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QUALITY CHOICE IN MODELS OF VERTICAL DIFFERENTIATION*

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In this note, we offer the complete characterization of quality choices in a duopoly model of vertical product differentiation where firms simultaneously choose the quality of the product and then compete in prices. We thereby give precise content to the "principle of differentiation" in models of vertical product differentiation, which completes and amends previous results on the subject.

I. INTRODUCTION

OLIGOPOLIES in which firms sell products of different qualities have been analyzed first in Gabszewicz and Thisse [1979]. They showed that price competition could yield equilibrium market outcomes where some consumers prefer to refrain from buying or outcomes where all consumers buy one of the two products. The degree of product differentiation and the extent of consumers' heterogeneity determine which alternative actually realizes. The intuition that firms are led to choose products of different qualities has been further investigated in Shaked and Sutton [1982]. Assuming that firms do not cover the market, Choi and Shin [1992] show that the lower quality firm will choose a quality level which is a fixed proportion of the higher quality firm's choice. Moorthy [1988] considers the problem of quality choice in a duopoly, assuming the existence of a quadratic cost function for quality. He also focuses on uncovered market configurations. On the contrary, Tirole [1988] assumes that firms cover the market and shows that, in this case, firms maximize product differentiation over the available range of qualities. In the present note, we provide a full characterization of quality choice, *without assuming ex ante that the market is, or is not, covered.* This allows us to show that covered or uncovered market are *endogenous outcomes* of the quality game. To this end, we

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consider a two-stage game where firms first choose the quality level simultaneously and then compete simultaneously in price.

II. PRICE COMPETITION

We assume that two firms are selling products of different qualities to a population of consumers differing in their "taste for quality".[1] Consumers' preferences are described as follows: a consumer, identified by θ, enjoys utility $U(\theta) = \theta s - p$ when consuming a product of quality s sold at a price p. His utility is zero if he refrains from buying. The population of consumers is described by the parameter θ which is uniformly distributed between θ^- and θ^+, with $\theta^+ > \theta^- > 0$ (the density in the interval $[\theta^-, \theta^+]$ is $\frac{1}{\theta^+ - \theta^-}$). Firm i produces at zero cost a good of quality s_i and sells it at price $p_i, i = 1, 2$. Qualities are exogenous in the price game and we assume $s_2 > s_1$. Firm 2 is thus the top quality firm.

Demand addressed to firm i is defined by the set of consumers who maximize utility when buying product i, rather than product j or refraining from buying. Given (p_1, p_2), we denote by $\bar{\theta}(p_1, p_2)$ the marginal consumer who is indifferent between consuming either of the two products. By definition he satisfies $\bar{\theta}(p_1, p_2)s_1 - p_1 = \bar{\theta}(p_1, p_2)s_2 - p_2$. Accordingly, consumers with $\theta > (<)\bar{\theta}(p_1, p_2)$ strictly prefer product 2 (1). Some consumers could also refrain from buying at prevailing prices. In particular, we denote by $\theta_1(p_1)$ the consumer who is indifferent between buying product 1 and refraining from buying. He is defined as the solution to $\theta_1 s_1 - p_1 = 0$; any consumer of type $\theta < \theta_1(p_1)$ refrains from buying. In this case, the market is uncovered. Finally, since $s_2 > s_1$, all consumers prefer product 2 to product 1 when $p_1 = p_2$. Firm 2 thus benefits from the possibility of preempting the market with a limit price, $p_2^l = p_1 + \theta^-(s_2 - s_1)$.

Three market configurations may arise at the price equilibrium.[2] They are characterized by the following demand functions:

(1) Uncovered market: $D_1(p) + D_2(p) < 1$, with $D_i(p) > 0$ for $i = 1, 2$

$$\begin{cases} D_1(p_1, p_2) = \dfrac{1}{\theta^+ - \theta^-}\left(\dfrac{p_2 - p_1}{(s_2 - s_1)} - \dfrac{p_1}{s_1}\right); \\ D_2(p_1, p_2) = \dfrac{1}{\theta^+ - \theta^-}\left(\theta^+ - \dfrac{p_2 - p_1}{(s_2 - s_1)}\right). \end{cases}$$

[1] In the original analysis of Gabszewicz and Thisse [1979] this difference rests on differences in income. The formulation used here was proposed by Mussa and Rosen [1978].

[2] See Gabszewicz and Thisse [1979] for the first characterization of these configurations.

(2) Covered market: $D_1(p) + D_2(p) = 1$, with $D_i(p) > 0$ for $i = 1, 2$

$$\begin{cases} D_1(p_1, p_2) = \dfrac{1}{\theta^+ - \theta^-}\left(\dfrac{p_2 - p_1}{(s_2 - s_1)} - \theta^-\right); \\ D_2(p_1, p_2) = \dfrac{1}{\theta^+ - \theta^-}\left(\theta^+ - \dfrac{p_2 - p_1}{(s_2 - s_1)}\right). \end{cases}$$

(3) Preempted market: $D_1(p) = 0, D_2(p) = 1$.

Nash equilibrium in the price subgame is obtained in two steps. First, we have to compute equilibrium candidates corresponding to each market configuration. Second, we identify the parameters constellations for which candidates effectively yield the corresponding market outcome. We identify four intervals for the values of $\dfrac{\theta^+}{\theta^-}$ whose bounds depend on (s_1, s_2), i.e. on the degree of product differentiation. Formally, we obtain the following equilibrium outcomes:

(A) The market is not covered at equilibrium whenever $\dfrac{\theta^+}{\theta^-} \in \left[\dfrac{4s_2 - s_1}{s_2 - s_1}, \infty\right[$.

The Nash equilibrium is given by

$$\begin{cases} p_1^{**} = \theta^+(s_2 - s_1)\,\dfrac{s_1}{4s_2 - s_1}; \\ p_2^{**} = \theta^+(s_2 - s_1)\,\dfrac{2s_2}{4s_2 - s_1}. \end{cases}$$

(B) The market is covered with firm 1 quoting the price which is just sufficient to cover the market[3] whenever $\dfrac{\theta^+}{\theta^-} \in \left[\dfrac{2s_2 + s_1}{s_2 - s_1}, \dfrac{4s_2 - s_1}{s_2 - s_1}\right[$.

The equilibrium is given by

$$\begin{cases} p_1^c = \theta^- s_1; \\ p_2^c = \dfrac{\theta^- s_1 + \theta^+(s_2 - s_1)}{2}, \end{cases}$$

(C) The market is covered in the usual sense whenever $\dfrac{\theta^+}{\theta^-} \in \left]2, \dfrac{2s_2 + s_1}{s_2 - s_1}\right[$.

The equilibrium is given by

[3] The presence of this corner solution might be explained as follows: there is a range of parameter values where neither condition $p_1^{**} > \theta^- s_1$ nor $p_1^* < \theta^- s_1$ holds. For these values of the parameters, a corner solution prevails where firm 1 quotes $p_1^c = \theta^- s_1$, the price which makes a consumer of type θ^- indifferent between buying product 1 and not buying at all.

$$\begin{cases} p_1^* = \dfrac{\theta^+ - 2\theta^-}{3}(s_2 - s_1); \\ p_2^* = \dfrac{2\theta^+ - \theta^-}{3}(s_2 - s_1). \end{cases}$$

(D) The market is preempted by firm 2 whenever $\dfrac{\theta^+}{\theta^-} \in]1, 2]$. The equilibrium is given by

$$\begin{cases} p_1^l = 0; \\ p_2^l = \theta^-(s_2 - s_1). \end{cases}$$

We summarize the outcomes of the price game in the following proposition:

Proposition 1 (Gabszewicz and Thisse [1979]). The Nash equilibrium in prices and the associated market outcomes are determined as a function of the degree of population heterogeneity (θ^+, θ^-) and the degree of product differentiation (s_1, s_2), in accordance with the domain defined in (A), (B), (C), (D).

III. CHOICE OF QUALITY LEVELS

Now, we consider the choice of quality levels by firms in an interval $[0, s^+]$.[4] We develop the analysis of firm 1's best replies against s_2, a symmetric analysis prevails for firm 2. The first intuition is that choosing $s_1 = s_2 < s^+$ cannot be a best reply since it yields Bertrand competition (i.e. zero profits in the price game). From the analysis of the price game, it is also clear that the best reply in s_1 will differ according to whether we consider a reply in $]s_2, s^+]$ or in $[0, s_2[$. In the former case firm 1 is the best quality firm and this allows her to sell to high θ, in the latter case, firm 1 is the low quality firm and sells to low θ.

Best replies in the domain $[s_2, s^+]$ are easily determined. Profits of the top quality firm are increasing in its quality, whatever the market configuration, so that the best reply for firm 1 against any $s_2 < s^+$ is $s_1 = s^+$, i.e., choosing the best available quality.

The analysis of the best reply in the domain $[0, s_2]$ is more involved. Indeed, when choosing its quality, the low quality firm will determine the relevant configuration in the price subgame. A low quality will tend to lead to an uncovered market configuration whereas a high quality will lead either to a corner solution or to a covered configuration. However, the critical values defining the limit for each configuration depend on the

[4] We concentrate on configurations where the two firms are active in the price game. A sufficient condition is $\dfrac{\theta^+}{\theta^-} > 2$.

distribution of consumers' tastes. This clearly appears by rearranging the conditions identified in (A), (B), (C) as follows.

(A') The market is not covered in the price game whenever $s_1 < s_2 \dfrac{\theta^+ - 4\theta^-}{\theta^+ - \theta^-}$.

(B') The market is covered with a corner solution whenever

$$s_1 \in \left[s_2 \frac{\theta^+ - 4\theta^-}{\theta^+ - \theta^-}, s_2 \frac{\theta^+ - 2\theta^-}{\theta^+ + \theta^-}\right].$$

(C') The market is covered with an interior solution whenever

$$s_1 > s_2 \frac{\theta^+ - 2\theta^-}{\theta^+ + \theta^-}.$$

Therefore the analysis proceeds in two steps. First we compute firm 1's best reply conditional to each of the three configurations for the price game and identify the range of population parameter values in which they are defined, according to (A'), (B') and (C'). Second, when (θ^+, θ^-) take values such that a reply is defined for two configurations, we compare corresponding profits in order to identify the best reply.

• The uncovered configuration: Against s_2, firm 1's best reply in the uncovered configuration, denoted by s_1^{**}, is[5]

$$s_1^{**} = \frac{4}{7} s_2$$

We may check that for $s_1^{**} = \frac{4}{7} s_2$, the condition (A') is met if and only if $\frac{\theta^+}{\theta^-} > 8$. Thus we conclude that the reply $s_1^{**} = \frac{4}{7} s_2$ yields an uncovered market equilibrium outcome in the price game only when $\frac{\theta^+}{\theta^-} > 8$.

• The corner solution: At the corner solution, firm 1's profits are given by

$$\pi_1^c = \frac{1}{\theta^+ - \theta^-} \left(\frac{\theta^+(s_2 - s_1) - \theta^-(2s_2 - s_1)}{2(s_2 - s_1)} \theta^- s_1\right)$$

The best reply function is derived from the first order condition and is given by

$$s_1^c = s_2 \left(1 - \frac{\sqrt{\theta^-}}{\sqrt{\theta^+ - \theta^-}}\right)$$

It is then a matter of computation, using (B'), to show that this choice of

[5] See Choi and Shin [1992].

quality yields a corner solution in the price game if and only if $\frac{\theta^+}{\theta^-} \in [5, 10]$.

- The covered configuration: In this case, firm 1's profits are given by

$$\pi_1^* = \frac{1}{\theta^+ - \theta^-}\left(\frac{(\theta^+ - 2\theta^-)^2}{9}(s_2 - s_1)\right)$$

Since it is strictly decreasing in s_1, maximal differentiation holds, subject to the restriction that the market is covered. From (C'), the best reply in the covered configuration, denoted by s_1^*, is thus

$$s_1^* = s_2\frac{\theta^+ - 2\theta^-}{\theta^+ + \theta^-}$$

This best reply is defined for all values of $\frac{\theta^+}{\theta^-} \in]2, \infty[$. Notice that $p_1(s_1^*) = \theta^- s_1^*$. Thus, choosing the lowest quality which preserves a covered market outcome in the price game amounts to choose the largest quality which corresponds to a corner solution.

Having considered quality choices for each market configuration separately, we can now compare these choices in order to determine the best reply of the low quality firm in the quality game.

Direct computations show that

1. $\pi_1(s_1^{**}, s_2) > \pi_1(s_1^*, s_2)$ whenever $\frac{\theta^+}{\theta^-} > 8$, i.e. whenever it defines an uncovered outcome in the price game, s_1^{**} beats s_1^*.
2. Whenever $\frac{\theta^+}{\theta^-} \in [8, 10]$, both s_1^c and s_1^{**} are defined. Computations show that $\pi_1(s_1^c, s_2) > \pi_1(s_1^{**}, s_2)$ whenever $\frac{\theta^+}{\theta^-} \in [8, \alpha[$, with α approximately equal to 8,6581.
3. Finally, since s_1^* corresponds to the upper bound for the low quality to define a corner solution, it is always beaten by s_1^c, whenever this last reply is defined, i.e., whenever $\frac{\theta^+}{\theta^-} \in [5, 10]$.

The best replies of firm 2 against s_1 is defined similarly. Given the preceding analysis, it is clear that any subgame perfect equilibrium entails one firm choosing s^+, the other one choosing either s_1^*, s_i^c or s_i^{**}, depending on the value of $\frac{\theta^+}{\theta^-} \in]2, \infty[$. Our findings about firm 1's best reply are summarized in the following.

1. When $\frac{\theta^+}{\theta^-} > 8.6581$, an equilibrium exhibits one firm choosing the best available quality, the other one choosing a fixed proportion of this quality (s_i^{**}). The market is uncovered in the price game (case (A)).

2. When $\frac{\theta^+}{\theta^-} \in [5, 8.6581]$, an equilibrium entails one firm choosing the best quality, the other firm choosing a lower quality, s_1^c, yielding a corner solution in the price game (case (B)).

3. When $\frac{\theta^+}{\theta^-} \in]2, 5[$, an equilibrium entails one firm choosing s^+ the other one choosing s_i^*. The market is covered at the equilibrium of the price game (case (C)).[6]

Our main findings are stated in the next proposition.[7]

Proposition 2. Any subgame perfect equilibrium at which the two firms enjoy positive market shares entails product differentiation. One firm chooses the best available quality and, depending on the population's characteristics, the other firm either chooses a fixed proportion of the best quality or a quality level which is determined by population characteristics. This determines whether the market is covered or not in the price game.

IV. COMMENTS

When studying the quality game, we have imposed no *a priori* restriction on the price game of the second stage. Allowing for either covered or uncovered market outcomes, we have shown that market outcomes at the price stage are endogenous outcomes of equilibrium quality choice. Moreover, equilibrium in the quality game may yield a corner solution in the price game. In this respect, our analysis differs from previous ones where attention was restricted to uncovered market configurations (Choi and Shin [1992], or covered ones (Tirole [1988]).

Choi and Shin [1992] study the equilibrium in a quality game similar to ours. However, they restrict attention to uncovered market outcomes. Formally, this amounts to restricting the set of strategies available for the low quality firm to $\left[0, s^+ \frac{\theta^+ - 4\theta^-}{\theta^+ - \theta^-}\right]$. Since they exclude the possibility of quality choices yielding covered outcomes or a corner solution in the price game, they fail to see that s_i^{**} is not a best reply for values of $\frac{\theta^+}{\theta^-} \in [8, 8.6581[$. Within this interval, the low quality firm gains when

[6] When $\frac{\theta^+}{\theta^-} \leq 2$, any pair of qualities entailing one firm choosing s^+ is an equilibrium pair. The market is preempted (case (D)).

[7] A detailed proof of proposition 2 is available from the author on request.

playing the corner solution. Thus, s_i^{**} cannot be part of an equilibrium for these values of the population's parameters. On the contrary, Tirole [1988] assumes that the market is covered in the price game and considers that quality choices are made in an interval $[s^-, s^+]$. Formally, this amounts to assuming that $s^- \geq s^+ \frac{\theta^+ - 2\theta^-}{\theta^+ + \theta^-}$. A principle of maximal differentiation is then obtained. However, when we allow for quality choice in $[0, s^+]$, we have shown that this solution is systematically dominated by the quality choice yielding a corner solution in the price game. It is only when the latter is not defined that we observe the quality choice identified in Tirole.

Equilibrium quality choices yielding a corner solution in the price game have not been identified previously. Interestingly enough, this is a new result precisely because previous analyses have restricted their scope to particular cases. It relates to the fact that the transition from uncovered market structures to covered ones is not smooth. The nature of competition changes when the market is covered. *This is so because price competition becomes a pure battle for market shares.* Accordingly, in such models of vertical differentiation, it should not be imposed *a priori* that the market is covered or not covered. Covering the market or not is at the heart of the strategic problem for firms.

With respect to the issue of product differentiation, our results may be summarized as follows. The decisive factor is the distribution of consumers' tastes. In markets where the distribution of tastes is broad, we expect that quality choices will result in the market being served partially. In these circumstances the quality differential does not depend on the population attributes. When consumers' tastes are concentrated, a fiercer price competition cannot be avoided by the choice of qualities. In these cases, relaxing price competition calls for an increase in product differentiation. Therefore, we expect here that the quality differential will be negatively related to the population dispersion. In this sense, the degree of heterogeneity in the population places an upper bound to the extent of product differentiation.

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