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Essay on the fairness of public pension systems

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Abstract

Pension system reform is a complex and challenging task, requiring a holistic approach that addresses the key conditions of financial sustainability, social adequacy and transparency. This thesis proposes a comprehensive framework for pay-as-you-go pension system reform that takes into account these essential conditions, as well as demographic changes and socio-economic disparities. Specifically, the thesis proposes an application of a system for Morocco that is based on retirement points and guided by the Musgrave rule. This system aims to improve both financial sustainability and inter-generational equity. The thesis also introduces progressive pension formulas to mitigate longevity disparities, providing higher benefits to individuals with lower lifetime incomes and potentially shorter life expectancies. To ensure financial sustainability, the thesis proposes automatic adjustment mechanisms that are integrated into the pension formula and steering mechanisms that are in accordance with the Musgrave rule. These mechanisms ensure a balanced distribution of demographic risk between working individuals and retirees, contributing to the overall stability of the pension system. Finally, the thesis synthesizes these concepts by incorporating double Automatic Adjustment Mechanisms and stochastic multi-population mortality modelling. This approach provides a more nuanced and equitable understanding of pension system design in the face of demographic changes and socio-economic disparities. It underscores the importance of addressing socio-economic differences in longevity. By integrating double Automatic Adjustment Mechanisms and multi-population mortality modelling, this research contributes to equitable and sustainable pension system design, which is essential in navigating demographic changes and socio-economic differences tied to longevity. In summary, this thesis provides a comprehensive and innovative approach to pension system reform that addresses the key challenges of financial sustainability, social adequacy, demographic changes and longevity inequality.

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Préface

In the realm of public pension systems, profound challenges emerge from the intricate interplay of factors such as population ageing and socio-economic disparities in longevity. This essay delves into the critical topic of fairness in public pension schemes, examining both intergenerational equity and intragenerational fairness among retirees of diverse socio-economic classes.

Chapter 1: The Landscape of Public Pension Systems

This opening chapter sets the stage, delving into the complex issues surrounding public pension systems. We explore the ramifications of an ageing population and the inherent socio-economic disparities in longevity that shape the landscape of pension schemes. The purpose of this thesis is illuminated, unveiling the quest for fairness and sustainability in the face of demographic shifts.

Chapter 2: Application of the point system on Moroccan Pension Data

In this chapter, we present the first published conference paper, where we apply the system proposed by the Belgian Pension Commission to real-world data from a Moroccan pension scheme. Through this application, we seek to assess the viability and effectiveness of the proposed fairness mechanisms in a specific context, shedding light on potential adaptations and enhancements.

Chapter 3: Longevity Heterogeneity and Progressive Pension Formula

Delving into the intricacies of longevity heterogeneity, this chapter presents a paper published in "Risks" journal. We unveil a progressive pension formula designed to address the disparities in life expectancies among diverse socio-economic segments. The paper elucidates the theoretical foundations, statistical evidence, and potential implications of adopting such progressive measures in pension design.

Chapter 4: Automatic Adjustment Mechanisms to Address Population Ageing and Socio-economic Disparities

A pivotal chapter in this essay, we explore the realm of automatic adjustment mechanisms in public pension schemes. Our proposed mechanisms aim to navigate the challenges posed by population ageing and socio-economic disparities in longevity. This chapter represents a submission that showcases our vision for sustainable and equitable pension systems in the face of dynamic demographic changes.

Chapter 5: Multi-population Mortality and Pension Design

Building on the exploration of multiple populations, this chapter delves into the intricate relationship between the stochastic modelling of mortality patterns and pension design. Here, we perform a risk analysis of the pension system based on the variability of the mortality model, shedding light on potential policy implications and avenues for further research.

Chapter 6: Discussions and Conclusions

In this concluding chapter, we synthesize the key findings, contributions, and insights gleaned throughout this essay. We revisit the quest for fairness in public pension systems, acknowledging the challenges and opportunities that lie ahead. Our discussions encompass intergenerational equity, intragenerational fairness, and the delicate balance required for sustainable and just pension systems. In conclusion, this essay seeks to contribute to the ongoing dialogue on shaping public pension schemes that embrace both compassion and sustainability, ensuring a dignified future for retirees across diverse socio-economic backgrounds.

Contents

Table of notations

Table 1: Notations

Abbreviations:	
$PAYG$	Pay-as-you-go
DB	Defined Benefit
DC	Defined Contributions
DM	Defined Musgrave
SES	Socio-economic status
Indices:	
m	number of socio-economic classes (j$\in \{1, ..., m\}$)
t	period of time (t $\in \{0, .., t, .., T, ..\}$)
x_0	age of entry
x_r	retirement age
ω	death age
x	age (x $\in \{x_0, .., x_r, .., \omega\}$)
Parameters:	
δ_t	Replacement rate in DB system (also the pension rate in progressive systems)
π_t	Contribution rate in the year t
λ_j^t	Progressive factor in the year t for class j
$\bar{\delta}_t$	Average benefit ratio in the PDM system
D_t	Dependency ratio in the year t
Indexes:	
$S_j^{(x,t)}$	Salary of an agent of class j aged x at time t
$X_j^{(x,t)}$	Progressive transformation of salary for an agent of class j aged x at time t
$m_{x,t}^{(j)}$	Central mortality rate at age x in the year t for class j

Chapter 1

General introduction

Over the past few decades, pension system reforms have been a focal point of discussions and debates worldwide, particularly in Europe. As societies grapple with the challenges of demographic ageing and changing economic landscapes, the sustainability and fairness of pension systems have come under scrutiny. In 2019, when this project commenced, the topic was already at the forefront of public discourse. Fast forward four years, and the discussions surrounding pension reforms have only intensified, sparking heated debates and social unrest, as witnessed in France during Macron's controversial proposal to increase the retirement age from 62 to 64 [Boulhol & Queisser, 2023]. These events highlight that the issue of social security is not merely a fiscal matter; it carries profound social implications that directly impact the lives of citizens and stakeholders.

Addressing the complexities of pension reform requires a multifaceted approach that goes beyond financial considerations. While ensuring the fiscal viability of pension systems is crucial, it is equally vital to recognize the social dimension of such reforms. Stakeholders, including retirees, workers, policymakers, and society at large, are profoundly affected by any changes to pension schemes. Consequently, a comprehensive understanding of the potential impacts at every step of the reform process is paramount to achieving social adequacy.

In this thesis, we try to dessicate the intricacies of pension system reforms, aiming to shed light on the path towards sustainable, fair, and socially adequate designs. As we embark on this journey, it becomes evident that the quest for a well-balanced pension system goes beyond economic models and financial equations. It is a pursuit that involves careful consideration of the needs and aspirations of all stakeholders, and a commitment to transparent and evidence-based decision-making. By critically examining existing pension designs, exploring innovative approaches, and acknowledging the social dimensions of reform, we aim to contribute to the broader discourse on social security and pave the way for pension systems that truly serve the well-being of all individuals in our society.

1.1 Ageing and longevity heterogeneity risks inside public pension systems

Pension systems around the world are typically structured into three pillars, each serving a distinct purpose in providing income security during retirement. The first pillar of the pension system is often referred to as the public or mandatory pillar. It is designed to provide a first level of income security for all individuals, typically through a pay-as-you-go system where current workers contribute to support current retirees. This pillar is crucial in ensuring that older adults have a reliable source of income during retirement, especially those with limited means or who are unable to accumulate significant personal savings. However, the financial sustainability of this pillar can be strained as the number of retirees grows in relation to the working-age population.

The second pillar is known as the occupational or company pension schemes. These schemes are typically provided by employers and aim to supplement the first pillar by offering additional retirement benefits based on an individual's employment history and contributions. The occupational pillar helps to bridge the gap between the basic level of income provided by the first pillar and the income needed to maintain a desired standard of living in retirement. These schemes often take the form of defined benefit or defined contribution plans, with benefits tied to factors such as salary, years of service, or investment returns. The majority of second pillar plan are fully funded.

The third pillar consists of individual or voluntary pension arrangements, which are primarily driven by personal savings and investments. Individuals are encouraged to contribute to private pension products, such as individual retirement accounts (IRAs) or personal pension funds, to build up a supplementary retirement nest egg. These plans offer individuals more flexibility and control over their retirement savings, allowing them to tailor their contributions and investment strategies to meet their specific needs and goals. The third pillar plays an essential role in diversifying sources of retirement income and providing individuals with greater financial autonomy.

These three pillars of the pension system play a critical role in ensuring the financial well-being of older adults. The combination of a public protection, occupational pension schemes, and individual savings aims to provide a comprehensive and sustainable approach to retirement income. However, as demographic shifts and economic challenges unfold, it becomes increasingly important to evaluate and adapt these pillars to ensure their continued effectiveness and adequacy in meeting the needs of future retirees. The implications of population ageing are profound, particularly for public pension systems in pay-as-you-go (PAYG) and the first pillar. With a smaller number of non-elderly workers supporting an increasing number of elderly retirees, there is mounting pressure on these systems to sustain benefit payments.

The 21st century is witnessing a significant shift in population dynamics, particularly in developed nations, characterized by declining fertility rates and increasing life expectancy [Roser & Ortiz-Ospina. 2017]. Projections by the United Nations indicate negative population growth rates in many developed countries, primarily driven by falling fertility rates below the replacement level. This demographic trend, coupled with rising life expectancy, is expected to result in a rapid ageing of the global population in the coming decades. Concurrently, life expectancy at age 65 is projected to rise by approximately one year per decade [Bryant & Velculescu, (2002)]. This demographic shift has significant implications for the old-age dependency ratio, defined as the proportion of the elderly population (65 years and older) relative to the working-age population (15-64 years). Projections indicate that by 2050, the average old-age dependency ratio will double. At present, Japan stands as the sole country with an old-age dependency ratio exceeding 40, but by 2050 , more than 55 countries are expected to surpass this threshold [UN, 2016].

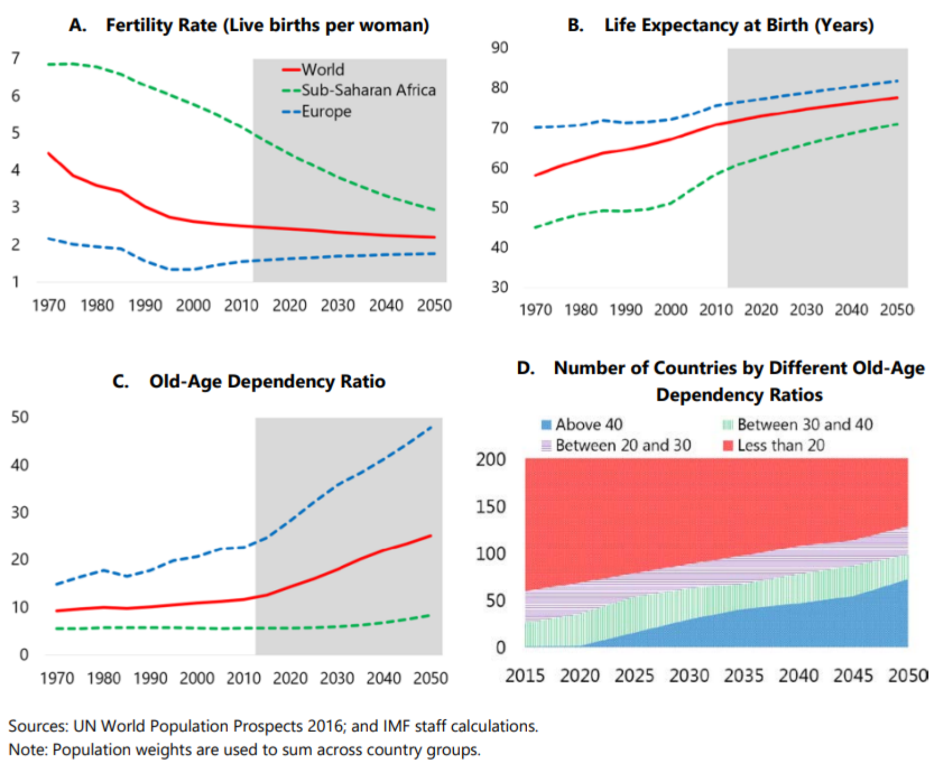


Figure 1.1: Demographic development [Amaglobeli & al, 2019]

The demographic challenges associated with population ageing have triggered widespread recognition among policymakers and administrators [Linz & Stula, 2010] of the significant hurdles they are currently facing. The sustainability and adequacy of public pension systems have become key concerns, as policymakers grapple with the task of ensuring that benefit payments for the growing number of elderly retirees can be met by the shrinking working-age population.

This recognition of the challenges posed by population ageing has set the stage for intense discussions and debates within political and administrative circles. Policymakers are actively seeking strategies to address the economic and social implications of an ageing population, with the aim of designing robust pension systems that can effectively support retirees and ensure intergenerational equity.

Therefore the current landscape of public pension systems is profoundly influenced by the formidable challenges presented by population ageing. In industrial countries, public pension systems predominantly operate under a pay-as-you-go (PAYG) framework, often supplemented by privately managed funded schemes. Under the PAYG system, the working population contributes through payroll taxes, which are then used to provide benefits to retirees. However, unlike fully funded schemes, PAYG systems generally lack a direct relationship between individual contributions and benefits. As these systems mature, the number of beneficiaries increases, leading to a potential imbalance where benefit payouts surpass contributions. This necessitates adjustments such as raising payroll taxes or resorting to budget transfers. Moreover, the inclusion of redistributive elements in PAYG systems can further exacerbate fiscal pressures, particularly in the face of population aging. Failure to address these challenges could have significant macroeconomic and structural implications, both nationally and globally.

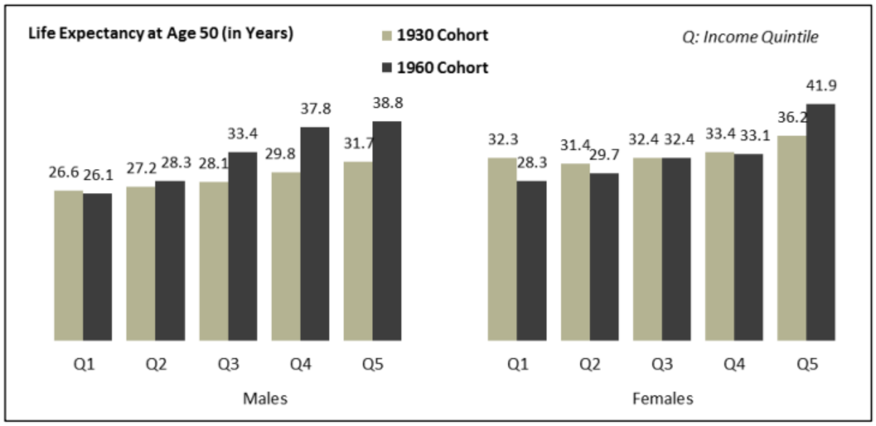
The implications of population ageing extend beyond fiscal and economic realms, encompassing complex social and political dimensions. Determining the distributional impact of public pension programs becomes increasingly contentious as the working-age population shrinks while the political influence of the elderly grows. Although individual responsibility for retirement provision is an ideal concept, public support for pension systems has been deemed necessary for various reasons. These systems have been instrumental in reducing poverty rates among the elderly [Shang, 2014]. However, determining equitable burden-sharing in supporting the ageing population becomes more intricate in this context.

To alleviate the fiscal pressures associated with public pension arrangements, several approaches have been proposed. Parametric adjustments, such as modifying contribution rates, retirement ages, and pension benefit indexation formulas, coupled with the establishment of financial reserves, represent one avenue. Another strategy involves systemic reforms, such such as point systems, NDC schemes, hybrid systems between DB and DC or development of DC fully funded pillars either within or alongside existing public pension schemes. Broader adjustments, including tax increases and expenditure cuts unrelated to public pensions, as well as macroeconomic modifications like stimulating labor force participation or encouraging immigration, have also been considered. While this study primarily focuses on parametric adjustments and systemic reforms, it acknowledges the relevance of macroeconomic factors in shaping the trajectory of public pension systems.

As the global population continues to age, it is essential to recognize that the demographic changes associated with population ageing can give rise to significant challenges and inherent unfairness within pension systems. While the three pillars of the pension system aim to provide income security in retirement, the reality is that the gains in life expectancy are not uniformly distributed across the population. Some individuals and socio-economic groups experience greater improvements in mortality rates, leading to longevity heterogeneity within the ageing population [Duggan & al, 2008], [Sheshinski & Caliendo, 2021]. This heterogeneity introduces a layer of complexity and potential unfairness in the distribution of retirement benefits, as individuals with longer life expectancies may receive proportionately higher government benefits than those with shorter life expectancies.

There is a growing recognition of the existence of socioeconomic differences in longevity, further complicating the fairness of public pension schemes. The emergence of longevity inequality among different socio-economic classes has gained attention in recent years, highlighting the need to address these disparities within pension systems. As societies grapple with the question of how to respond to these inequalities, the issue of whether individuals with longer life expectancies should receive higher government benefits becomes a crucial consideration. The intersection of population ageing and longevity heterogeneity underscores the need for a deeper examination of the implications for pension systems, ensuring that they address both the overall fairness and adequacy of retirement benefits in light of these demographic dynamics. Those two effects (ageing and unequal gain in longevity within socio-economic classes) are not exclusive and they evolve through time as presented in [NASEM, 2015].

According to the National Academy of Sciences (NAS), there is a notable relationship between income and life expectancy, particularly when examining the 1930 and 1960 birth cohorts. For men, as income increased, life expectancy at age 50 also increased. The disparity between the bottom and top income quintiles more than doubled between these two cohorts. The NAS findings, depicted in Figure 1, illustrate that men in the bottom income quintile born in 1930 had an average additional life expectancy of 26.6 years at age 50 (expected age of death at 76.6). However, there has been no improvement in life expectancy for men in the bottom quintile born in the 1960 cohort, with a life expectancy of 26.1 years at age 50 (expected age of death at 76.1). On the other hand, the top income quintile of men experienced an increase in life expectancy. For the 1930 cohort, life expectancy at age 50 was an additional 31.7 years, while for the 1960 cohort, it rose to 38.8 additional years. Consequently, the gap in life expectancy at age 50 between men in the lowest and highest income quintiles has significantly widened from 5.1 years for the 1930 cohort to 12.7 years for the 1960 cohort. This increase



Source: National Academy of Sciences, *The Growing Gap in Life Expectancy by Income: Implications for Federal Programs and Policy Responses* (Washington, DC: The National Academies Press, 2015), Figure 3-2.

Figure 1.2: Longevity heterogeneity gap by cohort in the United States

in the gap is primarily driven by the longevity gains among men in the top income quintile, although a slight decline in longevity among men in the bottom quintile also contributes to this trend.

The evaluation of pension systems is often centered around the concept of actuarial fairness, which asserts that individuals should receive benefits with a present value equal to the contributions collected in their name. However, demographic trends indicate that current and future retirees are expected to experience rates of return below the actuarially fair level, reflecting the cost of transfers to earlier cohorts. Within each cohort, it is essential to carefully assess the equity of expected rates of return for different population segments, such as income groups, to ensure fairness.

Balancing equity in rates of return with considerations of redistribution is a key challenge. The objective is to establish a sufficient safety net for individuals in the lower income distribution by implementing transfers from those with higher lifetime incomes. While fairness, political economy, and efficiency justify the examination of equitable rates of return, the visibility of such redistribution within the pension system may vary. The design of the benefit formula, which converts average indexed monthly earnings into a primary insurance amount, and other provisions like survivors' benefits, play a critical role in striking a balance between equity and redistribution.

Effectively navigating the complexities of mortality inequalities and their impact on public pension schemes requires a careful examination of actuarially fair rates of return, the provision of a safety net for lower-income individuals, and a thoughtful consideration of the trade-offs between equity and redistribution. These considerations serve as guiding principles in shaping the structure and principles of pension systems, ensuring the delivery of fair and adequate benefits for all individuals.

1.2 Purpose of the thesis

This thesis places itself in an interdisciplinary project[1] that aims to critically assess the key conditions that a public pension system should fulfil to be successfully reformed. Our hypothesis is that there are three such conditions: financial sustainability, social adequacy, and safe governance.

Financial sustainability and risk management are critical considerations for the reform of public pension systems. Financial sustainability refers to achieving a fiscal and financial balance between the revenues and liabilities of the pension system. It entails ensuring that the income generated by the system, such as contributions from workers and investment returns, is sufficient to cover the long-term obligations and benefit payments to retirees. Effective risk management strategies are essential to identify and mitigate potential risks and uncertainties, such as changes in life expectancy or economic recessions, that could jeopardize the stability and solvency of the pension system.

Ensuring financial sustainability, even in the broad sense described above, is not enough. No pension reform, particularly in a context where the latter may imply making the system less generous, can do without an in-depth examination of what justice requires, both between and within generations. Social adequacy is another critical condition that must be addressed in pension system reform. It focuses on ensuring that pension benefits provide a sufficient standard of living for retirees, taking into account factors such as income replacement ratios, poverty alleviation, and the ability to maintain a decent quality of life in retirement. Social adequacy aims to promote a more inclusive and equitable pension system that adequately supports retirees and reduces inequalities in old-age income security.

In addition to being financially sustainable and socially adequate, pension systems also need to be buttressed by good governance. Safe governance is a fundamental requirement for the successful reform of public pension systems. It encompasses the processes, structures, and mechanisms that support the effective and efficient management of the pension system. Good governance ensures transparency, integrity, and ethical practices in decision-making, financial management, and the administration of pension benefits. It also involves the development of appropriate governance structures, such as independent regulatory bodies or pension oversight committees, to oversee the operation of the system and safeguard against potential risks and contingencies [Hindriks, 2014].

[1] ARC Research project ≪ Sustainable, Adequate and Safe Pensions (SAS Pensions) ≫ 2018-2023 funded by the Fédération Wallonie Bruxelles

By comprehensively addressing the key conditions of financial sustainability and social adequacy this research seeks to provide insights into the reform of public pension systems. It aims to contribute to the development of strategies and policies that promote the long-term viability, fairness, and effectiveness of pension schemes in meeting the retirement income needs of individuals and ensuring the overall well-being of societies.

The second chapter of this thesis focuses on intergenerational fairness mechanisms through the prism of the challenges faced by the pay-as-you-go pension schemes in Morocco, primarily driven by the demographic trend and the characteristics of the labor market. The demographic trend, characterized by increased longevity and declining birth rates, poses a significant challenge to the sustainability of the pension funds in Morocco. Additionally, the labor market is dominated by a large share of informal sector employment, further complicating the pension system's stability and adequacy.

The mandatory Moroccan pension system operates on a PAYG basis with a buffer fund and is financed by defined benefits. Over time, the various pension plans have accumulated substantial financial reserves due to surplus situations. However, the existing defined benefit management model faces structural challenges, necessitating parametric reforms aimed at postponing the depletion of reserves. These past reforms have focused on adjusting contribution rates and extending careers. However, projections indicate that future parametric reforms alone will not be sustainable in terms of contribution rates or career extensions.

Given the unsustainability of these parametric reforms, it is evident that a structural overhaul of the Moroccan pension system is imperative. In this chapter, we propose a new system that restructures the current system into one based on retirement points. This proposed system will be piloted using the Musgrave rule [Hindriks & al, 2017], which takes into account the ability-to-pay principle in determining pension benefits. By adopting a retirement points system and incorporating the Musgrave rule, we aim to create a more sustainable and equitable pension system for Morocco.

Throughout this chapter, we will examine the rationale behind this proposed restructuring, examining the shortcomings of the current system and the potential benefits of the retirement points system piloted with the Musgrave rule. Additionally, we will discuss the potential implications and challenges associated with implementing such a structural overhaul. This chapter focuses on financial sustainability and fairness between active workers and retirees, while leaving aside considerations of longevity heterogeneity. Subsequent chapters will look deeper into addressing social adequacy and its implications in the context of pension design.

The third chapter of this thesis delves into the concept of progressive pension formulas and explores the issue of intragenerational fairness, specifically addressing the relationship between lifetime income, life expectancy at retirement, and the resulting actuarial unfairness within a cohort of retirees.

Numerous empirical studies have revealed a positive correlation between lifetime income and life expectancy at retirement. This indicates that individuals with higher lifetime incomes tend to have longer life expectancies, leading to potential disparities in retirement benefits. Such disparities can result in actuarial unfairness within a single cohort of retirees.

Recognizing the paramount significance of addressing fairness and ensuring the long-term sustainability of pension systems, our study takes an orthogonal point of view by focusing on longevity heterogeneity rather than ageing and intergenerational fairness. Building upon a DB framework, we establish a compensation mechanism that considers life expectancy heterogeneity during an individual's active years of their career. The ultimate goal is to effectively mitigate unfairness and promote equity in pension benefits upon retirement, fostering a more just and sustainable pension landscape for individuals from diverse socio-economic backgrounds.

The proposed compensation mechanism is based on the progressivity of pension benefit formula, aiming to provide higher benefits to individuals with lower lifetime incomes and potentially shorter life expectancies, while ensuring the sustainability of the pension system. To capture the constraints and complexities associated with the model, we implement these ideas within a simplified demographic context.

Within this chapter, we will study the theoretical foundations of progressive pension formulas and the rationale behind incorporating life expectancy heterogeneity into the pension benefit calculation. We will explore the potential impact of this approach on intragenerational fairness and the challenges associated with its implementation. By studying these concepts within a defined demographic context, we aim to gain insights into the practical implications of adopting progressive pension formulas and addressing life expectancy heterogeneity for the design of more equitable and sustainable pension systems.

The fourth chapter of this thesis introduces a novel approach for designing a pension system by incorporating automatic adjustment mechanisms. In many countries, the long-term financial sustainability of pension systems is a pressing concern, necessitating reforms to ensure their viability. However, these reforms often overlook the issue of longevity heterogeneity, leading to a trade-off between financial sustainability and intragenerational actuarial fairness.

To address this challenge, we propose a comprehensive pension system management system that integrates two adaptation mechanisms. The first mechanism is embedded directly into the pension formula and aims to correct for the heterogeneity in life expectancies among individuals within the pension scheme. By adjusting pension benefits based on individual longevity, this mechanism promotes fairness within the cohort of retirees.

The second mechanism involves a steering mechanism for the contribution rate and the mean benefit ratio, following the principle of Musgrave's rule. This mechanism ensures a balanced distribution of the demographic risk between working individuals and retirees, contributing to the overall stability of the pension system.

In order to capture the combined effects of these adjustment mechanisms and incorporate the mortality component into the pension formula, we develop a model that takes into account historical data on longevity heterogeneity and ageing. By analysing past trends and patterns, we can gain valuable insights into the potential impacts of these mechanisms on the overall pension system.

The proposed approach represents a significant contribution to the scientific understanding of pension system design. By addressing longevity heterogeneity and incorporating automatic adjustment mechanisms, our research aims to enhance the financial sustainability and intragenerational fairness of pension systems.

The fifth chapter of this thesis serves as a comprehensive synthesis of the concepts discussed in the previous chapters, focusing on the risk analysis of pension schemes with double Automatic Adjustment Mechanisms (AAM) and a stochastic multi-population mortality modelling. The chapter begins by exploring the motivations and principles underlying the study, emphasizing the significance of addressing socio-economic disparities in longevity and The subsequent sections investigate the intricacies of pension schemes with double AAM, examining the assumptions and mechanisms that account for aging and longevity heterogeneity. This analysis sheds light on how these mechanisms contribute to a more comprehensive and adaptive pension system.

Moving on to stochastic multi-population mortality modeling, the chapter provides a comprehensive definition of the models utilized and the data sources employed for comparison and validation. The aim is to develop a robust framework that accurately captures the complexities of mortality patterns across diverse populations, enabling a more nuanced understanding of future mortality disparities.

The chapter also includes a risk analysis of Automatic Adjustment Mechanisms within pension systems, specifically focusing on the double AAM approach in the context of ageing and longevity heterogeneity. Value-at-Risk analysis is utilized to assess the potential risks and uncertainties associated with these mechanisms, providing insights into the potential impact on pension systems.

Throughout the chapter, critical discussions are conducted to analyze the findings, acknowledge potential limitations, and consider the implications for policymakers and stakeholders. The chapter concludes by summarizing the key insights gleaned from the study and emphasizing the importance of addressing socioeconomic differences in longevity while striving for fairness within pension systems.

By integrating double AAM and multi-population mortality modelling, this research contributes to the advancement of our understanding of pension schemes. It provides a foundation for the development of more equitable and sustainable approaches to pension system design, particularly in the face of demographic changes and socio-economic disparities.

Bibliography

[Amaglobeli & al, 2019] Amaglobeli, M. D., Chai, H., Dabla-Norris, M. E., Dybczak, M. K., Soto, M., & Tieman, A. F. (2019). The future of saving: The role of pension system design in an aging world. International Monetary Fund.

[Boulhol & Queisser, 2023] Boulhol, Hervé and Queisser, Monika. "The 2023 France Pension Reform" Intereconomics, vol.58, no.3, 2023, pp.130-131. https://doi.org/10.2478/ie-2023-0025

[Bryant & Velculescu, (2002)] Bryant, R. C., & Velculescu, D. (2002). Population Aging and Public Pension Systems: A First Look at the Cross-Border and Global Effects. Brookings Institution.

[Chand, S. K., & Jaeger,1996] Chand, S. K., & Jaeger, M. A. (1996). *Aging populations and public pension schemes.* International Monetary Fund.

[Duggan & al, 2008] Duggan, J. E., Gillingham, R., & Greenlees, J. S. (2008). Mortality and lifetime income: evidence from US social security records. IMF Staff Papers, 55(4), 566-594.

[Hindriks, 2014] Hindriks, J. (2014). Un contrat social performant et fiable: propositions de la Commission de réforme des pensions 2020-2040 pour une réforme structurelle des régimes de pension (No. UCL-Université Catholique de Louvain).

[Hindriks & al, 2017] Hindriks, J., Devolder, P., Schokkaert, E., & Vandenbroucke, F. (2017). Réforme des pensions légales: le système de pension à points. Regards économiques, 130.

[Linz & Stula, 2010] LINZ, Kathrin et STULA, Sabrina. Demographic change in Europe-an overview. Observatory for Sociopolitical Developements in Europe, 2010, vol. 4, no 1, p. 2-10.

[NASEM, 2015] National Academies of Sciences, Engineering, and Medicine, & Committee on Population. (2015). *The growing gap in life expectancy by* *income: Implications for federal programs and policy responses*. National Academies Press.

[Roser & Ortiz-Ospina. 2017] Roser, M., and E. Ortiz-Ospina. 2017. Published online at OurWorldInData.org. Retrieved from: `https://ourworldindata.org/world-population-growth`

[Sheshinski & Caliendo . (2021)] Sheshinski, E., & Caliendo, F. N. (2021). Social security and the increasing longevity gap. Journal of Public Economic Theory, 23(1), 29-52.

[UN, 2016] United Nations. 2016. World Population Prospects: The 2015 Revision. New York: United Nations, Department of Economic and Social Affairs. `http://esa.un.org/wpp`

[Shang, 2014] Shang, B. (2014). Pension Reform and Equity: the impact on poverty of reducing pension benefits. Equitable and Sustainable Pensions: Challenges and Experience, 87.

Chapter 2

Introduction of reserves in self adjusting steering the parameters of a pay-as-you-go pension plan

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2.1 Introduction

The Defined Benefit pension system in Morocco is confronted with significant demographic challenges resulting from the increase in life expectancy at birth and the simultaneous decline in the total fertility rate. Over the past decades, life expectancy has steadily risen, leading to longer retirement periods for individuals. Concurrently, the total fertility rate has experienced a substantial decrease, with figures dropping from 7.7 in 1962 to 2.49 in 2016 [HCP,2018]. Projections indicate a further decline in the coming years, potentially exacerbating the challenges faced by the pension system. As long as the fertility rate remains below the population renewal threshold, a shrinking workforce will be tasked with financing an increasing number of pension benefits. This demographic shift is evident when considering the active worker-to-pensioner ratio, which has declined from 6 in 2000 to 2.23 in 2016 [ACAPS,2017].

Adding to the complexity of the demographic problem, the economic context poses additional hurdles to the sustainability of the pension system. The financial well-being of such systems relies on the contributions received, which are typically proportional to the wage bill. However, the labor market in Morocco is marked by a significant unemployment rate of 9.7%. Moreover, there has been a shift in the nature of employment, with a growing trend towards self-employment and entrepreneurship, resulting in a decline in the number of employee contributors. In fact, the informal sector accounts for a substantial portion, around 40%, of employment in 2016, as reported by the CESE annual report [CESE,2017].

Given the challenges posed by increasing longevity and the changing labor market dynamics, many countries worldwide have embarked on, or are considering, reforms of their mandatory pension schemes. These reforms have often taken the form of parametric adjustments, involving changes to various parameters such as retirement age, benefit rates, and early retirement conditions. However, these incremental changes, known as parametric reforms, may offer short-term viability but are proving to be insufficient in addressing the magnitude of the challenges at hand.

This paper proposes an alternative approach to reforming the Moroccan pension system by advocating for a transformation from the current pay-as-you-go system with defined benefits to a pension points system. Central to this transformation is the incorporation of the Musgrave rule, an automatic piloting rule that governs the different parameters of the pension system over time. The Musgrave rule provides a framework for managing and controlling the system, ensuring its financial sustainability and adaptability to demographic changes.

To explore the implications of this proposed reform, the paper will present a comprehensive analysis. First, an overview of the architecture of the Moroccan pension system as a whole will be provided, highlighting its characteristics and parameters. Subsequently, the theoretical framework of the Musgrave rule will be presented, focusing on its management and control of the pension system through the allocation of pension points. The paper will further examine the effect of introducing the Musgrave rule on the depletion date of the reserves through simulation modeling.

Finally, the current pension system will be compared with the simulated pension points system incorporating the Musgrave rule. This comparison will consider the impact of the proposed transformation on benefit levels and contributions, assessed through contribution rates and replacement rates. By evaluating these factors, the paper aims to offer a comprehensive understanding of the potential benefits and challenges associated with the proposed reform.

2.2 The pension system

The retirement system in Morocco is primarily based on a contributory pillar funded by provisioned distribution. It consists of three mandatory basic schemes and two conventional supplementary schemes, differentiated by professional categories. Self-employed workers, including artisans, liberal professionals, farmers and mobile workers, cannot enrol in these schemes.

According to the 2017 activity report of ACAPS [ACAPS,2017], the number of beneficiaries of retirement schemes in Morocco amounted to 1.4 million individuals at that time. Among them, 72.3% were primary retirees, while 27.7% were beneficiaries of survivor pensions, including surviving spouses and orphans.

"Caisse Marocaine des Retraites" (CMR) plays a central role in managing these retirement schemes in Morocco. In 2017, the CMR was responsible for paying approximately 52.7% of the retirement benefits provided by the basic pension schemes. The plan is funded on a pay-as-you-go basis. The contribution rate is set at 28% of base salary, bonuses and other allowances. The contribution rate is shared equally between employees and the state employer. The plan is based on the principle of the laddered premium which sets an equilibrium contribution rate for a minimum period of 10 years.
The pension is calculated as such:

$$P = N \times A \times SR, \tag{2.1}$$

where

- N is the number of years contributed.
- A is the annuity rate.
- SR is the reference salary.

Before 2016, the annuity rate was 2.5% , the reference salary was the last salary and the legal retirement age was 60 years, this caused the scheme to be too generous. It offered for 40 years of contribution replacement rate of 100%. The parametric reform tried to correct this generosity. Thus the maximum contribution period within the system is 40 years and the reference salary for the calculation of the pension is now the average of the last eight (8) earnings preceding the date of retirement. The annuity rate is now 2% [CMR,2016], thus the system offers a maximum of 80% replacement rate for a complete career. The legal retirement age is 63 years.

The system's surplus position in the past has allowed it to accumulate significant reserves. Today these reserves make it possible to fill the technical deficit. However, the evolution of the declining population ratio has accelerated the depletion of these reserves. The evolution of the demographic ratio is presented here:

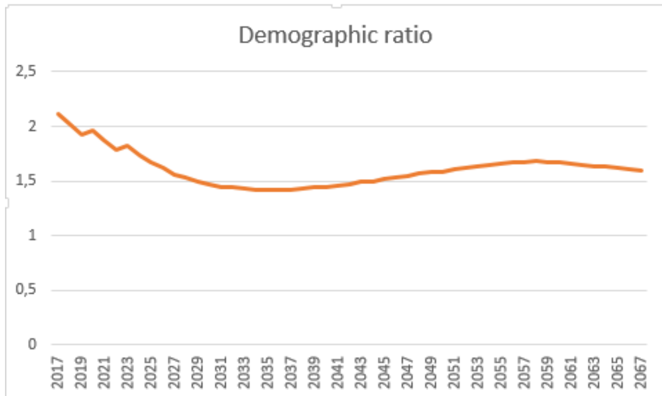

Figure 2.1: Projection of the demographic ratio of CMR

The main assumption used in this projection is the replacement of the workers, their number remain the same on the projection horizon. We used a deterministic projection for the retiree population.

We will use 5 professional categories representing career trajectories for our simulations, we present the average wages by age in for the following categories, the wages are in Moroccan Dirham (MAD):

- Administrators
- Engineers
- Grade A professors
- Secretaries
- Grade B teachers

the selected categories are representative of wage developments within the system.

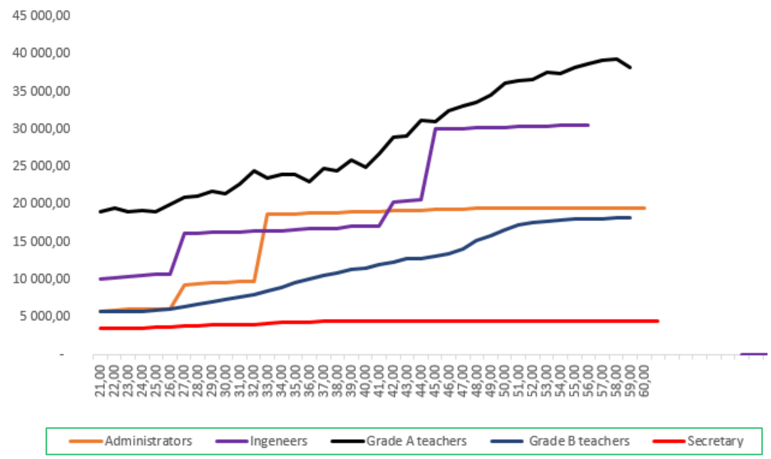

Figure 2.2: Evolution of Wage trajectory by age

The secondary school teachers careers trajectories (grade B teachers), administrators, higher education professors (Grade A professors) and state engineers have the same increasing pace depending on the length of their career. Holders of these occupations have long careers in the scheme (between 35 and 40 years). Salaries are changing gradually. Secretaries have "flat" careers and benefit from a very slight evolution. We show the distribution of the workforce in those categories:

	Admin	Engi	A Prof	B teach	Secretaries
Distribution	24%	8%	3%	63%	2%

Table 2.1: Distribution of the workforce between the categories

Administrators and Grade B teachers represent more than 80% of the members of the scheme.

Using the current parameters, we are going to simulate on these trajectories the contributions throughout their career, and determine the replacement rates associated with average contribution period in year for each trajectory as well as the ratio between expected benefits and contribution through the career.

	Admin	Engi	A Prof	B teach	Secretaries
Mean contribution period	40	35	38	38	40
Replacement rate	80%	75%	76%	75%	80%
Benefits/contributions	0,94	1,20	1,03	1,25	0,81

Table 2.2: Replacement rates and ratio between benefits and contributions

The individuals having contributed 40 years have a maximal replacement rate. The current system only rewards long haul. Only the contribution period increases the replacement rates. As a result, the ratio of benefits paid on contributions is better for careers with a high indexation rate, careers with low indexation rate are disadvantaged. The scheme pays on average for each monetary unit contributed 1.15 in return.
After describing the functioning of the CMR, we will present in what follows the theoretical framework of the rule of piloting the new regime that we will put in place

2.3 Theoretical framework of the Musgrave rule

When the demographic indicators deteriorate, depending on the management method, parameters such as contribution and replacement rates adjust to compensate for the decrease. We will present this mechanism when the regime is managed in defined benefits and then we will introduce a new management mode driven by the Musgrave rule .
We model the demographic risk by assuming a stable state (noted in state 1) composed of representative agents (same salary and same career) receiving a pension based on a replacement rate δ_1 and a contribution rate π_1, the dependency ratio (ratio of number of retirees to number of contributors) is D_1. The balance of the regime is characterized with the following system where P_1 is the average pension paid and S_t the average salary:

- Budget equation : $D_1 \cdot P_t = \pi_1 \cdot S_1$
- Pension equation : $P_1 = \delta_1 \cdot S_1$

The equilibrium is obtained when:

$$\pi_1 = D_1 \cdot \delta_1 \tag{2.2}$$

We suppose now that the system moves to another stage characterized by another dependence ratio $D_1 \rightarrow D_2$. We assume that $D_2 > D_1$. We want to find the contribution rate and the replacement rate in the second state linked by:

$$\pi_2 = D_2 \cdot \delta_2 \tag{2.3}$$

In a DB framework, there is an absolute guarantee for the retirees (fixed replacement rate) and the contributors must support the risks:

$$\Rightarrow \delta_2 = \delta_1 = \delta,$$

$$\pi_2 = \pi_1 \cdot \frac{D_2}{D_1}. \tag{2.4}$$

The demographic changes taking place within the regime are one of the main causes of the depletion of reserves. The defined benefit system puts the burden on the workers (contribution rate). Thus, according to the projections, when the demographic ratio decrease to 1.37 in 2050 the contribution rate should grow from 28% to 40%. Such contribution rates are too burdensome for the contributors only .

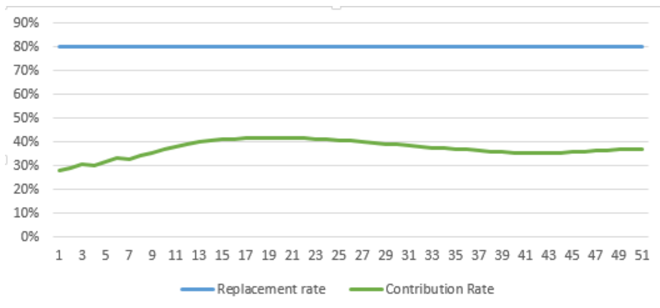

Figure 2.3: Projection of contribution rates and replacement rate in DB

Musgrave [Musgrave, 1981] has proposed another invariant leading to a form of sharing of the risk between the two generations. Let us define the Musgrave ratio as the ratio between the pension and the salary net of pension contributions :

$$M_1 = \frac{P}{S(1-\pi_1)} = \frac{\delta_1}{1-\pi_1} \tag{2.5}$$

Using the previous situation, when D_1 becomes D_2, we want to stabilize this coefficient :

$$M_1 = \frac{\delta_1}{1-\pi_1} = \frac{\delta_2}{1-\pi_2} = M_2 \tag{2.6}$$

Using 2.2 and2.3:

$$\frac{\delta_1}{1-\pi_1} = \frac{\delta_2}{1-\pi_2} \Rightarrow \delta_2 = \delta_1 \cdot \frac{1-\pi_2}{1-\pi_1}$$

$$\delta_2 = \delta_1 \cdot \frac{1-D_2 \cdot \delta_2}{1-D_1 \cdot \delta_1}$$

We deduce δ_2 and comes:

$$\boxed{\delta_2 = \delta_1 \cdot \frac{1}{1+\delta_1(D_2-D_1)}} \tag{2.7}$$

The contribution rate in the second state is determined in the same way and we have:

$$\boxed{\pi_2 = \pi_1 \cdot \frac{D_2}{D_1 + \pi_1(D_2 - D_1)}} \tag{2.8}$$

We present here the evolution of the contribution rate and the replacement rate using the Musgrave rule

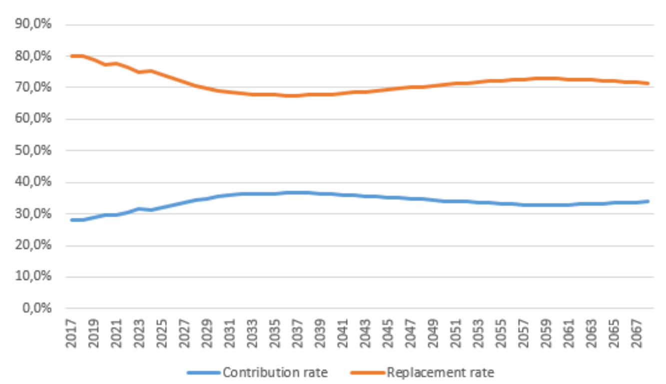

Figure 2.4: Projection of contribution rates and replacement rate under the Musgrave rule

It can be seen that the wage contribution that makes it possible to balance the plan from an actuarial point of view goes from 28% to 35% when the dependency ratio is the most deteriorated. The replacement rate drops to 69% at this period. The evolution of the rates follow the tendency of the dependency ratio (2.1), when it deteriorates the contribution rates rises and the replacement rate lowers . The consequences of the deterioration of the demographic ratio are shared by both the active workers and the retirees. In the next section we will apply this piloting rule one the parameters and transform the scheme.

2.4 Transformation of the retirement fund

After showing the problems related to the Defined benefits management on the pensions, we proposed the transformation on the current DB system of the CMR in a new one financed in Pay As You GO (PAYG), the benefits are calculated using a points system where the risks are shared between the retiree and the contributors.Pension benefits can be computed using points in a variety of ways. We adopted the [**?**] approach to transform our system. The transformed system is described as such:

Each year, the payment by the affiliate of his contribution entitles him to a certain number of retirement points. The number of points given is the ratio between the salary of the affiliate and an identical reference salary for all, called acquisition value of the point.

The monetary counterpart of these points is only known on the liquidation date, depending on the value of service of the point on that date.
The number of points earned at the time of retirement is the sum of the points earned during the career. The pension at retirement age is given by the formula:

$$P = N_T \cdot V_T \cdot \sigma_T, \tag{2.9}$$

where N_T represents the number of points earned at retirement age T, V_T the value of the point and σ_T an actuarial coefficient that depends on the length of career and the generation.

In order to determine the liquidation value of the point, we consider an individual who has contributed for a period N with a salary each year equals to the reference salary. The actuarial coefficient equals 1 for this individual. The amount of the pension P_T can be written according to a replacement rate δ and the reference wage S_T^r.

$$P_T = \delta \cdot S_T^r \tag{2.10}$$

and according to the number of points and the value of one point

$$P_T = N \cdot V_T \tag{2.11}$$

we deduce the value of the point:

$$\boxed{V_T = \frac{\delta \cdot S_T^r}{N}} \tag{2.12}$$

The point system presented in this section is a very flexible architecture and can be modeled using various calibrations. We can fix the value of the point and adapt it automatically through the evolution of the replacement rate.

We simulate through the 5 wage trajectories presented in section 2 the transformation of the current scheme in a new one managed with the point system we have just introduced. For individuals joining the plan today we calculate their pension entitlements and the ratio of the present value between the benefits (B) and the contributions (C). We will compare those indicators in the current system and after transformation

	Admin	Engi	A Prof	B teach	Secretaries
Replacement rate	78%	55%	69%	60%	99%
B/C	0,9199	1,0110	0,8843	0,9832	0,8858

Table 2.3: Replacement rates and contribution ratio in the new system

Indexing the pension in relation to the evolution of the average salary of the scheme has the immediate effect of improving the value of the pension for "flat" trajectories, trajectories with evolution rates higher than the evolution of the average wage have replacement rates lower than the target replacement rate of the scheme, the redistribution of wealth in the scheme is done more uniformly across the types of trajectories.

Replacement rates are not capped, thus trajectories with pay decreases at the end of the career have better replacement rates, they are also better for long careers (contribution period greater than the reference period) and do not penalize the fact of having a flat salary evolution. The scheme is more generous for trajectories having little revaluation throughout the career this is the case for secretary.
The second indicator measures the performance of the scheme, for each monetary unit paid in the form of a contribution, the CMR pays an average of 0.96. Its 0.19 less than the current system, the system is in average less generous. The new system benefits contributors with wage developments that are lower than the average wage in the scheme, so there is a different distribution of wealth in the scheme .

After transforming the pension plan we are interested in the impact on the level of the reserves as well as on the horizon of viability. Here is how we model the reserves:

$$Reserves_{t+1} = Reserves_t \cdot (1 + r_t) + Contributions_t - (Pension\ Expenditures)_t \ , \tag{2.13}$$

where r_t is the rate of return of the reserves . The rate of return is supposed constant. This rate corresponds to the average value of the rate observed over the last 10 years. The increase in contribution rates and the decline in replacement rates should slow down the rate of exhaustion of the fund, as the level of implicit debt is very high because the system has operated in the past with generous parameters.

Before transforming the scheme we present a projection of the reserve fund. There is no adjustment.

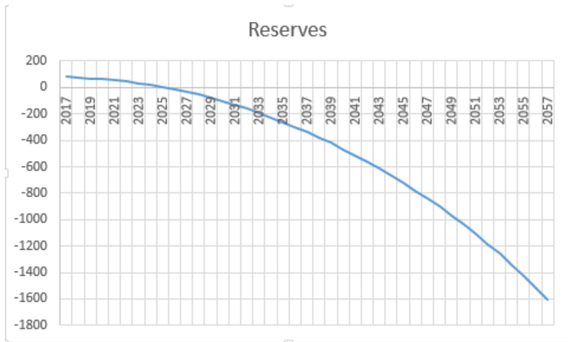

Figure 2.5: Projection the reserve fund without adjustment

We observe that with the current operating parameters of the fund it is possible to maintain a positive level of reserves only until 2027. The management is not financially sustainable in the long term. The deficit continues to grow until the horizon of projection.
After transformation, we introduce Musgrave rule which allows to control the level of contributions and benefits, we measure the impact on the level of reserves in the medium and long term.

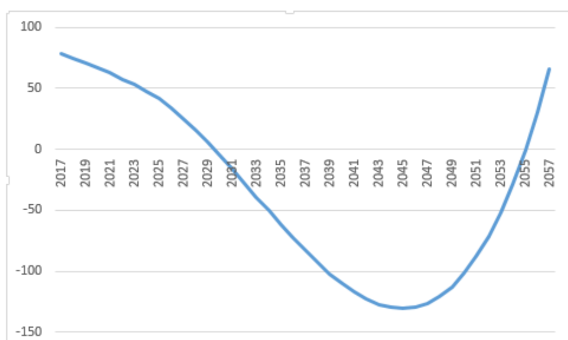

Figure 2.6: Projection the reserve fund with adjustment

The transformation in points pushes the date of exhaustion of the reserves to 2031. That is a gain of 4 years of operation of the regime in addition compared to the management in defined benefits. The system's pricing should be able to balance the technical result in the medium-term regime, at the end of the projection horizon the level of the reserves is again positive due to the piloting of the contribution and replacement rates with the Musgrave rule.

2.5 Conclusion

Facing the failing situation of pay-as-you-go financed pension plans managed in defined-benefit in Morocco , we examined through this work a management model, the points system and a steering mechanism of the plan that would make it possible to overcome the shortcomings of the current system. Our purpose was to determine whether the new scheme is financially sustainable and what is the impact on the standard of living of the contributors and retirees. We first drew a portrait of the current situation of the Moroccan retirement system. This analysis allowed us to identify the diverging parameters from one career to another as well as the problems related to the defined benefit pension management method. We also highlighted the actions taken to solve these problems. Then we presented the theoretical model of the points and Musgrave's rule to control the value of the point as well as contribution rates and equity in the distribution of the costs through active workers and retirees. Under pressure from the deterioration of the demographic ratio, the solutions to maintain pay-as-you-go pension system are becoming fewer and fewer. The point system allowed this load to be distributed with equity.
In this paper we have considered a deterministic approach to model the evolution of the demography and the rates of return. Future extensions will examine stochastic models for the rate of return of reserves.

Annex

Hypotheses for projections of the reserves and the population

In our analysis of the civil pension scheme under an open group framework, we make several key actuarial assumptions to project the evolution of reserves over a 50-year period. These assumptions are in line with the actuarial principles of the Caisse Marocaine des Retraites (CMR).

- Continuity of New Affiliations: We assume the continued inclusion of new affiliations into the pension scheme throughout the entire projection period of 50 years. This assumption reflects the ongoing nature of pension scheme enrolments.
- Investment Rate: The reserves are assumed to be invested with an annual rate of return of 4.25%. This rate is consistent with the actuarial practices outlined in the CMR's actuarial report [CMR,2016].
- Active Worker Replacement: We assume that the scheme anticipates replacing retirees with new incoming active workers. This assumption ensures that the number of active workers remains relatively stable over time, which, in turn, affects the overall scheme's financial stability.
- Mortality Assumption: The mortality table used is the TD 88-90. For individuals aged 60 and beyond the mortality rate are aggravated.
- Pension Revaluation: Pension benefits are assumed to be revalued at a constant rate of 1%.
- Treatment of Pre-Transformation Pension Benefits: It's important to note that under our analysis, the transformation of the system via the Musgrave rule does not retroactively change pension benefits that were already paid out. This rule primarily affects new benefits granted after its implementation.

These actuarial hypotheses provide the foundation for our analysis of the evolution of reserves within the civil pension scheme, ensuring that our projections align with established actuarial standards and the practices of the CMR.

Bibliography

[ACAPS,2017] Autorité de Contrôle des Assurances et de la Prévoyance Sociale, *Rapport annuel, secteur de la prevoyance sociale 2017.* 2017

[Blanchard, 2017] M. Blanchard , Pilotage et gestion d'un regime de retraite et impact sur sa situation financiere suite au decret 2017-887, 2017.

[Comission 2020-2040,(2014)] Comission 2020-2040,(2014) *Un contrat social performant et fiable*, http://pension2040.belgium.be/fr/

[CESE,2017] Conseil Economique, Social et Environnemental, Annual report 2017 `https://www.cese.ma/docs/rapport-annuel-2017/`

[CMR,2016] Caisse Marocaine des Retraite, Activity report, 2016 `https://www.cmr.gov.ma/wps/portal/PortalCMR/publication/rapports-activiteshttps://www.cmr.gov.ma/wps/portal/PortalCMR/publication/rapports-activites`

[CMR,2018] Cour des comptes, *Rapport sur Caisse Marocaine des Retraites* , 2018

[Devolder, 2010] P. Devolder, Perspectives pour nos regime de pensions legales.*Revue Belge de Securite Sociale*,4 , p597-614, 2010.

[Devolder ,2015] DEVOLDER, P. (2015). Pension Reform in Belgium: a new points system between DB and DC. In Proceedings of Colloquium of the International Actuarial Association-LIFE/PBSS/IACA (pp. 1-20).

[HCP,2018] AU PLAN, Haut Commissariat. *Les indicateurs sociaux du Maroc 2018.* 2018.

[Musgrave, 1981] R. Musgrave A reappraisal of social security finance.*Social Security financing*, Cambridge, MIT p89-127, 1981.

[Palier, 2014] B. Palier, *La reforme des retraites*, Chapter 1, Presses Universitaires de France, 2012

[Schokkaert & al,2017] Schokkaert2017 Schokkaert, E., Devolder, P., Hindriks, J., & Vandenbroucke, F. (2020). Towards an equitable and sustainable points system. A proposal for pension reform in Belgium. Journal of Pension Economics & Finance, 19(1), 49-79.

Chapter 3

Progressive Pension Formula and Life Expectancy Heterogeneity

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3.1 Introduction

Many studies have observed different tendencies in the relationship between mortality and income. A cross-country study covering 28 major western countries, [Neumayer, 2016] showed that income inequality before taxes and transfers was positively associated with inequality in the number of years lived. Unlike earlier times, the current increase in life expectancy of European nations seems to have been influenced by the development of the economy, as [Mackenbach, 2013] showed .
This trend was also observed in Denmark in a 2008 study by [Bronnum & al, 2008], where their results showed that the gap in health expectancy between persons with low and high educational levels was increasing with startling constancy.

It has been shown that there are many factors influencing the differences in life expectancy, with income gained during the period of professional activity being one of them ([Kreiner et al, 2018], [Kinge et al, 2019]). On the same track, [Walczak & al, 2021] demonstrated that richer people live longer and that income has a considerably greater influence on life expectancy among men than among women. [Blanpain, 2018] found that in France, people with better quality of life have higher life expectancy. For the period 2012-2016 , among the wealthiest people, men had a life expectancy at birth of 84.4 years. In contrast, among the lowest 5% of people, men had a life expectancy of 71.7 years. The wealthiest men therefore live on average 13 years longer than the poorest [Blanpain, 2016]. Among women, this gap is smaller: the life expectancy at birth of women among the wealthiest 5% of people reaches 88.3 years, compared to 80.0 years among the lowest 5%, i.e., 8 years apart.

Across societies, life expectancy is significantly linked to income ([Currie & Schwandt, 2016], [Mackenbach et al, 2018]). The relationship between a society's socioeconomic class and life expectancy is critical for measuring equality and assessing the risks and benefits of health-care and social security policies,([Snyder, 2006], [Waldron, 2007]). The relationship between income and life expectancy is well documented, ([Currie & Schwandt, 2016b], [Waldron, 2007]). Using tax data, [Chetty & al, 2016] calculated non parametric estimates of the relationship between income class and life expectancy.

When ranked by socio-economic level (SES), [Bosworth & al, 2016] discovered considerable death rate inequalities among elderly Americans, and these differences have expanded dramatically in recent years. It makes little difference whether we assess SES using education or midcareer earnings, though earnings are more directly linked to the income concept used to compute Social Security payments.

These studies demonstrate the influence of wealth disparity on life expectancy. When we apply the relationship between income and remaining life expectancy to public pension systems, we want to know what implications this relationship has on the contribution–benefit connection.

Public pension schemes transfer wealth over generations and, in some cases, among individuals. Individual pension benefits are not perfectly locked down by contributions, resulting in the latter sort of redistribution. Because individual pensions are given out as annuities regardless of life expectancy, the social security system transfers wealth from those with short lives to those with better life expectancies, according to [Borck, 2007] and also [Belloni & al 2013]. When considering the life expectancy difference, Borck's paper may be viewed as an investigation of the impact of pension schemes' wealth transfer characteristics.

Pension systems may be progressive because lower-income persons often contribute less than higher-income people in order to gain a similar pension. Individuals with higher incomes, on the other hand, live longer. As a result, if the longer life expectancy outweighs the greater payments, the system may be regressive. There is evidence that actual pension plans can be regressive if individual variations in life expectancy are considered. (e.g., [Coronado et al, 2000]; [Lefebvre, 2007]; [Reil-Held, 2000]). [Mitchell, 1996] explain that : "Despite its intent, the pension system is less progressive than it might seem, because there is a positive correlation between lifetime earnings and length of life". It is worth noting that this conclusion may be seen in both more Bismarckian systems, like the German system, and more Beveridgean systems, including the United States. More recently , [Economic Policy Committee, 2020] stated that unless alternative sources of funding are found, a shrinking working-age population and an increasing number of retirees will place a double burden on future workers: greater contribution rates while working and smaller pensions when they retire. Furthermore, pension systems must ensure pension fairness and redistribution across income levels, as a shift in funding from social security contributions to taxation, as well as changes in progressivity rules, limits pension systems' redistributive capacity and raises questions about their fairness across income classes.

Heterogeneity in longevity and the underlying unfairness issue in pension schemes motivated [Holzmann & al, 2017] to propose redesigning the Notional Defined Contribution (NDC) scheme. Here, two promising design alternatives were briefly presented: individualized annuities and the multiple contribution model. In comparison to the current system, both conceptual models succeed in minimizing tax disparities.

Longevity heterogeneity and pension fairness have been studied quantitatively since [Ayuso & al 2017], [Ayuso & al 2021] , [Bravo & al, 2021]. In order to measure the overall intensity of the transfer mechanism, profiles for tax/subsidy rates are computed from variances in life expectancy. [Culotta, 2021] quatified this intensity in Italy between 1995 and 2019 by gender and region. This research not only provided an up-to-date picture of differences in lifespan among Italian regions, but it also calculated the implications of such disparities in terms of an implicit transfer of pension resources. His conclusion was that the conventional architecture of Italian public pension systems has to be changed to differentiate structural characteristics, such as the longevity factor used to calculate pension annuities. In principle, this would diminish the intensity of an implicit but persistent tax/subsidy mechanism. The choice of the socioeconomic factor to tag will be critical in this case. Nonetheless, in order to make completely transparent redistributive performances of public pension systems, a closer and updated monitoring of lifespan heterogeneity along important socioeconomic characteristics is required.

Another approach would be taking into account the life expectancy differences directly in the pension formula, as [Breyer & Hupfeld, 2008] did . In their work, they established the notion "distributive neutrality", which considers disparities in life expectancy based on income group., and then they empirically analyzed the relationship between annual wages and life expectancy among retirees and demonstrated how the mechanism that ties benefits to contributions would need to be adjusted to ensure distributive neutrality.

In this chapter, we consider a defined benefit pension system as well as a stationary demographic framework in which agents are differentiated in terms of salary and life expectancy. To study the redistributive features of such a pension system, we consider the actuarial fairness ratio. This ratio for an agent is the discounted value of pension benefit over the agent's contributions. We introduce a progressive transformation of the pension formula that takes into account the life expectancy differential in order to satisfy two conditions: the sustainability condition of a PAYG scheme, and the actuarial fairness ratio equality condition for agents of different salary classes. The main contribution of this chapter is to obtain the explicit form of this progressive transformation. We show in particular that under very special conditions (canonical case), the correction to apply in the pension formula is based on a simple ratio of life expectancies. However, in more general cases, especially when the discount rate used to estimate the level of fairness does not correspond to the actuarial return of the PAYG system, we use a more general transformation formula, based on two multiplicative factors: the ratio of annuities (longevity effect) and the ratio of aggregated salaries (salary effect). The study is structured as follows: in section 2, we show the pension issue in our defined benefit framework as well as introducing the progressive concept, and we study the fairness properties in the general case. In section 3, we apply our model to some specific cases before illustrating it with a simple numerical example. Section 4 allows us to conclude and discuss our results and highlight the questions that our study gives rise to.

3.2 The pension unfairness issue

3.2.1 Defined benefits system pension calculation

Let us imagine a career average reevaluated earnings (CARE) defined benefits scheme . A CARE scheme normally offers an income at retirement based on a proportion of your average earnings, after adjusting these for inflation, during the whole period of membership of the scheme.

The pension benefit formula for a complete career at time t is :

$$P^{(x_r,t)} = \frac{\delta}{N} \cdot \sum_{x=x_0}^{x_r-1} S^x \prod_{i=1}^{x} (1+\gamma_i)^{x_r-i}$$

Where:

x_0 is the age of entry in the scheme and x_r is the retirement age

N is the length of contribution.

$P^{(x_r,t)}$ is the Pension at retirement for a contribution period of $x_r - x_0$ years

γ_i is the indexation rate during the period i.

δ is the target replacement rate of the scheme.

S^x is the individual salary taken into account at age x.

- **At retirement**

We assume that pensions in payment are adjusted each year according to the same coefficient as the indexation rate applied to the salary indexation of active workers:

$$P^{(x,t)} = P^{(x,t-1)}.(1+\gamma_t) \tag{3.1}$$

- **Actuarial Inequality tied to life expectancy disparity**

Let us imagine a scheme with only two salary classes, low incomes and high incomes ($S_1 < S_2$), associated with low and high life expectancy at retirement ($e_1 < e_2$).
In this simplistic framework there is no revaluation and salaries are constant. There is no interest rate.

In the simplistic DB framework, the pension formula at retirement is computed as proportion of the salary:

$$P_j = \delta \cdot S_j \tag{3.2}$$

For each class j the probability of surviving t years at age x is given by

$$_{x+t}p_x^j \tag{3.3}$$

No salary indexation implies that the present value of benefits at retirement B_j until death age noted ω is given by:

$$B_j = \sum_{x=x_r}^{\omega} P_j \cdot {}_{x_r}p_x^j = P_j \cdot \sum_{x=x_r}^{\omega} {}_{x_r}p_x^j \tag{3.4}$$

$$B_j = P_j \cdot e_j, \tag{3.5}$$

where e_j is the remaining life expectancy at retirement.
The present value of contributions with zero discount rate is the sums of all careers contributions, with π being the contribution rate.

$$C_j = \sum_{x=x_0}^{x_r-1} \pi \cdot S_j \tag{3.6}$$

$$C_j = N \cdot \pi \cdot S_j. \tag{3.7}$$

This leads to the the following results in term of fairness ratio for low incomes :

$$\frac{B_1}{C_1} = \frac{P_1.e_1}{N.\pi.S_1} = \frac{\delta}{N.\pi} \cdot e_1 \tag{3.8}$$

and for high incomes :

$$\frac{B_2}{C_2} = \frac{P_2.e_2}{N.\pi.S_2} = \frac{\delta}{N.\pi} \cdot e_2$$

The actuarial fairness ratio for an agent in this scheme is a proportion of the ratio between the remaining life expectancy at retirement of this agent and his contribution period.

For the same contribution period between the two agents, this ratio grows with life expectancy. This means that agents with higher life expectancy are expected to get more benefits due to the fact that they receive their pension benefit for a longer period. The positive relationship of income with life expectancy lead us to think that low incomes get penalized for not living long enough.
This mechanism is a result of the proportionality of the pension benefit with the salary in the considered DB scheme.
The following part introduce the progressive transformation of the pension formula that take into account the life expectancy inequality in order to reach actuarial fairness for all salary class.

3.2.2 Progressivity

[Biggs & al, 2009] defined progressivity as "The degree to which benefits are higher relative to lifetime payroll contributions for lower contributors than for higher contributors". This is a translation of [Musgrave & Thin, 1948] approach regarding income tax.

The characterization of progressivity in pension benefits formulae is given in an OCED pension report [OCED, 2011]. In this study, we show two pension systems operating according to a progressive formula: Switzerland and the USA.

- USA

The pension formula is progressive. Earning USD 895 a month gives a 90% replacement rate.

In 2018, the first 895 USD a month of relevant earnings attracts a 90% replacement rate. The income range is replaced at 32% from 895 USD and 5397 USD. The 2018 average national pay index's upper and lower bounds are 20% and 118%, respectively. Between the earlier cutoff and the earnings limitation, a replacement rate of 15% is applied.

$$P = \begin{cases} 0.9 \cdot S & \text{if } S < 10740 \\ 9666 + 0.32 \cdot (S - 10740) & \text{if } 10740 < S < 64764 \\ 20725 + 0.15 \cdot (S - 5397) & \text{if } S > 64764 \end{cases} \tag{3.9}$$

- Switzerland

The public earnings-related pension benefit is based on average lifetime earnings. Benefit payments include both superior and inferior limitations. The "two-branch" pension calculation benefits average earnings between these two levels. The pension computation tends to transfer income from rich to poor agents. Pension payouts ranged from CHF 14,100 to CHF 28,200 in 2016 with complete contributions. These figures correspond to 16% and 33% of mean worker wages, respectively. When the average income is CHF 84,600, the maximum profit is 99% of the national economy average income. The formula for the complete contribution period is as follows:

$$P = \begin{cases} 0.84 \cdot 16800 & \text{if } S < 16800 \\ 14113 + 0.56 \cdot (S - 16800) & \text{if } 16800 < S < 84600 \\ 0.33 \cdot 84600 & \text{if } S > 84600 \end{cases} \tag{3.10}$$

The existing system calibrates the progressive factors such that the pension benefit reaches certain thresholds such as quantiles of average salary; this approach only serves the accounting purpose of progressivity.

Our aim in the following part is to link the value of the progressive coefficients to the differences in life expectancy between salary classes in order to satisfy actuarial and accounting equilibrium.

3.2.3 Progressive pension model and fairness

The defined benefit system is one of the most popular Pay-As-You-Go pension schemes, and it has been shown, under the premise of life expectancy inequalities related to income, that this system is unfair to low-income individuals.

Our aim is to use progressivity to calibrate the progressive coefficients with two conditions: actuarial fairness and Pay-As-You-Go equilibrium

Model hypothesis

- The contribution period is noted N.
- There are m working classes determined upon entry into the scheme characterized by their salary $(S_1; ..; S_m)$ and their proportion $(w_1, .., w_m)$
- The age of entry in the scheme is noted x_0 and the age of retirement is noted x_r
- $L(x,t)$ is the population function giving at date t the size of the population of age x.

  We can write $L(x,t) = \sum_{i=1}^{m} L_i(x,t)$ where $L_i(x,t) = w_i.L(x,t)$ is the size of the population aged x at time t in the class i.
- Let us initially consider a population in an absolute stationary state. The population function is therefore independent of time .
- We ignore the effects of mortality before retirement. The population function at any age before retirement $x < x_r$ is given by :

$$L(x,t) = L(x) = L \tag{3.11}$$

- After retirement we consider that each class has a different mortality evolution after retirement age. ${}_{x+t}p^j_x$ represents the probability of an agent aged x and belonging to class j to survive t years. The population of class j is given by:
$$L_j(x,t) = L_j(x_r,t) \cdot {}_{x_r}p^j_x \tag{3.12}$$
- At any time t, let us imagine that the salary of and agent depends on his age (x) this gives
$$S_j^{(x,t)} = S_j^{(x_0,0)} \cdot (1+s_j)^{x-x_0} \cdot (1+g)^t \tag{3.13}$$
Where s_j is the career growth effect for the group j salary and g the inflation effect.
- The progressive salary transformation is given by:
$$X(S_1) = \lambda_1.S_1$$
$$X(S_j) = \lambda_1.S_1 + \sum_{i=2}^{j} \lambda_i \cdot (S_i - S_{i-1})$$
The general formula is:
$$X(S) = \sum_{i=1}^{m} \lambda_i \cdot \max(\min(S,S_i) - S_{i-1};0), \tag{3.14}$$
where $\lambda_1, ..., \lambda_m$ are the progressive coefficients.
- After retirement pension benefits are indexed with the same rate γ , we have the following relationship at any age $x > x_r$:
$$P^{(x,t)} = P^{(x_r,t)}.(1+\gamma)^{(x-x_r)} \tag{3.15}$$

Pension formula

We have implemented the progressive mechanism in the pension formula aiming to correct the underlying unfairness issue. In order to transform a DB system into a progressive one we chose to transform the salary in the pension formula using salary bandwidths and on those thresholds we apply the progressive coefficients.

We can write the salary transformation for any class j :

$$X(S_j^{(x,t)}) = X_j^{(x,t)} = \sum_{i=1}^{j} \lambda_i \cdot (S_i^{(x,t)} - S_{i-1}^{(x,t)}).$$

For any $T > t, X_j^{(x,T)} = X_j^{(x,t)} \cdot (1+g)^{T-t}$ we obtain the pension at retirement time $T = N$ with:

$$\begin{aligned}
P_j^{(x_r,N)} &= \sum_{t=0}^{N-1} \frac{\delta}{N} \cdot X_j^{(x_0+t,t)} \cdot (1+\gamma)^{N-t} \\
&= \sum_{t=0}^{N-1} \sum_{i=1}^{j} \frac{\delta}{N} \cdot \lambda_i \cdot (S_i^{(x_0+t,t)} - S_{i-1}^{(x_0+t,t)}) \cdot (1+\gamma)^{N-t} \\
&= \frac{\delta}{N} \cdot \sum_{i=1}^{j} \lambda_i \sum_{t=0}^{N-1} (S_i^{(x_0+t,t)} - S_{i-1}^{(x_0+t,t)}) \cdot (1+\gamma)^{N-t}
\end{aligned}$$

We have also

$$\begin{aligned}
\widehat{S_j^N}(\gamma,g) &= \sum_{t=0}^{N-1} (S_i^{(x_0+t,t)}) \cdot (1+\gamma)^{N-t} \\
&= \sum_{t=0}^{N-1} \cdot S_j^{(x_0,0)} \cdot (1+g)^t . (1+s_j)^t \cdot (1+\gamma)^{N-t} \\
&= S_j^{(x_0,0)} \cdot (1+\gamma) \cdot \frac{[(1+g).(1+s_j)]^N - (1+\gamma)^N}{(1+g).(1+s_j) - (1+\gamma)}
\end{aligned}$$

The last term is the future value of a geometric annuity with growth $(1+g).(1+s_j)$ indexed at rate $1+\gamma$ over a period of time N. It is noted

$$Gs_j^{(\gamma)} = (1+\gamma). \frac{[(1+g).(1+s_j)]^N - (1+\gamma)^N}{(1+g).(1+s_j) - (1+\gamma)}, \tag{3.16}$$

this leads to:

$$P_j^{(x_r,N)} = \frac{\delta}{N} \cdot \sum_{i=1}^{j} \lambda_i \cdot (\widehat{S_i^N}(\gamma,g) - \widehat{S_{i-1}^N}(\gamma,g)) = \frac{\delta}{N} \cdot \widehat{X_j^N}(\gamma,g) \tag{3.17}$$

with :

$$\widehat{X_j^N}(\gamma,g) = \sum_{i=1}^{j} \lambda_i \cdot (\widehat{S_i^N}(\gamma,g) - \widehat{S_{i-1}^N}(\gamma,g)) \tag{3.18}$$

Here we give an explanation on the notations we have introduced so far:

- $\widehat{S_j^N}(\gamma,g)$ is the sum of all salaries through a career for an agent of class j indexed at rate γ , with g being the inflation effect.
- $\widehat{X_j^N}(\gamma,g)$ is the progressive transformation applied to the indexed sum of salary.

3.2.4 Pay-as-you-go equilibrium

The number of active workers in the scheme is :

$$\sum_{x=x_0}^{x_r-1} L(x,t) = \sum_{x=x_0}^{x_r-1} L = N \cdot L \tag{3.19}$$

The total amount of contributions in the scheme for the agents of class i is :

$$\begin{aligned}\sum_{x=x_0}^{x_r-1} L_i(x,t).\pi.S_i^{(x,t)} &= \sum_{x=x_0}^{x_r-1} w_i.L.\pi.S_i^{(x,t)} \\ &= w_i.L.\pi.\sum_{x=x_0}^{x_r-1} S_i^{(x_0,t)} \cdot (1+s_i)^{x-x_0} \\ &= w_i.L.\pi.S_i^{(x_0,t)}.\sum_{x=x_0}^{x_r-1} (1+s_i)^{x-x_0}\end{aligned}$$

Where $S_i^{(x_0,t)}.\sum_{x=x_0}^{x_r-1}(1+s_i)^{x-x_0}$ is the sum of all salaries of class i from age x_0 to x_{r-1} at time t.

For all retired agent of class i the total of pension benefits paid is:

$$\begin{aligned}\sum_{x=x_r}^{\omega} L_i(x,t)P_i^{(x,t)} &= L.w_i.P_i^{(x_r)}.\sum_{x=x_r}^{\omega} {}_x p_{x_r}^i (\frac{1+\gamma}{1+g})^{(x-x_r)} \\ &= L.w_i.P_i^{(x_r,t)} \cdot a_j^{(\gamma,g)}\end{aligned}$$

Where $a_j^{(\gamma,g)} = \sum_{x=x_r}^{\omega} {}_x p_{x_r}^i (\frac{1+\gamma}{1+g})^{(x-x_r)}$ is a life annuity discounted with rate $\frac{(1+g)}{(1+\gamma)}$.
The pay-as-you-go equilibrium is characterized by the following relationship:

$$Contributions_t = Benefits_t$$

We are looking for the π value that satisfy this equilibrium for any given replacement rate.

$$\sum_{x=x_0}^{x_r-1}\sum_{i=1}^{m} L_i(x,t).\pi.S_i^{(x,t)} = \sum_{x=x_r}^{\omega}\sum_{j=1}^{m} L_j(x,t)P_j^{(x,t)}$$
$$\pi.\sum_{x=x_0}^{x_r-1}\sum_{i=1}^{m} w_i.L.S_i^{(x,t)} = \sum_{j=1}^{m} w_j.L.a_j^{(\gamma,g)}\frac{\delta}{N}.\widehat{X_j}(\gamma,g).$$

We divide this expression by L.

$$\pi.\sum_{x=x_0}^{x_r-1}\sum_{i=1}^{m} w_i.S_i^{(x,t)} = \frac{\delta}{N}.\sum_{j=1}^{m} w_j.\widehat{X_j}(\gamma,g).a_j^{(\gamma,g)}$$
$$\pi.\sum_{i=1}^{m} w_i.\sum_{x=x_0}^{x_r-1} S_i^{(x,t)} = \frac{\delta}{N}.\sum_{j=1}^{m} w_j.\widehat{X_j}(\gamma,g).a_j^{(\gamma,g)}$$

Here we introduce $\bar{S} = \sum_{i=1}^{m} w_i.\sum_{x=x_0}^{x_r-1} S_i^{(x,t)}$ being the average salary of all agents in the scheme .

This gives us for the sustainability condition between the contribution rate, the replacement rate and progressive factors.

$$\delta = \frac{N.\pi.\bar{S}}{\sum_{i=1}^{m}\lambda_1 w_i.a_i^{(\gamma,g)}.\widehat{S_1^N}(\gamma,g) + \sum_{i=2}^{m}\lambda_i\sum_{j=i}^{m} w_j.a_j^{(\gamma,g)}.(\widehat{S_i^N}(\gamma,g) - \widehat{S_{i-1}^N}(\gamma,g))} \tag{3.20}$$

The equilibrium is characterized by this equation for $(\lambda_1;..\lambda_m;\delta)$ where $\delta > 0$

$$\pi = \frac{\delta\cdot\sum_{i=1}^{m}\lambda_i.b_i}{N.\bar{S}} \tag{3.21}$$

with $b_j = \sum_{j=i}^{m} w_j.a_j^{(\gamma,g)}.(\widehat{S_i^N}(\gamma,g) - \widehat{S_{i-1}^N}(\gamma,g))$

This relationship express the fact that for a given replacement rate δ and a vector of progressive factors $\lambda' = (\lambda_1;..;\lambda_m)$, the contribution rate that allows for the sustainability of the scheme is given by the previous relationship 3.21 .
Using this, we implement our second criteria that is the actuarial fairness for every agent in order to find the value of the progressive factors $(\lambda_1, ..., \lambda_m)$.

Inter-class fairness conditions

We reach inter-class fairness when the actuarial fairness ratio is the same for every agent regardless of his salary class.

The PAYG condition allowed us to find a value for the system target contribution rate. In order to calibrate the progressive coefficients we need to solve the inter-class fairness conditions :

- Present value of contributions

The present value of contributions is computed with the discount factor r. It is not necessarily an actuarial rate of return.

$$\begin{aligned} C_j^N &= \sum_{t=0}^{N-1} \pi \cdot S_j^{(x_0+t,t)} \cdot (1+r)^{N-t} \\ &= \sum_{t=0}^{N-1} \pi \cdot S_j^{(x_0,0)} \cdot (1+g)^t.(1+s_j)^t \cdot (1+r)^{N-t} \\ C_j^N &= \pi \cdot S_j^{(x_0,0)} \cdot (1+r) \cdot \frac{[(1+g).(1+s_j)]^N - (1+r)^N}{(1+g).(1+s_j) - (1+r)} \end{aligned}$$

Using the geometric annuity form we can write the present value of contributions as:

$$C_j^N = \pi \cdot S_j^{(x_0,0)} \cdot Gs_j^{(r)} = \pi \cdot \widehat{S_j^N}(r,g) \tag{3.22}$$

- Present value of benefits

At retirement time N the pension benefit for the class j is $P_j^{(x_r,N)}$. The present value of benefits for the class j is indexed with rate $1+\gamma$ is computed with the survival probability p_j for the class j:

$$\begin{aligned} B_j^N &= \sum_{x=x_r}^{\omega} P_j^{(x_r,N)} \cdot {}_{x_r}p_x^j . (\frac{1+\gamma}{1+r})^{x-x_r} \\ B_j^N &= P_j^{(x_r,N)} \cdot a_j^{(\gamma,r)} \end{aligned}$$

Where $a_j^{(\gamma,r)} = \sum_{x=x_r}^{\omega} {}_{x_r}p_x^j . (\frac{1+\gamma}{1+r})^{x-x_r}$ is a life annuity calculated for the population with survival probability ${}_{x+t}p_x^j$ [1]

- Fairness condition

[1] in particular when $\gamma = r$, $a_j^{(\gamma,r)} = e_j$

We need to obtain the values of λ for each career by solving the following inter-class fairness conditions :

$$\frac{B_1^N}{C_1^N} = ... = \frac{B_j^N}{C_j^N} = ... = \frac{B_m^N}{C_m^N}$$

This leads to the following system

$$\begin{cases} \dfrac{B_1^N}{C_1^N} = \dfrac{\delta.\lambda_1}{N.\pi}.\dfrac{\widehat{S_1^N}(\gamma,g)}{\widehat{S_1^N}(r,g)}.a_1^{(\gamma,r)} \\ ... \\ \dfrac{B_j^N}{C_j^N} = \dfrac{\delta}{N.\pi.\widehat{S_j^N}(r,g)}.(\lambda_1.\widehat{S_1^N}(\gamma,g) + \sum_{i=2}^{j} \lambda_i.(\widehat{S_i^N}(\gamma,g) - \widehat{S^N}_{i-1}(\gamma,g))).a_j^{(\gamma,r)} \\ ... \\ \dfrac{B_m^N}{C_m^N} = \dfrac{\delta}{N.\pi.\widehat{S_m^N}(r,g)}.(\lambda_1.\widehat{S_1^N}(\gamma,g) + \sum_{i=2}^{m} \lambda_i.(\widehat{S_i^N}(\gamma,g) - \widehat{S^N}_{i-1}(\gamma,g))).a_m^{(\gamma,r)} \end{cases} \tag{3.23}$$

The solution is given by:

$$\lambda' = (\lambda_1, ..., \lambda_m)$$

λ is thus the vector of solution for our $(m-1)$ conditions. The coordinates are all a scalar times λ_1.

$$\lambda = \lambda_1 \cdot \frac{\widehat{S_1^N}(\gamma,g)}{\widehat{S_1^N}(r,g)} \cdot \left| \begin{array}{c} 1 \\ \dfrac{\widehat{S_2^N}(r,g) \cdot \dfrac{a_1^{(\gamma,r)}}{a_2^{(\gamma,r)}} - \widehat{S_1^N}(r,g) \cdot \dfrac{a_2^{(\gamma,r)}}{a_2^{(\gamma,r)}}}{\widehat{S_2^N}(\gamma,g) - \widehat{S_1^N}(\gamma,g)} \\ . \\ . \\ . \\ \dfrac{\widehat{S_i^N}(r,g) \cdot \dfrac{a_1^{(\gamma,r)}}{a_i^{(\gamma,r)}} - \widehat{S^N}_{i-1}(r,g) \cdot \dfrac{a_1^{(\gamma,r)}}{a_{i-1}^{(\gamma,r)}}}{\widehat{S_i^N}(\gamma,g) - \widehat{S_{i-1}^N}(\gamma,g)} \\ . \\ . \\ . \\ \dfrac{\widehat{S_m^N}(r,g) \cdot \dfrac{a_1^{(\gamma,r)}}{a_m^{(\gamma,r)}} - \widehat{S^N}_{m-1}(r,g) \cdot \dfrac{a_1^{(\gamma,r)}}{a_{m-1}^{(\gamma,r)}}}{\widehat{S_m^N}(\gamma,g) - \widehat{S_{m-1}^N}(\gamma,g)} \end{array} \right| \tag{3.24}$$

The progressive fair pension is then expressed as:

$$P_j^{(x_r,N)} = \frac{\delta}{N} \cdot \widehat{X_j^N} = \frac{\delta}{N} \cdot \sum_{i=1}^{j} \lambda_i \cdot (\widehat{S_i^N}(\gamma, g) - \widehat{S_{i-1}^N}(\gamma, g))$$

We then use the expression of the progressive factors solution of the inter-class fairness condition in the pension formula, this allows us to reduce the telescopic sum in this way for any class $j > 1$:

$$P_j^{(x_r,N)} = \delta \cdot \lambda_1 \cdot \frac{\widehat{S_1^N}(\gamma, g)}{\widehat{S_1^N}(r, g)} . \frac{\widehat{S_j^N}(r, g)}{N} \cdot \frac{a_1^{(\gamma,r)}}{a_j^{(\gamma,r)}} \tag{3.25}$$

Here we introduce

$$\beta_j = \frac{\widehat{S_j^N}(r, g)}{\widehat{S_j^N}(\gamma, g)} \cdot \frac{\widehat{S_1^N}(\gamma, g)}{\widehat{S_1^N}(r, g)}, \tag{3.26}$$

being the correction term for indexation for the classes. The fair pension becomes then:

$$\boxed{P_j^{(x_r,N)} = \delta \cdot \lambda_1 \cdot \frac{\widehat{S_j^N}(\gamma, g)}{N} \cdot \beta_j \cdot \frac{a_1^{(\gamma,r)}}{a_j^{(\gamma,r)}}} \tag{3.27}$$

This quantity can be interpreted as a replacement rate multiplied by the average career salary with two corrections, the first one is an economic correction linked to the distortion between the evolution of the agent salary during his career and the scheme indexation rate and the second one is a correction tied to the mortality differential between the classes.

Proposition 3.2.1 *For any couple (δ, λ_1) and any discount rate r there is a vector $\lambda = (\lambda_1; ..; \lambda_m)$ satisfying at the same time the PAYG and inter-class fairness conditions given by 3.21 and 3.24.*

The theoretical expression of the actuarial ratio is the same for every agent.

$$\frac{B_j}{C_j} = \frac{\delta \cdot \lambda_1}{N.\pi} \cdot . \frac{\widehat{S_1^N}(\gamma, g)}{\widehat{S_1^N}(r, g)} . a_1^{(\gamma,r)} \tag{3.28}$$

This result displays the fact the progressive fair pension makes it possible to obtain the same ratio of benefits to contributions for all classes. But this ratio is not necessarily equal to one. In the following part we are going to explore the condition on the discount rate for the system to be fair and reach equilibrium.

3.3 Canonical actuarial and indexation rates

Aaron-Samuelson [Aaron, 1966] linked the equilibrium 'return'(actuarial rate) on unfunded social security to the rate of growth of earnings, being the sum of earnings growth per head and growth of population.

In our framework, the canonical case will correspond to the two following conditions :

- The revaluation of pensions is consistent with the increase of salaries ($g = \gamma$);
- The discount rate r used to estimate the level of fairness is equal to the rate of return of the PAYG system. In the demographic stationary situation ,this rate of return is equal to the rate of increase of the salaries (g).

Therefore , the canonical case leads to : $r = g = \gamma$

3.3.1 Canonical model

We chose the canonical values of the discount factor and the indexation rate of the scheme. In a stationary demographic framework they are equal . We compute here the progressive coefficients in the canonical case.

- **Pension formula and present values**.

The sum of the salaries becomes:

$$\widehat{S_j^N} = S_j^{(x_0,0)} \cdot \frac{1-(1+s_j)^N}{s_j}$$

Thus the pension formula 3.17 transforms into :

$$P_j^N = \frac{\delta}{N} \cdot \sum_{i=1}^{j} \lambda_i \cdot (\widehat{S_i^N} - \widehat{S_{i-1}^N}) = \frac{\delta}{N} \cdot \widehat{X_j^N} \tag{3.29}$$

with :

$$\widehat{X_j^N} = \sum_{i=1}^{j} \lambda_i \cdot (\widehat{S_i^N} - \widehat{S_{i-1}^N}) \tag{3.30}$$

The present value of benefits is at retirement time N is :

$$\begin{aligned} B_j^N &= \sum_{x=x_r}^{\omega} P_j^N \cdot {}_x p_{x_r}^j \cdot (\frac{1+\gamma}{1+r})^{x-x_r} \\ &= P_j^N \sum_{x=x_r}^{\omega} \cdot {}_x p_{x_r}^j \\ B_j^N &= P_j^N \cdot e_j \end{aligned}$$

The present value of contributions at retirement time N is given by:

$$C_j^N = \sum_{t=0}^{N-1} \pi \cdot S_j^{(x_0+t,t)} \cdot (1+r)^{N-t}$$

$$C_j^N = \pi \cdot \widehat{S_j^N} \tag{3.31}$$

- **PAYG equilibrium**.

The PAYG equilibrium expression on the target replacement rate and contribution rate becomes independent of (γ, g) and becomes

$$\pi = \frac{\delta \cdot \sum_{i=1}^{m} w_i.\widehat{X}_i^N.e_i}{N.\bar{S}} \tag{3.32}$$

- **Inter-class fairness**.

We express the actuarial fairness condition equality between the classes. This lead to the condition on the progressive factors:

$$\lambda = \lambda_1 \cdot \left| \begin{array}{c} 1 \\ \dfrac{\widehat{S_2^N} \cdot \dfrac{e_1}{e_2} - \widehat{S_1^N} \cdot \dfrac{e_2}{e_2}}{\widehat{S_2^N} - \widehat{S_1^N}} \\ . \\ . \\ . \\ \dfrac{\widehat{S_i^N} \cdot \dfrac{e_1}{e_i} - \widehat{S^N}_{i-1} \cdot \dfrac{e_1}{e_{i-1}}}{\widehat{S_i^N} - \widehat{S_{i-1}^N}} \\ . \\ . \\ . \\ \dfrac{\widehat{S_m^N} \cdot \dfrac{e_1}{e_m} - \widehat{S^N}_{m-1} \cdot \dfrac{e_1}{e_{m-1}}}{\widehat{S_m^N} - \widehat{S_{m-1}^N}} \end{array} \right| \tag{3.33}$$

The progressive fair pension calculated with the progressive coefficients is then :

$$P_j^N = \frac{\delta}{N} \cdot \widehat{X_j^N} = \frac{\delta}{N} \cdot \sum_{i=1}^{j} \lambda_i \cdot (\widehat{S_i^N} - \widehat{S_{i-1}^N})$$

We compute the pension with the progressive factors obtained via the inter-class fairness condition :

$$\boxed{P_j^N = \delta \cdot \lambda_1 \cdot \frac{\widehat{S_j^N}}{N} \cdot \frac{e_1}{e_j}} \tag{3.34}$$

In this case , since the discount rate and the indexation rate are the same, we notice that the fair pension has only one correction term which is the mortality correction expressed as the ratio between two remaining life expectancy; the salary term has no effect with canonical rates :

$$\beta_j = 1 \tag{3.35}$$

Using 3.34 and 3.31, the actuarial ratio becomes :

$$\begin{aligned}\frac{B_j^N}{C_j} &= \frac{\delta.\lambda_1.e_1}{N.\pi} \\ &= \frac{\lambda_1.e_1.\bar{S}}{\sum_{i=1}^m w_i.\widehat{X}_i.e_i}\end{aligned}$$

this sum is telescopic, we then obtain :

$$\sum_{i=1}^{m} w_i.\widehat{X}_i.e_i = \sum_{i=1}^{m} w_i.\lambda_1.S_i.e_1 = \lambda_1.e_1.\bar{S}$$

Thus with canonical rates the system reaches perfect actuarial fairness for all the classes :

$$\frac{B_j^N}{C_j^N} = 1 \tag{3.36}$$

3.3.2 Salary indexation assumptions and illustration

We will consider here a special case of canonical environment where $g = 0$ and $s_j = 0$,

- **Salaries are constant during the career**.
- There is no indexation.
- The annuity used to compute the present value of benefits is simply the life

expectancy at retirement:

$$
\begin{aligned}
B_j &= \sum_{x=x_r}^{\omega} P_j \cdot {}_x p_{x_r}^j \\
&= P_j \sum_{x=x_r}^{\omega} \cdot {}_x p_{x_r}^j \\
B_j &= P_j \cdot e_j
\end{aligned}
$$

At any time t, the salary in a class j is $S_j^t = S_j$.

For an individual in a salary class j, his pension at retirement for a complete career is computed as such:

$$
\begin{aligned}
P_j &= \sum_{t=1}^{N} \frac{\delta}{N} \cdot X_j \\
P_j &= \delta \cdot X_j
\end{aligned}
$$

The PAYG equilibrium becomes:

$$\sum_{i=1}^{m} w_i.P_i^N.e_i = \sum_{i=1}^{m} w_i.C_i^N \tag{3.37}$$

This leads to the condition on the contribution rate:

$$\pi = \frac{\delta \cdot \sum_{i=1}^{m} w_i.X_i.e_i}{N.\bar{S}} \tag{3.38}$$

With the inter-class fairness we have the following condition for the progressive factors for $j > 1$:

$$\lambda_j = \lambda_1 \cdot \frac{S_j \cdot \frac{e_1}{e_j} - S_{j-1} \cdot \frac{e_1}{e_{j-1}}}{S_j - S_{j-1}}$$

The fair pension is given by :

$$P_i = \delta \cdot \lambda_1 \cdot S_i \cdot \frac{e_1}{e_i} \tag{3.39}$$

This expression is really intuitive and underlines the fact that in this particular situation we can correct unfairness using the life expectancy ratio in the pension formula.

- **Common wage indexation effect**

It is also possible to isolate the longevity heterogeneity effect. Indeed the indexation of the wage during the career affects the average wage in the progressive factors expression. Combined with the actuarial fairness principle, the progressive pension formula in a career average scheme corrects the career differences as well as the longevity differences. Let us assume that across the salary classes the wage indexation is the same during the career, for an agent of class j

$$\widehat{S_j^N} = S_j^{(x_0,0)} \cdot \frac{1-(1+s)^N}{s},$$

where s is the indexation rate of the wage during the career common to every agent.
We obtain the progressive coefficients via 3.33 and the progressive coefficients are given by :

$$\lambda_j = \lambda_1 \cdot \frac{S_j^{(x_0,0)} \cdot \frac{e_1}{e_j} - S_{j-1}^{(x_0,0)} \cdot \frac{e_1}{e_{j-1}}}{S_j^{(x_0,0)} - S_{j-1}^{(x_0,0)}}$$

The progressive pension becomes :

$$P_j = \frac{\delta}{N} \cdot \lambda_1 \cdot S_j^{(x_0,0)} \cdot \frac{1-(1+s)^N}{s} \cdot \frac{e_1}{e_j} \tag{3.40}$$

$$P_j = \frac{\delta}{N} \cdot \lambda_1 \cdot \widehat{S_j^N} \cdot \frac{e_1}{e_j} \tag{3.41}$$

Since the wage indexation rate is common in every class, there is no specific career effect in the pension formula from one class to another only the starting salary and differences in life expectancy across classes affect the progressive pension benefit. $\frac{1}{N} \cdot \frac{1-(1+s)^N}{s}$ is the wage indexation effect common to every class and $\frac{e_1}{e_j}$ the longevity heterogeneity correction.

Numerical illustration

Here we used French data on Distribution of net monthly salaries in 2016 from " Tableaux de l'économie française Édition 2018 "[2]. We chose 8 quantiles of salary to create our classes, with each salary is associated a life expectancy at 65. We assume there is no salary indexation. In our defined Benefit scheme the target

Table 3.1: Life expectancy with salary level.

Social class	**Salary**	**Life expectancy(65)**
Class 1	1189	19,47
Class 2	1346	20,36
Class 3	1479	21,26
Class 4	1621	22,18
Class 5	1995	23,1
Class 6	2273	24,03
Class 7	2709	24,98
Class 8	3576	25,93

replacement rate is fixed : $\delta = 0.8$.
We use the conditions of (4.1) in order to generate our progressive coefficients:

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
1,00	0,63	0,51	0,48	0,69	0,58	0,62	0,66

This allow us to find the equilibrium contribution rate : $\pi = 0.346$.

If we apply the correction of those progressive factors on the salary thresholds in the pension formula.The inter-class fairness condition is respected also the actuarial fairness ratio reaches 1 because there is no indexation.

3.4 Discussion

Income during one's career and the remaining life expectancy at retirement are inextricably linked. In pension schemes, this correlation induces certain inequalities. The average life expectancy tends to increase with increasing income, meaning that high-status agents obtain more benefits relative to their contributions in most Bismarckian pension designs [Hachon, 2009].
In many nations, income distributions are getting increasingly uneven. As mentioned before, mortality and lifespan inequalities have been widening in many, but not all, nations. While broadening income distributions appear to be an apparent explanation for growing Socioeconomic Status (SES) mortality disparities, most

[2]https://www.insee.fr/fr/statistiques/3303417?sommaire=3353488

empirical research is unable to throw light on this issue because they measure income or education by quantiles, which do not represent broadening or narrowing of distributions. Our methodology for the agents classification also relies on quantiles income. Although secular trends in differential mortality are considerable, their impact on the length of time people get benefits is muted by the fact that low-SES individuals tend to claim Social Security at earlier ages, and high-SES employees are more inclined to postpone retirement and benefit claiming. Differences in mortality throughout the earnings distribution negate part of the Social Security benefit formula's progressivity, but the pattern of lifetime benefits remains progressive. [Bosworth & al, 2016]

This chapter sought to introduce a progressive component in the pension formula of a defined-benefit scheme, offering a correction to the fairness linked to life expectancy inequalities in a stationary demographic framework. In comparison with [Breyer & Hupfeld, 2008] and their "Distributive Neutrality" approach in relation to pension benefits, our method introduces an additional correction term, the mortality correction, which is quite similar. The progressive pension formula that we proposed used factors that are the solution of actuarial equilibria conditions on salary level thresholds that can be chosen. The main result is that under the absolute stationary hypothesis, fairness between the classes can be obtained for any discount rate.

Our contribution is based on a theoretical discussion of the redistributive features of pension systems when life expectancy is linked to salary levels. We expand on the sentence from [Mitchell, 1996] that was mentioned in the beginning. PAYG DB pension systems are demonstrated to be less progressive than they appear to be. Furthermore, we demonstrate that, when the progressive pension formula is used, an equivalent redistribution of resources by wage distribution may be achieved.

Our progressive pension model links the pension formula to demographic and economical factors, with the immediate result being that the pension is reduced as life expectancy increases. This is an especially important issue for low-wage workers. As life expectancy rises, cutting their already inadequate payments could lead to a revival of old-age poverty. This link was studied in [OCED, 2011], where they demonstrated that there must be limits to tying benefit amounts to life expectancy. If benefit cuts force low-income workers to rely on social assistance and other safety-net programs in retirement, the savings from the life-expectancy link in public earnings-related benefits will be negated in part or entirely (notional accounts, defined-benefit or points). With private defined-contribution plans, it will mean more public spending. Parts of the pension system may have enhanced financial stability, but retirement income provision as a whole will not.

Our proposal introduces a modification of the benefit formula ,it's vital to remember that modifying the retirement pension formula can have significant incentive consequences, which can affect resource allocation efficiency. In a dynamically efficient economy, every PAYG obligatory pension scheme imposes an implicit tax on labor supply since the present value of future retirement benefits is lesser than the matching payments. [Breyer & Hupfeld, 2008] previously stated that increasing the retirement pension for poor workers and cutting it for rich workers will would boost (uncompensated) labor supply while reducing the tax burden. Then as result, the proposed revision of the retirement benefit formula's allocation impacts would, if anything, be beneficial.

It is often preferred to relate the pension age or career contributing requirements to life expectancy rather than just to pension benefits because the latter usually entails lower retirement income, raising pension adequacy and old-age poverty problems. In this context, it's important to remember that typical working lives are getting longer, and that extending them is one method to reconcile pension system sustainability and adequacy in the face of aging populations by balancing the time spent working and retired. Knowing ahead of time that living longer implies working longer to provide adequate pension payments generates substantial incentives for people to postpone retirement. Furthermore, the level of retirement benefits may have an indirect effect on mortality, particularly among the lower income categories. Since lower pension is a process that influences mortality itself, the proposed increase in pension savings in favor of low-income agents may improve life span within those categories,weakening the empirical association.

The demographic framework we used, although very simple, was necessary to understand how the correction was possible. The absolute stationary hypothesis forces the use of a periodic mortality table. Period life expectancy measurements have two drawbacks: first, they do not account for the likelihood that people' socioeconomic characteristics develop over time, namely, because membership in a particular demographic subgroup may change over time, so may the corresponding mortality rates. Except for education, this is usually the true for any metric of socioeconomic level. Second, life expectancy does not take into account the fact that mortality rates fall with time and at varying rates for people in different socioeconomic groups . Recent empirical evidence [Ayuso & al 2021] , [Bravo & al, 2021] show that, at retirement ages, most countries have a significant and systematic divergence in cohort and period life expectancy measures, resulting in significant ex-ante tax/subsidies from generations to come to present generation and, as a result, an unjust actuarial relationship between contributions and pension benefits. In addition, this affects labor supply decisions, resulting in macroeconomic inefficiencies, and wrongly communicates solvency expectations, delaying pension changes. We are considering creating an extension of the model with a stochastic model of the effect of mortality and dynamic economic conditions on salaries and discount rates as well as including incomplete contribution periods.

Bibliography

[Aaron, 1966] Aaron, H. J.,The Social Insurance Paradox.*Canadian Journal of Economics and Political Science* **1966**, ,32, 371-374.

[Ayuso & Al, 2017] Ayuso, Mercedes, Jorge Miguel Bravo, and Robert Holzmann. 2017 . On the Heterogeneity in Longevity among Socioeconomic Groups. Global Journal of Human Social Science Research 17: 33–58

[Ayuso & al, 2021(a)] Ayuso, Mercedes, Jorge M. Bravo, and Robert Holzmann. 2021. Getting Life Expectancy Estimates Right for Pension Policy: Period versus Cohort Approach. *Journal of Pension Economics and Finance* 20: 212–31.

[Ayuso & al, 2021(b)] Ayuso, M., Bravo, J. M., Holzmann, R. & Palmer, E. (2021). Automatic indexation of pension age to life expectancy: When policy design matters. Risks, 9(5), 96.

[Bravo & al, 2021] Bravo, J. M., Ayuso, M., Holzmann, R. & Palmer, E. (2021b). Addressing the Life Expectancy Gap in Pension Policy. Insurance: Mathematics and Economics, 99, 200-221.

[Belloni & al, 2013] Belloni, M., Maccheroni, C. Actuarial Fairness When Longevity Increases: An Evaluation of the Italian Pension System. *Geneva Pap Risk Insur Issues Pract*,38, 638–674.**2013**

[Biggs & al, 2009] Biggs Andrew G., M. Sarney, and R. Tamborini ≪ A Progressivity Index for Social Security ≫. *Social security office of Retirement and Disability Policy* **Issue Paper No. 2009-01**.

[Blanpain, 2016] Blanpain Nathalie., L'espérance de vie par niveau de vie - méthode et principaux résultats. *Insee report F1801* **2016**,

[Blanpain, 2018] Blanpain Nathalie.,Tables de mortalité par niveau de vie. *Insee Résultats* **2008**, .

[Borck, 2007] Borck R. , "On the Choice of Public Pensions when Income and Life Expectancy are Correlated", *Journal of Public Economic Theory*,J, 9, 711-725.**2007**

[Breyer & Hupfeld, 2008] Friedrich Breyer ,Stefan Hupfeld, 2008. "Fairness of Public Pensions and Old-Age Poverty," *Discussion Papers of DIW Berlin 817, DIW Berlin, German Institute for Economic Research.* , **2008**

[Bosworth & al, 2016] Bosworth, B., Burtless, G., Zhang, K., 2016. Later retirement, inequality in old 735 age, and the growing gap in longevity between rich and poor. (The Brookings Institution)

[Bronnum & al, 2008] Brønnum-Hansen Henrik, Mikkel Baadsgaard, T. Increase in social inequality in health expectancy in Denmark. *Scandinavian Journal of Public Health* **2008**,

[Chetty & al, 2016] Chetty R, & al. The association between income and life expectancy in the United States, 2001-2014. *J Am Med Assoc* **2016**, 315:1750–1766.

[Coronado et al, 2000] Coronado, J. L., Fullerton, D. and Glass, T. The progressivity of social security, *NBER working paper*,7520.**2000**

[Culotta, 2021] Culotta, F. Life Expectancy Heterogeneity and Pension Fairness: An Italian North-South Divide. Risks 2021, 9, 57.

[Currie & Schwandt, 2016] Currie J, Schwandt H (2016) Inequality in mortality decreased among the young while increasing for older adults, 1990-2010. *Science* 352:708–712 **2016**.

[Currie & Schwandt, 2016b] Currie J, Schwandt H (2016) Mortality inequality: The good news from a county-level approach. *J Econ Perspect* 30:29–52. **2016**.

[Devolder, 2019] Devolder P., T. Une alternative à la pension à points : le compte individuel pension en euros. *Regards economiques* **2019**,

[Economic Policy Committee, 2020] European Economic Policy Committee 2020 , Joint Paper on Pensions 2019 ,Ref. Ares(2020)393902 - 22/01/2020

[Gil, 1997] Gil, J. and Lopez-Casasnovas, G. . Life-time redistribution effects of the spanish public pension system, *University Pompeu Fabra working paper.*,**1997**

[Hachon, 2009] Hachon C., Who Really Benefits from Pension Systems ? When Life Expectancy Matters. *Revue d'économie politique* **2009**, vol. 119, no. 4, , pp. 613-632.

[Holzmann & al, 2017] Holzmann, Robert & Alonso-García, Jennifer & Labit-Hardy, Heloise & Villegas, Andres M., 2017., ≪ NDC Schemes and Heterogeneity in Longevity: Proposals for Redesign≫, In *IZA Discussion Papers*; 11193, Institute of Labor Economics (IZA).

[Kinge et al, 2019] Kinge, Jonas Minet, Jørgen Heibø Modalsli, Simon Øverland, Håkon Kristian Gjessing, Mette Christophersen Tollånes, Ann Kristin Knudsen, Vegard Skirbekk, Bjørn Heine Strand, Siri Eldevik Håberg, and Stein Emil Vollset. 2019. Association of Household Income with Life Expectancy and Cause-Specific Mortality in Norway, 2005–2015. *JAMA* , 321: 1916–23. . **2019**

[Kreiner et al, 2018] Kreiner, Claus T., Torben H. Nielsen, and Benjamin L. Serena. 2018. Role of Income Mobility for the Measurement of Inequality in Life Expectancy. *Proceedings of the National Academy of Sciences*, 115: 11754–59. **2018**

[Lefebvre, 2007] Mathieu Lefebvre, 2007. "The Redistributive Effects of Pension Systems in Europe: A Survey of Evidence," LIS Working papers 457, LIS Cross-National Data Center in Luxembourg.

[Mackenbach et al., 2008] Mackenbach JP, et al. Socioeconomic inequalities in health in 22 European countries. *New Engl J Med* 358:2468–2481.**2008**.

[Mackenbach et al, 2018] Mackenbach JP, et al. (2018) Trends in health inequalities in 27 European countries. *Proc Natl Acad Sci USA* 115:201800028. **2018**.

[Mackenbach, 2013] Mackenbach JP, Looman CW (2013) Life expectancy and national income in Europe, 1900-2008: An update of Preston's analysis. *International Journal of Epidemiology* Issue 4, Pages 1100–1110**2013**. https://doi.org/10.1093/ije/dyt122

[Mitchell, 1996] Mitchell Olivia and Zeldes, Stephen, Social Security Privatization: A Structure for Analysis. *American Economic Review* **1996**, , vol:86, 363-67

[(Musgrave 1948)] Musgrave R. A. and Tun Thin T. ≪Income Tax Progression, 1929-48≫. *Journal of Political Economy* **1948**.

[Neumayer, 2016] Neumayer Eric., Plumper T. Inequalities of Income and Inequalities of Longevity: A Cross-Country Study. *American journal of public health* **2016**.

[Samuelson (1958)] Samuelson, Paul A. 1958. An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy* 66: 467–82.

[OCDE, 2005] OCDE(2005), . In *OECD Pensions at a Glance 2005*; Éditions OCDE, Paris.

[OCED, 2011] OCDE (2011), ≪ Progressivity of Pension Benefit Formulae ≫, In *Pensions at a Glance 2011: Retirement-income Systems in OECD and G20 Countries*; Éditions OCDE, Paris.

[OCED, 2011] OCDE (2011), ≪ "Linking Pensions to Life Expectancy ≫, In *Pensions at a Glance 2011: Retirement-income Systems in OECD and G20 Countries*; Éditions OCDE, Paris.

[OCED, 2019] OCDE (2019), *Pensions at a Glance 2019 : OECD and G20 Indicators*, Éditions OCDE, Paris, https://doi.org/10.1787/b6d3dcfc-en.

[Preston, 1975] Preston SH The changing relation between mortality and level of economic development.*Popul Stud*, 29:231–248. **1975**

[Reil-Held, 2000] Reil-Held, A. (2000). Einkommen und Sterblichkeit in Deutschland: Leben Reiche langer?, *Discussion paper*,00-14, SFB 504, Mannheim.**2000**

[Snyder, 2006] Snyder SE, Evans WN ; The effect of income on mortality: Evidence from the social security Notch. *Rev Econ Stat* , 88:482–495. **2006**

[Walczak & al, 2021] Walczak, D.; Wantoch-Rekowski, J.; Marczak, R. Impact of Income on Life Expectancy: A Challenge for the Pension Policy., *Risks 2021*, , 9, 65. https://doi.org/10.3390/risks9040065 .

[Waldron, 2007] Waldron H, Trends in mortality differentials and life expectancy for male Social Security-covered workers, by socioeconomic status. *Social Security Bulletin*, 67:1–28. **2007**

Chapter 4

Automatic Adjustment Mechanisms in Public Pension Schemes to Address Population Ageing and Socio-economic Disparities in Longevity

This paper has been submitted K.Diakite , P.Devolder

4.1 Introduction

As the population ages and life expectancy continues to increase, countries around the world have been forced to confront the fiscal challenges posed by their pension systems. To ensure their long-term financial sustainability, many countries have implemented systemic and/or gradual parametric reforms to their pension schemes. These reforms have typically aimed to achieve solvency and enhance fiscal sustainability, while also introducing adequacy safeguards through automatic adjustments that keep pace with increasing life expectancy.

Parametric reforms have taken many different forms, including modifying the pension system rules and parameters such as the standard and early retirement rules, qualifying conditions, the contribution rate, the benefit formula, the index for accrued rights and indexation of benefits, and pension decrements and increments. Other reforms have focused on increasing pre-funding (reserve funds), adjusting early-retirement options to enhance work incentives, expanding contribution options, expanding the coverage of private (mandatory or voluntary) pensions, and developing auto-enrolment schemes [OECD 2019]. Some countries have also reformed first-tier social/guarantee pensions, brought public-sector pension benefits more in line with private-sector benefits, and reformed the taxation of pensions.

While some pension reform approaches have proposed introducing an automatic link between future pensions and developments in life expectancy [Turner, 2009] it is important to note that this is not the definitive solution. One of the most common responses to population ageing has been to increase standard and early retirement ages in an automatic or scheduled way as life expectancy at the pension age progresses. However, raising the retirement age can have undesirable effects on actuarial fairness. This is because it could exacerbate existing inequalities between those with lower life expectancies and those with higher life expectancies. As a result, policy-makers may need to consider other options, such as increasing contributions or adjusting benefit formulas, to ensure that pension systems remain equitable and sustainable in the face of demographic changes. Countries have pursued various retirement policy strategies, including implementing fixed schedules, targeting a constant expected period in retirement, targeting a constant balance between time spent in work and retirement, targeting a constant ratio of adult life spent in retirement, targeting a stable old-age dependency ratio, setting a target age for retirement, following simple ad-hoc rules to share the longevity risk burden between workers and pensioners, and linking the eligibility age for pensions to the eligibility age for other benefits such as public health care. The common thread among these various retirement age policies is their utilization of automatic adjustment mechanisms to effectively address the negative impact of economic and demographic trends on the financial stability of national pension plans. Although these policies are typically intended to align with the ultimate goals of these pension plans, such as ensuring adequacy, long-term sustainability, and intergenerational fairness, this alignment is often insufficient.

Moreover, income and socio-economic status (SE) are important determinants of health and longevity, with income being a commonly used indicator of material resources that is positively associated with longevity [Bosworth 2018]. While education and social class are also relevant SE indicators, income provides a better long-term measure of SE status due to its wider range of variation. The link between SE status and mortality has implications for social security programs, especially in light of increasing longevity and population ageing. OECD countries have responded to these challenges by increasing the retirement age, but this approach does not necessarily take into account SE differences in mortality. Differences in life expectancy (LE) between high and low SE groups affect the actuarial fairness and progressivity of public pension systems.

The literature on the subject is growing, with researchers increasingly interested in mortality and LE inequalities related to SE status and their impact on social security programs. Evidence suggests that there is a sizeable and possibly growing disparity in late-life longevity in high-income countries. More notably, disparities in old-age mortality measured by SE status have widened in recent years in the Netherlands [Kalwij & al 2013] [Wouterse & al 2021] ,Germany [Wenau & al (2019)],the UK [Longevity Panel (2020)], Italy [Belloni & al 2013] [Lallo & al (2018)] [Ardito et al 2020], Sweden [Fors & al 2021], Canada [Kleinow & Cairns (2020)] and the USA [Waldron, 2007] [Goldman & Orszag 2014] [Bosley & al (2018)].

In most European countries, there has been a decline in mortality in lower

SE groups, but relative inequalities in mortality have increased due to smaller percentage declines in these groups. These trends have important implications for pension reform and scheme design, as taxes and subsidies may not adequately address the effects of a closer contribution-benefit link, a later formal retirement age, and more individual funding and private annuities. As such, it is important for policy-makers to consider the welfare implications of growing inequality in LE by SE status when designing and implementing social security programs.

The distribution of lifespan among socio-economic classes is not uniform or random [Donkin et al 2002]. Despite considerable improvements in mortality rates over the years, the health gap between diverse groups has risen, leading to rising inequality. A large body of literature reveals the unequal distribution of health and lifespan among persons of different socio-economic classes, which appears to be rising over time, perpetuating inequality [Marmot(2015)]. This calls the fairness of the social security system into question. Fairness is a complicated notion that incorporates subjective distributive justice values.

The notion of actuarial fairness has recently emerged as a benchmark in the field of pension economics [Borsch-Supan 2006]. Essentially, it involves ensuring that the internal rate of return for all individuals is equal, regardless of the amount of contributions paid or benefits received. This implies that if two individuals who belong to the same birth cohort paid identical amounts of contributions during their working lives, they should receive equal pension wealth over their lifespan. Similarly, if two individuals paid different contribution amounts, the internal rate of return on their contributions should be equivalent. However, actuarial fairness alone may not suffice in addressing distributive justice, since it does not account for factors beyond an individual's control, such as their gender, family background, or health status. Thus, it may be necessary to introduce a progressive redistribution scheme that allocates more resources to individuals who made lower contributions during their working lives, compensating them for their disparate outcomes and opportunities.

In this paper, we propose a pension system based on two automatic adaptation mechanisms (AAM):

- The first dynamic mechanism is integrated directly into the pension formula and corrects the heterogeneity of longevity when it exists between socio-economic classes (intragenerational fairness principle).
- A steering mechanism for both the contribution rate and the replacement rate respects Musgrave's rule, which makes it possible to distribute the demographic risk between working people and retirees (intergenerational fairness principle).

The pension formula operates on a longitudinal axis and integrates longevity heterogeneity correction via the progressivity in the formula, and the financial sustainability is assured on a transverse axis by piloting the contribution rate and the replacement rate using a risk-sharing invariant inspired by the Musgrave rule.

It must be said that other risk sharing mechanisms than the Musgrave rule are possible.

Our main contributions are the dynamic longevity heterogeneity correction directly integrated in the progressive pension formula through an intragenerational correction as well as the design of the Progressive Musgrave system, which operates on both intergenerational and intragenerational levels.

The paper is divided into five main sections. The first section describes the progressive longevity heterogeneity correction and its generalization in a dynamic environment. The second section presents the defined Musgrave system as introduced by Devolder and De Valeriola [Devolder & De Valeriola (2020)]. The third section combines the two mechanisms into a unique plan and shows the rules for calculating pension benefits as well as the evolution of the plan's parameters. The fourth section introduces a useful tool that should aid in the transition from traditional pension systems to those outlined in the preceding part and the final section presents the results of the analysis, including the computation of the integrated plan based on historical data gathered from the United States. Overall, the paper aims to provide a comprehensive analysis of the integrated pension plan and its potential benefits for retirees based on historical mortality data.

4.2 Intra-generational Automatic Adjustment Mechanism

Although not in all nations, the differences in mortality and life expectancy have been growing as global wealth inequality has become more lopsided. The use of income or education quantiles, which do not signal distributional widening or narrowing, has made it challenging to connect widening income distributions to mortality disparities. Ex-ante disparities and ex-post disparities are two categories for lifetime differences. While the latter reveals the random element of mortality occurrences, the former displays variations in the chance of death. Ex-ante inequalities in longevity may cast doubt on the fairness of risk-sharing financial instruments like annuities and pensions. Ex-post inequalities in death ages, however, may lead to substantial variations in the compensation received but do not represent the same threat.

When life expectancy inequalities are linked to income distribution, it leads to an unavoidable regressivity in defined benefit(DB) pension systems. This underlying actuarial unfairness is a major concern, particularly if the public pension scheme aims for redistribution. To address this issue, a progressive factor mechanism was proposed in a previous paper [Diakite & Devolder2021], for a specific pension plan in a static environment. In this section, the study extends this progressivity mechanism for a DB final salary plan in a dynamic environment, presenting the main results for pension transformation, progressive factors, and pension benefits.

4.2.1 Progressive formula for a final salary plan in a static environment

We consider a defined-benefit pension system in which agents are differentiated in terms of salary and life expectancy. To study the redistributive features of such a pension system, we consider the lifetime benefit ratio. This ratio for an agent is the discounted value of the pension benefit divided by their last salary. We introduce a progressive transformation of the pension formula that takes into account the life expectancy differential in order to satisfy the lifetime replacement rate equality condition for agents of different salary classes. In the previous paper, we were able to explicitly describe a method to determine the progressive factors in a career average defined benefit scheme.

We implemented the progressive mechanism in the pension formula, aiming to correct the underlying unfairness issue. In order to transform a DB system into a progressive one, we chose to transform the salary in the pension formula using salary bandwidths, and based on those thresholds, we applied the progressive coefficients.

We apply this technique here for a final salary plan in a static environment.

In this plan, the final salary is noted S^{x_r}.

The following are the main assumptions about the salary class and mortality:

- Salary levels differentiate the agent classes. There are m classes. We take note of S_j, which is the salary for class j. Let us imagine that the salary of an agent depends on his age (x). This gives :

$$S_j^x = S_j^{x_0} \cdot (1 + s_j)^{(x-x_0)}$$

  Where s_j is the salary growth effect for group j.

  We can write the salary transformation for any class j as such(with $S_0^x = 0$):

$$X(S_j^{x_r}) = X_j^{x_r} = \sum_{i=1}^{j} \lambda_i \cdot (S_i^{x_r} - S_{i-1}^{x_r})$$

- The pension at age x_r for class j agents is noted $P_j^{x_r} = \delta \cdot X_j^{x_r}$ instead of $P^{x_r} = \delta \cdot S^{x_r}$ when we do not take longevity heterogeneity into account.

- Present value of benefit are discounted with the factor $\dfrac{1}{1+r}$

- We compute the total population periodic mortality table and using the survival probabilities ${}_x p_{x_r}$ we obtain the average annuity rate :

$$\ddot{a}_{x_r} = \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r} \cdot (\frac{1}{1+r})^{x-x_r}$$

- Once the classes are formed, we can compute the periodic mortality table for each class $j \in [1 : m]$ and obtain the stratified survival probabilities ${}_{x}p_{x_r}^{(j)}$ used to compute the stratified annuity rates

$$\ddot{a}_{x_r}^{(j)} = \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r}^{(j)} \cdot (\frac{1}{1+r})^{x-x_r}$$

- We define the inter-class fairness condition using lifetime replacement rate equality.
The lifetime replacement rate measures the present value of benefits divided by the last salary. This indicator is equivalent to the lump sum at retirement expressed in the number of final salaries. Since the progressive transformation affects only pension benefits, this indicator compares efforts between retirees of different classes.
Before taking into account longevity heterogeneity, the LR based on the mean longevity rates for the whole population in the regime is written as follows:

$$\begin{aligned} LR^* &= \frac{1}{S^{x_r}} \sum_{x=x_r}^{\omega} P^{x_r} \cdot {}_{x-x_r}p_{x_r} \cdot (\frac{1}{1+r})^{x-x_r} \\ &= \frac{P^{x_r}}{S^{x_r}} \cdot \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r} \cdot (\frac{1}{1+r})^{x-x_r} \\ LR^* &= \delta \cdot \ddot{a}_{x_r} \end{aligned}$$

This value of the lifetime replacement rate is by definition the same for every agent; this is the target LR value when ignoring longevity heterogeneity.

- When taking into account longevity heterogeneity, we can write the lifetime replacement rate for each class as follows:

$$LR_j = \delta \cdot \frac{X_j^{x_r}}{S_j^{x_r}} \cdot \ddot{a}_{x_r}^{(j)} \tag{4.1}$$

- We obtain the values of progressive coefficients when the LR equality condition is satisfied :

$$LR_1 = ... = LR_j = ... = LR_m = LR^*$$

Solving this system for $\lambda = (\lambda_1; ...; \lambda_m)$ gives the following form for the progressive coefficients :

$$\lambda_1 = \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(1)}} \tag{4.2}$$

$$\lambda_j = \frac{S_j^{x_r} \cdot \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j)}} - S_{j-1}^{x_r} \cdot \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j-1)}}}{S_j^{x_r} - S_{j-1}^{x_r}} \tag{4.3}$$

The pension for a complete contribution for an agent of class j is given by:

$$P_j^{x_r} = \delta \cdot X_j^{x_r} = \delta \cdot S_j^{x_r} \cdot \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j)}}$$
$$P_j^{x_r} = \delta \cdot S_j^{x_r} \cdot \theta_j$$

where $\theta_j = \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j)}}$. The longevity heterogeneity correction for an agent of class j will be denoted as θ_j.

We notice that if all the classes are identical in terms of mortality after retirement, we come back to a classical DB on a final salary. In this system, the pension benefit would be equal to:

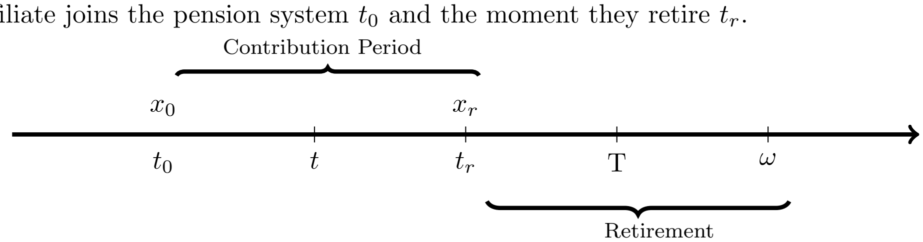
$$P_j^{x_r} = \delta \cdot S_j^{x_r}$$

In this first stationary framework ,the progressive formula only transforms the salary taken into account in the pension benefit calculation; the replacement rate stays the same. The solution to the lifetime replacement rate fairness condition yields the progressive coefficient value. This condition states that the present value of benefits divided by the last salary of every agent in the regime is the same.

4.2.2 Progressive formula in a dynamic environment

The estimation of the length of the remaining years of life is evolving and is different between classes; therefore, introducing static mechanisms in the pension formula that incorporate correction based on these estimations is prone to being inaccurate over time and has to be corrected.

We will introduce a dynamic progressive formula that takes into account the evolution of the annuity rate to correct for inaccuracies between the moment an affiliate joins the pension system t_0 and the moment they retire t_r.



- Before retirement we define the salary at age $x < x_r$ for a class j :

$$S_j^{(x,t)} = S_j^{(x_0,t_0)} \cdot (1 + s_j)^{(x-x_0)} \cdot (1 + \gamma)^{(t-t_0)}$$

  Where s_j is the career growth effect for the group j salary and γ the inflation effect depending on time t.

- After retirement, we define $S_j^{(x,t)} = S_j^{(x_r,t)} \cdot (1+\gamma)^{x-x_r}$ for $x > x_r$.
  Because the concept of salary no longer makes sense after retirement, we consider the retired "notional salary" to be the indexed last salary, with γ representing the indexation rate of pension benefits (wage growth rate).

- The pension benefit at retirement is given by : $P_j^{(x_r,t_r)} = \delta \cdot X_j^{(x_r,t_r)}$

  The pension benefit formula is made up of two parts: the scheme pension rate δ, and a progressive longevity heterogeneity component, which is determined by the salary transformation $X_j^{(x_r,t_r)}$

- Pension benefit after retirement are indexed with γ as such for $x > x_r$: $P_j^{(x,T)} = P_j^{(x_r,t_r)} \cdot (1+\gamma)^{x-x_r}$

- The progressive factors are solutions to the lifetime replacement rate equality condition. The present value of the pension benefit computed at retirement divided by the last salary should be the same for all agents.

- The progressive transformation at retirement is expressed this way:

$$X_j^{(x_r,t_r)} = \sum_{i=1}^{j} \lambda_i^{t_r} \cdot (S_i^{(x_r,t_r)} - S_{i-1}^{(x_r,t_r)})$$

- Pension formula

The dynamic coefficient formula every period t_j between t_0 and t_r :

$$\lambda_i^{t_j} = \frac{S_i \cdot \dfrac{\ddot{a}_{x_r}(t_j)}{\ddot{a}^i_{x_r}(t_j)} - S_{i-1} \cdot \dfrac{\ddot{a}_{x_r}(t_j)}{\ddot{a}^{i-1}_{x_r}(t_j)}}{S_i - S_{i-1}} \tag{4.4}$$

Where $\ddot{a}_{x_r}(t_j) = \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r}(t_j) \cdot (\frac{1+\gamma}{1+r})^{x-x_r}$ represents the life annuity at retirement computed using estimated survival probabilities at time t_j, Every period, the progressive coefficients are communicated to affiliates of the regime. Active workers can evaluate their pension at retirement based on the annual update of the coefficients.

The pension benefit estimated at time t ($t_0 < t < t_r$) is :

$$\bar{\bar{P}}_j^{(x,t)} = \delta \cdot \sum_{i=1}^{j} \lambda_i^t \cdot (S_i^{(x,t)} - S_{i-1}^{(x,t)})$$
$$\bar{\bar{P}}_j^{(x,t)} = \delta \cdot S_j^{(x,t)} \cdot \frac{\ddot{a}_{x_r}(t)}{\ddot{a}^j_{x_r}(t_0)} = \delta \cdot S_j^{(x,t)} \cdot \theta_j^t$$

The pension benefit at retirement is :

$$P_j^{(x_r,t_r)} = \delta \cdot X_j^{(x_r,t_r)}$$
$$P_j^{(x_r,t_r)} = \delta \cdot S_j^{(x_r,t_r)} \cdot \frac{\ddot{a}_{x_r}(t_r)}{\ddot{a}^j_{x_r}(t_r)} = \delta \cdot S_j^{(x_r,t_r)} \cdot \theta_j^{t_r}$$

The lifetime replacement rate is then given by

$$LR_j(t_r) = \delta \cdot \ddot{a}_{x_r}(t_r)$$

The inter class fairness condition is met : the present value of pension benefits over the last salary (lifetime replacement rate) is the same across all the classes.

- After retirement

In order to avoid too much uncertainty on the amount to pay after retirement, we assume that the progressive factors are fixed at their value at the retirement age for all the agents. The pension benefit at a time $T > t_r$ for an agent of age $x > x_r$ is :

$$P_j^{(x,T)} = \delta \cdot X_j^{(x,T)}$$

, the progressive transformation of the pension benefit is written:

$$X_j^{(x,T)} = \sum_{i=1}^{j} \lambda_i^{t_r} \cdot (S_i^{(x,T)} - S_{i-1}^{(x,T)})$$
$$X_j^{(x,T)} = X_j^{(x_r,t_r)} \cdot (1+\gamma)^{x-x_r}$$

This means that every new cohort of retirees will have their own pension benefit formula at retirement, but in the year following retirement, the progressive part of the pension benefit is fixed for them.

Remark 1: Fully dynamic system

It is also possible for the progressive system to be fully dynamic even after retirement; such a system does not respect the lifetime replacement rate equality condition and introduces potential uncertainty after retirement. In this scheme, the progressive factors for the entire retiree population evolve dynamically, and even after retirement, the pension level is adjusted with the estimation of the life annuity at retirement.

$$X_j^{(x,T)} = \sum_{i=1}^{j} \lambda_i^{T} \cdot (S_i^{(x,T)} - S_{i-1}^{(T;x)})$$

Remark 2: Communication strategy
Before retirement, the plan doesn't have to be fully dynamic, from an operational standpoint, we present an equivalent system able to reproduce the same pension benefit at retirement with fixed parameters and an ex-post correction at retirement age.

- Pension formula:

The progressive pension benefit formula corrects for longevity heterogeneity ex ante. The progressive factors are computed using mortality data at inception . The estimated pension benefit at time t ($t_0 < t < t_r$) and communicated to the affiliate as the target retirement age is:

$$\begin{aligned}\bar{P}_j^{(x_r,t)} &= \delta \cdot \bar{X}_j^{(x_r,t)} \\ &= \delta \cdot S_j^{(x_r,t)} \cdot \frac{\ddot{a}_{x_r}(t_0)}{\ddot{a}^j_{x_r}(t_0)}\end{aligned}$$

- Balance mechanism:

Once one reaches retirement we apply a ex post correction based on the ratios between the final mortality table and the initial one.

The correction factor is given for an agent of class j by

$$g_j(t_0;t_r) = \frac{\ddot{a}_{x_r}(t_r)}{\ddot{a}^j_{x_r}(t_r)} \cdot \frac{\ddot{a}^j_{x_r}(t_0)}{\ddot{a}_{x_r}(t_0)}$$

The pension benefit is obtained via the correction with:

$$P_j^{(x_r,t_r)} = \bar{P}_j^{(x_r,t_r)} \cdot g_j(t_0;t_r) = \delta \cdot S_j^{(x_r,t)} \cdot \frac{\ddot{a}_{x_r}(t_r)}{\ddot{a}^j_{x_r}(t_r)}$$

The lifetime replacement rate:

$$LR_j(t_r) = \delta \cdot \ddot{a}_{x_r}(t_r)$$

The lifetime replacement rate is the same for every agent, the system respects our fairness criteria.

4.3 Intergenerational Automatic Adjustment Mechanism

In order to introduce a first form of risk sharing, let us first set up a simple deterministic two-period framework without longevity heterogeneity in which the

Musgrave rule [Musgrave & Thin, 1948] can be easily defined. The framework is set up as a two-period model with no longevity heterogeneity. In period 0, the system is stable, and each active and retiree individual receives a homogeneous salary S and pension P, respectively. The replacement rate, denoted by δ_0, is the percentage of a retiree's pre-retirement income that is replaced by their pension. The contribution rate, denoted by π_0, is the percentage of an active individual's salary that goes toward the pension system. The dependence ratio, denoted by D_0, is the ratio of retirees to contributors. The pension system is characterized by the budget equation and the pension equation.

The budget equation states that the total amount paid in pensions must equal the total amount contributed to the pension system

$$D_0 \cdot P = \pi_0 \cdot S \tag{4.5}$$

The pension equation states that the pension received by each retiree (P) is equal to the replacement rate (δ_0) times their pre-retirement income (S).

$$P = \delta_0 \cdot S \tag{4.6}$$

Leading to the condition

$$\pi_0 = \delta_0 \cdot D_0 \tag{4.7}$$

Now, suppose a demographic shock occurs in period 1, which changes the dependence ratio from D_0 to D_1 (with $D_1 > D_0$, indicating an ageing population). The impact of this shock on δ_1 and π_1 will depend on the pension architecture.

In a pure defined benefit (DB) scheme, the replacement rate is constant, and the active population bears the whole burden of the demographic shock through an increase in the contribution rate. In this case, δ_1 remains equal to δ_0, while π_1 increases by a factor of (D_1/D_0).

$$\pi_1 = \pi_0 \cdot \frac{D_1}{D_0}$$

On the other hand, in a pure defined contribution (DC) scheme, the contribution rate is constant, and the retiree population bears the whole burden of the demographic shock through a decrease in benefits. In this case, π_1 remains equal to π_0, while δ_1 decreases by a factor of (D_0/D_1).

$$\delta_1 = \delta_0 \cdot \frac{D_0}{D_1}$$

The defined Musgrave architecture aims to provide a form of risk sharing between the active and retired generations by fixing the Musgrave ratio, which represents the ratio between the pension and the salary net of pension contributions. By fixing this ratio, the burden of the demographic shock is shared between the two generations.

To achieve this, the values of δ and π are adjusted in such a way that the Musgrave ratio remains constant at time t=1. The evolution of the replacement and contribution rates is obtained by using the Musgrave ratio equality:

$$M_0 = M_1 = M = \frac{\delta_0}{1-\pi_0} = \frac{\delta_1}{1-\pi_1}$$

Using the budget equation , we can rewrite π_0 in terms of D_0 and P:

$$\pi_0 = \frac{D_0 \cdot P}{S}$$

Substituting this into the expression for M_0, we obtain:

$$M_0 = \frac{\delta_0}{1 - \frac{D_0 \cdot P}{S}}$$

Solving for P and simplifying, we get:

$$P = \frac{S\delta_0}{1 + \pi_0 \cdot \delta_0}$$

Using this expression for P, we can rewrite the Musgrave ratio as:

$$M_0 = \frac{\delta_0}{1-\pi_0} = \frac{\delta_0}{1 - \frac{D_0\delta_0}{1+\pi_0 \cdot \delta_0}}$$

Simplifying, we obtain:

$$M_0 = \frac{\delta_0}{1 - \frac{D_0}{1+\pi_0\delta_0}} = \frac{\delta_0(1 + \pi_0 \cdot \delta_0)}{1 + \pi_0 \cdot \delta_0 - D_0}$$

Setting this expression equal to M_1 and solving for π_1, we obtain:

$$\pi_1 = \pi_0 \cdot \frac{D_1}{D_0 + \pi_0 \cdot (D_1 - D_0)}$$

Similarly, setting the expression for P at time $t = 1$ equal to the expression for P at time $t = 0$, we obtain:

$$\delta_1 = \frac{\delta_0}{1 + \delta_0 \cdot (D_1 - D_0)}$$

Therefore, the Defined Musgrave architecture provides a way to share the risk of demographic shocks between the active and retired generations by adjusting the replacement and contribution rates in such a way that the Musgrave ratio remains constant over time.

The Musgrave rule is appealing for two reasons. To begin, it means that demographic or economic shocks cause equiproportional changes in pensions and labor wages net of contributions, to the extent that these changes are defined by pension policy. As a result, despite these shocks, inter-generational income inequality will remain unaltered [Schokkaert & al (2020)]. This could be regarded as good in terms of equity. Second, the Musgrave rule can be viewed as a pragmatic interpretation of an optimal insurance policy in terms of allocating resources to one cohort over its own life cycle [1] . Given reasonable assumptions about individual utility functions, effective inter-generational risk sharing requires that shocks have no effect on the ratio of old to young consumption levels [Mankiw 2007].
In the event of population ageing, the dependency ratio D rises, therefore the replacement rate lowers, which might make political acceptance harder. This will uniformly cut the level of pensions without taking into account disparities in lifespan, particularly for the lowest pensions, which may lead to non-social acceptability due to injustice and unfairness. In the next section, we propose a mechanism that reconciles risk sharing between generations and social class equity.

[1] For instance, Ball and Mankiw (2007) propose that social security should match this conclusion and derive this result as the anticipated outcome in a hypothetical scenario with fully developed insurance markets.

4.4 Inter-generational and Intra-generational Automatic Adjustment Mechanisms

This section describes a pay-as-you-go pension system combining a progressive pension benefit formula as well as a demographic risk sharing mechanism piloting the contribution rate and the pension rate.

- Agents only enter the regime at age x_0 .
- Active workers can only exit the regime by dying.
- The exit condition is death.
- The effective of workers aged x at period t of class j is given by $Na_j^{(x,t)}$, the number of retirees aged y is $Nr_j^{(y,t)}$
- The effective are given via the relationship $Na_j^{(x+1,t)} = Na_j^{(x,t)} \cdot {}_1p_x^j(t)$

  ${}_1p_x^i(t)$ is the one year survival probability of individuals of age x at time t in class j . We obtain the retiree's number following the same procedure. Since there is no migration, the population function at year t can be obtained from the entry function and a periodic mortality table.
- We assume that there is only on way to enter the regime at age x_0. The entry function is constant,it gives the number of workers entering the regime for each class j is written $E_j = L_j(x_0, t)$
- The total number of active workers at time t is given by Na^t and the total number of retirees Nr^t

4.4.1 Two period model

This part introduces a mix of intergenerational ageing risk sharing using a Musgrave-like invariant and an intragenerational correction factor for longevity heterogeneity through a progressive pension formula.
In this framework, we are displaying the two mechanisms in action. We imagine a system functioning for a long period of time with the initial parameters for contribution rate replacement and progressive factors, until we add a disturbance in ageing and longevity heterogeneity at time $t = 1$ and observe both of the risk sharing mechanisms in action[2].

- $t = 0$

[2]This implies that before $t = 0$ we are in a DB system

When we initialize the system, we start computing the progressive transformation for all retirees using the first mortality table. For all ages $x \geq x_r$ the progressive transformation is:

$$X_i^{(x,0)} = \sum_{j=1}^{i} \lambda_j^0 \cdot (S_j^{(x,0)} - S_{j-1}^{(x,0)})$$

The pension benefit for the retirees of class j after retirement is written:

$$P_j^{(x,0)} = \delta_0 \cdot X_j^{(x,0)}$$

The budget equation is

$$\pi_0 \cdot \sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(x,0)} \cdot S_i^{(x,0)} = \delta_0 \sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,0)} \cdot X_i^{(x,0)}$$

We introduce the notation for the average benefit ratio $\bar{\delta}$, which is the ratio between the pension benefits and the retiree's notional salary:

$$\bar{\delta_0} \cdot \sum_{i=1}^{m} \sum_{x=x}^{\omega} Nr_i^{(x,0)} \cdot S_i^{(x,0)} = \delta_0 \sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,0)} \cdot X_i^{(x,0)} \tag{4.8}$$

The previous expression becomes :

$$\pi_0 = \bar{\delta_0} \cdot \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,0)} \cdot S_i^{(x,0)}}{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(x,0)} \cdot S_i^{(x,0)}}$$

We define the weighted dependency ratio as :

$$D_0^* = \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,0)} \cdot S_i^{(x,0)}}{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(x,0)} \cdot S_i^{(x,0)}}$$

then the budget equation is given by

$$\pi_0 = \bar{\delta_0} \cdot D_0^* \tag{4.9}$$

Musgrave proposed an invariant that results in risk sharing between active workers and retirees. Let us define the Musgrave invariant (or the invariant risk sharing condition) as the ratio between the average pension and the average salary net of pension contributions:

$$M_0 = \frac{\bar{\delta_0} \cdot \dfrac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,0)} \cdot S_i^{(x,0)}}{Nr^0}}{(1-\pi_0) \cdot \dfrac{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(x,0)} \cdot S_i^{(x,0)}}{Na^0}}$$

We define :

$$\mu_0 = \frac{\frac{\sum_{i=1}^{m}\sum_{x=x_r}^{\omega} Nr_i^{(x,0)} \cdot S_i^{(0,x)}}{Nr^0}}{\frac{\sum_{i=1}^{m}\sum_{x=x_0}^{x_{r-1}} Na_i^{(x,0)} \cdot S_i^{(x,0)}}{Na^0}} = \frac{D_0^*}{D_0}$$

μ_t is the ratio between the weighted dependency ratio and the dependency ratio at time t

The Musgrave ratio becomes :

$$M_0 = \frac{\bar{\delta_0}}{1-\pi_0} \cdot \mu_0 \tag{4.10}$$

- $t = 1$

The system progresses to the next stage, which is distinguished by another weighted dependency ratio, D_1^*.We want to define the relationship between the parameters $\bar{\delta_1}$ and π_1 and $(\lambda_1^1, .., \lambda_m^1)$so that the risk sharing condition stays invariant and the budget equation and the intergenerational fairness condition are respected.

The budget equation becomes:

$$\pi_1 = \bar{\delta_1} \cdot D_1^*$$

and the Musgrave ratio stays invariant, leading to this condition:

$$M_1 = \frac{\bar{\delta_1}}{1-\pi_1} \cdot \mu_1 = \frac{\bar{\delta_0}}{1-\pi_0} \cdot \mu_0 = M_0$$

We obtain the values of the contribution rate and average benefit ratio solving the previous system of equations:

$$\bar{\delta_1} = \bar{\delta_0} \cdot \frac{\frac{\mu_0}{\mu_1}}{1 + \bar{\delta_0} \cdot (D_1^* \cdot \frac{\mu_0}{\mu_1} - D_0^*)} \tag{4.11}$$

$$\pi_1 = \pi_0 \cdot \frac{\frac{\mu_0}{\mu_1} \cdot D_1^*}{D_0^* + \pi_0 \cdot (D_1^* \cdot \frac{\mu_0}{\mu_1} - D_0^*)} \tag{4.12}$$

4.4.2 Multi-period model

We can extend those results to any given period of time based on the recurrence property tying the parameters. Here is presented the procedure for the adjustment of the regime parameters at any given time t:

- Intragenerational correction:
  The first step is to adjust the longevity heterogeneity correction . The lifetime replacement rate equality condition allows us to obtain the progressive coefficient. They only depend on the final salary and on the annuity computed with the year's survival probability .

$$\lambda_i(t) = \frac{S_i^{(x_r,t)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^i_{x_r}(t)} - S_{i-1}^{(x_r,t)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^{i-1}_{x_r}(t)}}{S_i^{(x_r,t)} - S_{i-1}^{(x_r,t)}}$$

  The progressive coefficients allow us to obtain the progressive transformation for all the classes.

- Intergenerational correction:
  The pay-as-you-go equilibrium for year t is obtained through recurrence with the previous year's equilibrium, and the risk sharing is constant. The relationship between the average benefit ratio and the contribution rate for those years is expressed as such:

$$\pi_t = \pi_{t-1} \cdot \frac{\dfrac{\mu_{t-1}}{\mu_t} \cdot D_t^*}{D_{t-1}^* + \pi_{t-1} \cdot (D_t^* \cdot \dfrac{\mu_{t-1}}{\mu_t} - D_{t-1}^*)} \tag{4.13}$$

$$\bar{\delta}_t = \bar{\delta}_{t-1} \cdot \frac{\dfrac{\mu_{t-1}}{\mu_t}}{1 + \bar{\delta}_{t-1} \cdot (D_t^* \cdot \dfrac{\mu_{t-1}}{\mu_t} - D_{t-1}^*)} \tag{4.14}$$

- Average pension rate:
  When we obtain the average benefit ratio, we can transform it back into a mean pension rate via equation 4.8 and use it in the pension benefit formula.

$$\delta_t = \bar{\delta}_t \cdot \frac{\sum_{i=1}^m \sum_{x=x_r}^{\omega} Nr_i^{(x,t)} \cdot S_i^{(x,t)}}{\sum_{i=1}^m \sum_{x=x_r}^{\omega} Nr_i^{(x,t)} \cdot X_i^{(x,t)}} \tag{4.15}$$

- Pension benefit for new retirees:
  In this scheme, once you reach retirement your progressive transformation is fixed.

$$P_i^{(x_r,t_r)} = \delta_{t_r} \cdot X_i^{(x_r,t_r)} = \delta_{t_r} \cdot \sum_{j=1}^{i} \lambda_j^{t_r} \cdot (S_j^{(x_r,t_r)} - S_{j-1}^{(t_r;x_r)}) \tag{4.16}$$

- Pension benefit for old retirees:
  The progressive transformation for already retired generations stays the same, for $x > x_r$, only the pension rate and the notional salary change .

$$P_j^{(x,T)} = \delta_T \cdot X_j^{(x,T)} = \delta_T \cdot \sum_{i=1}^{j} \lambda_i^{T-x+x_r} \cdot (S_i^{(x,T)} - S_{i-1}^{(T;x)}) \tag{4.17}$$

Remark 3 : Fully dynamic system

It is also possible to extend those results in the system that does not guarantee the progressive transformation after retirement, we present the results in terms of pension rate in the following.

The progressive transformation after retirement in the fully dynamic progressive factors scheme is:

$$P_j^{(x,T)} = \delta_T \cdot X_j^{(x,T)} = \delta_T \cdot \sum_{i=1}^{j} \lambda_i^{T} \cdot (S_i^{(x_r,T)} - S_{i-1}^{(x_r,T)})$$

The pension rate becomes :

$$\delta_T = \bar{\delta_T} \cdot \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,T)} \cdot S_i^{(x,T)}}{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,T)} \cdot X_i^{(x,T)}}$$

This system, although simpler to implement and explain lacks of inter-generational fairness since the lifetime replacement rate equality is never verified when there is longevity heterogeneity.

4.5 Numerical application

In this section, we aim to project the population by utilizing mortality rates from the United States spanning from 1982 to 2019, segmented by socio-economic quintiles [Barbieri 2020]. We will further analyze the evolution of pension system parameters, accounting for potential heterogeneity in longevity and demographic risk sharing. Initially, the population was distributed using the mortality rates of 1982. However, we will subsequently introduce demographic modifications to incorporate differences in survival probabilities and mortality rates for active workers and retirees in each salary class, which will be informed by mortality rate estimates segmented by socio-economic quintiles for the United States from 1982 through 2019.

The mortality data highlight two important effects on the population: first, the increase in longevity across all classes; and, most importantly, the fact that the gain in longevity is not equally distributed between poor and rich agents in the regime. These two effects motivate the introduction of an intergenerational risk-sharing mechanism to spread the longevity risk between active workers and retirees and an intragenerational correction mechanism for longevity heterogeneity.

We will start with a defined benefit system offering a replacement rate $\delta^{DB} = 60\%$ of the last working salary and observe its reaction to the two previous effects. The objective is to compare this system to one that integrates longevity heterogeneity correction and automatic adaptation mechanisms via the progressivity of the pension formula and a risk sharing rule that steers the contribution rate and the mean benefit ratio.

We divided the population in three salary categories, and we suppose that the population is entirely characterized by these three classes.

- Low income class agents represent the first quintile Q_1 of the population.

- Average class composed of the next three quintiles(Q_2, Q_3, Q_4) .

- High income class agent represent the 5th quintile Q_5 of income distribution.

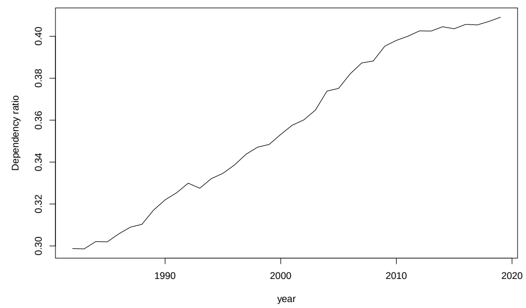
As starting salary for each class we used income quintiles from the United States [US Census Bureau (2023)]

$$
\begin{aligned}
S_1^{(1982;x_0)} &= 4790 \\
S_2^{(1982;x_0)} &= 20675 \\
S_3^{(1982;x_0)} &= 54720
\end{aligned}
$$

The methodology for projecting the salaries, the population effective and computing the life annuities is explained in 4.6.

Figure 4.1 presents the evolution of the Dependency ratio (ratio between the number of retirees and the number of active workers)

Figure 4.1: Evolution of the dependency ratio.



The starting and last values are summarized in **Table 4.1**

Table 4.1: Dependency ratio values

	D
1982	0.2987
2019	0.4091

We also compute the annuity rates by class, the life annuity for the entire population and their evolution. The values for the discount factor and indexation rate are specified in the Appendix 1.

$$\ddot{a}^{j}_{x_r}(t) = \sum_{x=x_r}^{\omega} {}_{x-x_r}p^{j}_{x_r}(t) \cdot (\frac{1+\gamma}{1+r})^{x-x_r}$$

We present in **Figure 4.2** the evolution of the longevity heterogeneity correction, defined as the ratio between the entire population life annuity and the life annuity of each class every year.

$$\theta^{t}_{j} = \frac{\ddot{a}_{x_r}(t)}{\ddot{a}^{j}_{x_r}(t)}$$

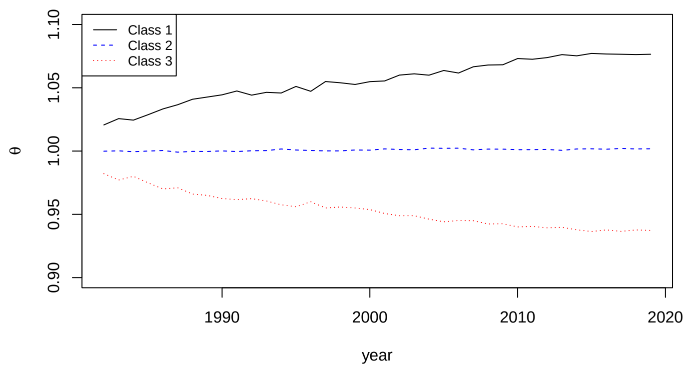


Figure 4.2: Evolution of the longevity heterogeneity correction by class.

This parameter describes how the gain in mortality is distributed every year among our three classes.

Table 4.2: Longevity gap evolution between a salary class and the general population

θ_j^t	1982	2019
Class 1	1.0206	1.0765
Class 2	0.9998	1.0018
Class 3	0.9821	0.9373

Table 4.2 shows that the longevity heterogeneity gap increases between the low income class and the high income class every year. Each year , θ_j^t is also the intensity of the correction on the pension benefit for class j.

We observe that in both years, Class 1 has the highest value for the correction, followed by Class 2, and then Class 3. This means that gains in mortality are distributed more heavily towards the higher income class (Class 3) compared to the lower income class (Class 1).

However, we can also see that the values for this parameter have changed over time, indicating that the gap in longevity heterogeneity between different income classes has increased. For example, in 1982, the value of the correction for class 1 was 1.0206, while in 2019, it had increased to 1.0765. In contrast, the value for class 3 decreased from 0.9821 in 1982 to 0.9373 in 2019.

We also describe in **Table 4.3** the evolution of the progressive factors at different periods. They follow the relationship:

$$\lambda_i(t) = \frac{S_i^{(x_r,t)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^i_{x_r}(t)} - S_{i-1}^{(x_r,t)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^{i-1}_{x_r}(t)}}{S_i^{(x_r,t)} - S_{i-1}^{(x_r,t)}}$$

Table 4.3: Progressive factors evolution

	λ_1	λ_2	λ_3
1982	1.0206	0.9966	0.9740
1992	1.0442	0.9934	0.9452
2002	1.0600	0.9920	0.925
2012	1.0738	0.9899	0.9112
2019	1.0765	0.9901	0.9080

Over time, the progressive factors decrease for the high income class and increase for the low income class, this suggests that the gain in longevity relative the salary differences are more important in the high income group than in the low income groups.

- **Remark : replacement rate, pension rate and mean benefit ratio**

The replacement rate is the ratio of a pensioner's retirement income to their pre-retirement earnings. In a DB system, the replacement rate is typically a percentage of final average earnings. For example, a replacement rate of δ would mean that a pensioner would receive a pension equal to $P_i = \delta \cdot S_i$.
In a progressive system, their pension benefit would become $P_i = \delta \cdot S_i \cdot \theta_i$, in such a system, δ is the pension rate and the replacement rate is $\delta \cdot \theta_i$. In a progressive system the replacement rate differs for each class.
The mean benefit ratio is the ratio between the average pension for retirees and the average salary of contributors.

4.5.1 Pure Defined Benefits system (DB)

In our initial DB system , the replacement rate is fixed at $\delta = 60\%$. There is no longevity heterogeneity correction strategy nor risk sharing mechanism. The replacement rate is constant and the demographic risk is supported by the active workers, we present in **Figure 4.3** the evolution of the contribution rate in this system ;

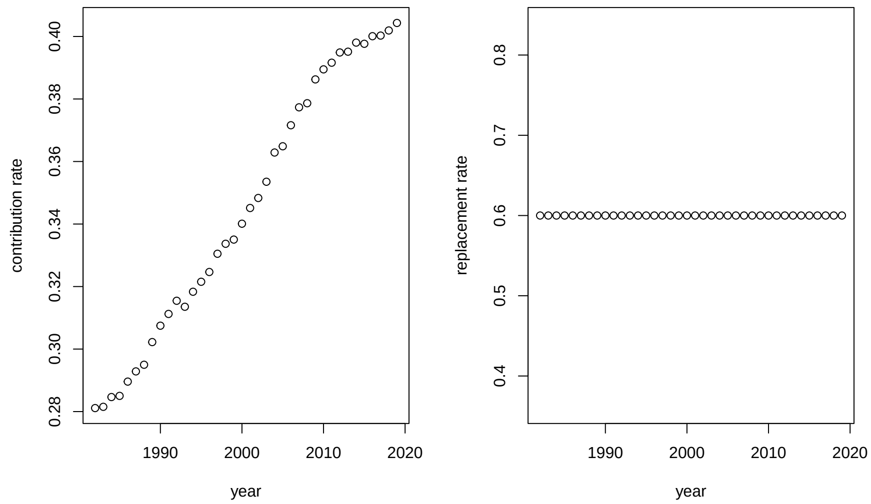


Figure 4.3: Contribution rate and replacement rate in a pure DB system.

The starting and final values of the contribution rate and replacement rate are displayed in **Table 4.4**:

	π
1982	0.2811
2019	0.4043

(a) Contribution rate

	δ
1982	0.6
2019	0.6

(b) Replacement rate

Table 4.4: Contribution and replacement rate in the pure DB system

We compute in **Figure 4.4** the lifetime replacement rate at retirement for the three classes, to observe unfairness after retirement between the three classes :

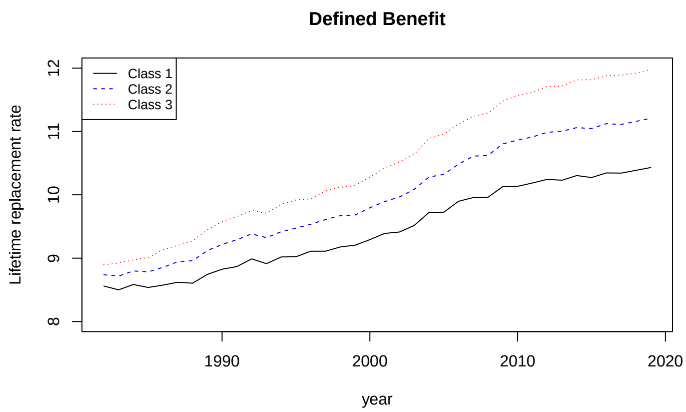


Figure 4.4: Lifetime replacement rate by salary class in a DB system.

We can compare the relative lifetime replacement rate compared to class 2, ($\frac{LR_j}{LR_2}$):

Table 4.5: Relative lifetime replacement rate in a DB system in 1982 and 2019

	1982	2019
Class 1	97.96%	93.06%
Class 3	101.81%	106.88%

Table 4.5 suggests that the lifetime pension benefits for Class 1 decreased more compared to Class 2, while the lifetime pension benefits for Class 3 increased more compared to Class 2, indicating a greater increase in lifetime replacement rate among high-income workers. In a Defined benefit system the demographic risk is solely supported by active workers. Since longevity gain are not uniformly distributed among all retirees classes we observe a greater increase in lifetime replacement rate among high income workers.
The base plan is a DB system on a final salary, providing a replacement rate of $\delta = 60\%$ and the dependency ratio in 1982 is $D = 0.27$, thus the equilibrium contribution rate is $\pi = 0.28$.

The dependency ratio increases every year, the pension contribution equilibrium is affected, and by design the replacement rate stays constant, therefore the contribution of active workers increases. In 2019, its final value was $\pi = 0.4$. The replacement rate of every retiree is the same, and the lifetime replacement rate for each class depends on the remaining life expectancy at retirement. Initially, since there is little longevity heterogeneity, we can observe from **Figure 4.4** that high incomes have a higher lifetime replacement rate than low incomes, but this difference in last salary received after retirement is less than a year. At the end of the projection , the overall longevity increased, since all the curves are increasing and the replacement rate is constant thus longevity is increasing. We also observe that the longevity gain is not evenly distributed among the salary class, meaning that the increase among high income agents is greater than that among low-income agents. This translates into more than a 1-year gap in the lifetime replacement rate differences between the classes.

4.5.2 Defined Musgrave system

In order to share the demographic risk between retirees and active workers, we introduced the Musgrave rule, as defined in the intergenerational automatic adjustment mechanism section. This rule allows an intergenerational risk sharing mechanism between workers and retirees. **Figure 4.5** describes the evolution of the contribution rate and replacement rate in this system:

Figure 4.5: Contribution rate and replacement rate in a DM system.

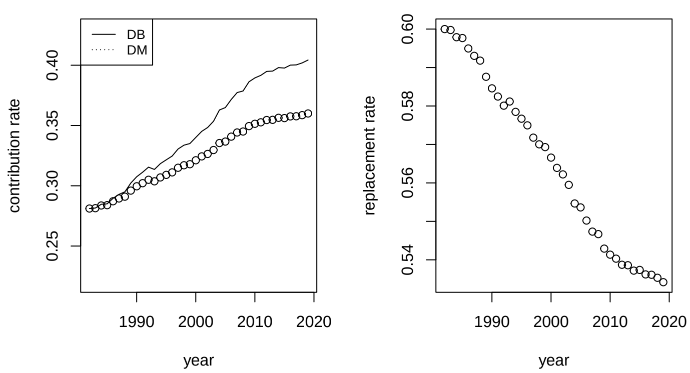


Table 4.6 shows that the replacement rate for retires decreases as well as the contribution rate increases such that the Musgrave ratio is constant the whole time.

	π
1982	0.2811
2019	0.3599

(a) Contribution rate

	δ
1982	0.6
2019	0.5364

(b) Replacement rate

Table 4.6: Contribution and replacement rate in the DM system

When taking into account longevity when computing the lifetime replacement rate we observe that the system is not fair. We are going to compare in **Figure 4.6** the lifetime replacement rate between the three classes in a DM system vs a DB system to assess intra-generational fairness between the classes.

Figure 4.6: Comparison of lifetime replacement rate in a DM system (left) vs DB system (right).

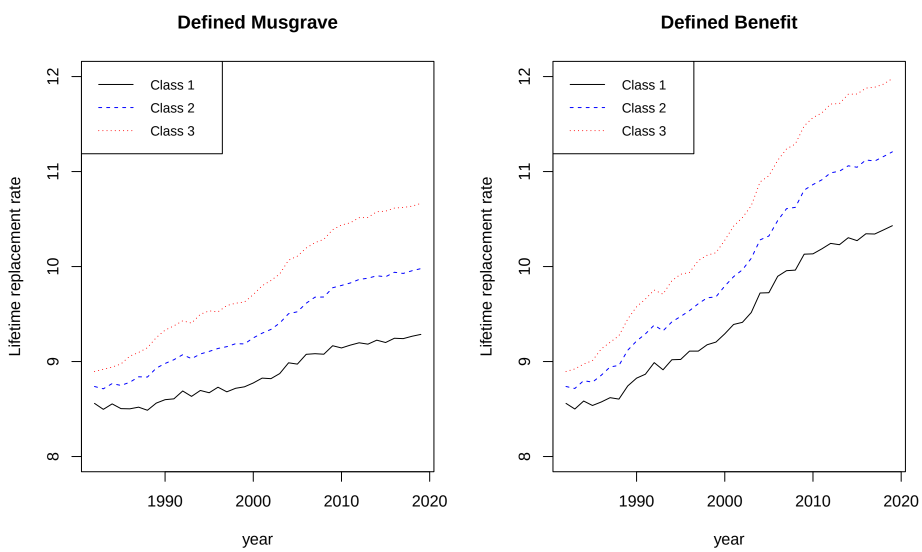


Table 4.7: Relative Lifetime replacement rate in a DM system

	1982	2019
Class 1	97.96%	93.06%
Class 3	101.81%	106.88%

Table 4.8: Lifetime replacement rate by class in a DM system

LR_i^t	1982	2019
Class1	8.5595	9.2859
Class2	8.7374	9.9782
Class3	8.8952	10.6650

From **Table 4.8**, we can see that the lifetime replacement rate in a DM system has increased for all three classes between 1982 and 2019. Additionally, in **Table**

4.7 the relative lifetime replacement rate compared to class 2 is highest for class 3 in both years. This suggests that high-income workers benefit more from a DM system than low-income workers, as they receive a higher lifetime replacement rate and also experience a smaller decrease due to longevity heterogeneity. Thus we need a longevity heterogeneity correction mechanism , as it could help to reduce the penalty for low-income workers. Moreover the values of relative lifetime replacement rate are equivalent in both systems (Table 4.5) because the benefits are not progressive.

In the Defined Musgrave system (DM), which includes a risk-sharing parameter M that allows for the evolution of both the replacement rate and the contribution rate in order to share the demographic risk between active workers and retirees. **Table 4.6** presents the initial and final values of those two parameters, and we observe that the increase in contribution rate is less intense for active workers, the final value of $\pi = 0.3599$ is inferior to the DB system, but the replacement rate in the counterpart also decreases, from 60% to 53%. The lifetime replacement are also affected by the piloting of the replacement rate, reducing the lifetime replacement rate in each class. This indicates that the inter-generational risk sharing piloting of the replacement rate penalizes the low-income workers twice, as they not only support the decrease in the replacement rate, but also the longevity heterogeneity.

4.5.3 Progressive DB system (PDB)

We introduce a progressive system without a demographic risk sharing mechanism to analyse the first effect on the pension benefit and the lifetime replacement rate. We define the PDB pension benefit formula as such:

$$P_j^{(x_r,t)} = \delta \cdot X_j^{(x_r,t)}$$

In this system only the progressive part of the pension formula is updated for every new generation that retires. The pension rate is constant, the demographic risk is transferred to the active workers hence the "defined benefit" nomenclature. In a progressive system δ becomes the **pension rate**.

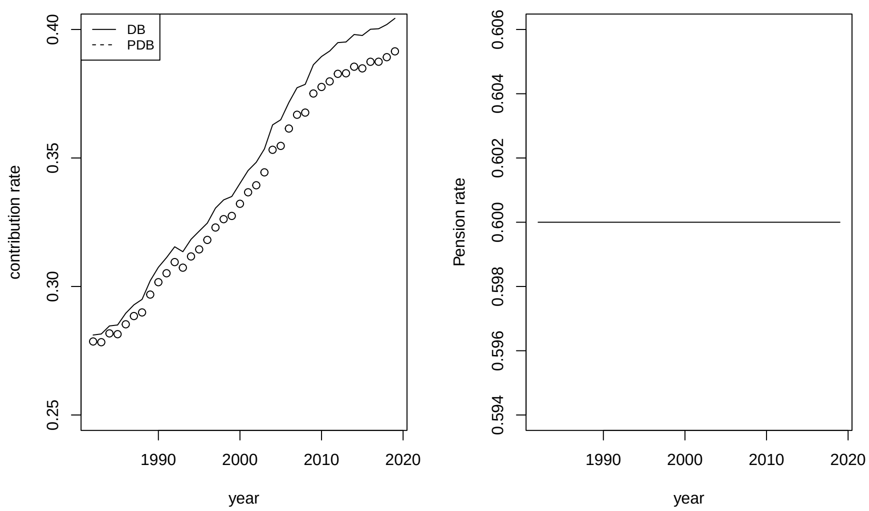


Figure 4.7: Evolution of the contribution rate and pension rate in the progressive DB system.

The starting and final values of the contribution rate and pension rate are :

Table 4.9: Contribution and pension rate in the progressive DB system

	π
1982	0.2786
2019	0.3914

(a) Contribution rate

	δ
1982	0.6
2019	0.6

(b) Pension rate

We observe in **Table 4.9** that the contribution rate in the DB system in 2019 stands at 0.4043, while that of the PDB system is at 0.3914 . The disparity in contribution rates is attributable to differences in redistribution mechanisms employed in the two systems.

The lifetime replacement rate is the same in every class due to the progressive transformation applied in the pension formula. We present the evolution of the lifetime replacement rate in **Table 4.10**:

Table 4.10: Lifetime replacement rate in a Progressive DB system

	1982	2019
LR_i	8.73644	11.22801

The intragenerational fairness condition is met although the demographic risk is only supported by active workers. In a progressive system, since the lifetime replacement rate is the same in all the classes the relative lifetime replacement rate is equal to 1. Because this system focuses solely on the intra-generational risk-sharing mechanism, we examined the progressive defined benefit system, which directly incorporates longevity heterogeneity correction in the pension formula. Like the defined benefit (DB) system, the PDB system lacks a demographic risk-sharing component, placing the demographic risk burden on active workers. We observed that the contribution rate in the PDB system increases from 27.86% to 39.14% while the pension rate remains constant. **Figure 4.7** depicts the similarity in the evolution of the contribution rate between the PDB and DB systems. However, the contribution rate in the PDB system is lower than that of the DB system due to the distinct redistribution approaches utilized by both systems, it is due to the fact that the initial pension benefit are adjusted with the longevity heterogeneity corrections (based on life annuities) in each class.**Table 4.9** shows that the lifetime replacement rate is equal in all three classes, with pension benefits adjusted for longevity heterogeneity correction factors. Although the PDB system would be a favourable alternative, the absence of a demographic risk-sharing component renders it unsuitable for consideration.

4.5.4 Progressive Defined Musgrave system (PDM)

We propose a system that combines the implementation of longevity heterogeneity correction techniques with intergenerational risk sharing. The risk-sharing parameters are in **Figure 4.8** as follows.

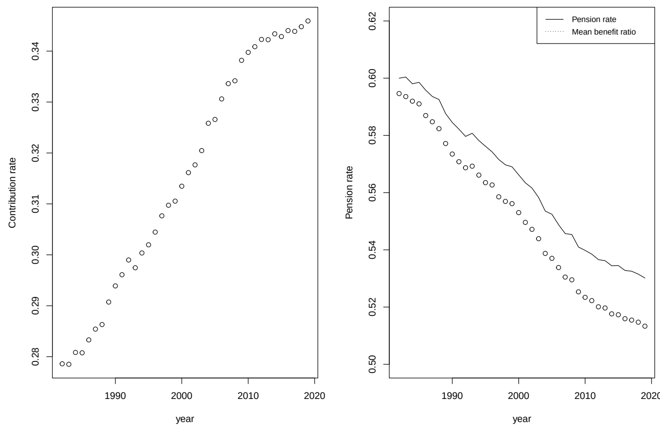


Figure 4.8: Contribution rate and pension rate in a PDM system.

There is a steering mechanism between the contribution rate, the mean benefit ratio and the risk sharing invariant.

	π
1982	0.2786
2019	0.3459

(a) Contribution rate

	δ
1982	0.6
2019	0.5301

(b) Replacement rate

Table 4.11: Contribution rate and pension rate in the PDM system

This system also integrates intragenerational fairness, the lifetime replacement rate at retirement should be the same for every agents in the regime. we present in **Table 4.12** the initial and final values for the lifetime replacement rate in this system.

	1982	2019
LR_j	8.73644	9.921065

Table 4.12: Lifetime replacement rate values in the PDM system

We define the class replacement rate at retirement as the ratio between the pension benefit over the last working salary:

$$Tr_j^{(x_r,t)} = \frac{P_j^{(t,x_r)}}{S_j^{(t,x_r)}} \tag{4.18}$$

We present across the projection some values for the risk sharing mean benefit ratio $\overline{\delta}$, the pension rate δ and the replacement rate at retirement for each class Tr_1, Tr_2, Tr_3.

	$\overline{\delta}$	δ	Tr_1	Tr_2	Tr_3
1982	0.5946298	0.6	0.6123982	0.5999337	0.5892852
1992	0.5686889	0.5796425	0.605263	0.5797883	0.5578465
2002	0.5471703	0.5616006	0.595314	0.5622799	0.5329082
2012	0.5200873	0.5365872	0.5762197	0.5372748	0.5040369
2019	0.5133252	0.5301598	0.5707188	0.5311234	0.4969196

Table 4.13: Comparative table of mean benefit ratio, pension rate, and replacement rate by class

- After retirement

After retirement , the pension benefit evolution is only driven by the evolution of the pension rate δ_t. The class replacement rate for every new generation retiring is:

$$Tr_j^{(x_r+(T-t_r),T)} = \delta_T \cdot \theta_j^{t_r} \tag{4.19}$$

We will compare in **Figure 4.9** the evolution of the replacement rate at 65 and 80 between the first generation to retire in 1982 and the one who retires in 2019 for low incomes and high incomes.

Figure 4.9: Evolution of the replacement rate at 65 and 80 for low incomes(top) and high income (bottom) in the PDM system.

The replacement rate in the high income class is higher for older retirees because the newer generation longevity correction increases.

$\delta_T \cdot \theta_j^{t_r}$	65	80
1982	0.5892852	0.5892852
2019	0.4948666	0.499557

Table 4.14: Replacement rate at 65 and 80 for high incomes in the double AAM system

The opposite effect appears in the low income class, older generation have a lower replacement rate because the longevity correction favours more the new retirees.

$\delta_T \cdot \theta_i^{t_r}$	65	80
1982	0.6123982	0.6123982
2019	0.5683609	0.5596287

Table 4.15: Replacement rate at 65 and 80 for low incomes in the double AAM system

We combined the two mechanisms into one system, the double AAM system, in order to correct for longevity heterogeneity and demographic risk sharing. In comparison with the previous intra-generational fair systems, the lifetime replacement rate is the lowest, and indeed the pension rate δ is decreasing, driving the LR for all classes down (4.12). The pension rate communicated to the affiliates is above the mean benefit ratio, $\overline{\delta}$ making it easier to understand since it applies directly to the transformed salary. Two effects influence the replacement rate, for low income agents, the decline in the pension rate is mitigated by an increase in the longevity heterogeneity correction, thereby reducing the loss in replacement rate. However, for high income class agents, both effects are negative, resulting in a lower replacement rate at the end of the projection.

The proposed formula would represent a significant departure from traditional pension systems, which typically use fixed benefit calculations. Our approach aims to introduce a more flexible system that can adapt to changing demographic trends and ensure the long-term sustainability of pension programs.

4.5.5 Transition to progressive systems

Starting from the ground up, such a system would be socially difficult to implement; however, a gradual transition from existing systems could be achieved through the use of a convex transformation of mortality intensity. To accomplish this, we propose in the following section a gradual transition mechanism based on a convex transformation of mortality intensity. This mechanism ensures a smooth and controlled transition from the current system to the proposed progressive formula, minimizing any social and economic disruptions. Using this mechanism, the proposed system can be implemented in a fair and transparent manner, with the potential to significantly improve the financial sustainability of pension programs.

The previous numerical application was done using the mortality intensities $\mu_{x,t}^{j}$ extracted from the tables by socio-economic groups. For the entire population, we used $\mu_{x,t}$, the mortality intensity for all the groups. We are going to test the results under different scenarios for longevity heterogeneity using a transformation inspired by credibility theory . The idea is to use a convex transformation of the mortality intensity specific to a socio-economic class and the mortality intensity of the entire population.

Let us consider the following expression for the estimate of the mortality intensity in each class j,

$$\tilde{\mu}^{j}_{x,t} = (1-\alpha)\cdot\mu^{j}_{x,t} + \alpha\cdot\mu_{x,t} \tag{4.20}$$

α is defined as the progressivity indicator of the transition. Each value of $\alpha \in [0;1]$ defines a new mortality intensity . Consequently each value of α defines a new progressive system given that the pension formula uses a previously defined progressive transformation. The application of the credibility transformation in the longevity adaptation and demographic risk sharing framework also allows for a parallel with the systems previously defined.

In a system without demographic risk sharing :

- $\alpha = 1$ is equivalent to a pure DB system
- $\alpha = 0$ is equivalent to the Progressive DB system.

In a system with demographic risk sharing :

- $\alpha = 1$ is equivalent to a Defined Musgrave system
- $\alpha = 0$ is equivalent a PDM system .

This means that we can allow for the parametrization of a large number of systems depending on the importance we give to mortality intensity relative to the socio-economic classes. The progressivity indicator, α, plays a crucial role in defining the degree of progressivity in the system. The previous numerical application used a fixed value of α, but it is important to note that the progressivity indicator can also evolve over time. In fact, a system described with a decreasing $\alpha(t)$ will become completely progressive when the progressivity indicator reaches its minimal value. Therefore, the proposed pension system can be customized and adapted to changing circumstances by adjusting the value of α.

We are going to present the evolution of the longevity correction, the mean benefit ratio,the replacement rates at retirement in each class and the contribution rates for each system for select values of α that we refer to as the progressivity indicator.

- Longevity heterogeneity correction

The previous application in systems that allowed longevity heterogeneity correction showed that most of the differences existed between the first and last socio-economic classes. We are displaying the evolution of the longevity heterogeneity correction , the average benefit ratio, and the contribution rate for different values of α.

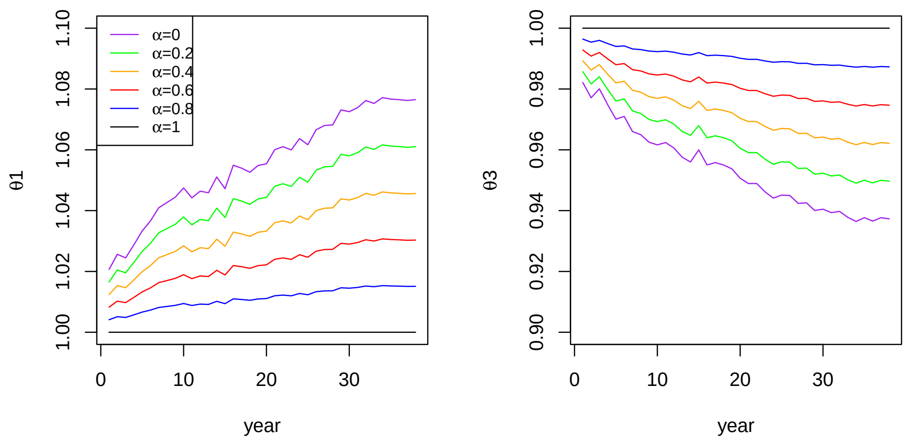


Figure 4.10: Longevity heterogeneity correction in the low income class (left) and high income class (right)

Figure 4.10 displays the evolution of the longevity heterogeneity correction for both low and high socio-economic classes. When α is close to 0 the correction is more important in both classes.

- Mean benefit ratio and contribution rate

Figure 4.11 displays the evolution of the average benefit ratio for different values of α. As expected, when $\alpha = 1$ the replacement rate is the highest for all socio-economic classes, which corresponds to a DM system. On the other hand, when $\alpha = 0$, the replacement rate is the lowest, which corresponds to the Progressive DM. The intermediate values of α allow for a parametrization of the system between these two extremes.

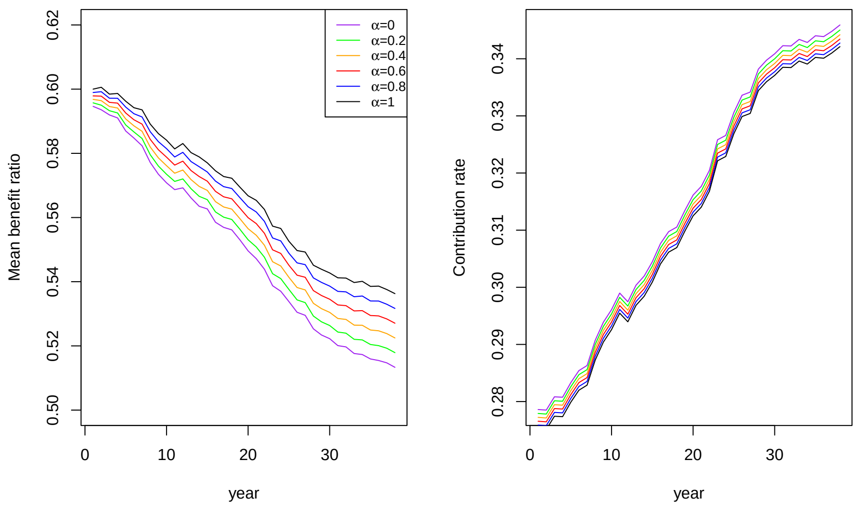


Figure 4.11: Mean benefit ratio and contribution rate for different values of α .

We present the evolution of the replacement rate for each class using a sequence for the value of the progressivity indicator

Table 4.16: Evolution of the class replacement rate for different values of the progressivity indicator

t	α	Tr_1	Tr_2	Tr_3	α	Tr_1	Tr_2	Tr_3
1982	0.8	0.6025	0.6000	0.5978	0	0.6124	0.5999	0.5893
1992	0.6	0.5908	0.5807	0.5718	0	0.6053	0.5798	0.5578
2002	0.4	0.5832	0.5633	0.5456	0	0.5953	0.5623	0.5329
2012	0.2	0.5691	0.5379	0.5112	0	0.5762	0.5373	0.5040
2019	0	0.5707	0.5311	0.4969	0	0.5707	0.5311	0.4969

From the previous tables, we can see the impact of longevity correction and progressivity on the replacement rates for each class. As the progressivity indicator increases, the replacement rates decrease for each class, indicating that a more progressive system leads to a lower replacement rate for the high earners. This is because a more progressive system places a larger burden on high earners to support low earners, which reduces the replacement rate for high earners. We can also see that the impact of progressivity on the replacement rate depends on the level of progressivity indicator and the longevity correction. For example, when the progressivity indicator decreases from one to zero , a less abrupt transition from a DM system to a fully progressive DM system is possible for different values of α. This suggests that a gradual transition to a more progressive system may be beneficial for minimizing the impact on replacement rates.

It is also possible not to completely correct longevity heterogeneity and only partially apply progressivity in the system. We showcase the example with $\alpha = 0.4$ throughout the evolution of the system.

Table 4.17: Evolution of the class replacement rate for $\alpha = 0.4$

t	α	Tr_1	Tr_2	Tr_3
1982	0.4	0.6074	0.5999	0.5935
1992	0.4	0.5956	0.5803	0.5671
2002	0.4	0.5832	0.5633	0.5456
2012	0.4	0.5620	0.5386	0.5185
2019	0.4	0.5566	0.5329	0.5120

We see that when only partial progressivity is applied (e.g., $\alpha = 0.4$), the replacement rates still decrease over time, but the rate of decrease is not as steep as in a fully progressive system. This indicates that a partial application of progressivity can still lead to a more adequate system while minimizing the impact on replacement rates.

Overall, the class replacement rates provide important information on the impact of progressivity and longevity correction on the replacement rates for each class, which is crucial for designing a sustainable and equitable pension system.

4.5.6 Key Parameter Comparison: A Revealing Table

We are going to compare, through the projection period (1982–2019), the evolution of the lifetime replacement rate and the pension benefit for the three classes between the previous system presented and also the contribution effort of active workers within those systems. The following **Table 4.18** summarises the contribution rate, pension rate and class replacement rate for each group in all the systems previously introduced.

	DB		DM		PDB		PDM	
	1982	2019	1982	2019	1982	2019	1982	2019
π_t	0.2811	0.4043	0.2811	0.3599	0.2786	0.3914	0.2786	0.3459
δ_t	0.6	0.6	0.6	0.5364	0.6	0.6	0.5997	0.5301
Tr_1	0.6	0.6	0.6	0.5364	0.6125	0.6452	0.6125	0.5707
Tr_2	0.6	0.6	0.6	0.5364	0.5997	0.6010	0.5997	0.5311
Tr_3	0.6	0.6	0.6	0.5364	0.5897	0.5630	0.5897	0.4969

Table 4.18: Contribution rate and pension rate comparison

Each system that follows the defined benefit framework includes a mechanism for sharing longevity heterogeneity and ageing risks. The defined Musgrave system distributes demographic risk between active workers and retirees by controlling the contribution and replacement rates. As the population ages, the replacement rate declines, leading to a higher share of benefits for the high-income class. On the other hand, the progressive defined benefit system addresses longevity heterogeneity by sharing risks between retirees of different socio-economic classes, with lifetime replacement rates being equalized across retirees. However, this system may result in a loss for active workers. The double Automatic adjustment mechanism system combines both approaches, facilitating wealth redistribution from high to low socio-economic classes while ensuring financial stability amidst population ageing.

4.6 Conclusion

The aim of this study is to address the challenges faced by national public pension systems in the context of demographic ageing and significant longevity inequalities between socio-economic classes. The chapter proposes a pension system that seeks to achieve intergenerational and intragenerational equity, while also ensuring financial sustainability and social adequacy. The first goal of the proposed system is to design a fair system that takes into account the differences in longevity among retirees. This will help to ensure that the system is fair to all categories of retirees. In addition, the system incorporates an intergenerational risk sharing mechanism that will distribute demographic risk more fairly among active workers and retirees. This will help to ensure that the system is financially sustainable over the long-term. Finally, the proposed system aims to be socially acceptable through transparent communication and easy to implement, mainly due to the fact that it is based on salaries.

This chapter emphasizes the importance of making changes to national public pension systems in order to ensure long-term sustainability in the face of demographic and economic events. To address financial imbalances, most pension schemes have implemented automatic adjustment or stabilization mechanisms, but a difficult trade-off must be made to balance intergenerational costs with intragenerational actuarial fairness and social adequacy. Moreover, we suggest a pension system based on two automatic adaptation mechanisms that ensure intergenerational and intragenerational equity. The first mechanism accounts for longevity heterogeneity among pension scheme agents, while the second distributes demographic risk among working people and retirees. The pension formula incorporates longevity heterogeneity correction and progressivity, while financial sustainability is ensured by the Musgrave rule-inspired risk-sharing invariant.

Finally, we argue that constructing pension schemes entirely on actuarial principles is challenging and that a balance must be achieved between intergenerational costs, intragenerational actuarial fairness, and social adequacy. The suggested pension system may provide a solution to these issues, but further research is needed to determine its viability and usefulness in various circumstances. The study underlines the significance of tackling the issues faced by the ageing population as well as preserving the long-term viability and fairness of public pension systems. Overall, this study highlights the importance of preserving the long-term viability and fairness of public pension systems, and suggests a promising path forward to achieve these goals. In conclusion, this chapter has proposed a pension system that seeks to achieve intergenerational and intragenerational equity, financial sustainability, and social adequacy in the face of demographic ageing and socio-economic disparities in longevity.

While the proposed system is based on a fixed retirement age and final salaries as a basis for benefits, further research could explore alternative rules of indexation and the potential benefits of incorporating progressivity into the system. For instance, alternative rules of indexation could include the use of average salaries or a different measure of inflation to adjust benefits, which could potentially impact the financial sustainability and social adequacy of the system. Furthermore, incorporating progressivity into the system could provide more flexibility in achieving greater equity by adjusting retirement age instead of pension benefits to equalize the lifetime replacement rate across different demographic groups. By continuing to refine and improve the proposed pension system, we can help ensure the long-term viability and fairness of public pension systems in the years to come.

Formula in a Average wage system

The mechanisms presented are based on a final salary formula. It is also possible to extend every results in a career average system. We use the same assumptions that the dynamic environment, we describe present the main results in the average wage environment:

- The average wage in a career for an agent of class j is noted $\bar{S}_j^t$
- The progressive transformation is written

$$X_j^{(x_r,t)} = \sum_{i=1}^{j} \lambda_i^t \cdot (\bar{S}_i^t - \bar{S}_{i-1}^t) \tag{4.21}$$

  and the pension benefit becomes $P_j^{(x_r,t)} = \delta_t \cdot X_j^{(x_r,t)}$

- The lifetime replacement rate in this system is expressed as the ratio between the present value of pension benefits at retirement and the average wage during the career. The target LR based on the mean longevity rates for the whole population is written as follows:

$$LR^*(t) = \frac{1}{\bar{S}^t} \sum_{x=x_r}^{\omega} P^{(x_r,t)} \cdot {}_{x-x_r}p_{x_r}(t) \cdot (\frac{1}{1+r})^{x-x_r} \tag{4.22}$$

$$= \frac{P^{(x_r,t)}}{\bar{S}^t} \cdot \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r} \cdot (\frac{1}{1+r})^{x-x_r} \tag{4.23}$$

$$LR^*(t) = \delta_t \cdot \ddot{a}_{x_r}(t) \tag{4.24}$$

  it is written in class j

$$LR_j(t) = \delta_t \cdot \frac{X_j^{x_r}}{\bar{S}_j^{x_r}} \cdot \ddot{a}_{x_r}^{(j)}(t) \tag{4.25}$$

- We obtain the values of progressive coefficients when the LR equality condition is satisfied :

$$LR_1(t) = ... = LR_j(t) = ... = LR_m(t) = LR^*(t).$$

  We follow the same method to obtain the progressive coefficients in this system,

$$\lambda_j^t = \frac{\bar{S}_j^t \cdot \frac{\ddot{a}_{x_r}(t)}{\ddot{a}_{x_r}^j(t)} - \bar{S}_{j-1}^t \cdot \frac{\ddot{a}_{x_r}(t)}{\ddot{a}_{x_r}^{j-1}(t)}}{\bar{S}_j^t - \bar{S}_{j-1}^t}$$

  we can then compute the intergenerational AAM using benefits based on those coefficients, the evolutions of the parameters as well as the policy implication will differ given the new information contained in the entire career.

Methodology for projection

We extracted the mortality rates by quintile of the population from the Mortality by Socio-economic Category in the United States study. The research stratifies the US population into socio-economic groups of similar population size using eleven county-wide factors on education, occupation, employment, income. The study produced a set of comprehensive life tables broken down by socio-economic group and year that may be utilized in mortality models. Details on the data and methodologies used in the development of the life tables are summarized in the research report. [Barbieri 2020]

- The mortality data range is from 1982 to 2019. From those periodic mortality tables we are able to extract each period the 1 year survival probabilities by socio-economic group j, is ${}_1p_x^j(t)$.
- Agents only enter the regime at age $x_0 = 25$. Retirement age is $x_r = 65$
- Active workers can only exit the regime by dying.
- The effective of workers aged x at period t is given by $Na_i^{(x,t)}$, the number of retirees aged y is $Nr_i^{(t;y)}$
- The effective are given via the relationship $Na_i^{(x+1,t)} = Na_i^{(t;x)} \cdot {}_1p_x^i(t)$

  ${}_1p_x^i(t)$ is the one year survival probability . We obtain the retirees effective following the same procedure. Since there is no migration the population function at year t can be obtained recursively from the entry function and the periodic mortality table.
- The total number of active workers at time t is given by Na^t and the total number of retirees Nr^t.
- The initial number of active workers corresponds to the entry function at age x_0 is $E = 100000$
- The distribution of workers within the salary classes stays the same throughout the whole projection :
  Quintile 1 (Q_1) of salary distribution represents the low salary workers.
  Quintiles 2,3,4 (Q_2, Q_3, Q_4) represent the average salary class.
  Quintile 5 (Q_5) of salary distribution represents the high salary class.
- Low income class agents represent 20% of the population , average class represent 60% ,and high income retiree represent 20% of the population.

- The dependency ratio measures the number of retirees over the number of active workers

- The initial dependency ratio depicts the demography of a system using the mortality table of 1982. Every following year the number of workers and retirees are obtained via the corresponding mortality table this year.

- Using the same data we compute each period the life annuity at retirement by socio-economic quintile using the following formula

$$\ddot{a}^i_{x_r} = \sum_{x=x_r}^{\omega} {}_{x-x_r}p^i_{x_r} \cdot (\frac{1+\gamma}{1+r})^{x-x_r}$$

- The life annuity indexation rate is the same as the pension indexation rate, we used the average rate of salary evolution.

- We defined salary in each class with

$$S_i^{(t;x)} = S_i^{(x_0,t_0)} \cdot (1+s_i)^{(x-x_0)} \cdot (1+h)^{(t-t_0)}$$

  As starting salary for each class we used data on household income quintiles [US Census Bureau (2023)] , in 1982

$$S_1^{(1982;x_0)} = 4790$$
$$S_2^{(1982;x_0)} = 20675$$
$$S_3^{(1982;x_0)} = 54720$$

- The career growth effect in each group is given by

$$s_1 = 0.1\% \ ; \ s_2 = 0.15\% \ ; \ s_3 = 0.2\%$$

- The time effect is obtained by averaging the yearly evolution of salary in the data through our on our projection horizon and we obtained $h = 2.5\%$

- Pension benefits are indexed with the yearly average salary growth rate $\gamma = 2.5\%$.

Bibliography

[Alho & al (2013)] J Alho, J M Bravo, E Palmer Non financial defined contribution Pension Schemes in a Changing Pension World, volume 2, p. 395 - 436

[Ardito et al 2020] Ardito C, D'Errico A, Leombruni R, Ricceri F, Costa G. Life expectancy inequalities and their evolution in Italy. How these impact on the equity of the pension system? **Eur J Pub Health**. 2020;30(5):ckaa165.764. https://doi.org/10.1093/eurpub/ckaa165.764.

[Ayuso & al 2017] Ayuso M, Bravo JM, Holzmann R. On the heterogeneity in longevity among socioeconomic groups: scope, trends, and implications for earnings-related pension schemes. **Glob J Hum Soc SciEcon**. 2016;17(1):33–58.

[Ayuso & al 2021] Ayuso, Mercedes, Jorge M. Bravo, and Robert Holzmann. 2021. Getting Life Expectancy Estimates Right for Pension Policy: Period versus Cohort Approach. **Journal of Pension Economics and Finance** 20: 212–31.

[Barbieri 2020] Barbieri Magali. Mortality by Socioeconomic Category in the United States. Society of Actuaries, November 2020.

[Belloni & al 2013] Belloni M, Alessie R, Kalwij ET,& al. Lifetime income and old age mortality risk in Italy over two decades. Demog Res. 2013;29:1261–98.

[Bosworth 2018] Bosworth B. Increasing disparities in mortality by socioeconomic status. **Annu Rev Public Health**. 2018;39:237–51.

[Bovenberg & al (2015)] L Bovenberg, S Bilsen, R Mehlkopf Personal pensions with risk sharing affordable, adequate and stable private pensions in Europe Netspar Discussion Paper Series, p. 3 - 2015

[Borsch-Supan 2006] Borsch-Supan A. What are NDC systems? What do they bring to reform strategies? In: Holzmann R, Palmer E, editors. Pension reform: issues and prospects for ndc schemes. Washington: World Bank Publishing; 2006.

[Bosley & al (2018)] Bosley T, Morris M, Glenn K. Mortality by Career-Average Earnings Level. In: Actuarial study, 124, Social Security Administration, Office of the Chief Actuary; 2018. $https://www.ssa.gov/OACT/NOTES/pdf_studies/study124.pdf$

[US Census Bureau 2023] US Census Bureau Historical Income tables. March 2023 https://www.taxpolicycenter.org/statistics/household-income-quintiles

[Donkin et al 2002] Donkin A, Goldblatt P, Lynch K. Inequalities in life expectancy by social class 1972–1999. **London: Health Statistics Quarterly** no. 15; 2002.

[Devolder & Melis (2015)] Devolder, Pierre ; Melis, Roberta. Optimal mix between pay as you go and funding for pension liabilities in a stochastic framework. In: **Astin Bulletin : the journal of the International Actuarial Association**, Vol. 45, no.3, p. 551-575 (2015)

[Devolder & De Valeriola (2020)] Devolder, Pierre ; De Valeriola, Sébastien. Pension Design and Risk Sharing: Mixed Solutions Between Defined Benefit and Defined Contribution for Public Pension Schemes. In: Marta Peris-Ortiz, José Álvarez-García, Inmaculada Domínguez-Fabián, Pierre Devolder (eds.), **Economic Challenges of Pension Systems**, Springer International Publishing : Cham 2020, p. 311-339

[Diakite & Devolder2021] Diakite, K.; Devolder, P. Progressive Pension Formula and Life Expectancy Heterogeneity. Risks 2021, 9, 127. https://doi.org/10.3390/risks9070127

[Fors & al 2021] Fors S, Wastesson JW, Morin L. Growing income-based inequalities in old-age life expectancy in Sweden, 2006-2015. Demography. 2021;58(6):2117–38.

[Holtzmann 2019] Ayuso M, Bravo JM, Holzmann R. On the heterogeneity in longevity among socioeconomic groups: scope, trends, and implications for earnings-related pension schemes. **Glob J Hum Soc SciEcon**. 2016;17(1):33–58.

[Jijiie & al (2021)] Jijiie A, Alonso-García J, Arnold S. Mortality by socio-economic class and its impact on the retirement schemes: how to render the systems fairer? **Eur Actuar J**. 2021.

[Kalwij & al 2013] Kalwij A, Alessie R, Knoef M. The association between individual income and remaining life expectancy at the age of 65 in the Netherlands. Demography. 2013;50(1):181–206.

[Kleinow & Cairns (2020)] Wen J, Kleinow T, Cairns AJG. Trends in Canadian mortality by pension level: evidence from the CPP and QPP. **N Am Act J**. 2020;24(4):533–61.

[Kibele & al(2013)] Kibele EUB, Jasilionis D, Shkolnikov VM. Widening socioeconomic differences in mortality among men aged 65 years and older in Germany. **J Epidemiol Community Health**. 2013;67:453–7.

[Longevity Panel 2018] Longevity Science Panel. Life expectancy: Is the socio-economic gap narrowing? 2018. http://www.longevitypanel.co.uk/viewpoint/life-expectancy-is-the-socio-economic-gap-narrowing/

[Longevity Panel (2020)] Longevity Science Panel. An analysis of mortality trends in developed countries, focusing on the recent slowdown in mortality improvements. 2020.`https://www.longevitypanel.co.uk/_files/LSP2020_Report.pdf`

[Lallo & al (2018)] Lallo C, Raitano M. Life expectancy inequalities in the elderly by socioeconomic status: evidence from Italy. **Popul Health Metrics**. 2018;16(1):7.

[Mackenbach & al 2016] Mackenbach JP, Kulhánová I, Artnik B,& al. Changes in mortality inequalities over two decades: register based study of European countries. **Brit Med J**. 2016;352(5):1732.

[Marmot(2015)] Marmot M. The health gap. The challenge of an unequal world. London: Bloomsbury Publishing; 2015.

[Musgrave & Thin (1948)] Musgrave, Richard, and Tun Thin. 1948. Income Tax Progression, 1929–1948. **Journal of Political Economy** 56: 498.

[Mankiw 2007] Mankiw NG, Ball L. Intergenerational Risk Sharing in the Spirit of Arrow, Debreu, and Rawls, with Applications to Social Security Design. Journal of Political Economy. 2007;115 (4) :523-547.

[Goldman & Orszag 2014] Goldman DP, Orszag PR. The growing gap in life expectancy: using the future elderly model to estimate implications for social security and medicare. Am Econ Rev. 2014;104(5):230–3.

[OECD 2019] Pensions at a Glance 2019: OECD and G20 Indicators. Paris: OECD Publishing.

[Schokkaert & al (2020)] Schokkaert Erik, Devolder Pierre, Hindriks Jean, Vandenbroucke Frank, 2020. "Towards an equitable and sustainable points system. A proposal for pension reform in Belgium," LIDAM Reprints CORE 3121, Université catholique de Louvain, Center for Operations Research and Econometrics (CORE).

[Turner, 2009] Turner, J. Social Security Financing: Automatic Adjustments to Restore Solvency. AARP Research Report 2009-01

[Waldron, 2007] Waldron H. Trends in mortality differentials and life expectancy for male social security-covered workers, by socioeconomic status. Soc Secur Bull. 2007;67(3):1–28.

[Wenau & al (2019)] Wenau G, Grigoriev P, Shkolnikov V. Socioeconomic disparities in life expectancy gains among retired German men, 1997-2016. **J Epidemiol Community Health**. 2019;73:605–11.

[Wouterse & al 2021] Wouterse B, Bär M, Riumallo Herl C, Van Ourti T, Van Doorslaer E. Diverging mortality inequality trends among young and old in the Netherlands. **Fisc Stud**. 2021;42:79–101.

Chapter 5

Multi-population mortality and pension design

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5.1 Introduction

Health inequalities have a lasting impact on retirement policies, as individuals often reach the statutory retirement age with varying work abilities and residual life expectancies [Strozza & al, 2022]. These disparities arise from unequal exposure to occupational risk factors, unfavorable living conditions, and adaptive behaviors resulting from chronic distress [Forster & al, 2018]. Such health inequalities pose a challenge to retirement policies, which must address the final health disadvantage that leads to greater economic disadvantage [Ardito & Costa, 2022]. The impact of longevity inequalities further complicates pension systems, resulting in a redistribution of resources from the less well-off to the better-off [Holzmann et al., 2020]. The relationship between life expectancy and pension policies has gained significant attention in recent years [Arnold & Jijie, 2020]. Life expectancy plays a crucial role in determining the number of years of pension receipt and serves as a key parameter in pension rules, affecting eligibility and benefit amounts in various countries.

Many OECD pension systems incorporate automatic adjustment mechanisms, such as linking benefits to life expectancy at retirement through longevity factors in notional defined contribution schemes or tying the statutory retirement age to changes in life expectancy within the population. The case for increasing the state pension age overlooks crucial factors, contrary to the assumption that individuals' work capacity will steadily improve, empirical evidence from the Trade Union Congress (TUC) research conducted in the United Kingdom reveals a decline in employment rates among women aged above the state pension age and between 50 and the state pension age since 2010. Moreover, the pension Bill that proposes to increase the retirement age to 67 reduces incentives for working beyond the pension age [TUC report, 2013]. Research also shows growing inequalities in disability-free life expectancy among different regions [Sundberg & al, 2023]; [Wilmoth & al, 2023], highlighting how some individuals face higher disability risks by the time they reach the state pension age. These blind spots reveal the gendered impact and health-related limitations that challenge the sustainability of extended working lives and call into question the rationale for increasing the pension age. However, these mechanisms have implications for pension wealth distribution. Individuals with lower-than-average life expectancy receive pension benefits for a shorter duration, resulting in a loss of pension wealth relative to what would be actuarially fair. Conversely, groups with above-average life expectancy enjoy a pension "premium," with their benefits financed by the most disadvantaged groups. Studies have shown that occupational inequality in mortality can offset a significant portion of income redistribution in pension systems. For instance, it has been estimated that occupational inequality neutralizes a third of the income redistribution in the French PAYG pension system and fully offsets it in Germany. OECD reports highlight the impact of the average gap in remaining life expectancy at retirement, which reduces total pensions received by low earners by 13% compared to high earners, independent of differences in earnings. Motivated by the within-generation inequalities and disparities in life expectancy associated with socio-economic status (SES), it is crucial to explore the intersection of multi-population mortality modelling and pension systems [Shi & Kolk, 2022]. The existing literature on pensions has extensively focused on mortality modelling and forecasting for single populations,but the inherent interconnections between mortality improvements across different populations are only taken into account with respect to Non-financial Defined Contribution (NDC) pension systems or adjusting the retirement age in Defined Benefit (DB) pension systems. This oversight ignores the potential underlying unfairness in DB systems arising from the diverse characteristics and needs of sub-populations. By incorporating multi-population models into mortality analysis, we can capture the cross-population dependence and observe the variations in mortality patterns among different socio-economic classes [Antonio & al, 2015] , [Jevtić & al, 2023]. The disparities in life expectancy between high and low socio-economic status groups have been well-documented [Majer & al, 2011], [Dudel & Schmied, 2019], revealing a clear correlation between longevity and social advantage. Individuals with higher SES tend to enjoy longer lifespans, thereby benefiting more from pension systems compared to their lower SES counterparts. Consequently, the existing pension systems may inadvertently perpetuate unfairness by disproportionately favoring certain socio-economic classes [Queisser & Whitehouse, 2006].

Some papers in the literature have studied the implications of heterogeneity in longevity on pension systems. [Holzmann et al., 2020] investigate the implications of longevity heterogeneity on an Non-financial Defined Contribution (NDC) system. They show how it makes losing the contribution-benefit link generally considered as the signature element of a NDC system. To overcome this issue, they present some alternative adjustment on the pension calculation by modifying either the annuity rate or the contribution rate. The intervention can operate at retirement or during the accumulation phase. [Arnold & Jijie, 2020] aim to determine the optimal retirement age in a PAYG-funded public pension system, considering both a defined benefit (DB) and a notional defined contribution (NDC) scheme. The authors consider both an utilitarian and an actuarial approach. In the second approach, the optimal retirement age is determined so that the accumulated value at the extreme age of the pensions received under each system is as close as possible to the value that would be accumulated if a theoretically fair pension was paid. The proposed approach makes it possible to set a lower retirement age for the lower socio-economic classes and a higher age for the upper classes, while leaving pension benefits unchanged. In [Jijiie & al., 2022] they measure the fairness of both a DB and a DC scheme and observe that in both systems there is a transfer from the poorer to the richer classes, which is exactly the opposite of the objective of a social security system. Starting from the observation that in practice mortality rates by socio-economic class are not adopted in pension systems, and that they are often not even known, they propose to adjust the parameters of each system to improve fairness. In particular, they suggest adjusting the interest rate used to define the theoretically fair pension, the accrual rate of DB pensions and the notional rate of NDC pensions. [Boado-Penas et al., 2022] study the lifetime redistribution of a generic NDC system using the ratio between the present value of expected pensions received and contributions paid and adopting two alternative types of annuity divisor to determine the initial amount of pension: the demographic one and the economic adopted in Sweden. They develop their analysis by adopting a stratified Lee-Carter model to project the mortality of sub-populations ([Delbon & al., 2011]). On one hand, their analysis shows that the adoption of a unisex demographic divider benefits subgroups of highly educated women and men. On the other hand, if gender and educational mortality differentials are adopted, the differences between the divisors are mainly produced by the gender-related longevity effect.

To address these inherent biases, throughout this paper we will investigate the impact of sub-population differences in mortality on a pension system and shed light on the implicit unfairness present within it. By employing multi-population mortality modelling techniques, we can uncover the nuances of mortality patterns across socio-economic classes and examine their implications for pension design. This analysis will not only facilitate a deeper understanding of the underlying unfairness in pension systems but also serve as a foundation for proposing more equitable approaches to pension design and distribution.

This paper aims to contribute to the understanding of pension system challenges by integrating double Automatic Adjustment Mechanisms (AAM) and multi-population mortality modelling, providing a framework to analyze and mitigate these complexities. The exploration of pension schemes, multi-population mortality modelling, and socio-economic differences in longevity offers valuable insights into the design of more equitable and robust pension policies. The paper's structure revolves around two main themes: pension schemes with double AAM and multi-population mortality modelling, delving into the motivations, principles, assumptions, and mechanisms underlying these approaches. The next section delves into multi-population mortality modelling, providing a comprehensive definition of the models used and the data sources employed to compare and validate these models. The aim is to develop a robust framework that captures the intricacies of mortality patterns across different populations. Furthermore, the paper conducts a risk analysis of Automatic Adjustment Mechanisms within pension systems, specifically examining the double AAM approach in light of aging and longevity heterogeneity. The concept of Value-at-Risk analysis is employed to assess the potential risks and uncertainties associated with these mechanisms. Throughout the paper, discussions are held to critically analyze the findings, address potential limitations, and consider implications for policy-makers and stakeholders. The paper concludes by summarizing key insights from the study, emphasizing the importance of considering socio-economic differences in longevity and addressing unfairness within pension systems. By integrating double AAM and multi-population mortality modelling, this research contributes to advancing our understanding of pension schemes and provides a foundation for more equitable and sustainable approaches in the face of demographic changes and socio-economic disparities.

5.2 Multi-population mortality modelling

In this chapter, we use a progressive pension formula as well as the inter-generational steering mechanism of the contribution rate and the benefit ratio, in order to highlight the need for a multi-population model to assess the risk of the system we propose. We follow the assumptions and hypotheses from Chapter 4, more precisely in a progressive defined Musgrave system. Through this section, we will explore various multi-population mortality models and their applicability in capturing the dynamics of mortality improvements among different sub-populations. By considering sub-population differences in mortality, we can develop a more comprehensive understanding of the complex interactions between mortality patterns and socio-economic factors, allowing for a more accurate assessment of fairness within pension systems.

5.2.1 The models definition

In order to represent the mortality evolution of pension system members, taking into account the heterogeneity between different economic classes, a multi-population mortality model should be selected. We consider different mortality models so that we can represent different types of heterogeneity in mortality between the reference sub-populations: absence of heterogeneity, level difference, trend difference. Let us denote with $D_{x,t}^{(i)}$ the number of deaths between age x and $x+1$ in the year t in the class i and with $E_{x,t}^{(i)}$ the corresponding exposed to risk, the central mortality rate at age x in the year t for class i, $m_{x,t}^{(i)}$, is given by:

$$m_{x,t}^{(i)} = \frac{D_{x,t}^{(i)}}{E_{x,t}^{(i)}} \tag{5.1}$$

We denote with $D_{x,t}^{(P)}$ the number of deaths between age x and $x+1$ in the year t in the total population ($D_{x,t}^{(P)} = \sum_i D_{x,t}^{(i)}$) and with $E_{x,t}^{(P)}$ the corresponding exposed to risk ($E_{x,t}^{(P)} = \sum_i E_{x,t}^{(i)}$). The central mortality rate at age x in the year t for the total population, $m_{x,t}^{(P)}$, is given by:

$$m_{x,t}^{(P)} = \frac{D_{x,t}^{(P)}}{E_{x,t}^{(P)}} \tag{5.2}$$

The first mortality model we consider is the modified Lee-Carter model proposed by Brouhns et al. ([Brouhns & al., 2002]), assuming absence of heterogeneity between classes, so that the same model is applied to all the m classes:

$$\log m_{x,t}^{(i)} = \alpha_x^{(P)} + \beta_x^{(P)} \kappa_t^{(P)} \tag{5.3}$$

It should be noted that by assuming this model any form of heterogeneity in mortality between classes is neglected. In the following we denote this model as CLC model.

The second model we considered is the common factor model introduced by Li and Lee ([Li & Lee, 2005])

$$\log m_{x,t}^{(i)} = \alpha_x^{(i)} + \beta_x^{(P)} \kappa_t^{(P)} \tag{5.4}$$

where the age-specific mortality pattern $\alpha_x^{(i)}$ is population specific while the time index driving the mortality change and the age-specific responses to changes in the level of mortality are the same for the sub-populations. The parameters $\alpha_x^{(i)}$ are estimated by:

$$\alpha_x^{(i)} = \frac{\sum_t \log m_{x,t}^{(i)}}{n_t} \tag{5.5}$$

where n_t is the number of periods available. Note that this model imply different mortality levels but the same mortality improvements for all sub-population at all times (see [Villegas & al., 2005]). In the following we denote this model as common Lee-Carter. In the following we denote this model as CF model.

In order to relax the assumption of identical mortality improvement we consider as third model the joint-κ model, which has been proposed as a possible way of extending the single-population Lee-Carter model already by Lee and Carter([Lee & Carter, 1992]).

$$\log m_{x,t}^{(i)} = \alpha_x^{(i)} + \beta_x^{(i)}\kappa_t^{(P)} \tag{5.6}$$

In this model the time index driving the mortality change is the same for all the sub-populations while the age-specific mortality pattern and the age-specific responses to changes in the level of mortality are population specific. Assuming this model, mortality improvements between populations are not equal (due to different value of $\beta_x^{(i)}$) but perfectly correlated (see [Villegas & al., 2005]). In the following we denote this model as JK model.

The fourth model considered is a simplified version of the Common-Age-Effect (CAE) model proposed by Kleinow ([Kleinow, 2015]).

$$\log m_{x,t}^{(i)} = \alpha_x^{(i)} + \beta_x^{(P)}\kappa_t^{(i)} \tag{5.7}$$

In this model the age-specific responses to changes in the level of mortality is the same for all the sub-populations while the age-specific mortality pattern and the time index driving the mortality change are population specific. Note that assuming this model, mortality improvements between populations are not perfectly correlated. In the following we denote this model as CAE model.

The fifth model considered is the so-called augmented common factor model proposed by Li and Lee ([Li & Lee, 2005]).

$$\log m_{x,t}^{(i)} = \alpha_x^{(i)} + \beta_x^{(P)}\kappa_t^{(P)} + \beta_x^{(i)}\kappa_t^{(i)} \tag{5.8}$$

In this model the term $\beta_x^{(i)}\kappa_t^{(i)}$ represents deviations in mortality evolution of the sub-population i from the general common trend represented by $\beta_x^{(P)}\kappa_t^{(P)}$. In the following we denote this model as ACF model.

The last approach considered is to use independent Lee–Carter models for each sub-population:

$$\log m_{x,t}^{(i)} = \alpha_x^{(i)} + \beta_x^{(i)}\kappa_t^{(i)} \tag{5.9}$$

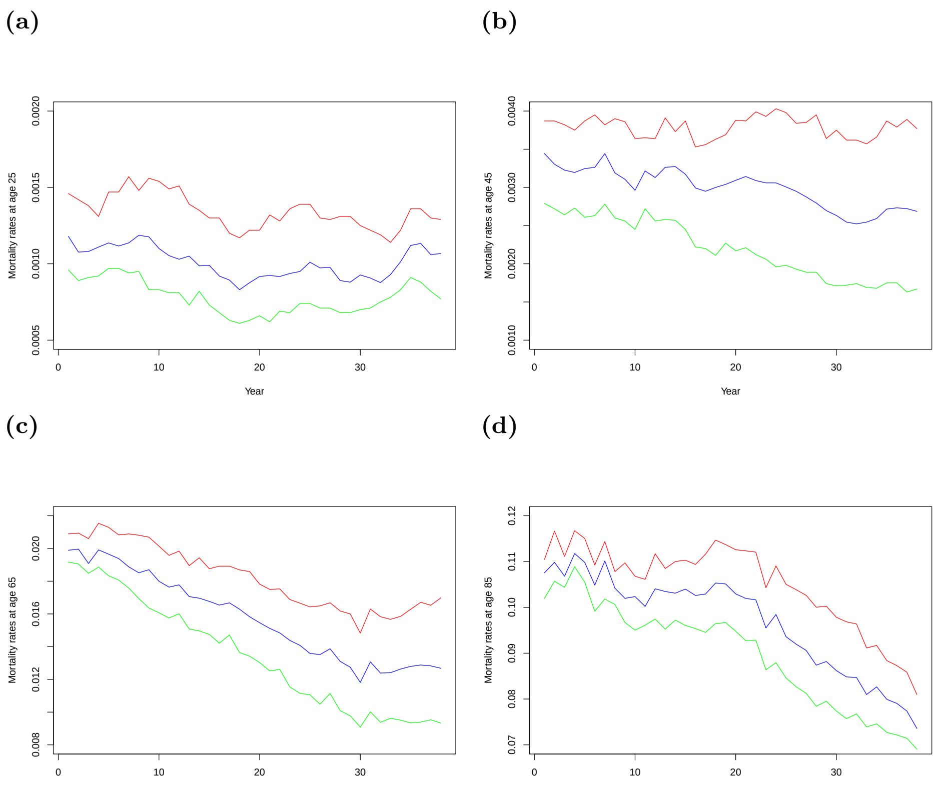
Note that under this approach the dependence between sub-populations can be represent by appropriately modelling the processes followed by the time indices $\kappa_t^{(i)}$. In the following we denote this model as ILC model.

5.2.2 Mortality data and model comparison

The six mortality models previously defined have been compared on mortality data by socio-economic category in the United States. Data are taken from a study realized by Barbieri and published by the Society of Actuaries (SOA) in 2021 (see [Barbieri, 2021]) that presents mortality rate estimates for the United States by year (from 1982 to 2019) separately by socio-economic quintile and decile. The mortality indicators are obtained "for groupings of U.S. counties based on their

socio-economic characteristics as measured by county-wide variables on education, occupation, employment, income and housing price and quality."([Barbieri, 2021]) In our application, we take the data by quantile and we group the quantiles from 2 to 4 in a single class so that 3 socio-economic classes are considered: the first class coincides with the first socio-economic quantile, in the second class there are the three socio-economic quintiles from the second to the fourth and the third class coincides with the fifth socio-economic quantile. Since we are interested in the mortality of people enrolled in a pension system we do not consider the mortality at young ages and we restrict our analysis to the age range 25-100. We fit the models on the total population.
The following figure shows the evolution over time of the observed mortality rates for the 3 considered classes at 4 ages.

Figure 5.1: Mortality rates by socio-economic classes (in red the first class, in blue the second, in green the third) at different ages.



We fit by maximum likelihood estimation (MLE) the six models described in subsection 5.2.1 on the mortality data previously described. The estimated parameters are reported in the appendix. In order to compare the models, since their number of parameters is different, we consider both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) with:

$$AIC = L - \nu$$
$$BIC = L - 0.5 \cdot \nu \cdot log(K)$$

where L is the log-likelihood and ν the number of parameters of the model while K is the number of data points. The log-likelihood, the AIC and the BIC for each model are reported in the following table.

Table 5.1: Log-likelihood and the BIC for the six mortality models (in bold the best values).

Model	CLC	CF	JK	CAE	ACF	ILC
Log-Like	-193,494	-82,518	-66,106	-68,979	**-55,495**	-65,383
AIC	-193,682	-82,934	-66,599	-69,396	**-56,171**	-67,940
BIC	-194,346	-84,404	-68,341	-70,869	**-58,559**	-67,940

We also perform a back-test analysis to compare the models. Specifically, we fit the models of the first 28 years data (from 1982 to 2009), then we forecast the death rates for the following 10 years and compare the forecasted values with the observed ones in the years 2010-2019. Mean error (ME), mean squared error (MSE), mean percentage error (MPE) and mean absolute percentage error (MAPE) are then determined to evaluate the goodness of the forecasts. The backtesting procedure as not applied to the 2 models that perform worse in terms of log-likelihood and BIC: the CLC and the CF models. In order to project the mortality rates, assumptions should be made on the processes followed by the time indexes. in particular:

- $\kappa_t^{(P)}$ is modelled as a random walk with drift (ARIMA(0,1,0)) in all the models;
- $\kappa_t^{(i)}$ is modelled as a random walk with drift (ARIMA(0,1,0)) in the CAE model and in the ILC model;
- in the ACF model we made two different assumptions for $\kappa_t^{(i)}$: in the first case it is modelled as a first-order autoregressive process (AR(1)), while in the second case it is modelled as an ARIMA(1,1,0) process.

The assumption of a random walk with drift process for $\kappa_t^{(P)}$ in all the models and $\kappa_t^{(i)}$ in the CAE and ILC models, is in line with the prevailing literature. With reference to $\kappa_t^{(i)}$ in the ACF model, Le and Lee ([Li & Lee, 2005]) suggest to assume

an AR(1) process in order to avoid possible divergence in the mortality evolution of the sub-populations. While it seems appropriate that a multi-population mortality model verifies this condition of no divergence in the mortality evolution of two distinct sub-groups of the same population, on the other hand increasing gaps in life expectancy between different socio-economic groups are observed in several countries: se e.g. [Mackenbach & al, 2003] for 6 Western European Countries including Finland, Sweden, Norway, Denmark, England/Wales, and Italy; [Cristia, 2009] for U.S.; [Villegas & al., 2005] for England; [Cairns & al., 2019] for Denmark). We therefore consider that limiting the analysis to the case of an AR(1) process alone is not appropriate. Moreover, the Augmented Dickey-Fuller test show that the $\kappa_t^{(i)}$ series in the ACF model is not stationary for the first and the third socio-economic groups and the auto.arima function in R suggests second-order ARIMA models for that series. Otherwise, [Haberman & Renshaw, 2009] suggests avoid the use of second order ARIMA(p,2,q) processes, because they could produce excessively wide prediction intervals. In light of these considerations, we found it useful to consider as an alternative hypothesis to the AR(1) process that of an ARIMA(1,1,0) process.
ME, MSE, MPE and MAPE for the five models tested are reported in the following table.

Table 5.2: ME, MSE, MPE and MAPE in backtesting (in bold the best values).

Model	JK	CAE	ACF.0	ACF.1	ILC
ME	-0.003560418	-0.003045175	-0.003147554	**-0.002560360**	-0.004690399
MSE	1.073900e-04	8.631294e-05	1.021048e-04	**8.547876e-05**	2.660421e-04
MPE	0.04356787	0.03288820	0.04304839	0.04646680	**0.02945959**
MAPE	0.09861886	0.11568775	0.09642061	**0.09477354**	0.13504576

The ACF.1 with the sub-population-specific time index modeled as a random walk with drift is the best mortality model for the populations considered for ME, MSE, and MAPE measures. while the ILC model is the best for the MPE one.

5.3 Risk analysis of Automatic Adjustment mechanisms

In this section, we conduct a comprehensive analysis of the Progressive Defined Musgrave pension system defined the previous chapter, focusing on its mean evolution and risk characteristics. We examine the contribution rate, the pension rate, and the class replacement rate to gain insights into the system's dynamics. Additionally, we assess the variability introduced by the longevity heterogeneity correction using multi-population models for socio-economic group mortality. Through Value-at-Risk (VaR) measurements, we analyze the potential variations in the correction at different probability levels and for different socio-economic classes. By visualizing density distributions and VaR values, we provide a comprehensive view of the system's risk exposure over short and long projection horizons.

Furthermore, we explore the relationship between the contribution rate, the mean benefit ratio, and the longevity heterogeneity correction. These factors, influenced by the dependency ratio, play a crucial role in the system's financial implications for retirees and contributors. We analyze their evolution over time and present their distribution after a 20-year projection period.

Moreover, we investigate the class replacement rate, a key indicator of the pension system's effectiveness in providing post-retirement income. By comparing the class replacement rates for different socio-economic classes under two alternative mortality forecast models (ACF and CAE[1]), we illustrate the system's resilience and adaptability, independent of the specific mortality model used.

Overall, our analysis aims to highlight the importance of considering risk measurement and variability in pension systems alongside mean evolution. By examining various risk indicators and projection scenarios, we contribute to a deeper understanding of the Progressive Defined Musgrave system's implications for different stakeholders.

5.3.1 Double AAM for Aging and longevity heterogeneity

We selected the ACF.1 model as the main model in order to explain the mortality of the sub-populations. The fitted model allows us to forecast the central death rates in each class over a period of 20 years. We construct the sub-population periodic mortality tables from the mortality rates. The 1 year survival probabilities used in the population effective function are obtained via:

$$ {}_kp_x^{(i)}(t) = \exp\,(\int_x^{x+k} m_{x+s,t}^{(i)} ds) \tag{5.10}$$

We kept the same socio-economic classification as section 5.2. The total population is composed of three socio-economic classes ranked by salary level. The corresponding salary for each quintile is obtained via the US census bureau [US Census Bureau (2023)]. The historical data stop in 2019, we summarize the system parameters at that period. The contribution rate, dependency ratio and pension rate are obtained from Chapter 4:

- The contribution rate $\pi_{2019} = 0.3667$ and pension rate $\delta_{2019} = 0.5435$
- The dependency ratio $D_{2019} = 0.4208$

[1] We opted for the CAE model to compare to the ACF model because it was the second best predicting model while having significant differences to the ACF models

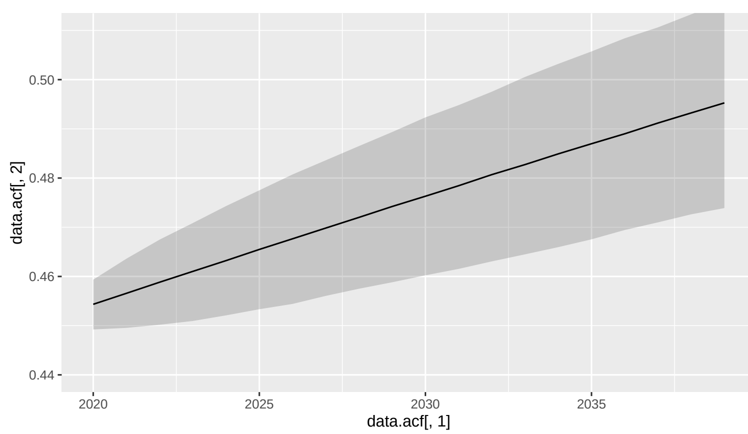
- The longevity heterogeneity correction in each class $\theta_1^{2019} = 1.0667$, $\theta_2^{2019} = 1.0007$ and $\theta_3^{2019} = 0.9454$

We present some results on the pension system.

- **Dependency ratio**

The effective number of active workers and retirees is a consequence of the population equation. The evolution of the dependency ratio is subject to the projection of mortality intensities forecasted by the ACF model. Here is the forecasted dependency ratio:

Figure 5.2: Forecast of the dependency ratio ACF model

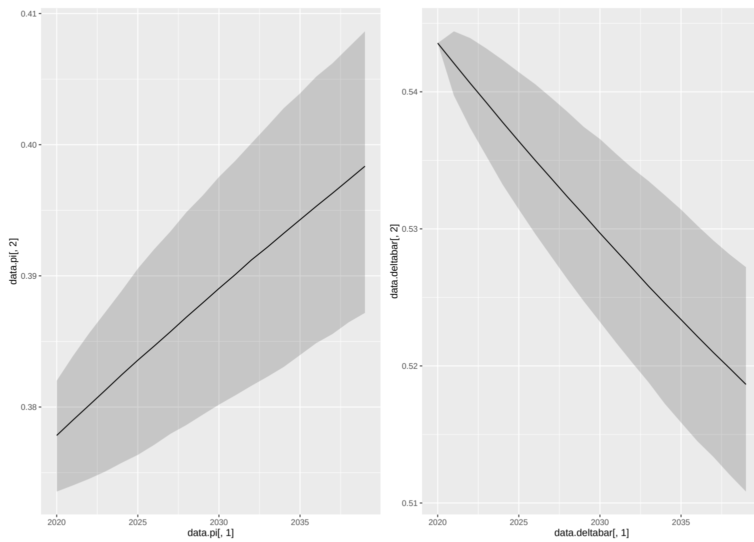


The dependency ratio is expected to increase over the projection horizon. This increase is driven by the mortality evolution as well as the constant entry function hypothesis. The greyed-out area limits represent the 2.5% and 97.5% quantiles of the dependency ratio distribution. Regardless of the evolution of mortality in the sub-populations, the dependency ratio is not very volatile; this is also a consequence of the demographic stability assumption. This has a direct effect on the variations in the inter-generational risk sharing parameters.

- Contribution rate and pension rate

The contribution and pension rates are both functions of the dependency ratio via 4.13 and 4.14.

Figure 5.3: Forecast of the contribution rate and pension rate ACF model



The contribution rate and pension rate describe the yearly cost of the system.They are driven by the evolution of the number of active workers and retirees. Here is their average value through the forecast.

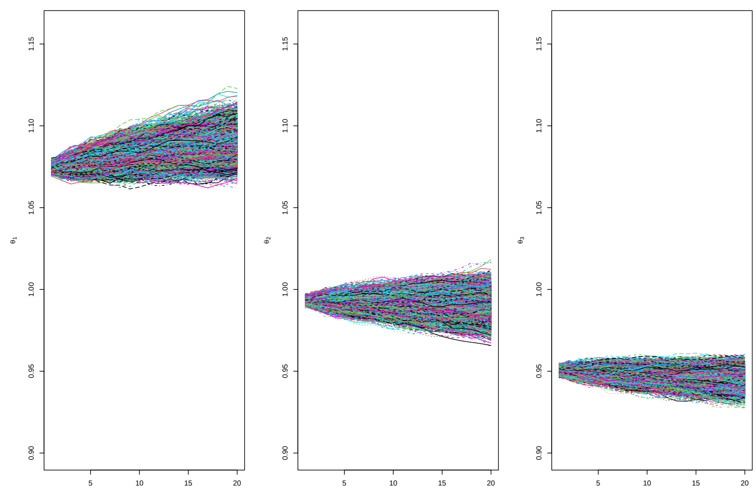
Table 5.3: Average contribution rate and pension rate evolution ACF model

t	π_t	δ_t
2020	0.3778363	0.5435501
2039	0.3982966	0.518698

- Longevity heterogeneity correction

We present the forecasted evolution of θ_j^t ,the longevity heterogeneity correction factor in each class

Figure 5.4: Forecast of the longevity heterogeneity correction ACF model



Each path represents a projection of the mortality intensity used in computing the longevity heterogeneity correction. The following table summarizes the average path in each class.

Table 5.4: Evolution of the average longevity heterogeneity correction in each class ACF model

	θ_1	θ_2	θ_3
2020	1.074335	0.993162	0.951015
2039	1.089464	0.990076	0.945154

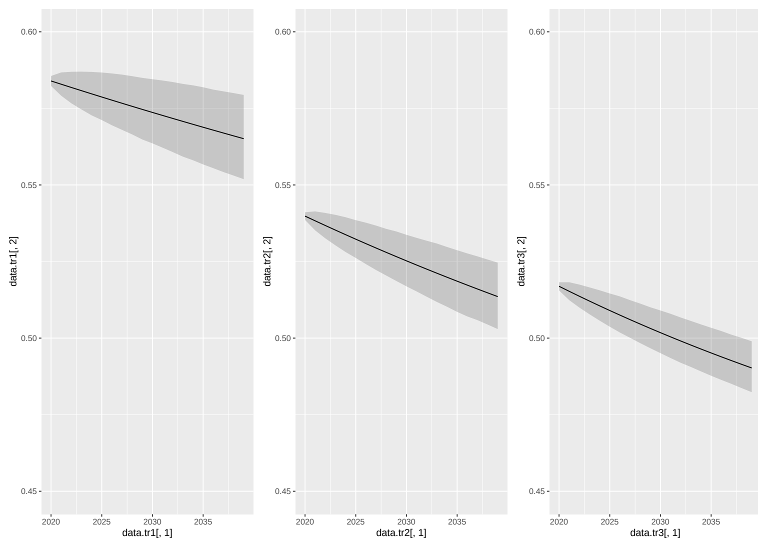
The mortality intensity model has more influence on the variations of the longevity heterogeneity correction than on the dependency ratio

- Class replacement rate

The class replacement rate at retirement for an agent of class i is defined as the ratio between the pension benefit and the last working salary:

$$Tr_i^{(x_r,t)} = \frac{P_i^{(x_r,t)}}{S_i^{(x_r,t)}} = \delta_{t_r} \cdot \theta_i^{t_r} \tag{5.11}$$

Figure 5.5: Forecast of the class replacement rate ACF model



The class replacement rate at retirement of each sub-population combines the two adjusting effects. Its distribution is directly influenced by the mortality model selected for the forecast, thus we are going to compare at the horizon of projection the average class replacement rates in each class for the ACF model.

Table 5.5: Average class replacement rate at 65

	ACF		
	$Tr_1^{(65,t)}$	$Tr_2^{(65,t)}$	$Tr_3^{(65,t)}$
2020	0.583955	0.539833	0.516924
2039	0.565103	0.513551	0.490249

In order to analyse the mortality model effect on the class replacement rate we are looking into the class replacement rate differences between two classes i and j presented as such :

$$Tr_i^{(x_r,t)} - Tr_j^{(x_r,t)} = \delta_t \cdot (\theta_i^t - \theta_j^t) \tag{5.12}$$

$$Tr_i^{(x_r,t)} - Tr_j^{(x_r,t)} = \delta_t \cdot d_{i,j}^t, \tag{5.13}$$

with $d_{i,j}^t = \theta_i^t - \theta_j^t$ measuring the difference in longevity heterogeneity correction between the two classes. we present the evolution of the differences between our three classes in the ACF model.

Figure 5.6: Forecast of the longevity heterogeneity correction distances ACF model

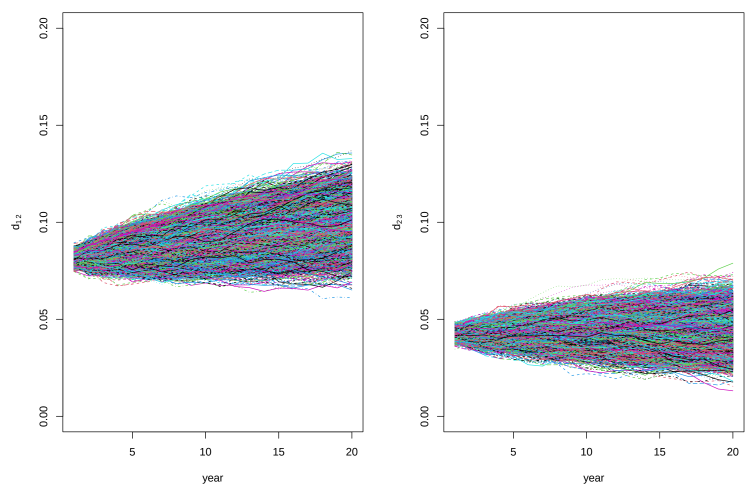


Figure 5.6 shows the projected difference in longevity heterogeneity correction between socio-economic class 1 and class 2 and between class 2 and class 3 in the ACF model. The trend for the low income class is that the difference is widening over time , meaning that the progressive nature of the pension system is becoming more pronounced. The differences for the high income group relative to the average group do not seem to be increasing.

5.3.2 Quantile analysis

The Progressive Defined Musgrave system is characterized every period t by the triplet $(\pi_t; \delta_t; \theta_t)$. In this stochastic situation, it becomes appropriate to examine not only mean evolution but also the risk measurement of three processes: the contribution rate ,the pension rate and the class replacement rates. More specifically, we wish to analyze and monitor the level of risk imposed by a progressive pension formula, as well as compare risk exposure for the two stakeholders (retirees and contributors).

- **Contribution rate and pension rate**

In the context of socio-economic group mortality, it is crucial to consider the variability introduced by multi-population models when estimating the contribution and pension rates. This variability directly impacts the future evolution of these rates and, consequently, the determination of pension benefits. Therefore, the objective of this analysis is to examine the potential variations in the contribution and pension rates by comparing the Value-at-Risk (VaR) at a specified yearly confidence level of $u = 0.995$.

To address this objective, we adopt a comprehensive perspective that accounts for the long-term nature of our projections. Following the Solvency 2 approach, which typically considers a one-year period, we extend our analysis to capture the long-term characteristics of the contribution and pension rates. We employ the maturity approach, as presented by [Devolder & Lebègue, 2016], which has also been incorporated within the SII (Solvency 2) legislation through the duration-based equity risk sub-module. This approach is particularly suited for the analysis of assets and liabilities with prolonged time horizons and distinctive characteristics.

In our analysis, parameter quantiles are computed employing a risk measure, while the confidence level employed is regarded as a probability, consistent with the Probabilistic criterion proposed by Hürlimann [Hürlimann, 2010]. To capture the projection period, we establish an integer $T > 0$ to denote the projection horizon, thus providing a framework for defining the adjusted quantiles of the parameters.

The corrected quantiles at maturity represent the probability that the longevity correction does not exceed a particular quantile, adjusted based on a confidence level of 0.995 raised to the power of T. This approach enables us to explore the potential variability in the longevity correction over time and its consequential impact on the determination of pension benefits.

Through an examination of the VaR at a confidence level of 0.995^T, we obtain valuable insights into the potential range of outcomes and associated risk levels in the contribution and pension rate. This analytical framework contributes to a deeper understanding of the implications of employing multi-population models in assessing fairness and stability within pension systems. By illuminating the potential variations in the correction and their implications for future pension benefits, this analysis aids in informing the design of pension systems that are more equitable and robust. The variability of the population's effective poses a financial burden on contributors and retirees. Contributors are sensitive to an increase in the contribution rate, and retirees are sensitive to a decrease of the pension rate. Through the projection, we evaluate the 5-year and the 20-year Value-at-Risk of the contribution rate via

$$VaR_{u^T}(\pi_T) = \inf_l \{\mathbb{P}[\pi_T \leq l] > u^T\} \tag{5.14}$$

Here the 1-year confidence level is $u = 0.995$

Table 5.6: Value-at-risk evolution for the contribution rate ACF model

$VaR_{0.975}(\pi_5)$	$VaR_{0.90}(\pi_{20})$
0.3888	0.4051

We observe that the 20 years VaR of the contribution rate deviates from the average forecast (Table 5.3) by 0.5%. This highlight the fact that the demographic stability hypothesis limit the impact of the mortality model variations on the increase of the contribution rate on active workers.

For the pension rate, we are looking at scenarios corresponding to a loss for the retirees. We evaluate the 5-year and the 20-year Value-at-Risk of the pension rate rate via :

$$VaR_{1-u^T}(\delta_T) = \inf_l \{\mathbb{P}[\delta_T \leq l] < (1 - u^T)\} \tag{5.15}$$

Table 5.7: Value-at-risk evolution for the pension rate

$VaR_{0.025}(\delta_5)$	$VaR_{0.1}(\delta_{20})$
0.5332	0.5133

We observe that the 20 years VaR of the pension rate deviates from the average forecast (Table 5.3) by 0.5%. This highlight the fact that the demographic stability hypothesis limit the impact of the mortality model variations on the increase of the pension rate on retirees.

- **Longevity heterogeneity correction**

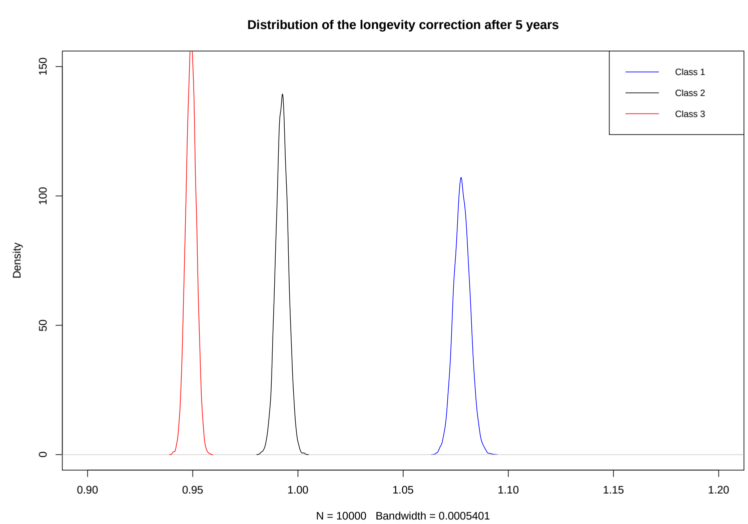
The variations of the longevity heterogeneity correction both on the left and right tail of the distribution affect the pension benefit level within all classes. This part will focus on the quantiles ($Q_{u^T/2}$ and $Q_{1-u^T/2}$)of their distribution for a probability level u^T given by the VaR adjusted confidence level.

Table 5.8: 0.025 and 0.975 Quantiles of the longevity correction ACF model

	$\theta_1(5)$	$\theta_2(5)$	$\theta_3(5)$
$Q_{0.025}$	1.071888	0.9907212	0.9437043
$Q_{0.975}$	1.085261	0.9983154	0.9544127

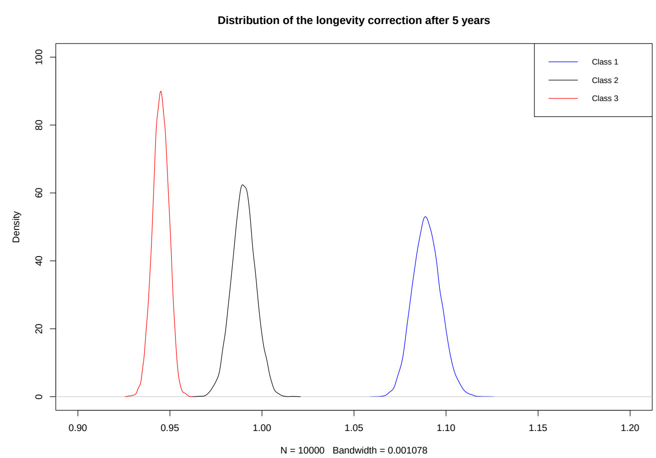
Table 5.8 shows the 0.025 and 0.975 quantiles of the longevity correction in the ACF model for three socio-economic classes: low-income, average and high-income. The inter-quantile range (difference between the quantiles of the distribution) in the low income group is more important. This means that the distribution of longevity heterogeneity is more spread out for the low-income class than for the average and high-income classes. We visualise the distribution of the longevity heterogeneity correction after 5 years for each group as well as the inter-quantile range in the following figure :

Figure 5.7: Distribution of the longevity heterogeneity correction after 5 years ACF model



In the PDM system, the variations in the longevity correction are more important for the low-income group, as the longevity gains in this group are less important relative to the total population. This translates into a better correction for the low-income group. At the projection horizon we present the distribution of correction in order to evaluate the evolution of the inter-quantile range order within each class :

Figure 5.8: Distribution of the longevity heterogeneity correction after 20 years, ACF model



Although the low-income group correction is more likely to vary in time, the magnitude of variation is not significantly more important that the correction in the other two classes at the projection horizon.

The quantile analysis also focuses on the distribution of longevity heterogeneity correction differences. We presented in figure 5.6 that over time the evolution differed between low and high income classes. We present in the following the distribution of the differences at the horizon of projection:

We observe from the evolutions of the distances in longevity heterogeneity correction between classes that the differences in longevity correction between the low income group and the total population increase faster than the differences between the high income group and the total population. For a given mortality model it is interesting to evaluate the consequences on the class replacement rate distribution.

- **Class replacement rate**

The class replacement rate derives from the product of the pension rate and the longevity heterogeneity correction for a given class, its distribution therefore is affected by the variations of the two adjustment mechanisms proposed, the following figure displays the distribution of the class replacement rate.

Figure 5.9: Longevity heterogeneity distance evolution ACF model

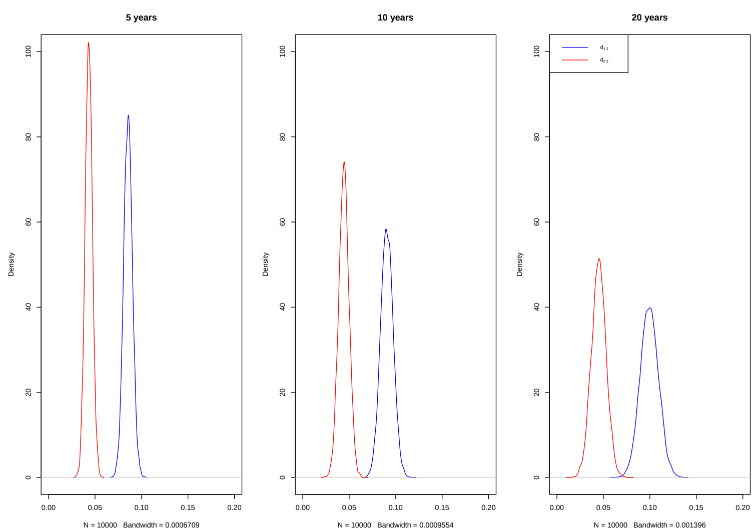
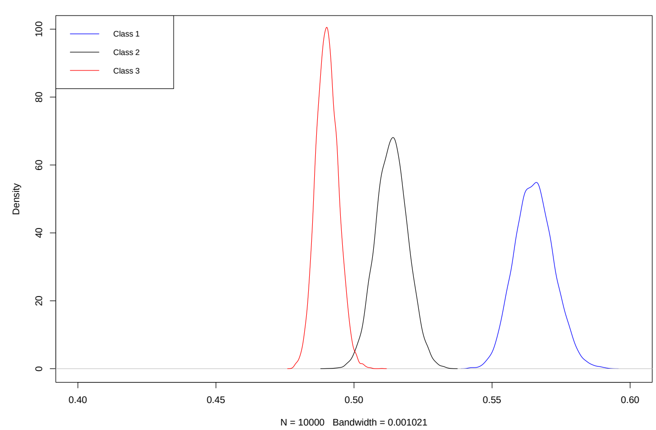


Figure 5.10: Class replacement rate distribution after 20 years, ACF model



When comparing the figure 5.8 and figure 5.10 we observe that the mode of each distribution are similar due to the proportionality between the longevity correction and the class replacement rate.

At the projection horizon we are also interested in looking into the inter-decile gap[2] within each class

Table 5.9: interdecile gap in the ACF model at the projection horizon

	$Q_{0.1}$	$Q_{0.9}$	$Q_{0.9} - Q_{0.1}$
$Tr_1^{(65,2039)}$	0.56366	0.58374	2.01%
$Tr_2^{(65,2039)}$	0.51406	0.53822	2.42%
$Tr_3^{(65,2039)}$	0.49121	0.51498	2.38%

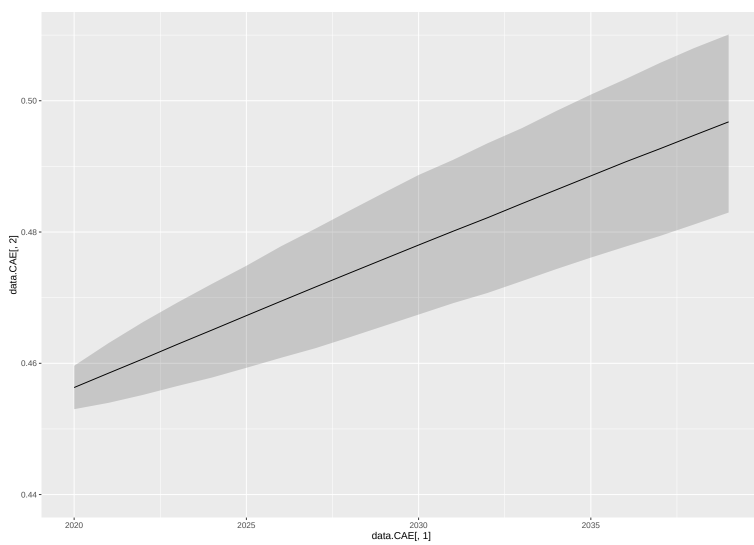
It is notable that the inter-decile gap does not increase for the class replacement rate, its order of magnitude is the same for every class.

5.3.3 Model comparison

We are going to compare the demographic projections for the two selected models (ACF and CAE) as well as the pension system parameters.

- **Dependency ratio**

Figure 5.11: Forecast of the dependency ratio CAE model



The comparison of the two mortality models, ACF and CAE, reveals that the demographic stability assumption restricts significant variability in the evolution

[2]The probability level is adjusted according to the horizon, $1 - 0.995^{20} \approx 0.1$

	ACF	CAE
Year	D_t	D_t
2020	0.454344	0.45631
2039	0.495271	0.496785

of the dependency ratio.The central forecasts exhibit similar trends due to their adherence to the same demographic hypothesis. As a result, the impact on the dependency ratio remains relatively consistent between the two models.

- **Contribution rate and pension rate**

Figure 5.12: Forecast of the contribution rate and pension rate CAE model

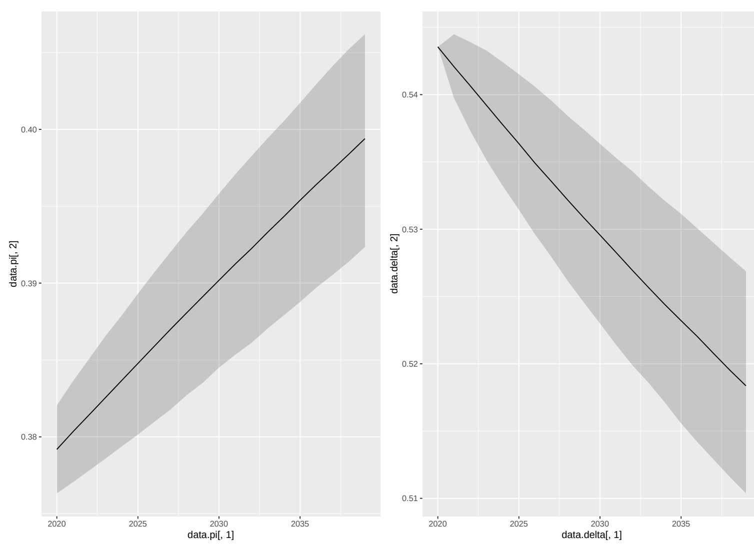


Table 5.10: Average contribution rate and pension rate comparison

	ACF		CAE	
Year	π_t	δ_t	π_t	δ_t
2020	0.377836	0.54355	0.377836	0.54355
2039	0.398297	0.518698	0.393458	0.52445

From the comparison of the contribution rate and pension rate in the central forecast between the ACF and CAE models, we observe that both models exhibit similar values. This suggests that the evolution of the dependency ratio has a direct impact on the contribution and pension rates, leading to comparable results in their central forecasts. We will display the 20 year quantile for the contribution and pension rate to compare their distribution

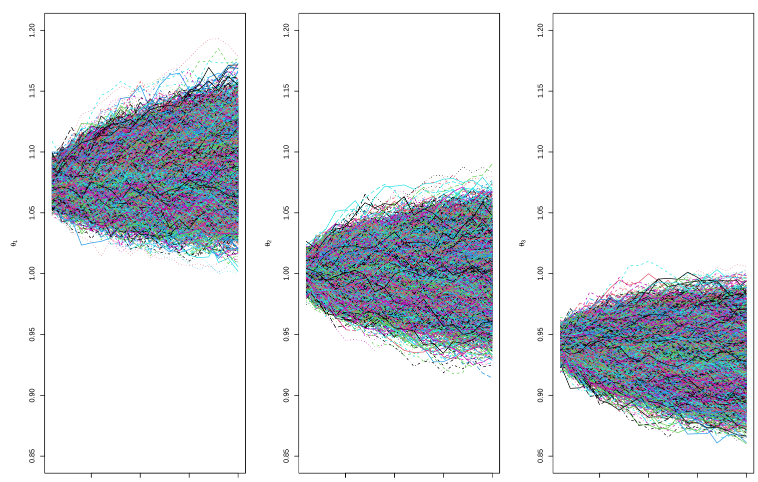
Table 5.11: 20-year quantiles for contribution and pension rate

	ACF		CAE	
	π_t	δ_t	π_t	δ_t
$Q_{0.025}$	0.387157	0.510733	0.392397	0.510298
$Q_{0.975}$	0.408405	0.527212	0.406252	0.526709

To gain further insights into the distribution of the contribution and pension rates, we analyze the 20-year quantiles for both variables. The quantiles show that the contribution and pension rates remain relatively close in terms of their distribution between the ACF and CAE models. The 20-year quantiles indicate that the models' predictions for these rates are within a narrow range, reinforcing the consistency between their forecasts.

- **Longevity heterogeneity correction**

Figure 5.13: Forecast of the longevity heterogeneity correction CAE model



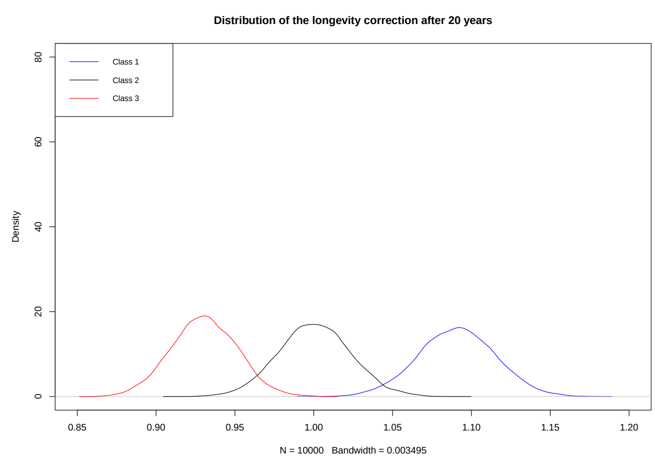
The forecast of the longevity heterogeneity correction in the CAE model, as shown in Figure 5.13, reveals a noticeable dispersion compared to the ACF model. Unlike the ACF model, the simulated paths of the correction in the CAE model appear to overlap at the projection horizon, indicating potential variations in the correction for each socio-economic class.

To further investigate these variations, we analyze the 20-year quantiles of the longevity correction for the CAE model, as presented in Table 5.12. The $Q_{0.05}$ and $Q_{0.95}$ quantile values provide insights into the potential range of losses or gains in the correction for each socio-economic class after 20 years. The quantile analysis highlights the possibility of different correction outcomes for different income classes, which may lead to less advantageous corrections for the average income population compared to the high-income class, and vice versa.

Table 5.12: 20-year Quantiles of the longevity correction CAE model

	$\theta_1(20)$	$\theta_2(20)$	$\theta_3(20)$
$Q_{0.05}$	1.076929	0.9782042	0.9357285
$Q_{0.95}$	1.103535	1.002691	0.9536671

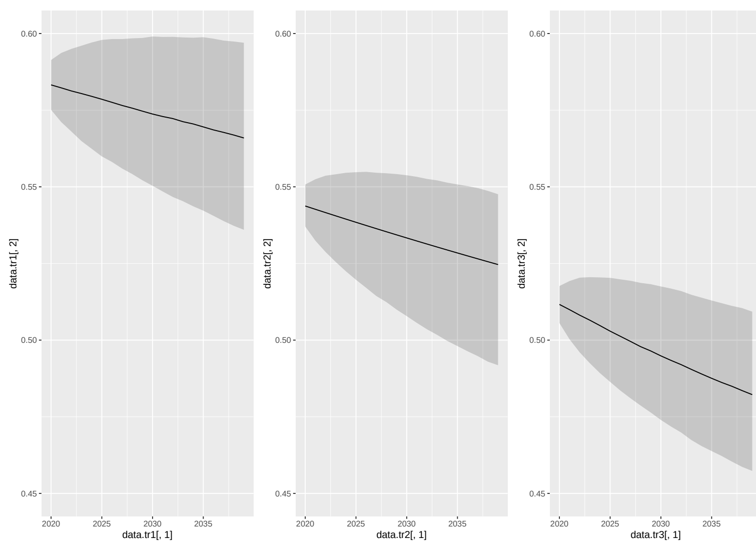
Figure 5.14: Distribution of the longevity heterogeneity correction after 20 years CAE model



The distribution of the longevity heterogeneity correction after 20 years, as depicted above, further illustrates the overlapping nature of the correction. This observation indicates that there is a non-negligible probability that the longevity correction order can change between two socio-economic classes. As a result, the CAE model introduces the potential for different socio-economic classes to experience varying levels of correction, with the average-income class potentially receiving less advantageous corrections than the high-income class and, conversely, more advantageous corrections compared to the low-income class. While the progressive transformation offers a more flexible approach to correcting longevity disparities, it also introduces the possibility of variations in the correction across different income classes depending on the mortality model selected.

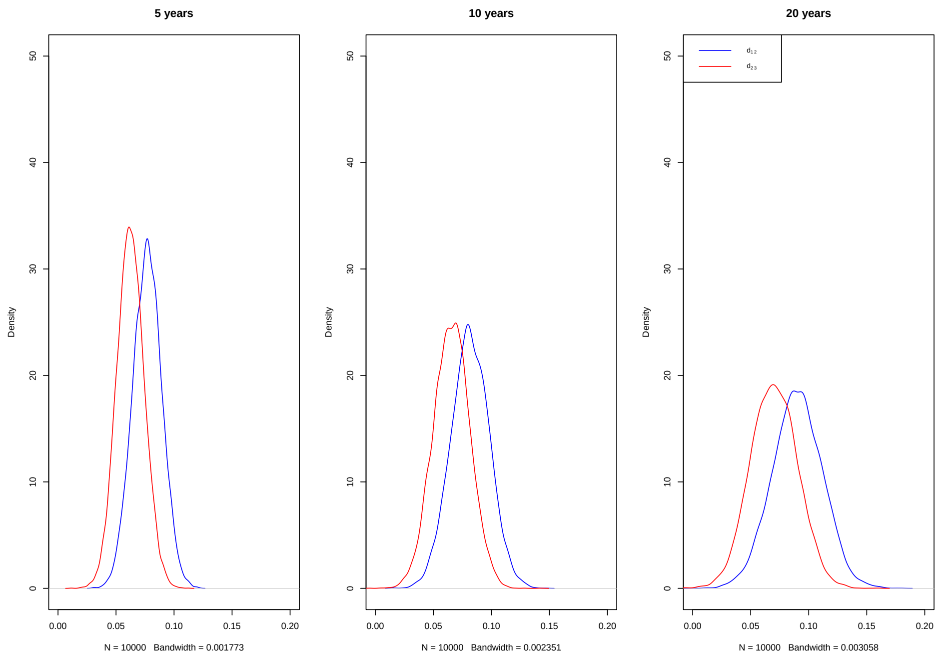
- **Class replacement rate**

Figure 5.15: Forecast of the class replacement rate CAE model



The CAE model underestimates the mortality for the low-income population relative to the total population and overestimates the mortality for the high-income population compared to the ACF model. Even though the average class replacement rate projected by each model is progressive in the salary class order. When observing the differences in class replacement rate projected in the CAE model in figure 5.16, the distances are not as pronounced for the low income group compared to the high income group

Figure 5.16: Class replacement rate distance distribution after 20 years, CAE model



This also indicates that the longevity gains combined with the evolution of the pension rate lead to a decrease of the distances within class replacement rate over time in the CAE model .

Table 5.13: interdecile gap in the CAE model at the projection horizon

	$Q_{0.1}$	$Q_{0.9}$	$Q_{0.9} - Q_{0.1}$
$Tr_1^{(65,2039)}$	0.55715	0.58963	3.25%
$Tr_2^{(65,2039)}$	0.51284	0.54661	3.38%
$Tr_3^{(65,2039)}$	0.4774537	0.51282	3.54%

The interdecile gap in the CAE model at the projection horizon is greater that the ACF model . This confirms the more important variability already observed in the longevity heterogeneity correction, combining this to the overlap in the distribution comforts the ACF model selection.

5.4 Discussion

- **Contribution and pension rate**

  The numerical application of the contribution rate and pension rate reveals important insights about the financial dynamics within the pension system. The contribution rate is expected to increase in the ACF average forecast slightly from 37.8% in 2020 to 39.8% by 2039, highlighting the growing financial burden on contributors over time. Similarly, the pension rate is projected to decrease from 54.4% to 51.9% by 2039, reflecting the sensitivity of retirees to changes in the pension rate and its potential impact on post-retirement income.

  However, it is worth noting that the contribution rate and pension rate are less sensitive to the model induced variations due to the evolution of the dependency ratio and the demographic assumptions. At the projection horizon, the difference between the value-at-Risk at confidence level 0.9 and the average projection is less than 1%. This suggests that the pension system's financial indicators are relatively stable and less susceptible to variations in demographic factors.

  Nevertheless, the variability of these rates over time poses financial challenges for both contributors and retirees. The analysis of VaR at different confidence levels provides insights into the potential range of outcomes and associated risk levels. These VaR analyses help us understand the potential financial risks and uncertainties associated with the contribution rate and pension rate within the pension system. They provide valuable information for policymakers and stakeholders in designing and managing a more robust pension system.

- **Longevity heterogeneity correction and class replacement rate**

  The variations in the longevity heterogeneity correction, particularly on the left and right tails of the distribution, have implications for the pension benefit levels across all classes. We focus on the quantiles ($Q_{u^T/2}$ and $Q_{1-u^T/2}$) of the correction distribution for a given probability level u^T, adjusted based on the VaR-adjusted confidence level. Additionally, we examine the absence of overlap in corrections between different socio-economic classes, particularly in worst-case scenarios. The tables and figures present the quantiles and distributions of the longevity heterogeneity correction, highlighting the potential variations in the correction for each socio-economic class and the non-overlapping nature of corrections between classes. We observed that the longevity heterogeneity correction is directly affected by the choice of mortality model. The ACF model, which we selected, shows limited overlaps in the distribution of the correction, indicating a progressive aspect of the class replacement rates. On the other hand, when comparing the results with the CAE model, the overlaps in the distribution become more evident. This highlights the risk associated with distribution overlap, as it leads to a loss of the progressive nature of the class replacement rates.

The variations in the longevity heterogeneity correction have important implications for the pension system. They can affect the benefit levels for different socio-economic classes, potentially leading to disparities in pension outcomes. By examining the quantiles and distributions of the correction, we gain a better understanding of the potential range of corrections and their impact on different population segments.

Similarly, the class replacement rate, which represents the replacement of income at retirement for each sub-population, is influenced by both the mortality model chosen and the adjustments made. By comparing the class replacement rates projected by the ACF and CAE models, we can assess the effects of different mortality assumptions on the retirement income for each socio-economic class.

Analyzing the distribution of class replacement rates at the projection horizon provides insights into the fairness and variability within the pension system. The interdecile gaps in the distributions indicate the potential range and spread of class replacement rates, reflecting the disparities that individuals may face in their retirement benefits.

Those observations underscore the importance of carefully selecting the mortality model and considering the implications of the longevity heterogeneity correction. It highlights the need for equitable pension systems that account for variations in longevity and socio-economic factors while ensuring fair and progressive class replacement rates for retirees.

5.5 Conclusion

In conclusion,this paper presents a comprehensive and insightful exploration of pension schemes with double Automatic Adjustment Mechanisms (AAM) and multi-population mortality modelling. The underlying motivation of this study lies in the pressing need to address socio-economic disparities in longevity, model it and ensure fairness within pension systems.

Throughout this chapter, we have looked into the principles and motivations guiding our investigation, emphasizing the significance of considering diverse demographic groups when designing pension systems. By accounting for socio-economic differences in longevity, we strive to create pension schemes that promote equitable benefit distribution and enhance overall societal well-being. The examination of pension schemes with double AAM sheds light on their adaptive nature and flexibility. These mechanisms play a pivotal role in responding effectively to demographic changes, ensuring the long-term sustainability and resilience of pension systems.

By embracing dynamic adjustments, we create systems that are better equipped to accommodate an ageing population with varying needs.

Additionally, the paper's exploration of multi-population mortality modelling has provided a robust framework for understanding the complexities of mortality patterns across diverse populations. This nuanced approach allows us to gain valuable insights into future mortality disparities, enabling more informed decision-making in pension design.

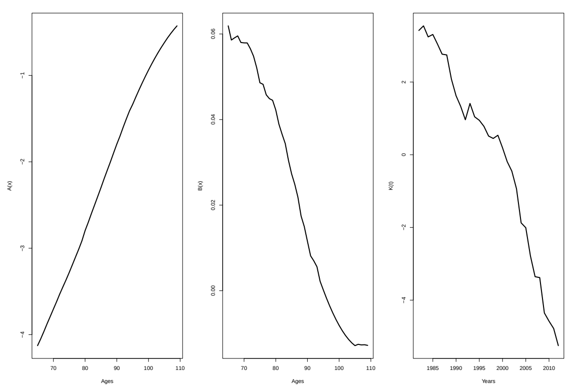
The risk analysis of Automatic Adjustment Mechanisms has been a crucial aspect of this chapter. By employing Value-at-Risk analysis, we have assessed potential risks and uncertainties associated with these mechanisms. This evaluation offers valuable insights for policymakers and stakeholders, providing a comprehensive perspective on the potential impacts of the proposed pension design approaches.

Appendix

We display the figures for the estimated parameters of each model, in the age range of [65 : 110] and the time period it [1982 : 2019]

- Lee carter estimates on the total population

Figure 5.17: $\alpha; \beta; \kappa$ LC estimate on the total population



- ACF estimates

Figure 5.18: κ estimate ACF

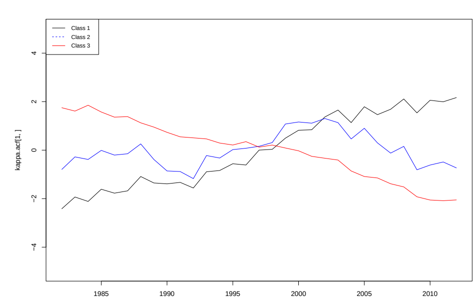


Figure 5.19: α estimate ACF

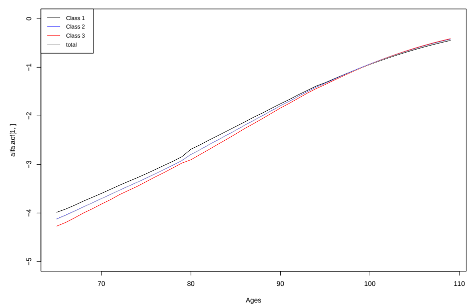
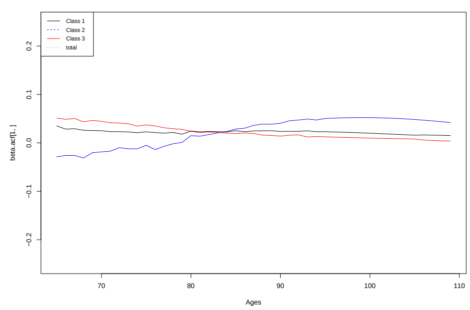


Figure 5.20: β estimate ACF



- Joint-Kappa estimates

Figure 5.21: α estimate JK

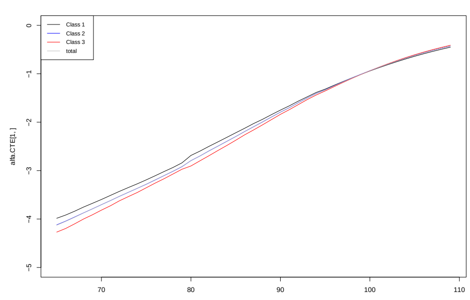
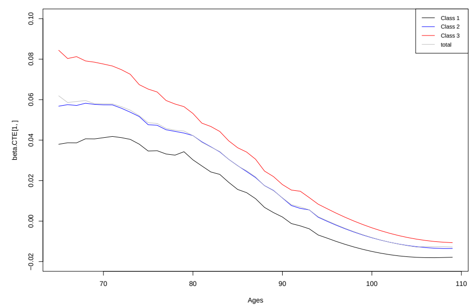


Figure 5.22: β estimate JK



- CAE estimates

Figure 5.23: α estimate CAE

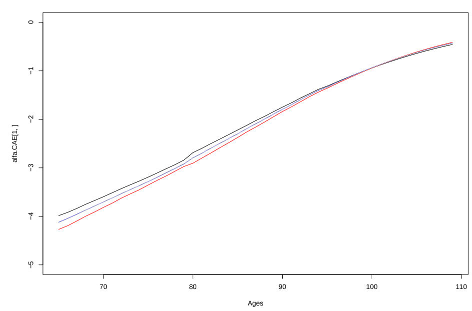
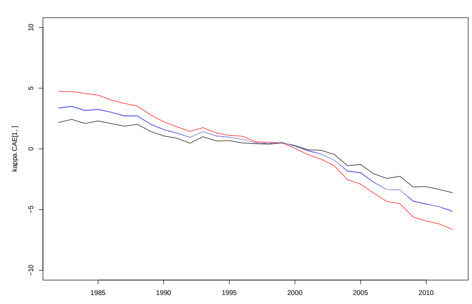


Figure 5.24: κ estimate CAE



- ILC estimates

Figure 5.25: α estimate ILC

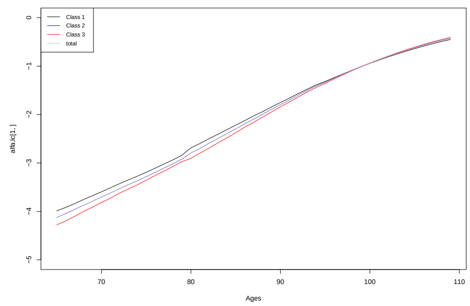


Figure 5.26: β estimate ILC

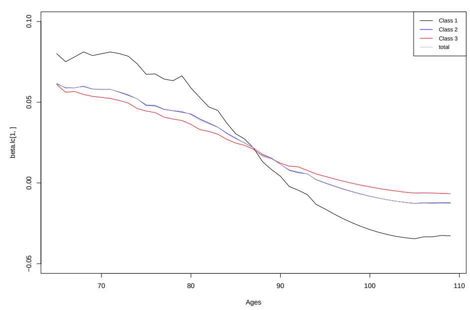
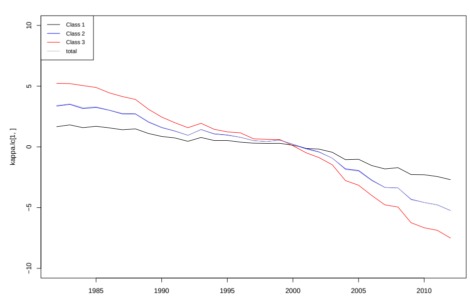


Figure 5.27: κ estimate ILC



Bibliography

[Antonio & al, 2015] Antonio, K., Bardoutsos, A., Ouburg, W.: Bayesian Poisson log-bilinear models for mortality projections with multiple populations. Eur. Actuar. J. 5(2), 245–281 (2015)

[Ardito & Costa, 2022] Ardito, C., & Costa, G. (2022). Could a fairer retirement age mitigate health inequalities? Evidence and decision-making. Frontiers in Public Health, 10, 965140.

[Arnold & Jijie, 2020] Arnold, Séverine, and Anca Jijiie. 2020. "Retirement Ages by Socio-Economic Class" Risks 8, no. 4: 102. https://doi.org/10.3390/risks8040102

[Balia & Jones, 2008] Balia, S., Jones, A.M. (2008). Mortality, lifestyle and socio-economic status. Journal of Health Economics, 27(1), 1–26.

[Barbieri, 2021] Barbieri, M., 2021. Mortality by socio-economic Category in the United States. SOA Research Institute.

[Boado-Penas et al., 2022] Boado-Penas, M., Haberman, S., Naka, P., 2022. Fairness and annuity divisors for notional defined contribution pension schemes. Journal of Pension Economics & Finance, 21(2), 143-167. doi:10.1017/S1474747220000311

[Brouhns & al., 2002] Brouhns, N., Denuit, M., Vermunt, J., 2002. A poisson log-bilinear regression approach to the construction of projected lifetables. *Insurance: Mathematics and Economics*, 31(3): 373–393

[Cairns & al., 2019] Cairns, A.J.G., Kallestrup-Lamb, M., Rosenskjold, C., Blake, D., Dowd, K. (2019). Modelling socio-economic differences in mortality using a new affluence index. ASTIN Bulletin, 49(3), 555–590.

[Cristia, 2009] Cristia, J.P. (2009). Rising mortality and life expectancy differentials by lifetime earnings in the United States. Journal of Health Economics, 28, 984–995.

[Delbon & al., 2011] Debón A, Montes F and Martínez-Ruiz F (2011) Statistical methods to compare mortality for a group with non-divergent populations: an application to Spanish regions. European Actuarial Journal 1, 291–308.

[Devolder & Lebègue, 2016] Devolder, P. & Lebègue, A. (2016), 'Iterated VaR or CTE measures: A false good idea?', *Scandinavian Actuarial Journal* pp. 1–32. DOI: 10.1080/03461238. 2015.1126343

[Devolder & Diakite , 2023] Diakite, K., & Devolder, P. (2023). Automatic Adjustment Mechanisms in Public Pension Schemes to Address Population Ageing and socio-economic Disparities in Longevity (No. UCL-Université Catholique de Louvain).

[Dudel & Schmied, 2019] Dudel, C., & Schmied, J. (2019). Pension adequacy standards: An empirical estimation strategy and results for the United States and Germany (MPIDR Working Paper No. WP 2019-003). Rostock, Germany: Max Planck Institute for Demographic Research.

[Forster & al, 2018] Forster, T., Kentikelenis, A., & Bambra, C. (2018). Health inequalities in Europe: setting the stage for progressive policy action.

[Haberman & Renshaw, 2009] Haberman, S., Renshaw, A. (2009). On Age-Period-Cohort Parametric Mortality Rate Projections. Insurance: Mathematics and Economics 45: 255–270.

[Holzmann et al., 2020] Holzmann, R., Alonso-Garcia, J., Labit-Hardy, H., Villegas, A., 2020. NDC schemes and heterogeneity in longevity: Proposals for redesign in Holzmann, R., Palmer, E., Palacios,R., & Sacchi, S. (Eds.) *Progress and Challenges of Nonfinancial Defined Contribution Pension Schemes, Volume 1: Addressing Marginalization, Polarization, and the Labor Market* (pp. 307-332). Washington, DC: World Bank. doi:10.1596/978-1-4648-1453-2.

[Hürlimann, 2010] Hürlimann, W. (2010). Solvency risk capital for the short and long term: probabilistic versus stability criterion. *Belg. Actuar. Bull*, 9(1), 8-16.

[Jevtić & al, 2023] Jevtić, P., Qin, C., & Zhou, H. (2023). Multi-population mortality modelling with Lévy processes. Decisions in Economics and Finance, 1-27.

[Jijiie & al., 2022] Jijiie, A., Alonso-García, J., & Arnold, S. (2002). Mortality by socio-economic class and its impact on the retirement schemes: how to render the systems fairer?. European Actuarial Journal, 12, 701–743.

[US Census Bureau (2023)] US Census Bureau Historical Income tables. March 2023 `https://www.taxpolicycenter.org/statistics/household-income-quintiles`

[Kleinow, 2015] Kleinow, T., 2015. A common age effect model for the mortality of multiple populations. Insurance: Mathematics and Economics, 63(1): 147–152. doi: 10.1016/j.insmatheco.2015.03.023

[Lee & Carter, 1992] Lee, R., Carter, L., 1992. modelling and forecasting U.S. mortality. Journal of the American Statistical Association, 87: 659–671.

[Li & Lee, 2005] Li, N., Lee, R.D., 2005. Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. Demography, 42(3): 575–594.

[Mackenbach & al, 1997] Mackenbach, J.P., Kunst, A.E., Cavelaars, A.E., Groenhof, F., Geurts, J.J. (1997). socio-economic inequalities in morbidity and mortality in western Europe. The Lancet, 349(9066), 1655–1659.

[Mackenbach & al, 2003] Mackenbach, J.P., Bos, V., Andersen, O., Cardano, M., Costa, G., Harding, S., Reid, A., Hemström, Ö., Valkonen, T., and Kunst, A.E. (2003). Widening socio-economic inequalities in mortality in six Western European countries. International Journal of Epidemiology, 32: 830-837.

[Majer & al, 2011] Majer, I. M., Nusselder, W. J., Mackenbach, J. P., & Kunst, A. E. (2011). socio-economic inequalities in life and health expectancies around official retirement age in 10 Western-European countries. Journal of Epidemiology and Community Health, 65, 972–979.

[Musgrave & Thin, 1948] Musgrave, R. A., & Thin, T. (1948). Income tax progression, 1929-48. Journal of political Economy, 56(6), 498-514.

[Queisser & Whitehouse, 2006] Queisser, M., & Whitehouse, E. (2006). Neutral or fair?: actuarial concepts and pension-system design.

[Shi & Kolk, 2022] Shi, J., & Kolk, M. (2022). How does mortality contribute to lifetime pension inequality? Evidence from five decades of Swedish taxation data. Demography, 59(5), 1843-1871.

[Sheshinski & Caliendo, 2021] Sheshinski, E., & Caliendo, F. N. (2021). Social security and the increasing longevity gap. Journal of Public Economic Theory, 23(1), 29-52.

[Strozza & al, 2022] Strozza, C., Vigezzi, S., Callaway, J., Kashnitsky, I., Aleksandrovs, A., & Vaupel, J. W. (2022). socio-economic inequalities in survival to retirement age: a register-based analysis

[Sundberg & al, 2023] Sundberg L, Agahi N, Wastesson JW, Fritzell J, Fors S. Increasing inequalities in disability-free life expectancy among older adults in Sweden 2002-2014. Scand J Public Health. 2023 Aug;51(6):835-842. doi: 10.1177/14034948211062309. Epub 2021 Dec 29. PMID: 34965792; PMCID: PMC10350709.

[TUC report, 2013] CONGRESS, Trades Union. Life expectancy inequalities and state pension outcomes. TUC, 2013.

[Villegas & al., 2005] Villegas, A., Haberman, S., Kaishev, V., Millossovich, P., 2017. A comparative study of two-population models for the assessment of basis risk in longevity hedges. ASTIN Bulletin: The Journal of the IAA, 47(3): 631-679. doi:10.1017/asb.2017.18

[Wen & al., 2020] Wen, J., Kleinow, T., Cairns, A.J.G. (2020). Trends in Canadian Mortality by Pension Level: Evidence from the CPP and QPP, North American Actuarial Journal. 24:4. 533-561. DOI: 10.1080/10920277.2019.1679190

[Wen & al., 2021] Wen, J., Cairns, A.J.G., Kleinow, T. (2021). Fitting multi-population mortality models to socio-economic groups. Annals of Actuarial Science 15, 144–172. DOI: 10.1017/S1748499520000184

[Wilmoth & al, 2023] Wilmoth, J. R., Bas, D., Mukherjee, S., & Hanif, N. (2023). World Social Report 2023: Leaving No One Behind in an Ageing World. UN.

Chapter 6

Discussion and extensions

6.1 General conclusion and perspectives

This thesis examines the challenges and implications of population ageing, longevity heterogeneity, and socio-economic inequalities on public pension systems. The study explores the need for reforms to ensure financial sustainability, social adequacy, and safe governance within pension systems. By proposing a new pension system design incorporating automatic adjustment mechanisms, progressive pension formulas, multi-population mortality modelling, and addressing the fairness and adequacy of benefits, this research aims to contribute to a more equitable and sustainable approach in the face of demographic changes and socio-economic disparities.

6.1.1 Summary of the main contributions

The objective of this thesis was to propose sustainable, fair, and safe pension designs. Each chapter focused on different aspects of pension systems in the context of demographic aging, longevity inequalities, and the need for financial sustainability.

In **Chapter 2**, we examined the failing situation of the pay-as-you-go defined-benefit pension system in Morocco. Our goal was to address the demographic challenges by introducing a new management model, the points system, and implementing Musgrave's rule. By providing an overview of the current state of the Moroccan retirement system and identifying the shortcomings of the defined-benefit method, we laid the foundation for proposing a more equitable and financially sustainable system.

Transitioning to **Chapter 3**, we shifted our attention to the correlation between income, life expectancy, and the fairness of pension systems. We introduced a progressive pension formula within a defined-benefit scheme to address the inequalities arising from this correlation. By considering the impact of life expectancy on benefit levels, particularly for different income groups, we aimed to achieve greater equity in pension provision. This chapter emphasized the importance of balancing intergenerational costs, intragenerational actuarial fairness, and social adequacy within pension system reforms.

In **Chapter 4**, our focus expanded to the challenges faced by national public pension systems in the context of demographic ageing and longevity inequalities. The proposed pension system aimed to achieve intergenerational and intragenerational equity, while ensuring financial sustainability and social adequacy. We introduced automatic adaptation mechanisms to address longevity heterogeneity and distribute demographic risk. By incorporating corrections for longevity heterogeneity and progressivity in the pension formula, we aimed to create a more balanced and secure system. This chapter emphasized the need to strike a balance between intergenerational costs, actuarial fairness, and social adequacy in designing pension systems. Additionally, we highlighted the potential for better social acceptance of pension reforms through the inclusion of social gradients measures. These measures offer a way to partially compensate or mitigate the limitations of benefits induced by financial sustainability concerns, ensuring that the pension system remains socially equitable and sustainable for all stakeholders.

The 5^{th}**chapter** serves as a concluding component of this thesis, consolidating the knowledge acquired from previous chapters and presenting an extended vision of the previous chapter by adding a stochastic dimension to the mortality modelling and the risk analysis of pension schemes with double AAM . By addressing socio-economic disparities in longevity and embracing adaptive mechanisms, we pave the way towards more equitable and sustainable pension systems. Our work underscores the significance of considering diverse demographic factors and fosters a deeper understanding of the complexities involved in designing pension schemes to meet the needs of an ageing population. As policymakers and researchers, our collective endeavor should be focused on creating pension systems that prioritize intergenerational and intragenerational fairness, ensuring the well-being and security of present and future retirees alike.

In conclusion, this thesis has proposed sustainable, fair, and safe pension designs in response to the challenges posed by demographic ageing, longevity inequalities, and financial sustainability. Chapter by chapter, we examined the current shortcomings of pension systems and introduced innovative approaches to address these issues. By considering factors such as the points system, progressive pension formulas, and automatic adaptation mechanisms, we aimed to create pension systems that are both equitable and viable in the long run.

6.1.2 Discussion and limitations

In the pursuit of developing more equitable and sustainable pension systems, we must acknowledge the inherent challenges that arise when predicting longevity and differentiating socioeconomic classes. While our proposal strives to address these issues by incorporating salary-based differentiations and progressive reforms within defined benefit (DB) schemes, it is crucial to recognize the limitations in predicting longevity solely based on salary. The complexities of human lifespan go beyond mere income levels, encompassing a myriad of factors that influence individual life expectancies. Moreover, our demographic hypotheses may warrant a more flexible approach, accounting for more dynamic population changes, migration and evolving societal conditions. By acknowledging these limitations, we pave the way for future research and advancements, fostering more comprehensive and inclusive solutions to the complexities of public pension systems and their impact on retirees of varying socioeconomic backgrounds.

While our salary-based approach for differentiating socioeconomic classes has its limitations in terms of predicting longevity accurately, it remains a central component of public pension systems and a relevant predictor of an individual's socioeconomic status. Salary plays a pivotal role in determining pension benefits and is a key consideration for pension planners when designing equitable pension policies. Despite its shortcomings as a predictor of longevity, salary provides valuable insights into an individual's financial well-being and overall socioeconomic standing, making it a practical and widely used criterion in pension systems.

However, it is crucial to recognize that using salary as the primary differentiating factor may not fully capture the complexities of lifespan heterogeneity within the population. Arno Baurin's cautionary note[1] about differentiating retirement age based on socioeconomic life expectancy gradients highlights the persistent challenges in addressing the entire distribution of lifespans. While salary is an essential indicator, it may not comprehensively account for the various factors that contribute to disparities in lifespans, such as individual health, lifestyle, and genetic factors.

As their analysis suggests, even with multiple differentiated retirement ages based on socioeconomic indicators, the reduction in inequality remains relatively modest. This underscores the need for caution in relying solely on salary-based differentiation to tackle lifespan heterogeneity within pension systems. Overlapping lifespan distributions across socioeconomic status groups add another layer of complexity, further emphasizing the potential limitations of our proposed progressive formula.

In light of these limitations, we acknowledge that our salary-based approach, while pragmatic and relevant for pension planning, may not be a panacea for achieving complete equity in pension systems, nevertheless it is a promising first step in the direction of social fairness. As we continue to explore progressive pension designs, future research should consider alternative predictors and additional socioeconomic factors to develop more comprehensive and effective strate-

[1] Baurin, A. (2022). Heterogeneity in pension policy (Doctoral dissertation, UCL-Université Catholique de Louvain)

gies for addressing lifespan heterogeneity and promoting fairness within pension systems. Combining multiple indicators and innovative modeling techniques may hold promise for achieving more equitable and robust pension policies that truly account for the diverse needs and characteristics of retirees.

It is also essential to acknowledge that the foundations of our proposals stem from the point system concept put forth by the Belgian pension committee. As a result, our transformations and systems proposed in this thesis operate within the framework of various pension systems, including DB, hybrid DB and DC systems. However, it is important to note that we have not covered notional defined contribution (NDC) configurations in this work, which represents a potential avenue for future research and exploration.

The NDC schemes have distinct features and mechanisms that require a separate analysis and adaptation of our proposed techniques. These systems often involve the distribution of notional accounts, and their operation differs significantly from the traditional DB approach. As a result, applying our progressive pension formula and other methodologies directly to NDC schemes may not be straightforward and may necessitate further research and adjustments.

To truly extend the scope and impact of our work, future endeavors should aim to delve into the complexities of NDC systems and explore the feasibility of implementing our techniques within such frameworks [2]. This may involve understanding the nuances of notional accounts, incorporating progressivity into NDC configurations, and evaluating the effects on income inequalities and retirement outcomes for individuals of varying socioeconomic classes.

While our thesis contributes valuable insights and advancements within the DB context, there is indeed more work to be done to broaden the applicability and relevance of our techniques to encompass various public pension system designs. By exploring and adapting our proposals to NDC and other configurations, we can create a more comprehensive and inclusive set of tools that address longevity heterogeneity and socioeconomic disparities in a broader range of pension systems, ultimately striving towards greater fairness and sustainability in public pension policies.

6.1.3 Future research and perspectives

Throughout this thesis, several research questions have been addressed, yet many avenues remain unexplored. The complexity of the subject and the scope of this study allowed for only a partial exploration of the potential research directions. Here, we propose three potential directions for future work to further advance the understanding and application of the proposed concepts:

[2] For example Holzmann et al& al proposals in "NDC schemes and heterogeneity in longevity: Proposals for redesign"

Extending Fairness Mechanisms in Pension Design

In the future work following this thesis, an exciting avenue for exploration lies in extending the salary-based approach to correct for longevity heterogeneity towards a retirement age-based approach. By utilizing the retirement age as a lever, we can potentially equalize the lifetime replacement rate with variable ages for each socioeconomic class, thereby further enhancing the fairness and social adequacy of pension systems. This topic is currently being discussed and explored in the field, and it holds great promise for applying our techniques to achieve more equitable pension designs.

Additionally, throughout our journey in this research, our primary focus has been on finding fairness mechanisms for Defined Benefit and hybrid DB/DC systems. However, there are other pension schemes, such as Notional Accounts and progressivity in NDC (Notional Defined Contribution) systems, that warrant further investigation. Exploring and applying our techniques in these alternative pension schemes could provide valuable insights and contribute to the development of more inclusive and socially just pension systems.

Gini index and measure of inequalities

The Gini coefficient, quantifies the degree of income or wealth inequality within the retiree population.The Gini coefficient, denoted as G, can be mathematically expressed as follows:

$$G = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|P_i - P_j|}{2\sum_{i=1}^{n}\sum_{j=1}^{n}P_j} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|P_i - P_j|}{2n\sum_{j=1}^{n}P_j} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|P_i - P_j|}{2n^2\bar{P}} \tag{6.1}$$

In this equation, the summations are performed over all individuals i and j in the population, and $|P_i - P_j|$ denotes the absolute difference between the pension benefit of person i and person j.

In a pure defined benefit (DB) system, income inequalities among retirees may lack any correction mechanisms, potentially leading to disparities in pension benefits. This initial analysis serves as a crucial benchmark, providing a clear understanding of the existing income inequalities within the retiree population.

However, following the implementation of the progressive defined Musgrave system, which incorporates corrections for longevity heterogeneity and progressivity in the pension formula, we anticipate observing a positive shift in the Gini index. As the reform is designed to promote fairness and equity within pension systems, we expect to see improvements in reducing income inequalities between retirees. The progressive nature of the pension formula, which takes into account factors such as longevity and socio-economic status, aims to ensure that pension benefits are distributed more fairly.

By examining the evolution of the Gini coefficient post-reform, we can gauge the effectiveness of the proposed pension design in achieving its goals of intergenerational and intragenerational fairness. A reduction in the Gini index would indicate a successful reduction in income inequalities among retirees, validating the efficacy of the progressive defined Musgrave system. This analysis, presented in Chapter 4, provides valuable evidence to support future policy decisions aimed at enhancing the fairness and sustainability of public pension systems. Here is presented the first evolution of the Gini index based the PDM system.

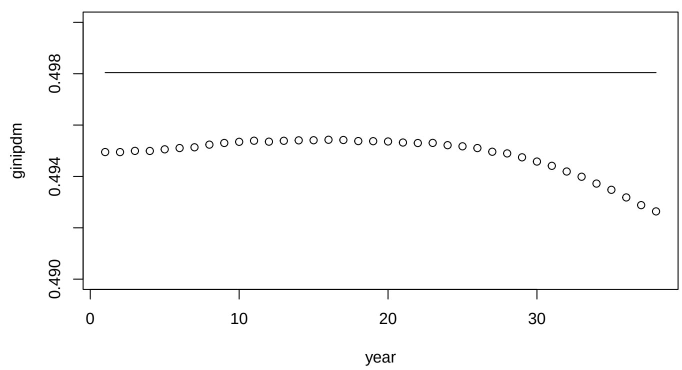


Figure 6.1: Gini index evolution

Additionally, the examination of the Gini index within the retiree population offers an opportunity to continuously refine the pension system. As we gain a deeper understanding of the factors influencing income inequalities, we can further tailor the pension design to address specific challenges and disparities. Policymakers can use these insights to make informed decisions and ensure that the pension system remains adaptive and effective in meeting the needs of retirees from diverse socio-economic backgrounds.

Tools for transition

As we promote reforms aiming to address socio-economic disparities in longevity and promote fairness, it is essential to ensure that their application reform does not lead to abrupt reductions in pension benefits or significant increases in contribution rates. Social acceptance of the reform is vital for its success. This is why we introduced in Chapter 4 the progressive indicator (α).

Inspired by credibility theory, this indicator incorporates mortality intensities from both the total population and specific subpopulations, weighted by α. As we have observed, the intensity of the correction provided by α depends on the existing longevity inequalities within the population and the state of the dependency ratio. One of the critical aspects of the progressive indicator is its potential role in smoothing the transition from traditional pension systems to progressive ones.

The thesis has already presented an example using increasing progressivity based on time, demonstrating the benefits of gradual adjustments. However, we can further explore the potential of optimizing the function of the progressive indicator to maximize utility during this transition period. By finding the optimal function, we can strike a balance between addressing inequalities and ensuring the financial sustainability and social acceptance of the reform.

Introducing a more dynamic and adaptive approach to the progressive indicator allows us to tailor the reform to the specific needs and characteristics of different populations. By accounting for variations in socio-economic factors, demographic trends, and other relevant parameters, the progressive indicator can evolve and adapt over time, ensuring the ongoing effectiveness of the pension system.

The use of credibility theory in the context of the progressive indicator opens up a rich area of research, offering opportunities for further theoretical developments and empirical investigations. Through rigorous analysis and empirical validation, we can fine-tune the progressive indicator and unlock its full potential in achieving intergenerational and intragenerational fairness within pension systems.

In conclusion, the progressive indicator represents a promising tool to support the implementation of progressive pension systems. By incorporating credibility theory and exploring dynamic optimization, we can design a more responsive and effective reform that addresses longevity inequalities while ensuring social acceptance and financial sustainability. This exploration of the progressive indicator adds depth to the thesis's contribution and points toward a path of continuous improvement in public pension design.

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