Optimal Population Growth and Social Security Reform with Heterogeneous Agents[*]

Gemma Abío
abio@eco.ub.es

Ció Patxot
patxot@eco.ub.es

Grup de Recerca en Economia de la Política Social (EPS)
Universitat de Barcelona
Facultat de Ciències Econòmiques i Empresarials
Departament de Teoria Econòmica
Avda. Diagonal, 690
08034 Barcelona (Spain)

Abstract

In this paper we propose a pension policy that would isolate the social security system from any financial crisis resulting from changes in population structure. This policy consists of linking social security benefits to the fertility behaviour of the individual. We present a theoretical analysis to show that this policy restores the optimality of the capital-labour ratio and the population growth rate in an overlapping-generations model with endogenous fertility. We extend this analysis to the case of heterogeneous agents with respect to their preferences towards children.

---

1. Introduction

In recent decades, declining fertility together with increasing longevity has resulted in population ageing in most OECD countries. This, in turn, has placed financial pressure on pay-as-you-go (PAYG) pension systems since any increase in the dependency ratio needs to be matched by a cut in pension expenditures[1] and/or an increase in payroll taxes. If neither of these reforms is implemented, the system starts to accumulate debt, and eventually a solution must be found in order to finance the system. This accumulation of debt will unavoidably appear in the following decades unless drastic reforms are undertaken. The current debate on social security reform is concerned with just these issues.

Indeed, the present demographic crisis facing the social security system is a direct result of the way in which the system is defined. A PAYG system in its purest form is one in which the pensions distributed to the retired population each period are derived from the contributions received from the active population in the same period. In this way the system is financially balanced every year. However, in most countries the system guarantees a pension benefit which is solely dependent on wages[2] and not on demographic evolution. This may never come to constitute a problem as long as the population structure remains stable. Yet when a negative demographic shock occurs, the system will not collect enough money to pay out all the pension benefits, and so it slides into crisis. To make it financially feasible, pension benefits should be defined in such a way that they satisfy the budgetary constraints of the system; that is, they should be proportional to future wages and to the population growth rate.

The role of intergenerational transfers in overlapping-generations (OLG) models was first analysed within an exogenous fertility framework. In such a framework, it has been well established that the *laissez-faire* equilibrium is generally different from the social optimum. A variety of instruments can be introduced in the competitive economy in order to achieve the optimal capital-labour ratio, defined by the golden rule.

Samuelson (1975) derived the optimal growth rate for population in the simple two-period Diamond OLG model, and showed that even if we satisfy the golden rule,

---

[1] Either by reducing pension benefits or by postponing the retirement age.
[2] In most countries, the pension formula establishes a replacement rate on an average of labour earnings of the last years of work, adjusting it for inflation.

we might not be in the social optimum as long as the population growth rate (n) differs from its optimal value. Hence, there is not only one but two potential inefficiencies that must be resolved in the competitive equilibrium. One is the achievement of the optimal population growth rate and the other is the achievement of the golden rule. In fact, Samuelson showed that if the population growth rate stands at its optimal value, the *laissez-faire* equilibrium and the social optimum will coincide[3].

The fact that the *laissez-faire* equilibrium is different from the social optimum in OLG models with exogenous population is after all perfectly reasonable. What would have been surprising is that for a given –non-chosen, thus most probably non-optimal– n, the social optimum was reached. Obviously, an endogenous fertility setting is needed to analyse policy instruments that lead the *laissez-faire* economy to the social optimum.

This paper undertakes an analysis of social security reform by using an OLG framework with endogenous fertility. Our proposal for reform is to link, within a PAYG system, pension benefits to the fertility behaviour of agents. We conduct a theoretical analysis of the faculty of this proposal to restore optimality in the competitive economy. We show that an economy with a social security system that links an individual's future pension benefit to the number of her children achieves the social optimum if the payroll tax is appropriately fixed by the government. Moreover, this policy isolates the pension system from any crisis due to demographic shocks. We approach optimality by maximising the steady state utility of a representative agent. Previous studies on optimal population growth in an OLG framework with endogenous fertility include Eckstein and Wolpin (1985) and Bental (1989). The latter reaches a similar result to that presented here, but in his model individuals have children only for the investment motive, while here agents derive utility from having descendants.

Next, we allow for heterogeneity in the preferences of individuals towards children. We assume that, while some individuals like children, others do not, implying that they will only have descendants for the investment motive. In this case, the implementation of the policy mentioned above also leads to the optimal rate of population growth and the optimal capital-labour ratio, but additional measures are needed to redistribute wealth among agents according to the social welfare function

---

[3] This is the so-called "serendipity theorem". Deardorff (1976) showed that under certain conditions Samuelson's optimal rate of population growth actually leads to a minimum of utility. See also Michel and Pestieau (1993) and Jaeger (1989).

chosen. Heterogeneity with respect to the ability of having children can also be introduced, with no effects on the results presented.

In section 2 we analyse the potential divergences between the *laissez-faire* economy and the social optimum in our model. In section 3 we focus on the case of homogeneous agents, and in section 4 on the case of heterogeneous agents. Section 5 concludes with some final remarks regarding implementation of the policy proposal.

2. Divergences between the laissez-faire economy and the first best in OLG models with endogenous fertility

Throughout the paper, we will employ a simple three-period overlapping-generations model. Agents consume $c_{1,t}$ and $c_{2,t+1}$ in the two periods of their adult life, and a fixed amount e of resources during childhood. The production function is given by $F(K_t, N_t)$, where K_t is the stock of capital and N_t the stock of labour. In intensive form, the production function is expressed as $f(k_t)$, with k_t being the capital-labour ratio.

The resource constraint of the economy in per capita terms of the working population is:

$$f(k_t) + (1-\delta)k_t = c_{1,t} + \frac{c_{2,t}}{1+n_{t-1}} + e(1+n_t) + k_{t+1}(1+n_t) \qquad (1)$$

where n_t is the growth rate of population in period t, and δ is the rate of capital depreciation.

Equation *(1)* is the constraint faced by the social planner. It says that production and non-depreciated capital in period t are devoted to consumption (of the three generations alive at time t) plus investment in next period's capital. A more detailed examination of this equation will help us understand why the *laissez-faire* economy may differ from the first best in this framework.

If population is increasing ($n_t > 0$ for all t), three things happen at the macro level: 1) children's consumption per worker becomes more expensive as there are more children to be raised; 2) accumulation of capital for the next generation becomes more expensive per worker –as long as the same capital-labour ratio is maintained; and 3) consumption of the elderly per worker becomes cheaper. Obviously, the opposite would be the case were the population to be decreasing ($n_t < 0$ for all t). Since agents choose the

number of children they want to have, their decisions determine the rate of population growth. The question is whether agents perceive the three effects of their fertility rate on the economy, so that they are internalised.

The first effect is generally taken into account at the micro level, since it is usually the parents who pay for the cost of raising their children. In the absence of altruism, however, the other two effects are not taken into account by the individual[4]. As a consequence, there are two differences between the planner's and the competitive economies in the absence of corrective policies. These are the second and third effects, which have been called respectively the "capital-dilution effect" and the "intergenerational transfer effect" (Michel and Pestieau, 1993)[5].

3. The model with homogeneous agents

The preferences of the agents take the form:

$$u_t = u(c_{1,t}, c_{2,t+1}, 1 + n_t) \qquad (2)$$

i.e. individuals derive utility from consumption in the two periods of their adult life and from the number of their children. We assume absence of altruism, as agents do not value the utility of their children. We suppose this function is increasing and concave in each of its arguments.

3.1. The planner

The problem faced by the planner is to maximise the utility function in equation *(2)* at the steady state subject to the resource constraint in equation *(1)* (without time subscripts) with respect to c_1, c_2, $1 + n$ and k. For an interior solution, this gives the following first order conditions:

$$\frac{u_1}{u_2} = 1 + n \qquad (3)$$

---

[4] If forward altruism was introduced, the second effect would be internalised, whereas if backward altruism was introduced the third effect would be internalised. If both backward and forward altruism were introduced, the resulting extended family would become no different than the planner and no externalities would remain. See Nerlove, Razin and Sadka (1987), chapter 7.

[5] According to this last effect, population growth is desirable as long as there are net intergenerational transfers from the young to the old.

$$\frac{u_3}{u_1} + \frac{c_2}{(1+n)^2} = e + k \qquad (4)$$

$$f'(k) - \delta = n \qquad (5)$$

together with equation *(1)*. The second order conditions of the problem must also be verified (see the Appendix).

3.2. The decentralised economy

In this section we decentralise the social optimum. In the first period, agents are assumed not to make economic decisions, but they consume a fixed amount *e* of resources provided by their parents. In the second period, agents work, earn a wage income w_t, and spend this money consuming $c_{1,t}$, saving s_t and raising $(1 + n_t)$ children. We assume that labour is supplied inelastically. In the third period, agents receive their capital income $(1 + r_{t+1})s_t$, and consume $c_{2,t+1}$.

To decentralise the first best, we introduce in the competitive economy a PAYG social security system that links pension benefits to the number of children as well as to the level of future wages, so that:

$$p_{t+1} = w_{t+1}\tau_{t+1}(1 + n_t)$$

where p_{t+1} is the pension benefit and τ_{t+1} is the payroll tax in period *t+1*. Our goal is to show that, if the payroll tax is properly chosen, this pension policy leads the economy to the optimal steady state. We assume that agents know this policy to be operative. In this way the social security system's budget constraint is introduced in the individual's problem.[6]

A representative agent maximises his or her utility function subject to the following constraints:

$$c_{1,t} + s_t + e(1 + n_t) = w_t(1 - \tau_t) \qquad (6)$$

$$c_{2,t+1} = w_{t+1}\tau_{t+1}(1 + n_t) + s_t(1 + r_{t+1}) \qquad (7)$$

with respect to $c_{1,t}$, $c_{2,t+1}$, $1 + n_t$ and s_t, where the pension benefit p_{t+1} has already been substituted by its value. Solving the problem gives the following conditions:

---

[6] Recall that the budget constraint of a PAYG social security system is $p_{t+1}N_t = w_{t+1}\tau_{t+1}N_{t+1}$, where N_t is the size of the generation that becomes adult at period *t*.

$$\frac{u_{t,1}}{u_{t,2}} = 1 + r_{t+1} \qquad (8)$$

$$\frac{u_{t,3}}{u_{t,1}} + \frac{w_{t+1}\tau_{t+1}}{1 + r_{t+1}} = e \qquad (9)$$

Equation *(9)* is the first order condition with respect to the number of children. The individual decides how many children to have by equalising the marginal cost of having an extra child −*e*− to the marginal benefit, which is the sum of two components: the marginal benefit in terms of utility plus the marginal benefit in terms of the higher pension the agent will receive from the social security system, if she has an additional child. In the centralised economy, we have seen that when deciding the optimal *n* −see equation *(4)*− the planner takes into account the three effects explained in Section 2.

From the representative firm's problem we also have the conditions:

$$r_t = f'(k_t) - \delta \qquad (10)$$

$$w_t = f(k_t) - k_t f'(k_t) \qquad (11)$$

Finally, the capital market clearing condition is given by:

$$k_{t+1}(1 + n_t) = s_t \qquad (12)$$

3.3. Comparison of the two steady state solutions

Using *(10)*, we can express *(8)* at the steady state as:

$$\frac{u_1}{u_2} = 1 + f'(k) - \delta \qquad (8')$$

At the same time, using *(7)*, *(10)* and *(12)*, equation *(9)* can be written at the steady state as:

$$\frac{u_3}{u_1} + \frac{c_2}{(1+n)(1+f'(k)-\delta)} = e + k \qquad (9')$$

Thus, comparing the equilibrium conditions in the steady state for both the centralised and decentralised economies, rearranged in such a way that they are more readily comparable, it is easy to see that if the interest rate in the market economy is equal to the rate of population growth, the two solutions will be exactly the same. That is, if the golden rule is satisfied in the market economy, the two solutions give the same steady state values of the capital-labour ratio, the growth rate of population, and consumption for the two periods. Then, the government can choose the *optimal* payroll

tax τ^* of the PAYG pension scheme that makes the capital stock in the decentralised economy equal to the optimal one, achieving optimality in the whole economy. For any other value of τ the capital stock will be different from the optimal one, and correspondingly all the other economic variables.

An example: Cobb-Douglas utility and production functions. In the case where preferences are log-linear and production is Cobb-Douglas, i.e.

$$u(c_{1,t}, c_{2,t-1}, 1+n_t) = \log c_{1,t} + \beta \log c_{2,t+1} + \theta \log(1+n_t)$$

$$y_t = f(k_t) = Ak_t^{\alpha}$$

with $\beta, \theta > 0$; $0 < \alpha < 1$. Assuming $\delta = 1$ and solving for the planner's optimum at the steady state gives the following capital-labour ratio:

$$k^* = \frac{(1+\theta+2\beta)e\alpha}{\theta+\beta-\alpha(1+\theta+2\beta)}$$

Here variables with a star refer to the planner's optimal values. From this we see that we need $(\theta+\beta) > \alpha(1+\theta+2\beta)$ in order for k^* to be positive. This inequality also ensures that the second order conditions of the problem are satisfied (see the Appendix).

Solving for the decentralised economy with the reform proposal, the capital-labour ratio at the steady state is given by:

$$\hat{k} = \frac{\alpha\beta e}{\alpha\theta + (1-\alpha)(\theta+\beta)\tau}$$

The value of τ such that the two capital-labour ratios coincide is:

$$\tau^* = \frac{\beta-\alpha(1+\theta+2\beta)}{(1-\alpha)(1+\theta+2\beta)}$$

So if the government chooses this value of the payroll tax, both the optimal k^* and the optimal $(1+n^*)$ are achieved in the decentralised economy. Note that τ^* is always lower than 1, and that it can be positive or negative depending on the sign of $\beta-\alpha(1+\theta+2\beta)$. Note also that, if $\beta-\alpha(1+\theta+2\beta) > 0$, the value of the capital stock in the *laissez-faire* economy (in which there is no social security system, so $\tau=0$) is higher than the optimal one, and the growth rate of population is lower than the optimal one. Moreover, the second order conditions of the problem are automatically satisfied if this is the case. On the other hand, if $\beta-\alpha(1+\theta+2\beta) < 0$, there will be under-accumulation of capital in the pure market economy, and too many children.

Since $\hat{k}$ depends negatively on τ, in the first case τ^* will have to be positive, whereas in the second case it will have to be negative. A negative payroll tax would imply a system that redistributed from the old to the young. The situation nowadays seems to be more closely represented by the first of these cases.

4. The model with heterogeneous agents

We will now assume there are two types of agent. A share γ of agents derive utility from the number of children they choose to have, while the rest of households do not. We will denote all the variables corresponding to the first type of agent by superscript A, and those corresponding to the second type by B. We will assume that the share of agents of each type remains constant over time[7].

Preferences of each type of agent are then:

$$u^A = u^A(c_{1,t}^A, c_{2,t+1}^A, 1 + n_t^A)$$
$$u^B = u^B(c_{1,t}^B, c_{2,t+1}^B)$$

As before, we assume these functions are increasing and concave in each of their arguments.

In this case, the growth rate of population in the economy is given by $\widetilde{n}_t$, with:

$$1 + \widetilde{n}_t \equiv \frac{N_{t+1}}{N_t} = \gamma(1 + n_t^A) + (1 - \gamma)(1 + n_t^B)$$

4.1. The planner

We assume that the planner maximises a weighted sum of the utilities of the representative agent of each type. Let μ be the weight assigned to the utility of type-A agents and $(1 - \mu)$ the weight assigned to type-B agents' utility. Then the problem of the planner is:

$$Max \quad \mu \cdot u^A(c_1^A, c_2^A, 1 + n) + (1 - \mu) \cdot u^B(c_1^B, c_2^B)$$

---

[7] If, instead of assuming a constant share of each type of agent, we assumed that children have the same preferences as their parents, the share of agents who like children would increase over time until heterogeneity disappeared.

subject to $f(k)+(1-\delta)k=\gamma c_1^A+(1-\gamma)c_1^B+\dfrac{\gamma c_2^A+(1-\gamma)c_2^B}{(1+\tilde{n})}+e(1+\tilde{n})+k(1+\tilde{n})$ *(13)*

with respect to c_1^A, c_1^B, c_2^A, c_2^B, $1+n^A$, $1+n^B$ and k. Given the assumptions made on preferences, we do not need to include non-negativity constraints for any variable except for $(1+n^B)$. The first order conditions for the planner are:

$$\frac{u_1^A}{u_2^A}=\frac{u_1^B}{u_2^B}=1+\tilde{n} \tag{14}$$

$$\frac{u_1^A}{u_1^B}=\frac{u_2^A}{u_2^B}=\frac{1-\mu}{\mu}\cdot\frac{\gamma}{1-\gamma} \tag{15}$$

$$\frac{u_3^A}{u_1^A}+\frac{\gamma c_2^A+(1-\gamma)c_2^B}{(1+\tilde{n})^2}=e+k \tag{16}$$

$$\frac{\gamma c_2^A+(1-\gamma)c_2^B}{(1+\tilde{n})^2}\leq e+k \tag{17}$$

$$f'(k)+(1-\delta)=1+\tilde{n} \tag{18}$$

By comparing equations *(16)* and *(17)*, it is easy to see that the latter must hold with strict inequality, since $\dfrac{u_3^A}{u_1^A}>0$, implying that $1+n^B=0$. So the planner decides that the more efficient solution is that only agents who derive utility from children have them. Hence, $1+\tilde{n}=\gamma(1+n^A)$ and we can substitute this into equations *(14), (16)* and *(18)*.

4.2. The decentralised economy

The budget constraints of a representative agent of type i ($i = A, B$) are:

$$c_{1,t}^i+s_t^i+e(1+n_t^i)\leq w_t(1-\tau_t)$$

$$c_{2,t+1}^i\leq w_{t+1}\tau_{t+1}(1+n_t^i)+s_t^i(1+r_{t+1})$$

Solving the problem for all agents gives the following conditions at equilibrium:

$$\frac{u_{t,1}^A}{u_{t,2}^A}=\frac{u_{t,1}^B}{u_{t,2}^B}=1+r_{t+1} \tag{19}$$

$$\frac{u_{t,3}^A}{u_{t,1}^A}+\frac{w_{t+1}\cdot\tau_{t+1}}{1+r_{t+1}}=e \tag{20}$$

$$1 + n_t^B = 0 \qquad (21)$$

The last equation says that individuals who do not like children will never have any, even with the presence of the proposed PAYG social security system that awards pension benefits in direct proportion to fertility behaviour. In fact, as can be seen from equation *(20)*, the system only pays a share of the cost of children, the part of the cost that is not compensated by the marginal willingness –of agents who like children– to have them. That is the reason why agents not deriving any utility from children will never want to have any. This result has yet another implication. If we compare the intertemporal budget constraints of the two types of agent, we will see that, while both have the same net wage, type-*A* agents consume less, as they devote a share of their resources to raising children and later they receive only a share of these resources back in their pension benefit.

The production side of the economy does not change, so the firm's conditions are the same as in the case of homogeneous agents.

The market clearing condition for the capital market is in this case:

$$k_t \gamma (1 + n_t^A) = \gamma s_t^A + (1 - \gamma) s_t^B \qquad (22)$$

4.3. Comparison of the two steady state solutions

As before, we can rearrange equations *(19)* and *(20)* at the steady state as:

$$\frac{u_1^A}{u_2^A} = \frac{u_1^B}{u_2^B} = 1 + f'(k) - \delta \qquad (19')$$

$$\frac{u_3^A}{u_1^A} + \frac{\gamma c_2^A + (1 - \gamma) c_2^B}{\gamma (1 + n^A)(1 + f'(k) - \delta)} = e + k \qquad (20')$$

to make them more readily comparable to equations *(14)* and *(16)* corresponding to the planner's optimum.

Again, if the golden rule is satisfied in the decentralised economy with the defined pension system, the optimal capital-labour ratio and growth rate of population are achieved, as can be seen by comparing the equations of the two solutions. So the government can choose the optimal payroll tax that leads to this outcome. However, this policy does not restore full optimality of the decentralised equilibrium, because it does not achieve an optimal redistribution of resources among agents. Suppose, for example, that the planner assigns a weight to the utility of agents of type *i* equal to its share in

population. That is, $\mu = \gamma$. If we assume that the utility functions satisfy the following conditions:

$$u^B(c_1, c_2) = u(c_1, c_2)$$
$$u^A(c_1, c_2, 1+n) = u(c_1, c_2) + v(1+n)$$

so that marginal utilities with respect to consumption are the same for both types of agent, the previous weights imply that consumption of the two types of agent should be the same in the optimal solution. As we have seen in Section 4.2., this will never be the case in the decentralised economy, as children are not entirely subsidised by the social security system[8].

If we want all agents to consume the same, we must introduce another instrument that transfers resources from agents who do not like children to agents who do. For example, we could redistribute wealth between agents via lump-sum taxes and transfers as follows[9]:

$$c_1^B + \frac{c_2^B}{1+r} = w(1-\tau) - \psi^B$$

$$c_1^A + \frac{c_2^A}{1+r} + e(1+n^A) = w(1-\tau) + \frac{w\tau(1+n^A)}{1+r} + \psi^A$$

with $\psi^A, \psi^B > 0$, where ψ^B is a lump-sum tax imposed on type-B agents and ψ^A is a lump-sum transfer given to type-A agents. As the size of each group of agents differs, it must be the case that:

$$\psi^A = \frac{1-\gamma}{\gamma} \cdot \psi^B$$

The value of this tax/transfer can be obtained through equating consumption in the two budget constraints, obtaining the following:

$$\psi^B = \gamma \cdot \left[e(1+n^A) - \frac{w\tau(1+n^A)}{1+r} \right]$$

The role of this tax/transfer policy is to reassign the utility that children generate in the economy as a whole to the agents that actually have these children.

Note that the transfer is given to agents who have children regardless of how many they have. As lump-sum policy instruments are non-distorting, this will not

---

[8] Note that, if they were, as in Bental (1989), type-A individuals would have as many children as possible as long as they were costless and gave them a positive marginal utility. In this case, there would be an overshooting of the number of children in the economy.

[9] Alternatively, we could do it by means of a surcharge/reduction in the payroll tax, which is equivalent to a lump-sum tax/transfer in this framework with exogenous labour supply.

change incentives for any of the agents, that is, type-*B* agents will remain without children and type-*A* agents will have the same amount.

Although this two-instrument policy restores optimality, the solution implies that type-*B* agents have less utility than type-*A* agents. This is due to the specific weights ($\mu=\gamma$) used in the social welfare function (which imply an utilitarian objective). If we thought that the welfare function should be such that the planner always assigned the same utility to all types of agent regardless of their preferences (i.e. an egalitarian objective), the tax/transfer policy would have to be defined in order to equalise utilities instead of consumptions of the two groups of agent. In this case, the direction of the transfer is not clear and would depend on the shape of the utility functions. The only thing we can say is that ψ^B will be lower with the egalitarian objective than with the utilitarian one.

From a normative point of view, we cannot discriminate between the two welfare functions mentioned above. Nevertheless, there is a specific case in which it would seem more appropriate to use an egalitarian welfare function. Imagine that some people would like to have children but cannot. It can be shown that in a similar model to the one in this section but in which all agents like children and only a constant fraction succeed in having them, the policies to restore optimality would be the same. In this case, with an utilitarian objective, the second instrument would remove utility from agents who are unfortunate in not being able to have children towards agents who are fortunate enough to have them. So notice that inability to have children would be penalised twice. First, agents who are unable to have children would receive no pension benefits despite contributing to the pension system, and second, they would have to pay a lump-sum tax. This is because, for this specific welfare function, the planner does not care about each individual but only about the representative agent.

5. Final remarks

To summarise, we have seen that a policy that links pension benefits to fertility as well as to future wages restores the optimal population growth rate and capital-labour ratio both in the case of homogeneous and heterogeneous preferences towards children.

However, in the latter case, the choice of the welfare function determines the particular distribution of utility between different types of agent.

In this section we outline some issues that should be taken into account in implementing this policy, although a full analysis of this matter goes beyond the scope of this paper.

Our policy proposal leads to a first best solution, but the question of whether a Pareto improving transition to the first best solution exists has not been addressed in this paper. However, our analysis allows us to make some recommendations regarding the transition.

The first thing to be borne in mind is that, in order to avoid hurting the initial generations, this policy should first be applied to households that are in the beginning of their fertile period at the time the policy is announced[10]. Hence, the financial stability of the pension system would be attained once the first generations affected by the policy reached retirement. In the meanwhile, a gap between pension claims and contributions would remain. In any case, this gap might be lower than in the absence of the policy, because there would be an incentive to increase fertility that would reduce the strength of the demographic transition. Furthermore, since this situation is expected to be transitory, there exists the possibility of avoiding a cut in pension benefits or a rise in the contribution rate during the transition period by issuing debt.

On the other hand, if agents are not fully rational and/or there exist liquidity constraints, it could be argued that in order to restore the optimal n it would be more effective to implement this policy as a reduction in the payroll tax. Nevertheless, by means of this measure the pension system loses its PAYG nature as long as the pension benefit of the parent is no longer raised from the contributions of his children's generation.

Along similar lines, Sinn (1997) proposes a transition to a hybrid system (PAYG and funding) that can be implemented either by linking pension benefits to fertility (as we have done here) or by reducing the contributions to the PAYG of those who have children. This proposal has the appeal of approaching the design of the transition by taking into account heterogeneity in fertility behaviour.

---

[10] In 1986, the German law started introducing measures that establish an equivalence between the number of years of female childcare and the number of years of contributions in the pension formula (See Weikard 2000). This is an appealing way of implementing this policy that would potentially help to overcome the pension differential between males and females.

Appendix: Second Order Conditions

Using the following assumptions for the utility and production functions:

$$u_i > 0 \quad i = 1,2,3$$

$$u_{ii} < 0 \quad i = 1,2,3$$

$$u_{ij} = 0 \quad i = 1,2,3; \ j = 1,2,3; \quad \forall j \neq i$$

$$f'(k) > 0$$

$$f''(k) < 0$$

and using the principal minors method, one can show that the Hessian matrix corresponding to the centralised problem evaluated at the *foc* is negative semi-definite if the following two conditions are satisfied:

$$f''(k) \cdot u_{33} > u_1 \qquad (28)$$

$$-2c_2 \left[u_{22} + \frac{u_{11}}{(1+n)^2} \right] > \frac{u_1}{1+n} \qquad (29)$$

which ensure that the solution is a local maximum since it is interior and the objective function is concave. These two equations give the second order conditions for the planner's problem.

Cobb-Douglas example. If we use Cobb-Douglas utility and production functions such as those in Section 3.3, equation *(29)* is automatically satisfied, and the only condition we need in order to have a maximum is equation *(28)*, which can be expressed as:

$$\alpha < \frac{\theta + \beta}{1 + \theta + 2\beta}$$

As can be seen in Section 3.3, this condition always holds for relevant values of the decision variables, i.e. positive values.

References

Bental, B. (1989), The Old Age Security Hypothesis and Optimal Population Growth, *Journal of Population Economics* 1, pp. 285-301.

Deardorff, A.V. (1976), The Optimum Growth Rate for Population: Comment, *International Economic Review 17,* pp.510-515.

Diamond, P. A. (1965), National Debt in a Neoclassical Growth Model, *American Economic Review 55, n. 5,* pp. 1126-1150.

Eckstein, Z. and K.I. Wolpin (1985), Endogenous Fertility and Optimal Population Size, *Journal of Public Economics 27,* pp. 93-106.

Jaeger, K. (1989) The Serendipity Theorem Reconsidered: The Three-Generations Case Without Inheritance, in: Klaus F. Zimmermann (Ed.) *Economic Theory of Optimal Population*, Springer-Verlag, pp. 75-87.

Michel, Ph. and P. Pestieau (1993), Population Growth and Optimality. When does serendipity hold?, *Journal of Population Economics* 6, pp. 353-362. Also *CORE Reprint* No. 1072.

Nerlove, M., A. Razin and E. Sadka (1987), *Household and Economy.* Academic Press, Inc.

Samuelson, P. A. (1975), The Optimum Growth Rate for Population, *International Economic Review 16,* pp. 531-538.

Sinn, H.-W. (1997), The Value of Children and Immigrants in a Pay-As-You-Go Pension System: A Proposal for a Partial Transition to a Funded System, *NBER Working Paper* No. 6229.

Weikard (2000), A Rights-egalitarian Pay-as-you-go Pension System, *mimeo.* Wageningen University, The Netherlands.