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Quality underprovision by a monopolist when quality is not costly

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Abstract

In the Mussa and Rosen model [J Econ Theory 18 (1978) 301] of vertical differentiation, a monopolist may optimally choose to underprovide quality if consumers are allowed to buy several units of the indivisible good, even if quality provision involves no cost of any sort.

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1. As is well known by now, the perverse effects of monopoly power are not limited to the price–quantity pair the monopolist sells to the market. The selection of products' attributes is often put forward as a major source of inefficiency. When the monopolist sells several variants of the same product, he sometimes offers too many variants, sometimes too few of them, as compared to first best. Regarding quality provision, two problems have to be considered. The range of qualities offered by the monopolist is one issue but at the same time the level of quality supplied matters per se.

Mussa and Rosen (1978) show that a monopolist facing consumers who are heterogeneous in their willingness to pay for quality will in general offer a menu of price–quality combinations. As compared to the first best outcome, Mussa and Rosen (1978) establish two main results: first, a monopolist tends to enlarge the range of qualities offered and, second, almost all consumers buy lower quality products at the monopolist's optimum. The monopolist thus sells too many products while consumers end up buying too low quality levels. In this note, we show that this last conclusion is quite dependent on the specification of demand. In particular, in the Mussa and Rosen (1978) framework, it

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hangs on the unit demand assumption. Specifically, we show that *the simple fact that consumers are allowed to buy several units of the indivisible good is likely to induce the monopolist to select a quality for his product which is not the highest one, even if no cost of any sort is attached to quality improvement.* In this case, the monopolist underprovides quality in a very well-defined sense.

Acharyya (1998) already showed that the 'menu' result of Mussa and Rosen (1978) was heavily dependent both on the cost of quality upgrading and on the distribution of consumers' type. In particular, if quality is not costly, the monopoly will offer only one quality, the best available one, as its optimal policy. In this case, the monopolist replicates the competitive outcome in terms of quality provision and the only source of inefficiency follows from the pricing policy.[1] If quality upgrading is not costly and consumers value quality upgradings, the efficient quality is the same for all consumers. It is therefore intuitive that the monopolist is likely to maximize quality as would a social planner. On the other hand, Sällström (1999) puts emphasis on the cost side and shows that technological progress may in fact lead to quality deterioration if the new technology introduces a bias towards large scale production.

2. We first recall the standard Mussa and Rosen model. Consider a market for some commodity which is bought in indivisible units by consumers represented as points θ in the unit interval [0,1] with unit density. If consumer θ buys one unit of the good at price p, his utility is given by $\theta u - p$, while, if he does not buy, his utility is given by 0. Market demand is derived by identifying the consumer θ_1 who is indifferent between the alternative of buying one unit of the product at price p or not buying at all. Solving $\theta u - p = 0$, we obtain $\theta_1 = p/u$. Market demand is then given by $D(p, u) = 1 - p/u$.

Suppose that the product is sold by a monopolist *who produces the good at zero cost*, and selects the market price p. Then revenue writes as $R(p, u) = (1 - p/u)p$ and is maximal when $p^* = u/2$ with corresponding revenue $R(p^*, u)$ equal to $u/4$

Assume now that the monopolist is also allowed to choose the quality of his product, and selects it within a given range of variants leading to utility indices u in the interval $[u^-, u^+]$. If no cost is attached to quality improvement, he will always select the top quality leading to the utility index u^+ since equilibrium revenue is increasing in u.

As shown in Acharyya (1998), the monopolist will never offer a menu of qualities in the present setting. The quality u^+ therefore defines *the* optimal choice of the monopolist. Hence, under zero cost assumption, consumers' welfare losses do not result from quality selection.

3. Let us then consider a variant of the standard model where consumers are allowed to buy multiple units of the good, i.e., we relax the unit demand assumption. More precisely we assume that the quantity decision set of each household is extended to also include the possibility of buying *two* units of the indivisible good, and denote by $\phi(u)$ the utility index associated with this new option. Excluding bundling strategies, if a consumer θ buys *two* units of the good at a unit price p, his utility is now assumed to be given by $\theta\phi(u) - 2p$. We shall assume throughout that $u < \phi(u) < 2u$, i.e., marginal utility is decreasing.

What is the optimal policy for the monopolist in this new setting? Denote by θ_2 the consumer who is indifferent between buying at unit price p two units of the good and not buying at all. Solving $\theta\phi(u) - 2p = 0$, we obtain

[1] Kim and Kim (1996) also point to cases where a monopolist offers only one quality in a Mussa and Rosen-like setting.

$$\theta_2 = \frac{2p}{\phi(u)}.$$

Similarly, denote by θ_{12} the consumer who is indifferent between buying at unit price p one unit of the good and two units of it. Solving $\theta\phi(u) - 2p = \theta u - p$, we obtain

$$\theta_{12} = \frac{p}{\phi(u) - u}.$$

Note that, under the assumption $\theta(u) < 2u$, we have $\theta_1 < \theta_2 < \theta_{12}$.

Given a unit price p, consumers in the interval $[\theta_1, \theta_{12}]$ buy only a single unit while those in the interval $[\theta_{12}, 1]$ buy two units. Notice that no consumer would like to buy two units when $p \geq \phi(u) - u$. Consequently, the demand function of the monopolist now becomes

$$D(p,u) = \begin{cases} 1 - \dfrac{p}{u} & \text{if } p \geq \phi(u) - u \\ \left(1 - \dfrac{p}{u}\right) + \left(1 - \dfrac{p}{\phi(u) - u}\right) & \text{if } 0 \leq p \leq \phi(u) - u \end{cases}$$

Fig. 1 depicts such a (kinked) demand function $D(p)$.

Keeping the assumption that production takes place at zero cost, the revenue function then writes as,

$$R(p,u) = \begin{cases} \left(1 - \dfrac{p}{u}\right) p & \text{if } p \geq \phi(u) - u \\ \left[\left(1 - \dfrac{p}{u}\right) + \left(1 - \dfrac{p}{\phi(u) - u}\right)\right] p & \text{if } 0 \leq p \leq \phi(u) - u \end{cases}$$

It is now easy to examine how, given the utility index u, the monopolist should revise his pricing decision when the market also includes consumers who may be considering buying more than a single unit of the good. When the price maximising revenue lies above $\phi(u) - u$, it must satisfy the standard first-order necessary condition $1 - 2p/u = 0$, in which case $p^* = u/2$, with a corresponding revenue $R(p^*, u)$ equal to $u/4$.

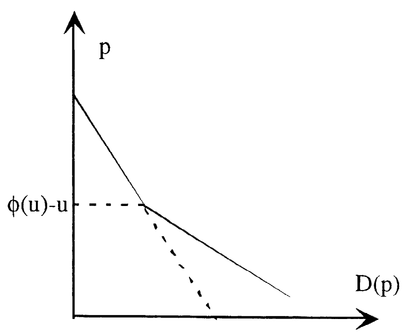

Fig. 1. A (kinked) demand function $D(p)$.

When, on the contrary, revenue is maximal at a price p^{**} which is less than $\phi(u) - u$, the first-order necessary condition then implies that

$$\frac{\partial R}{\partial u} \left| p \leq \phi(u) - u \right| = 2 - \frac{2p}{u} - \frac{2p}{\phi(u) - u} = 0$$

which in turn implies

$$p^{**} = \frac{u[\phi(u) - u]}{\phi(u)}.$$

In that case, it is easy to check that the corresponding demand $D(p^{**}, u)$ is equal to 1, so that the resulting revenue $R(p^{**}, u)$ is equal to p^{**}. From a direct comparison of $R(p^{*}, u)$ and $R(p^{**}, u)$, we see that the monopolist will select a price which induces some customers to buy two units of the good whenever

$$\phi(u) \geq \frac{4u}{3}.$$

We summarize this preliminary finding in the following proposition:

Proposition 1. *Define two subdomains*:

$$D_1 = \left\{ (u, \phi(u)) \colon u < \phi(u) \leq \frac{4u}{3} \right\}, \quad D_2 = \left\{ (u, \phi(u)) \colon \frac{4u}{3} \leq \phi(u) < 2u \right\}$$

Then, p^{*} defines the monopoly solution in D_1, and p^{**} in D_2.

4. Now that we have characterised the *price* monopoly solution when allowing households to buy also two units of the good, we can examine the problem of *quality* selection by the monopolist in $[u^-, u^+]$. Notice first that if, for all values of u in this domain, $(u, \phi(u))$ belongs to D_1, the monopolist chooses the highest quality.

If on the other hand $(u, \phi(u))$ belongs to D_2 for all values of u, $R(p^{**}, u)$ is defined by

$$\frac{u[\phi(u) - u]}{\phi(u)}.$$

The first-order condition can be rearranged as follows:

$$\phi(u)(\phi(u) - u) + u\left(u \frac{\partial \phi(u)}{\partial u} - \phi(u) \right) = 0$$

Second-order conditions being satisfied under our assumptions, the first-order condition immediately shows that the revenue may not be monotone in quality, so that the monopolist may, indeed, select an interior solution in the quality range.

Under which condition should we expect quality underprovision to occur? Note that $\theta(u) - u$ defines the marginal utility derived from the consumption of the second unit. Let us denote it by $v(u)$. Accordingly, we may reexpress $R(p^{**}, u)$ as $(uv(u))/(\phi(u))$ The first-order condition may then be rewritten as

$$v(u)\left[\phi(u) - u\,\frac{\partial\phi(u)}{\partial u}\right] + u\phi(u)\frac{\partial v(u)}{\partial u} = 0$$

For an interior solution to occur within the available quality range, we need that the above expression is negative at u^+. No intuitive expression can be identified for this property to hold true. However, a negative third term should clearly help, i.e., quality underprovision is more likely to occur if marginal utility is decreasing in quality

$$\left(\frac{\partial v(u)}{\partial u} < 0\right).$$

If this is not the case, we need

$$\frac{u}{\phi u}\,\frac{\partial\phi(u)}{\partial u} > 1.$$

However, neither of these conditions are sufficient in isolation, nor are they necessary.[2] The problem of existence of an interior solution leading to quality underprovision has been also identified in Maskin and Riley (1984), in the context of optimal auction theory. In particular, these authors consider the case of a seller facing a unique risk averse buyer. Under conditions on the buyer's utility function which are akin to ours, they show that there may exist consumers? types for which the optimal auction design can be interpreted as quality underprovision, ''even though quality is costless to provide'' (Maskin and Riley, p. 1510).[3]

In order to illustrate the previous discussion, let us consider the following example. Let the domain $[u^-, u^+]$ be the interval $[\frac{2}{3}, \frac{6}{3}]$ and assume further that $\phi(u) = 1 + (u/2)$.

It is easy to check that, for all u in the admissible domain $[\frac{2}{3}, \frac{6}{5}]$, the corresponding pair $(u, \phi(u)) \in D_2$. Therefore, it is always optimal to select the price p^{**} so that $R(p^{**}, u)$ is defined $(u(2-u))/(2+u)$.

Since

$$\frac{\partial^2 R(p^{**}, u)}{\partial u^2} < 0,$$

the necessary and sufficient condition for $R(p^{**}, u)$ to reach a maximum in u is that

$$\frac{\partial R(p^{**}, u)}{\partial u} = 0.$$

Denoting by $\hat{u}$ the solution of this equation, we have

$$\hat{u} = 2(\sqrt{2} - 1) < \tfrac{6}{5}.$$

The following proposition summarizes our main finding:

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[2] Obviously, the quality range may also allow for quality selections leading to D_1 and D_2. Then, the monopolist may still underprovide quality provided u^+ is not too high.

[3] We are grateful to an anonymous referee for attracting our attention on this analogy.

Proposition 2. *In the Mussa and Rosen model of vertical differentiation where consumers may buy several units of the products, the monopolist may not find it optimal to choose the best available quality even though all consumers value quality positively and quality upgrading is not costly.*

The intuition underlying our result should be clear by now. When consumers are allowed to buy two units of the product within the standard Mussa and Rosen's framework, the monopolist faces a new trade-off when selecting quality: when quality increases some consumers who were previously inclined to buy two units now prefer to buy only one of it, so that, given price, aggregate demand would decrease. In order to keep the demand level unchanged the monopolist must therefore decrease its price. Recalling that $\phi(u) - u$ measures the marginal utility of the second unit, we may note that in our example, the marginal utility is in fact decreasing in quality. Accordingly, raising quality increases demand for the first unit but lowers it for the second one.[4]

5. Two objections naturally come to mind when considering the construction of Proposition 2. The first one is that the result may not be robust to bundling and the second one is that it is not robust to quality menus. We briefly discuss these objections hereafter.

Let us first consider the quality menus argument. Intuition suggests indeed that the monopolist could try to offer several qualities, including the best available one, in order to solve the trade-off he faces when upgrading quality. He would thereby try to induce consumers to buy the best quality as a first unit and a lower quality (and thus cheaper) one as a second unit. Recall however that in the standard model with unit demand, the monopolist never finds it optimal to offer several qualities (Acharyya, 1998). Suppose then that the monopolist contemplates the possibility of offering two qualities, u_1 and u_2 in our multiple unit model. From the consumers' viewpoint, there are now five purchasing options: One unit of u_1, one unit of u_2, two units of one of them and (u_1, u_2) jointly at price p_1, p_2, $2p_1$, $2p_2$, $p_1 + p_2$, respectively. Extending our decreasing marginal utility assumption $(\phi_i < 2u_i)$ to the joint purchase case by assuming that $\phi(u_1, u_2) < u_1 + u_2$, we immediately notice that from the consumers viewpoint, offering two qualities does not enlarge the range of net utilities that can be derived from the quality range $[u^-, u^+]$. Accordingly, from the monopolist's viewpoint, the prospect cannot be better than what could be derived by selecting five qualities from the quality range in the unit demand model.[5] From Acharyya (1998), we may then conclude that offering a quality menu cannot increase profits.

Let us now turn to the second objection: selling bundles. This objection is of course more compelling and indeed the monopolist would use quantity discrimination if he was allowed to do so.

Denote by π the price proposed for the bundle. Monopolist's revenue then writes as

$$R(\pi, \phi(u)) = \left(1 - \frac{\pi}{\phi(u)}\right)\pi$$

and reaches its maximal value at $\pi^* = (\phi(u))/2$ with a corresponding equilibrium revenue $R(\pi^*,$

[4]Note that this last property is not necessary for our result to hold true: similar results would obtain for instance with $\phi(u) = u(2 - (u/\alpha))$ with $\alpha > 1$.

[5]The prospect is in fact even worse since it faces additional constraints (on price and quality derived from the consumption of two units).

$\phi(u)$) equal to $\phi(u)/4$. It is easy to see that it is more profitable for the monopolist to sell bundles only.[6]

Since in that case revenue $R(\pi^*, \phi(u))$ is again monotonically increasing in the quality u, quality underprovision disappears. Notice however that this is achieved at the cost of another inefficiency: that associated with the fact that the good is priced in such a way that only bundles are sold. Moreover, this presumes that bundling itself is robust to arbitrage while in the present setting, arbitrage might be in fact very easy to perform.

Consider the consumer $\theta = \frac{1}{2}$ who is indifferent between buying a bundle of two units at price π^* and not buying at all. This consumer would be willing to resell one of the two units in the bundle at any price which would exceed the price p^0 defined by the condition

$$\frac{u}{2} - \frac{\phi(u)}{2} + p^0 = 0:$$

indeed, the left-hand term of this equality is his utility level after buying the bundle at price π^* and reselling one unit of it at price p^0, while the second term is his utility level after the purchase of the bundle at price π^*. On the other hand, any consumer θ in the interval

$$\left[\frac{\phi(u) - u}{2u}, \frac{1}{2}\right]$$

would be willing to buy such a unit at price $p^0 = \frac{1}{2}[\phi(u) - u]$, because he would then reach a utility level equal to $\theta u - \frac{1}{2}[\phi(u) - u]$, which is positive if

$$\theta \in \left[\frac{\phi(u) - u}{2u}, \frac{1}{2}\right].$$

Since

$$\frac{\phi(u) - u}{2u} < \frac{1}{2}$$

due to the assumption $\phi(u) < 2u$, the set of consumers willing to buy a unit of the good at price p^0 is non-empty, giving rise to an advantageous transaction between consumer $\theta = \frac{1}{2}$ and any consumer θ in the interval

$$\left[\frac{\phi(u) - u}{2u}, \frac{1}{2}\right].$$

Certainly the monopolist is willing to avoid such a threatening competition; this should prevent him to restrict the choice of consumers to the sole purchase of bundles, thus opening also the faculty of buying a single unit of the good as well. This is probably why the sale of bundles may not be so pervasive in market for durable goods.

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[6] Notice that selling bundles only is not surprising in view of Acharyya's (1998) result. Indeed, selling bundles only amounts, de facto, to offer only one product (the bundle).

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