On price competition with complementary goods

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Abstract

We consider a duopoly industry with two separate firms each selling an indivisible product. The joint consumption of these goods has a specific value for the consumers which exceeds the mere addition of utilities when products are consumed in isolation : the higher this excess, the larger the complementarity between the goods. We analyse price equilibria in this market as related to the degree of complementarity existing between the two products.

1. Consider a market with two separate firms each offering an indivisible good. Each good can be consumed separately but joint consumption is also an available alternative for the consumer. Furthermore, we assume that products are complements, in the sense that joint consumption yields a larger utility than the mere addition of utilities obtained when products are consumed in isolation. On the other hand, it is as well meaningful to consume one of them at the exclusion the other.

As an example, consider the following situation. When buying a new dress, a woman can be thought of as deriving utility from the comfort obtained from wearing the dress. Should we therefore expect a woman to buy only a single dress among the existing ones, possibly the most comfortable one, and wear it out? This possibility could make sense to an economist but certainly not to a "real- life" woman. Should she buy two identical dresses, and wear them alternatively? Again this is a possibility, a bit more sensible. It is intuitive however that there is something to be gained when buying two different dresses: namely the possibility of choosing between the two dresses and, eventually, combining each of them with different shoes and clothes. In this example, we see distinct goods whose isolated consumption is an available option, but whose joint purchase defines a third good, choosing among the two dresses, in addition to simply wearing them, which may be valuable per se. Therefore the utility accruing from the purchase of the two goods is higher than the mere "addition" of the utilities obtained from their separate consumption.

Another example is as follows.[1] Consider an advertising agency wishing to buy ad-spots to be inserted in newspapers. Buying one spot in a wide-audience newspaper is an option, as well as buying one in a more specialized journal. However, buying spots in both newspapers may also be highly valuable, even if their audiences overlap. In particular, if advertising efficiency is correlated with the repetition of the messages, the benefit of simultaneously placing two ad-spots may exceed the sum of the benefits which would obtain under each separate option.

These two examples share a common feature: joint consumption has a specific value, which is larger than the mere addition of that of separate purchases, but the purchase of one good at the exclusion of the other is also a non-trivial alternative.

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[1]This example is drawn from Gabszewicz, Laussel and Sonnac (1999)

The early contribution of Cournot (1838) already pointed out that, in cases where only joint consumption makes sense, firms set an identical price based only on the value of the joint consumption. The recent literature on competition among complementary components of composite goods (see Matutes and Regibeau (1989), Economides and Salop (1992)) is also related to our context. The main difference with ours is that separate consumption of individual components is nil while that of the composite good is significant. Each firm is accordingly assumed to produce all components, while we assume that each good is produced by a separate firm. The key issue studied in this literature is whether firms should sell separate, fully compatible components or supply the market with a composite good whose components are incompatible with those of the rival firms.

Because, in our model, each product can be consumed at the exclusion of the other, each firm faces a trade-off when choosing its price: either it behaves as a pure monopolist, neglecting the presence of the complementary good, or it designs its pricing strategy taking into account consumers' valuation for joint consumption. Intuition suggests that, because joint consumption is valued beyond mere addition, the firm can charge a higher price if it sells only to those who consume both products. As a counterpart, the firm's sales depend on the total price of the bundle, i.e. on the other's price as well. Clearly, the issue of the trade-off will depend on the relative valuation of separate consumption, as compared with that of the "joint" product, i.e. on the degree of product complementarity. In the proposition below, we show that when the complementarity is weak, only asymetric equilibria exist at which one firm behaves as a pure monopolist while the other benefits from the complementarity of the two goods. In this case, there are consumers who buy only the monopoly product and others who buy both. When complementarity is strong, a unique symmetric equilibrium prevails where both firms coordinate on the value of the bundle. All consumers then buy simultaneously the two products: This is the Cournot result referred to above. For intermediate values, both types of equilibria coexist.

In order to establish our results we rely on a standard modelling tool for analyzing markets with vertically differentiated products, based on the preferences "à la Mussa-Rosen". The only difference is that we now allow the consumers to buy simultaneously the two products while it is generally assumed that consumers make mutually exclusive purchases. As it will soon appear,

this is sufficient to dramatically alter the nature of price competition.

2. Consider a model with two products indexed by their quality u_i, $i = 1, 2$. Without loss of generality, we assume that $u_2 \geq u_1$. Firms sell at zero cost. They choose prices non-cooperatively in order to maximize profits.

Consumers are indexed by a parameter θ, which expresses the intensity of their preferences for buying a unit of the good. The utility derived by consumer θ when buying one unit of good i is given by

$$u_i\theta - p_i, \tag{1}$$

where p_i denotes the price of good i. When buying the two goods simultaneously, a consumer can be viewed as consuming a new good of quality u_3. Our key assumption in the following is that

$$u_3 \geq u_1 + u_2.$$

We suppose that consumer θ, if he buys, buys either one unit of a good, or one unit of the "bundle".[2] In what follows, the difference between u_3 and $u_1 + u_2$ will be used as a proxy for the degree of complementarity.[3]

The problem of a consumer therefore summarizes as

$$Max\{0, u_1\theta - p_1, u_2\theta - p_2, u_3\theta - (p_1 + p_2)\}. \tag{2}$$

Let us denote by $\overline{\theta_i}$ the consumer who is indifferent between buying good i or not buying. Using (2) we obtain:

$$\overline{\theta_1} = \frac{p_1}{u_1} \; , \overline{\theta_2} = \frac{p_2}{u_2} \; , \overline{\theta_3} = \frac{p_1 + p_2}{u_3} \tag{3}$$

Demand addressed to each firm is then easily derived. By definition, the set of consumers willing to buy product i, given p_i, is defined by the interval $[\overline{\theta_i}, 1]$. It may be the case however that among these consumers who are willing to buy i, some are also willing to buy j, in which case they buy both because $u_3 \geq u_1 + u_2$. Consider for instance p_2 such that $\overline{\theta_2} < 1$, i.e. some

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[2]Notice that we do not allow for the purchase of two units of an identical product. This assumption reflects the idea that what is specifically valued when two products are bought is precisely that they are different.

[3]In the extreme case where $u_3 = u_1 + u_2$, products can even be viewed as independent. Caillaud,Grilo and Thisse (1999) consider a case where $u_3 < u_1 + u_2$.

consumers are willing to buy product 2. The demand addressed to firm 1 is constructed as follows. When setting a large p_1, firm 1 first sells its product to those consumers with high θ, i.e. those consumers who exhibit a strong taste for the good. They are also those who are willing to buy product 2. Accordingly, firm 1's sales are defined by those consumers who buy the bundle. Demand is thus defined by $\overline{\theta_3}$. Eventually, for sufficiently low prices, firm 1 will sell not only to those interested in buying the bundle but also to some consumers who are willing to buy product 1 only, given p_2. This will happen whenever $\overline{\theta_1} < \overline{\theta_3}$. Demand addressed to firm i is therefore defined either by $1 - \overline{\theta_3}$ or by $1 - \overline{\theta_i}$. The latter definition holds whenever $\overline{\theta_i} < \overline{\theta_3}$. Formally we obtain:

$$D_1(p_1, p_2) = \begin{cases} 1 - \frac{p_1}{u_1} & \text{if } p_1 \leq p_2 \frac{u_1}{u_3 - u_1} \\ 1 - \frac{p_1 + p_2}{u_3} & \text{if } p_1 \geq p_2 \frac{u_1}{u_3 - u_1} \end{cases} \tag{4}$$

$$D_2(p_1, p_2) = \begin{cases} 1 - \frac{p_2}{u_2} & \text{if } p_2 \leq p_1 \frac{u_2}{u_3 - u_2} \\ 1 - \frac{p_1 + p_2}{u_3} & \text{if } p_2 \geq p_1 \frac{u_2}{u_3 - u_2} \end{cases} \tag{5}$$

Notice that these demands are kinked but not concave. Accordingly payoffs might not be concave in own price. In order to characterize the set of Nash equilibria of the pricing game, we first identify three critical regions in the space of prices, according to their resulting demands. This is done in Figure 1.
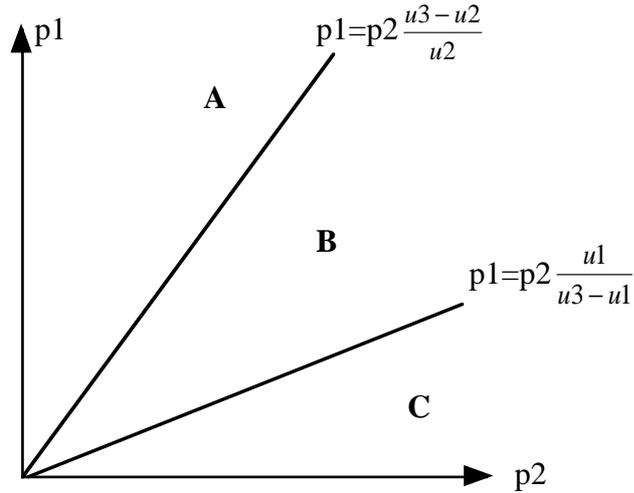

Figure 1

In region A, demand addressed to firm 1 is given by $1 - \frac{p_1 + p_2}{u_3}$ whereas firm 2's demand

is given by $1 - \frac{p_2}{u_2}$, i.e. in region A firm 1 sells only to consumers buying the two products whereas firm 2 enjoys in addition "exclusive" buyers. Formally, firm 2's demand is that of a pure monopolist. In region C, the contrary prevails and, in region B, both firms' sales are defined by the demand for the bundle. Two remarks are in order here. First, whenever u_3 tends to $u_1 + u_2$ region B shrinks and eventually disappears for the case where $u_3 = u_1 + u_2$, i.e. when buying the two products has no value per se. Alternatively, should isolated consumptions yield negligible utilities, only region B would be relevant. Second, in region A and C, one of the firm's payoffs does not depend on the rival's price.

Let us now derive best replies. Due to the fact that payoffs are not quasiconcave, best replies exhibit discontinuties which reflect the two possible strategy profiles available to the firms: either it plays "on the value" of the bundle with high prices or it acts as a monopolist over a larger market with lower prices. We identify the first case as the "complementarity regime" and the other as the "monopoly regime".

Consider firm 1. Playing on the complementarity of the products yields a payoff equal to $p_1(1 - \frac{p_1+p_2}{u_3})$. This payoff is valid only in regions A,B. Using the first order necessary condition we identify the candidate best reply $b_1(p_2) = \frac{u_3-p_2}{2}$. It yields a payoffs $\pi_1^c(p_2) = \frac{(u_3-p_2)^2}{4u_3}$. Playing as a monopolist amounts to quote $p_1^M = \frac{u_1}{2}$, yielding a payoff $\pi_1^M = \frac{u_1^2}{4}$. This payoff is valid only in region C. In order to identify the "true" best reply of firm 1, it remains to compare the payoffs in these two cases. Solving $\pi_1^c(p_2) = \pi_1^M$ we identify a critical price for firm 2, denoted $\hat{p_2}$, at which firm 1 switches from the complementarity regime to the monopoly regime. A similar argument applies for firm 2. We may therefore summarize firms' best replies ϕ_i as follows

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\phi_i(p_i, p_j) = \begin{cases} \frac{u_i}{2} & \text{if } p_j \geq \hat{p_j} = \sqrt{u_3}(\sqrt{u_3} - \sqrt{u_i}) \\ b_i(p_j) = \frac{u_3-p_j}{2} & \text{if } p_j \leq \hat{p_j} = \sqrt{u_3}(\sqrt{u_3} - \sqrt{u_i}) \end{cases} \tag{6}
$$

In order to identify the set of price equilibria, notice first that for firm 1 to be on the first branch of its best reply, it must be that the pair of prices is in region C on figure 1. At the same time, firm 2's best reply in region C is defined by the second branch of (6). There exists therefore an asymmetric Nash equilibrium candidate with firm 1 playing "monopoly" and firm 2 playing "complementarity"; it is given by $(p_1^M, b_2(p_1^M)) = (\frac{u_1}{2}, \frac{2u_3-u_1}{4})$. Obviously there exists a similar

asymmetric candidate where firm 2 plays "monopoly" in region A, namely $(\frac{2u_3-u_2}{4}, \frac{u_2}{2})$. Finally, there is a symmetric Nash candidate equilibrium with both firms playing "complementarity", in region B, which is $(p_1^*, p_2^*) = (\frac{u_3}{3}, \frac{u_3}{3})$. It is then a matter of computation to identify under which parametric constellations each of these candidates is indeed an equilibrium. As summarized in Proposition 1, one, two or three equilibria may coexist.

Proposition 1 *When goods are weakly complementary ($u_3 \in [u_1 + u_2, \frac{9}{4}u_2]$), only asymmetric "monopoly" equilibria exist. When goods are strongly complementary ($u_3 \geq (\frac{3}{2} + \sqrt{2})u_2$), the unique equilibrium is the symmetric "complementarity" one. For intermediate complementarity, the two types of equilibria coexist.*

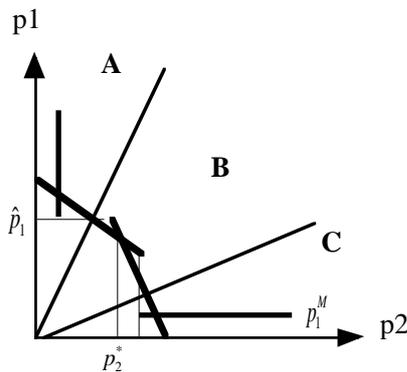Figure 2 illustrates our Proposition. A formal proof of it can be found in the appendix.

Figure 2

3. As stated above, it is known since Cournot (1838) that when two distinct goods are useful only as components of a third one, firms set a unique price, reflecting only the value of the composite good, irrespective of the specific contribution of each good to this value. This conclusion also holds in our setting when goods are strongly complementary: even though products do not contribute in the same proportion to the value of the bundle, firms quote identical prices at equilibrium. At the other extreme, when the value of joint consumption is simply defined as the sum of the separate consumption values, there is nothing specifically valuable in consuming the bundle. In this case, firms strategies are fully independent and each

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sets its monopoly price defined with reference to its own value u_i. This occurs when $u_1 + u_2 = u_3$. Our analysis has focused on all the cases of intermediate complementarity that lie between these two extremes. Its outcome may then be summarized as follows. Because the bundle exhibits a specific value, it may be profitable for the firm to design its strategy on the value of the bundle. However, because this strategy implies using a higher price, it has a cost: that of reducing market share by loosing that part of the market which is willing to consume only one of the two products (low θ's). When complementarity is high, this dilemma is easily solved because the value of the bundle is significantly higher so that the price effect dominates the market sales effect. For intermediate complementarity, the dilemma cannot be solved so easily. The surplus obtained from joint consumption is not large enough to be shared between the two firms, so that asymmetric equilibria arise at which only one firm appropriates the value attached to the bundle leaving no other choice to the other but to maximize its monopoly profit. There is a coordination problem here because each firm would like to benefit from the externality that arises from the existence of the complementary product. On the contrary, when the externality is large, one firm cannot expect the other to let him capture alone the full benefit of the complementarity. As a consequence, only the symmetric equilibrium survives.

It is straightforward to show that whenever they coexist (intermediate complementarity), the aggregate profits under asymmetric equilibria dominate aggregate profits at the symmetric one. Therefore, although the symmetric equilibrium may seem collusive because firms coordinate their decisions on the value of the bundle, the symmetric equilibrium does not maximize joint profits. Notice also that the asymmetric equilibria look like Stackelberg equilibria. Think for instance of a time sequence where good 1 is introduced first in the market, firm 1 enjoying accordingly a monopoly position. Then a second, complementary product is introduced by firm 2. A typical prediction of proposition 1 is the following: if complementarity is not too high, the second mover is likely to take full advantage of the complementarity, irrespective of the value of its good when considered in isolation. This is exactly what happens at the asymmetric equilibria.

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A Appendix: Proof of Proposition 1

There are three candidate Nash equilibria, which we identify according to the region in which they are defined.

- candidate A: $p_1^A = \frac{u_3 - p_2^M}{2} = \frac{2u_3 - u_2}{4}$, $p_2^M = \frac{u_2}{2}$

- candidate B: $p_1^* = p_2^* = \frac{u_3}{3}$

- candidate C: $p_1^M = \frac{u_1}{2}$, $p_2^C = \frac{u_3 - p_1^M}{2} = \frac{2u_3 - u_1}{4}$

A necessary and sufficient condition for candidate A to be a Nash equilibrium is that $p_1^A \geq \hat{p_1}$. Computations indicate that this is satisfied whenever $u_3 \in [u_1 + u_2, (\frac{3}{2} + \sqrt{2})u_2]$.

A necessary and sufficient condition for candidate C is that $p_2^A \geq \hat{p_2}$. Computations indicate that this is satisfied whenever $u_3 \in [u_1 + u_2, (\frac{3}{2} + \sqrt{2})u_1]$

A necessary and sufficient condition for candidate B is that $p_i^* \leq Min\{\hat{p_1}, \hat{p_2}\}$. Since $u_1 \leq u_2$, $Min\{\hat{p_1}, \hat{p_2}\} = \hat{p_2}$. Computations indicate that candidate B is a Nash equilibrium whenever $u_3 \geq \frac{9}{4}u_2$.

Taking into account our assumptions $u_1 \leq u_2$ and $u_3 \geq u_1 + u_2$, we can identify four critical regions as a function of u_3.

1. Whenever $u_3 \in [u_1 + u_2, \frac{9}{4}u_3[$, only candidate A and C are Nash equilibria.

2. Whenever $u_3 \in [\frac{9}{4}u_3, (\frac{3}{2} + \sqrt{2})u_1[$, candidate A,B and C are Nash equilibria

3. Whenever $u_3 \in [(\frac{3}{2} + \sqrt{2})u_1, (\frac{3}{2} + \sqrt{2})u_2[$, candidate A and B are Nash equilibria

4. Whenever $u_3 \geq (\frac{3}{2} + \sqrt{2})u_2$, the unique Nash equilibrium is candidate B.