}^{-}(u_1), \dots, F_{\epsilon,p}^{-}(u_p))$, where $F_{\epsilon,j}^{-}(\cdot)$ is the generalized inverse of $F_{\epsilon,j}(\cdot)$, $j = 1, \dots, p$. The model (2.2) is equivalent to assuming that the true residual copula belongs to a parametric family, which is known but the parameter θ_0 is unknown. Although there are some available multivariate copulas for the model (2.2), the choice of multivariate copulas is rather limited in contrast to the bivariate case. The so-called pair copula constructions overcome this issue. Regular vines are a convenient graphical model to classify such pair copula constructions and hierarchical in nature. Each level only involves the specification of arbitrary bivariate copulas as building blocks and hence allows for very flexible models exhibiting effectively various dependence structures.

2.5 Regular vine copula

We specify a family of copulae within which we will search for an appropriate copula linking the assets of interest. To that end, we consider a class of the regular vine copula proposed by Dissmann et al. (2013), which is a flexible construction that models multivariate dependence with a cascade of bivariate copulae.

According to Definition 4.4 of Kurowicka and Cooke (2006), a regular vine on p variables consists first of a sequence of linked trees (connected acyclic graphs) $T_1, \ldots, T_{p-1}$ with nodes N_i and edges E_i for $i = 1, \ldots, p-1$, where T_1 has nodes $N_1 = \{1, \ldots, p\}$ and edges E_1, and for $i = 2, \ldots, p-1$ tree T_i has nodes $N_i = E_{i-1}$. Moreover, the proximity condition requires that two edges in tree T_i are joined in tree T_{i+1} only if they share a common node in tree T_i. It is shown in Bedford and Cooke (2001) and Kurowicka and Cooke (2006) that the edges in an R-vine tree can be uniquely identified by two nodes, the conditioned nodes, and a set of conditioning nodes, i.e., edges are denoted by $e = j(e), k(e)|D(e)$ where $D(e)$ is the conditioning set. The multivariate copula associated to tree $T_1, \ldots, T_{p-1}$ is then built up by associating each edge $e = j(e), k(e)|D(e)$ in E_i with a bivariate copula density $c_{j(e),k(e)|D(e)}$. According to Theorem 4.2 of Kurowicka and Cooke (2006) the R-vine copula density is uniquely determined and given by

$$c(F_1(r_1), \ldots, F_p(r_p)) = \prod_{i=1}^{p-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}(F(r_{j(e)}|\mathbf{r}_{D(e)}), F(r_{k(e)}|\mathbf{r}_{D(e)})), \tag{2.3}$$

where $\mathbf{r}_{D(e)}$ denotes the subvector of $\mathbf{r} = (r_1, \ldots, r_p)^\top$ indicated by the indices contained in $D(e)$. In our analysis, we use six kinds of bivariate copulae, which are Gaussian, Student t, Clayton, Gumbel, Frank and Joe copulae. For the estimation and model selection for R vine copulas, we follow the method described in Dissmann et al. (2013)

2.6 Optimal allocation based on the copula-GARCH model

To find the optimal asset allocation in the Mean-CVaR model, we generate a random sample of moderate size drawn from the distribution of a multivariate time series using the copula-GARCH model describes in Sections 2.4 and 2.5. The details of the estimation of the copula-GARCH model is given as follows:

– For each asset return $R_{j,t}$ (j is an index for denoting an asset), we fit a GARCH(1,1) model of the form $R_{j,t} = \mu + h_{j,t}\,\epsilon_{j,t}$ with variance equations $h_{j,t}^2 = \alpha_{j,0} + \alpha_{j,1} R_{j,t-1}^2 + \beta_{j,1} h_{j,t-1}^2$ and obtain the residuals $\hat{\epsilon}_{j,t}$ from each marginal time series.

– As we are not interested in parametric models for the marginal distributions of $\epsilon_{j,t}$, all residuals are transformed into uniform ones by means of the empirical probability transform, *i.e.*

$$\hat{U}_{j,t} = \hat{F}_j(\hat{\epsilon}_{j,t}),$$

where $\hat{F}_j(x)$ is the empirical distribution of $\epsilon_{j,t}$ based on the residuals $\hat{\epsilon}_{j,t}$.

– Using the transformed residual $\hat{U}_{j,t}$, we estimate the copula function $C(U_{1,t}, \ldots, U_{p,t})$ which links $\epsilon_{1,t}, \ldots, \epsilon_{p,t}$ together and hence $R_{1,t}, \ldots, R_{p,t}$ as well. Here, p is the number of the assets of interest.

Once the appropriate copula-GARCH model is estimated, we find the efficient frontier of asset allocations in the Mean-CVaR model following the method of Rockafellar and Uryasev (2000). Specifically, using their idea we will find the optimal allocation which minimizes CVaR-risk of confidence level β guaranteeing that we have returns exceeding a certain threshold ρ. Here, β represents the level of risk that we would like to control. To have more understanding about their method, let us briefly review the idea of the method of Rockafellar and Uryasev (2000) .

Assuming that there are p assets in a portfolio, $\mathbf{w} = (w_1, \ldots, w_p)^\top$ is the long-only position for each asset, $w_j \geq 0,\ j = 1, \ldots, p$ and $\sum_{j=1}^p w_j = 1$. Hence, if the asset returns are $\mathbf{R} = (R_1, \ldots, R_p)^\top$ with density $p(\mathbf{r})$, the loss function of the portfolio is $f(\mathbf{w}, \mathbf{R})$. Rockafellar and Uryasev (2000) show that a continuous differentiable and convex function $F_\beta(\mathbf{r}, \alpha)$ can be used to express the constraint problem of minimizing the C-VaR values for the loss random variable associated with returns $\mathbf{R}$: $\phi(\mathbf{R}) = \frac{1}{1-\beta}\int_{f(\mathbf{w},\mathbf{R})\geq\alpha} f(\mathbf{w}, \mathbf{R})p(\mathbf{r})d\mathbf{r}$ where α is the Value-at-Risk:

$$\begin{aligned} F_\beta(\mathbf{r}, \alpha) &= \alpha + (1-\beta)^{-1}\int_{\mathbf{r}\in\mathbb{R}^p} [f(\mathbf{w}, \mathbf{r}) - \alpha]^+ p(\mathbf{r})\, d\mathbf{r}; && (2.4) \\ \mathrm{V}aR_\beta(\mathbf{w}) &\in \mathrm{a}rgmin_{\alpha\in\mathbb{R}} F_\beta(\mathbf{r}, \alpha); && (2.5) \\ \mathrm{C}VaR_\beta(\mathbf{w}) &= \min_{\alpha\in\mathbb{R}} F_\beta(\mathbf{r}, \alpha) = F_\beta(\mathbf{r}, \mathrm{V}aR_\beta(\mathbf{w})), && (2.6) \end{aligned}$$

where $[U]^+ = \max(U, 0)$. Note that $F_\beta(\mathbf{r}, \alpha)$ is a continuous differentiable and convex function. In practice, the density of $\mathbf{R}$ is usually unknown hence we generate a random sample of size n, $\mathbf{R}^1, \ldots, \mathbf{R}^n$, from the estimated distribution of $\mathbf{R}$ and consider the following approximation to $F_\beta(\mathbf{r}, \alpha)$:

$$\tilde{F}_\beta(\mathbf{w}, \alpha) = \alpha + \frac{1}{n(1-\beta)}\sum_{i=1}^n [-\mathbf{w}^\top\mathbf{R}^i - \alpha]_+$$

Let us define $u_i = [-\mathbf{w}^\top \mathbf{R}^i - \alpha]$, the excess loss with respect to Value at Risk α. The expected shortfall, or CVaR (Conditional Value at Risk), is the expected value of this conditional loss. In our case, we generate a collection of n possible scenarios of $\mathbf{R}_t = (R_{1,t}, \ldots, R_{p,t})^\top$ at time t using the estimated copula-GARCH model, which we denote by $\{\mathbf{R}_t^1, \ldots, \mathbf{R}_t^n\}$. Based on this collection, if we consider CVaR at confidence level β (in our analysis, we set $\beta = 0.95$), the optimal position for each asset, $w_j \geq 0$ $(j = 1, \ldots, p)$ and $\sum_{j=1}^p w_j = 1$, is obtained through the following linear programming problem:

$$
\begin{aligned}
(\hat{\alpha}, \hat{\mathbf{w}}, \hat{\mathbf{u}}) = \arg \min_{\alpha, \mathbf{w}, \mathbf{u}} \quad & \alpha + \frac{1}{n(1-\beta)} \sum_{i=1}^n u_i \\
\textit{subject to} \quad & \\
& \mathbf{w}^\top \mathbf{R}_t^i + \alpha + u_i \geq 0, i = 1, ..., n; \\
& u_i \geq 0, \ i = 1, \ldots, n; \\
& n^{-1} \sum_{i=1}^n \mathbf{w}^\top \mathbf{R}_t^i \geq \rho; \\
& \mathbf{w}^\top \mathbf{1} = 1; \\
& \mathbf{w} \geq 0 \ \ (\textit{elementwise});
\end{aligned}
$$

where $\mathbf{u} = (u_1, \ldots, u_n)^\top$, $\mathbf{w} = (w_1, \ldots, w_p)^\top$ and ρ is the investor's expected return. Since $\hat{\mathbf{w}}$ (the optimal allocation vector), $\hat{\alpha}$ and $\hat{\mathbf{u}}$ depend on ρ, we have the efficient frontier function at time t:

$$
(\mathrm{C}VaR_\beta(\rho), \mathrm{E}xpected\ Returns(\rho)) = \left(\hat{\alpha}(\rho) + \frac{1}{n(1-\beta)} \sum_{i=1}^n \hat{u}_i(\rho), n^{-1} \sum_{i=1}^n \hat{\mathbf{w}}(\rho)^\top \mathbf{R}_t^i \right).
$$

Since we have to choose just one optimal allocation for our analysis, we use the STARR ratio, which was proposed by Stoyanov, Rachev, and Fabozzi (2007) and is defined by

$$
\mathrm{S}TARR(\rho) = \frac{\mathrm{E}xpected\ Returns(\rho)}{\mathrm{C}VaR_\beta(\rho)},
$$

and find the ρ which maximizes the STARR ratio.

Table 2.2: The variables and their estimated coefficients from the regressions to compute the art index.

	Coefficient QR 5%	Coefficient QR 50%	Coefficient QR 95%	Coefficient OLS	p-value (OLS)			Coefficient QR 5%	Coefficient QR 50%	Coefficient QR 95%	Coefficient OLS	p-value (OLS)	
(Intercept)	8.1E+00	9.4E+00	1.2E+01	1.0E+01	0.00E+00	***	(artist)Jean Dubuffet	1.9E-02	5.7E-01	-4.7E-02	1.1E-01	1.15E-01	
height	4.3E-04	4.8E-03	1.1E-02	1.9E-03	8.08E-19	***	(artist)Joan Miro	-2.3E-01	-1.3E+00	-3.3E-02	-1.3E+00	1.56E-140	***
width	6.2E-04	8.2E-03	1.1E-02	5.1E-03	6.22E-129	***	(artist)Joseph Mallord William Turner	-1.7E-01	1.2E+00	1.9E+00	8.2E-01	1.72E-10	***
fee	4.9E-01	7.7E-01	9.8E-01	9.4E-01	0.00E+00	***	(artist)Juan Gris	5.6E-01	2.0E+00	2.5E+00	1.4E+00	1.09E-21	***
(artist)Alexej Jawlensky	2.6E-02	1.8E+00	1.6E+00	1.2E+00	3.63E-36	***	(artist)Jules Rene Herve	-1.3E+00	-1.9E+00	-3.6E+00	-2.4E+00	1.69E-213	***
(artist)Alfred Sisley	1.8E+00	3.0E+00	1.3E+00	2.3E+00	1.48E-74	***	(artist)Karel Appel	-4.5E-01	-4.0E-01	-1.6E+00	-7.5E-01	1.61E-32	***
(artist)Andre Derain	-2.1E+00	-2.1E+00	-2.0E+00	-2.2E+00	3.69E-252	***	(artist)Lucas Cranach	-8.1E-02	2.3E-01	1.9E+00	2.4E-01	1.54E-01	
(artist)Andy Warhol	-8.7E-02	-5.7E-01	3.9E-02	-5.5E-01	6.68E-33	***	(artist)Lucian Freud	2.5E-01	1.9E-01	2.0E+00	2.6E-01	6.74E-03	**
(artist)Annibale Carracci	-6.1E-01	-8.8E-02	2.0E+00	-2.9E-01	4.55E-01		(artist)Lucien Adrion	-1.1E+00	-1.7E+00	-3.1E+00	-2.1E+00	2.54E-65	***
(artist)Antoni Tapies	-3.2E-01	8.9E-02	-1.2E+00	-4.1E-01	4.69E-07	***	(artist)Marc Chagall	-1.4E-01	-8.0E-01	1.6E-01	-8.2E-01	3.51E-65	***
(artist)Arman	-6.8E-01	-1.0E+00	-2.3E+00	-1.4E+00	1.00E-141	***	(artist)Marcel Broodthaers	-1.0E-01	-8.7E-01	-1.5E+00	-1.1E+00	6.35E-10	***
(artist)Berthe Morisot	7.8E-02	6.7E-01	7.5E-01	1.8E-01	2.79E-01		(artist)Marsden Hartley	-8.4E-01	1.4E+00	1.1E+00	6.9E-01	3.51E-06	***
(artist)Camille Pissarro	-5.0E-01	-1.2E-01	1.2E+00	-7.6E-02	2.76E-01		(artist)Mary Stevenson Cassatt	-4.3E-01	-5.0E-01	9.5E-01	-7.9E-01	1.02E-13	***
(artist)Canaletto	-3.5E-01	-1.1E+00	1.7E+00	-9.0E-01	3.20E-11	***	(artist)Maurice de Vlaminck	-7.7E-02	4.5E-01	-5.0E-01	7.2E-03	9.15E-01	
(artist)Chaim Soutine	2.1E+00	2.6E+00	2.1E+00	2.2E+00	1.24E-48	***	(artist)Max Liebermann	-1.3E+00	-2.2E-01	3.6E-02	-6.1E-01	6.74E-15	***
(artist)Charles Angrand	-1.7E+00	-2.1E+00	-1.5E+00	-2.2E+00	1.71E-11	***	(artist)Max Slevogt	-1.9E+00	-9.6E-01	-1.4E+00	-1.5E+00	5.58E-18	***
(artist)Claude Monet	8.4E-01	3.5E+00	2.5E+00	2.6E+00	6.74E-142	***	(artist)Morgan Russell	-2.2E+00	-2.5E+00	-1.9E+00	-2.6E+00	3.18E-17	***
(artist)Clyfford Still	2.6E+00	3.3E+00	1.4E+00	3.4E+00	2.02E-17	***	(artist)Natalia Goncharova	-1.2E+00	-6.1E-01	-1.7E-01	-8.7E-01	4.00E-24	***
(artist)Cy Twombly	-4.7E-02	8.4E-01	4.1E-01	4.1E-01	9.93E-06	***	(artist)Nicolas de Stael	2.2E-01	8.4E-01	8.4E-01	5.9E-01	4.50E-06	***
(artist)David Hockney	-2.5E-01	-1.1E+00	-1.3E+00	-1.3E+00	1.66E-109	***	(artist)Nicolas Poussin	-2.1E+00	6.7E-01	8.7E-01	-2.5E-01	5.55E-01	
(artist)David Teniers	-6.5E-01	8.2E-01	1.6E-01	1.9E-01	1.77E-01		(artist)Odilon Redon	-1.6E-01	4.5E-01	5.4E-01	2.7E-02	7.96E-01	
(artist)Diego Rodriguez de Silva y Velasquez	7.0E-01	-6.9E-01	2.9E+00	1.9E-01	7.92E-01		(artist)Oskar Kokoschka	-6.4E-01	-3.3E-01	-2.1E-01	-5.8E-01	2.39E-08	***
(artist)Donald Judd	1.0E-01	8.8E-01	3.5E-01	4.7E-01	1.36E-07	***	(artist)Pablo Picasso	-1.7E-01	-8.1E-01	5.2E-01	-9.2E-01	7.70E-93	***
(artist)Edouard Manet	-2.6E-01	-3.4E-01	1.9E+00	-2.6E-01	8.02E-02	.	(artist)Paul Cezanne	7.0E-01	1.3E+00	2.6E+00	1.2E+00	5.04E-31	***
(artist)Edouard Vuillard	-8.5E-01	7.9E-02	4.3E-01	-3.5E-01	6.35E-06	***	(artist)Paul Gauguin	-1.5E-01	3.3E-01	1.9E+00	2.1E-01	3.31E-02	*
(artist)Edvard Munch	4.3E-02	6.2E-01	1.2E+00	4.1E-01	6.55E-06	***	(artist)Paul Klee	4.6E-01	8.7E-01	1.5E+00	4.9E-01	2.53E-09	***
(artist)Egon Schiele	2.1E-01	2.1E+00	2.5E+00	1.5E+00	2.28E-63	***	(artist)Paul Ranson	-3.9E-01	-3.4E-01	-1.2E+00	-5.3E-01	1.05E-01	
(artist)Emil Nolde	6.0E-02	8.2E-01	5.2E-01	2.2E-01	8.99E-04	***	(artist)Paul Serusier	-1.5E+00	-3.2E-01	-5.9E-01	-8.3E-01	2.45E-10	***
(artist)Emile Bernard	-1.8E+00	-1.8E+00	-1.5E+00	-2.0E+00	7.04E-73	***	(artist)Paul Signac	2.2E-03	8.5E-02	6.8E-01	-2.1E-01	4.51E-03	**
(artist)Eugene Boudin	-4.9E-01	7.5E-01	2.2E-01	4.2E-03	9.54E-01		(artist)Pavel Filonov	2.1E+00	2.2E+00	2.0E+00	1.7E+00	3.99E-03	**
(artist)Felix Vallotton	-1.9E+00	-1.6E+00	-5.9E-01	-1.7E+00	1.64E-30	***	(artist)Piero Manzoni	3.5E-01	2.0E+00	9.4E-01	1.3E+00	1.05E-30	***
(artist)Fernand Leger	-1.1E-01	7.7E-01	1.1E+00	4.7E-01	4.38E-13	***	(artist)Pierre-Auguste Renoir	-1.2E-01	1.1E+00	1.3E+00	4.1E-01	1.01E-11	***
(artist)Francis Bacon	-9.7E-02	-1.2E+00	1.5E+00	-9.2E-01	2.41E-32	***	(artist)Pierre Alechinsky	-6.9E-01	-8.9E-01	-2.3E+00	-1.1E+00	5.42E-38	***
(artist)Franz Marc	-2.7E-01	1.1E-01	1.6E+00	-1.4E-01	4.27E-01		(artist)Pierre Bonnard	-8.1E-01	-4.3E-01	9.7E-01	-3.1E-01	3.46E-05	***
(artist)Georges Braque	-2.8E-01	-9.2E-01	1.9E-01	-8.6E-01	1.28E-32	***	(artist)Piet Mondrian	5.0E-01	1.7E+00	3.0E+00	1.5E+00	7.29E-16	***
(artist)Georges Lemmen	-1.9E+00	-1.6E+00	-1.7E+00	-1.9E+00	1.36E-53	***	(artist)Raoul Dufy	-1.7E+00	-7.3E-01	-3.6E-01	-1.0E+00	5.25E-72	***
(artist)Georges Seurat	1.2E-02	2.9E+00	2.7E+00	1.8E+00	1.75E-15	***	(artist)Richard Prince	-5.9E-02	5.3E-01	-5.0E-01	1.6E-01	3.48E-02	*
(artist)Georgia O Keeffe	1.6E+00	3.0E+00	2.0E+00	2.3E+00	1.70E-26	***	(artist)Robert Delaunay	3.9E-01	6.1E-01	-2.8E-01	2.4E-01	1.69E-01	
(artist)Gerhard Richter	-6.8E-02	4.0E-01	3.2E-01	2.6E-01	5.11E-05	***	(artist)Roy Lichtenstein	-1.6E-01	-1.2E+00	-7.5E-01	-1.1E+00	5.94E-100	***
(artist)Giacomo Balla	-4.0E-01	6.0E-01	3.7E-01	3.2E-02	7.81E-01		(artist)Salvador Dali	-1.1E+00	-7.3E-01	-2.5E-01	-1.0E+00	9.94E-72	***
(artist)Gustav Klimt	4.2E-01	3.7E-01	1.0E-01	1.2E-01	1.59E-01		(artist)Sam Francis	-2.0E-01	-4.4E-01	-1.2E+00	-8.4E-01	4.92E-45	***
(artist)Gustave Caillebotte	2.8E+00	2.6E+00	2.2E+00	2.4E+00	3.60E-30	***	(artist)Sandro Botticelli	-1.1E+00	2.1E+00	2.2E+00	1.7E+00	4.65E-03	**
(artist)Hans Hofmann	-1.9E-01	2.4E-01	-9.2E-01	-1.2E-01	2.98E-01		(artist)Space Invader	-2.0E+00	-2.1E+00	-3.7E+00	-2.5E+00	2.43E-19	***
(artist)Hans Memling	3.9E+00	2.0E+00	-7.7E-01	1.4E+00	3.66E-01		(artist)Theo van Rysselberghe	-1.3E+00	-4.8E-01	-2.5E-01	-8.3E-01	1.20E-13	***
(artist)Henri de Toulouse-Lautrec	-1.7E-01	-7.4E-01	-7.8E-01	-1.0E+00	2.83E-64	***	(artist)Victor Vasarely	-6.9E-01	-5.5E-01	-2.1E+00	-1.1E+00	1.73E-64	***
(artist)Henri Fantin-Latour	-1.5E+00	1.5E-01	8.7E-01	-2.2E-01	5.92E-02	.	(artist)Vincent van Gogh	2.7E+00	3.3E+00	3.3E+00	2.9E+00	2.78E-54	***
(artist)Henri Matisse	-2.3E-02	-4.8E-01	1.0E+00	-5.2E-01	1.75E-21	***	(artist)Willem de Kooning	-1.2E-01	5.8E-01	1.4E+00	3.4E-01	1.51E-05	***
(artist)Jackson Pollock	-1.3E-01	6.1E-01	1.8E+00	3.5E-01	5.49E-02	.	(artist)Yves Klein	1.4E-01	7.9E-01	1.0E+00	3.8E-01	2.82E-05	***
(artist)Jean-Michel Basquiat	4.0E-01	1.0E+00	-4.4E-02	7.3E-01	1.05E-22	***	(artist)Zao Wou-Ki	-1.9E-01	8.0E-01	-4.6E-01	2.4E-01	1.12E-02	*
(artist)Jean Arp	-6.7E-01	-3.1E-01	2.5E-01	-5.7E-01	9.98E-09	***	(artist)Zhang Daqian	-1.2E+00	-1.1E-02	-2.6E-02	-5.2E-01	1.90E-02	*
							(support)paper	-5.6E-02	-1.3E-01	-7.7E-01	-3.2E-01	1.05E-120	***

Table 2.3: The estimates of the time dummy variables from the regressions (Base date: 1st semester 2001)

	Coefficient QR 5%	Coefficient QR 50%	Coefficient QR 95%	Coefficient OLS	p-value (OLS)	
2001 2nd semester	1.7E-02	-3.1E-01	-3.2E-01	-3.6E-01	6.0E-13	***
2002 1st semester	7.8E-02	-1.7E-01	-2.1E-01	-2.1E-01	7.6E-05	***
2002 2nd semester	5.5E-02	-1.8E-01	-4.3E-01	-3.1E-01	1.5E-10	***
2003 1st semester	1.1E-01	-3.8E-02	-1.7E-01	-1.0E-01	5.0E-02	*
2003 2nd semester	1.3E-01	-5.0E-02	-1.0E-01	-1.5E-01	1.8E-03	**
2004 1st semester	2.2E-01	1.5E-01	6.7E-02	8.3E-02	9.6E-02	.
2004 2nd semester	2.5E-01	6.5E-02	-1.8E-01	-1.1E-01	2.6E-02	*
2005 1st semester	2.2E-01	1.1E-01	1.6E-01	3.4E-02	4.7E-01	
2005 2nd semester	2.8E-01	1.2E-01	1.6E-01	9.4E-03	8.4E-01	
2006 1st semester	3.4E-01	3.9E-01	5.1E-01	3.2E-01	3.0E-11	***
2006 2nd semester	2.7E-01	2.8E-01	4.3E-01	1.2E-01	8.0E-03	**
2007 1st semester	1.5E-01	3.1E-01	9.7E-02	3.3E-02	5.1E-01	
2007 2nd semester	5.8E-02	-4.6E-02	-1.5E-01	-3.1E-01	7.6E-11	***
2008 1st semester	-3.1E-01	-3.7E-01	-2.9E-01	-6.7E-01	4.6E-35	***
2008 2nd semester	-1.7E-01	-3.3E-01	-6.4E-01	-6.5E-01	3.0E-37	***
2009 1st semester	-4.2E-02	-3.0E-02	-2.5E-01	-2.4E-01	1.2E-05	***
2009 2nd semester	-1.4E-01	-2.5E-01	-4.7E-01	-5.0E-01	1.1E-22	***
2010 1st semester	1.9E-02	5.7E-02	-2.5E-02	-6.5E-02	2.2E-01	
2010 2nd semester	-5.9E-02	-2.1E-01	-3.4E-01	-4.3E-01	3.2E-17	***
2011 1st semester	-6.1E-02	-4.1E-03	-9.1E-02	-1.5E-01	6.1E-03	**
2011 2nd semester	2.5E-02	8.8E-03	1.2E-01	-1.1E-01	7.9E-02	.

2.7 Empirical results

Our empirical analysis aims at identifying differences or similarities between returns of an investment in physical art, returns of financial indices, and returns of a basket of art companies. We then analyze the diversification benefits of holding such a portfolio of art companies during the financial crises.

We start this analysis by describing the correlation between the three semi-annual hedonic price indices for physical art (HPI) and liquid quoted financial indices. The results are displayed in Table 2.5. Firstly, we focus on the hedonic price indices that target median and top art prices. The analysis tends to point out that correlation between returns of physical art and returns of most financial indices is close to zero. This supports the hypothesis that art as a physical asset is relatively uncorrelated to equities in general but also to art-related companies in particular. Nevertheless, the correlation between the median hedonic price index and gold bullion was of +25%. This positive relationship between physical art and gold could be explained by the status of art as a safe haven against crises and inflation, a feature shared with gold. Oosterlinck (2010) already showed evidence of such positive relationship during the second world war.

On the other hand, the HPI that focuses on the 5% cheapest artworks exhibits completely different characteristics: the price of these artworks seem to be positively correlated (+67%) with the MSCI World Real Estate. This is also expected: growth of real-estate stock may be the sign of a growing real estate market, hence growing needs to decorate newly built walls. Because the index targets the 5% cheapest artworks, we can possibly see this price index as the one that best reflects the characteristic of art as a consumption good rather than a pure investment vehicle. It is known that growth in real estate positively influences demand for decoration: these results were highlighted by Hiraki et al. (2009) and Bocart, Bastiaensen, and Cauwels (2011) who drew these conclusions observing cointegration between the real estate bubble and the art bubble of the 1980s in Japan. We also observe that cheaper art is correlated to equities (+62%). This confirms the tight relationship between very affordable art and the general economy reflected by the global stock markets. The ACI is also positively correlated to cheap physical art, a sign that art-related companies may be more dependent on art as consumption than on expensive art as investment. Mandel (2009) states that physical art overwhelmingly stands as a conspicuous consumption good rather than an investment. According to our results, it seems that this is also reflected in business activity of art companies.

Table 2.4: Correlation of art-related stocks with semi-annual Hedonic Price Indices (HPI)

Correlation of semi-annual returns	HPI. Tau = 0.05	HPI. Tau = 0.50	HPI. Tau = 0.95	Δ auction volumes (6 months lag)
ACI	51%	-6%	-8%	65%
Sotheby's	48%	-4%	-6%	69%
Art Vivant	22%	21%	19%	39%
Artnet	56%	2%	1%	56%
Artprice	27%	-4%	-8%	49%

We want to make sure that our analysis at the level of the ACI is still valid at the level of each stock. To verify it, we compute the correlation of art companies constituting the ACI with the HPIs[3] in Table 2.4. The returns of the individual companies exhibit indeed a pattern similar to the art composite index: no linear relationship with median and top prices, but a positive correlation with returns of low prices. Artnet' stock returns are sensitive to the returns of the 5% cheapest works (+56% correlation). Sotheby's stock returns are also much more correlated to price level of the 5% cheapest artworks (+48%) than to the median (-4%) and top market (-6%), which can be surprising considering that the auction house generally does not accept to auction items worth less than 5000 USD. A plausible explanation is that Sotheby's turnover could be driven by cheaper artworks rather than expensive ones. Also, an increase of prices of cheaper artworks means more items to

[3]Weng Fine Art and Seoul Auction are discarded because their quotation covers too few semesters

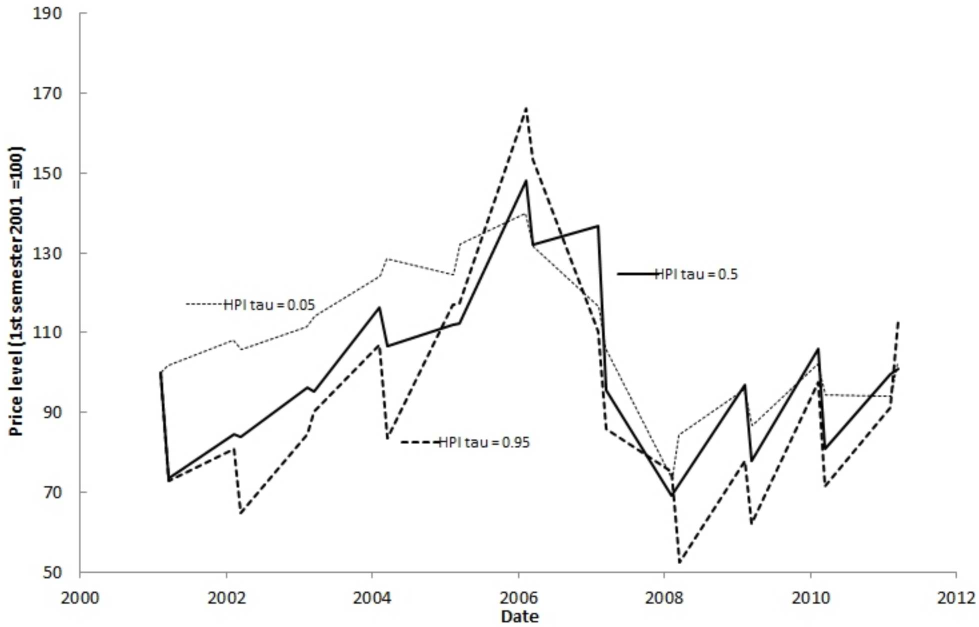

Figure 2.2: Semi-annual Hedonic Price Indices for physical art

auction for Sotheby's, since more objects are more likely to pass the mark of 5000 USD, hence driving Sotheby's future volumes upward. To confirm this hypothesis that art stocks may be driven by volumes rather than prices, we include in the correlation analysis the semi-annual change, in percentage, of volumes of our set of 99 artists sold at auction. This factor is well correlated with ACI's semi-annual returns (+65%), provided that one takes a 6 months lag into account. A reasonable explanation for this lag is that most auction sales are planned at least three to six months in advance whereas, under the efficient market hypothesis, the stock market immediately incorporates expectations of future sales in the stock prices. A first conclusion from this descriptive analysis is that the ACI and art-related companies covered by this analysis reflect more the anticipation of volumes exchanged in the art market than the general level of art prices.

Before turning to the benefits of the ACI in a diversified portfolio, we investigate its absolute performance. In Table 2.6, we compare the performance of the ACI with other financial indices. With a Compound Annual Growth Rate (CAGR) of +24%, the index of art-related stocks strongly outperformed other asset classes over the course of the years. The CAGR of the ACI is of +67% between the 15th of September 2008 (date of the collapse of Lehman Brothers) until end of October 2012. Gold prices exhibited a CAGR of +45% during the same period.

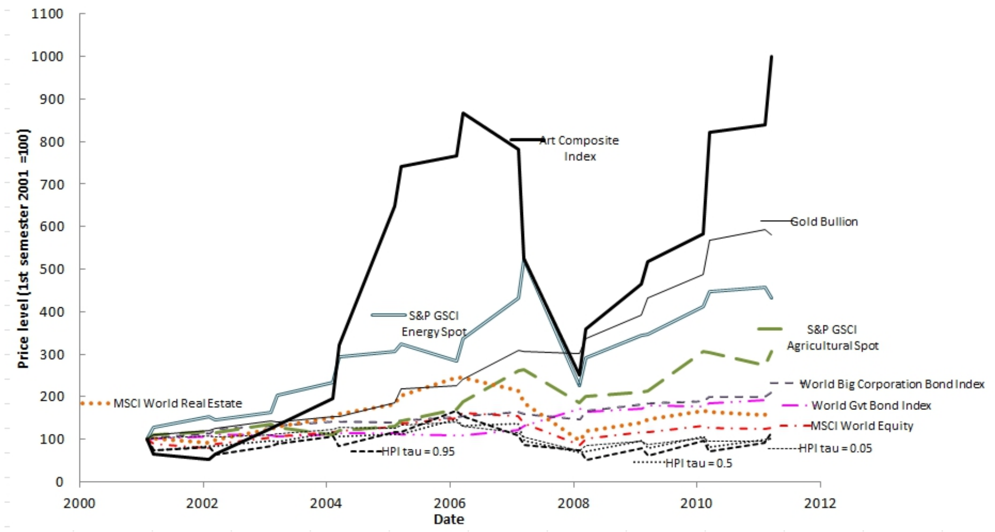

Figure 2.3: Semi-annualized financial indices and semi-annual HPI

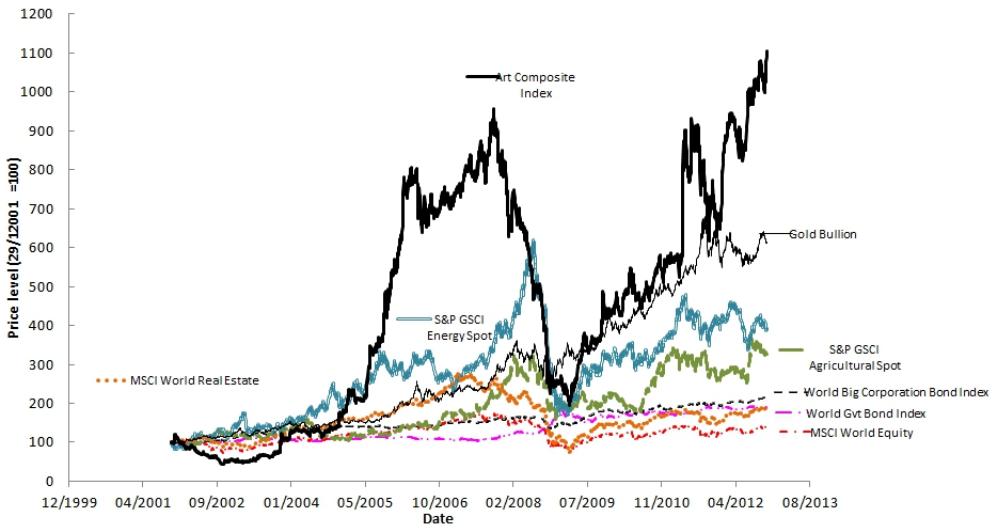

Figure 2.4: Daily financial indices

Table 2.5: Correlation of financial indices with semi-annual Hedonic Price Indices (HPI)

Correlation of semi-annual returns	HPI. Tau = 0.05	HPI. Tau = 0.50	HPI. Tau = 0.95
Art Composite Index	51%	-6%	-8%
World Gvt Bond Index	-67%	18%	18%
World Big Corporation Bond Index	42%	-6%	-10%
MSCI World Equity	62%	2%	0%
S&P GSCI Agricultural Spot	48%	13%	12%
S&P GSCI Energy Spot	49%	-14%	-16%
MSCI World Real Estate	67%	-5%	-6%
Gold Bullion	28%	25%	21%

Table 2.6: Compound Annual Growth Rate

CAGR	29/10/2001 - 29/10/2012	15/09/2008 - 29/10/2012
World Gvt Bond Index	6%	9%
World Big Corporation Bond Index	7%	12%
MSCI World Equity	3%	0%
S&P GSCI Agricultural Spot	11%	24%
S&P GSCI Energy Spot	13%	30%
MSCI World Real Estate	6%	9%
Gold Bullion	18%	45%
Art Composite Index	24%	67%

Table 2.7: Correlation between financial indices

Correlation 29/10/2001 - 29/10/2012	World Gvt Bond Index	World Big Corporation Bond Index	MSCI World Equity	S&P GSCI Agricultural Spot	S&P GSCI Energy Spot	MSCI World Real Estate	Gold Bullion	Art Composite Index
World Gvt Bond Index	100%	5.98%	-40.87%	-17.99%	-21.64%	-36.67%	-5.14%	-17.06%
World Big Corporation Bond Index		100%	11.64%	10.07%	11.51%	18.37%	33.94%	13.39%
MSCI World Equity			100%	25.97%	33.40%	79.37%	9.40%	39.02%
S&P GSCI Agricultural Spot				100%	34.62%	19.67%	17.14%	11.14%
S&P GSCI Energy Spot					100%	26.93%	21.91%	12.77%
MSCI World Real Estate						100%	11.19%	35.42%
Gold Bullion							100%	5.23%
Art Composite Index								100%

Table 2.8: Correlation between financial indices after Lehman's collapse

Correlation 15/09/2008 - 29/10/2012	World Gvt Bond Index	World Big Corporation Bond Index	MSCI World Equity	S&P GSCI Agricultural Spot	S&P GSCI Energy Spot	MSCI World Real Estate	Gold Bullion	Art Composite Index
World Gvt Bond Index	100%	5.98%	-40.87%	-17.99%	-21.64%	-36.67%	-5.14%	-13.96%
World Big Corporation Bond Index		100%	11.64%	10.07%	11.51%	18.37%	33.94%	24.87%
MSCI World Equity			100%	25.97%	33.40%	79.37%	9.40%	54.18%
S&P GSCI Agricultural Spot				100%	34.62%	19.67%	17.14%	11.14%
S&P GSCI Energy Spot					100%	48.89%	24.15%	28.96%
MSCI World Real Estate						100%	10.27%	46.95%
Gold Bullion							100%	1.22%
Art Composite Index								100%

Table 2.7 shows that over the 11 years of our analysis, daily returns of ACI are most correlated with the ones of the MSCI World Equity Index (39%), and then with the MSCI World Real Estate (35%). Interestingly, despite their relatively comparable long run performances, returns of art companies are least correlated to daily returns of gold prices over the course of the entire period (5%). Table 2.8 that focuses on the post-Lehman era shows that correlation between art companies and stocks increased during this period.

The ACI is the most volatile index among all, with an annualized volatility of +51.14% between October 2001 and October 2012. This high volatility compares to the one of the Gold Bullion (+35.28%). The ACI is strongly skewed to the right (+1.08), a characteristic only shared with government bonds (+0.58). Also, our pool of art-related companies has the biggest kurtosis: +9.0, ahead of the MSCI World Equity Index (+7.6).

Table 2.9: Moments

Moments 29/10/2001 - 29/10/2012	World Gvt Bond Index	World Big Corporation Bond Index	MSCI World Equity	S&P GSCI Agricultural Spot	S&P GSCI Energy Spot	MSCI World Real Estate	Gold Bullion	Art Composite Index
Volatility (annualized)	5.91%	17.87%	24.43%	31.82%	19.29%	18.95%	35.28%	51.14%
Skewness	0.58	0.00	- 0.18	0.00	- 0.04	- 0.21	- 0.29	1.08
Kurtosis	6.34	4.40	7.60	1.99	2.03	7.52	3.74	8.98

To sum up, amongst our selected indices, the ACI cumulates many superlatives: the most volatile, the most skewed with the fattest tails. Given this fact, the second question that needs to be answered is whether or not a basket of art-related companies was an effective diversification tool during the financial crises. If yes, then it would mean that it exhibits what can be expected from a safe haven such as physical art itself. If no, then it would mean that holding a basket of art related companies does not offer a hedge against crises, unlike physical art, eg, during the Second World War.

This is further investigated by observing the evolution of an optimal portfolio through the last decade. The portfolio is rolled every 30 trading days and is calibrated on 500 trading days. Figure 2.5 presents the results of this optimal portfolio rolling throughout the entire period[4].

The evolution of the portfolio is in line with well-known developments of financial markets during the last decade. We identify three periods: the real estate boom of

[4]For the sake of clarity of the plot, we sum optimal weights of government bonds and corporate bonds to create a "Fixed Income" category. Similarly, energy and agricultural commodities are gathered under a category "Commodities".

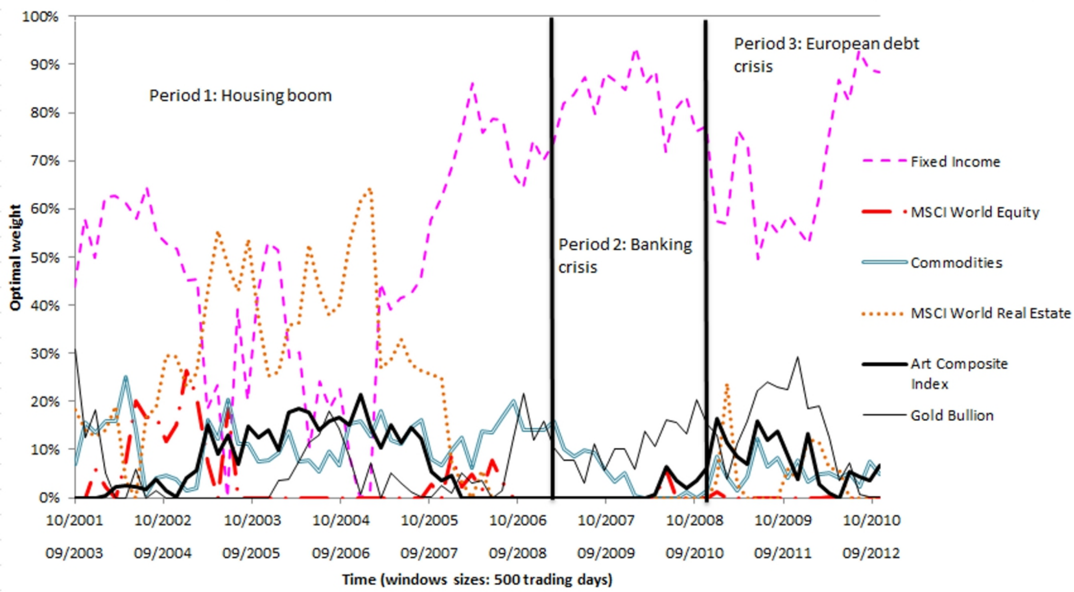

Figure 2.5: Optimal weights of the assets on the basis of a rolling window of 30 trading days. Calibration period: 500 trading days. Vertical lines are placed with respect to the center of time windows.

2001 - 2008, the banking crisis in 2008 - 2009, and the European debt crisis of 2009 - 2012.

The first period highlights the developments during the U.S. housing boom. We define the first period as a time frame between October 2001 and March 2008, a month hit by the bail-out of Bear Stearns by the Federal Reserve. During this first period, the MSCI World Real Estate overwhelms the optimal portfolio: at one point (between February 2005 and January 2007) it is supposed to have accounted for 64% of the entire portfolio. However, equity completely disappeared from the optimal portfolio covering the periods September 2003 to July 2005. Shares of other assets are evenly split. The Art Composite Index has a 21.5% share in the time window ranging from January 2005 to December 2006. This level was never reached again. During this first period, the average optimal weight for the ACI is of 7.3% against 5.4% for gold and 4.2% for the MSCI World Equity Index.

The second period ranges from the Bear Stearn's bail-out until the start of the European debt crisis in October 2009. Only three types of assets held their ground in the optimal portfolios during these challenging times: commodities, gold and fixed income. Art stocks' weight completely dropped to zero, revealing the uselessness of holding art-related stock during the banking crisis. Take note that commodities and gold are physical assets. Since we have shown that returns of art stocks are not directly correlated to returns of physical art, our analysis does not tell much about a possible role of physical artworks during the banking crisis. The analysis only shows that investments in art companies were a poor choice during that crisis. This second period ends in October 2009 that marks the start of the European debt crisis (Lane, 2012). Indeed, this month corresponds to the general Greek election that emphasized the sovereign risk of Greece and the possibility of a default inside the Euro Area.

During the third period, the MSCI World Equity still does not recover, to the contrary of art-related stocks that benefit from a rally. Fixed Income drops to a 50% allocation, simultaneously, art-stocks take over commodities to top 16% allocation between July 2009 and June 2011. Gold's share in the optimal portfolio reaches a maximum of 29% in the period covering December 2009 to November 2011.

Table 2.10: Descriptive statistics of optimal weights in rolling portfolio

Average weight	World Gvt Bond Index	World Big Corporation Bond Index	MSCI World Equity	S&P GSCI Agricultural Spot	S&P GSCI Energy Spot	MSCI World Real Estate	Gold Bullion	Art Composite Index
Housing boom	17.8%	29.1%	4.2%	6.8%	4.6%	24.8%	5.4%	7.3%
Banking crisis	49.4%	34.4%	0.4%	1.4%	2.4%	0.0%	10.8%	1.2%
European debt crisis	32.8%	36.4%	0.1%	4.1%	1.2%	4.2%	13.4%	7.8%
Maximum weight								
Housing boom	74.2%	76.4%	26.3%	22.3%	16.0%	64.3%	30.8%	21.5%
Banking crisis	81.4%	62.9%	5.5%	7.0%	9.3%	0.0%	20.4%	6.4%
European debt crisis	55.5%	68.0%	1.2%	12.2%	7.6%	23.7%	29.3%	16.4%

Increase of the weight of art-related stocks in an optimal portfolio can reasonably be explained by anticipation of higher volumes in the art market that may have boosted returns of art companies. This is confirmed by data on observed volumes at auction (see Figure 6). It remains to be unveiled why the volumes of physical art went up during the European debt crisis. We identify three paths for further research. Firstly, quantitative easing by central banks may have directly or indirectly fuelled liquidity of markets of certain goods that can be perceived as a hedge against inflation, such as art or gold. Secondly, fears related to the Greek crisis could have triggered a flight to assets less exposed to geographical or political risks. Under that perspective, physical art could be a hedge against political risks, in a fashion similar to what Oosterlinck (2010) describes for the Second World War. Thirdly, following a severe art market disruption in 2008, sellers may have tried to postpone their sales, waiting for better market conditions. The increase in 2009 would then reflect the fact that sellers cannot postpone their sales indefinitely.

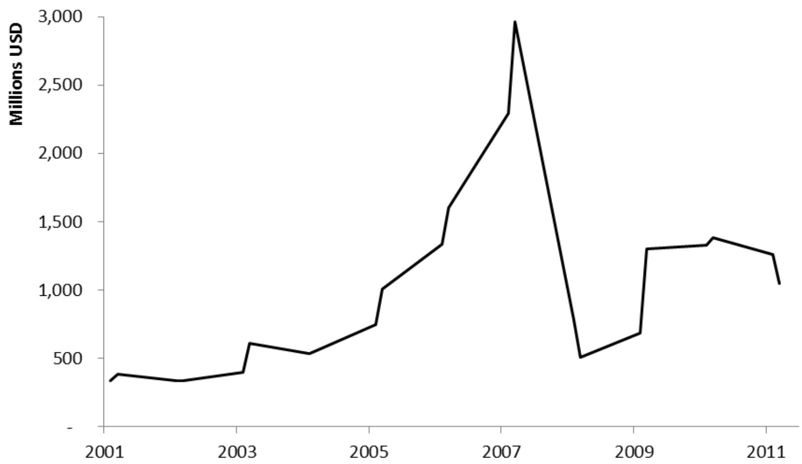

Figure 2.6: Semi-annual volumes (in USD) of sales at auction for 99 artists

2.8 Conclusion

Our analysis suggests that holding art-related stocks is not an approximation for holding physical art: art-linked equities exhibit no correlation with median and top art prices, but are instead positively correlated to cheaper, bottom 5% artworks and real estate equities. This highlights the close relationship between art companies and physical art as a consumption good. More strikingly, prices of art companies depend on changes in expected volumes at auction, so that a long position in a basket of art-related stocks yields an exposure to market activity, not to price levels. This contrasts with oil or gold companies that are closely related to the price of the underlying assets they mine. Such discrepancy can be easily understood, since art companies do not produce art but depend on existing flows in the market. Using a new framework for portfolio optimization based on R-Vine copulae, we confidently confirm that holding a composite index of these companies was useless during the banking crisis, though it was beneficial during the European debt crisis thanks to a corresponding increase in volumes at auction. These results lead to suggestions for further research: why art volumes took off during the European debt crisis? Was this directly related to new monetary policies and increased liquidity supply? Or did art simply benefit from economic recovery led by these policies? Does the art market activity specifically soar during political distress ?

Chapter 3

Econometric analysis of volatile art markets

Adapted from Bocart and Hafner (2012a)

3.1 Abstract

A new heteroskedastic hedonic regression model is suggested. It takes into account time-varying volatility and is applied to a blue chips art market. Furthermore, a nonparametric local likelihood estimator is used. This estimator is more precise than the often used dummy variables method. The empirical analysis reveals that errors are considerably non-Gaussian, and that a student distribution with time-varying scale and degrees of freedom does well in explaining deviations of prices from their expectation. The art price index is a smooth function of time and has a variability that is comparable to the volatility of stock indices.

3.2 Introduction

It is well documented that volatility of many commodities and stocks displays a certain degree of time variation. This feature has important consequences for economists, policy makers, economic agents, actors in the financial and commodity markets. Since Engle (1982)'s ARCH model, several models have been built to investigate volatility of commodities, and a large literature is now dedicated to its time-varying structure.

Surprisingly, while considerable efforts have been devoted to assess returns in the art market, few studies attempt to investigate the volatility structure of art as a function of time. Yet, volatility of fine art is worth investigating, and a better understanding of its structure may be of practical use for market participants, more particularly for participants exposed to derivatives on art. Such derivatives include price guarantees underwritten by auction houses Greenleaf, Rao, and Sinha (1993) that are similar to short positions in put options. Volatility of fine art also plays a role when pieces of art are used as collateral for loans McAndrew and Thompson (2007). Campbell and Wiehenkamp (2008) illustrate the mechanism of another art-based option: the Art Credit Default Swap: A bank lends money to an entity on the one hand, and buys an option (the Art Credit Default Swap) from a third party -the seller of protection- on the other hand. This option gives the bank the right to swap the art object against cash, would the borrower default. Many other derivatives, sensitive to volatility, abound in the market for insurance on luxury goods and art. Unlike commodities exchanged on organised platforms, a common complication in analysing the market for art and antiques is the heterogeneity of exchanged goods. This feature prevents the observer from directly estimating returns and volatility of the market. As far as returns are concerned, two main methodologies have been developed to cope with this issue: the repeat sale methodology (RSM) and the hedonic regression. RSM is based on various goods that have been sold several times in different periods, so as to compute an average rate of return. RSM has been used by Baumol (1986), Goetzmann (1993), Pesando (1993), as well as Mei and Moses (2002). A major critique against RSM is that it focuses on a small, biased sample of goods Collins, Scorcu, and Zanola (2009) that have been resold through time.

This paper focuses on the hedonic regression methodology (HRM) that is further detailed in Section 2. Hedonic regression has been favoured to study the art market by Chanel, Gérard-Varet, and Ginsburgh (1996), Hodgson and Vorkink (2004), Collins, Scorcu, and Zanola (2009), Oosterlinck (2010), Renneboog and Spaenjers (2013) and Bocart, Bastiaensen, and Cauwels (2011). Hedonic regression has the advantage of using all goods put for sale. The approach is to regress a function of the price of each good on its characteristics, including time dummy variables whose coefficients will constitute the basis for building an index. The main disadvantage of hedonic regression methodology is that the index depends on the explanatory variables. Ginsburgh, Mei, and Moses (2006) discuss the main problems of hedonic regression applied to the art markets, such as the choice of a functional form, the specification bias and the “revision volatility” -that is, as new data are included in the dataset, the price index changes. Methodology-wise, ordinary least squares are usually employed to estimate parameters. Recent research aims at correcting methodological flaws in hedonic regression: Collins, Scorcu, and Zanola (2009) introduce the Heckman procedure to take into account a sample selection bias linked to unsold artworks as well as a Fisher index to cope with time instability of parameters. Jones and Zanola (2011) detail the use of a so-called smearing factor to correct for a retransformation bias when a log scale of prices is used in the regression. Scorcu and Zanola (2011) suggest a quantile regression to take into account the fact that parameters depend on price levels. Hodgson and Vorkink (2004), highlight that for the art market, non-Gaussianity is an issue that needs to be treated, since OLS estimates are not efficient. They assume an i.i.d. error term with nonparametric density function, and suggest Bickel's adaptive estimation to obtain efficient estimates. While this is an important improvement of standard OLS estimation in this framework, the assumption of i.i.d. errors may seem too restrictive for markets which exhibit time-varying features such as changing uncertainty concerning the evaluation of art. In particular, we show in this paper that art markets can be heteroskedastic.

We recommend a local maximum likelihood procedure to obtain time-varying estimates of higher moments, i.e., variance, skewness and kurtosis. The time-varying variance is later used to derive what we call “volatility of predictability”. It can also be used to obtain more efficient parameter estimates by using weighted least squares. However, our main interest lies in volatility in itself, as it can be used further e.g. for derivative pricing. Modelling unconditional volatility as a deterministic function of time has become popular recently in financial markets, starting from Engle and Rangel (2008) who use a spline estimator for unconditional volatility combined with a classical GARCH model for conditional volatility. Our research follows the same spirit but allows moreover for time-varying skewness and kurtosis.

The paper is organised as follows: Section 2 introduces the data we use to build a blue-chips art index and presents the HRM methodology and a time-dependent estimator for variance. Section 3 illustrates our results. Concluding remarks are presented in section 4.

3.3 Data and methodology

Data

We choose to restrict our analysis to two-dimensional artworks, excluding works on paper and photographs, made by artists ranked amongst the top 100 sellers (in sales revenue in auction houses, according to Artprice, a company specialized in publishing auction results), both in 2008 and 2009. The rationale behind this choice is that large volumes of sales may signal a particular interest from the market for these artists.

We believe that these artists are more likely to be seen both as consumption goods and investment goods unlike many little traded artists whose objects are more likely to be bought as pure consumption goods. Indeed, Frey and Eichenberger (1995) state that "pure speculators" who consider art as an investment may avoid markets presenting too much uncertainty (such as financial risk or attribution risk).

Furthermore, Goetzmann (1993) emphasizes that art prices are influenced by "stylistic risk", that is the risk of having not enough bidders when reselling the artwork. Mei and Moses (2002) compare stylistic risk in art markets to liquidity risk in financial markets. Unknown and relatively little traded artists are typically cursed by considerable financial and liquidity risk, as it can be difficult to realize a sale in a market where demand is weak.

On the other hand, buyers of liquid artists – with a low stylistic risk – know ex-ante that they will be able to re-sell artworks, which might attract speculators and investors. In practice, art is actually traded as an investment. This is empirically confirmed by activity from dealers, funds, foundations and private individuals who store artworks in warehouses, bank vaults, or in Switzerland's port-franc containers, where obviously the aesthetic return is null.

Based on the assumption that liquid artists can be seen as an investment, we focus on "Blue Chips Artists": we need to select artists who stay in the top 100 of best sellers two consecutive years, in order to avoid bias from exceptional or unusual sales. Forty artists correspond to this description, out of which 32 stayed in the top 100 from 2005 to 2010 in a row. We record auction data from January 2005 to June 2010. 5612 sold pieces are recorded. An auction process is an opportunity to record information. Auction houses announce weeks to months in advance the dates when auctions will occur. Sometimes, a single auction is split into several days. In most cases, the sale is organized around a certain theme ("Impressionist art" for instance). Prior to the auction, a catalogue is published by the auction house. In this catalogue, each artwork is linked to a lot number, a price estimate, and a description. The length of the description differs from one artwork to another, but key variables are systematically recorded. For each sale, we gather the following information: the price in USD, and whether it is a hammer price (that is, the price reached at auction), or a premium price (the price including the buyer's premium), the sales date, the artist's name, the width and height of the painting in inches, the year it has been made, the painting's title, the auction house and city where the sale occurred and the title of the auction house's sales theme. From this information we extract additional variables, such as the subject of the painting (derived from the title), the theme of the auction (modern, contemporary, impressionist, etc.), derived from the sale's title, the artist's birthday, at what age he painted the piece and whether he was alive or dead at the time of the auction. We also derive the weekday of the sale. Some factors are omitted that may influence the final price for a painting, such as exhibition costs, transaction costs, and transport. All variables are presented in tables 3.3, 3.4 and 3.6.

Hedonic Regression Methodology

Hedonic regression is a common tool to estimate consumer price indices (see e.g. Ginsburgh et al., 2006) and has been widely used in real estate and art markets. Let p_i denote the price of sale i. The logarithm of this price is usually modelled by the following hedonic regression model,

$$\log p_i = \nu + \sum_{t=1}^{T} \beta_t d_{i,t} + \sum_{k=1}^{K} \alpha_k v_{i,k} + u_i, \quad i = 1, ..., N. \tag{3.1}$$

$d_{i,t}$ is a dummy variable taking the value 1 if the artwork i was sold in period t, and 0 otherwise. ν is a constant term. The time index $t = 1$ corresponds to the very first period of the series and is used as benchmark. In our case, it would be the first quarter of 2005. For identification, we set β_1 equal to zero. The K variables $v_{i,k}$ are all other characteristics of the piece of art i (for instance: the height, surface, and dummies for the artists, subject, etc.). The index, with base 100 in $t = 1$, using a bias correction factor based on Duan (1983) is then defined as follows, see Jones and Zanola (2011). The idea behind the smearing factor is that the estimated residuals can be used to estimate the distribution-robust retransformation factor $\frac{1}{N}\sum \exp(\hat{u}_i)$, so that $E[p_i \mid p_i > 0, \beta_t d_{i,t}, v_{i,k}]$ is estimated as $\frac{1}{N}\sum \exp(\hat{u}_i)\exp(\widehat{\log(p_i)})$. In particular, to estimate the relative difference between the prices of a characteristics-neutral artwork at time t and at time 1:

$$\text{Index}_t = 100 \times e^{\beta_t} \times \frac{\frac{1}{N_t}\sum_{i=1}^{N} d_{i,t}e^{\hat{u}_i}}{\frac{1}{N_1}\sum_{i=1}^{N} d_{i,1}e^{\hat{u}_i}}, \tag{3.2}$$

where $N_t = \sum_{i=1}^{N} d_{i,t}$ is the number of observations at time t. Regression (4.1) is generally estimated using Ordinary Least Squares (OLS). OLS estimators are efficient when errors u_i are normally distributed with constant variance, i.e., $u_i \sim N(0, \sigma_u^2)$.

Data from art sales, however, often violate this assumption. Hodgson and Vorkink (2004) and Seckin and Atukeren (2006) focus on the normality part and propose a semiparametric estimator of the index based on a nonparametric error distribution, while maintaining the assumption that u_i is i.i.d. and, hence, homoskedastic.

Furthermore, indices based on the OLS methodology suffer from a sample selection bias. Indeed, only sold paintings are taken into account, whereas unsold paintings carry important information as well. Collins, Scorcu, and Zanola (2009) suggest a two-stage estimation to cope with the issue. Let S_i denote a dummy variable taking value 1 if the painting i has been sold and 0 otherwise. The first stage involves a probit estimation:

$$P(S_j = 1 \mid w_j) = \Phi\left(\sum_{p=1}^{P} \delta_p w_{j,p}\right),\ j = 1, ..., N + U, \tag{3.3}$$

where Φ is the cumulative distribution of the standard normal, N is the number of pieces sold and U is the number of unsold pieces. The P variables $w_{j,p}$ are characteristics of the piece of art j (for instance: the height, surface, and dummies for the artists, subject, etc.), and $\delta = (\delta_1, \ldots, \delta_P)'$ is a parameter vector.

The second stage involves an OLS estimation similar to equation (4.1), but only for the sold pieces ($S_i = 1$):

$$\log p_i = \nu + \sum_{t=1}^{T} \beta_t d_{i,t} + \sum_{k=1}^{K} \alpha_k v_{i,k} + \kappa\zeta_i + u_i,\ i = 1, ..., N. \tag{3.4}$$

The term ζ_i is a correcting variable, based on parameters of the probit estimation and found using the procedure of Heckman (1979).

We now propose to modify the time component, replacing the time dummies $d_{i,t}$ by a smooth unknown function of time, and allowing for heteroskedasticity of unknown form. An important advantage of choosing a continuous function $\beta(t)$ rather than time dummies is that one avoids gathering paintings sold at different periods in a single variable. We also remove the normality assumption, allowing for skewness and leptokurtosis. In particular, we assume that residuals are distributed according to a student-skewed distribution. Our semiparametric heteroskedastic model can then be written as

$$\log p_i = \nu + \sum_{k=1}^{K} \alpha_k v_{i,k} + \kappa\zeta_i + \beta(t_i) + \sigma(t_i)\varepsilon_i, \quad i = 1, ..., N, \tag{3.5}$$

or, alternatively:

$$\log p_i = \sum_{m=1}^{M=2+K} \gamma_m x_{i,m} + \xi_i, \quad i = 1, ..., N, \tag{3.6}$$

where $x_i = (1, v_{i,1}, ...v_{i,k}, ..., v_{i,K}, \zeta_i)$, and

$$\xi_i = \beta(t_i) + \sigma(t_i)\epsilon_i = \beta(t_i) + u_i, \quad u_i = \sigma(t_i)\epsilon_i. \tag{3.7}$$

The function $\sigma(t)$ is a smooth function of time, t_i is the selling time of the i-th sale, $\beta(t)$ is the trend component of the log price at time t, and for identification we restrict its mean to zero. The error term ε is independent, but not identically distributed, with mean zero and variance one, given by a standardized student skewed distribution. The probability density function of the student skewed distribution $t(\eta, \lambda)$ with mean zero and variance equal to one is provided by Hansen (1994):

$$g(\varepsilon \mid \lambda, \eta) = bc\left(1 + \frac{1}{\eta - 2}(\frac{\varepsilon + a}{1 - \lambda})^2\right)^{\frac{-(\eta+1)}{2}} \quad \forall\varepsilon < -a/b, \tag{3.8}$$

and

$$g(\varepsilon \mid \lambda, \eta) = bc\left(1 + \frac{1}{\eta - 2}(\frac{\varepsilon + a}{1 + \lambda})^2\right)^{\frac{-(\eta+1)}{2}} \quad \forall\varepsilon \geq -a/b, \tag{3.9}$$

where η stands for the degrees of freedom with $2 < \eta < \infty$, and λ is a parameter characterizing the skewness of the distribution, with $-1 < \lambda < 1$. The constants are given by

$$a = 4\lambda c\frac{\eta - 2}{\eta - 1}, \quad b^2 = 1 + 3\lambda^2 - a^2, \quad c = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta - 2)}\Gamma(\frac{\eta}{2})}. \tag{3.10}$$

A first stage estimation of γ is a prerequisite. We suggest constructing feasible weighted least squares (FWLS) estimators of γ:

$$\hat{\gamma} = (X'\hat{W}X)^{-1}X'\hat{W}Y, \tag{3.11}$$

where X is the N×M matrix of observed independent variables, Y is the $N \times 1$ vector of observed log-prices, $\hat{W}$ is an N×N diagonal matrix with $w_{ii} = \hat{\sigma}^{-2}(t_i)$. $\hat{\sigma}^{-2}(t_i)$ can be iteratively derived, starting, for instance, with the standard OLS estimator. If a nonparametric Nadaraya-Watson estimator is used for $\hat{\sigma}^2$, then the estimator (3.11) has been first proposed by Rose (1978). For the case of a volatility function depending on an i.i.d. random variable, Carroll (1982) showed that it is asymptotically equivalent to the WLS estimator with known volatility function. We can consistently estimate the variance of the FWLS estimator by

$$\widehat{\mathrm{Var}}(\hat{\gamma}) = (X'\hat{W}X)^{-1}. \tag{3.12}$$

Because it yields more precision in parameter estimates, FWLS may lead to a better selection of explanatory variables, as compared to the OLS methodology. This is important since *"choosing the functional form and the variables that represent quality are pervasive in hedonic indexing, and can lead to all the problems linked to mis-specification"* Ginsburgh, Mei, and Moses (2006).

Conditional on this first stage estimate of γ, we use a nonparametric estimation, introducing a kernel function K and a bandwidth h. We suggest estimating the local parameter vector $\theta = (\beta, \sigma, \eta, \lambda)'$ by local maximum likelihood. One advantage of considering θ as a function of continuous time is the improved stability of estimation compared to ordinary least squares with time dummies. Indeed, we avoid all risks linked to the inversion of a near singular matrix, a problem often met when few data are available in a given period. Smoothing over several adjacent time periods allows to stabilize the estimation of a parameter at a given time. Formally, the local likelihood estimator of θ is defined as

$$\widehat{\theta}(\tau) = \mathrm{argmax}[l(\theta \mid \xi, \tau, h)], \tag{3.13}$$

where

$$l(\theta \mid \xi, \tau, h) = \sum_{i=1}^{N} \log[g_s(\xi_i \mid \theta)] K(\frac{t_i - \tau}{h}), \tag{3.14}$$

$$g_s(\xi_i \mid \theta) = \frac{1}{\sigma} g\left(\frac{\xi_i - \beta}{\sigma} \mid \lambda, \eta\right), \tag{3.15}$$

and where $g(\cdot)$ is the standardized skewed student t density given in (3.8) and (3.9). No closed form solution for (3.13) is available in the general case, but numerical methods can be employed in a straightforward way to maximize the local likelihood function and thus obtain local parameter estimates.

In order to construct pointwise confidence intervals, we proceed as in Staniswalis (1989). Let υ denote one of the four local parameters $(\beta, \sigma, \eta, \lambda)$ and ι, the three others. An expression for the variance $\mathrm{Var}(\hat{\upsilon})$ is given by

$$\mathrm{Var}(\hat{\upsilon}) = \frac{|| K ||^2}{NhI(\upsilon)f(t)}, \tag{3.16}$$

where

$$I(\upsilon) = E\left\{ \left(\frac{\partial log[g_s(\upsilon \mid \iota)]}{\partial \upsilon} \right)^2 \mid u \right\}, \tag{3.17}$$

$f(t)$ is the density of the time of sales, and $|| K ||^2$ is the L_2 norm of the kernel used in equation (3.14). Based on the asymptotic normality of the estimator of $\theta(\tau)$, one can then construct pointwise confidence intervals as usual.

The special case where $\lambda(t) = 0$ and $\eta(t) = \infty$ yields the Gaussian likelihood, for which the maximizer is available in closed form and given by the Nadaraya-Watson estimator Hardle (1990). Hence, our estimator nests the Nadaraya-Watson estimator as a special case.

Bandwidth selection can be based on a classical plug-in methodology for bandwidth selection, following Boente, Fraiman, and Meloche (1997):

$$h = N^{-1/5} \frac{|| K ||^2 \, \sigma^2}{C_2^2(K) \int_0^1 m''(u)^2 du}, \tag{3.18}$$

where $C_2(K) = \int_{-\infty}^{+\infty} x^2 K(x) dx = 1$, $m''(u) = \sum_{i=1}^{n} \frac{1}{Nh_0^3} K''(\frac{u-u_i}{h_0}) u_i$, σ^2 is the empirical variance of ξ and h_0 is a pilot bandwidth. We follow this procedure in the empirical example of the following section.

3.4 Results

Hedonic Regression

We first build a quarterly index using time dummies, using an OLS methodology with Heckman correction. The variables selected in the probit equation (3.3) are presented in Table 3.2. Variables included in regression (3.4) are selected following a backward selection methodology: they are kept in the model if significant at a level of 5% using OLS regression. Advantages and disadvantages of backward selection are discussed e.g. in Hendry (2000). As compared to other selection methodologies such as forward selection, backward selection suffers from the fact that the initial model may be inadequate. Indeed, non-orthogonality of variables may lead to erroneously eliminate variables, or wrongly keep colinear variables. To avoid this problem, we run different initial models, separating variables that share a high degree of colinearity. The model presenting the highest adjusted-R^2 has been kept.

Table 3.1 summarizes results from the regression and Figure 3.5 presents the resulting OLS-based price index. The need to correct for time dependent error variance is indicated by a Breusch-Pagan test for heteroskedasticity on OLS residuals, which delivers a p-value of 0.02. We hence reject the null hypothesis of homoskedasticity at a level of 5%. The quantile plot in Figure 3.7 highlights that normality of residuals is an unrealistic assumption.

We then proceed with our methodology: we discard time-dummy variables and select explanatory variables with a backward selection methodology at a 5% level, this time using FWLS regression. Table 3.1 compares results from OLS with those from FWLS. As we expected, the model changes as some variables prove not significantly different from zero at a 5% level with the FWLS estimation. These four variables are Mark Rothko and Camille Pissarro, pieces sold in Tokyo and artworks sold at Bonhams.

The 23 artists (out of 40 available) present in the table have a significant impact on price, ceteris paribus, compared to the other 17 that were not included. However, one should not try to draw a ranking from this table, as difference between artists would not always be statistically significant. Some other results from Table 3.1 are in line with existing literature: the size (width) has a positive effect on price, but the surface has a negative one, reflecting the idea that a bigger artwork is more expensive, up to the point that it is too big to hang. Prestigious auction houses, like Sotheby's or Christie's are also statistically different from the other ones. Surprisingly, Villa Grisebach (in Germany) stands in the same category. The negative sign linked to the age of the artist reveals that the market prefers, on average, earlier works of the artist whereas untitled artworks are less favoured by the public. Interestingly, mentioning a collection in the title of the sale (for instance: "Important works from the collection of...") leads to higher price. We believe this may be linked to a signal of "good provenance". Also, evening sales tend to exhibit more expensive paintings than day sales.

The second stage of our methodology consists of estimating four continuous time dependent parameters: $\beta(t)$, that will be used to create a price index, $\sigma(t)$, a heteroskedastic term, $\eta(t)$ and $\lambda(t)$ are the parameters that shape the student-skewed distribution of residuals of regression (3.5). We estimate numerically the parameters by finding the values that maximize the local log-likelihood function in

Table 3.1: Parameters estimates of regression 3.5. Variables are selected by backward selection at a level of 5% with an OLS and a FWLS estimation, respectively.

	Estimate OLS	Estimate FWLS	Std. Error (OLS)	Std. Error (FWLS)
(Intercept)	10.85 ***	11.03 ***	0.16	0.12
AgePainted	-0.004 ***	-0.004 ***	0.001	0.001
Alexander.Calder	-3.24 ***	-3.28 ***	0.17	0.17
Alexej.Jawlensky	-0.38 ***	-0.34 ***	0.09	0.09
Andy.Warhol	-0.75 ***	-0.74 ***	0.07	0.07
Bonhams	-0.42 ***	-	0.19	-
Camille.Pissarro	-0.18 ***	-	0.11	-
Childe.Hassam	-0.58 ***	-0.53 ***	0.16	0.16
Christies	0.19 ***	0.24 ***	0.06	0.07
Collection	0.38 ***	0.34 ***	0.11	0.11
Contemporary	-0.23 ***	-0.17 ***	0.06	0.06
Damien.Hirst	-0.76 ***	-0.68 ***	0.10	0.11
DaySales	-0.28 ***	-0.2 ***	0.05	0.06
Dead	0.68 ***	0.67 ***	0.09	0.09
Donald.Judd	-1.66 ***	-1.53 ***	0.37	0.39
Edgar.Degas	-0.82 ***	-0.78 ***	0.20	0.21
Edouard.Vuillard	-1.10 ***	-1.11 ***	0.11	0.11
Evening	1.37 ***	1.45 ***	0.05	0.05
Georges.Braque	-0.68 ***	-0.68 ***	0.12	0.12
Gerhard.Richter	-0.32 ***	-0.29 ***	0.10	0.10
Hammer	-0.20 ***	-0.27 ***	0.06	0.07
Henri.de.Toulouse.Lautrec	-0.71 ***	-0.65 ***	0.19	0.20
Henri.Matisse	0.47 ***	0.56 ***	0.13	0.14
Henry.Moore	-2.94 ***	-3.07 ***	0.52	0.55
HongKong	1.13 ***	1.17 ***	0.14	0.14
Impressionist	-0.11 ***	-0.16 ***	0.06	0.06
Jean.Michel.Basquiat	-0.61 ***	-0.59 ***	0.10	0.11
Kees.van.Dongen	-0.51 ***	-0.54 ***	0.09	0.09
KollerAuktionen	0.54 ***	0.74 ***	0.20	0.21
London	0.93 ***	0.91 ***	0.07	0.07
Mark.Rothko	0.32 ***	-	0.16	-
Maurice.de.Vlaminck	-1.40 ***	-1.44 ***	0.07	0.07
Max.Ernst	-0.89 ***	-0.85 ***	0.09	0.10
Milan	0.38 **	0.5 **	0.15	0.15
NY	0.87 ***	0.91 ***	0.06	0.07
Pablo.Picasso	0.36 ***	0.39 ***	0.08	0.08
Paris	0.44 ***	0.52 ***	0.07	0.07
Pierre.Auguste.Renoir	-0.43 ***	-0.44 ***	0.07	0.07
Raoul.Dufy	-1.00 ***	-1.02 ***	0.09	0.09
SaintCyr	-0.51 ***	-0.52 ***	0.13	0.14
Sam.Francis	-2.00 ***	-2.06 ***	0.07	0.07
Sothebys	0.13 ***	0.21 ***	0.06	0.06
Surface	-0.00002 ***	-0.00002 ***	0.000001	0.000001
ThemeWord	0.01 ***	0.009 ***	0.002	0.002
Tokyo	-2.79 ***	-	0.23	-
Untitled	-0.44 ***	-0.43 ***	0.06	0.06
VillaGrisebach	0.71 ***	0.83 ***	0.15	0.16
Width	0.02 ***	0.02 ***	0.0007	0.0007
Woman	0.19 ***	0.2 ***	0.06	0.06
Yayoi.Kusama	-1.46 ***	-1.47 ***	0.10	0.11
Heckman Correction	0.10	0.12	0.18	0.20
Adjusted R^2	68%	65%		
Maximum VIF (Variance Inflation Factor)		6.53		
Median VIF (Variance Inflation Factor)		1.46		
Maximum Cook's distance		0.04		
Median Cook's distance		0.0005		
Standard Deviation of Residuals		1.07		

Table 3.2: Variables and estimators of parameters of probit equation (3.3) -first stage for Heckman procedure-

	Estimate	Std. error
(Intercept)	0.77 ***	0.04
Contemporary	0.16 **	0.06
Impressionist	-0.15 **	0.07
DaySales	-0.25 ***	0.06
Sam.Francis	-0.14 *	0.08
Kees.van.Dongen	-0.26 **	0.11
Georges.Braque	-0.44 ***	0.14
Edouard.Vuillard	-0.33 **	0.13
Andy.Warhol	-0.42 ***	0.08
Christies	1.04 ***	0.07
Sothebys	0.64 ***	0.06
McFadden Pseudo R2	14%	

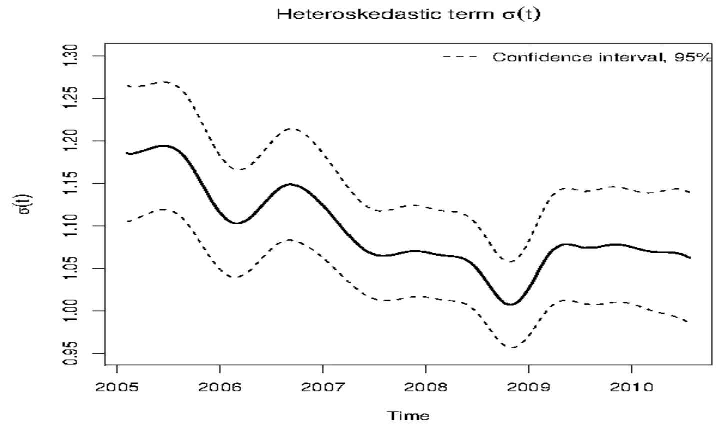

Figure 3.1: Local maximum likelihood estimator of the heteroskedastic term $\sigma(t)$ from equation (3.5).

equation (3.14).

In order to be as precise as possible, we use the day as unit for t. For the local likelihood estimation, we choose a Gaussian kernel and a bandwidth of $h = 88$ following the plug-in method described above, where the pilot bandwidth h_0 was chosen in the range $h_0 = [1; 30]$.

Figures 3.1, 3.2 and 3.3 plot the estimates of $\sigma(t)$, $\lambda(t)$ and $\eta(t)$, respectively. In order to obtain pointwise confidence intervals, it is necessary to estimate their variance. Figure 3.8 in appendix illustrates the estimated function $f(t)$ used in equation (3.16). Practically, this function is estimated by a Nadaraya-Watson estimator.

When considering the parameters, one can first conclude from Figure 3.2 that we cannot reject the null hypothesis that $\lambda(t) = 0$. In other words, the skewness

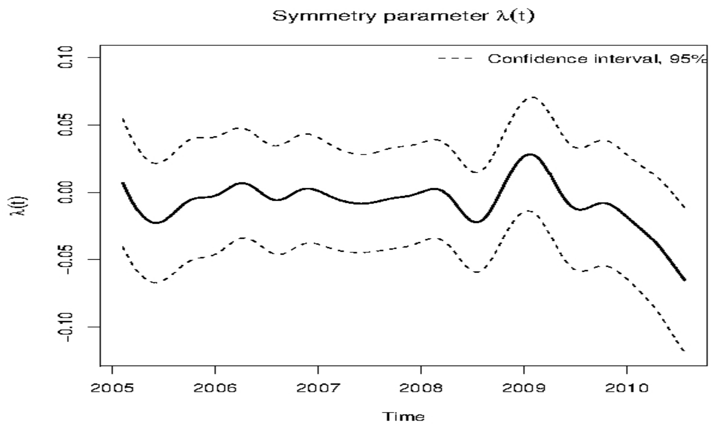

Figure 3.2: Local maximum likelihood estimator of the symmetry parameter $\lambda(t)$ of equations (3.5)

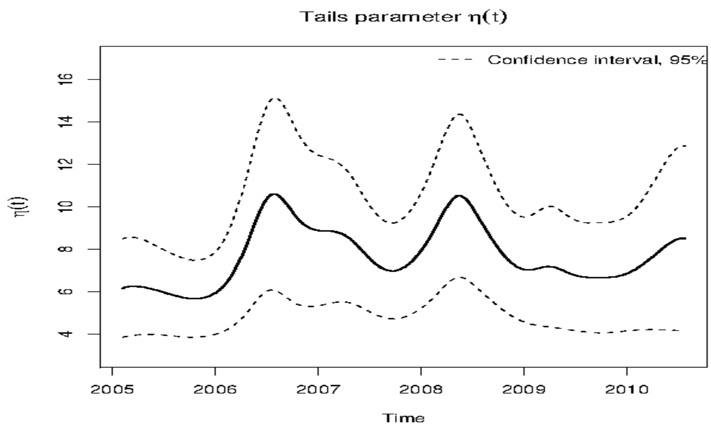

Figure 3.3: Local maximum likelihood estimator of the degrees of freedom parameter of equations (3.5)

parameter does not prove useful for this precise example. Nevertheless, we believe one should not draw the conclusion that asymmetry of residuals is typically an unrealistic assumption. For instance, with the same data, we observed that $\lambda(t)$ is significantly different from zero when the Heckman correction is neglected. Concerning the tail parameter $\eta(t)$, it is clear from Figure 3.3 that tails are fat and that a student distribution better fits data than the Gaussian. For both parameters, however, we cannot conclude that time dependency significantly adds value to the model as compared to a constant term.

On the other hand, it is indispensable to allow for heteroskedasticity through a time dependent scaling function. Furthermore, the behaviour of $\sigma(t)$ has an economic meaning: $\sigma(t)$ can be interpreted as the degree of deviation of the realized logged-price of a given painting from the rest of the art market. We call it the "volatility of predictability". In other words, a high $\sigma(t)$ means that is more difficult to predict an artwork's price. A low $\sigma(t)$ corresponds to a more precise estimation of a painting's value. Predictability of prices is vital for auction houses and their clients, especially when guarantees are involved. From Figure 3.2, we observe that this uncertainty steadily decreased from January 2005 to October 2008. Then, it increased again, or at least stabilized according to the lower confidence interval. It is interesting that the trough of the volatility function occurs at the end of 2008, at about the same time as the peak of the financial crisis with the collapse of Lehman Brothers (September 2008). It also coincides with the drop of the art price index, see Figure 3.4. This suggests that the precision of the art index has increased during the crisis of 2008/09. An explanation could be the asymmetry of art sales: while there is no upper bound, there is very often a lower bound through a reserve price below which sales are not allowed. Thus, in boom periods there may be a large dispersion due to extreme prices, while in crisis periods, dispersion is smaller since masterpieces are sold at lower values.

The $\beta(t)$ parameters stand for the difference between the returns of a painting cleansed of all its characteristics at a time t, and the average return of this painting through time. This must be compared with the time dummies methodology, where the parameters represent the returns with respect to a given period. We propose a continuous version of Duan (1983)'s and Jones and Zanola (2011)'s smearing estimate. In this framework, a price index whose base value at time $t = 1$ is equal to 100 is given by:

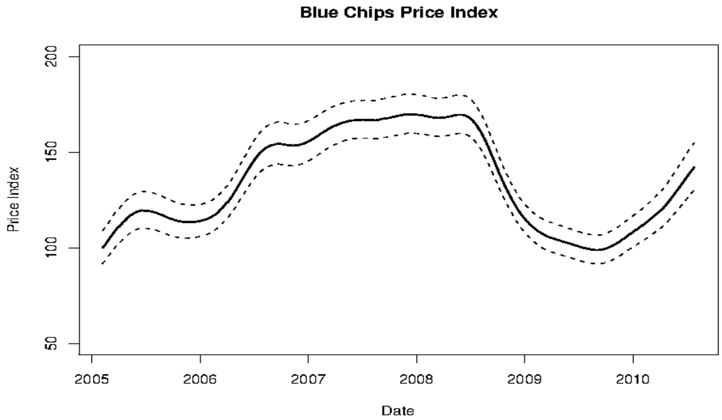

Figure 3.4: Price index resulting from equation (3.19): $Price\ Index(t) = 100e^{\beta(t)-\beta(1)} \times S$ where S is a smearing factor and $\beta(t)$ originates from equation (3.5) and is estimated by maximum likelihood (with local non parametric correction), as shown in equation (3.13).

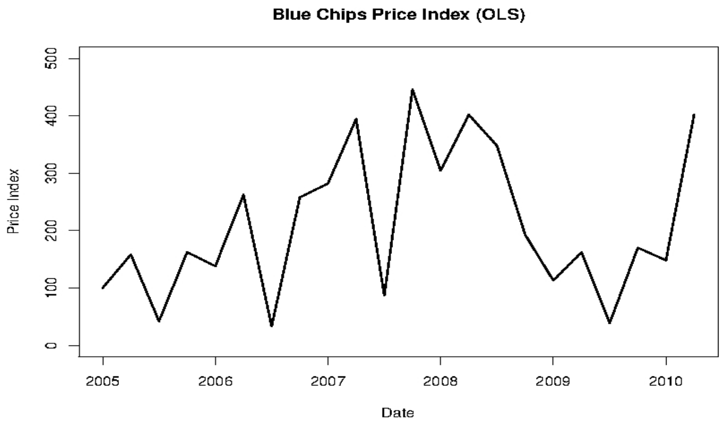

Figure 3.5: Price index resulting from equation (4.2), based on time dummies and estimated by Ordinary Least Squares.

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$$Price\ Index(t) = 100e^{\beta(t)-\beta(1)} \times \frac{w_t^{-1}\sum_{i=1}^{N} K(\frac{t_i-t}{h})\exp(\hat{u}_i)}{w_1^{-1}\sum_{i=1}^{N} K(\frac{t_i-1}{h})\exp(\hat{u}_i)}, \qquad (3.19)$$

where $w_t = \sum_{i=1}^{N} K(\frac{t_i-t}{h})$. Note that for the degenerate case $K(\frac{t_i-t}{h}) = d_{i,t}$ we obtain Jones and Zanola (2011) discrete smearing factor. The price index constructed in this way is plotted in Figure 3.4.

In addition to a daily resolution of time parameters and a higher precision than OLS, we empirically observe that the semi-parametric regression is also less sensitive to lack of data in certain time clusters: as seen in Figure 3.5, the OLS estimation suggests a 87% drop in price in the summer of 2006 and another crash in the summer of 2007. Such impressive drops in prices do not appear in the continuous index in Figure 3.4. More generally, there is to our knowledge no economic rationale, nor empirical evidence to support the idea that the general level of prices collapsed during the summers of 2006 and 2007. We believe this drop in price shown by the OLS estimation is due to a bias caused by the absence of important sales during summer. Such local flaws are naturally smoothed away by the semi-parametric regression.

Volatility of index returns

As far as the Blue Chips Index is concerned, it seems an improved methodology based on local maximum likelihood estimation yields more robust results than the traditional OLS methodology. Furthermore, the new regression form presented in equation (3.5) introduces the concept of volatility of predictability, a measurement that proves useful to better apprehend the discrepancy of valuation of artworks through time.

However, we are also interested in the volatility of the price of a basket of paintings. A possible method to derive volatility of, for instance, quarterly returns when using prior OLS estimation is to consider that the estimated $\beta(t)$ in equation (3.1) parameters are the "true" observed returns, and compute their volatility, as for any other good quoted in the stock market (see for example Hodgson and Vorkink, 2004). If volatility is assumed constant, then it could be estimated by the sample standard deviation of $\hat{\beta}(t)$, otherwise using e.g. GARCH-type models fitted to the $\hat{\beta}(t)$ process.

Note, however, that the underlying object, $\beta(t)$, is not a stochastic process but rather a deterministic function. It is the expectation of the log-price of a "neutral" painting at time t, and as such does not have a variance. Any attempt to fit time series models designed for stochastic processes to the estimates of $\beta(t)$ is theoretically flawed. What we can do, however, is to assess a degree of variability of this function. Rescaling time as $\tau = t/T$ to map the sample space into the interval $[0, 1]$, the total variation of $\beta(t)$, assuming that $\beta(t)$ is differentiable, is given by

$$TV(\beta) = \int_0^1 |\beta'(\tau)| d\tau,$$

where $\beta'(\tau) = d\beta(\tau)/d\tau$. $TV(\beta)$ is a measure of the overall variability of a function on an interval. On a discretized scale, it could be calculated as the sum of absolute returns, recalling from (3.19) that log returns over the interval $[t, t+1]$ can be expressed as $\beta(t+1) - \beta(t)$.

Since $TV(\beta)$ is linear in time, it can be further decomposed to obtain, for example, total variations for each year. In our case, we obtain 12.67 % for 2005, 27.90% for 2006, 10.75% for 2007, 59.05% for 2008, and 18.63% for 2009. One can also define $|\beta'(t)|$ as the instantaneous variability of $\beta(t)$, and regard this instantaneous variability as the volatility of the art index, which is time-varying.

Figure 3.6 plots the estimated instananeous volatility of art along with the VIX index (an index of implied volatility of the S&P 500). Both indices are annualized such that the scales are comparable. The overall level of art and VIX volatilities is about the same, but the art index volatility shows larger swings at the beginning of the sample. One directly observes that, in addition to the change in regime of volatility of predictability as seen previously, the art market suffered from a shock in volatility of prices, linked to a severe drop in returns. This period coincides with the financial crisis in 2008 and the peak in the VIX index. Although the two indices are not directly comparable as the VIX concerns implied volatility whereas our index concerns instantaneous variability of the index, it seems that the VIX index also suffered from a shock end of 2008, a timing corresponding to Lehman Brothers' bankruptcy.

On the other hand, the apparent high variability of art returns in 2006-2007 is not accompanied by high levels of the VIX. It is our understanding that this variability apparently independent from the stock market was triggered by booming prices of post-war and contemporary art at the time. We believe that the biggest increase in historical volatility of art prices may be linked to the financial crisis, end of 2008. The surge in volatility had serious impact on market participants: in its 2008 third quarter release, Sotheby's affirmed *"These third quarter figures reflect a significant level of losses on our guarantee portfolio principally for fourth quarter sale events including this week's USD10 million Impressionist guarantee losses as well as our estimate of USD17 million on probable guarantee losses in next week's Contemporary sales. We have reduced our guarantee position by 52% as compared*

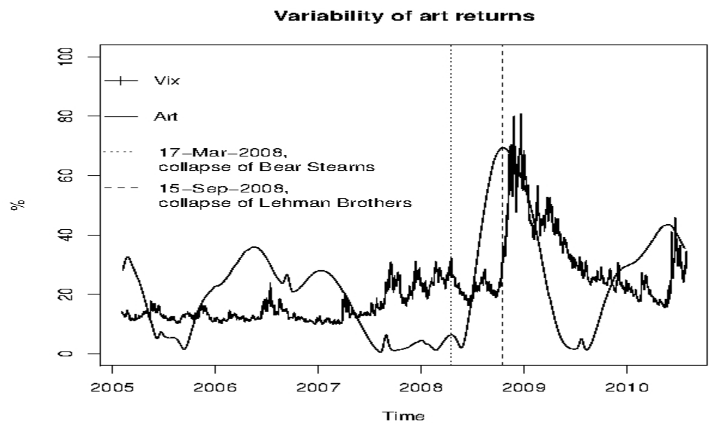

Figure 3.6: Historical volatility of the art market as compared to implied volatility of S&P 500 options -VIX Index-

to last year and our net guarantee exposure is USD114 million. In this period of considerable economic instability, we will dramatically reduce the guarantees and other special concessions we grant to sellers [...]".

Emitting guarantees is equivalent to shorting put options on art. Since the evaluation of such options crucially depends on volatility measures as discussed in this paper, our results may contribute to this new direction of research.

3.5 Conclusions and outlook

In this paper we have discussed the construction of volatility indices for the art market. In a classical hedonic regression framework, we estimate local parameters, in particular the scale, using a local likelihood approach, which contrasts with the typical OLS estimation method. Our results for a data set comprising blue chip auction data show that the scale parameter is indeed time-varying, which means that the predictability of prices is low when the scale is large, and vice versa. We find that during the financial crisis in 2008/09, this volatility of predictability has been smaller than before, meaning that during this period, price predictions were more precise.

Furthermore, we have considered volatility of the art price index as explained by the variability of the estimated index. We suggest a measure for the degree of variability of the art index and show that for our data, it has about the same magnitude as an implied volatility index on the S&P 500. Art volatility increases similar to the stock index volatility during the financial crisis. Thus, unlike the volatility of predictability, it co-moves with the stock market.

Several applications of these results are possible. For example, to evaluate derivatives on art, such as options, one would have to consider volatility of predictability if the underlying is a single painting, or rather volatility of the art index if the underlying is a large basket or a collection of paintings. For both cases, we have provided suggestions for the evaluation of volatility, but a concise investigation of option pricing on the art market is delegated to future research.

Acknowledgement

We are grateful for helpful comments of participants at the 4th CSDA International Conference on Computational and Financial Econometrics (CFE'10) in London, December 2010. Financial support from the contract *Projet d'Actions de Recherche Concertées* nr. 07/12/002 of the *Communauté francaise de Belgique*, granted by the *Académie universitaire Louvain*, is gratefully acknowledged.

3.6 Description of data

Table 3.3: Description of data available in the database, per artist. Variables with a "***" are variables whose explanatory power is significant in equation (3.5) (see Table 3.1 for more details)

Variable	Description	Number of observations	Proportion
Alexander.Calder ***	Dummy variable: the artist is Alexander Calder (1) or not (0)	41	0.7%
Alexej.Jawlensky ***	id.	159	2.8%
Alfred.Sisley	id.	89	1.6%
Andy.Warhol ***	id.	545	9.7%
Camille.Pissarro ***	id.	110	2.0%
Childe.Hassam ***	id.	52	0.9%
Claude.Monet	id.	143	2.5%
Damien.Hirst ***	id.	328	5.8%
Donald.Judd ***	id.	8	0.1%
Edgar.Degas ***	id.	29	0.5%
Edouard.Vuillard ***	id.	111	2.0%
Edvard.Munch	id.	36	0.6%
Egon.Schiele	id.	17	0.3%
Emil.Nolde	id.	40	0.7%
Ernst.Ludwig.Kirchner	id.	32	0.6%
Georges.Braque ***	id.	90	1.6%
Gerhard.Richter ***	id.	295	5.3%
Henri.de.Toulouse.Lautrec ***	id.	33	0.6%
Henri.Matisse ***	id.	64	1.1%
Henry.Moore ***	id.	4	0.1%
Jean.Michel.Basquiat ***	id.	171	3.0%
Joan.Miro	id.	86	1.5%
Kees.van.Dongen ***	id.	167	3.0%
Lucio.Fontana	id.	172	3.1%
Marc.Chagall	id.	236	4.2%
Mark.Rothko ***	id.	50	0.9%
Maurice.de.Vlaminck ***	id.	325	5.8%
Max.Ernst ***	id.	138	2.5%
Pablo.Picasso ***	id.	222	4.0%
Paul.Gauguin	id.	33	0.6%
Paul.Klee	id.	29	0.5%
Pierre.Auguste.Renoir ***	id.	363	6.5%
Raoul.Dufy ***	id.	167	3.0%
Rene.Magritte	id.	61	1.1%
Richard.Prince	id.	107	1.9%
Sam.Francis ***	id.	482	8.6%
Wassily.Kandinsky	id.	43	0.8%
Willem.de.Kooning	id.	117	2.1%
Yayoi.Kusama ***	id.	225	4.0%
Zao.Wou.Ki	id.	192	3.4%

Table 3.4: Description of the qualitative data available in the database. Variables with a "***" are variables whose explanatory power is significant in equation (3.5) (see Table 3.1 for more details)

Variable	Description	Num. of obs.	Proportion
Dead ***	Dummy variable: the artist is dead (1) or not (0)	4,465	79.6%
DaySales ***	Dummy variable: the auction is a "Day Auction" (1) or not (0)	1,115	19.9%
Morning	Dummy variable: the auction is a "Morning Auction" (1) or not (0)	389	6.9%
Evening	Dummy variable:the auction is an "Evening Auction" (1) or not (0)	1,353	24.1%
Christies ***	Dummy variable: the auction house is Christie's (1) or not	2,198	39.2%
Artcurial	id.	121	2.2%
Bonhams ***	id.	30	0.5%
Dorotheum	id.	17	0.3%
KettererKunst	id.	32	0.6%
KollerAuktionen	id.	29	0.5%
Tokyo	id.	25	0.4%
Phillips	id.	171	3.0%
PierreBerge	id.	9	0.2%
SaintCyr	id.	86	1.5%
Sothebys ***	id.	2,246	40.0%
VillaGrisebach	id.	56	1.0%
Nineteenth	Dummy variable: the auction's theme is based on 19th century art (1) or not (0)	65	1.2%
Collection ***	Dummy variable: the auction's theme is based on a collection (1) or not (0)	114	2.0%
Asian	Dummy variable: the auction's theme is based on Asian art (1) or not (0)	132	2.4%
Contemporary	Dummy variable: the auction's theme is based on contemporary art (1) or not (0)	2,226	39.7%
Impressionist ***	Dummy variable: the auction's theme is based on impressionist art (1) or not (0)	2,134	38.0%
Modern	Dummy variable: the auction's theme is based on modern art (1) or not (0)	2,602	46.4%
PostWar	Dummy variable: the auction's theme is based on post-war art (1) or not (0)	581	10.4%
Surreal	Dummy variable: the auction's theme is based on surrealist art (1) or not (0)	43	0.8%
London ***	Dummy variable: the city where the sales occur is London (1) or not (0)	1,945	34.7%
HongKong ***	id.	111	2.0%
Milan	id.	63	1.1%
NewYork ***	id.	2,196	39.1%
Paris ***	id.	517	9.2%
Monday	Dummy variable: the day of the auction is Monday (1) or not (0)	581	10.4%
Tuesday	id.	1,241	22.1%
Wednesday	id.	1,765	31.5%
Thursday	id.	1,166	20.8%
Friday	id.	470	8.4%
Saturday	id.	200	3.6%
Sunday	id.	189	3.4%
Untitled ***	Dummy variable: the painting's is untitled (1) or not (0)	586	10.4%
Landscape	Dummy variable: the painting's title makes reference to a landscape (1) or not (0)	726	12.9%
Portrait	Dummy variable: the painting's title makes reference to a portrait (1) or not (0)	233	4.2%
StillLife	Dummy variable: the painting's title makes reference to a still life (1) or not (0)	217	3.9%
Animal	Dummy variable: the painting's title makes reference to an animal (1) or not	117	2.1%
Woman ***	Dummy variable: the painting's title makes reference to women (a woman) (1) or not (0)	393	7.0%
Hammer ***	Dummy variable: the price is a hammer price (1), or a premium price (0)	3,223	57.4%

Table 3.5: Description of time dummy variables

Time dummy	Description	Num. of obs.	Proportion
Y2005Q1	Dummy variable: the quarter of the sale is the first quarter of 2005 (1) or not (0)	150	2.7%
Y2005Q2	id.	420	7.5%
Y2005Q3	id.	42	0.7%
Y2005Q4	id.	320	5.7%
Y2006Q1	id.	175	3.1%
Y2006Q2	id.	519	9.2%
Y2006Q3	id.	25	0.4%
Y2006Q4	id.	396	7.1%
Y2007Q1	id.	234	4.2%
Y2007Q2	id.	539	9.6%
Y2007Q3	id.	38	0.7%
Y2007Q4	id.	463	8.3%
Y2008Q1	id.	248	4.4%
Y2008Q2	id.	426	7.6%
Y2008Q3	id.	229	4.1%
Y2008Q4	id.	274	4.9%
Y2009Q1	id.	141	2.5%
Y2009Q2	id.	348	6.2%
Y2009Q3	id.	35	0.6%
Y2009Q4	id.	313	5.6%
Y2010Q1	id.	151	2.7%
Y2010Q2	id.	126	2.2%

Table 3.6: Description of the quantitative data available in the database. Variables with a "***" are variables whose explanatory power is significant in equation (3.5) (see Table 3.1 for more details)

Variables	Description	Average	Standard Deviation	Min	Max
Height	Height of the painting, in inches	28	21.12	1	195
Width ***	Width of the painting, in inches	28	25.10	1.57	421
Surface ***	The surface of the painting, in inches square				
Lot	Lot Number of the painting	320	321.15	1	7,299
ThemeWord ***	Number of letters for the auction's theme	34	11.51	7	103
WordTitle	Number of letters for the painting's title	20	13.19	2	225
AgePainted ***	The age at which the artist painted the artwork	49	15.99	13	97
AgePainting	The age of the artwork the day of its sale	59	38.44	1	159
Born	The artist's year of birth	1,898	36.73	1831	1,965
Price	The price of the artwork, in USD	1,222,838	3,558,190.72	258	85,000,000
YearPainted	The year the painting was made	1954.77	37.40	1854	2009

3.7 QQ-Plot

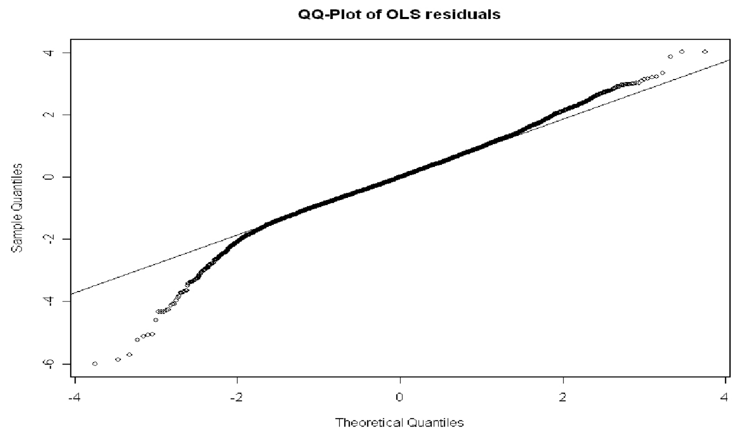

Figure 3.7: QQ Plot of residuals of regression (4.1)

3.8 Density of transactions

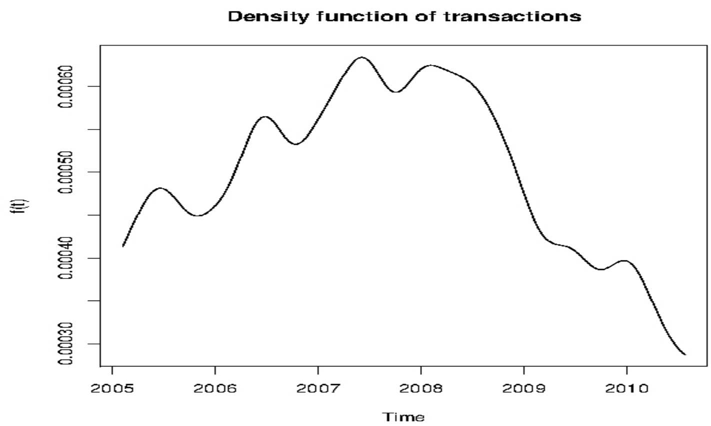

Figure 3.8: Estimation of the density of transactions $f(t)$ using a Nadaraya Watson estimator

Chapter 4

Volatility of price indices for heterogeneous goods with applications to the fine art market.

Adapted from Bocart and Hafner (2013c)

4.1 Abstract

Price indices for heterogenous goods such as real estate or fine art constitute crucial information for institutional or private investors considering alternative investment decisions in times of financial markets turmoil. Classical mean-variance analysis of alternative investments has been hampered by the lack of a systematic treatment of volatility in these markets. In this paper we propose a hedonic regression framework which explicitly defines an underlying stochastic process for the price index, allowing to treat the volatility parameter as the object of interest. The model can be estimated using maximum likelihood in combination with the Kalman filter. We derive theoretical properties of the volatility estimator and show that it outperforms the standard estimator. We show that extensions to allow for time-varying volatility are straightforward using a local-likelihood approach. In an application to a large data set of international blue chip artists, we show that volatility of the art market, although generally lower than that of financial markets, has risen after the financial crisis 2008/09, but sharply decreased during the recent debt crisis.

4.2 Introduction

Over the last two decades, there has been a growing interest among scholars, business practitioners, and policy makers in price indices tracking the financial performance of a basket of heterogeneous goods. These price indices have typically been developed for physical assets that can be considered as investments, such as housing, art, wine, as well as many other collectibles (musical instruments, watches, jewelry, etc.). In addition to managing all risks specific to physical assets (forgery, theft, destruction, etc.), investors in physical assets must deal with the risks common to all financial investments: market risks, liquidity risks and counterparty risks. Obviously, prior to modelling and managing these financial risks, a pre-requisite is to have an estimate of the underlying time series of prices and volatility of returns.

Returns of baskets of physical assets need to be indirectly estimated because of the presence of heterogeneity in the series. Generally, two methodologies are used to cope with this problem: the repeat sale methodology (RSM) and the hedonic regression. Some advantages and disadvantages of hedonic regression as compared to RSM for estimating returns in the art market are discussed in Ginsburgh, Mei, and Moses (2006). Robert et al. (2010) discuss hedonic versus repeat-sales indices in the real estate market of Los Angeles and San Diego metropolitan areas. RSM can be viewed as a nested case of hedonic regression and consists of computing average returns of identical goods sold through time. A major critique is that RSM focuses on a small, biased sample of goods (Collins, Scorcu, and Zanola, 2009). RSM has been used to develop real-estate price indices by Case and Shiller (1987) and Goetzmann (1987). Pesando (1993), Goetzmann (1993) and Mei and Moses (2002) use RSM to estimate returns in the art market.

The hedonic approach is to regress the price of each good on its characteristics, in order to control for variations due to observable differences between heterogeneous goods. The classical approach is to include time dummy variables in the regression, whose coefficients constitute the basis for building an index. Hedonic regression has been extensively used to build price indices. A few examples are Barre, Docclo, and Ginsburgh (1994), Collins, Scorcu, and Zanola (2009), Hodgson and Vorkink (2004), Renneboog and Spaenjers (2013) and Bocart and Hafner (2012a) for art markets, Engle, Lilien, and Watson (1985), Schulz and Werwatz (2004) and Gouriéroux and Laferrère (2009) for real estate, Combris, Lecocq, and Visser (1997) and Fogarty (2006) for wine and Graddy and Margolis (2011) for violins.

The choice of an initial functional form is frequently debated in the literature.

Empirically, Hansen (2009) finds that hedonic and repeat-sales methods provide similar estimates of price growth of Australian real estate when the sample is large. Robert et al. (2010) suggest that hedonic regression methods perform better at a local level to track prices of real estate in Los Angeles and San Diego. For the art market, Ginsburgh, Mei, and Moses (2006) show that hedonic regression performs better than RSM when the sample size is small, while giving very similar results in large samples.

The goal of this article is to challenge the classical methodology of constructing the index using ordinary least squares (OLS), implicitly assuming deterministic prices, which is incoherent with a subsequent modelling of prices and returns as a stochastic process. Similar to the discussion of fixed versus random effects in the literature on panel data, we show that parameter estimation is more efficient exploiting the structure of a hypothesized random process. In particular, for an assumed random walk or stationary autoregressive process for the underlying market index, we derive explicitly the efficiency gains that can be achieved with maximum likelihood estimation compared to OLS. The parameters of interest are the variances of the two random components, i.e. the variance of unobserved market returns, which we call market volatility, and the variance of the object-specific error terms, which we call idiosyncratic volatility. Efficiency gains using MLE therefore imply a more precise estimation of idiosyncratic and market volatility.

Interpreting the hedonic regression as an unbalanced panel model with time effects rather than individual effects, we further show that the fact of having an unbalanced panel deteriorates the properties of the estimators compared to the case of balanced panels with the same average number of observations. Nevertheless, this negative effect disappears as the average number of observations per period increases. It should be understood that data of heterogeneous asset prices are typically highly unbalanced. In art markets, for example, sales are concentrated in spring and fall, with very few observations in summer.

Having in mind the large swings of volatility in financial markets, especially during crises, it is doubtful whether markets for heterogeneous goods have constant volatility. In the nonparametric framework of Bocart and Hafner (2012a), volatility is treated in an *ad hoc* way without an explicit model for it. In this paper, we suggest an extension of a state-space model allowing idiosyncratic and market volatility to be smooth functions of time that capture long-run trends in volatility. These functions can be conveniently estimated by local maximum likelihood.

We apply our methodology to the market of highly traded artworks in the period from 2000 to 2012. An ongoing debate about the diversification benefits of art in a portfolio has been taking place since Baumol (1986), and we contribute to this literature by explicitly delivering information about the associated risks of investing in this market. Our results suggest that the idiosyncratic risks, i.e. the prediction uncertainty of the price of an individual asset, increased after the recent financial crisis 2008/09, while the market volatility has sharply decreased.

The remainder of the paper is organized as follows. In Section 2, the basic model is presented. The third section introduces maximum likelihood estimation and compares efficiency of MLE with OLS. Section 4 discusses three extensions of the basic model, and Section 5 elaborates the results by applying the methodologies to empirical data on the art market. The last section closes this paper with final conclusions.

4.3 The model

As hedonic regression can be viewed as a generalization of the RSM approach, we consider an initial model that complies with the definition of a fully specified hedonic regression. However, the proposed estimation procedure can equally be applied to the RSM case.

Let there be N observed transactions and p_i denote the price of sale i. The logarithm of this price is usually modelled by the following hedonic regression model,

$$Y_i = \log p_i = \sum_{t=1}^{T} \beta_t d_{it} + \sum_{k=1}^{K} \alpha_k X_{ik} + u_i, \quad i = 1, ..., N. \tag{4.1}$$

The variable d_{it} is a dummy taking the value 1 if the object i was sold in period t, and 0 otherwise. The parameters β_t will be used to construct the pricing index. The parameters α_k are the coefficients of the explanatory variables, including a constant intercept term.

The time index $t = 1$ corresponds to the first period of the series and is used as benchmark. For identification, we set β_1 equal to zero. The K variables X_{ik} are all characteristics of the object i that have an impact on its price. For example, for a housing price index these would be variables such as the number of bathrooms and a dummy for a swimming-pool, for an art price index it would be the height, surface, and dummies for the artists, subject, etc. The price index, with base 100 in $t = 1$ is then defined as

$$\text{Index}_t = 100 \exp(\beta_t), \tag{4.2}$$

possibly corrected by a bias correction factor (see Jones and Zanola, 2010).

The regression (4.1) is generally estimated using Ordinary Least Squares (OLS). OLS estimators are efficient when errors u_i are normally distributed with constant variance, i.e., $u_i \sim N(0, \sigma_u^2)$. Empirical data, however, often violate this assumption.Hodgson and Vorkink (2004) and Seckin and Atukeren (2006) focus on the normality part and propose a semiparametric estimator of the index based on a nonparametric error distribution, while maintaining the assumption that u_i is i.i.d. and, hence, homoskedastic.

Furthermore, β_t is, by model assumption, a deterministic parameter rather than a stochastic process. To that extent, price indices built using OLS procedure cannot be interpreted as a random motion such as stock indices observed in financial markets. Nevertheless, it is standard practice to estimate β_t as if it were a deterministic parameter, and then continue working with the estimated β_t as if it was a realization of a stochastic process. As we will see, this methodological incoherence has important consequences for the properties of volatility estimators.

Note that model (4.1) can be written equivalently in the form

$$Y_{it} = \beta_t + X_{it}'\alpha + u_{it}, \quad t = 1, \ldots, T; \quad i = 1, \ldots n_t, \tag{4.3}$$

where Y_{it} is the log price of the i-th sale at time t, and n_t is the number of sales at time t. The vector X_{it} contains the K explanatory variables of the i-th sale at time t, and α is a $(K \times 1)$ parameter vector. This model can be viewed as an unbalanced panel model with time effects. Individual effects are absent because the object of the i-th transaction at time t is not necessarily the same as the object of the i-th transaction at time t', $t' \neq t$. In fact, the ordering of the sales at a given time t is irrelevant as long as the error term u_{it} is i.i.d. across sales.

As is well known from the panel literature, the common OLS estimator of the hedonic regression (4.1) is equivalent to the fixed effects estimators $\hat{\alpha}_{FE}$ and $\hat{\beta}_{FE}$ of (4.3). Defining the $(n_t \times 1)$ vector $a_t = (1, \ldots, 1)'$, these are given by $\hat{\alpha}_{FE} = (\sum_t X_t Q_t X_t')^{-1} \sum_t X_t Q_t Y_t$ and

$$\hat{\beta}_t = \frac{1}{n_t} \sum_{i=1}^{n_t} (Y_{it} - X_{it}'\hat{\alpha}_{FE}), \quad t = 2, \ldots, T \tag{4.4}$$

where $Q_t = I_{n_t} - a_t a_t'/n_t$ is the projection matrix taking deviations with respect to time means. For example, a typical element of the matrix $Q_t X_t'$ is $X_{it} - \bar{X}_t$, where $\bar{X}_t = \sum_{i=1}^{n_t} X_{it}/n_t$. The fixed effects estimator has the advantage of being consistent even if X_t is endogenous with respect to time. However, it is inefficient under random effects, and as we will see this inefficiency is particularly strong for our object of interest, i.e., the volatility of β_t.

As an alternative, a random effects approach would assume that $\beta_t \sim N(0, \sigma_\beta^2)$, which yields the possibility to directly estimate volatility σ_β^2 of the underlying random process. As the explanatory variables X_{it} contain a constant, identification is achieved by setting the expectation of β_t to zero, so that the restriction $\beta_1 = 0$ is not needed. Stacking for each t the observations Y_{it} into a $(n_t \times 1)$ column vector Y_t, and the explanatory variables into a $(K \times n_t)$, matrix X_t, the model can be written compactly as

$$Y_t = X_t'\alpha + a_t\beta_t + u_t, \quad t = 1, \ldots, T \tag{4.5}$$

where $u_t = (u_{1t}, \ldots, u_{n_t,t})'$. As in classical random effects models, we now need to impose exogeneity of the regressors with respect to the time component, i.e., $\mathrm{E}[\beta_t|X] = 0$. This allows us to consider $\eta_t = a_t\beta_t + u_t$ as a composite error term with variance $\Omega_t = a_t a_t'\sigma_\beta^2 + \sigma_u^2 I_{n_t}$, and estimate α in the regression $Y_t = X_t'\alpha + \eta_t$ by feasible GLS, $\hat{\alpha}_{GLS} = (\sum_t X_t\hat{\Omega}_t^{-1}X_t')^{-1}\sum_t X_t\hat{\Omega}_t^{-1}Y_t$, where $\hat{\Omega}_t$ is a consistent estimator of Ω_t. In order to test the validity of the exogeneity assumption, a Hausman-type test statistic can be constructed as

$$H = (\hat{\alpha}_{FE} - \hat{\alpha}_{GLS})'(V_{FE} - V_{GLS})^{-1}(\hat{\alpha}_{FE} - \hat{\alpha}_{GLS}), \tag{4.6}$$

where $V_{GLS} = (\sum_t X_t\hat{\Omega}_t^{-1}X_t')^{-1}$, and $V_{FE} = \hat{\sigma}_u^2(\sum_t X_t Q_t X_t')^{-1}$. Under the null hypothesis, H has an asymptotic χ^2 distribution with K degrees of freedom. If the null is not rejected, then the exogeneity assumption of X would appear reasonable and $\hat{\alpha}_{GLS}$ is consistent and efficient.

In a second step, the realizations of β_t can be estimated by $\hat{\beta}_t = \frac{1}{n_t}\sum_{i=1}^{n_t}(Y_{it} - X_{it}'\hat{\alpha}_{GLS})$. These $\hat{\beta}_t$ will have a mean close to zero, but $\hat{\beta}_1$ is not necessarily close to zero. One can apply the adjustment $\hat{\beta}_t - \hat{\beta}_1, t = 1, \ldots, T$, if the usual standardization $\beta_1 = 0$ is required in order to obtain an index value of 100 at the beginning of the sample.

We now extend the classical random effects model by introducing assumptions about the dynamics of β_t. In particular, we will assume an autoregressive process or order one, AR(1), including the random walk as a special case:

$$\beta_t = \phi\beta_{t-1} + \xi_t, \tag{4.7}$$

with $|\phi| \leq 1$, $\beta_0 = 0$, and ξ_t is an i.i.d. error term with mean zero and variance σ_ξ^2. If $\phi = 1$, then ξ_t can be interpreted as returns to a market portfolio, and σ_ξ^2 as market volatility. On the other hand, the variance of the object-specific error terms u_{it}, σ_u^2, is interpreted as idiosyncratic volatility, since it reflects the variation around the predicted value using the market index and object characteristics. If $\phi < 1$ the model induces a mean-reversion factor that can be useful to track mean-reverting commodities.

The system (4.5)-(4.7) is a state space representation. If one imposes a normality assumption on both error terms, maximum likelihood and the Kalman filter can be applied to efficiently estimate the state variables β_t, which will be discussed in Section 3. For the case of a balanced panel with random walk, Chang, Miller, and Park (2009) derive asymptotic properties of the maximum likelihood estimator using the Kalman filter. The proposed model is similar in spirit to the real-time macroeconomic monitoring approach of Aruoba and Diebold (1987), whose latent real activity factor is comparable to our index β_t.

Let us first discuss in this dynamic framework the properties of the fixed effects estimator for β_t and the implied estimators of σ_u^2 and σ_ξ^2. Let us assume for simplicity that ϕ is known. For example, a typical choice would be to set $\phi = 1$, meaning that log-prices follow a random walk, and the sequence ξ_t represents the returns. One could estimate ϕ, assuming stationarity, in a two step procedure where in a first step, consistent fixed effects estimates of β_t are obtained, and in a second step, the AR(1) model (4.7) is estimated. It is however more common to directly assume a random walk for log-prices, which also simplifies the analysis of volatility estimators. Possible model extensions, allowing e.g. for autocorrelations of returns ξ_t, are delegated to Section 4.

Our assumptions are summarized in the following.

(A1) The error terms u_{it} and ξ_t are mutually independent, i.i.d. with mean zero, variances σ_u^2 and σ_ξ^2, respectively, and finite fourth moments.

(A2) The number of observations, n_t, is a positive integer i.i.d. random variable, satisfying $P(n_t \geq 2) > 0$.

Consider the following estimator of σ_u^2:

$$\hat{\sigma}_u^2 = \left(1 - \frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\right)^{-1} \frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}(Y_{it} - \hat{\beta}_t - X_{it}'\hat{\alpha})^2.$$

If $n_1 = n_2 = \ldots = n_T = N$, then the estimator is given by

$$\hat{\sigma}_u^2 = \frac{1}{(N-1)T}\sum_{t=1}^{T}\sum_{i=1}^{N}(Y_{it} - \hat{\beta}_t - X_{it}'\hat{\alpha})^2.$$

Estimated returns, $\hat{\xi}_t$ say, are obtained by $\hat{\xi}_t = \hat{\beta}_t - \phi\hat{\beta}_{t-1}$, and the variance of returns is estimated by

$$\hat{\sigma}_\xi^2 = \frac{1}{T}\sum_{t=1}^{T}(\hat{\xi}_t - T^{-1}\sum_{j=1}^{T}\hat{\xi}_j)^2 - (1+\phi^2)\hat{\sigma}_u^2\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}.$$

For the particular case $n_t = N, t = 1, \ldots, T$, and $\phi = 0$, this estimator becomes

$$\hat{\sigma}_\xi^2 = \frac{1}{T}\sum_{t=1}^{T}(\hat{\xi}_t - T^{-1}\sum_{j=1}^{T}\hat{\xi}_j)^2 - \frac{1}{N}\hat{\sigma}_u^2,$$

which is the well known variance estimator in panel data analysis with time and cross section units reversed, see e.g. equation (3.10) of Arellano (2003).

Proposition 1. *Under (A1) and (A2), $\hat{\sigma}_u^2$ and $\hat{\sigma}_\xi^2$ are $\sqrt{T}$-consistent estimators of σ_u^2 and σ_ξ^2, respectively.*

We can further derive the asymptotic distribution of the OLS estimator of $\theta = (\sigma_u^2, \sigma_\xi^2)$, but need an additional distributional assumption.

(A1') The error terms u_{it} and ξ_t are mutually independent with $u_{it} \sim N(0, \sigma_u^2)$ and $\xi_t \sim N(0, \sigma_\xi^2)$.

(A3) Assume that θ_0, the true parameter vector, is an interior point of Θ.

Clearly, (A1') encompasses and substitutes (A1).

Proposition 2. *Under (A1'), (A2) and (A3),*

$$\sqrt{T}(\hat{\theta} - \theta_0) \to_d N\left(0, 2\sigma_u^4 \lim_{T\to\infty} E[\Sigma_T]\right), \quad \Sigma_T = \begin{pmatrix} \Sigma_{uu,T} & \Sigma_{uv,T} \\ \Sigma_{uv,T} & \Sigma_{vv,T} \end{pmatrix},$$

where

$$\Sigma_{uu,T} = \frac{T^{-1}\sum_{t=1}^{T}(n_t-1)/n_t^2}{\left\{T^{-1}\sum_{t=1}^{T}(n_t-1)/n_t\right\}^2}, \tag{4.8}$$

$$\Sigma_{uv,T} = -(1+\phi^2)\Sigma_{uu,T}\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}, \tag{4.9}$$

$$\Sigma_{vv,T} = \left(\frac{\sigma_\xi^2}{\sigma_u^2} + \frac{1}{n_t} + \frac{\phi^2}{n_{t-1}}\right)^2 + \frac{2\phi^2}{n_{t-1}^2} + (1+\phi^2)^2\Sigma_{uu,T}\left(\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\right)^2 + Var\left(\frac{1}{n_t}\right) \tag{4.10}$$

For the balanced case, i.e., $n_t = N, a.s., t = 1, \ldots, T$, this result reduces to

$$\Sigma_T = \Sigma = \begin{pmatrix} \frac{1}{N-1} & -\frac{(1+\phi^2)}{N(N-1)} \\ -\frac{(1+\phi^2)}{N(N-1)} & \left(\frac{\sigma_\xi^2}{\sigma_u^2} + \frac{1+\phi^2}{N}\right)^2 + \frac{2\phi^2}{N^2} + \frac{(1+\phi^2)^2}{N^2(N-1)} \end{pmatrix}.$$

Note that for the large N, large T case, we would obtain $\sqrt{NT}(\hat{\sigma}_u^2 - \sigma_u^2) \rightarrow_d N(0, 2\sigma_u^2)$ and $\lim_{N,T\to\infty} \mathrm{Cov}(\hat{\sigma}_u^2, \hat{\sigma}_\xi^2) = 0$. Hence, both variance estimators are independent if sufficient cross-sectional data is available. However, $\sqrt{NT}(\hat{\sigma}_\xi^2 - \sigma_\xi^2)$ diverges since additional cross-sectional data does not increase the information about σ_ξ^2.

In order to assess the effects of an unbalanced panel on efficiency compared with the balanced panel case, let us assume that $n_t - 1$ follows a Poisson distribution with parameter λ, $Po(\lambda)$. Figure 4.1 plots the relative efficiencies of the estimators of σ_u^2 and σ_ξ^2, calculated as the ratio of the asymptotic variances under the assumption of a fixed design with $N = 1 + \lambda$ (numerator), and an unbalanced $Po(\lambda)$ design (denominator). While this relative efficiency only depends on the distribution of n_t for σ_u^2, it depends on the population parameters σ_u^2 and σ_ξ^2 for the estimation of σ_ξ^2. For the calculation, we used $\sigma_u^2 = 1$ and $\sigma_\xi^2 = 0.01$, which corresponds to typical empirical estimates (see Section 5). Clearly, the unbalanced design decreases the efficiency of both estimators, but the relative inefficiency disappears as the average number of observations, given by $1 + \lambda$, increases.

4.4 Maximum likelihood estimation

To estimate model (4.5)-(4.7), we propose a maximum likelihood estimator combined with the Kalman filter to recover the underlying state variables.

The composite error term $\eta_{it} = u_{it} + \beta_t$ can be obtained as $\eta_{it} = Y_{it} - X_{it}'\hat{\alpha}_{GLS}$. One can write the model (4.5) as

$$Y_{it} = X_{it}'\alpha + \eta_{it}, \quad t = 1, \ldots, T; i = 1, \ldots n_t \tag{4.11}$$

The joint model (4.5)-(4.7) then reads compactly

$$\eta_t = a_t\beta_t + u_t \tag{4.12}$$
$$\beta_t = \phi\beta_{t-1} + \xi_t, \tag{4.13}$$

where $\eta_t = (\eta_{1t}, \ldots, \eta_{n_t,t})'$. This linear Gaussian state space representation (4.12)-(4.13) allows us to estimate the underlying β_t, for given parameter estimates, using

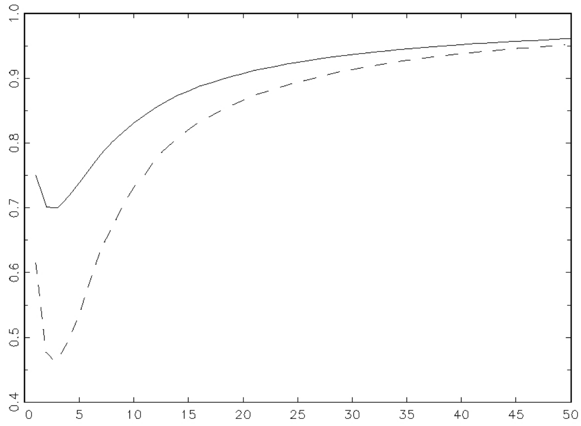

Figure 4.1: *R*elative efficiency of the estimators of σ_u^2 (solid line) and σ_ξ^2 (dashed line), calculated as the ratio of the asymptotic variances under the assumption of a fixed design (numerator), and an unbalanced design with Poisson distribution (denominator). The abscissa represents the parameter λ of the Poisson distribution.

the Kalman filter. This will be shown in Appendix B. Extensions to allow for time-varying volatility will be discussed in Section 4.

Parameter estimation can be achieved in an efficient and straightforward way by maximum likelihood. Denote the parameter vector by $\theta = (\sigma_\xi^2, \sigma_u^2)$ and define the parameter space $\Theta = \{\theta : \sigma_\xi^2 > 0, \sigma_u^2 > 0\}$. If stationarity is imposed on the AR(1) model in (4.7), that is, $|\phi| < 1$, then ϕ could be included in θ and be jointly estimated with σ_ξ^2 and σ_u^2. We do not discuss this possibility further, however, since we want to explicitly allow for the unit root case, $\phi = 1$.

Denote by $\eta_{t|t-1}$ and $\Sigma_\eta(t|t-1)$ conditional mean and variance, respectively, of η_t conditional on the information generated by $\eta_{t-1}, \eta_{t-2}, \ldots$, and let $e_t(\theta) = \eta_t - \eta_{t|t-1}$ and $\Sigma_t(\theta) = \Sigma_\eta(t|t-1)$. Then, the log-likelihood, up to an additive constant, can be written as

$$L(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left\{ \ln(|\Sigma_t(\theta)|) + e_t(\theta)' \Sigma_t(\theta)^{-1} e_t(\theta) \right\}, \tag{4.14}$$

and the maximum likelihood estimator is defined as

$$\hat{\theta} = \arg\max_{\theta \in \Theta} L(\theta),$$

with parameter space $\Theta = \mathcal{R}_+^2$. The maximization problem has no analytical solution, but numerical methods can be used conveniently. In large dimensions, computational problems may arise due to the optimization of a function that involves frequent calculation of the determinant and inverse of high dimensional matrices. We can exploit however the particular structure of Σ_t to obtain explicit formulas that largely facilitate the optimization. It can easily be shown that $|\Sigma_t| = \sigma_u^{2(n_t-1)}(n_t\zeta_{t-1} + \sigma_u^2)$ and $\Sigma_t^{-1} = (n_t\zeta_{t-1} + \sigma_u^2)^{-1}a_t a_t'/n_t + (I_{n_t} - a_t a_t'/n_t)/\sigma_u^2$, where $\zeta_t = \phi^2\sigma_\beta^2(t|t) + \sigma_\xi^2$. Using these expressions in (4.14) reduces computational costs substantially.

We now have the following proposition.

Proposition 3. *Under (A1'), (A2) and (A3), the MLE of θ is consistent and asymptotically normally distributed,*

$$\sqrt{T}(\hat{\theta} - \theta_0) \to_d N\left(0, \lim_{T\to\infty}\left(\frac{I(\theta)}{T}\right)^{-1}\right),$$

where

$$\begin{aligned} I(\theta) &= -E\left[\frac{\partial^2 \ln L(\theta)}{\partial\theta\partial\theta'}\right] \\ &= \frac{1}{2}\sum_{t=1}^{T}\left\{\frac{\partial vec(\Sigma_t)'}{\partial\theta}(\Sigma_t^{-1}\otimes\Sigma_t^{-1})\frac{\partial vec(\Sigma_t)}{\partial\theta'} + 2E\left(\frac{\partial e_t'}{\partial\theta}\Sigma_t^{-1}\frac{\partial e_t}{\partial\theta'}\right)\right\}. \end{aligned}$$

Analytical expressions for the derivatives used to calculate $I(\theta)$ are provided in the appendix.

Finally, for the special case $\phi = 0$, it is straightforward to show that the MLE estimator is asymptotically equivalent to the OLS estimator, as in that case the information matrices $I(\theta)$ coincide. If $\phi \neq 0$, however, the MLE and OLS estimators are different, and in the following we discuss their relative efficiency.

We consider several scenarios in order to compare the efficiency of the OLS and maximum likelihood estimators of volatility. Since log prices are usually assumed to follow a random walk, we set $\phi = 1$. Moreover, we assume that η_{it} are observed directly, in order to focus on the estimation of θ without the need to estimate α. It may be expected that MLE of θ is even more efficient relative to OLS if α is estimated jointly with θ.

To further simplify the analysis, note that only the ratio of σ_u^2 and σ_ξ^2 is of interest, since the scaling of the data η_{it} is irrelevant. Hence, we set σ_u to one without loss of generality. We assume a balanced panel with $N = 5, 10, 20$ and 50 observations per period. Define the asymptotic relative effiency as

$$\lim_{T\to\infty} \frac{\mathrm{Var}(\hat{\theta}_{MLE})}{\mathrm{Var}(\hat{\theta}_{OLS})},$$

which, if MLE is more efficient than OLS, is a number between 0 and 1. Table 4.1 reports the asymptotic relative efficiencies.

Table 4.1: Asymptotic relative efficiency of $\hat{\theta}_{OLS}$ w.r.t. $\hat{\theta}_{MLE}$.

σ_ξ^2	$N=5$		$N=10$		$N=20$		$N=50$	
	$\hat{\sigma}_\xi^2$	$\hat{\sigma}_u^2$	$\hat{\sigma}_\xi^2$	$\hat{\sigma}_u^2$	$\hat{\sigma}_\xi^2$	$\hat{\sigma}_u^2$	$\hat{\sigma}_\xi^2$	$\hat{\sigma}_u^2$
0.1	0.0225	0.9341	0.0588	0.9912	0.1213	0.9993	0.2725	1.0000
0.2	0.1162	0.9880	0.2258	0.9998	0.3880	0.9998	0.6617	0.9998
0.3	0.2481	0.9998	0.4175	0.9991	0.6241	0.9992	0.8538	0.9997
0.4	0.3806	0.9975	0.5801	0.9973	0.7728	0.9987	0.9272	0.9997
0.5	0.4958	0.9919	0.6973	0.9956	0.8543	0.9984	0.9558	0.9996
0.6	0.5881	0.9865	0.7749	0.9944	0.8975	0.9981	0.9680	0.9996
0.7	0.6582	0.9819	0.8241	0.9935	0.9208	0.9980	0.9738	0.9996
0.8	0.7098	0.9782	0.8550	0.9928	0.9337	0.9979	0.9766	0.9996
0.9	0.7469	0.9752	0.8745	0.9923	0.9411	0.9978	0.9782	0.9996
1	0.7733	0.9729	0.8870	0.9919	0.9455	0.9977	0.9791	0.9996

Note that in all situations, the OLS estimator of σ_u^2 is almost as efficient as the ML estimator. However, this is not the case for our parameter of interest, the variance of index returns, σ_ξ^2. Here, the efficiency loss of OLS is remarkable in cases where σ_ξ is small, even if N is large. Figure 4.2 depicts the relative efficiencies of the estimator of σ_ξ^2. Clearly, for σ_ξ^2 close enough to zero, the relative efficiency is arbitrarily small no matter how large N. This motivates the ML estimator, knowing that small values of σ_ξ are empirically relevant as we will see in Section 5.

Our efficiency analysis assumes random walk dynamics of β_t and normality of error terms ξ_t and u_{it}. The reported relative efficiencies in Table 4.1 are to be understood as best case scenarios under correct specification of the model and the distributions. If the distributional assumptions are violated, then Gaussian MLE generally retains consistency, but is no longer efficient. We expect the reported relative efficiencies to be less in favor of MLE if the true error distributions are skewed or fat-tailed, or both. Alternatively, one may assume a specific class of distributions, e.g. skewed student-t, establish the likelihood based on this distribution, and use a general filtering algorithm such as MCMC to obtain predicted and updated values of

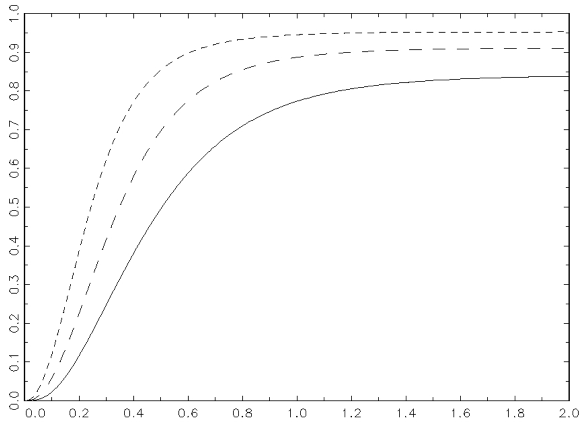

Figure 4.2: *A*symptotic relative efficiency of the estimator of σ_ξ^2 using OLS versus MLE. The value of σ_ξ^2 is on the abscissa, σ_u^2 and ϕ are fixed at 1. The curves are for $N = 5$ (solid), $N = 10$ (long dashed) and $N = 20$ (short dashed).

β_t. This procedure would be consistent and asymptotically efficient if the assumed distribution corresponds to the true one, but may not be consistent if they are different, see e.g. Newey and Steigerwald (1997).

4.5 Model extensions

In this section we will discuss three possible extensions of the model: First, the inclusion of a drift term in the random walk characterizing market prices. Second, the possibility of autocorrelation in returns. And finally, allowing for time-varying volatility.

Non-zero mean of returns

Instead of assuming a random walk with mean zero for β_t, we could add a constant drift parameter γ and replace (4.13) by $\beta_t = \gamma + \phi\beta_{t-1} + \xi_t$. The only change in the Kalman filter would be in equation (7.2) in the appendix, which would be replaced by $\beta_{t|t-1} = \gamma + \phi\beta_{t-1|t-1}$. The drift γ would have to be estimated by MLE, jointly with σ_u and σ_ξ. Alternatively, one could detrend the data in a first step and instead of (4.11) estimate $Y_{it} = X_{it}'\alpha + \gamma t + \eta_{it}$ by OLS. The composite error $\eta_{it} = \beta_t + u_{it}$

would then have, by construction, mean zero without linear time trend. Returns would be estimated by adding the OLS estimate of γ to the residuals $\hat{\xi}_t$. This latter procedure would be convenient but less efficient than the former.

Rather than explicitly modelling non-zero means of returns, it should be noted that the Kalman filter of the model without drift at least partially captures a potential non-zero mean of returns, which would end up in a non-zero mean of residuals $\hat{\xi}_t$. To see this, consider the updating equation for β_t, (7.3). If the Kalman filter without drift is used but the true model contains a drift, then the prediction error $\eta_t - \eta_{t|t-1}$ is equal to $\gamma + u_t$. Straightforward calculations show that the second term on the right hand side of (7.3) would be given by

$$\frac{n_t \sigma_\beta^2(t|t-1)\gamma + a_t' u_t}{n_t \sigma_\beta^2(t|t-1) + \sigma_u^2},$$

which, conditional on n_t and letting n_t increase, converges to γ in probability. Hence, (7.3) corrects the predicted β_t by the neglected γ, if the cross-section information is sufficiently large. For the estimated β_t it therefore does not make a difference whether or not a trend is included. An explicit estimation of γ would have the advantage of possible inference concerning the drift term, but it does not matter for the subsequent modelling and estimation of volatility.

Autocorrelation of returns

Markets for heterogenous goods may deliver returns that are serially correlated. For real estate markets, this has been motivated by Schulz and Werwatz (2004). It is possible to extend our basic model to account for serial correlation. Consider, for example, the random walk $\beta_t = \beta_{t-1} + \xi_t$, where now ξ_t itself follows an AR(1) model, $\xi_t = \rho \xi_{t-1} + v_t$, with $|\rho| < 1$ and v_t white noise. This can be written as an AR(2) model with parameter constraints, i.e., $\beta_t = (1+\rho)\beta_{t-1} - \rho\beta_{t-2} + v_t$. We can then define a new state vector $(\beta_t, \beta_{t-1})'$ and a transition equation

$$\begin{pmatrix} \beta_t \\ \beta_{t-1} \end{pmatrix} = \begin{pmatrix} 1+\rho & -\rho \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_{t-1} \\ \beta_{t-2} \end{pmatrix} + \begin{pmatrix} v_t \\ 0 \end{pmatrix}.$$

The Kalman filter equations can then be extended easily to this case. The parameter ρ could be estimated jointly with the other model parameters by maximum likelihood.

In the empirical part, we will estimate the model without autocorrelation of returns, and then test for residual autocorrelation using standard Portmanteau-type tests.

Time-varying volatility

With the enormous experience on time-varying volatility in financial and other markets, it seems doubtful that markets with heterogenous goods have constant volatility. Having information on possibly time-varying volatility, for example by rejecting the hypothesis of an absence of structural breaks, one may want to generalize the above model to allow for time-varying volatility. It is a priori difficult to guess which pattern volatility may follow. One could assume, asHodgson and Vorkink (2004) suggest, that returns follow a GARCH type process, as has been standard for financial markets. There are however three drawbacks of this approach. First, data sets of heterogenous markets typically have a much smaller time dimension, which renders estimation imprecise and highly dependent on starting values. Second, due to the high degree of time-aggregation, short term fluctuations of volatility may have been averaged out such that GARCH effects become insignificant, as it is also the case in Hodgson and Vorkink (2004). Third, estimation of the GARCH part could only feasibly be done in a second step, having estimated first the index returns, e.g. by OLS. This two-step procedure is inefficient, and it would be desirable to develop a framework where the model components can be estimated in one step.

In what follows, we propose a nonparametric extension of the model presented in Section 3, letting both market and idiosyncratic volatility be unknown functions of time that can be estimated with nonparametric methods. The approach is similar in spirit to the estimation of long-run trends of volatility in financial markets, as in the spline GARCH model of Engle and Rangel (2008).

We can regard $\theta = (\sigma_u^2, \sigma_\xi^2)'$ as a smooth function of time, $\theta(\tau)$, and obtain an estimate thereof via the local maximum likelihood approach, which has been discussed in a unified framework by Fan, Farmen, and Gijbels (1998). We apply their approach to our problem.

The local likelihood estimator is defined as

$$\widehat{\theta}(\tau) = \mathrm{argmax}_\theta \{L(\theta \mid \tau)\},$$

where $\theta = \theta(\tau)$ and

$$L(\theta \mid \tau) = -\frac{1}{2} \sum_{t=1}^{T} \left\{ \ln(|\Sigma_t(\theta)|) + e_t(\theta)'\Sigma_t(\theta)^{-1} e_t(\theta) \right\} K\left(\frac{t-\tau}{h}\right), \tag{4.15}$$

which gives estimates of time-varying idiosyncratic and market volatility. This approach fits locally a constant to the unknown volatilities, weighted by a kernel function K and bandwidth h. The kernel is typically a symmetric weight function such as the Gaussian density, but for predictive purposes a one-sided kernel function could be used as well. One could extend this approach to local polynomial fitting, often giving more precise estimates especially at the boundaries of the support.

Bias and variance of this local likelihood estimator can be calculated following the lines of Fan, Farmen, and Gijbels (1998), which also allows us to obtain pointwise confidence intervals by invoking asymptotic normality. This procedure will be used in the empirical example of Section 5.

Seasonality

We can extend the basic model (4.5) to allow for seasonal effects introducing a vector of dummy variables, z_t, of length s, where s is the number of seasons. The model including seasonal effects can then be written as

$$Y_t = X_t'\alpha + a_t(\beta_t + \gamma' z_t) + u_t, \quad t = 1, \ldots, T \tag{4.16}$$

with parameter vector γ of length s. Identification is achieved by imposing a suitable restriction, for example $\sum_{j=1}^{s} \gamma_j = 0$. This model can be estimated as before using a fixed effects estimator, which is equivalent to the OLS estimator of the hedonic regression extended by the seasonal dummies. Letting $\beta_t \sim N(0, \sigma_\beta^2)$, a random effects estimator can also be applied as before. In that case, the model (4.16) would have fixed seasonal effects and a stochastic price index evolution over time. Both versions will be applied in the empirical part of the paper, to which we turn in the following section.

4.6 Volatility of the art market

Data provided by Artnet AG[1] and Tutela Capital S.A.[2] is used to illustrate the methodology. The dataset concerns artworks sold at auction between January 2000 and May 2012 and consists of 12,643 paintings made by 40 artists who had the biggest volume of sales at auction in 2008 and 2009.[3] The choice of artists is similar

[1]A provider of data related to art. www.artnet.com

[2]A company specialized in managing art as an asset class. www.tutelacapital.com

[3]These artists are Jean-Michel Basquiat (1960-1988), Georges Braque (1882-1963), Alexander Calder (1898-1976), Mark Chagall (1887-1985), Edgar Degas (1834-1917), Kees van Dongen (1877-1968), Raoul Dufy (1877-1953), Max Ernst (1891-1976), Lucio Fontana (1899-1968), Sam Francis (1923-1994), Paul Gauguin (1848-1903), Childe Hassam (1859-1935), Damien Hirst (1965-

to that of Bocart and Hafner (2012a). It could occur that these painters differ from those with the biggest volumes in the previous years. Derivation of a methodology that can cope with evolving constituents of the index could be a future line of research. The typical frequency used in this literature is semi-annual, which avoids periods with only few observations and highly unstable OLS estimator. Using our methodology, we will be able to construct monthly price and volatility indices, and show that the MLE-Kalman estimator is more stable than the OLS estimator. The average number of observations per month is highly unbalanced, as reported in Table 4.2.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Min	1	5	7	10	136	132	1	1	3	44	179	39
Max	31	258	41	117	293	267	80	3	182	141	362	102
Avg	6	139	23	28	216	194	17	2	32	67	234	62

Table 4.2: *A*verage, minimum and maximum number of observations per month.

First, logged prices are regressed on available characteristics using ordinary least squares (OLS) without time dummies. The explanatory variables are the artist's name (40 levels), the medium used by the artist (35 levels), the height and width of the artwork in cm, the nationality of the artist (14 levels), the estimated date when the artwork was realized, the auction house where the sale took place (97 levels), whether the price in the database includes the buyer's premium or not, and the country in which the sale happened. Also, monthly seasonal dummies are included to account for the high seasonality of art transactions, so that we estimate the extended hedonic regression model (4.16).

We applied three methods to select variables: stepwise forward, stepwise backward and autometrics[4], all with a 5% significance level. The backward selection kept 119 variables in the model, the forward selection 102, and the autometrics procedure

), Alexej von Jawlensky (1864-1941), Donald Judd (1928-1994), Wassily Kandinsky (1866-1944), Ernst Ludwig Kirchner (1880-1938), Paul Klee (1879-1940), Willem de Kooning (1904-1997), Yayoi Kusama (1929-), René Magritte (1898-1967), Henri Matisse (1869-1954), Joan Miró (1893-1983), Claude Monet (1840-1926), Henry Moore (1831-1895), Edvard Munch (1863-1944), Emil Nolde (1867-1956), Pablo Picasso (1881-1973), Camille Pissarro (1831-1903), Richard Prince (1949-), Pierre-Auguste Renoir (1841-1919), Gerhard Richter (1932-), Mark Rothko (1903-1970), Egon Schiele (1890-1918), Alfred Sisley (1839-1899), Henri de Toulouse-Lautrec (1864-1901), Maurice de Vlaminck (1876-1958), Edouard Vuillard (1868-1940), Andy Warhol (1928-1987), Zao Wou Ki (1921-).

[4]See Doornik (2009) and the working paper version, Bocart and Hafner (2012b), for an explanation of the autometrics procedure.

kept 111 variables. 89 variables are common to the forward and backward selection, 80 variables are common to the forward and autometrics selection procedures, while 90 variables are common to the backward and autometrics procedures. Results of estimated returns and volatilities are robust to the choice of the selection procedure, and we therefore only report the results for the autometrics procedure. The adjusted R^2 for all three selected models is about 60%. 93% of the p-values in the final model are smaller than 1%. The residuals have a skewness of 0.23 and a kurtosis of 7.28, which leads to a tiny p-value of the Jarque-Bera test for normality. The final estimation results are reported in the working paper version, Bocart and Hafner (2012b). For the selected model, we also calculated the fixed effects estimator, i.e., the OLS estimator of the model including monthly time dummies. The Hausman test in (4.6) takes the value 2.28, which is insignificant at 1%, hence supporting our assumption of exogeneity of X. Furthermore, the maximum variance inflation factor of 16.1 for the final model does not indicate a severe problem due to multicollinearity.

The estimated β_t are computed using the fixed effects (OLS) and MLE estimators. Figure 4.3 plots the index on a monthly basis with both methodologies. The estimated index $100\exp(\beta_{t|T})$ of the Kalman filter using MLE is set to 100 in January 2000, while the fixed effects index is adjusted to have the same overall mean as the MLE index. The index estimated by OLS is much more erratic than the one estimated by MLE, which confirms our theoretical findings. The intuitive explanation is that OLS does not exploit the information of adjacent time periods, so that the estimator has a large variance especially in months with few observations, where spikes are likely to occur. On the other hand, the MLE-Kalman estimator effectively uses this information to smooth over different time periods, assuming a particular dynamic process for the index. Figure 4.4 depicts the returns corresponding to the index estimated by MLE, calculated as $\beta_{t|T} - \beta_{t-1|T}$, $t = 2\dots,T$.

The mean of estimated returns, $T^{-1}\sum_{t=1}^{T}\hat{\xi}_t$, is 0.0029 for OLS and 0.0042 for MLE, corresponding to annualized returns of about 3.5% (OLS) and 5% (MLE), higher than the mean annualized returns for the S&P 500 over the same period (about 0% annual return).

The pattern of the estimated index and its returns is remarkable. Negative returns of 2008 to 2009 reflect the direct impact of the banking crisis on the art market. Mechanisms that may link the financial markets to the art market have been highlighted by Goetzmann, Renneboog, and Spaenjers (2011) who have shown the positive relationship between top-income and art prices. On the other hand, positive returns during the European debt crisis may relate to conclusions of Oosterlinck

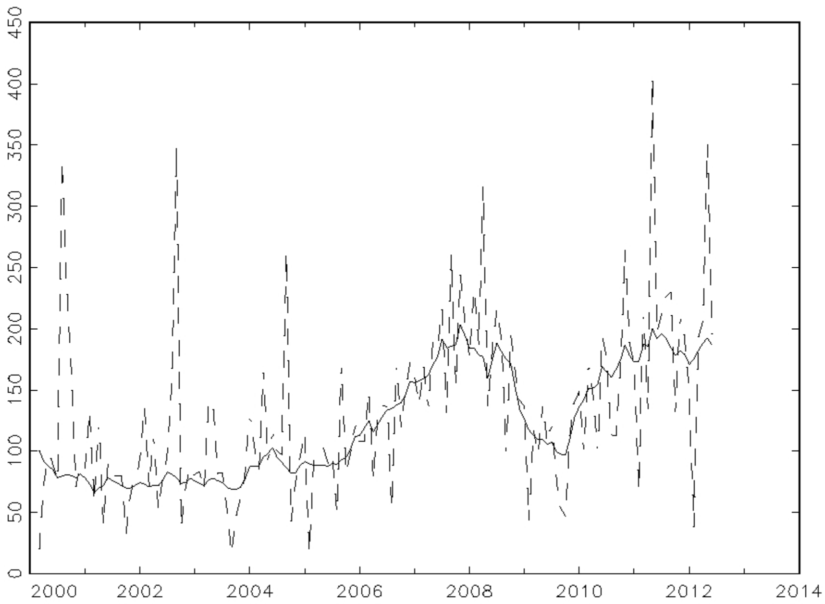

Figure 4.3: Monthly price index for blue chip artists. The dashed line corresponds to the fixed effects estimator (4.4), the solid one to the smoothed estimator $\beta_{t|T}$ of the Kalman filter using MLE. The horizontal axis is the time period January 2000 to May 2012, the vertical one is the price index.

(2010) who analysed the role of art as an alternative to government bonds during the Second World War when European countries faced high political risk.

We now turn to the volatility estimation. Table 4.3 reports the estimation results for the full sample. As expected, market volatility is much lower than idiosyncratic volatility, but the OLS estimate of market volatility is about 8% higher than the corresponding MLE estimate. It is likely that OLS, being less efficient than MLE and not taking into account the time variation of β_t, overestimates market volatility. In order to see whether our distributional assumptions of the error terms are reasonable, we show in Table 4.4 summary statistics of the estimated residuals. The Jarque-Bera normality test does not reject normality for ξ_t estimated by MLE, it does so however for OLS (p-value 0.004). For the idiosyncratic residuals u_{it}, both estimates reject normality, mainly due to the high kurtosis. This is similar to financial markets, where leptokurtosis is often still present in residuals, even after standardizing with

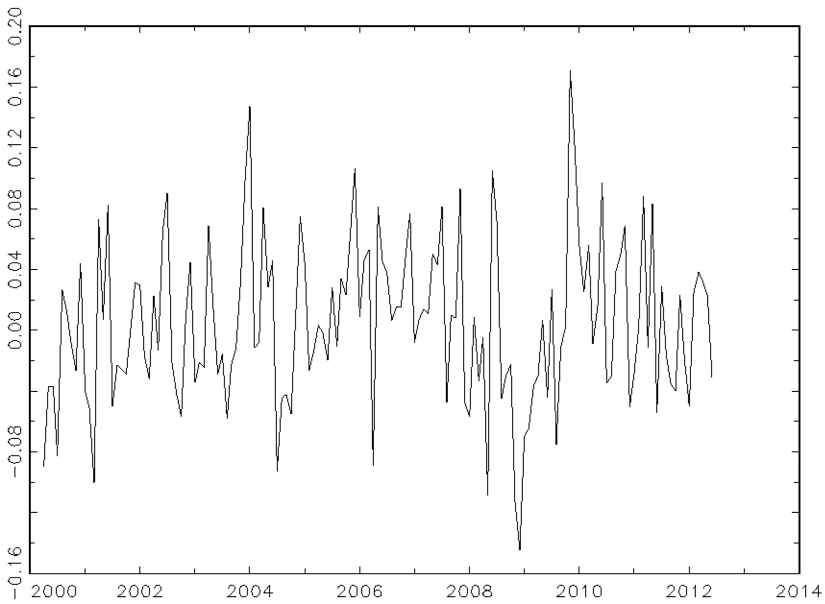

Figure 4.4: Monthly returns for blue chip artists, calculated as $\beta_{t|T} - \beta_{t-1|T}$, where $\beta_{t|T}$ is the Kalman smoother using maximum likelihood estimates. The horizontal axis is the time period January 2000 to May 2012, the vertical one is the monthly returns.

volatility estimates. In our case, the non-normality of u_{it} implies that the Kalman filter used in MLE is not fully efficient. Even though we do not expect major gains in efficiency using more general filtering algorithms, this may be a line of future research.

Table 4.5 reports empirical autocorrelations $\hat{\rho}(h)$ of estimated residuals $\hat{\xi}_t$ and portmanteau statistics of order h, $Q(h) = T^2 \sum_{i=1}^{h} (T-i)^{-1} \hat{\rho}(i)^2$. Under H_0 of white noise, $Q(h)$ has an asymptotic χ^2 distribution with h degrees of freedom. The empirical p-values indicate that we do not reject the null at 1%, which corroborates our decision not to model autocorrelation of returns explicitly.

In order to gauge parameter stability, we estimate the model for two additional subsamples whose results are reported in Table 4.3. Obviously, both estimated idiosyncratic and market volatilities are lower in the first subsample than in the second. A formal test of parameter constancy, $H_0 : \sigma^2_{u,1} = \sigma^2_{u,2}, \sigma^2_{\xi,1} = \sigma^2_{\xi,2}$, is the likelihood ratio test. Let L_i^* denote the log-likelihood of the ith subsample. Then, the LR statistic is given by $LR = 2(L_1^* + L_2^* - L)$ and has under the null an asymptotic χ^2 distribution with two degrees of freedom. In our case, the LR statistic takes the value 93.58 with corresponding p-value smaller than 1E-20, and parameter stability is clearly rejected. We therefore turn to extensions of the basic model allowing for time-varying volatilities.

		σ_u	σ_ξ	log likelihood
Full sample	OLS	1.1621 (0.2085)	0.0820 (0.0633)	
	MLE	1.2476 (0.0332)	0.0090 (0.0052)	-90.9893
first half	OLS	1.0705 (0.1935)	0.1467 (0.0827)	
	MLE	1.1851 (0.0408)	0.0109 (0.0085)	-17.9140
second half	OLS	1.2342 (0.2187)	0.0392 (0.0558)	
	MLE	1.2776 (0.0480)	0.0213 (0.0128)	-26.2833

Table 4.3: *P*arameter estimates of the static model using OLS and MLE. Asymptotic standard errors are given in parentheses.

		mean	std.dev.	skewness	kurtosis	JB
ξ_t	OLS	0.0029	0.6632	-0.3605	4.1253	11.01
	MLE	0.0042	0.0510	0.1218	2.9272	0.3962
u_{it}	OLS	0.0000	1.1062	-0.1478	6.2224	5517.94
	MLE	0.0343	1.1284	-0.1715	6.2681	5689.79

Table 4.4: *S*ummary statistics for $\hat{\xi}_t$ and $\hat{u}_{it}$ in the constant volatility model. JB is the Jarque-Bera test statistic, which under normality has an asymptotic χ_2^2 distribution.

h	1	2	3	4
ACF(h)	0.1983	-0.0181	-0.0540	0.1415
$Q(h)$	5.8591	5.9079	6.3424	9.3257
p-value	0.0154	0.0521	0.0960	0.0534

Table 4.5: *A*utocorrelation function of order h of residuals $\hat{\xi}_t$, corresponding Portmanteau statistics $Q(h)$ and p-values.

We estimate a model with smoothly time-varying idiosyncratic and market volatilities using the local likelihood estimator of Section 4.5 with Gaussian kernel and bandwidth chosen as the minimizer of the estimated mean integrated squared error.

Figure 4.5 depicts the estimate of idiosyncratic volatility, $\sigma_u(\tau)$, which shows an increasing trend in the second part of the sample. In relative terms, however, the variation of estimated idiosyncratic volatility is rather weak. The increase from 2006 to 2011 is about 20%.

Figure 4.6 shows the local likelihood estimate of market volatility, $\sigma_\xi(\tau)$. Recall from Table 4.3 that the constant likelihood estimate is 0.009. We basically see three periods of relatively high volatility: beginning of the sample in 2000, around 2004 and 2010, with the volatility estimate attaining 0.03 in the latter period, about three times higher than the average level over the sample period.

While market volatility seems to increase jointly with idiosyncratic volatility in the period 2008-2010, it drops drastically in 2011 (after the financial crisis) whereas idiosyncratic volatility continues to rise during the European debt crisis. This result means that as a whole, market risk of bearing a diversified investment in art has declined during the debt crisis, as expected from a safe haven asset. However, the increase in idiosyncratic volatility reveals that prices of individual artworks became less predictable than before.

4.7 Conclusion

The widespread use of the hedonic regression methodology in the economics of heterogeneous goods has led academics and business practitioners to devise risk metrics from price indices as if they were directly measured. We have shown that the standard deviation of estimated returns overestimates market volatility and needs to be corrected by taking into account the idiosyncratic volatility. We have further shown that in a framework where the market index follows a random walk, or a stationary autoregressive process, important efficiency gains of the volatility estimator can be obtained by using maximum likelihood in combination with the Kalman filter. As an extension, we propose a nonparametric approach to allow for time-varying volatility.

The application to a blue chips art market has shown that returns declined during the financial crisis 2008/09 but increased during the recent European debt crisis. We may suspect art to be considered as an alternative safe haven asset in crisis times, but our dataset needs to be augmented to confirm this for the recent debt crisis.

The behavior of idiosyncratic and market volatility of the art market is remark-

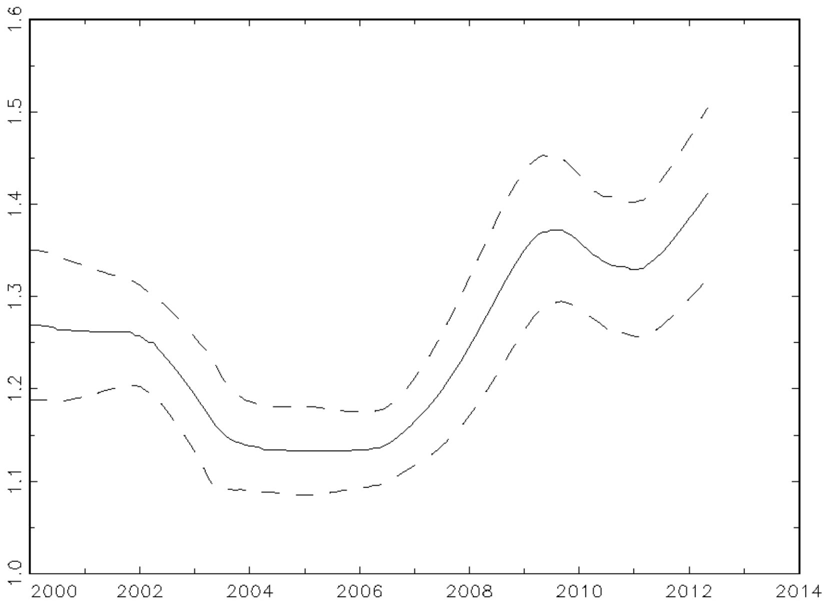

Figure 4.5: Idiosyncratic volatility of the blue chips art market, estimated by local maximum likelihood. The horizontal axis is the time period January 2000 to May 2012, the vertical one is the estimated idiosyncratic volatility.

able. While idiosyncratic volatility remained on a high level throughout the recent crises, market volatility increased after the financial crisis 2008/09, but decreased sharply during the recent debt crisis in 2011/12.

In future work, one may model explicitly time-varying correlations between art, financial and other assets to gauge the diversification benefits of including alternative assets in the portfolio. The modelling framework developed in this paper naturally permits to include other assets and estimate time-varying correlations, which is not feasible in classical OLS estimation.

On the econometrics side, future work may consider more general filters than the Kalman filter to accomodate departures from normality in the error terms. Further efficiency gains may be expected.

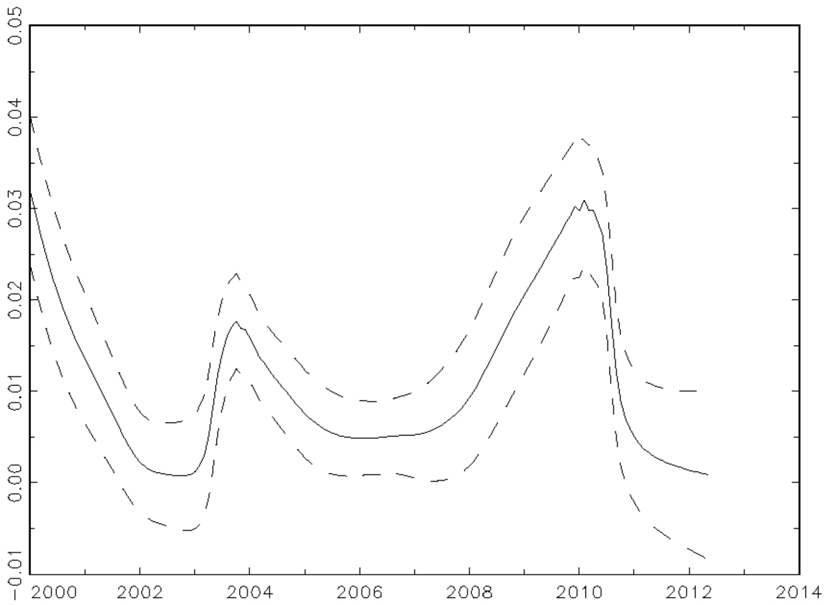

Figure 4.6: Market volatility of the blue chips art market, estimated by local maximum likelihood. The horizontal axis is the time period January 2000 to May 2012, the vertical one is the estimated market volatility.

Chapter 5

Fair re-valuation of wine as an investment

Adapted from Bocart and Hafner (2013b)

5.1 Abstract

The prices of wine is a key topic for market participants interested in valuing their stock, including dealers, restaurants or consumers who may be interested in optimizing their purchases. As a closely related issue, re-valuation is the need to regularly update the value of a stock. This need is especially met by fund managers in the growing industry of wine as an investment. In this case, fair-value measurement is compulsory by law. We briefly review methods available to funds and introduce a new quantitative method aimed at meeting IFRS 13 compliance for fair valuation. Using 70,000 auction data, we apply this method to compute current fair value of a basket of 368 different wines.

5.2 Introduction

Although consumers generally hold bottles of wine in view of drinking it, some hold it also for the investment it may represent. Recent literature has highlighted the direct benefits of wine investment and the positive diversification effects wine can offer to a portfolio of standard assets (Sanning, Shaffer, and Sharratt, 2008). Indeed, wine shares many characteristics with other agricultural goods considered as investments, not least an active auction market that offers transparency and liquidity to market participants. Wine funds in particular have industrialized the art of speculating in wine, offering the possibility to actively invest in this alternative asset.

Measuring performance of wine investment funds is needed to properly compute performance fees of managers, assess fair value of a share in the fund, and, more generally, provide accurate reporting to all stakeholders involved. Traditional valuation of physical assets by independent appraisers is slowly rendered obsolete by increasing access to data and automation capabilities. Furthermore, the growing level of stocks held by wine funds makes a regular “manual” valuation by experts if not impossible, at least very difficult to achieve. As a consequence, the adoption of IFRS 13 (effective since January 2013) by regulated wine funds requires significant changes to traditional procedures for determining fair value. To the contrary of stocks and bonds, a wine bottle does not yield any coupon or dividend, and unlike other conspicuous assets such as art that perpetually yield aesthetic dividends (Baumol, 1986), wine cannot be consumed without destroying its value. For the same reason, cash-flows cannot be obtained from renting, or leasing bottles of wine, so that any type of net-present-value valuation cannot be applied. This research addresses the question of valuation of wine in the context of wine funds valued in going-concern and that are subject to IAS-IFRS regulation. We first review the existing literature on quantitative methods for wine valuation and application of IAS-IFRS in the wine industry.

Valuation of wine generally relates to the application of hedonic regression. Hedonic regression was popularized by Rosen (1974) who suggested that consumers pay a marginal price for each characteristic of a given good with the sum of these implicit prices consisting in the observed market price. Golan and Shalit (1993) apply hedonic regression to assess impact of characteristics of Israeli wines on prices. They created a pricing system based on grape variety. Oczkowski (1994) focuses on Australian wines and included new variables, such as vintage and region. Nerlove (1995) rather regresses quantity sold on price and quality attributes, since supply of varieties may not be exogeneously determined. Using data on Bordeaux wines, Combris, Lecocq, and Visser (1997) include in the hedonic regression not only the information appearing on the label of the bottle, but also the sensory characteristics of the wine. They showed that the market price is mainly determined by objective characteristics. Yoo, Florkowski, and Carew (2011) use hedonic regression to price wines supplied in British Columbia.

Priilaid and Rensburg (2012) identify four categories of explanatory variables: objective (such as vintage or geographical location), sensory (for instance taste, bouquet), climatic and chemical wine characteristics (concentration in sugar and alcohol). They use a hedonic regression methodoloy to assess consumer prices in the South African market.

The question of IAS-IFRS compliance in agricultural markets is discussed by Marsh and Fischer (2013) The authors mention that wine, as a processed product, is typically excluded from IAS 41 for agriculture. Azevedo (2007) precisely focuses on the impact of IAS 41 in the viticulture industry. The author highlights that fair value can be determined based on the price of active market when it exists but in the case of the vine-growing industry, this exercise is rendered difficult by heterogeneity of wines accross regions. The author suggests valuing an agricultural stock of vines by expressing it in litres of wine. Bohusova, Svoboda, and Nerudova (2012) review possibilites for SMEs active in the vine growing industry to properly implement provisions in an IFRS framework for vines as biological assets.

The remainder of the paper is organized as follows. Section 2 presents the current environment and methods presently used by some wine funds. Section 3 introduces a new methodology to estimate returns of a fund using either the hedonic or repeated-sales approach. In the subsequent section, we illustrate the hedonic method using 70311 lots sold at auction at Christie's between January 2007 and October 2013. The last section concludes.

5.3 Current environment

Since 2005, compliance with IAS-IFRS is compulsory for all investment vehicles quoted on European stock exchanges, including wine funds. Furthermore, the recent European directive 2011/61/EU on Alternative Investment Fund Managers (AIFM) highlights a growing interest by supranational bodies to improve transparency in the market of alternative strategies, including funds that used to be less regulated. "IFRS 13 Fair Value Measurement" has become effective in January 2013. In this framework, fair-value is defined as "*the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date*". For non-financial assets, the selected valuation method must be appropriate for the measurement consistent with their "highest and best use". While this notion makes sense for physical assets such as real estate or machinery (that can be rented or exploited), the "highest and best use" of a stock held by a wine fund is limited to store it in a well-tempered cellar or wine-refridgerator. As a consequence, the fair-value of a wine stock must necesseraly rely on [IFRS 13:24]: it should correspond to a transaction taking place in the principal market for the asset or liability or, in the absence of a principal market, in the most advantageous market for the asset or liability. Since there is no centralized, or principal, market for wine, the most advantageous market is defined as the one that maximizes the amount that would be received to sell the asset after taking into account transaction costs and transport costs ([IFRS 13:A1]).

Wine funds currently implement various methods to value their stocks. Table 5.1 presents some funds of wine as an investment and which valuation they use, if published. None of the funds appears to use a historical cost approach, where inventories are valued at acquisition price. On the contrary, several funds already

Table 5.1: List of wine investment funds.

Name	Location	Valuation
The wine investment fund	Bermuda	Liv-ex system
Nobles Crus	Luxemburg	Average of dealers and auction prices
The vintage wine fund	Cayman Islands	Auction data and independent valuation
Wine Growth Fund	Luxemburg	Unknown
Lunzer Wine Fund	British Virgin Islands	Liv-ex system and independent valuation
Curzon Cap Fine Wine Geared Growth Fund	Guernsey	Unknown
SPL Fine Wine NR2 IC Ltd	Guernsey	Unknown
Patrimoine Grands Crus	France	Liv-ex system

rely on a market approach to value their stocks, even though IFRS 13 compliance is not obvious in that case. Interestingly, some funds seem to use the “Liv-ex” valuation methodology promoted by the Liv-ex, an internet and telephone transaction platform for wine professionals. The company brands itself “industry standard” and “the official valuer for a number of leading wine funds”.

The Liv-ex platform is organized in a similar fashion as a stock exchange: bids and/or offers are put on the platform by professionals. In case of a trade happening, both counterparties are notified of the transaction. The seller then delivers within 14 days the wine to the Liv-ex warehouse that is verified by Liv-ex. Simultaneously, the buyer sends the funds to Liv-ex that transfers the money to the seller within three weeks, whereas the buyer can either collect the wine at the warehouse, or be delivered.

The Liv-ex exploits available information on its platform to produce valuations of wines. The valuation method is the following: upon submission of a list of wines to be valued, the exchange verifies the current best offer for each wine in its own system and at other dealers. The valuer then observes the best bid on the platform and looks at the most recent transaction (within the last 30 days). If it lies within the bid-offer spread, then this transaction is used for valuation, otherwise, the mid-price is computed as the average between the bid and the offer. The scenario becomes more complex when no offer is available. In this case, Liv-ex relies on an undisclosed list of offer prices by merchants “identified as the major stockholders of wine”. In case no offer was available in the last 30 days neither at a dealer or on the Liv-ex, then the valuation is performed by a “valuation committee” that uses “off-market bids and offers, historical list prices and transaction data”. In case no bid is available, the bid is estimated from the average spreads to “orphan offers”, defined as “an offer price where [Liv-ex has] no corresponding bid. Orphans can be both live exchange offers or merchant list prices”. The Liv-ex does not include auction prices in calculation “due to a lack of standardization of auction lots” that can make weekly prices “very volatile with large swings” and also because “auction commissions can vary”.

Despite being an interesting approach, the method seems to fail meeting requirements for fair-value computation of wine as a financial asset, especially in the IFRS 13 sense. First, despite being a very successful venture with 400 members and more than 1000 transactions per month, there is little evidence that Liv-ex is the most advantageous market to sell any type of wine that could be held by a fund. According to Liv-ex, in 2010, Bordeaux wines accounted for 95% of its exchanges, with five Premiers Crus standing for 61% of Liv-ex trades by value: Château Lafite-Rothschild (Pauillac), Château Latour (Pauillac), Château Margaux (Margaux), Château Haut-Brion (Pessac, Graves), Château Mouton-Rothschild (Pauillac).

In 2011, more than GBP100m worth of wine were exchanged on the Liv-ex platform, which is a considerable amount in absolute value but indeniably smaller than the yearly USD397m+ worth of transactions reached the same year at major auction houses Acker Merrall and Condit, Christies International, Sothebys, Zachys and Hart Davis Hart Wine Co. In some cases, favouring ask prices of dealers to estimate a bid price instead of favouring auction house transactions publicly available seems an unreasonable choice given the opacity of dealer prices and the relative importance of large auction houses in the secondary market for wine (auctions would account for roughly 10% of the market according to Liv-ex), especially as far as old vintages and collectible wines are concerned.

As stated by Jones and Storchmann (2001), wines are “*traded all over the world in established wine auctions. The system guarantees, similar to a stock market, a comparatively high price transparency. Therefore, it can be assumed that auction prices indicate the relative (economic) scarcity and therefore the international esteem for those wines”*. Second, unlike auctions, the Liv-ex is based on standard contracts that assume similar quality for wines presenting similar features. This approach, well suited to recent vintages, prevents investors from gaining complementary information about condition of older wines. In the case of auctions, on the other hand, Ashenfelter (1989) highlights that at wine auction, "*revealing information tends to remove uncertainty and make low bidders more aggressive; this puts upward pressure on the bidding of others, which is in the interest of the auctioneer*". Similarly, Muth et al. (2008) showed that in the market for fed cattle, auction barn prices are higher than equivalent forward prices. Pagano and Röell (1996) proved that "the implicit bid-ask spread in a transparent auction is tighter than in a less transparent dealer market". For the art market, Bocart and Oosterlinck (2011) showed that large auction houses act as agents mitigating authenticity issues.

Finally, one can reasonably question the independence of an exchange excluding its competitors (auction houses) but including data from its clients or prospects (dealers). The inclusion of a valuation committee in case of absence of data lets stakeholders clueless about the methodology and data eventually used to perform valuation. In any case, a conflict of interest is possible between an exchange that simultaneously acts as intermediary and expert and a fund whose fee, like the exchange, depends on the price level.

5.4 New approach to valuation of wine as an investment

IFRS 13 provides three degrees of hierarchy in inputs that can be used for fair value measurement. The idea behind the hierarchy is that lower levels should be preferred: Level 1 inputs are "quoted prices in active markets for identical assets or liabilities that the entity can access at the measurement date. [IFRS 13:76]". Level 2 inputs "are derived mainly from or corroborated by observable market data by correlation or other means ('market-corroborated inputs') [IFRS 13:81]". Level 3 inputs are unobservable inputs used "*with the best information available in the circumstances, which might include the entity's own data, taking into account all information about market participant assumptions that is reasonably available*" [IFRS 13:87-89].

In the case of wine, Level 1 inputs are not readily available, especially considering the fact that available exchanges (Hong-Kong Wine Exchange, BWinex in the Bordeaux region, Vinetrade in Japan, BBX and Liv-ex in the U.K. to name but a few) are highly specialized and do not represent the market with the greatest volume and level of activity for the asset or liability. Level 2 inputs, on the other hand, are accessible to wine funds since, first, they observe their own transactions, and second, they observe prices reached at auction and also on electronic platforms. Level 3 inputs are also of significant importance for wine funds since it concerns their intrinsic qualities to generate profit. Indeed, their strategies often involve acquisition and selling tactics that best exploits their positioning in the market since they can benefit from significant economies of scale. Furthermore, they can act as liquidity providers and rip a liquidity premium. They can best adjust their movements in a market prone to dysfunctionalities, as mentioned by Ashenfelter (1989): *"at the first wine auction I ever attended, I saw the repeal of the law of one price"*, referring to the declining price anomaly in wine auctions provoked by non-optimal absentee bidders (Ginsburgh, 1998). Naturally, funds' strategies differ from each other. Some specifically focus on heavily traded Bordeaux wines and try to track the overall price levels, whereas others play in niche markets of collectibles. They trade intensively in the OTC (Over The Counter) market for restaurants, dealers and collectors. Our approach to fair-valuation of a wine fund combines level 2 and level 3 inputs.

Level 2 inputs consist of observed transactions, both made by the fund itself and observable prices reached at auction, buyer's premium included, for identical wines. The auction market can be considered as the most advantageous market since it is open to all and applies an English auction system, known to be the one that maximizes seller revenues amongst auction mechanisms (Lopomo, 1998). Unfortunately, because of heterogeneity at auction, several prices for seemingly identical wines (with respect to domain, vintage and format for instance) are simultaneously observed. A straight approach would consist in averaging prices observed simultaneously, so as to obtain, through time, an evolution of the average price of a given wine.

$$w_{it} = \frac{1}{N_{it}} \sum_{j}^{N_{it}} p_{itj}, \tag{5.1}$$

where w_{it} is the value of wine i at time t, p_{itj} is the jth transaction of wine i sold at time t and N_{it} is the amount of identical wines i sold at time t.
This naive methodology suffers from various drawbacks, including sensitivity to outliers. Also, as (5.1) can be seen as a particular case of hedonic regression whose explanatory variables would consist only of a single constant term and time dummies, Bocart and Hafner (2013c) showed that the traditional estimator of volatility $\sqrt{\frac{1}{N_i-1}\sum_t (w_{it} - \bar{w}_i)^2}$ is a fixed effect, biased estimator. We rather suggest constructing a price index based on a random effect estimator of each type of wine through time. Such estimator is naturally more robust to outliers thanks to a smoothing

effect. The model is:

$$\log(p_{itj}) = C_i + \beta_{it} + \nu_{ijt}. \tag{5.2}$$

$$\beta_{it} = \beta_{it-1} + \xi_{it}. \tag{5.3}$$

The term C_i is a constant, β_{it} is the marginal impact of time on prices of wine i and $\nu_{ijt} \sim N(0, \sigma^2_{\nu_i})$. In (5.3), we suppose that β_{it} is a random walk, with $\xi_{it} \sim N(0, \sigma^2_{\xi_i})$ and which, for identification, is restricted to have a mean of zero. Finally, ν_{ijt} is a Gaussian error term with mean zero.
The model can be estimated similar to Bocart and Hafner (2013c). At a first stage, estimate

$$\log(p_{itj}) = C_i + \eta_{itj} \tag{5.4}$$

by OLS, where $\eta_{itj} = \beta_{it} + \nu_{itj}$. At a second stage, estimate β_{it} by a Kalman filter. A wine i's fair value w_{iT} at time T is then estimated:

$$\tilde{w_{iT}} = \sum_{\tau}^{T} \sum_{j}^{N_{i\tau}} \exp(\widehat{\beta_{iT}} - \widehat{\beta_{i\tau}}) p_{i\tau j} \lambda_{i\tau} \tag{5.5}$$

$N_{i\tau}$ is the total amount of transactions observed at auction of wine i at time τ, $\lambda_{i\tau}$ is the weight allocated to the τth period, allowing, for instance, to give more weight to more recent observations. The particular case $\lambda_{i\tau} = \frac{1}{N_{i\tau}}\delta_{\tau T}$ (where $\delta_{\tau T}$ is the Kronecker delta) yields the classical approach of equation (5.1). Alternatively, $\lambda_{i\tau} = \frac{1}{TN_{i\tau}}$ gives all past time periods an equal weight in the current valuation. Another direct extension of the model is the possibility to aggregate wines per domain and/or per vintage by including additional variables in (5.4). Also, a predictor of future prices can be derived from the Kalman filter's approach:

$$\log(\widehat{p_{ij(T+1)}}) = \widehat{C}_i + \hat{\beta}_{i(T+1)}. \tag{5.6}$$

5.5 Empirical results

To test the suggested method, we gather all sales of the most popular[1] wines sold at auction at Christie's and Sotheby's between February 2007 and December 2013. The database consists of 26640 observed transactions on 232 different wines whose list is available in appendix. Selling prices are converted in USD/bottle. In order to mimick a wine fund's portfolio, we simulate a portfolio made of one bottle of each of the 232 different wines available in our sample.

[1]that is, wines that appeared at auction at least 50 times

Each wine is reevaluated monthly. In case no observation is available a given month, the estimated fair value of the previous month is forwarded. Annualized volatility of each wine is also computed. It averages 25%.
In appendix, individual monthly valuation of each of the 232 wines is provided. As expected, the price average method resulting from equation (5.1) yields an unstable valuation. At individual wine levels, unreasonable spikes can be observed. Figure 5.1 compares the filtered valuation of the wine with the most transaction (Mouton-Rothschild 1982) with the price average through time. A clear spike is visible, due to an abnormal transaction recorded at auction. Such effects are smoothed in the filtered version of the valuation.

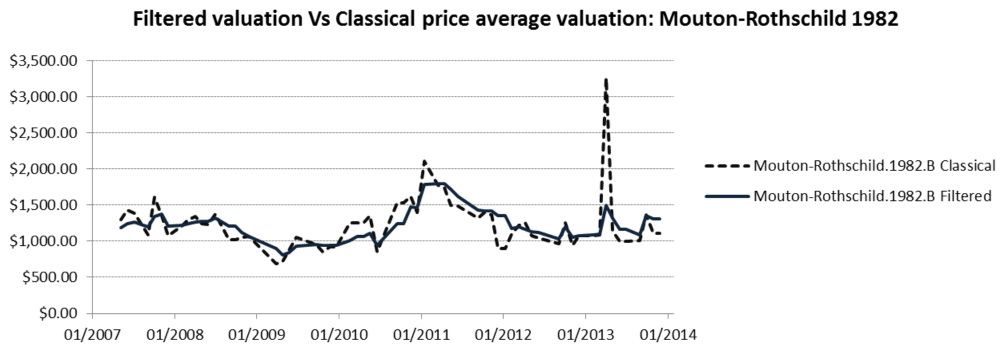

Figure 5.1: Valuations of Mouton-Rothschild 1982. Dotted line stands for the classical price average methodology, whereas the plain line represents the filtered version of the valuation

At portfolio level, Figure 5.2 plots the two types of valuation for our virtual portfolio made of the cumulated value of the 232 wines through time. Dotted line stands for the classical price average methodology, whereas the plain line comes from the filtered version of the valuation. The filtered version exhibits a smoother progression.

Finally, a more precise estimation of returns can also be considered to appreciate relationships and dynamics between different wines. For instance, Figure 5.3 illustrates a tight relationship between 1996 Bordeaux wines whose domain is situated between St-Estephe and Margaux in the Medoc region. These similar patterns are in sharp contrast with other Bordeaux wines (see Figure 5.4), such as those situated further south in the Pessac-Leognan area (Haut-Brion and Mission Haut-Brion), or even further East in the Pomerol area. These nuances are not captured by the

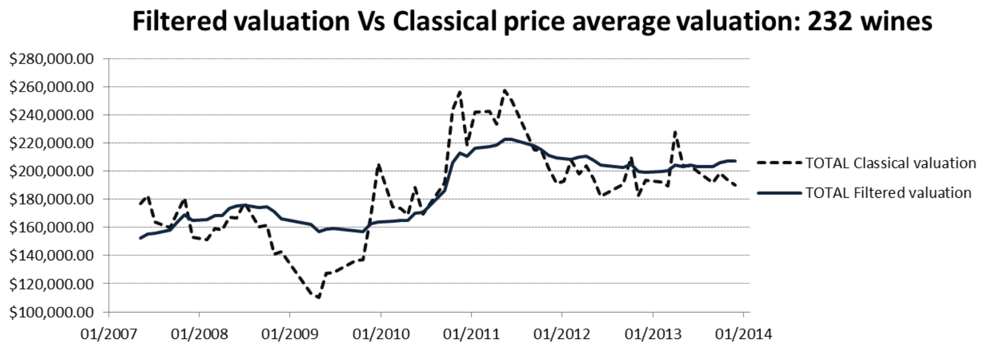

Figure 5.2: Valuations of a portfolio made of each of the 232 surveyed. Dotted line stands for the classical price average methodology, whereas the plain line represents the filtered version of the valuation

classical estimator 5.1. In the context of valuing a portfolio, such analysis can be useful, for instance, to create peer groups with corresponding price indices.

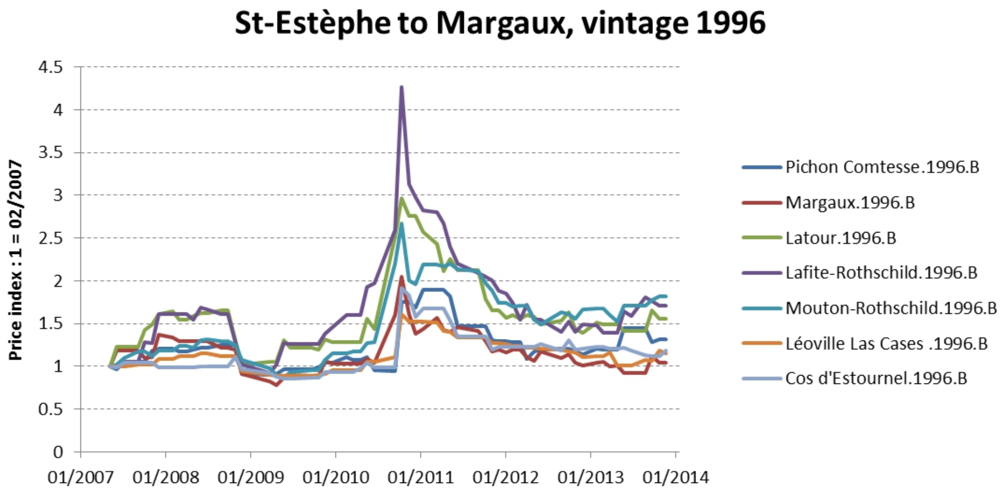

Figure 5.3: Monthly price index of Bordeaux wines situated in the narrow area St-Estephe to Margaux, vintage 1996

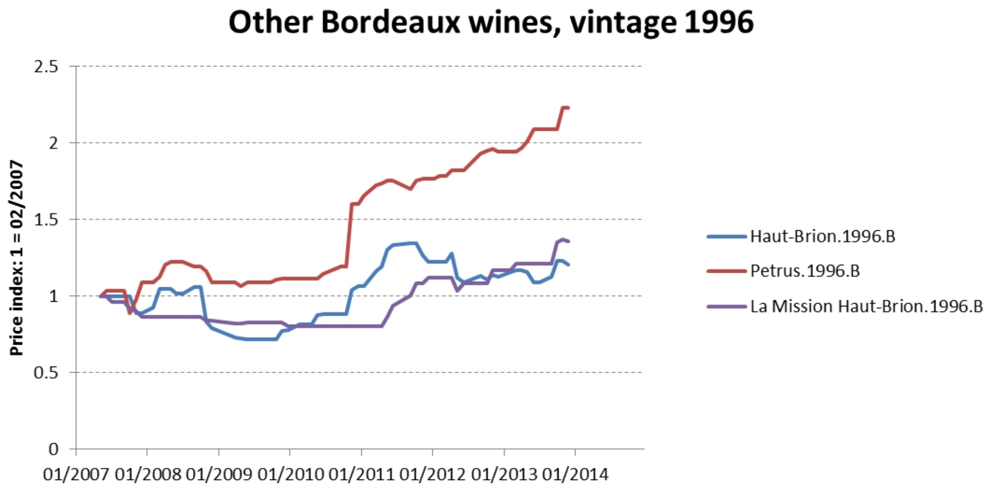

Figure 5.4: Monthly price index of some Bordeaux wines situated outside the narrow area St-Estephe to Margaux, vintage 1996

Figure 5.5: Map of Bordeaux region. Courtesy: Financial Times

5.6 Conclusion

IFRS 13 compliant revaluation of wine as an investment is an important topic for fund managers, investors and fiscal authorities. Since the notion of "highest and best use" for non-financial assets is difficult to apply for bottles of wine, a fair valuation can only rely on a market approach. Unfortunately, wines are heterogeneous goods that are not traded continuously. Furthermore, they can be traded at different places: dealers, local exchanges and auction houses. Wine funds use independent valuation, auction and dealers based methodology, or the so-called "Liv-ex" method. We argue that none of these fully satisfy the stringent requirements of IFRS 13. They either fail to justify the origin of data (such as in the case of independent expertise), hence the type of input, or are calibrated on markets that are not principal or most advantageous (such as the Liv-ex). We suggest estimating returns of a wine portfolio by applying Kalman filtering on price progression of individual wines. We advocate that data used to calibrate the model should be the fund's own transactions married with data from auction houses, the latter being the biggest observable market for wine transaction, and the one that best fits the definition of "most advantageous market". Naturally, a possible extension can further discriminate amongst auction houses, geographical location, etc. Our empirical results show superior performance than traditional average of prices that yields distorted results and fails to properly capture the market's dynamics, including market's volatility.

Chapter 6

Conclusion

Price indices are widely used by both academics and business practitioners to study market movements as if they were directly measured. In certain cases, financial derivatives are constructed based on their values. We have confirmed in Chapter 2 that expected characteristics of physical assets such as art cannot be easily replicated using stocks of companies. Using a new framework for portfolio optimization based on R-Vine copulae, we confidently confirm that holding a composite index of art-related companies was useless during the banking crisis, though it was beneficial during the European debt crisis thanks to a corresponding increase in volumes at auction. Indeed, stocks of art companies offer an exposure to volumes at auction, but not to prices. Turning to a continous framework in Chapter 3, we present the construction of volatility indices for the art market. In a classical hedonic regression framework, we estimate local parameters using a local likelihood approach, which contrasts with the typical OLS estimation method. We find that during the financial crisis in 2008/09, the volatility of predictability has been smaller than before, meaning that during this period, price predictions were more precise. Using total variation of returns, we compare variability of prices of artworks to the vix index and find similar behaviour during the financial crisis. Chapter 4 presents evidence that standard deviation of returns estimated with OLS tends to overestimate market volatility and needs to be corrected. In a framework where the market index follows a random walk, or a stationary autoregressive process, important efficiency gains of the volatility estimator can be obtained by using maximum likelihood in combination with the Kalman filter. Higher frequency of estimation of returns is

also achieved. This method is applied in Chapter 5, where we illustrate how it can be used by alternative investment funds to mark-to-market a portfolio. We discuss more particularly the case of fine wine and introduce a new database made of 26,640 auction data. As an extension to the methodology, we suggest a nonparametric approach to allow for time-varying volatility. Another possible extension, as in Bocart and Hafner (2013a), consider time-varying correlations with other assets.
Future research shall involve more general filters. More specifically: filters that waive the normality assumption in the error terms.

Chapter 7

Appendix

7.1 Appendix to Chapter 2

Table 7.1: Optimal Portfolio Weights

	Return	C-VaR	GOVIES	CREDIT	STOCKS	AGRI	ENERGY	REALESTATE	GOLD	ART
From 10/2001 to 9/2003	0.000548	0.007309	0	0.439348	0	0.037259	0.031848	0.183168	0.308377	0
From 12/2001 to 11/2003	0.000478	0.005889	0.147832	0.433125	2.64E-05	0.072665	0.083701	0.139326	0.123324	0
From 1/2002 to 12/2003	0.000611	0.00645	0	0.498231	0.058054	0.067553	0.067058	0.127524	0.181581	0
From 3/2002 to 1/2004	0.000581	0.006285	0	0.62322	0.022984	0.148155	0.009479	0.14165	0.050902	0.003611
From 4/2002 to 3/2004	0.00062	0.006545	0.02937	0.598176	0	0.15318	0.005023	0.191261	0	0.022989
From 5/2002 to 4/2004	0.000595	0.007853	0	0.614081	0.074785	0.222638	0.028304	0.035845	0	0.024347
From 7/2002 to 6/2004	0.000504	0.006984	0.009287	0.570352	0.201179	0.121621	0.018322	0	0.057809	0.02143
From 8/2002 to 7/2004	0.000548	0.006245	0	0.645048	0.167341	0	0.005346	0.16541	0	0.016855
From 9/2002 to 8/2004	0.000628	0.007192	0	0.548778	0.168936	0	0.041282	0.187766	0.015646	0.037593
From 11/2002 to 10/2004	0.000647	0.006709	0.149621	0.378677	0.115719	0	0.046131	0.294267	0	0.015584
From 12/2002 to 11/2004	0.000649	0.006326	0.279501	0.23786	0.149685	0	0.038406	0.292174	0	0.002375
From 2/2003 to 12/2004	0.000826	0.007099	0.023452	0.427421	0.263499	0	0.013203	0.231805	0	0.040619
From 3/2003 to 2/2005	0.000891	0.007953	0.045443	0.408205	0.212229	0	0.01992	0.258431	0	0.055772
From 4/2003 to 3/2005	0.001229	0.011858	0.075701	0.107557	0.072899	0	0.160235	0.431585	0	0.152024
From 6/2003 to 5/2005	0.000909	0.011606	0.110364	0.123517	0	0	0.122287	0.554481	0	0.089351
From 7/2003 to 6/2005	0.001201	0.012254	0	0	0.184421	0.086712	0.117485	0.480669	0	0.130713
From 9/2003 to 7/2005	0.000906	0.008341	0.274845	0.115698	0	0.0006	0.112109	0.427749	0	0.068999
From 10/2003 to 9/2005	0.001102	0.00965	0.203087	0	0	0	0.112373	0.535665	0	0.148875
From 11/2003 to 10/2005	0.000856	0.008093	0.37455	0.051418	0	0	0.07565	0.375044	0	0.123339
From 1/2004 to 12/2005	0.000861	0.0078	0.531084	0	0	0	0.076724	0.252246	0	0.139947
From 2/2004 to 1/2006	0.000796	0.006681	0.515137	0	0	0	0.091947	0.261294	0.034495	0.097127
From 3/2004 to 2/2006	0.001208	0.009689	0.290928	0	0	0	0.136516	0.357326	0.038227	0.177004
From 5/2004 to 4/2006	0.001302	0.008649	0.302101	0	0	0	0.074054	0.364031	0.074389	0.185425
From 6/2004 to 5/2006	0.001346	0.01004	0.105456	0	0	0	0.076399	0.526403	0.114441	0.1773
From 8/2004 to 6/2006	0.001142	0.008827	0.24218	0	0	0.008096	0.045085	0.433602	0.129633	0.141404
From 9/2004 to 8/2006	0.001204	0.010436	0.187189	0	0	0	0.094907	0.378878	0.18082	0.158205
From 10/2004 to 9/2006	0.00107	0.009708	0.226776	0	0	0.065812	0	0.398794	0.141701	0.166916
From 12/2004 to 11/2006	0.001083	0.009413	0.08069	0	0	0.095966	0.057333	0.528791	0.085837	0.151384
From 1/2005 to 12/2006	0.00136	0.012039	0	0	0	0.123318	0.036169	0.618607	0.007395	0.21451
From 2/2005 to 1/2007	0.001205	0.010306	0.010151	0.002711	0	0.126129	0	0.643161	0.071655	0.146194
From 4/2005 to 3/2007	0.000781	0.007973	0.1856	0.261304	0	0.18006	0	0.268868	0	0.104168
From 5/2005 to 4/2007	0.0009	0.008716	0.027852	0.362701	0	0.116932	0.003638	0.286581	0.052147	0.150149
From 7/2005 to 6/2007	0.000723	0.008806	0	0.413974	0	0.111792	0	0.327992	0.031252	0.11499
From 8/2005 to 7/2007	0.0007	0.008599	0	0.421412	0	0.142985	0.00256	0.276353	0.011616	0.145073
From 9/2005 to 8/2007	0.000647	0.007436	0	0.453354	0	0.160346	0	0.263299	0.000237	0.122765
From 11/2005 to 10/2007	0.000573	0.005404	0.102374	0.47395	0.028863	0.075866	0.00519	0.256555	0.002377	0.054825
From 12/2005 to 11/2007	0.000524	0.004965	0.005576	0.621932	0	0.055433	0.010631	0.246776	0.024615	0.035037
From 1/2006 to 12/2007	0.0004	0.004797	0.149168	0.539436	0.088845	0.097665	0	0.070454	0.008585	0.045848
From 3/2006 to 2/2008	0.000476	0.00486	0.099636	0.669328	0.023301	0.101406	0.024221	0.031965	0.050144	0
From 4/2006 to 3/2008	0.000401	0.003962	0.135171	0.724679	0.047707	0.061724	0	0	0.030719	0
From 6/2006 to 5/2008	0.000417	0.0048	0.15179	0.607134	0.013467	0.091887	0.045703	0.055211	0.034809	0
From 7/2006 to 6/2008	0.000453	0.005227	0.023077	0.763903	0.078253	0.099179	0.035589	0	0	0
From 8/2006 to 7/2008	0.000432	0.005585	0.17051	0.614794	0.03264	0.106817	0.059725	0	0.015514	0
From 10/2006 to 9/2008	0.000588	0.007752	0.444563	0.228636	0	0.120203	0.081163	0	0.125435	0
From 11/2006 to 10/2008	0.000562	0.008406	0.643779	0	0	0.118792	0.021228	0	0.216201	0
From 1/2007 to 11/2008	0.000691	0.008565	0.742129	0	0	0.111525	0.028263	0	0.118083	0
From 2/2007 to 1/2009	0.000783	0.01002	0.70153	0	0	0.098044	0.042178	0	0.158248	0
From 3/2007 to 2/2009	0.000795	0.01102	0.726085	0.011266	0	0.096551	0.06344	0	0.102657	0
From 5/2007 to 4/2009	0.000837	0.012075	0.813635	0.005606	0	0.044793	0.057327	0	0.078639	0
From 6/2007 to 5/2009	0.000837	0.008578	0.744784	0.092647	0	0.070278	0.014447	0	0.077844	0
From 7/2007 to 6/2009	0.000615	0.008406	0.591582	0.281	0	0.030687	0.067375	0	0.029355	0
From 9/2007 to 8/2009	0.000656	0.011074	0.678289	0.118485	0	0	0.092897	0	0.110328	0
From 10/2007 to 9/2009	0.000591	0.011591	0.585319	0.297185	0	0	0.059811	0	0.057685	0
From 12/2007 to 10/2009	0.000602	0.013596	0.545877	0.320422	0	0	0.032057	0	0.101643	0
From 1/2008 to 12/2009	0.000496	0.013168	0.432558	0.415109	0	0.041236	0.009014	0	0.102083	0
From 2/2008 to 1/2010	0.000435	0.010314	0.368104	0.568081	0	0	0.004188	0	0.059627	0
From 4/2008 to 3/2010	0.000453	0.011279	0.393511	0.467775	0	0	0	0	0.138714	0
From 5/2008 to 4/2010	0.00042	0.008633	0.439959	0.446644	0	0	0	0	0.106402	0.006996
From 6/2008 to 5/2010	0.000529	0.008675	0.553971	0.16582	0.055231	0	0	0	0.160725	0.064252
From 8/2008 to 7/2010	0.000495	0.006614	0.337393	0.471141	0	0	0	0	0.156042	0.035424
From 9/2008 to 8/2010	0.000564	0.006043	0.295499	0.538502	0	0.012441	0	0	0.132583	0.020975
From 11/2008 to 10/2010	0.000816	0.006397	0.131992	0.628816	0	0	0	0	0.204388	0.034805
From 12/2008 to 11/2010	0.000732	0.006515	0.092376	0.679724	0	0.013253	0	0	0.152707	0.061939
From 1/2009 to 12/2010	0.000784	0.008554	0.184804	0.388553	0.011959	0.084945	0	0.035742	0.129696	0.164302
From 3/2009 to 2/2011	0.00082	0.007681	0.22378	0.345215	0	0.042513	0	0.236709	0.038421	0.113362
From 4/2009 to 3/2011	0.000648	0.007125	0.132641	0.630378	0	0.008235	0.005146	0.026404	0.112648	0.084548
From 6/2009 to 4/2011	0.000599	0.006905	0.230027	0.498712	0	0.043093	0.001132	0	0.158403	0.068635
From 7/2009 to 6/2011	0.000863	0.009205	0.336527	0.159806	0	0.122305	0	0	0.222167	0.159196
From 8/2009 to 7/2011	0.000712	0.007171	0.36642	0.209998	0	0.063336	0	0	0.240392	0.119853
From 10/2009 to 9/2011	0.000741	0.009272	0.499529	0.048664	0	0.08382	0	0	0.228998	0.138988
From 11/2009 to 10/2011	0.000545	0.007794	0.479411	0.108375	0	0.041881	0	0.058189	0.224239	0.087906
From 12/2009 to 11/2011	0.000515	0.008376	0.555066	0	0	0	0.076314	0.038624	0.293069	0.036927
From 2/2010 to 1/2012	0.000581	0.008287	0.52441	0.002736	0	0	0.033468	0.123181	0.184777	0.131428
From 3/2010 to 2/2012	0.000501	0.007397	0.386674	0.233438	0	0.047454	0	0.114853	0.190718	0.026862
From 5/2010 to 3/2012	0.000392	0.00586	0.252709	0.496815	0.002344	0.051008	0.000815	0.061194	0.125521	0.009593
From 6/2010 to 5/2012	0.000356	0.004743	0.226149	0.641119	0	0.033158	0.007795	0.055619	0.03616	0
From 7/2010 to 6/2012	0.000344	0.005701	0.435117	0.389086	0	0.050459	0	0	0.071705	0.053633
From 9/2010 to 8/2012	0.000296	0.00544	0.33183	0.596963	0	0.021888	0	0	0.006113	0.043206
From 10/2010 to 9/2012	0.000256	0.005327	0.338066	0.551441	0	0.035152	0.039537	0	0.00109	0.034715
From 11/2010 to 10/2012	0.00032	0.005994	0.314767	0.570222	0	0.002153	0.04589	0	0.000297	0.066671

Table 7.2: Semi-annual art indices based on quantile regression

	HPI q = 0.05	HPI q = 0.5	HPI q = 0.95
2001.1	1.00	1.00	1.00
2001.2	1.02	0.74	0.73
2002.1	1.08	0.85	0.81
2002.2	1.06	0.84	0.65
2003.1	1.11	0.96	0.85
2003.2	1.14	0.95	0.90
2004.1	1.24	1.16	1.07
2004.2	1.29	1.07	0.83
2005.1	1.24	1.12	1.17
2005.2	1.32	1.12	1.17
2006.1	1.40	1.48	1.66
2006.2	1.32	1.32	1.53
2007.1	1.16	1.37	1.10
2007.2	1.06	0.96	0.86
2008.1	0.73	0.69	0.75
2008.2	0.84	0.72	0.52
2009.1	0.96	0.97	0.78
2009.2	0.87	0.78	0.62
2010.1	1.02	1.06	0.98
2010.2	0.94	0.81	0.71
2011.1	0.94	1.00	0.91
2011.2	1.03	1.01	1.13

7.2 Appendix to Chapter 4

Appendix A: Proofs

Proof of Proposition 1. The first part of the proposition concerns the estimator of σ_u^2. We have $\hat\beta_t = \beta_t + \bar u_t$, with $\bar u_t = \frac{1}{n_t}\sum_{i=1}^{n_t} u_{it}$, and hence $Y_{it} - X_{it}'\alpha - \hat\beta_t = u_{it} - \bar u_t$. Consider the expression

$$\begin{aligned}
&\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}(Y_{it} - X_{it}'\alpha - \hat\beta_t)^2 \\
= &\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}(u_{it} - \bar u_t)^2 \\
= &\sigma_u^2 + \frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}(u_{it}^2 - \sigma_u^2) - \frac{2}{T}\sum_{t=1}^{T}\frac{1}{n_t^2}\sum_{i=1}^{n_t}u_{it}^2 - \frac{2}{T}\sum_{t=1}^{T}\frac{1}{n_t^2}\sum_{i=1}^{n_t}\sum_{j\neq i}u_{it}u_{jt} + \frac{1}{T}\sum_{t=1}^{T}\bar u_t^2.
\end{aligned}$$

The second and fourth terms are the means of independent r.v. with mean zero and finite variance by Assumption (A1). Hence, the Chebychev weak law of large numbers applies to these terms, which are $O_p(T^{-1/2})$. In the same vein, $\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t^2}\sum_{i=1}^{n_t}u_{it}^2 = \sigma_u^2\mathrm{E}[n_t^{-1}] + O_p(T^{-1/2})$ and $\frac{1}{T}\sum_{t=1}^{T}\bar u_t^2 = \sigma_u^2\mathrm{E}[n_t^{-1}] + O_p(T^{-1/2})$. Thus, we have $\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}(Y_{it} - \hat\beta_t)^2 = \sigma_u^2(1 - \mathrm{E}[n_t^{-1}]) + O_p(T^{-1/2})$. It immediately follows that $\hat\sigma_u^2 = \sigma_u^2 + O_p(T^{-1/2})$, as stated.

The second part of the proposition concerns the estimator of σ_ξ^2. Note that $\hat\xi_t = \xi_t + \bar u_t - \phi\bar u_{t-1}$, where we denote $\bar u_t = \frac{1}{n_t}\sum_{i=1}^{n_t}u_{it}$, with $\bar u_t \sim N(0, \sigma_u^2/n_t)$. Consider the naive estimator $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}(\hat\xi_t - \frac{1}{T}\sum_{j=1}^{T}\hat\xi_j)^2 = \frac{1}{T}\sum_{t=1}^{T}\xi_t^2 + \frac{1}{T}\sum_{t=1}^{T}\bar u_t^2 + \frac{\phi^2}{T}\sum_{t=2}^{T}\bar u_{t-1}^2 + O_p(T^{-1/2})$. Since $\frac{1}{T}\sum_{t=1}^{T}\bar u_t^2 \to_p \sigma_u^2\mathrm{E}[n_t^{-1}]$, we have

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}(\hat\xi_t - \frac{1}{T}\sum_{j=1}^{T}\hat\xi_j)^2 = \sigma_\xi^2 + (1+\phi^2)\sigma_u^2\mathrm{E}[n_t^{-1}] + O_p(T^{-1/2}),$$

and it follows using the first part of the proposition that the estimator $\hat\sigma_\xi^2$ is $\sqrt{T}$-consistent, as stated. Q.E.D.

Proof of Proposition 2.

Asymptotic normality follows from an application of the Liapounov central limit theorem. By Proposition 1, the estimators are asymptotically unbiased. It remains to compute the asymptotic variance. Define the information set generated by the number of observations by $\mathcal{N}_T = \sigma(n_1, n_2, \ldots, n_T)$. Then, we have the variance decomposition $\mathrm{Var}(\sqrt{T}\hat\sigma_u^2) = \mathrm{Var}(\mathrm{E}[\sqrt{T}\hat\sigma_u^2 \mid \mathcal{N}_T]) + \mathrm{E}[\mathrm{Var}(\sqrt{T}\hat\sigma_u^2)|\mathcal{N}_T]$. The first term is zero since $\mathrm{E}[\hat{\sigma}_u^2 \mid \mathcal{N}_T] = \sigma_u^2$. The second term is

$$\mathrm{E}[\mathrm{Var}(\sqrt{T}\hat{\sigma}_u^2)|\mathcal{N}_T] = 2\sigma_u^4 \frac{T^{-1}\sum_{t=1}^T (n_t-1)/n_t^2}{\left\{T^{-1}\sum_{t=1}^T (n_t-1)/n_t\right\}^2},$$

which delivers $\Sigma_{uu,T}$. Next, the covariance is given by

$$\mathrm{Cov}(\sqrt{T}\hat{\sigma}_u^2, \sqrt{T}\hat{\sigma}_\xi^2) = \mathrm{Cov}(\sqrt{T}\hat{\sigma}_u^2, \frac{1}{\sqrt{T}}\sum_{t=1}^T \hat{\xi}_t^2) - (1+\phi^2)\mathrm{Cov}(\sqrt{T}\hat{\sigma}_u^2, \hat{\sigma}_u^2 \frac{1}{\sqrt{T}}\sum_{t=1}^T \frac{1}{n_t}).$$

It is straightforward to show that the first term on the right hand side is zero. The covariance in the second term is given by

$$\mathrm{Cov}(\sqrt{T}\hat{\sigma}_u^2, \hat{\sigma}_u^2 \frac{1}{\sqrt{T}}\sum_{t=1}^T \frac{1}{n_t}) = \mathrm{E}[\mathrm{Var}(\sqrt{T}\hat{\sigma}_u^2|\mathcal{N}_T)\frac{1}{T}\sum_{t=1}^T \frac{1}{n_t}] =$$

$$2\sigma_u^4 \mathrm{E}\frac{T^{-1}\sum_{t=1}^T (n_t-1)/n_t^2}{\left\{T^{-1}\sum_{t=1}^T (n_t-1)/n_t\right\}^2}\frac{1}{T}\sum_{t=1}^T \frac{1}{n_t},$$

which gives $\Sigma_{uv,T}$.

We now prove the expression for the asymptotic variance of $\hat{\sigma}_\xi^2$. First note that $\frac{1}{\sqrt{T}}\sum_{t=1}^T(\hat{\xi}_t - \frac{1}{T}\sum_{j=1}^T \hat{\xi}_j)^2 = \frac{1}{\sqrt{T}}\sum_{t=1}^T \hat{\xi}_t^2 - \sqrt{T}(\frac{1}{T}\sum_{j=1}^T \hat{\xi}_j^2)^2$, where the second term on the right hand side is $O_p(T^{-1/2})$ and, hence, asymptotically negligible. We have

$$\lim_{T\to\infty} \mathrm{Var}(\sqrt{T}\hat{\sigma}_\xi^2) = \lim_{T\to\infty} \mathrm{Var}(\frac{1}{\sqrt{T}}\sum_{t=1}^T \hat{\xi}_t^2) + (1+\phi^2)^2 \lim_{T\to\infty} \mathrm{Var}(\hat{\sigma}_u^2 \frac{1}{\sqrt{T}}\sum_{t=1}^T \frac{1}{n_t}), \quad (7.1)$$

since again the covariance term disappears. Note first that $\hat{\xi}_t^2 = \xi_t^2 + \bar{u}_t^2 + \phi^2\bar{u}_{t-1}^2 + 2\xi_t\bar{u}_t - 2\phi\xi_t\bar{u}_{t-1} - 2\phi\bar{u}_t\bar{u}_{t-1}$. All terms are mutually orthogonal and, by assumption, $\xi_t \sim N(0, \sigma_\xi^2)$ and $\bar{u}_t \sim N(0, \sigma_u^2/n_t)$. Hence, $\mathrm{Var}(\hat{\xi}_t^2|\mathcal{N}_T) = 2\sigma_\xi^4 + 2\sigma_u^4/n_t^2 + 2\phi^4\sigma_u^4/n_{t-1}^2 + 4\sigma_\xi^2\sigma_u^2/n_t + 4\phi^2\sigma_\xi^2\sigma_u^2/n_{t-1} + 4\phi^2\sigma_u^4/(n_t n_{t-1}) = 2\sigma_u^4(\sigma_\xi^2/\sigma_u^2 + 1/n_t + \phi^2/n_{t-1})^2$. Next, $\mathrm{Cov}(\hat{\xi}_t^2, \hat{\xi}_{t-1}^2|\mathcal{N}_T) = 2\sigma_u^4\phi^2/n_{t-1}^2$, and $\mathrm{Cov}(\hat{\xi}_t^2, \hat{\xi}_{t-\tau}^2|\mathcal{N}_T) = 0$, $|\tau| \geq 2$. Therefore, we have

$$\mathrm{Var}(\frac{1}{\sqrt{T}}\sum_{t=1}^T \hat{\xi}_t^2|\mathcal{N}_T) = 2\sigma_u^4 \frac{1}{T}\sum_{t=2}^T \left\{\left(\frac{\sigma_\xi^2}{\sigma_u^2} + \frac{1}{n_t} + \frac{\phi^2}{n_{t-1}}\right)^2 + \frac{2\phi^2}{n_{t-1}^2}\right\}.$$

Furthermore, $\mathrm{Var}(\mathrm{E}[\frac{1}{\sqrt{T}}\sum_{t=1}^T \hat{\xi}_t^2|\mathcal{N}_T]) = \mathrm{Var}(\frac{1}{\sqrt{T}}\sum_{t=1}^T(\sigma_\xi^2 + \sigma_u^2/n_t + \phi^2\sigma_u^2/n_{t-1})) = \sigma_u^4(1+\phi^2)^2\mathrm{Var}(1/n_t)$. Hence,

$$\mathrm{Var}(\frac{1}{\sqrt{T}}\sum_{t=1}^T \hat{\xi}_t^2) = 2\sigma_u^4 \frac{1}{T}\sum_{t=2}^T \mathrm{E}\left\{\left(\frac{\sigma_\xi^2}{\sigma_u^2} + \frac{1}{n_t} + \frac{\phi^2}{n_{t-1}}\right)^2 + \frac{2\phi^2}{n_{t-1}^2}\right\} + \sigma_u^4(1+\phi^2)^2\mathrm{Var}(1/n_t).$$

Finally, we have for the variance in the second term of (7.1):

$$\mathrm{Var}(\hat{\sigma}_u^2 \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{1}{n_t} | \mathcal{N}_T) = 2\sigma_u^4 \frac{\frac{1}{T}\sum_{t=1}^{T} \frac{n_t - 1}{n_t^2}}{\left\{\frac{1}{T}\sum_{t=1}^{T} \frac{n_t-1}{n_t}\right\}^2} \left(\frac{1}{T}\sum_{t=1}^{T} \frac{1}{n_t}\right)^2,$$

and $\mathrm{Var}(\mathrm{E}[\hat{\sigma}_u^2 \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{1}{n_t} | \mathcal{N}_T]) = \sigma_u^4 \mathrm{Var}(1/n_t)$. Putting the pieces of (7.1) together, we obtain the stated result for $\lim_{T\to\infty} \mathrm{Var}(\sqrt{T}\hat{\sigma}_\xi^2)$. Q.E.D.

Proof of Proposition 3.

Since ϕ is known but possibly equal to one, in which case $\{\beta_t\}$ would be non-stationary, classical theory on the estimation of state space models does not directly apply. As noted by Pagan (1980), however, the theory remains valid in the unit root case if the model is locally asymptotically identified in the sense of Kohn (1978). A necessary and sufficient condition for θ_0 to be locally asymptotically identified is that $\lim_{T\to\infty}(I(\theta_0)/T)^{-1}$ be non-singular. In our model we can directly check for this condition to hold. The sum in $I(\theta)$ contains two terms, the second of which, $2\mathrm{E}\left(\frac{\partial e_t'}{\partial \theta} \Sigma_t^{-1} \frac{\partial e_t}{\partial \theta'}\right)$ is positive semi-definite. It suffices to show that the first term, $\frac{\partial \mathrm{vec}(\Sigma_t)'}{\partial \theta}(\Sigma_t^{-1} \otimes \Sigma_t^{-1})\frac{\partial \mathrm{vec}(\Sigma_t)}{\partial \theta'}$, is positive definite. Similar to Proposition 1 of Pagan (1980), we have

$$\frac{\partial \mathrm{vec}(\Sigma_t)'}{\partial \theta}(\Sigma_t^{-1} \otimes \Sigma_t^{-1})\frac{\partial \mathrm{vec}(\Sigma_t)}{\partial \theta'} \geq \lambda_{min}^2(\Sigma_t^{-1})\Omega_t,$$

where $\Omega_t = \frac{\partial \mathrm{vec}(\Sigma_t)'}{\partial \theta}\frac{\partial \mathrm{vec}(\Sigma_t)}{\partial \theta'}$, $\lambda_{min}(\cdot)$ denotes the smallest eigenvalue, and where $\geq$ means that the left hand side matrix minus the right hand side matrix is p.s.d. Due to the particular structure of our model, we have $\lambda_{min} = \sigma_u^2 > 0$, by assumption. Hence, it suffices to show that Ω_t is p.d. Using the expressions in Appendix B, and denoting $X_t = \partial\sigma_t^2/\partial\sigma_u^2$ and $Y_t = \partial\sigma_t^2/\partial\sigma_\xi^2$, we can write

$$\Omega_t = n_t \begin{pmatrix} n_t X_t^2 + 2X_t + 1 & (Y_t+1)(n_t X_t + 1) \\ (Y_t+1)(n_t X_t+1) & n_t(Y_t+1)^2 \end{pmatrix}.$$

It follows immediately that $|\Omega_t| > 0$ if and only if $n_t > 1$, which happens with positive probability by assumption. Hence, $\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}\Omega_t$ is positive definite almost surely, which implies that $\lim_{T\to\infty}(I(\theta_0)/T)^{-1}$ is positive definite almost surely. This shows asymptotic local identifiability of the model.

The remaining conditions of Theorem 4 of Pagan (1980) hold trivially, as a_t is uniformly bounded from above and non-stochastic, the state space form is uniformly completely observable and uniformly completely controllable. Finally, for asymptotic normality we need that θ_0 is an interior point of Θ. Then, consistency and asymptotic normality follow by Theorem 4 of Pagan (1980). The form of the information matrix is standard for state space models, see e.g. Lütkepohl (1993, p.437).

Appendix B: The Kalman recursions and the information matrix

Note that

$$
\begin{aligned}
(\beta_t|\eta_1,\ldots,\eta_{t-1}) &\sim N(\beta_{t|t-1},\sigma_\beta(t|t-1)),\\
(\beta_t|\eta_1,\ldots,\eta_t) &\sim N(\beta_{t|t},\sigma_\beta(t|t)),\\
(\eta_t|\eta_1,\ldots,\eta_{t-1}) &\sim N(\eta_{t|t-1},\Sigma_\eta(t|t-1)).
\end{aligned}
$$

For a given set of parameters, the conditional means and variances can be obtained using the following Kalman recursions:

1. Prediction step ($t = 1,\ldots,T$)

$$
\begin{aligned}
\beta_{t|t-1} &= \phi\beta_{t-1|t-1}, \qquad (7.2)\\
\sigma^2_\beta(t|t-1) &= \phi^2\sigma^2_\beta(t-1|t-1)+\sigma^2_\xi,\\
\eta_{t|t-1} &= a_t\beta_{t|t-1},\\
\Sigma_\eta(t|t-1) &= a_t\sigma^2_\beta(t|t-1)a_t'+\sigma^2_u I_{n_t}.
\end{aligned}
$$

2. Correction step ($t = 1,\ldots,T$)

$$
\begin{aligned}
\beta_{t|t} &= \beta_{t|t-1}+\sigma^2_\beta(t|t-1)a_t'\Sigma_\eta^{-1}(t|t-1)(\eta_t-\eta_{t|t-1}), \qquad (7.3)\\
\sigma^2_\beta(t|t) &= \sigma^2_\beta(t|t-1)-\sigma^4_\beta(t|t-1)a_t'\Sigma_\eta^{-1}(t|t-1)a_t.
\end{aligned}
$$

3. Smoothing step ($t = T-1, T-2,\ldots,1$)

   To estimate the underlying state β_t, one uses the full sample information ($t = 1,\ldots,T$).

$$
\begin{aligned}
\beta_{t|T} &= \beta_{t|t}+\phi\frac{\sigma^2_\beta(t|t)}{\sigma^2_\beta(t+1|t)}\left\{\beta_{t+1|T}-\beta_{t+1|t}\right\},\\
\sigma^2_\beta(t|T) &= \sigma^2_\beta(t|t)+\phi^2\frac{\sigma^4_\beta(t|t)}{\sigma^4_\beta(t+1|t)}\left\{\sigma^2_\beta(t+1|T)-\sigma^2_\beta(t+1|t)\right\}.
\end{aligned}
$$

We now calculate derivatives appearing in the information matrix of the MLE. The model in (4.5)-(4.7) can be written as

$$
\begin{aligned}
e_t &= \eta_t - a_t\phi\beta_{t-1|t-1}, \\
\beta_{t|t} &= \phi\beta_{t-1|t-1} + \zeta_{t-1}a_t'\Sigma_t^{-1}e_t, \\
\Sigma_t &= a_t\zeta_{t-1}a_t' + \sigma_u^2 I_{n_t}, \\
\sigma_t^2 &= \zeta_{t-1} - \zeta_{t-1}^2 a_t'\Sigma_t^{-1}a_t,
\end{aligned}
$$

where $\zeta_t = \phi^2\sigma_\beta^2(t|t) + \sigma_\xi^2$. Let $\theta = (\sigma_u^2, \sigma_\xi^2)'$ and $\delta = (1,0)'$. Then,

$$
\begin{aligned}
\frac{\partial e_t}{\partial\theta} &= -a_t\phi\frac{\partial\beta_{t-1|t-1}}{\partial\theta}, \\
\frac{\partial\beta_{t|t}}{\partial\theta} &= \phi\frac{\partial\beta_{t-1|t-1}}{\partial\theta} + \frac{\partial\zeta_{t-1}}{\partial\theta}a_t'\Sigma_t^{-1}e_t - \zeta_{t-1}\left\{\frac{\partial\mathrm{vec}(\Sigma_t)'}{\partial\theta}(\Sigma_t^{-1}e_t\otimes\Sigma_t^{-1}a_t) - a_t'\Sigma_t^{-1}\frac{\partial e_t}{\partial\theta}\right\}, \\
\frac{\partial\sigma_\beta^2(t|t)}{\partial\theta} &= \frac{\partial\zeta_{t-1}}{\partial\theta}\left(1 - 2\zeta_{t-1}a_t'\Sigma_t^{-1}a_t\right) + \zeta_{t-1}^2\frac{\partial\mathrm{vec}(\Sigma_t)'}{\partial\theta}(\Sigma_t^{-1}a_t\otimes\Sigma_t^{-1}a_t),
\end{aligned}
$$

$$
\frac{\partial\mathrm{vec}(\Sigma_t)}{\partial\theta'} = (a_t\otimes a_t)\frac{\partial\zeta_{t-1}}{\partial\theta'} + \mathrm{vec}(I_{n_t})\delta'.
$$

Some expressions can be simplified by observing that, due to the particular structure of the model, $a_t'\Sigma_t^{-1}a_t = n_t/(n_t\zeta_{t-1} + \sigma_u^2)$. For example, this leads to

$$
\frac{\partial\sigma_\beta^2(t|t)}{\partial\theta} = \frac{\partial\zeta_{t-1}}{\partial\theta}\frac{\sigma_u^4}{(n_t\zeta_{t-1} + \sigma_u^2)^2} + a_t\Sigma_t^{-2}a_t\zeta_{t-1}^2\delta.
$$

7.3 Appendix to Chapter 5

Table 7.3: Valuation of wines through time

Wine	02/2007	03/2007	04/2007	05/2007	06/2007	07/2007	09/2007	10/2007	11/2007	12/2007	02/2008	03/2008	04/2008	05/2008	06/2008	07/2008
Angelus.2000.B							245	245	245	245	245	245	245	259	259	259
Ausone.1998.B		458	458	458	458	458	458	458	458	530	530	530	530	530	540	540
Ausone.2000.B				1,695	1,695	1,695	1,674	1,674	1,717	1,717	1,717	1,717	1,717	1,795	1,775	1,775
Batard-Montrachet Leflaive.2002.B				489	489	489	489	489	193	229	229	229	229	229	229	229
Calon-Segur.1982.B		146	147	151	151	151	178	179	190	190	190	190	190	190	190	190
Calon-Segur.1995.B	90	90	90	89	89	87	87	85	87	86	85	85	85	85	86	86
Calon-Segur.1996.B		77	75	78	78	78	78	78	79	79	79	79	79	81	83	83
Calon-Segur.2000.B	78	80	80	80	80	80	80	80	80	80	80	80	80	80	80	81
Carruades de Lafite Lafite-Rothschild.1996.B					159	159	159	159	159	159	185	185	185	185	441	441
Carruades de Lafite Lafite-Rothschild.2000.B			94	94	94	94	94	94	94	113	113	113	113	113	113	113
Cheval Blanc.1982.B		1,041	1,043	1,424	1,362	1,362	1,346	1,361	1,251	1,211	1,208	1,208	1,208	1,209	1,222	1,277
Cheval Blanc.1985.B		588	588	493	493	493	492	496	494	485	485	485	485	488	488	488
Cheval Blanc.1989.B				386	380	380	376	386	386	391	402	402	402	408	408	421
Cheval Blanc.1990.B	255	721	721	968	1,001	997	997	997	997	997	1,085	1,061	1,061	1,073	1,081	1,081
Cheval Blanc.1995.B		299	290	309	309	309	309	309	309	334	334	334	334	334	334	346
Cheval Blanc.1996.B		276	276	284	301	301	280	280	280	296	275	275	275	277	277	277
Cheval Blanc.1998.B	552	548	548	566	682	682	682	713	713	735	735	735	735	735	737	737
Cheval Blanc.1999.B				248	248	249	249	249	249	249	249	249	249	262	262	262
Cheval Blanc.2000.B		745	745	745	745	745	745	745	745	745	745	930	930	992	965	965
Cheval Blanc.2003.B				242	242	242	242	242	242	242	242	342	342	342	342	342
Cheval Blanc.2004.B																
Cheval Blanc.2005.B																
Cos d'Estournel.1982.B			178	178	178	301	311	323	335	321	321	316	317	319	319	322
Cos d'Estournel.1985.B		172	170	155	155	155	154	155	155	154	153	151	151	151	151	154
Cos d'Estournel.1986.B		178	162	162	162	162	159	163	162	157	158	158	159	161	161	161
Cos d'Estournel.1988.B		79	79	79	52	52	59	59	78	89	91	97	101	104	104	104
Cos d'Estournel.1990.B		201	193	206	208	197	197	204	206	215	215	215	215	218	218	218
Cos d'Estournel.1995.B	149	149	135	147	147	150	150	150	150	150	150	152	152	154	154	154
Cos d'Estournel.1996.B	113	117	117	129	128	135	135	134	134	128	128	127	127	127	129	129
Cos d'Estournel.2000.B				144	144	144	139	139	139	139	139	144	144	144	144	144
Cos d'Estournel.2003.B		168	168	198	198	198	198	198	207	206	206	209	209	209	209	209
Cos d'Estournel.2005.B																
Dom Perignon Moet & Chandon.1990.B		273	237	236	236	246	246	250	248	249	249	251	251	253	252	252
Dom Perignon Moet & Chandon.1996.B	184	184	184	262	257	257	257	257	257	259	259	259	259	246	246	246
Ducru Beaucaillou .1982.B		245	245	256	256	256	250	260	243	242	239	239	239	239	239	239
Ducru Beaucaillou .1990.B	90	94	97	108	107	108	107	107	108	108	108	105	107	107	107	107
Ducru Beaucaillou .1995.B	113	125	125	129	129	135	135	134	134	136	136	136	136	136	136	136
Ducru Beaucaillou .1996.B	139	137	137	135	135	140	140	140	142	143	143	144	144	145	147	147
Ducru Beaucaillou .2000.B	128	117	117	123	123	123	123	123	126	126	126	128	128	136	136	136
Echezeaux DRC.1990.B	1,076	1,076	614	614	600	693	693	595	595	649	693	693	693	741	741	741
Echezeaux DRC.1996.B	234	473	337	281	296	296	296	269	269	208	208	208	208	181	197	253
Echezeaux DRC.1999.B	1,421	1,421	1,421	860	860	305	309	353	415	415	415	415	415	452	452	452
Echezeaux DRC.2002.B	189	183	183	183	183	183	224	232	232	232	205	205	205	253	253	253
Echezeaux DRC.2005.B																
Gruaud-Larose.1982.B			225	307	307	307	278	302	286	310	310	310	310	316	305	305
Gruaud-Larose.1986.B		92	92	116	116	116	128	134	141	145	145	150	150	150	150	151
Gruaud-Larose.2000.B		88	88	100	100	100	100	100	100	100	100	99	99	107	107	107
Haut-Brion.1966.B		397	263	263	263	263	279	275	275	275	275	275	263	263	266	274
Haut-Brion.1982.B			675	825	777	777	701	762	764	777	777	798	798	790	860	860
Haut-Brion.1985.B	490	448	448	412	412	412	414	414	412	401	401	401	401	407	403	403
Haut-Brion.1986.B		295	295	295	295	295	473	461	398	398	398	398	398	363	363	363
Haut-Brion.1988.B				266	266	266	266	266	261	302	302	302	302	302	302	302
Haut-Brion.1989.B		1,180	1,180	1,204	1,204	1,204	1,196	1,196	1,287	1,311	1,311	1,534	1,495	1,373	1,376	1,376
Haut-Brion.1990.B	705	701	701	705	705	705	717	715	667	723	692	692	692	741	802	802
Haut-Brion.1994.B						215	223	227	233	232	232	231	205	212	212	212
Haut-Brion.1995.B				312	330	351	351	351	372	409	419	442	442	403	382	382

Table 7.4: Valuation of wines through time

	08/2008	09/2008	10/2008	11/2008	03/2009	04/2009	05/2009	06/2009	09/2009	10/2009	11/2009	12/2009	02/2010	03/2010	04/2010	05/2010
Angelus.2000.B	259	259	256	256	256	256	256	256	230	238	238	238	238	238	238	238
Ausone.1998.B	540	540	540	540	540	540	540	540	540	540	559	559	559	559	559	581
Ausone.2000.B	1,775	1,775	1,775	1,595	1,595	1,494	1,494	1,494	1,494	1,494	1,647	1,647	1,621	1,621	1,621	1,621
Batard-Montrachet Leflaive.2002.B	229	229	240	237	237	238	238	238	230	224	224	224	224	224	224	224
Calon-Segur.1982.B	181	181	181	181	181	179	179	179	177	170	172	172	172	168	168	168
Calon-Segur.1995.B	84	84	83	78	79	79	81	81	81	83	83	83	83	83	83	83
Calon-Segur.1996.B	83	83	83	82	73	73	75	75	77	77	77	78	78	78	78	78
Calon-Segur.2000.B	81	83	83	79	81	80	80	80	80	80	83	83	83	83	83	83
Carruades de Lafite Lafite-Rothschild.1996.B	441	441	441	376	302	309	292	292	292	303	309	309	309	309	309	309
Carruades de Lafite Lafite-Rothschild.2000.B	113	276	253	183	185	185	185	185	185	185	225	225	225	225	225	225
Cheval Blanc.1982.B	1,189	1,189	1,156	1,015	815	789	853	853	874	871	868	868	868	866	866	866
Cheval Blanc.1985.B	488	488	488	474	474	474	451	451	448	448	443	442	442	442	442	442
Cheval Blanc.1989.B	421	421	421	404	404	404	341	336	332	336	336	336	336	336	336	336
Cheval Blanc.1990.B	1,112	1,112	1,034	1,026	1,009	993	993	993	1,039	1,039	1,034	1,034	1,034	1,034	1,034	1,034
Cheval Blanc.1995.B	346	346	336	327	315	315	312	312	312	312	314	316	316	316	316	316
Cheval Blanc.1996.B	277	277	270	268	255	255	254	254	254	233	233	238	238	238	238	238
Cheval Blanc.1998.B	737	737	722	722	722	614	594	572	570	562	565	565	551	551	551	552
Cheval Blanc.1999.B	262	262	262	249	249	247	271	267	259	259	273	273	273	273	273	298
Cheval Blanc.2000.B	965	965	932	860	860	761	854	850	853	659	776	799	837	837	837	837
Cheval Blanc.2003.B	342	342	342	342	342	342	342	308	303	303	303	292	292	292	292	292
Cheval Blanc.2004.B				325	325	325	325	325	325	325	338	324	324	324	324	324
Cheval Blanc.2005.B							701	701	664	664	664	664	664	664	664	664
Cos d'Estournel.1982.B	322	322	321	310	295	285	290	290	291	283	282	282	282	282	282	282
Cos d'Estournel.1985.B	154	154	153	152	152	152	152	152	152	152	150	150	150	150	150	150
Cos d'Estournel.1986.B	160	160	160	151	148	148	146	146	146	146	146	146	146	146	146	146
Cos d'Estournel.1988.B	104	104	104	102	108	106	105	105	105	101	103	103	103	103	103	103
Cos d'Estournel.1990.B	221	221	217	206	186	186	186	186	186	187	188	188	188	188	188	188
Cos d'Estournel.1995.B	154	154	154	149	156	156	154	154	154	155	156	156	156	156	156	157
Cos d'Estournel.1996.B	129	129	144	125	119	114	111	111	112	112	120	121	121	121	127	134
Cos d'Estournel.2000.B	144	137	137	125	109	108	108	108	121	121	122	126	126	126	127	127
Cos d'Estournel.2003.B	209	209	209	205	205	193	189	189	186	186	184	184	184	184	184	184
Cos d'Estournel.2005.B	216	216	216	188	188	188	188	188	193	193	193	193	193	193	193	193
Dom Perignon Moet & Chandon.1990.B	252	252	252	249	248	242	240	238	238	238	238	227	227	229	229	229
Dom Perignon Moet & Chandon.1996.B	246	246	246	247	247	247	247	246	235	241	247	247	265	265	265	265
Ducru Beaucaillou .1982.B	247	247	244	239	226	223	222	222	224	224	224	224	224	227	227	228
Ducru Beaucaillou .1990.B	107	107	107	106	106	106	106	106	106	106	106	106	106	106	106	106
Ducru Beaucaillou .1995.B	136	136	135	131	123	123	122	122	122	117	118	120	120	120	120	120
Ducru Beaucaillou .1996.B	147	147	145	149	140	140	133	133	133	134	134	134	134	134	134	142
Ducru Beaucaillou .2000.B	136	133	132	117	115	115	113	113	116	116	120	119	119	119	119	124
Echezeaux DRC.1990.B	741	741	799	799	799	799	983	1,019	1,036	977	1,131	1,131	1,131	1,131	1,092	1,092
Echezeaux DRC.1996.B	275	275	259	259	256	256	256	321	321	321	361	361	361	384	384	384
Echezeaux DRC.1999.B	437	437	437	437	437	430	430	430	430	430	522	522	522	522	522	522
Echezeaux DRC.2002.B	253	253	253	253	253	253	256	256	256	256	325	325	325	325	325	325
Echezeaux DRC.2005.B				123	123	123	123	123	221	221	221	221	221	221	221	221
Gruaud-Larose.1982.B	300	300	297	241	210	211	216	216	216	221	230	230	232	238	238	238
Gruaud-Larose.1986.B	150	150	150	146	145	145	145	145	145	150	151	151	151	151	151	151
Gruaud-Larose.2000.B	107	107	107	101	98	98	98	98	98	98	100	101	101	101	101	101
Haut-Brion.1966.B	274	274	274	274	274	274	274	274	263	254	254	254	256	256	256	256
Haut-Brion.1982.B	862	862	803	686	650	611	621	621	591	589	661	640	640	640	640	689
Haut-Brion.1985.B	402	402	402	402	402	397	384	384	382	382	390	390	390	390	390	389
Haut-Brion.1986.B	363	363	363	329	322	322	303	303	298	298	302	313	312	312	312	322
Haut-Brion.1988.B	302	302	302	260	231	223	223	223	230	230	246	250	250	250	250	262
Haut-Brion.1989.B	1,376	1,376	1,148	1,184	1,138	1,058	1,040	1,050	1,057	1,101	1,222	1,222	1,222	1,222	1,222	1,387
Haut-Brion.1990.B	747	747	627	627	627	627	574	574	560	564	564	564	564	564	564	564
Haut-Brion.1994.B	212	212	209	203	200	200	200	200	200	200	205	205	205	205	197	203
Haut-Brion.1995.B	403	403	371	271	259	256	273	273	273	273	273	291	307	307	307	331

Table 7.5: Valuation of wines through time

	06/2010	09/2010	10/2010	11/2010	12/2010	01/2011	03/2011	04/2011	05/2011	06/2011	09/2011	10/2011	11/2011	12/2011	01/2012	02/2012	03/2012	04/2012
Angelus.2000.B	245	249	249	249	249	337	337	337	337	337	337	337	332	335	335	336	336	359
Ausone.1998.B	581	762	762	762	762	761	761	761	761	761	761	761	761	761	761	753	753	753
Ausone.2000.B	1,621	2,075	2,075	2,127	2,127	2,351	2,351	2,351	2,351	2,351	2,146	1,992	1,992	1,992	1,992	1,990	1,990	1,890
Batard-Montrachet Leflaive.2002.B	220	220	220	224	224	255	255	253	251	251	251	255	261	261	261	268	268	268
Calon-Segur.1982.B	168	168	168	168	168	168	168	168	171	168	171	171	170	170	170	170	170	172
Calon-Segur.1995.B	78	79	79	81	80	108	108	108	108	109	109	109	117	117	117	116	116	116
Calon-Segur.1996.B	78	78	78	78	80	80	80	80	80	84	84	84	84	84	84	84	87	87
Calon-Segur.2000.B	87	89	89	89	92	183	183	183	183	177	177	177	160	160	160	146	146	141
Carruades de Lafite Lafite-Rothschild.1996.B	326	349	349	399	399	399	399	399	529	581	565	573	578	578	578	578	578	578
Carruades de Lafite Lafite-Rothschild.2000.B	225	442	442	543	549	606	597	597	584	609	587	574	545	545	545	478	478	468
Cheval Blanc.1982.B	816	948	948	1,602	1,602	1,574	1,187	1,095	1,095	1,079	976	958	941	955	955	939	939	966
Cheval Blanc.1985.B	435	435	435	440	440	445	445	445	445	445	440	440	442	443	443	443	443	443
Cheval Blanc.1989.B	332	330	330	348	351	375	375	381	400	400	403	403	396	397	397	402	402	402
Cheval Blanc.1990.B	1,034	1,043	1,123	1,134	1,134	1,201	1,207	1,201	1,204	1,202	1,193	1,186	1,192	1,189	1,189	1,189	1,188	1,182
Cheval Blanc.1995.B	322	329	329	383	381	385	388	392	407	412	412	412	405	406	406	407	406	403
Cheval Blanc.1996.B	246	252	252	252	260	260	260	260	284	317	328	339	334	334	334	339	339	339
Cheval Blanc.1998.B	534	534	664	680	680	635	635	623	623	619	611	611	595	584	584	582	582	556
Cheval Blanc.1999.B	298	298	298	323	323	323	323	323	323	323	333	333	347	352	352	371	371	371
Cheval Blanc.2000.B	852	1,365	1,760	1,523	1,349	1,655	1,584	1,455	1,455	1,407	1,309	1,135	983	927	927	927	921	976
Cheval Blanc.2003.B	303	388	458	458	458	433	433	433	436	433	424	424	407	392	392	392	392	395
Cheval Blanc.2004.B	324	346	346	368	368	434	434	434	440	459	459	459	462	456	456	456	456	456
Cheval Blanc.2005.B	664	623	623	656	656	723	723	723	723	731	731	700	700	700	700	724	724	565
Cos d'Estournel.1982.B	286	280	280	280	281	290	303	308	308	308	318	318	320	320	320	321	321	325
Cos d'Estournel.1985.B	150	150	150	150	150	150	150	151	151	151	151	151	151	151	151	151	151	152
Cos d'Estournel.1986.B	145	145	145	147	144	144	149	149	149	152	152	152	151	151	151	157	157	158
Cos d'Estournel.1988.B	102	102	102	111	111	111	111	111	111	111	111	111	119	119	119	120	120	120
Cos d'Estournel.1990.B	185	185	185	187	187	187	189	190	191	192	193	193	197	198	198	198	198	199
Cos d'Estournel.1995.B	156	156	156	163	163	163	163	163	163	166	167	167	168	169	171	172	172	172
Cos d'Estournel.1996.B	127	127	248	237	204	217	217	217	199	175	175	175	155	158	158	158	158	158
Cos d'Estournel.2000.B	125	164	164	160	156	264	251	251	251	236	217	224	198	194	194	182	182	184
Cos d'Estournel.2003.B	183	189	197	197	197	203	204	204	205	206	207	207	207	207	207	207	207	208
Cos d'Estournel.2005.B	193	184	184	184	184	206	206	206	206	208	207	207	207	207	205	205	206	206
Dom Perignon Moet & Chandon.1990.B	229	232	232	252	253	253	257	258	258	262	260	260	263	263	263	262	262	262
Dom Perignon Moet & Chandon.1996.B	267	262	262	323	317	317	316	316	316	309	309	309	309	309	309	311	311	309
Ducru Beaucaillou .1982.B	231	234	234	246	249	288	293	319	328	338	335	335	323	323	323	318	318	327
Ducru Beaucaillou .1990.B	105	105	105	105	105	105	108	113	117	119	121	121	124	124	124	128	128	128
Ducru Beaucaillou .1995.B	118	122	122	122	123	126	126	126	126	130	132	133	134	134	135	139	139	140
Ducru Beaucaillou .1996.B	141	143	143	144	144	149	152	154	162	162	162	163	165	165	165	165	165	169
Ducru Beaucaillou .2000.B	124	126	126	129	130	194	194	191	190	193	210	210	194	187	187	190	190	196
Echezeaux DRC.1990.B	958	958	958	958	958	958	1,047	1,292	1,292	1,292	1,334	1,423	1,546	1,546	1,546	1,765	1,920	1,924
Echezeaux DRC.1996.B	384	384	384	502	502	563	563	598	598	598	589	555	576	609	609	601	601	601
Echezeaux DRC.1999.B	469	459	459	555	589	632	614	632	614	623	623	631	631	631	631	614	614	614
Echezeaux DRC.2002.B	312	399	399	399	404	418	418	451	451	429	429	434	427	427	427	409	409	409
Echezeaux DRC.2005.B	189	439	439	485	485	485	485	536	591	566	566	566	546	546	546	589	589	581
Gruaud-Larose.1982.B	252	262	262	274	277	277	297	297	302	315	283	283	277	277	277	277	298	311
Gruaud-Larose.1986.B	152	152	152	163	163	163	164	164	164	164	165	165	163	163	163	164	165	167
Gruaud-Larose.2000.B	99	102	102	102	102	105	105	105	105	106	106	106	106	109	109	109	109	110
Haut-Brion.1966.B	256	256	256	256	260	260	260	283	286	286	287	287	288	288	288	288	288	288
Haut-Brion.1982.B	650	701	701	822	815	872	952	885	892	869	831	831	770	770	770	772	772	762
Haut-Brion.1985.B	386	385	385	387	387	387	392	392	395	395	396	399	401	401	401	401	401	400
Haut-Brion.1986.B	322	322	322	348	348	398	406	406	406	406	407	407	389	389	389	396	396	403
Haut-Brion.1988.B	262	262	262	272	272	272	272	272	272	292	292	346	363	363	363	363	363	358
Haut-Brion.1989.B	1,290	1,418	1,418	1,691	1,691	1,759	1,829	1,650	1,636	1,636	1,504	1,516	1,460	1,475	1,486	1,528	1,522	1,290
Haut-Brion.1990.B	554	832	1,205	936	936	911	931	885	885	844	771	771	673	703	703	707	703	703
Haut-Brion.1994.B	212	212	212	222	222	222	222	234	234	234	274	274	281	288	288	308	308	345
Haut-Brion.1995.B	324	356	356	383	383	406	474	477	477	473	471	511	479	465	465	425	425	451

Table 7.6: Valuation of wines through time

	05/2012	06/2012	09/2012	10/2012	11/2012	12/2012	02/2013	03/2013	04/2013	05/2013	06/2013	07/2013	09/2013	10/2013	11/2013	12/2013	N	Annualized Volatility
Angelus.2000.B	359	355	355	364	380	380	383	382	382	382	382	382	382	384	384	386	57	26%
Ausone.1998.B	753	744	744	781	781	781	765	765	764	759	761	761	761	745	745	745	52	0%
Ausone.2000.B	1,772	1,725	1,619	1,481	1,448	1,113	1,134	1,147	1,147	1,147	1,199	1,199	1,199	1,183	1,183	1,183	67	39%
Batard-Montrachet Leflaive.2002.B	268	263	263	270	265	269	277	277	277	277	277	281	281	281	281	281	67	14%
Calon-Segur.1982.B	172	172	170	170	171	179	179	179	179	179	179	179	179	178	178	178	63	0%
Calon-Segur.1995.B	112	103	102	102	102	103	103	104	104	104	102	102	102	94	94	94	65	24%
Calon-Segur.1996.B	87	86	89	89	92	91	91	91	92	92	92	92	92	95	96	96	59	15%
Calon-Segur.2000.B	141	120	120	118	116	116	119	119	107	107	104	104	111	111	103	103	71	55%
Carruades de Lafite Lafite-Rothschild.1996.B	578	578	578	578	495	455	455	426	426	426	365	365	365	365	365	365	65	72%
Carruades de Lafite Lafite-Rothschild.2000.B	447	447	417	371	323	323	323	340	343	322	326	326	326	326	326	326	92	81%
Cheval Blanc.1982.B	919	899	911	899	864	837	837	815	815	836	867	867	812	909	886	886	263	40%
Cheval Blanc.1985.B	453	451	447	447	447	447	447	444	444	441	439	439	439	439	439	439	66	0%
Cheval Blanc.1989.B	442	442	442	474	458	460	460	455	455	453	453	453	441	475	475	457	82	17%
Cheval Blanc.1990.B	1,186	1,183	1,175	1,169	1,152	1,140	1,139	1,138	1,138	1,138	1,117	1,117	1,115	1,172	1,165	1,165	168	0%
Cheval Blanc.1995.B	437	429	423	422	413	406	406	403	404	404	397	397	398	416	419	419	83	12%
Cheval Blanc.1996.B	363	367	368	368	371	371	371	371	371	371	367	367	374	381	381	381	73	22%
Cheval Blanc.1998.B	650	612	605	597	546	537	533	533	528	536	540	540	548	521	519	519	120	15%
Cheval Blanc.1999.B	391	394	394	401	380	381	381	382	382	387	387	387	387	419	419	419	54	23%
Cheval Blanc.2000.B	1,116	954	874	881	789	777	770	771	866	866	903	903	903	995	995	995	218	46%
Cheval Blanc.2003.B	372	364	367	363	362	360	360	360	360	360	366	366	369	370	370	370	82	15%
Cheval Blanc.2004.B	452	450	487	487	482	478	477	477	477	477	477	477	478	481	481	481	74	0%
Cheval Blanc.2005.B	654	640	632	632	585	527	525	530	530	550	536	536	543	532	520	520	66	24%
Cos d'Estournel.1982.B	323	324	323	331	336	338	342	341	345	347	349	350	350	349	350	339	131	8%
Cos d'Estournel.1985.B	151	151	151	150	150	151	151	150	150	150	151	151	151	151	151	151	54	0%
Cos d'Estournel.1986.B	161	161	161	163	161	165	165	165	165	167	170	170	172	172	172	172	75	8%
Cos d'Estournel.1988.B	120	120	120	120	120	121	121	121	121	121	125	125	125	125	125	125	58	20%
Cos d'Estournel.1990.B	199	200	200	201	202	202	202	203	203	203	201	201	201	201	201	201	68	0%
Cos d'Estournel.1995.B	173	172	172	175	175	175	175	177	177	177	177	177	177	177	177	177	76	6%
Cos d'Estournel.1996.B	158	163	153	169	154	156	159	159	156	155	158	154	147	145	145	153	133	39%
Cos d'Estournel.2000.B	184	186	161	193	186	172	172	172	172	157	169	169	169	165	165	165	106	42%
Cos d'Estournel.2003.B	207	209	208	209	208	207	206	206	206	206	206	206	206	204	204	203	83	0%
Cos d'Estournel.2005.B	206	200	196	199	195	195	193	192	192	192	192	192	189	189	188	188	67	6%
Dom Perignon Moet & Chandon.1990.B	262	262	261	264	267	268	269	269	269	272	272	272	277	287	286	286	124	8%
Dom Perignon Moet & Chandon.1996.B	312	312	314	313	311	312	312	316	320	323	323	324	320	315	319	322	109	3%
Ducru Beaucaillou .1982.B	327	330	330	331	351	350	350	343	357	357	350	350	350	354	351	351	89	20%
Ducru Beaucaillou .1990.B	127	128	129	129	129	129	129	129	129	128	128	128	128	128	128	128	62	12%
Ducru Beaucaillou .1995.B	133	133	134	135	135	137	137	139	144	144	144	144	144	144	147	147	80	11%
Ducru Beaucaillou .1996.B	166	166	168	166	171	172	172	173	172	172	175	175	177	177	181	181	104	11%
Ducru Beaucaillou .2000.B	198	197	197	197	190	190	190	191	188	188	184	184	186	186	185	184	134	25%
Echezeaux DRC.1990.B	2,021	2,139	1,649	1,649	1,649	1,835	1,835	1,835	1,835	1,733	1,733	1,653	1,653	1,933	1,963	1,859	70	44%
Echezeaux DRC.1996.B	488	517	449	482	482	482	482	482	482	482	482	445	554	645	740	700	65	56%
Echezeaux DRC.1999.B	614	627	644	656	656	678	669	657	657	657	657	657	679	646	646	650	57	0%
Echezeaux DRC.2002.B	435	435	435	487	487	559	538	538	542	542	542	542	595	657	657	689	52	44%
Echezeaux DRC.2005.B	615	540	635	697	697	755	755	871	851	699	730	730	786	939	1,001	986	53	54%
Gruaud-Larose.1982.B	328	314	327	327	326	343	343	339	347	342	334	334	334	303	308	301	121	24%
Gruaud-Larose.1986.B	166	164	163	163	167	167	167	167	167	167	167	167	169	172	167	167	63	8%
Gruaud-Larose.2000.B	110	110	109	109	111	112	112	114	114	114	117	117	117	118	120	119	55	8%
Haut-Brion.1966.B	282	281	281	276	273	255	255	260	263	263	263	263	263	261	262	262	54	0%
Haut-Brion.1982.B	739	675	665	707	705	715	728	702	731	759	746	746	744	859	846	846	200	23%
Haut-Brion.1985.B	400	401	400	400	401	400	400	401	407	412	410	411	411	419	420	420	85	4%
Haut-Brion.1986.B	394	379	371	387	380	378	378	378	398	424	424	424	416	447	445	445	83	24%
Haut-Brion.1988.B	353	362	362	375	373	363	362	363	378	379	374	374	374	390	398	398	76	29%
Haut-Brion.1989.B	1,205	1,210	1,201	1,296	1,259	1,254	1,280	1,307	1,299	1,313	1,337	1,337	1,331	1,884	1,684	1,684	364	32%
Haut-Brion.1990.B	605	596	647	672	673	557	573	604	631	631	564	564	622	726	727	727	188	44%
Haut-Brion.1994.B	345	353	351	351	347	347	347	348	348	347	347	347	347	360	360	360	75	28%
Haut-Brion.1995.B	399	403	387	409	404	444	432	420	418	418	403	401	416	500	489	489	223	33%

Table 7.7: Valuation of wines through time

Wine	02/2007	03/2007	04/2007	05/2007	06/2007	07/2007	09/2007	10/2007	11/2007	12/2007	02/2008	03/2008	04/2008	05/2008	06/2008	07/2008
Haut-Brion.1996.B				332	332	332	332	332	295	295	307	347	347	348	337	337
Haut-Brion.1997.B				219	219	221	221	221	218	216	216	216	222	217	217	217
Haut-Brion.1998.B	377	261	166	191	302	302	302	364	358	408	408	450	450	417	417	417
Haut-Brion.1999.B				211	211	211	211	215	215	213	213	213	213	243	243	243
Haut-Brion.2000.B		517	517	556	556	556	556	556	556	647	665	796	796	805	805	805
Haut-Brion.2001.B				175	175	175	175	175	175	194	194	194	194	194	194	194
Haut-Brion.2002.B				138	138	138	138	138	138	138	170	170	170	170	170	170
Haut-Brion.2003.B				295	295	295	308	308	308	308	308	332	332	332	332	327
Haut-Brion.2005.B																
Hermitage La Chapelle Jaboulet.1978.B	818	818	818	976	1,011	1,011	1,082	2,092	2,194	2,117	2,117	2,117	2,117	2,117	2,117	2,117
Hermitage La Chapelle Jaboulet.1989.B		257	257	268	268	268	241	261	261	254	254	250	250	250	256	256
Hermitage La Chapelle Jaboulet.1990.B		460	460	581	581	581	576	573	573	595	595	595	597	597	590	599
La Conseillante.2000.B							232	232	232	232	232	240	240	273	273	273
La Mission Haut-Brion.1982.B							806	848	882	924	924	924	924	924	924	929
La Mission Haut-Brion.1989.B			678	825	879	879	879	879	909	961	1,004	1,004	1,004	1,004	1,010	1,010
La Mission Haut-Brion.1990.B								618	618	570	570	570	570	570	570	570
La Mission Haut-Brion.1995.B			144	146	146	151	151	155	151	151	151	151	151	151	151	158
La Mission Haut-Brion.1996.B				158	158	153	153	145	143	137	137	137	137	137	137	137
La Mission Haut-Brion.1998.B					195	189	189	191	165	165	165	165	165	165	165	165
La Mission Haut-Brion.2000.B		764	764	764	764	764	733	733	726	802	802	811	811	806	806	806
La Mission Haut-Brion.2005.B																
La Tache DRC.1990.B		5,268	5,268	4,952	5,099	4,986	4,986	4,945	5,185	5,091	5,081	5,081	5,081	5,272	5,273	5,273
La Tache DRC.1996.B				1,848	1,905	1,905	1,923	1,923	1,923	1,923	1,923	1,923	1,923	1,923	1,923	1,923
La Tache DRC.2000.B				1,545	1,545	1,545	1,536	1,495	1,495	1,461	1,461	1,461	1,461	1,501	1,501	1,501
Lafite-Rothschild.1961.B		1,491	928	1,046	1,046	1,046	1,046	1,046	1,100	1,100	1,100	1,100	1,100	1,103	1,103	1,131
Lafite-Rothschild.1966.B			211	238	238	238	239	239	239	236	236	236	236	250	250	279
Lafite-Rothschild.1970.B		216	167	167	167	167	167	167	167	176	190	191	195	195	204	217
Lafite-Rothschild.1975.B		243	243	243	243	243	271	271	259	262	262	237	237	237	274	274
Lafite-Rothschild.1978.B			169	199	212	212	212	212	218	218	218	218	225	225	210	238
Lafite-Rothschild.1981.B	184	184	184	200	200	200	200	200	200	200	200	200	200	317	323	323
Lafite-Rothschild.1982.B	471	471	873	1,226	1,226	1,479	1,552	1,816	2,113	2,097	2,210	2,356	2,369	2,648	2,911	2,924
Lafite-Rothschild.1983.B	782	454	454	455	455	455	455	455	452	435	435	435	435	449	457	490
Lafite-Rothschild.1985.B		370	335	367	389	389	389	389	382	399	399	399	406	458	475	486
Lafite-Rothschild.1986.B	146	146	438	805	877	877	930	1,059	1,136	1,164	1,164	1,161	1,161	1,134	1,238	1,283
Lafite-Rothschild.1988.B		409	409	409	427	427	427	427	472	489	450	457	457	475	475	475
Lafite-Rothschild.1989.B				439	468	519	519	497	486	481	481	502	502	488	520	520
Lafite-Rothschild.1990.B		507	507	636	636	636	591	591	549	567	585	628	628	613	671	671
Lafite-Rothschild.1993.B				293	293	340	340	340	354	364	364	364	364	368	368	368
Lafite-Rothschild.1994.B	256	252	252	252	252	252	252	252	307	307	307	307	324	360	391	391
Lafite-Rothschild.1995.B	418	418	386	410	410	419	419	462	464	471	471	471	471	544	557	557
Lafite-Rothschild.1996.B	762	708	708	730	730	766	766	933	931	1,177	1,177	1,177	1,177	1,129	1,234	1,218
Lafite-Rothschild.1997.B		251	251	251	251	295	295	317	366	378	378	378	378	440	567	567
Lafite-Rothschild.1998.B	415	424	424	424	507	507	514	547	547	579	579	579	579	622	614	614
Lafite-Rothschild.1999.B	242	242	242	242	242	242	242	242	280	345	357	357	347	379	394	403
Lafite-Rothschild.2000.B	863	851	851	1,268	1,268	1,268	1,316	1,124	1,229	1,307	1,366	1,493	1,493	1,599	1,601	1,601
Lafite-Rothschild.2001.B	231	231	231	231	231	231	231	312	312	345	345	345	345	345	345	345
Lafite-Rothschild.2002.B								371	371	352	352	352	352	371	386	394
Lafite-Rothschild.2003.B	602	629	629	694	694	694	776	776	1,680	1,311	1,264	1,176	1,176	1,039	1,069	1,069
Lafite-Rothschild.2004.B																
Lafite-Rothschild.2005.B																
Lafite-Rothschild.2006.B																
Lafite-Rothschild.2007.B																
Lafite-Rothschild.2008.B																
Lascombes.2000.B	47	51	53	53	53	53	54	54	54	55	55	54	54	54	54	54
Latour.1959.B	1,182	1,343	1,437	1,770	1,770	1,770	1,864	2,097	2,398	2,421	2,421	2,402	2,402	2,433	2,406	2,435
Latour.1961.B	3,096	3,013	2,274	2,412	2,462	2,462	2,492	2,600	2,829	2,829	2,719	2,706	2,706	2,706	2,689	2,756
Latour.1966.B	414	524	512	507	507	507	530	593	581	580	568	573	573	588	578	578

Table 7.8: Valuation of wines through time

	08/2008	09/2008	10/2008	11/2008	03/2009	04/2009	05/2009	06/2009	09/2009	10/2009	11/2009	12/2009	02/2010	03/2010	04/2010	05/2010
Haut-Brion.1996.B	352	352	277	263	242	240	238	238	238	238	256	259	271	271	271	290
Haut-Brion.1997.B	217	217	200	170	167	167	180	180	180	182	182	182	182	182	182	182
Haut-Brion.1998.B	397	397	380	299	288	275	279	284	302	310	317	319	319	319	319	354
Haut-Brion.1999.B	243	243	230	228	228	228	222	222	222	222	222	229	229	229	229	229
Haut-Brion.2000.B	777	816	789	675	675	581	610	582	585	580	580	598	598	598	598	598
Haut-Brion.2001.B	194	194	194	217	223	223	227	227	232	232	232	239	239	239	239	239
Haut-Brion.2002.B	170	170	170	170	158	158	168	168	173	173	173	185	185	185	185	185
Haut-Brion.2003.B	327	327	327	288	260	259	261	261	274	274	289	307	316	316	316	316
Haut-Brion.2005.B					564	575	587	587	587	587	634	634	634	634	634	634
Hermitage La Chapelle Jaboulet.1978.B	2,117	2,117	2,038	1,935	1,935	1,935	1,935	1,935	1,935	1,935	1,990	1,990	1,990	1,990	1,990	1,990
Hermitage La Chapelle Jaboulet.1989.B	256	256	256	249	241	241	241	241	241	229	229	229	228	228	228	228
Hermitage La Chapelle Jaboulet.1990.B	597	597	606	590	585	585	553	553	553	553	570	570	570	570	570	570
La Conseillante.2000.B	273	273	266	254	252	245	245	245	245	245	245	245	245	245	245	245
La Mission Haut-Brion.1982.B	922	922	922	856	856	856	837	809	808	812	815	815	815	815	815	853
La Mission Haut-Brion.1989.B	992	992	992	956	956	956	950	950	950	877	881	881	881	881	881	881
La Mission Haut-Brion.1990.B	570	570	570	525	525	525	460	460	460	460	460	460	460	460	460	460
La Mission Haut-Brion.1995.B	158	158	158	157	157	156	156	156	156	156	156	156	156	160	160	167
La Mission Haut-Brion.1996.B	137	137	133	133	130	130	131	131	131	131	131	127	127	127	127	127
La Mission Haut-Brion.1998.B	165	165	165	165	163	163	175	176	177	177	231	231	231	231	231	231
La Mission Haut-Brion.2000.B	806	854	854	827	827	666	676	644	637	645	637	646	646	646	646	643
La Mission Haut-Brion.2005.B					451	451	451	451	451	451	451	451	451	451	451	451
La Tache DRC.1990.B	5,273	5,273	5,273	5,273	4,830	4,830	4,830	4,830	4,724	4,724	4,565	5,092	5,044	5,044	5,044	4,976
La Tache DRC.1996.B	1,923	1,923	1,923	1,923	1,922	1,922	1,866	2,039	2,039	2,039	1,989	1,989	1,989	1,989	1,989	2,102
La Tache DRC.2000.B	1,459	1,459	1,459	1,459	1,446	1,402	1,362	1,362	1,362	1,362	1,377	1,377	1,377	1,377	1,377	1,377
Lafite-Rothschild.1961.B	1,131	1,131	1,113	1,073	1,050	1,050	1,050	1,050	1,050	1,050	1,112	1,112	1,112	1,126	1,126	1,126
Lafite-Rothschild.1966.B	285	285	285	289	285	285	285	285	293	357	357	357	397	397	397	437
Lafite-Rothschild.1970.B	217	217	191	185	195	195	188	188	188	197	231	231	231	231	231	231
Lafite-Rothschild.1975.B	274	274	273	273	273	273	276	276	276	297	347	347	386	386	386	419
Lafite-Rothschild.1978.B	246	246	246	210	210	210	209	208	205	261	277	277	349	349	349	379
Lafite-Rothschild.1981.B	323	323	302	297	299	299	305	305	305	305	343	343	381	438	438	438
Lafite-Rothschild.1982.B	2,897	2,897	2,689	2,336	2,266	2,400	2,602	2,749	2,758	2,442	2,965	2,932	3,036	3,015	3,015	3,262
Lafite-Rothschild.1983.B	490	490	461	484	483	483	474	474	474	474	511	511	511	538	538	538
Lafite-Rothschild.1985.B	486	486	486	437	429	429	427	427	438	447	477	477	477	477	477	477
Lafite-Rothschild.1986.B	1,247	1,247	1,247	957	957	966	845	845	870	928	1,104	1,134	1,134	1,134	1,134	1,134
Lafite-Rothschild.1988.B	475	475	475	454	429	429	433	433	465	478	514	514	511	511	533	533
Lafite-Rothschild.1989.B	501	501	501	411	389	384	402	402	430	455	589	589	589	589	589	589
Lafite-Rothschild.1990.B	432	432	432	434	432	445	476	476	593	596	698	698	698	759	759	844
Lafite-Rothschild.1993.B	368	368	318	310	310	364	365	365	396	396	396	456	456	456	456	456
Lafite-Rothschild.1994.B	391	391	391	391	368	376	376	376	396	422	440	511	511	511	511	511
Lafite-Rothschild.1995.B	557	557	522	442	409	430	452	452	464	492	546	546	580	580	580	683
Lafite-Rothschild.1996.B	1,174	1,174	933	748	676	743	926	923	923	923	1,011	1,065	1,166	1,166	1,166	1,406
Lafite-Rothschild.1997.B	567	567	521	521	465	476	476	476	521	552	552	552	552	552	552	661
Lafite-Rothschild.1998.B	614	614	552	449	450	464	568	546	570	570	644	656	656	656	656	824
Lafite-Rothschild.1999.B	403	403	403	386	386	386	444	444	444	444	482	501	501	501	501	501
Lafite-Rothschild.2000.B	1,601	1,680	1,664	1,276	1,276	1,248	1,408	1,452	1,467	1,486	1,713	1,736	1,736	1,806	1,806	1,877
Lafite-Rothschild.2001.B	345	345	345	316	300	300	337	337	356	356	380	412	412	412	412	412
Lafite-Rothschild.2002.B	394	394	394	303	296	296	365	365	365	390	426	457	457	502	502	502
Lafite-Rothschild.2003.B	1,069	1,069	1,021	831	831	747	799	799	827	827	892	892	892	892	892	1,055
Lafite-Rothschild.2004.B			214	301	301	301	301	301	301	301	363	363	363	363	363	363
Lafite-Rothschild.2005.B				932	932	932	858	858	879	879	931	942	942	942	942	1,418
Lafite-Rothschild.2006.B											589	589	589	589	589	577
Lafite-Rothschild.2007.B																641
Lafite-Rothschild.2008.B																
Lascombes.2000.B	54	64	63	63	63	63	63	63	63	63	63	63	63	63	63	63
Latour.1959.B	2,435	2,435	2,435	2,484	2,484	2,442	2,452	2,452	2,452	2,482	2,517	2,517	2,517	2,517	2,517	2,565
Latour.1961.B	2,756	2,756	2,726	3,752	3,623	3,623	3,623	3,623	3,542	3,540	3,635	3,635	3,635	3,635	3,635	3,635
Latour.1966.B	571	571	586	579	579	579	579	579	566	566	566	566	566	560	560	560

Table 7.9: Valuation of wines through time

	06/2010	09/2010	10/2010	11/2010	12/2010	01/2011	03/2011	04/2011	05/2011	06/2011	09/2011	10/2011	11/2011	12/2011	01/2012	02/2012	03/2012	04/2012
Haut-Brion.1996.B	293	293	293	346	353	353	385	397	432	442	447	447	420	407	407	407	407	424
Haut-Brion.1997.B	182	190	190	190	190	190	190	222	222	242	252	252	252	257	257	275	275	340
Haut-Brion.1998.B	355	383	560	544	568	578	578	530	530	525	520	518	503	486	478	463	463	459
Haut-Brion.1999.B	233	233	233	245	245	245	245	245	263	263	290	290	302	299	299	316	326	348
Haut-Brion.2000.B	582	714	763	874	850	949	938	928	928	926	908	832	792	763	763	765	765	757
Haut-Brion.2001.B	324	342	342	356	356	356	356	347	398	412	426	429	436	451	451	451	448	457
Haut-Brion.2002.B	190	248	248	264	264	264	264	278	327	351	351	351	366	342	342	342	346	349
Haut-Brion.2003.B	303	512	530	512	464	464	464	464	454	462	453	453	424	403	403	403	400	408
Haut-Brion.2005.B	634	930	930	972	972	903	904	870	870	847	828	823	821	735	735	735	735	643
Hermitage La Chapelle Jaboulet.1978.B	1,990	1,885	1,885	1,882	1,882	1,882	1,882	1,882	1,882	1,882	1,758	1,758	1,947	1,947	1,947	1,947	1,947	1,947
Hermitage La Chapelle Jaboulet.1989.B	227	227	227	226	226	226	226	226	226	226	224	224	232	232	232	232	232	232
Hermitage La Chapelle Jaboulet.1990.B	570	570	570	570	566	566	569	569	569	569	557	557	572	571	571	572	572	569
La Conseillante.2000.B	244	245	245	249	251	251	251	252	252	255	255	255	283	284	284	285	285	285
La Mission Haut-Brion.1982.B	864	894	894	989	1,011	1,011	1,062	1,062	1,062	1,062	1,063	1,069	1,059	1,059	1,050	1,076	1,076	1,081
La Mission Haut-Brion.1989.B	881	907	907	925	925	951	951	951	954	954	969	985	999	1,000	1,000	1,000	1,005	1,001
La Mission Haut-Brion.1990.B	430	435	435	447	447	465	465	475	473	473	470	467	467	468	468	468	468	468
La Mission Haut-Brion.1995.B	163	171	171	187	190	190	190	190	194	194	209	209	216	214	214	214	214	214
La Mission Haut-Brion.1996.B	127	127	127	127	127	127	127	127	137	149	159	172	172	178	178	178	178	178
La Mission Haut-Brion.1998.B	214	215	215	215	214	214	214	222	222	222	222	222	229	235	235	243	243	243
La Mission Haut-Brion.2000.B	643	751	751	752	747	747	770	770	770	818	818	751	767	767	767	765	765	679
La Mission Haut-Brion.2005.B	451	575	575	684	684	673	673	673	673	650	650	630	518	518	503	497	497	489
La Tache DRC.1990.B	4,976	4,965	4,965	5,088	5,088	5,148	5,148	5,068	5,068	5,068	5,041	5,041	5,089	5,089	5,089	5,057	5,057	5,057
La Tache DRC.1996.B	2,060	2,060	2,060	2,263	2,263	2,347	2,347	2,416	2,416	2,416	2,438	2,438	2,443	2,443	2,443	2,443	2,594	2,623
La Tache DRC.2000.B	1,377	1,370	1,370	1,758	1,758	1,774	1,774	1,785	1,785	1,793	1,795	1,799	1,798	1,798	1,796	1,796	1,796	1,811
Lafite-Rothschild.1961.B	1,158	1,187	1,316	1,360	1,360	1,360	1,364	1,434	1,477	1,489	1,489	1,489	1,589	1,589	1,589	1,589	1,589	1,612
Lafite-Rothschild.1966.B	461	517	615	643	713	713	713	756	784	850	850	886	897	897	897	910	910	910
Lafite-Rothschild.1970.B	261	261	261	289	289	289	289	391	461	515	562	610	590	590	590	574	587	582
Lafite-Rothschild.1975.B	419	444	444	489	489	489	489	642	642	642	642	642	660	660	660	660	660	660
Lafite-Rothschild.1978.B	393	490	490	519	519	519	519	576	655	655	658	658	637	637	637	638	577	577
Lafite-Rothschild.1981.B	438	598	598	755	755	755	783	783	872	900	810	810	745	745	745	745	745	745
Lafite-Rothschild.1982.B	3,355	3,403	4,164	5,734	5,525	5,491	5,122	5,179	4,855	4,770	4,339	4,215	4,050	3,956	3,967	3,882	3,882	3,973
Lafite-Rothschild.1983.B	551	581	581	620	662	662	744	801	883	903	895	895	827	795	795	795	786	786
Lafite-Rothschild.1985.B	477	520	714	798	798	825	879	913	966	956	956	972	981	981	981	949	949	949
Lafite-Rothschild.1986.B	1,249	1,429	1,848	2,580	2,428	2,445	2,392	2,310	2,190	2,195	1,913	2,088	1,918	1,835	1,786	1,614	1,586	1,459
Lafite-Rothschild.1988.B	552	666	840	918	918	918	918	918	967	993	1,000	985	961	961	961	967	966	977
Lafite-Rothschild.1989.B	651	716	910	1,258	1,193	1,199	1,161	1,116	1,079	1,114	973	991	949	900	900	894	894	932
Lafite-Rothschild.1990.B	862	1,433	1,957	1,437	1,437	1,491	1,460	1,275	1,299	1,177	993	1,082	978	931	931	907	895	882
Lafite-Rothschild.1993.B	456	510	510	737	737	737	737	737	814	869	801	882	847	847	847	829	832	832
Lafite-Rothschild.1994.B	539	539	620	782	735	770	770	805	805	805	805	921	900	900	900	835	835	840
Lafite-Rothschild.1995.B	732	1,026	1,684	1,477	1,357	1,441	1,344	1,312	1,227	1,188	1,131	1,075	951	972	939	885	885	896
Lafite-Rothschild.1996.B	1,437	1,895	3,114	2,284	2,173	2,065	2,044	1,948	1,750	1,605	1,530	1,497	1,463	1,376	1,352	1,269	1,132	1,259
Lafite-Rothschild.1997.B	699	747	1,134	1,174	1,174	1,174	1,174	1,174	1,239	1,276	1,218	1,178	1,086	1,038	1,044	992	992	976
Lafite-Rothschild.1998.B	792	1,135	1,523	1,290	1,231	1,357	1,357	1,273	1,249	1,255	1,094	1,092	919	901	901	973	873	878
Lafite-Rothschild.1999.B	634	719	1,306	1,162	1,162	1,162	1,162	1,155	1,164	1,175	1,115	1,066	911	922	922	922	922	877
Lafite-Rothschild.2000.B	1,936	2,576	3,746	3,444	3,377	3,417	3,417	3,417	3,079	3,092	2,620	2,710	2,400	2,376	2,357	2,232	2,232	2,199
Lafite-Rothschild.2001.B	585	678	1,113	1,030	1,022	1,118	1,118	1,112	1,080	1,077	1,044	1,004	944	886	886	886	886	860
Lafite-Rothschild.2002.B	575	671	1,338	1,105	1,105	1,105	1,108	1,108	1,085	1,082	1,024	987	932	881	874	785	785	825
Lafite-Rothschild.2003.B	1,091	1,474	2,649	2,129	1,914	1,727	1,817	1,817	1,796	1,751	1,617	1,536	1,414	1,281	1,281	1,188	1,145	1,018
Lafite-Rothschild.2004.B	490	577	1,615	1,221	1,221	1,220	1,220	1,220	1,220	1,172	1,119	965	889	901	901	868	868	884
Lafite-Rothschild.2005.B	1,418	1,423	2,753	2,464	2,464	2,005	2,006	2,006	2,006	1,782	1,585	1,260	1,318	1,222	1,222	1,222	1,145	1,277
Lafite-Rothschild.2006.B	577	919	1,359	1,293	1,293	1,293	1,282	1,282	1,282	1,132	1,132	1,006	1,006	927	927	927	927	871
Lafite-Rothschild.2007.B	641	641	1,428	1,428	1,428	1,428	1,428	1,428	1,428	1,428	1,347	1,152	1,116	1,043	1,043	1,043	891	884
Lafite-Rothschild.2008.B			2,428	2,428	2,428	2,428	2,428	2,428	2,223	2,223	2,223	1,903	1,701	1,701	1,701	1,490	1,490	1,490
Lascombes.2000.B	63	66	66	69	69	69	69	71	71	71	71	71	71	71	71	71	71	71
Latour.1959.B	2,498	2,498	2,498	2,498	2,515	2,557	2,580	2,589	2,767	2,767	2,767	2,767	2,769	2,769	2,769	2,769	2,769	2,752
Latour.1961.B	3,606	3,548	3,548	3,824	3,824	4,007	4,248	4,260	5,894	5,894	5,894	5,745	5,077	5,077	5,077	5,077	5,077	4,757
Latour.1966.B	560	572	572	572	572	572	572	636	643	642	648	648	644	644	644	644	645	645

Table 7.10: Valuation of wines through time

	05/2012	06/2012	09/2012	10/2012	11/2012	12/2012	02/2013	03/2013	04/2013	05/2013	06/2013	07/2013	09/2013	10/2013	11/2013	12/2013	N	Annualized Volatility
Haut-Brion.1996.B	372	361	376	368	378	373	385	389	389	384	361	361	374	408	408	399	188	29%
Haut-Brion.1997.B	330	330	330	325	335	335	335	334	334	334	334	334	334	366	366	366	59	41%
Haut-Brion.1998.B	459	452	471	461	427	423	426	426	420	420	415	415	413	453	402	402	184	42%
Haut-Brion.1999.B	362	360	360	360	363	363	364	371	371	371	371	371	371	392	391	391	71	28%
Haut-Brion.2000.B	748	694	690	683	662	633	638	656	676	676	681	678	675	761	745	745	246	27%
Haut-Brion.2001.B	457	457	463	456	446	428	432	431	434	423	420	420	420	420	420	420	58	34%
Haut-Brion.2002.B	349	340	340	343	342	353	353	347	347	347	341	341	341	353	350	350	62	39%
Haut-Brion.2003.B	413	417	386	385	353	372	372	374	374	374	374	374	374	368	369	369	141	35%
Haut-Brion.2005.B	643	653	634	644	613	580	585	578	586	590	610	610	612	633	633	633	94	29%
Hermitage La Chapelle Jaboulet.1978.B	1,885	1,885	1,885	1,885	1,784	1,776	1,776	1,776	1,786	1,786	1,759	1,759	1,864	1,730	1,624	1,586	59	25%
Hermitage La Chapelle Jaboulet.1989.B	223	223	222	222	226	233	233	233	233	230	230	229	235	237	234	234	60	0%
Hermitage La Chapelle Jaboulet.1990.B	565	565	555	557	556	565	565	565	565	564	564	566	577	584	586	586	80	0%
La Conseillante.2000.B	285	285	285	285	286	286	286	283	285	285	285	285	285	285	286	286	68	5%
La Mission Haut-Brion.1982.B	1,025	1,010	1,026	1,039	1,011	1,007	1,010	996	996	991	991	991	999	1,112	1,095	1,095	89	12%
La Mission Haut-Brion.1989.B	1,001	992	990	988	986	986	988	976	977	977	977	977	978	1,030	1,027	1,027	115	4%
La Mission Haut-Brion.1990.B	474	466	475	475	472	472	475	469	469	472	470	470	470	479	482	482	53	0%
La Mission Haut-Brion.1995.B	209	216	216	216	216	216	216	225	229	229	229	229	238	278	278	278	60	28%
La Mission Haut-Brion.1996.B	164	171	171	171	185	185	185	192	192	192	192	192	192	214	217	215	51	29%
La Mission Haut-Brion.1998.B	243	251	278	281	270	283	291	304	304	304	317	317	324	364	364	364	85	33%
La Mission Haut-Brion.2000.B	651	639	670	660	594	570	564	592	547	547	547	547	548	702	680	680	133	24%
La Mission Haut-Brion.2005.B	489	469	469	451	440	437	454	454	454	454	438	438	432	453	446	446	73	34%
La Tache DRC.1990.B	5,092	5,078	5,078	5,074	5,100	5,132	5,132	5,135	5,174	5,170	5,163	5,163	5,151	5,214	5,215	5,215	66	0%
La Tache DRC.1996.B	2,639	2,596	2,950	3,015	3,386	3,348	3,348	3,348	3,348	3,315	3,332	3,332	3,282	3,480	3,477	3,477	58	27%
La Tache DRC.2000.B	1,833	1,824	1,861	1,909	1,917	1,923	1,923	1,985	2,028	2,028	2,040	2,040	2,040	2,037	2,037	2,037	55	12%
Lafite-Rothschild.1961.B	1,613	1,613	1,613	1,613	1,558	1,608	1,608	1,589	1,589	1,589	1,620	1,620	1,620	1,620	1,572	1,572	50	22%
Lafite-Rothschild.1966.B	861	861	861	861	861	813	813	813	813	813	778	778	797	782	782	782	54	59%
Lafite-Rothschild.1970.B	582	563	584	559	559	505	505	505	505	505	505	505	505	428	428	428	65	65%
Lafite-Rothschild.1975.B	632	567	567	572	547	494	494	501	501	501	497	490	490	444	444	444	59	48%
Lafite-Rothschild.1978.B	532	528	516	516	501	462	462	462	462	462	462	462	462	381	381	381	63	56%
Lafite-Rothschild.1981.B	707	707	693	724	688	637	637	637	637	638	632	632	632	506	506	506	62	60%
Lafite-Rothschild.1982.B	3,646	3,679	3,315	3,416	3,183	3,196	3,204	3,189	3,427	3,425	3,835	3,835	3,852	3,395	3,395	3,395	398	35%
Lafite-Rothschild.1983.B	786	765	765	731	625	583	583	584	584	538	549	560	560	469	457	457	103	36%
Lafite-Rothschild.1985.B	946	868	861	861	817	806	806	806	806	813	813	800	800	761	761	761	76	41%
Lafite-Rothschild.1986.B	1,369	1,297	1,297	1,292	1,127	1,168	1,180	1,180	1,486	1,393	1,509	1,368	1,333	1,203	1,192	1,192	297	49%
Lafite-Rothschild.1988.B	930	930	912	868	801	794	796	777	775	788	766	749	749	723	723	723	105	37%
Lafite-Rothschild.1989.B	814	770	761	728	675	664	664	666	666	574	611	618	618	639	691	691	167	48%
Lafite-Rothschild.1990.B	852	803	775	808	787	762	762	785	800	814	691	691	755	801	796	796	288	55%
Lafite-Rothschild.1993.B	832	832	817	802	802	802	802	802	802	785	780	780	780	780	780	780	55	56%
Lafite-Rothschild.1994.B	824	797	797	789	747	725	725	725	735	731	731	731	734	734	734	734	77	45%
Lafite-Rothschild.1995.B	864	808	799	798	766	752	752	752	1,072	1,072	1,072	1,072	945	752	731	731	246	55%
Lafite-Rothschild.1996.B	1,127	1,131	1,024	1,110	1,027	1,087	1,077	1,012	1,012	1,012	1,200	1,156	1,323	1,289	1,247	1,247	355	56%
Lafite-Rothschild.1997.B	963	895	895	891	887	868	868	868	868	868	868	868	868	868	868	868	71	56%
Lafite-Rothschild.1998.B	866	821	817	847	767	765	772	775	775	768	768	768	782	840	840	840	270	45%
Lafite-Rothschild.1999.B	870	790	780	776	735	755	755	755	766	756	758	758	758	687	687	687	113	56%
Lafite-Rothschild.2000.B	2,132	2,012	1,904	1,917	1,818	1,774	1,741	1,930	1,936	1,936	2,079	2,079	2,050	1,892	1,861	1,861	302	43%
Lafite-Rothschild.2001.B	821	762	771	789	692	716	700	700	700	641	673	673	673	677	677	677	121	57%
Lafite-Rothschild.2002.B	745	726	702	705	685	751	739	739	783	766	736	726	723	765	784	784	154	53%
Lafite-Rothschild.2003.B	1,018	984	963	986	921	991	1,004	1,001	1,025	1,038	1,044	1,044	981	1,095	1,044	997	258	62%
Lafite-Rothschild.2004.B	853	761	773	733	686	714	712	712	712	712	738	707	707	725	727	727	80	85%
Lafite-Rothschild.2005.B	1,183	1,091	1,028	998	1,025	1,113	1,041	1,016	1,016	1,016	1,079	1,079	990	962	943	943	156	58%
Lafite-Rothschild.2006.B	871	869	869	767	706	728	726	726	726	726	758	731	726	731	744	744	88	52%
Lafite-Rothschild.2007.B	866	826	757	691	666	660	659	659	659	659	659	630	637	674	674	674	55	36%
Lafite-Rothschild.2008.B	1,314	1,240	1,118	991	938	938	938	918	968	873	859	834	813	744	732	732	52	44%
Lascombes.2000.B	71	70	70	70	89	90	90	91	93	94	95	95	97	97	97	102	55	35%
Latour.1959.B	2,712	2,712	2,683	2,649	2,668	2,657	2,657	2,657	2,676	2,692	2,708	2,708	2,708	2,669	2,669	2,669	76	0%
Latour.1961.B	3,776	3,728	3,728	3,728	3,589	3,367	3,367	3,530	3,646	3,555	3,565	3,579	3,579	3,579	3,550	3,550	74	35%
Latour.1966.B	629	629	630	630	630	637	637	641	635	635	635	635	635	646	647	647	69	0%

Table 7.11: Valuation of wines through time

Wine	02/2007	03/2007	04/2007	05/2007	06/2007	07/2007	09/2007	10/2007	11/2007	12/2007	02/2008	03/2008	04/2008	05/2008	06/2008	07/2008
Latour.1970.B	711	604	483	500	505	505	523	523	523	511	511	511	511	522	523	524
Latour.1975.B	162	162	162	162	162	162	179	179	179	171	171	171	177	177	177	184
Latour.1982.B	1,696	1,759	1,545	1,657	1,653	1,653	1,713	1,718	1,744	1,796	1,796	1,833	1,833	1,624	1,665	1,471
Latour.1983.B			199	305	303	303	303	308	308	323	323	323	323	326	323	323
Latour.1985.B	373	373	373	394	405	405	405	405	419	411	399	399	399	394	387	387
Latour.1986.B	509	509	398	398	426	426	426	432	432	430	430	445	445	452	515	515
Latour.1988.B				333	348	348	356	347	357	366	366	352	352	353	353	353
Latour.1989.B				340	356	381	389	395	358	364	364	386	386	365	384	384
Latour.1990.B		212	486	531	566	601	618	680	784	804	730	730	730	767	698	638
Latour.1993.B				240	240	248	248	248	246	246	246	247	230	257	268	268
Latour.1994.B		248	213	247	257	257	259	259	255	263	263	263	263	271	275	275
Latour.1995.B		358	372	420	428	473	473	486	473	490	508	508	508	493	489	489
Latour.1996.B	598	639	639	512	629	629	629	729	760	822	843	792	792	813	827	827
Latour.1997.B				237	243	253	253	270	239	248	248	248	248	272	293	293
Latour.1998.B	209	221	221	221	254	257	259	270	230	250	250	250	262	279	285	285
Latour.1999.B	235	202	202	202	202	226	226	226	226	255	268	268	268	332	332	332
Latour.2000.B	1,078	978	978	978	978	978	978	1,183	1,183	1,276	1,276	1,150	996	1,099	1,099	1,099
Latour.2001.B	91	91	91	180	180	180	180	180	180	209	209	209	209	209	209	209
Latour.2002.B				303	303	303	303	342	342	357	357	357	357	357	342	342
Latour.2003.B	705	705	705	864	864	864	864	864	914	950	950	986	986	1,011	1,011	1,011
Latour.2004.B																
Latour.2005.B																
Leoville Poyferre.2000.B										132	128	128	128	128	128	128
Leoville Las Cases .1982.B	591	406	386	408	408	408	417	466	506	541	540	530	530	541	544	546
Leoville Las Cases .1986.B				527	343	343	343	347	347	379	384	384	384	384	339	345
Leoville Las Cases .1989.B			163	163	163	163	155	155	155	156	156	156	163	163	160	161
Leoville Las Cases .1990.B		427	355	355	355	355	355	355	355	343	343	311	301	309	307	307
Leoville Las Cases .1995.B	161	161	161	154	154	154	154	154	158	158	159	159	159	159	167	167
Leoville Las Cases .1996.B	281	204	204	229	229	229	234	234	234	249	249	257	257	257	263	263
Leoville Las Cases .2000.B	327	228	228	258	265	265	271	271	271	299	299	312	312	324	324	324
L'Evangile.1990.B			358	370	370	370	364	357	357	344	334	334	334	339	337	337
Margaux.1982.B			791	958	797	797	847	882	893	966	973	983	983	992	974	1,008
Margaux.1983.B		597	467	499	499	499	449	465	465	540	543	546	546	549	543	557
Margaux.1985.B		499	358	429	429	429	431	444	444	457	457	460	460	443	443	450
Margaux.1986.B		557	557	619	622	645	570	596	586	560	560	560	565	563	518	534
Margaux.1988.B		297	297	325	325	325	325	325	299	291	291	291	291	295	295	295
Margaux.1989.B	343	343	334	454	450	456	456	453	404	400	400	408	408	416	416	423
Margaux.1990.B		311	534	605	663	663	663	728	901	962	986	1,029	1,029	1,100	1,107	1,107
Margaux.1994.B				199	199	215	215	223	231	234	234	240	247	272	271	271
Margaux.1995.B		389	385	405	413	457	457	456	480	493	493	470	470	472	472	472
Margaux.1996.B		596	596	604	715	715	715	661	670	831	809	783	783	785	783	783
Margaux.1996.IMP				9,135	9,135	9,135	9,135	9,135	9,135	7,135	7,135	8,370	8,370	11,112	11,112	11,112
Margaux.1997.B				236	236	253	253	288	246	252	252	252	252	243	251	251
Margaux.1998.B	234	224	224	198	209	209	215	223	207	241	241	241	241	277	277	277
Margaux.1999.B	243	243	243	243	256	256	256	256	256	269	269	269	269	310	316	316
Margaux.2000.B	836	814	814	850	875	875	875	875	999	1,100	1,076	1,076	1,076	1,088	1,088	1,088
Margaux.2003.B		538	538	589	589	589	589	589	589	638	638	660	660	660	660	660
Margaux.2004.B														381	381	381
Margaux.2005.B																
Mouton-Rothschild.1945.B	7,717	7,717	7,717	6,319	6,324	6,324	6,324	6,574	8,046	6,196	6,196	6,196	6,196	6,196	6,196	6,339
Mouton-Rothschild.1961.B			939	1,269	1,269	1,269	1,269	1,590	1,721	1,713	1,713	1,677	1,677	1,677	1,714	1,923
Mouton-Rothschild.1970.B			172	161	161	161	158	160	164	164	164	167	166	160	155	152
Mouton-Rothschild.1978.B			129	132	132	132	132	142	153	153	153	153	153	153	153	155
Mouton-Rothschild.1982.B	1,025	1,041	1,077	1,185	1,238	1,266	1,199	1,346	1,376	1,211	1,226	1,242	1,267	1,275	1,272	1,322
Mouton-Rothschild.1983.B	237	239	207	235	235	235	247	252	242	241	236	236	236	243	239	244
Mouton-Rothschild.1985.B		291	291	303	301	284	284	290	306	302	302	302	302	289	281	281
Mouton-Rothschild.1986.B	815	843	716	846	846	846	778	811	1,097	976	985	933	933	890	997	985

Table 7.12: Valuation of wines through time

	08/2008	09/2008	10/2008	11/2008	03/2009	04/2009	05/2009	06/2009	09/2009	10/2009	11/2009	12/2009	02/2010	03/2010	04/2010	05/2010
Latour.1970.B	524	524	536	523	520	509	519	516	516	516	517	517	517	517	517	517
Latour.1975.B	184	184	199	226	226	221	221	221	221	221	224	224	230	241	241	241
Latour.1982.B	1,496	1,496	1,441	1,365	1,366	1,340	1,344	1,374	1,400	1,410	1,482	1,482	1,482	1,462	1,462	1,538
Latour.1983.B	323	323	293	279	279	279	279	279	270	270	279	279	279	290	290	290
Latour.1985.B	387	387	376	371	365	365	359	359	329	329	329	329	329	331	331	331
Latour.1986.B	505	505	505	485	471	471	454	454	452	452	448	448	448	448	448	448
Latour.1988.B	353	353	334	348	320	320	319	319	320	322	334	335	335	335	335	335
Latour.1989.B	384	384	384	337	293	293	278	278	282	297	312	318	318	318	318	342
Latour.1990.B	648	648	648	667	636	629	623	625	621	623	700	700	700	700	700	780
Latour.1993.B	268	268	234	211	211	211	211	211	211	211	218	218	218	218	218	218
Latour.1994.B	275	275	267	256	223	223	223	223	223	223	230	230	230	230	230	230
Latour.1995.B	489	489	489	417	347	340	366	366	376	373	479	456	456	456	456	456
Latour.1996.B	846	846	666	524	536	539	665	624	622	614	671	656	656	656	656	797
Latour.1997.B	292	292	292	292	292	292	291	291	291	291	291	301	301	301	301	301
Latour.1998.B	285	285	256	243	223	227	226	226	240	240	265	266	266	266	266	331
Latour.1999.B	332	332	282	280	280	280	284	284	284	284	284	287	287	287	287	287
Latour.2000.B	1,099	1,208	1,170	972	972	898	849	849	851	853	1,094	1,072	1,072	1,072	1,072	1,270
Latour.2001.B	209	209	215	240	240	240	267	267	296	296	316	326	326	326	326	326
Latour.2002.B	342	342	342	321	302	302	303	303	303	303	308	308	308	308	308	308
Latour.2003.B	1,011	1,011	1,011	873	873	801	768	768	741	741	802	802	802	802	802	937
Latour.2004.B				267	225	225	234	234	234	234	261	261	261	261	261	288
Latour.2005.B				1,193	1,142	1,142	1,208	1,208	1,149	1,149	1,199	1,199	1,160	1,160	1,160	1,160
Leoville Poyferre.2000.B	128	117	117	112	108	108	108	108	108	108	108	110	110	110	110	110
Leoville Las Cases .1982.B	535	535	508	434	434	411	363	363	363	363	363	363	363	373	373	396
Leoville Las Cases .1986.B	345	345	348	328	328	320	298	298	298	295	295	295	295	295	295	297
Leoville Las Cases .1989.B	161	161	159	157	157	157	159	159	158	157	157	157	157	157	157	157
Leoville Las Cases .1990.B	305	305	308	291	286	287	281	281	281	281	280	280	280	280	280	280
Leoville Las Cases .1995.B	167	167	167	167	166	166	166	166	166	155	155	155	155	155	155	155
Leoville Las Cases .1996.B	256	256	256	216	206	204	203	204	204	206	208	217	217	217	217	242
Leoville Las Cases .2000.B	324	350	345	286	283	279	278	278	280	281	280	284	284	284	284	284
L'Evangile.1990.B	335	335	335	328	321	314	309	309	308	307	307	307	307	307	307	307
Margaux.1982.B	772	772	793	729	701	648	673	702	701	706	725	715	715	734	734	823
Margaux.1983.B	549	549	540	494	489	482	489	489	475	472	469	469	469	469	469	475
Margaux.1985.B	450	450	450	431	425	419	403	403	400	399	399	399	399	399	399	399
Margaux.1986.B	527	527	506	459	405	414	388	388	389	388	404	409	403	403	403	478
Margaux.1988.B	295	295	295	292	284	280	272	272	271	271	274	274	274	274	274	274
Margaux.1989.B	423	423	423	388	379	379	358	358	353	360	354	352	342	342	342	342
Margaux.1990.B	1,112	1,112	958	902	807	791	835	835	845	860	919	919	928	928	928	928
Margaux.1994.B	271	271	244	236	235	234	228	228	228	228	228	228	228	228	228	228
Margaux.1995.B	462	462	452	447	347	338	360	360	359	359	360	360	360	360	360	360
Margaux.1996.B	737	737	722	553	500	472	525	541	539	529	641	621	621	621	621	656
Margaux.1996.IMP	9,838	9,838	9,838	9,838	8,951	7,904	7,904	7,904	6,745	6,745	6,306	6,306	6,306	6,306	6,306	6,306
Margaux.1997.B	251	251	236	236	231	225	230	228	229	229	232	232	232	232	232	232
Margaux.1998.B	277	277	257	260	234	228	238	234	234	234	250	254	254	254	254	299
Margaux.1999.B	316	316	268	252	246	246	246	246	246	258	258	258	258	266	266	266
Margaux.2000.B	1,088	1,088	962	705	705	713	758	803	825	825	942	954	988	988	988	1,076
Margaux.2003.B	660	660	660	636	636	521	544	544	509	509	498	498	498	498	498	498
Margaux.2004.B	381	381	381	352	352	352	352	352	329	336	335	331	331	331	331	331
Margaux.2005.B				787	833	833	907	907	872	872	872	872	872	850	850	850
Mouton-Rothschild.1945.B	6,339	6,339	6,533	4,994	4,994	4,994	4,994	4,994	5,039	5,039	6,284	6,284	6,284	6,284	6,284	6,284
Mouton-Rothschild.1961.B	1,923	1,923	1,923	1,796	1,796	1,796	1,796	1,796	1,796	1,826	1,845	1,845	1,845	1,845	1,845	1,958
Mouton-Rothschild.1970.B	164	164	162	165	165	165	164	164	164	162	165	165	165	168	168	168
Mouton-Rothschild.1978.B	155	155	155	145	144	145	145	145	145	145	145	145	145	145	145	153
Mouton-Rothschild.1982.B	1,207	1,207	1,122	1,070	902	815	849	932	960	940	944	944	1,005	1,064	1,064	1,112
Mouton-Rothschild.1983.B	244	244	240	227	219	219	219	219	217	221	224	224	224	224	224	232
Mouton-Rothschild.1985.B	281	281	281	262	253	243	243	244	244	244	256	256	256	256	256	275
Mouton-Rothschild.1986.B	858	858	714	640	542	550	552	552	598	720	728	740	740	740	740	799

Table 7.13: Valuation of wines through time

	06/2010	09/2010	10/2010	11/2010	12/2010	01/2011	03/2011	04/2011	05/2011	06/2011	09/2011	10/2011	11/2011	12/2011	01/2012	02/2012	03/2012	04/2012
Latour.1970.B	512	524	524	524	524	532	531	541	555	555	555	552	554	554	554	555	558	558
Latour.1975.B	241	250	250	261	261	261	277	328	436	438	438	438	438	438	438	433	433	433
Latour.1982.B	1,568	1,631	1,631	2,132	2,114	2,336	2,377	2,390	2,685	2,685	2,351	2,361	2,267	2,267	2,252	2,193	2,187	2,152
Latour.1983.B	290	290	290	316	316	316	360	377	417	467	465	465	460	460	460	460	460	460
Latour.1985.B	331	332	332	332	334	335	335	345	363	365	378	378	383	385	385	386	386	388
Latour.1986.B	450	450	450	451	452	456	458	463	474	475	475	475	477	476	476	476	476	476
Latour.1988.B	335	335	335	469	469	469	469	469	572	572	557	557	559	549	549	549	549	556
Latour.1989.B	347	373	373	423	424	439	450	457	542	546	485	485	482	482	482	483	483	505
Latour.1990.B	775	878	878	882	873	962	975	971	1,038	1,031	1,000	963	924	920	920	916	916	875
Latour.1993.B	218	219	219	243	243	243	243	260	300	300	312	312	347	347	347	356	363	374
Latour.1994.B	230	230	230	249	264	264	264	307	365	365	380	401	401	401	401	417	417	425
Latour.1995.B	456	633	828	865	756	764	795	736	742	724	635	647	605	586	586	546	549	549
Latour.1996.B	732	1,286	1,517	1,412	1,412	1,316	1,241	1,081	1,154	1,088	1,088	914	847	849	802	816	798	818
Latour.1997.B	298	322	322	322	322	322	322	340	426	426	431	431	431	425	425	437	437	448
Latour.1998.B	326	326	326	343	343	400	400	418	491	491	496	501	483	476	476	478	478	485
Latour.1999.B	293	308	308	308	324	324	324	351	479	479	491	500	472	465	465	471	476	483
Latour.2000.B	1,162	1,609	2,279	1,846	1,680	1,750	1,750	1,750	1,737	1,682	1,562	1,323	1,242	1,189	1,189	1,189	1,238	1,152
Latour.2001.B	332	358	358	389	409	409	409	409	497	577	577	577	599	596	596	596	596	603
Latour.2002.B	308	338	338	391	391	391	391	391	411	431	456	456	465	461	459	475	475	495
Latour.2003.B	912	1,602	1,588	1,432	1,374	1,266	1,108	1,108	1,287	1,314	1,252	1,169	1,092	978	978	978	978	871
Latour.2004.B	288	394	394	455	455	455	455	455	455	545	538	538	538	530	530	522	486	491
Latour.2005.B	1,160	1,108	1,108	1,182	1,182	1,147	1,145	1,145	1,169	1,169	1,149	1,075	1,016	979	979	983	935	928
Leoville Poyferre.2000.B	108	108	108	117	121	143	143	146	146	148	154	154	155	158	158	166	166	162
Leoville Las Cases .1982.B	353	398	398	435	419	490	652	625	599	576	562	562	468	468	468	479	479	416
Leoville Las Cases .1986.B	297	297	297	300	300	302	305	305	310	307	308	308	306	306	307	308	308	308
Leoville Las Cases .1989.B	157	157	157	161	161	165	167	170	172	172	174	174	172	172	173	176	176	176
Leoville Las Cases .1990.B	280	280	340	340	340	340	341	341	342	338	337	337	336	333	333	332	332	330
Leoville Las Cases .1995.B	158	161	161	164	163	163	163	164	171	172	172	172	176	176	176	174	174	174
Leoville Las Cases .1996.B	242	254	368	348	348	348	345	323	320	306	306	306	291	291	289	284	284	279
Leoville Las Cases .2000.B	282	359	359	352	347	384	387	384	384	384	380	382	343	338	334	327	323	318
L'Evangile.1990.B	306	303	302	303	303	309	309	307	306	307	303	303	303	303	303	303	303	303
Margaux.1982.B	803	846	846	955	958	1,031	1,045	1,090	1,080	1,104	1,070	1,070	978	985	985	971	972	956
Margaux.1983.B	445	447	447	454	458	471	487	496	504	509	512	514	514	519	520	520	520	523
Margaux.1985.B	397	397	397	401	401	407	407	407	409	413	417	417	412	414	414	414	414	413
Margaux.1986.B	475	505	505	512	518	630	686	652	593	588	554	554	509	506	506	540	540	567
Margaux.1988.B	276	283	283	283	283	283	283	283	293	293	293	303	317	317	317	326	331	341
Margaux.1989.B	342	359	359	380	380	391	391	441	450	452	470	458	442	442	442	444	444	451
Margaux.1990.B	923	1,193	1,639	1,414	1,323	1,530	1,471	1,420	1,348	1,343	1,286	1,208	1,080	1,079	1,079	1,069	1,059	1,012
Margaux.1994.B	232	232	232	241	244	244	244	312	312	343	343	343	350	346	346	353	353	362
Margaux.1995.B	368	519	641	593	555	570	578	574	574	572	570	515	496	489	489	479	479	479
Margaux.1996.B	632	968	1,239	973	837	870	944	863	850	880	851	784	711	723	701	723	723	672
Margaux.1996.IMP	6,306	6,306	6,306	6,584	6,368	6,368	7,089	6,816	6,830	6,830	6,830	6,213	6,213	6,213	6,213	6,213	6,213	6,545
Margaux.1997.B	232	236	236	243	243	243	243	254	265	279	311	311	312	309	309	314	314	334
Margaux.1998.B	301	301	301	330	330	330	330	350	372	380	394	394	399	385	385	393	398	406
Margaux.1999.B	266	266	266	318	324	324	324	324	356	356	380	380	380	381	381	389	389	404
Margaux.2000.B	1,076	1,733	1,872	1,463	1,344	1,355	1,341	1,290	1,266	1,249	1,132	1,142	1,004	1,022	1,059	1,006	1,012	1,023
Margaux.2003.B	498	1,319	1,316	1,006	908	1,102	1,102	1,102	1,017	917	814	701	701	618	618	644	644	547
Margaux.2004.B	331	354	354	380	380	380	380	380	380	454	459	459	459	463	463	456	456	456
Margaux.2005.B	850	877	877	927	927	943	955	955	955	958	940	900	798	781	773	775	787	703
Mouton-Rothschild.1945.B	6,284	6,284	6,284	7,396	7,779	7,779	7,779	7,826	8,731	9,034	9,034	9,536	9,536	9,536	9,536	9,536	11,519	11,967
Mouton-Rothschild.1961.B	1,958	1,967	1,967	2,214	2,214	2,214	2,214	2,246	2,246	2,246	2,246	2,246	2,206	2,206	2,206	2,206	2,206	2,206
Mouton-Rothschild.1970.B	186	192	192	192	192	192	192	211	216	216	216	216	217	217	217	217	217	217
Mouton-Rothschild.1978.B	158	158	158	178	178	178	178	194	203	216	212	225	230	230	230	225	225	225
Mouton-Rothschild.1982.B	944	1,247	1,247	1,471	1,465	1,787	1,798	1,797	1,722	1,616	1,429	1,426	1,417	1,355	1,355	1,179	1,198	1,164
Mouton-Rothschild.1983.B	238	238	238	268	269	269	269	280	319	319	322	322	330	327	327	333	333	335
Mouton-Rothschild.1985.B	268	268	268	288	304	323	384	422	435	426	418	418	363	363	363	359	377	381
Mouton-Rothschild.1986.B	807	961	961	1,325	1,188	1,216	1,313	1,275	1,183	1,143	1,027	998	848	840	840	818	818	891

Table 7.14: Valuation of wines through time

	05/2012	06/2012	09/2012	10/2012	11/2012	12/2012	02/2013	03/2013	04/2013	05/2013	06/2013	07/2013	09/2013	10/2013	11/2013	12/2013	N	Annualized Volatility
Latour.1970.B	553	554	554	554	539	540	540	537	535	535	529	523	516	522	522	522	100	0%
Latour.1975.B	372	371	362	362	350	361	361	361	361	361	361	361	361	390	390	390	55	40%
Latour.1982.B	2,051	1,986	1,955	1,940	1,844	1,774	1,764	1,748	1,775	1,788	1,774	1,764	1,775	1,978	1,971	1,971	251	24%
Latour.1983.B	466	455	453	454	444	435	435	440	440	444	444	444	444	434	429	429	74	27%
Latour.1985.B	392	390	389	389	386	387	387	387	387	387	386	386	386	384	384	384	73	3%
Latour.1986.B	478	471	471	472	471	471	471	471	471	473	470	470	470	469	469	469	136	0%
Latour.1988.B	555	555	555	555	505	496	496	496	496	483	483	483	513	511	511	511	68	35%
Latour.1989.B	468	468	456	456	404	420	420	431	431	428	428	437	442	480	479	481	123	27%
Latour.1990.B	842	826	804	817	796	769	765	736	796	782	764	764	765	774	774	774	282	20%
Latour.1993.B	374	362	369	373	373	376	376	390	390	390	390	390	390	390	390	390	63	37%
Latour.1994.B	426	432	446	451	434	434	434	434	434	434	436	436	444	452	453	453	70	31%
Latour.1995.B	484	486	511	543	516	517	517	517	540	540	582	582	574	491	503	503	215	41%
Latour.1996.B	808	760	785	837	732	710	776	762	764	764	725	725	725	846	798	798	331	46%
Latour.1997.B	447	447	447	447	447	443	443	454	454	454	454	454	454	467	467	467	76	31%
Latour.1998.B	482	468	462	473	466	466	466	467	484	469	489	489	489	474	474	474	154	32%
Latour.1999.B	495	489	489	489	482	485	485	485	491	483	464	464	464	488	485	485	78	35%
Latour.2000.B	1,102	981	982	1,111	1,060	1,003	1,003	1,005	1,008	901	966	966	1,003	1,303	1,199	1,199	252	45%
Latour.2001.B	603	605	605	613	604	608	608	605	657	647	631	631	632	599	599	599	50	53%
Latour.2002.B	485	470	461	465	474	484	484	484	484	496	502	502	502	502	505	505	56	24%
Latour.2003.B	888	866	843	854	821	884	884	887	977	999	999	999	999	927	873	873	178	43%
Latour.2004.B	491	489	489	469	467	467	471	495	495	510	510	510	510	510	510	496	56	49%
Latour.2005.B	928	908	908	865	865	839	837	839	906	884	884	884	902	856	857	843	99	17%
Leoville Poyferre.2000.B	162	162	162	162	162	162	162	160	151	151	154	154	157	157	156	156	56	23%
Leoville Las Cases .1982.B	406	369	368	390	406	381	385	387	410	406	398	398	398	436	437	437	199	33%
Leoville Las Cases .1986.B	308	305	305	306	306	306	304	304	304	299	299	299	299	300	300	300	75	0%
Leoville Las Cases .1989.B	173	173	173	173	173	170	170	170	170	170	170	170	170	170	169	168	66	0%
Leoville Las Cases .1990.B	329	328	328	327	326	324	324	324	324	324	323	323	322	327	327	327	96	0%
Leoville Las Cases .1995.B	173	175	175	176	176	176	176	176	177	177	177	177	177	178	179	179	61	0%
Leoville Las Cases .1996.B	279	276	274	267	266	254	255	255	266	231	231	231	245	244	270	263	116	30%
Leoville Las Cases .2000.B	310	311	320	328	328	317	310	307	308	308	305	304	316	309	306	303	167	19%
L'Evangile.1990.B	298	289	289	289	289	288	288	288	288	286	288	288	289	287	287	287	67	4%
Margaux.1982.B	930	878	867	873	854	872	865	853	858	870	903	903	899	764	767	767	276	22%
Margaux.1983.B	501	487	483	490	466	457	457	460	460	468	464	464	473	450	457	457	207	9%
Margaux.1985.B	414	414	414	414	414	414	414	414	414	416	414	414	415	413	413	413	96	0%
Margaux.1986.B	529	516	503	513	472	501	516	519	533	549	502	502	502	490	490	490	216	24%
Margaux.1988.B	329	329	334	334	340	340	340	337	347	347	347	346	346	359	360	360	60	14%
Margaux.1989.B	451	453	452	462	460	443	443	444	451	453	445	445	445	446	446	446	107	14%
Margaux.1990.B	958	695	748	790	761	823	849	859	889	885	856	850	892	1,035	997	997	267	39%
Margaux.1994.B	363	350	357	357	355	349	351	355	355	355	354	354	358	358	358	358	68	25%
Margaux.1995.B	471	447	456	467	441	427	427	447	449	447	456	438	442	462	450	450	215	30%
Margaux.1996.B	643	708	663	687	629	613	629	634	602	607	559	559	559	676	629	629	377	39%
Margaux.1996.IMP	6,545	6,268	6,268	6,106	6,106	5,747	5,747	5,747	5,546	5,546	5,546	5,546	5,546	5,301	5,301	5,301	73	35%
Margaux.1997.B	298	301	313	316	316	316	316	316	323	340	340	340	340	343	343	343	96	21%
Margaux.1998.B	409	386	386	386	391	391	391	391	391	395	402	402	402	404	409	409	114	28%
Margaux.1999.B	407	412	412	416	409	413	413	413	413	416	421	421	421	421	430	430	68	28%
Margaux.2000.B	971	925	884	1,019	853	836	849	831	924	924	1,084	1,084	1,070	1,120	1,120	1,120	278	44%
Margaux.2003.B	552	561	578	623	554	523	529	529	529	495	523	523	523	577	582	582	175	67%
Margaux.2004.B	456	452	452	451	451	453	492	492	492	492	492	493	493	488	487	487	50	0%
Margaux.2005.B	703	707	708	705	702	690	709	709	724	724	716	716	685	685	687	686	97	17%
Mouton-Rothschild.1945.B	12,661	12,661	12,661	12,661	12,002	12,374	12,374	12,374	12,261	12,261	11,807	11,807	11,051	11,051	11,051	11,051	55	46%
Mouton-Rothschild.1961.B	2,206	2,201	2,207	2,224	2,267	2,327	2,327	2,361	2,361	2,361	2,329	2,356	2,429	2,429	2,429	2,429	51	22%
Mouton-Rothschild.1970.B	217	221	221	221	221	221	221	221	218	218	218	218	218	221	221	221	90	19%
Mouton-Rothschild.1978.B	233	233	248	248	249	247	247	247	251	256	256	250	250	218	218	218	50	26%
Mouton-Rothschild.1982.B	1,131	1,117	1,036	1,196	1,056	1,081	1,084	1,095	1,494	1,320	1,163	1,163	1,088	1,352	1,312	1,312	500	34%
Mouton-Rothschild.1983.B	335	281	287	287	284	292	294	304	304	319	319	319	319	299	299	299	112	17%
Mouton-Rothschild.1985.B	380	352	359	361	355	355	355	349	341	356	350	345	371	340	347	347	143	24%
Mouton-Rothschild.1986.B	789	815	818	903	722	738	740	832	1,170	1,032	898	754	829	971	959	959	387	46%

Table 7.15: Valuation of wines through time

Wine	02/2007	03/2007	04/2007	05/2007	06/2007	07/2007	09/2007	10/2007	11/2007	12/2007	02/2008	03/2008	04/2008	05/2008	06/2008	07/2008
Mouton-Rothschild.1988.B	226	226	235	258	262	262	264	264	267	262	262	268	268	270	268	265
Mouton-Rothschild.1989.B				294	294	313	259	252	316	294	294	304	306	306	321	322
Mouton-Rothschild.1990.B	248	250	238	282	289	289	292	290	282	276	279	282	278	289	291	291
Mouton-Rothschild.1993.B		191	191	184	187	194	197	207	203	204	203	203	203	206	206	206
Mouton-Rothschild.1994.B	173	176	176	174	192	192	200	204	197	202	202	202	203	212	212	214
Mouton-Rothschild.1995.B		311	306	300	299	299	299	352	389	380	380	325	331	345	358	358
Mouton-Rothschild.1996.B	298	275	275	275	280	300	322	322	307	325	325	340	340	336	360	363
Mouton-Rothschild.1997.B					199	199	199	200	201	209	209	209	209	219	219	219
Mouton-Rothschild.1998.B		232	232	298	299	299	296	312	284	300	300	300	300	327	327	327
Mouton-Rothschild.1999.B	190	189	189	189	194	194	222	191	191	203	203	203	203	241	242	245
Mouton-Rothschild.2000.B		635	635	614	614	614	655	714	714	756	756	779	779	815	818	818
Mouton-Rothschild.2001.B	189	189	189	187	192	192	192	193	203	212	212	212	212	212	212	212
Mouton-Rothschild.2002.B				185	185	215	215	234	234	232	242	242	242	264	261	261
Mouton-Rothschild.2003.B		66	66	296	308	308	308	324	324	324	324	346	348	353	353	353
Mouton-Rothschild.2005.B																
Palmer.1983.B	321	328	328	345	345	345	345	352	391	381	382	382	382	385	396	405
Palmer.1989.B		288	288	293	295	295	276	282	303	305	305	305	303	303	303	316
Palmer.1990.B	197	197	197	202	203	211	211	206	219	215	218	218	218	218	216	216
Palmer.2000.B	218	218	218	225	225	225	221	221	226	183	183	185	185	185	185	185
Palmer.2005.B																
Pavie .2000.B							391	391	391	387	387	388	388	388	382	382
Pavie .2003.B		171	171	171	171	171	171	171	171	171	171	171	171	193	193	193
Petrus.1970.B	1,474	1,474	1,515	1,543	1,543	1,607	1,607	1,607	1,593	1,578	1,659	1,659	1,655	1,658	1,658	1,725
Petrus.1975.B		1,558	1,558	1,414	1,414	1,414	1,414	1,457	1,461	1,450	1,450	1,450	1,534	1,515	1,515	1,567
Petrus.1978.B			694	757	757	757	757	759	765	711	677	677	677	696	696	725
Petrus.1979.B			460	546	546	546	546	546	628	664	664	672	672	707	693	693
Petrus.1982.B	3,758	3,758	3,758	6,292	6,292	5,958	5,958	5,958	5,487	5,185	5,146	5,079	5,027	5,033	5,058	5,131
Petrus.1983.B				957	957	957	957	957	861	861	861	866	923	973	973	973
Petrus.1985.B		1,077	979	979	979	1,033	1,033	1,083	1,041	1,064	1,079	1,107	1,107	1,169	1,169	1,167
Petrus.1986.B		985	985	1,015	1,042	1,042	1,042	1,042	952	981	1,007	1,007	1,007	1,067	1,067	1,104
Petrus.1988.B		1,016	1,016	1,016	1,119	1,119	1,119	1,119	1,064	1,077	1,077	1,078	1,076	1,119	1,119	1,119
Petrus.1989.B		3,000	3,000	3,085	3,173	3,173	3,173	3,173	3,419	3,561	3,561	3,778	3,778	3,815	3,855	3,855
Petrus.1990.B		3,014	3,141	3,274	3,414	3,414	3,414	3,414	3,683	3,816	3,816	3,916	3,916	4,072	4,142	4,142
Petrus.1994.B	992	992	992	929	920	920	920	920	884	889	917	910	940	944	965	965
Petrus.1995.B		1,213	1,213	1,394	1,442	1,442	1,442	1,442	1,562	1,695	1,709	1,735	1,760	1,775	1,775	1,775
Petrus.1996.B		944	944	940	973	973	973	833	921	1,025	1,025	1,056	1,136	1,151	1,151	1,151
Petrus.1997.B		909	909	876	890	890	890	806	806	838	838	890	903	978	989	989
Petrus.1998.B		2,660	2,660	2,549	2,679	2,679	2,679	3,084	3,141	3,204	3,204	3,283	3,250	3,207	3,080	3,080
Petrus.1999.B				777	963	963	963	963	1,008	1,074	1,137	1,159	1,200	1,200	1,200	1,200
Petrus.2000.B		3,171	3,171	3,171	3,335	3,335	3,335	3,298	3,298	3,496	3,570	3,722	3,722	3,722	3,722	3,722
Pichon Baron.1989.B					300	300	300	300	308	306	290	285	285	285	285	293
Pichon Baron.1990.B			228	228	228	228	228	230	233	233	233	228	228	228	228	228
Pichon Baron.1996.B		112	97	97	97	97	97	97	97	92	92	89	89	89	89	89
Pichon Baron.2000.B				121	121	121	129	129	129	140	141	141	141	137	137	137
Pichon Comtesse.1982.B		404	540	514	514	514	514	553	576	567	557	558	558	566	579	579
Pichon Comtesse.1986.B	201	210	204	208	208	208	199	197	197	197	197	197	198	200	200	200
Pichon Comtesse.1989.B		179	169	174	172	172	171	171	171	172	172	172	175	176	176	186
Pichon Comtesse.1990.B	164	151	151	153	152	152	152	152	152	152	150	148	148	148	148	148
Pichon Comtesse.1995.B				175	175	178	178	178	178	178	177	178	178	180	178	178
Pichon Comtesse.1996.B		150	150	163	157	172	172	171	193	196	196	191	191	195	197	197
Pichon Comtesse.2000.B		175	175	185	185	185	193	193	202	211	211	211	211	214	214	214
Pichon Comtesse.2003.B		108	108	117	117	117	117	117	117	117	117	117	117	117	117	117
Richebourg DRC.1990.B		363	363	747	929	929	929	929	1,221	1,345	1,497	1,497	1,497	1,645	1,645	1,645
Romanee St. Vivant DRC.2002.B								777	777	777	777	777	777	827	827	827
Romanee-Conti DRC.1990.B				21,996	21,996	21,996	21,996	21,996	22,088	19,948	19,948	19,948	19,948	19,948	19,948	19,948
Yquem.1967.B		1,143	1,234	1,307	1,318	1,318	1,326	1,326	1,326	1,362	1,360	1,360	1,346	1,325	1,293	1,302
Yquem.1975.B			709	709	709	709	822	877	953	885	885	885	885	872	865	865
Yquem.1983.B	455	466	466	466	466	466	466	427	431	431	431	439	439	434	423	424
Yquem.1986.B		457	453	453	453	453	453	430	430	453	453	453	453	453	440	440
Yquem.1988.B		508	508	508	591	569	569	569	520	458	458	458	458	454	454	454
Yquem.1990.B	434	402	416	412	439	439	439	439	439	410	410	410	410	436	438	438
Yquem.2001.B		667	667	667	667	667	667	667	667	709	709	709	709	709	709	709

Table 7.16: Valuation of wines through time

	08/2008	09/2008	10/2008	11/2008	03/2009	04/2009	05/2009	06/2009	09/2009	10/2009	11/2009	12/2009	02/2010	03/2010	04/2010	05/2010
Mouton-Rothschild.1988.B	265	265	265	226	217	213	212	212	214	242	258	261	261	273	273	273
Mouton-Rothschild.1989.B	322	322	309	238	232	233	240	242	239	252	259	259	252	249	249	249
Mouton-Rothschild.1990.B	289	289	289	232	221	221	220	225	223	226	251	251	257	257	257	289
Mouton-Rothschild.1993.B	208	208	208	201	197	197	197	197	197	197	198	198	198	198	198	208
Mouton-Rothschild.1994.B	217	217	180	157	156	165	168	168	168	168	184	184	184	199	199	199
Mouton-Rothschild.1995.B	358	358	305	292	257	246	281	278	278	279	268	276	276	303	303	380
Mouton-Rothschild.1996.B	356	356	345	295	267	248	239	257	263	263	294	317	317	322	322	349
Mouton-Rothschild.1997.B	219	219	195	183	175	179	179	179	185	185	185	185	185	176	176	176
Mouton-Rothschild.1998.B	327	327	327	248	257	252	234	243	251	265	288	298	298	298	298	451
Mouton-Rothschild.1999.B	245	245	209	179	179	179	183	183	183	183	189	189	189	189	189	189
Mouton-Rothschild.2000.B	818	857	857	744	728	698	688	688	725	762	803	848	848	862	862	862
Mouton-Rothschild.2001.B	212	212	212	213	214	214	216	216	221	221	232	232	232	232	232	232
Mouton-Rothschild.2002.B	261	261	261	264	231	231	223	223	226	239	239	243	243	243	243	259
Mouton-Rothschild.2003.B	353	353	353	314	314	312	290	290	284	284	287	287	287	287	287	292
Mouton-Rothschild.2005.B				523	523	523	531	531	535	535	535	533	533	533	533	539
Palmer.1983.B	405	405	399	380	380	366	363	360	360	360	356	356	356	356	356	356
Palmer.1989.B	316	316	315	304	269	269	269	269	268	271	266	266	266	266	266	266
Palmer.1990.B	216	216	205	191	181	182	179	179	179	179	179	182	182	182	182	182
Palmer.2000.B	185	185	185	176	176	176	176	179	185	185	185	185	185	185	185	185
Palmer.2005.B				398	398	398	398	398	352	352	352	311	311	311	311	311
Pavie .2000.B	382	382	356	295	295	289	289	289	301	301	301	326	326	326	326	359
Pavie .2003.B	193	193	193	206	206	193	193	193	186	186	186	186	186	186	186	186
Petrus.1970.B	1,725	1,725	1,725	1,620	1,620	1,620	1,663	1,677	1,677	1,673	1,720	1,720	1,720	1,720	1,720	1,720
Petrus.1975.B	1,567	1,567	1,559	1,492	1,492	1,492	1,497	1,497	1,499	1,477	1,462	1,462	1,467	1,467	1,467	1,467
Petrus.1978.B	725	725	717	717	661	608	605	605	605	622	642	642	642	642	642	642
Petrus.1979.B	693	693	693	673	673	673	691	691	691	723	768	768	768	768	768	768
Petrus.1982.B	5,131	5,131	5,033	5,088	4,795	4,599	4,599	4,588	4,582	4,471	4,469	4,469	4,469	4,469	4,469	4,469
Petrus.1983.B	973	973	973	973	973	907	907	907	907	907	900	900	900	900	900	900
Petrus.1985.B	1,167	1,167	1,155	1,155	1,155	1,155	1,083	1,083	1,057	1,017	1,038	1,038	1,038	1,038	1,038	1,038
Petrus.1986.B	1,104	1,104	1,104	984	984	984	952	939	939	943	943	943	943	954	954	954
Petrus.1988.B	1,119	1,119	1,119	1,080	1,080	1,031	1,003	1,003	999	997	1,010	1,010	1,010	1,010	1,010	1,010
Petrus.1989.B	3,855	3,855	3,828	3,905	3,806	3,478	3,387	3,387	3,363	3,351	3,338	3,304	3,304	3,304	3,304	3,304
Petrus.1990.B	4,121	4,121	4,077	4,069	4,069	4,021	3,734	3,692	3,678	3,678	3,660	3,646	3,646	3,646	3,646	3,681
Petrus.1994.B	965	965	965	988	952	909	828	828	828	828	818	825	825	825	825	873
Petrus.1995.B	1,775	1,775	1,775	1,725	1,463	1,418	1,427	1,442	1,442	1,426	1,431	1,431	1,431	1,431	1,431	1,566
Petrus.1996.B	1,123	1,123	1,095	1,022	1,022	1,004	1,022	1,022	1,022	1,041	1,047	1,047	1,047	1,047	1,047	1,047
Petrus.1997.B	989	989	977	1,002	1,002	986	967	967	967	967	967	967	967	967	967	967
Petrus.1998.B	3,080	3,080	3,080	2,955	2,955	2,919	2,798	2,763	2,763	2,763	2,685	2,685	2,685	2,685	2,685	2,685
Petrus.1999.B	1,156	1,156	1,156	1,156	1,017	836	873	873	873	873	873	873	873	873	873	873
Petrus.2000.B	3,722	3,722	3,659	3,496	3,496	3,496	3,543	3,543	3,543	3,525	3,633	3,691	3,691	3,691	3,691	3,691
Pichon Baron.1989.B	293	293	273	263	265	265	266	266	266	265	267	267	268	268	267	267
Pichon Baron.1990.B	250	250	242	235	224	224	223	223	221	221	221	221	222	222	222	223
Pichon Baron.1996.B	89	89	89	92	89	90	90	91	90	87	87	87	87	87	88	94
Pichon Baron.2000.B	137	144	144	142	140	140	138	138	138	141	141	145	145	145	147	147
Pichon Comtesse.1982.B	587	587	587	517	466	443	441	441	446	449	446	460	460	460	460	483
Pichon Comtesse.1986.B	202	202	202	197	197	195	192	192	191	191	194	194	194	194	194	194
Pichon Comtesse.1989.B	186	186	181	181	177	177	177	177	175	175	175	177	177	177	177	177
Pichon Comtesse.1990.B	150	150	148	147	140	140	141	141	141	141	141	141	141	141	141	141
Pichon Comtesse.1995.B	178	178	178	173	164	164	162	162	162	162	162	162	162	162	162	168
Pichon Comtesse.1996.B	203	203	203	149	147	148	158	157	157	160	170	170	180	174	174	180
Pichon Comtesse.2000.B	214	218	218	217	199	193	191	179	186	192	192	200	200	200	201	201
Pichon Comtesse.2003.B	117	117	117	117	103	108	108	105	104	104	104	104	104	104	104	104
Richebourg DRC.1990.B	1,706	1,706	1,706	1,706	1,706	1,710	1,823	1,874	1,896	1,896	1,896	1,896	1,896	1,896	1,896	1,896
Romanee St. Vivant DRC.2002.B	827	827	827	788	788	788	788	788	673	584	605	605	621	621	621	621
Romanee-Conti DRC.1990.B	19,948	19,948	19,948	17,692	17,692	16,116	16,116	16,116	15,393	15,393	14,903	14,903	15,284	15,284	15,284	15,714
Yquem.1967.B	1,302	1,302	1,302	1,265	1,247	1,247	1,241	1,241	1,241	1,251	1,249	1,249	1,249	1,249	1,249	1,249
Yquem.1975.B	842	842	810	777	777	777	777	777	765	709	713	713	713	713	713	713
Yquem.1983.B	418	418	418	388	388	388	400	400	401	401	401	397	392	392	392	392
Yquem.1986.B	440	440	440	398	398	377	426	426	426	426	426	426	426	426	386	386
Yquem.1988.B	454	454	454	427	427	427	427	427	427	414	414	409	409	409	409	409
Yquem.1990.B	438	438	432	378	369	369	335	341	341	349	359	359	359	359	359	380
Yquem.2001.B	709	709	709	682	463	463	443	443	454	465	490	490	490	490	490	490

Table 7.17: Valuation of wines through time

	06/2010	09/2010	10/2010	11/2010	12/2010	01/2011	03/2011	04/2011	05/2011	06/2011	09/2011	10/2011	11/2011	12/2011	01/2012	02/2012	03/2012	04/2012
Mouton-Rothschild.1988.B	276	288	288	298	308	308	308	321	340	355	389	392	358	351	351	356	356	374
Mouton-Rothschild.1989.B	258	280	280	326	337	337	358	390	400	414	419	418	397	394	394	390	390	403
Mouton-Rothschild.1990.B	281	281	281	324	324	340	357	386	408	408	412	412	402	395	395	390	371	374
Mouton-Rothschild.1993.B	226	226	226	260	269	269	269	282	290	321	333	333	350	350	350	356	356	381
Mouton-Rothschild.1994.B	210	215	215	253	253	253	253	273	273	303	311	311	330	326	326	334	334	351
Mouton-Rothschild.1995.B	359	566	814	572	544	544	582	584	573	551	513	542	467	456	456	501	501	449
Mouton-Rothschild.1996.B	353	605	735	550	539	602	602	596	604	588	583	541	519	480	480	468	469	470
Mouton-Rothschild.1997.B	190	250	250	260	260	260	260	276	310	310	338	352	346	350	350	350	350	370
Mouton-Rothschild.1998.B	400	450	715	574	542	552	552	529	530	530	525	525	468	459	459	421	418	416
Mouton-Rothschild.1999.B	214	227	227	270	270	270	270	270	307	309	326	326	326	330	330	344	344	388
Mouton-Rothschild.2000.B	893	1,668	1,790	1,484	1,440	1,752	1,752	1,712	1,703	1,660	1,686	1,464	1,214	1,243	1,244	1,106	1,199	1,299
Mouton-Rothschild.2001.B	330	343	343	382	382	382	382	382	431	455	455	457	446	428	428	428	428	428
Mouton-Rothschild.2002.B	259	314	314	436	436	436	436	436	446	453	457	457	441	426	423	417	408	421
Mouton-Rothschild.2003.B	287	620	820	712	641	639	633	633	595	587	558	535	509	481	481	488	488	454
Mouton-Rothschild.2005.B	539	561	561	683	683	707	707	707	707	707	707	729	688	637	637	626	589	575
Palmer.1983.B	356	356	356	373	373	379	384	389	389	390	391	392	391	391	391	391	391	392
Palmer.1989.B	266	271	271	284	287	295	298	321	321	322	320	320	322	322	322	329	329	329
Palmer.1990.B	179	179	179	194	194	194	196	196	199	199	201	206	215	215	215	215	215	220
Palmer.2000.B	192	186	186	228	231	285	290	289	289	289	290	304	297	291	291	306	272	274
Palmer.2005.B	304	294	294	239	214	259	259	260	261	264	264	264	268	269	269	269	269	268
Pavie .2000.B	359	409	409	410	414	518	514	502	502	500	481	496	498	485	485	469	457	453
Pavie .2003.B	183	205	205	203	203	203	203	203	211	210	215	215	210	205	205	210	211	259
Petrus.1970.B	1,725	1,735	1,735	1,745	1,749	1,766	1,766	1,786	1,786	1,786	1,786	1,801	1,777	1,777	1,778	1,787	1,804	1,804
Petrus.1975.B	1,467	1,493	1,493	1,501	1,507	1,507	1,544	1,578	1,578	1,590	1,585	1,585	1,582	1,588	1,588	1,584	1,596	1,596
Petrus.1978.B	651	744	744	791	805	849	849	920	1,010	1,025	1,038	1,038	1,072	1,072	1,072	1,072	1,072	1,080
Petrus.1979.B	768	894	894	923	923	1,034	1,077	1,077	1,102	1,102	1,081	1,081	1,119	1,119	1,119	1,168	1,205	1,223
Petrus.1982.B	4,453	4,454	4,454	4,635	4,618	4,736	4,760	4,767	4,785	4,879	4,865	4,864	4,892	4,892	4,896	4,903	4,872	4,888
Petrus.1983.B	899	936	936	954	954	954	1,001	1,034	1,065	1,065	1,065	1,089	1,138	1,161	1,161	1,214	1,254	1,262
Petrus.1985.B	1,060	1,089	1,089	1,089	1,089	1,089	1,089	1,089	1,089	1,180	1,180	1,234	1,263	1,273	1,293	1,324	1,324	1,387
Petrus.1986.B	961	961	961	989	989	989	1,085	1,349	1,349	1,349	1,349	1,349	1,389	1,389	1,389	1,386	1,386	1,378
Petrus.1988.B	1,010	1,034	1,034	1,061	1,061	1,061	1,061	1,079	1,131	1,166	1,222	1,233	1,313	1,342	1,342	1,342	1,373	1,400
Petrus.1989.B	3,299	3,299	3,299	3,402	3,393	3,475	3,540	3,539	3,553	3,559	3,566	3,583	3,567	3,565	3,573	3,571	3,571	3,588
Petrus.1990.B	3,677	3,694	3,694	3,888	3,887	3,934	3,948	3,948	3,952	3,952	3,948	3,950	3,953	3,958	3,958	3,958	3,978	3,984
Petrus.1994.B	887	887	887	923	923	997	997	997	1,034	1,079	1,079	1,079	1,232	1,245	1,245	1,245	1,245	1,232
Petrus.1995.B	1,566	1,742	1,742	2,580	2,498	2,456	2,129	2,137	2,045	1,970	1,970	1,863	1,846	1,782	1,782	1,868	1,868	1,864
Petrus.1996.B	1,073	1,120	1,120	1,503	1,503	1,556	1,620	1,633	1,648	1,647	1,599	1,648	1,662	1,662	1,662	1,678	1,678	1,711
Petrus.1997.B	967	967	967	982	982	982	982	1,014	1,014	1,014	1,014	1,046	1,046	1,046	1,046	1,046	1,046	1,140
Petrus.1998.B	2,685	2,694	2,694	2,972	2,920	3,123	3,152	3,152	3,152	3,133	3,133	3,133	3,075	3,004	3,000	3,000	3,000	3,017
Petrus.1999.B	873	934	934	934	934	1,180	1,180	1,180	1,180	1,180	1,241	1,241	1,406	1,406	1,429	1,429	1,429	1,547
Petrus.2000.B	3,687	3,687	3,687	3,891	3,897	4,582	4,618	4,618	4,618	4,618	4,583	4,584	4,459	4,377	4,351	4,351	4,351	4,446
Pichon Baron.1989.B	266	266	266	270	271	271	273	273	273	273	273	273	271	271	271	271	271	270
Pichon Baron.1990.B	223	223	223	249	249	260	272	271	271	275	268	268	255	255	255	254	254	253
Pichon Baron.1996.B	94	94	94	97	97	97	97	97	97	123	126	127	127	127	127	127	127	127
Pichon Baron.2000.B	147	149	149	157	157	200	200	200	200	204	206	206	206	207	207	207	207	209
Pichon Comtesse.1982.B	480	477	477	485	486	557	563	555	560	560	561	561	534	540	546	546	546	546
Pichon Comtesse.1986.B	192	191	191	193	195	195	195	195	195	196	195	195	189	190	190	190	190	188
Pichon Comtesse.1989.B	176	175	175	177	177	179	179	182	182	182	182	182	183	183	183	183	183	183
Pichon Comtesse.1990.B	141	140	140	141	141	141	143	144	144	145	146	145	145	145	145	145	145	145
Pichon Comtesse.1995.B	168	168	168	172	172	172	172	173	176	177	177	177	177	177	177	177	177	178
Pichon Comtesse.1996.B	156	153	285	285	274	309	309	309	296	238	238	239	212	210	210	208	208	177
Pichon Comtesse.2000.B	200	238	238	238	238	258	258	255	257	257	257	241	229	236	238	239	240	240
Pichon Comtesse.2003.B	107	108	257	212	212	169	169	158	152	155	155	142	126	123	129	129	129	129
Richebourg DRC.1990.B	1,896	1,896	1,896	1,896	1,896	1,995	1,995	2,505	2,505	2,505	2,524	2,377	2,511	2,641	2,641	2,641	2,641	2,607
Romanee St. Vivant DRC.2002.B	632	719	719	746	746	701	701	727	727	727	728	728	736	736	736	736	736	747
Romanee-Conti DRC.1990.B	15,714	15,552	15,552	15,943	15,943	15,849	15,849	16,032	16,032	16,055	15,847	15,847	16,091	15,920	15,920	15,920	15,920	15,927
Yquem.1967.B	1,241	1,241	1,241	1,243	1,241	1,249	1,249	1,263	1,267	1,267	1,267	1,267	1,263	1,263	1,263	1,263	1,263	1,263
Yquem.1975.B	700	670	670	649	649	626	626	626	633	633	625	625	613	613	613	617	617	617
Yquem.1983.B	390	390	390	395	395	395	395	395	395	395	374	374	371	371	371	371	371	371
Yquem.1986.B	386	386	386	377	381	384	385	385	386	386	384	384	383	383	383	385	385	385
Yquem.1988.B	409	409	409	395	390	390	390	388	386	386	377	377	385	385	385	387	387	387
Yquem.1990.B	380	388	388	376	367	400	405	443	443	464	449	449	450	450	450	450	450	449
Yquem.2001.B	490	523	523	532	532	532	532	545	545	545	545	556	556	556	556	580	580	604

Table 7.18: Valuation of wines through time

	05/2012	06/2012	09/2012	10/2012	11/2012	12/2012	02/2013	03/2013	04/2013	05/2013	06/2013	07/2013	09/2013	10/2013	11/2013	12/2013	N	Annualized Volatility
Mouton-Rothschild.1988.B	369	342	347	364	330	337	337	337	345	349	354	344	356	345	346	346	153	21%
Mouton-Rothschild.1989.B	400	352	358	368	357	364	364	368	362	359	377	376	383	403	403	403	173	25%
Mouton-Rothschild.1990.B	374	376	362	361	363	361	361	364	354	363	371	366	403	403	396	396	168	24%
Mouton-Rothschild.1993.B	380	382	383	387	398	400	400	396	386	382	382	382	382	368	378	378	101	27%
Mouton-Rothschild.1994.B	353	323	323	326	325	325	325	336	353	349	333	333	333	333	340	340	138	35%
Mouton-Rothschild.1995.B	429	405	429	456	391	402	402	429	443	443	439	416	448	473	468	457	330	46%
Mouton-Rothschild.1996.B	426	411	448	444	427	459	461	461	441	420	470	470	470	489	500	500	355	41%
Mouton-Rothschild.1997.B	370	356	375	369	364	364	364	364	364	364	364	364	364	364	364	364	68	39%
Mouton-Rothschild.1998.B	368	355	361	364	371	355	377	396	403	376	437	437	437	463	442	442	216	47%
Mouton-Rothschild.1999.B	387	384	384	386	386	386	388	394	394	394	402	402	414	494	489	489	102	43%
Mouton-Rothschild.2000.B	1,272	1,206	1,199	1,227	1,182	1,188	1,187	1,222	1,245	1,229	1,340	1,340	1,374	1,779	1,697	1,643	319	41%
Mouton-Rothschild.2001.B	428	412	412	412	399	410	410	410	435	435	443	443	443	443	455	455	65	42%
Mouton-Rothschild.2002.B	414	372	378	378	377	405	405	419	419	413	437	437	431	443	449	449	119	33%
Mouton-Rothschild.2003.B	449	433	421	424	389	410	410	410	416	416	418	418	433	500	480	480	183	52%
Mouton-Rothschild.2005.B	575	571	548	548	510	510	510	537	539	539	566	566	550	526	521	521	127	21%
Palmer.1983.B	392	387	386	386	387	392	392	392	392	390	390	390	387	410	411	411	88	6%
Palmer.1989.B	330	327	327	327	329	332	332	329	329	329	329	329	330	329	329	329	93	9%
Palmer.1990.B	222	219	219	217	217	220	220	220	220	228	228	228	229	229	229	229	61	14%
Palmer.2000.B	274	260	278	285	281	281	281	280	277	235	241	241	256	250	265	265	106	24%
Palmer.2005.B	268	267	268	268	267	266	266	266	266	266	265	265	265	263	263	263	59	0%
Pavie .2000.B	448	440	441	454	442	425	425	443	443	443	450	450	475	495	501	501	143	24%
Pavie .2003.B	260	260	260	260	246	254	254	252	270	270	270	270	270	269	268	268	79	21%
Petrus.1970.B	1,796	1,796	1,775	1,775	1,778	1,801	1,801	1,817	1,817	1,817	1,823	1,823	1,823	1,834	1,834	1,834	63	0%
Petrus.1975.B	1,596	1,595	1,595	1,609	1,606	1,601	1,601	1,601	1,601	1,601	1,601	1,601	1,600	1,600	1,606	1,606	59	0%
Petrus.1978.B	1,089	1,117	1,151	1,280	1,280	1,244	1,244	1,244	1,244	1,244	1,244	1,287	1,287	1,287	1,397	1,397	59	34%
Petrus.1979.B	1,298	1,210	1,245	1,245	1,247	1,258	1,258	1,256	1,256	1,256	1,256	1,256	1,285	1,251	1,251	1,251	51	32%
Petrus.1982.B	4,806	4,763	4,726	4,725	4,670	4,688	4,688	4,666	4,686	4,687	4,748	4,748	4,735	4,735	4,759	4,759	139	0%
Petrus.1983.B	1,262	1,262	1,224	1,224	1,218	1,196	1,196	1,204	1,216	1,229	1,210	1,205	1,233	1,257	1,478	1,478	51	23%
Petrus.1985.B	1,438	1,406	1,406	1,440	1,443	1,434	1,434	1,434	1,434	1,393	1,393	1,393	1,393	1,393	1,563	1,563	58	18%
Petrus.1986.B	1,353	1,345	1,359	1,416	1,416	1,489	1,489	1,486	1,614	1,614	1,614	1,614	1,614	1,617	1,766	1,766	55	28%
Petrus.1988.B	1,382	1,393	1,418	1,496	1,496	1,496	1,496	1,499	1,509	1,509	1,567	1,567	1,567	1,575	1,575	1,575	64	21%
Petrus.1989.B	3,584	3,541	3,520	3,535	3,538	3,525	3,516	3,515	3,537	3,537	3,587	3,587	3,577	3,574	3,581	3,581	125	0%
Petrus.1990.B	3,975	3,964	3,973	3,968	3,916	3,905	3,905	3,899	3,973	3,973	3,970	3,970	3,966	3,967	4,018	4,018	129	2%
Petrus.1994.B	1,239	1,248	1,264	1,302	1,316	1,316	1,316	1,316	1,316	1,356	1,356	1,374	1,390	1,450	1,678	1,678	84	28%
Petrus.1995.B	1,855	1,807	1,813	1,913	1,846	1,883	1,883	1,883	2,855	2,855	2,880	2,699	2,497	2,251	2,274	2,274	128	38%
Petrus.1996.B	1,711	1,711	1,813	1,834	1,841	1,827	1,827	1,827	1,846	1,887	1,963	1,963	1,963	1,962	2,093	2,093	72	29%
Petrus.1997.B	1,140	1,157	1,181	1,222	1,222	1,222	1,240	1,261	1,194	1,220	1,220	1,220	1,262	1,283	1,316	1,316	50	21%
Petrus.1998.B	2,928	2,914	2,949	2,971	2,928	2,773	2,773	2,773	2,797	2,797	3,013	3,013	2,985	2,766	3,719	3,719	80	19%
Petrus.1999.B	1,547	1,547	1,646	1,646	1,604	1,692	1,692	1,692	1,692	1,692	1,711	1,711	1,750	1,797	1,797	1,797	51	45%
Petrus.2000.B	4,446	4,260	4,260	4,300	4,292	4,286	4,286	4,226	4,188	4,188	4,182	4,182	4,182	4,129	4,129	4,129	75	13%
Pichon Baron.1989.B	269	268	268	268	279	279	279	279	279	279	279	279	279	281	282	282	73	0%
Pichon Baron.1990.B	256	266	263	263	264	259	262	272	272	273	273	273	273	296	295	295	82	12%
Pichon Baron.1996.B	127	126	126	126	126	126	126	126	125	125	125	125	126	125	126	126	67	20%
Pichon Baron.2000.B	198	198	195	195	191	194	195	191	191	194	201	201	201	217	207	205	98	19%
Pichon Comtesse.1982.B	511	512	515	515	526	518	525	531	531	540	542	542	555	550	556	554	133	13%
Pichon Comtesse.1986.B	186	186	186	186	182	182	184	184	184	184	184	184	184	184	184	184	109	3%
Pichon Comtesse.1989.B	183	182	182	182	182	184	184	184	184	185	185	185	185	185	186	186	112	0%
Pichon Comtesse.1990.B	145	144	144	144	144	144	144	144	144	143	143	143	144	143	143	143	75	0%
Pichon Comtesse.1995.B	174	174	174	174	174	175	175	175	175	175	175	175	175	175	177	177	66	0%
Pichon Comtesse.1996.B	191	195	196	196	191	185	196	195	195	194	235	235	236	208	214	214	122	39%
Pichon Comtesse.2000.B	238	238	238	250	246	238	233	220	226	225	219	219	221	206	212	212	155	17%
Pichon Comtesse.2003.B	129	129	129	129	126	126	125	125	133	133	133	133	127	114	122	122	89	65%
Richebourg DRC.1990.B	2,650	2,690	2,332	2,332	2,434	2,638	2,638	2,638	2,638	2,677	2,677	2,677	2,787	2,645	2,734	2,734	56	29%
Romanee St. Vivant DRC.2002.B	756	756	770	787	824	826	819	819	819	819	819	819	819	679	679	683	50	0%
Romanee-Conti DRC.1990.B	15,990	16,005	16,030	16,359	15,662	15,708	15,708	15,708	15,876	15,876	15,876	15,876	15,975	16,618	16,668	16,668	52	0%
Yquem.1967.B	1,300	1,299	1,299	1,299	1,313	1,316	1,316	1,316	1,316	1,316	1,316	1,316	1,316	1,316	1,316	1,316	71	0%
Yquem.1975.B	578	578	566	566	561	551	551	551	551	551	550	550	550	493	493	493	58	22%
Yquem.1983.B	363	362	365	365	353	358	358	358	363	363	362	362	362	361	360	360	53	3%
Yquem.1986.B	391	388	387	387	383	380	380	378	389	389	389	389	388	389	388	388	55	0%
Yquem.1988.B	398	400	401	401	397	397	394	394	398	398	395	395	395	395	395	395	54	0%
Yquem.1990.B	476	467	443	443	434	416	416	404	436	436	426	426	421	459	431	431	154	18%
Yquem.2001.B	628	626	628	628	628	627	627	628	637	637	637	637	636	627	627	627	80	0%

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