Measuring Well-Being and Lives Worth Living*

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Abstract

We study the measurement of well-being when individuals have heterogeneous preferences, including different conceptions of a life worth living. When individuals differ in the conception of a life worth living, the equivalent income can regard an individual whose life is not worth living as being better off than an individual whose life is worth living. In order to avoid this paradoxical result, we reexamine the ethical foundations of well-being measures in such a way as to take into account heterogeneity in the conception of a life worth living. We derive, from simple axioms, an alternative measure of well-being, which is an equivalent income net of the income threshold making lifetime neutral. That new well-being index always ranks an individual whose life is not worth living as worse-off than an individual with a life worth living.

Keywords: Well-being, measurement, equivalent income, lifetime, value of life.

JEL classification codes: I31, J17.

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Introduction

In recent years, equivalent income has become an increasingly used indicator of well-being, with various applications for well-being measurement and comparisons within and between countries.[1] Defined as the hypothetical income which, combined with references on non-monetary dimensions of life, would make an individual indifferent to his current situation, the equivalent income constitutes a preference-based indicator of well-being that is inclusive of (potentially) all non-monetary dimensions of well-being (Fleurbaey and Blanchet 2013, Fleurbaey 2016). Its specificity is to allow for the weighting of the different dimensions of life, while being respectful of how individuals evaluate these dimensions.

Given that lifetime is a central dimension of human well-being (Sen 1998), it does not come as a surprise that many studies using the equivalent income approach focused on the (income, lifetime) space, or on the (income, life expectancy) space. Such studies include Usher (1973, 1980), Williamson (1984), Crafts (1997) and Costa and Steckel (1997), as well as Nordhaus (2003) and Becker et al (2005). These studies construct equivalent income indexes while assuming a reference level for the lifetime dimension. The reference lifetime is defined as the particular level of lifetime at which interpersonal comparisons of well-being can be carried out by focusing only on the income dimension.

In order to compute equivalent incomes based on real-world data, existing studies assume some structure for individual preferences in the (income, lifetime) space. When modeling how individuals weight life-years against income, it is often assumed that there exists a unique strictly positive critical income making lifetime neutral. That critical income is the income threshold below which lifetime is an undesirable good, and above which lifetime is a desirable good (Becker et al 2005). The critical income making lifetime neutral is an aspect of individual preferences over all possible lives that accounts for the person's conception of a life worth living.[2]The existence of a unique strictly positive finite critical income level making lifetime neutral can be defended as follows. If such a critical income level did not exist, it would be the case either that *any* life-period, whatever living conditions are (even extremely miserable conditions), would be worth living, or, alternatively, that *any* life-period, whatever standards of living are (even excellent living conditions), would be not worth living. Those

[1]Recent applications include Decoster and Haan (2015), Carpentier and Sapata (2016), Decancq and Neumann (2016), Ponthiere (2016), Decancq et al (2017) and Onder et al (2018).

[2]The concept of critical income making lifetime neutral is the equivalent, in income terms, of Broome's concept of a utility level neutral for the continuation of existence (Broome 2004).

two implications are implausible.

When comparing the well-being of individuals who do not share the same conception of a life worth living, the equivalent income can have unattractive implications. A paradoxical result, first identified by Onder et al (2018), is the following: the equivalent income associated to the life of a person who regards his life as *not worth living* can, in some cases, *exceed* the equivalent income associated to the life of another person, who regards his life as *worth living*.

This possibility is illustrated on Figure 1, where individuals i and j have distinct indifference maps in the (income, lifetime) space, including different critical income levels $\widetilde{y}_i$ making lifetime neutral (corresponding to a vertical indifference line). Individual i has lower living standards than individual j, but he regards his life as worth living. On the contrary, individual j, who has a more demanding conception of a life worth living (i.e., $\widetilde{y}_j > \widetilde{y}_i$), considers that his life is not worth living. When computing the equivalent incomes for a particular reference lifetime level, we see that the equivalent income associated to the life of person i (denoted by EI_i) is lower than the equivalent income associated to the life of person j (denoted by EI_j). This inequality holds despite the fact that individual j regards his life as not worth living, whereas individual i regards his life as worth living. Thus the equivalent income considers that the individual whose life is not worth living is *better off* than the one whose life is worth living.

This result is somewhat paradoxical. Regarding one's life as worth living or not is a central component of individual preferences. Hence, it is paradoxical that a preference-based indicator of well-being such as the equivalent income regards a person whose life is not worth living as strictly better off than a person whose life is worth living. Of course, this paradox does not arise when individuals share the *same* conception of a life worth living. But there is no reason to assume *a priori* that all individuals share the same conception of a life worth living. If there exist as many preferences as there are individuals, for sure this aspect of life valuation is not uniform across all persons.

Hence, in the light of this paradoxical result, one may want to reexamine the construction of equivalent income well-being indexes in the context of distinct conceptions of a life worth living.[3] The goal of this paper is precisely to

[3]When facing this problem, one may argue that the person with a more demanding conception of a life worth living exhibits some form of "expensive tastes" (i.e., preferences that require more resources in order to achieve a given level of well-being). But that response, too, invites a revision of the well-being measure that is used in the context of unequal conceptions of life worth living. This paper examines how to do interpersonal comparisons of well-being

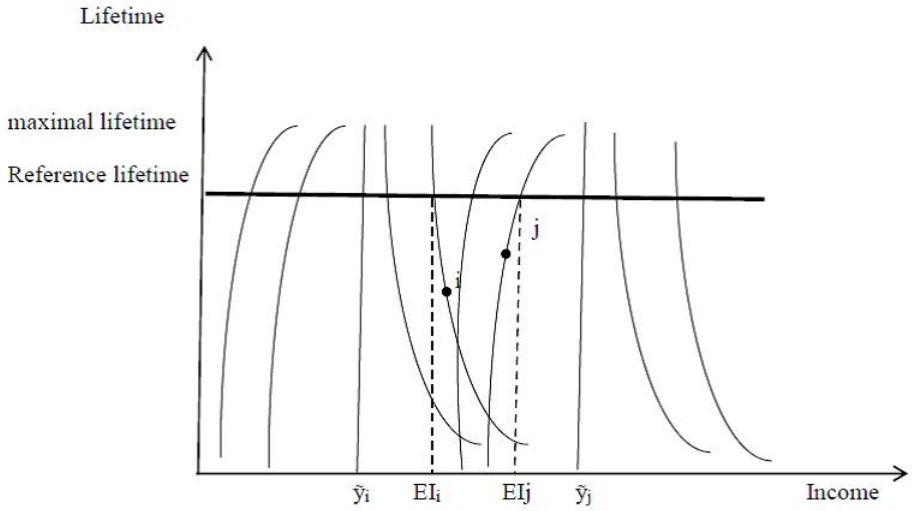

Figure 1: A paradox for the equivalent income.

reexamine the construction of a measure of well-being when individuals differ regarding their conception of a life worth living. For this purpose, we consider a model of the life cycle, where individuals differ in preferences on bundles in the (income, lifetime) space, in particular concerning the definition of a life worth living. Then, we propose to build a well-being index on the basis of several intuitive properties.

Throughout this paper, it is assumed that individual preferences should be respected by the social evaluator, and should therefore be taken into account in the construction of a well-being measure. This general assumption, which we will call the Sovereignty axiom throughout the paper, is quite standard in the literature on the construction of well-being indexes. However, one may argue that, in the specific context studied in this paper, not all individual preferences should necessarily be respected. In particular, mental diseases like depression could make all potential lives look not worth living for the depressed person. In the rest of this paper, we will consider the construction of an index of well-being while assuming that individual preferences—including the conception of a life worth living—are respectable, and, hence, constitute a relevant informational

when individuals have various, more or less demanding, conceptions of a life worth living.

basis for the construction of a measure of well-being.

Anticipating our results, we first show that the Sovereignty axiom, combined with two other axioms, suffices to fully characterize a new measure of well-being, which is the equivalent income index net of the critical income making lifetime neutral. This new well-being measure is immunized against the paradoxical result highlighted at the very outset of this paper. Two new axioms are used for its characterization. Conditional Priority states conditions on bundles under which the fact of caring more about lifetime than about income makes a person either better off or worse off. The Translation axiom states that a translation of all indifference curves along the income dimension does not affect the individual's well-being if her bundle is similarly translated. In a second stage, we characterize an alternative equivalent income index, also net of the critical income making lifetime neutral, but relying on different reference lifetime levels. This alternative well-being index relies on a variant of the Conditional Priority axiom, which regards individuals more concerned with lifetime as always worse off than individuals less concerned with lifetime, contrary to the initial Conditional Priority axiom.

This paper contributes to several branches of the literature. First, by revisiting the measurement of well-being in the (income, lifetime) space, this paper contributes to the literature on the measurement of well-being (Adler and Fleurbaey 2016), and, in particular, to the increasingly large literature on the construction of equivalent incomes (Fleurbaey and Blanchet 2013; Decancq and Neumann 2016; Decancq and Schokkaert 2016; Onder et al 2018). This paper contributes to that literature by revisiting the normative foundations of well-being measurement when individuals do not have the same conception of a life worth living, and by providing characterizations of two new well-being indexes. Second, this paper contributes also to the normative literature on the fair allocation of resources under unequal lifetime, such as Fleurbaey and Ponthiere (2013) and Fleurbaey et al (2014). These papers examined the design of optimal policy in the context of unequal lifetime, and dealt with interpersonal well-being comparisons by relying on standard consumption-equivalent indexes. The well-being measures used in these papers are subject to the paradoxical result discussed in Section 1 of the present paper. Here we propose a more attractive way of dealing with interpersonal well-being comparisons under distinct preferences in the (income, lifetime) space. This provides also more solid normative foundations for the design of optimal policies in the context of unequal lifetimes.

This paper is organized as follows. The model is presented in Section 2. Section 3 introduces the axioms. The characterization of the equivalent income net of the critical income making lifetime neutral is developed in Section 4. Section 5 examines the characterization of an alternative well-being index, based on a variant of the Conditional Priority axiom. Section 6 concludes.

The model

The population is a set N of individuals $i, j, ...$ Given that we want to examine interpersonal well-being comparisons, we assume that the number of individuals in N is at least two.[4]

Each individual life is characterized by a (constant) income per period $y_i \in \mathbb{R}_+$ and a lifetime $L_i \in \left[0, \overline{L}\right]$. Focusing on constant intertemporal income profiles is not restrictive, because under mild assumptions every sequence of income levels over the years of a lifetime is as good as some sequence of equal lifetime that yields the same satisfaction, at a constant level between the lowest and the greatest levels in the previous sequence. The two-dimensional space we retain here is very convenient to depict how each conception of a "life worth living" specifies a relation between the prevailing "quality" of life and the desirability of a larger "quantity" of life.[5]

Each individual i has well-defined preferences $\succeq_i$ on bundles (y, L) that are composed of (constant) income per period y and of lifetime L. As usual, the preference relation $\succeq_i$ is assumed to be complete, reflexive and transitive. Strict preference is denoted by $\succ_i$, while indifference is denoted by $\sim_i$.

The indifference curve of individual i containing the bundle (y, L) is defined as follows:

$$IC\left(y, L, \succeq_i\right) = \left\{(y', L') : (y', L') \sim_i (y, L)\right\}.$$

We say that $IC\left(y, L, \succeq_i\right)$ is *steeper* than $IC\left(y, L, \succeq_j\right)$ if for all $(y', L') \in IC\left(y, L, \succeq_i\right)$, one has $(y', L') \succ_j (y, L)$ if $y' < y$ and $(y, L) \succ_j (y', L')$ if $y' > y$. This local information on preferences captures which of income and lifetime an individ-

[4] Note that this paper considers heterogeneity in the conception of a life worth living while assuming a constant population size. As such, this paper does not explore the consequences of heterogeneity of conceptions of life worth living for choices of population size.

[5] Introducing non-constant income profiles would not fundamentally affect our results. Actually, this would lead us to characterize constant equivalent income indexes of well-being (in the same vein as in Fleurbaey et al 2014) instead of equivalent income indexes. This would not bring much extra value for the issue at stake in this paper: the measurement of well-being under different conceptions of a life worth living.

ual cares about more. Note that this implies single-crossing of the two curves: $IC\left(y,L,\succeq_i\right)\cap IC\left(y,L,\succeq_j\right)=\{(y,L)\}$.

For each individual i, the indifference map $IM(\succeq_i)$ is defined as the set of all indifference curves $IC\left(y,L,\succeq_i\right)$. The indifference map $IM\left(\succeq_i\right)$ is a *translation* of $IM\left(\succeq_j\right)$ whenever there is $z\in\mathbb{R}_+$ such that for all $(y,L),(y',L')$, one has $(y,L)\succeq_i(y',L')$ if and only if $(y+z,L)\succeq_j(y'+z,L')$.

For an individual i, we define the lower and upper contour sets at the bundle (y,L) as:

$$\mathbb{L}(y,L,\succeq_i)=\{(y',L'):(y,L)\succeq_i(y',L')\}$$

$$\mathbb{U}(y,L,\succeq_i)=\{(y',L'):(y',L')\succeq_i(y,L)\}$$

Preferences $\succeq_i$ on bundles (y,L) are assumed to be order-dense, that is, the set of all bundles $\mathbb{R}_+\times\left[0,\overline{L}\right]$ contains a countable order-dense subset. A set $S\subset\mathbb{R}_+\times\left[0,\overline{L}\right]$ is order-dense if and only if for any two bundles (y,L), $(y',L')\in\mathbb{R}_+\times\left[0,\overline{L}\right]\setminus S$ such that $(y,L)\succ_i(y',L')$, there is a bundle $(y'',L'')\in S$ such that: $(y,L)\succ_i(y'',L'')\succ_i(y',L')$.[6]

Preferences $\succeq_i$ on bundles (y,L) are supposed to be monotonic in income. However, preferences are not monotonic in lifetime. It is assumed that, for each individual i, there exists a unique positive critical income level $\widetilde{y}_i>0$ such that for all bundles with $y>\widetilde{y}_i$, lifetime is a desirable good, whereas for all bundles with $y=\widetilde{y}_i$, lifetime is a neutral good, and for all bundles with $y<\widetilde{y}_i$, lifetime is an undesirable good. Observe that the subset $\{(y,L):y=\widetilde{y}_i\}$ is an indifference curve for every i.

This assumption excludes the possibility that lifetime is always bad, no matter how affluent life is, as well as the possibility that it is always desirable, no matter how destitute.[7] It also excludes cases in which the desirability of ex-

[6] Restricting the set of preferences to order-dense preferences amounts to focusing on numerically representable preferences. This assumption excludes lexicographic preferences, for which no numerical representation exists. See Fishburn (1970).

[7] One might question the existence of lives not worth living, by arguing that individuals with such lives would commit suicide. But such a criticism is not well founded. Our analysis is about the measurement of personal well-being, not about a comprehensive assessment of people's reasons to live. It is possible to have a life not worth living in terms of personal well-being while being committed to live in the service of one's family or community. It is also possible that individuals in extreme poverty do consider that their life is not worth living, and stop caring for their health. This absence of health investment is not a violent suicide, but it is equivalent, at the end of the day, to some (more or less hidden) form of suicide, which is caused by extremely bad living conditions making a life not worth living.

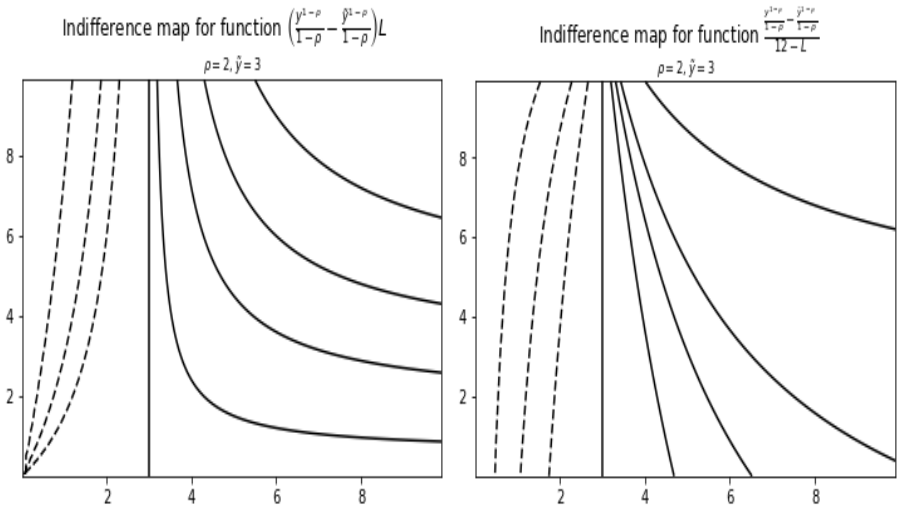

Figure 2: Illustration of indifference maps

tending life might depend both on income and lifetime, e.g., cases in which at certain levels of income there is an optimal lifetime and such that beyond this optimal level, making life desirable requires extra income. Our analysis could be extended to these more complex cases, by redefining the critical level $\widetilde{y_i}$ as the minimal income level that makes lifetime desirable from 0 up to L. But in this paper, we restrict attention to the case in which the desirability of extending life depends on income only and not on the current lifetime.

This assumption is commonplace in the literature. For instance, Becker et al (2005) and Onder et al (2019) rely on preferences represented by the utility function: $U(y,L) = L\left(\frac{y^{1-\sigma}}{1-\sigma} - \frac{\widetilde{y}^{1-\sigma}}{1-\sigma}\right)$. The indifference map for such a function is shown on the left panel of Figure 2. One notices that below the critical level, preferences are not convex. The right panel displays the indifference map for a function of the type $U(y,L) = \left(\frac{y^{1-\sigma}}{1-\sigma} - \frac{\widetilde{y}^{1-\sigma}}{1-\sigma}\right)/(\alpha\bar{L}-L)$, for $\alpha > 1$.[8] In this case, preferences are convex on both sides of the critical level. Both types of preferences are allowed in our study, as preferences are not assumed to be convex.

One can also interpret the critical income level making lifetime neutral as the equivalent, in terms of income, of Broome's concept of the "critical utility level neutral for the continuation of existence" (see Broome 2004).

[8] We thank a referee for suggesting this type of functional form.

Let $\mathcal{R}$ denote the set of preferences over bundles (y, L) that are order-dense, monotonic in income and that admit a unique positive critical income level making lifetime neutral.

In the rest of the paper, we want to construct a measure of individual well-being. Such a measure, denoted by $M(y, L, \succeq_i)$, assigns a real number to all bundles (y, L), assessing those bundles from the perspective of well-being for the preferences $\succeq_i$. We thus have: $M(y, L, \succeq_i) : \mathbb{R}_+ \times \left[0, \overline{L}\right] \times \mathcal{R} \to \mathbb{R}$. This measure is defined for all bundles, and for all preferences in the domain $\mathcal{R}$, and allows for the comparison of well-being across individuals with potentially different bundles and different preferences. Anonymity is built in the measure, since $(y_i, L_i, \succeq_i) = (y_j, L_j, \succeq_j)$ implies $M(y_i, L_i, \succeq_i) = M(y_j, L_j, \succeq_j)$.

Axioms

This section presents properties that we will impose on the well-being measure $M(y, L, \succeq_i)$. First, the Sovereignty axiom is standard: It merely states that a measure of well-being should respect individual preferences.

Sovereignty $\forall i \in N$, if $(y, L) \sim_i (y', L')$ then $M(y, L, \succeq_i) = M(y', L', \succeq_i)$, and if $(y, L) \succ_i (y', L')$ then $M(y, L, \succeq_i) > M(y', L', \succeq_i)$.

The possibility for a measure $M(y, L, \succeq_i)$ to satisfy this axiom for all preferences $\succeq_i$ comes from the assumption that preferences are order-dense (Fishburn 1970).

The next axiom, Conditional Priority,[9] states that, when comparing individuals enjoying the same bundle and sharing the same critical level but otherwise having different preferences, whether social priority should be given to individuals who are more concerned with income or with lifetime depends on whether their life is worth living, and on whether lifetime is above or below some reference threshold. More precisely, it stipulates that if the common bundle renders life not worth living, there exists a lifetime threshold $0 < \overline{L^1} < \overline{L}$ above which the individual who cares more about lifetime is regarded as the worst-off, and below which the opposite occurs. On the contrary, when lives are worth living, there exists another lifetime threshold $0 < \overline{L^2} < \overline{L}$ above which the individual who cares more about income is regarded as the worst-off, and below which the opposite occurs.[10]

[9] This axiom is called "Conditional" Priority, because it states that social priority should be assigned conditionally on the levels of individual achievements in terms of income and lifetime.

[10] In the Conditional Priority axiom, the reference lifetime can take only two distinct levels,

Conditional Priority $\forall i, j \in N$, with $\widetilde{y_i} = \widetilde{y_j} = \widetilde{y}$, whenever $(y_i, L_i) = (y_j, L_j) = (y, L)$ and $IC(y, L, \succeq_i)$ is steeper than $IC(y, L, \succeq_j)$ at (y, L):

- if $L \leq \overline{L^1}$ and $y < \widetilde{y}$, then $M(y, L, \succeq_i) \leq M(y, L, \succeq_j)$.
- if $L \geq \overline{L^1}$ and if $y < \widetilde{y}$, then $M(y, L, \succeq_i) \geq M(y, L, \succeq_j)$.
- if $L \leq \overline{L^2}$ and if $y > \widetilde{y}$, then $M(y, L, \succeq_i) \geq M(y, L, \succeq_j)$.
- if $L \geq \overline{L^2}$ and if $y > \widetilde{y}$, then $M(y, L, \succeq_i) \leq M(y, L, \succeq_j)$.

Figure 3 illustrates the requirements of Conditional Priority in terms of well-being comparisons. Figure 3 shows the four areas of the (income, lifetime) space that are defined in the Conditional Priority axiom. Those areas are delimited by the critical income making lifetime neutral, as well as by the two thresholds for lifetime $\overline{L^1}$ and $\overline{L^2}$. The little arrow indicates the individual who is considered to have social priority under Conditional Priority.

The Conditional Priority axiom can be justified as follows. When comparing the situations of two persons enjoying the same life but with different preferences (except their conception of a life worth living), it would be hard to consider that the person who cares more, at the margin, about one dimension should *necessarily* be regarded as worse-off or better off than the other person, independently of the levels of achievements along the two dimensions of life. Assigning such an "unconditional" priority is hardly defendable. On the contrary, it makes a lot of sense to consider that the worst-off person should be identified *conditionally* on the levels of achievements along the two dimensions of life. The Conditional Priority axiom provides conditions on those achievements determining the identification of the person who should have priority.

The two lifetime thresholds $\overline{L^1}$ and $\overline{L^2}$ are ethical parameters that have a clear meaning. When considering lives not worth living, one can acknowledge that, when life is sufficiently short, the worst-off individual is the one who cares more about income, and less about lifetime, whereas the opposite holds when the life not worth living is too long. The ethical parameter $\overline{L^1}$ acts as a threshold separating what is regarded as a life not worth living that is "sufficiently short" (so that social priority goes to the individual with more concern for income), or

depending on whether the income per period is above or below the critical income making lifetime neutral. Alternatively, one might prefer other versions of this axiom for which the lifetime threshold would be varying with the income per period and take more than two values. Determining how the lifetime threshold would vary as a function of income per period is not trivial. Hence we prefer here to keep the two-value function stated in the axiom of Conditional Priority.

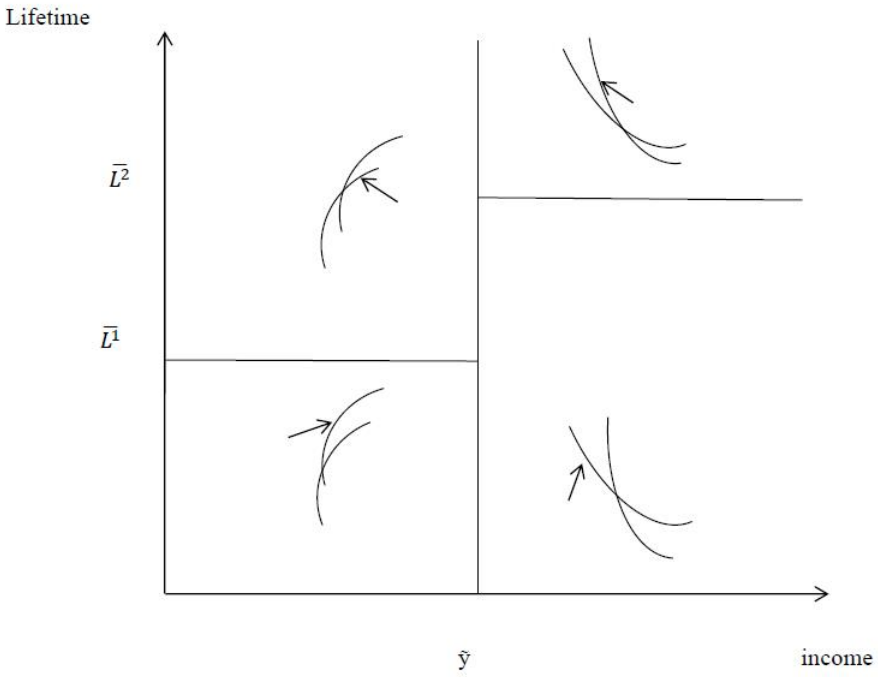

Figure 3: Conditional Priority

"too long" (so that social priority goes to the individual with more concern for lifetime). In a similar way, the threshold $\overline{L^2}$ separates, on the one hand, a life worth living that is "insufficiently long", and which leads thus to give priority to individuals who care more about lifetime, and, on the other hand, a life worth living that is "sufficiently long", so that social priority must be given to individuals who care more about income.[11]

Finally, regarding the ethical attractiveness of the Conditional Priority axiom, it should be stressed that interpersonal well-being comparisons carried out in the upper-left quadrant and the lower-right quadrant of Figure 3 can hardly be questioned. Indeed, it seems almost natural to regard a life not worth living as "insufficiently short", so that the person who cares more about longevity is considered, in that case, to be the worst-off. Moreover, when considering a life worth living, it is also quite intuitive to regard that life as "insufficiently long", and to think that the person who cares more about longevity is, here again, the worst-off. However, things might be somewhat less intuitive when considering interpersonal well-being comparisons in the two other quadrants of Figure 3. For instance, one might question the fact that some lives not worth living are "sufficiently short". One might also doubt about the fact that some lives worth living are "sufficiently long". This issue will be examined further in the next section of this paper, which introduces another specification of the Conditional Priority axiom, leading to alternative identifications of worst-off persons.

Let us now consider a third axiom, the Translation axiom, which concerns the comparison of well-being between individuals who differ regarding the critical income level making lifetime neutral, but share all other dimensions of the indifference map. In that case, individuals under comparison share the same indifference map up to a translation along the income dimension. This axiom states that when the indifference map of an individual is a translation of the indifference map of another individual, and if their bundles exhibit the same lifetime and lie at the same distance of their individual-specific critical income levels making lifetime neutral, then these individuals are deemed *equally well-off*. The Translation axiom states that, when measuring the well-being of individuals whose indifference maps are mere translations, what matters is the distance to the vertical indifference line associated to the critical income level.

[11] Obviously those two lifetime thresholds are ethical parameters embodying value judgments. One could think, for instance, about levels such as 30 years and 70 years. 30 years would be the threshold below which lives not worth living are sufficiently short, whereas 70 years would be the threshold above which lives worth living are sufficiently long.

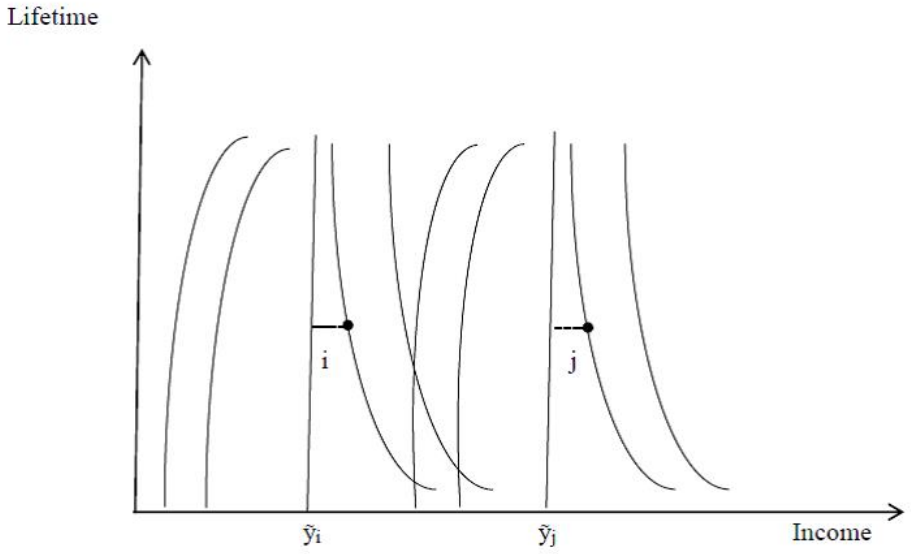

Figure 4: Translation axiom

Translation $\forall i, j \in N$ such that $IM(\succeq_i)$ is a translation of $IM(\succeq_j)$, with $\widetilde{y_j} = \widetilde{y_i} + x$ with $x > 0$, if $L_i = L_j$ and if $y_i - \widetilde{y_i} = y_j - \widetilde{y_j}$, then $M(y_i, L_i, \succeq_i) = M(y_j, L_j, \succeq_j)$.

Figure 4 illustrates the Translation axiom in a simple two-person case. The indifference map of individual j is a mere translation of the indifference map of individual i. Since the bundles of individuals i and j involve the same lifetime and are equi-distant with respect to their critical income making lifetime neutral (respectively $\widetilde{y_i}$ and $\widetilde{y_j}$), the Translation axiom considers that individuals i and j are equally well-off.

The Translation axiom can be justified as follows. The capacity of a given amount of income to increase the well-being of a person is neither absolute nor unconditional, but depends on the conception of a "life worth living" of that person, and, hence, varies across individuals who have unequal critical income levels. For individuals who have a more demanding conception of a "life worth living", a given amount of income will bring less well-being than for persons who have a less demanding conception of a "life worth living". The capacity of income to increase the well-being of different persons is thus relative

to the conception of a "life worth living" to which these persons adhere. The Translation axiom does justice to this idea, by stating that the relevant piece of information for interpersonal well-being comparisons is not the individual's absolute income, but the distance between that income and the critical income making lifetime neutral. Based on that axiom, having a larger income does not give any advantage to an individual over another individual, as long as the distance with respect to their critical income level remains the same.

The Translation axiom has several attractive implications. First, the Translation axiom implies that all individuals who enjoy a neutral life (i.e., a life with $y_i = \widetilde{y}_i$) are considered to be in an equally good situation, that is, have an equal level of well-being, whatever their precise conception of a "life worth living" is. Second, and more importantly, the Translation axiom allows the well-being measure $M(y, L, \succeq_i)$ to avoid the problem mentioned at the very outset of this paper, concerning unattractive interpersonal well-being comparisons between individuals who differ on their conception of a "life worth living". Actually, under well-being measures that do not satisfy the Translation axiom, it can be the case that a person whose life is not worth living is regarded as strictly better off than a person whose life is worth living. This kind of paradoxical result is necessarily avoided when one requires that the well-being measure satisfies the Translation axiom. Under a well-being measure satisfying the Translation axiom, a person whose life is not worth living is necessarily regarded as strictly worse off than a person whose life is worth living. This attractive implication of the Translation axiom provides some support for that axiom.[12]

Characterization of the well-being index

Theorem 1 states the logical implications of the Sovereignty, Conditional Priority and Translation axioms for the well-being measure $M(y, L, \succeq_i)$.

Let $\overline{L^1}, \overline{L^2}$ be given. The equivalent income $EI(y, L, \succeq_i)$ is defined as follows:

- If $y_i < \widetilde{y}_i$, $(y_i, L_i) \sim_i (EI(y_i, L_i, \succeq_i), \overline{L^1})$;
- If $y_i = \widetilde{y}_i$, $EI(y_i, L_i, \succeq_i) = \widetilde{y}_i$;

[12] It should be stressed, however, that this attractive implication could also be obtained by making well-being depend not on the *absolute* distance between the income and the critical income making lifetime neutral (as assumed in the Translation axiom), but on the *relative* distance between these. The Translation axiom provides thus one simple way - among others - to avoid the paradox mentioned in the introduction of this paper, but other, possibly more complex, axioms could also have the same implication.

- If $y_i > \widetilde{y}_i$, $(y_i, L_i) \sim_i (EI(y_i, L_i, \succeq_i), \overline{L^2})$.

Note that the equivalent income is not defined for indifference curves that lie everywhere above the thresholds $\overline{L^1}, \overline{L^2}$.

THEOREM 1 Consider the set of all $(y, L, \succeq_i)$ for which $EI(y, L, \succeq_i)$ is defined. On this domain, a well-being measure $M(y, L, \succeq_i)$ satisfies the axioms Sovereignty, Conditional Priority and Translation if and only if, up to an increasing transform, it takes the form:

$$M(y, L, \succeq_i) = EI(y, L, \succeq_i) - \widetilde{y}_i.$$

PROOF: See the Appendix.

Theorem 1 states that a well-being measure that satisfies the three axioms presented above—Sovereignty, Conditional Priority and Translation—must necessarily take (up to an increasing transform) the form of the equivalent income net of the critical income making lifetime neutral, where the equivalent income is defined for two particular reference lifetime levels (one for lives worth living, and one for lives not worth living).

The well-being index characterized in Theorem 1 shares some features with standard equivalent income indexes. First, the well-being index is defined only for bundles that are located on indifference curves that cross, at some point, a reference lifetime level. This incompleteness of the well-being measure is standard for all equivalent income measures of well-being relying on indifference maps. The reference level of lifetime is, by definition, the particular level of lifetime at which interpersonal well-being comparisons are driven only by income comparisons. Thus, it follows that bundles that lie on indifference curves that do not cross the reference lifetime level cannot be related to such interpersonal comparisons, and, hence, cannot be assigned numbers by the equivalent income index. Second, the well-being index characterized in Theorem 1 is not defined uniquely, but only up to an increasing transform, since such a transform would not alter interpersonal comparisons and would not interfere with the faithful representation of preferences.

However, in comparison to the standard equivalent income, the well-being index proposed here differs on two main grounds. First, the reference lifetime level is no longer unique for all lives, but is specific to whether the life is worth living or not, as required by Conditional Priority. A second difference lies in

the subtraction of the critical income making lifetime neutral, an implication of the Translation axiom. Subtracting the (individual-specific) critical income making lifetime neutral escapes the counterintuitive result of Figure 1. With this measure, material achievements are not valued absolutely, but relatively to the individuals' own conceptions of a life worth living. In particular, under this new measure of well-being, a person who regards his life as not worth living is always regarded as *worse-off* than a person who regards his life as worth living, since the equivalent income associated to a life not worth living is necessarily lower than the critical income making lifetime neutral. Hence, as a consequence, the well-being measure takes, for lives not worth living, a negative value, which is necessarily smaller than the values taken by the well-being index in case of lives worth living (since in that case the well-being index takes a strictly positive level).

However, this is not the only possible way to measure well-being when individuals differ in their conception of the life worth living. In the next section, we propose to consider an alternative well-being measure, which is still based on the Translation axiom, but relies on a modified form of the Conditional Priority axiom.

An alternative index

This section explores the characterization of an alternative measure of well-being, which is also an equivalent income net of the critical income level making lifetime neutral, but relies on alternative reference lifetime levels, and, hence, defines social priority in a different manner. For this purpose, we propose to adapt the Sovereignty axiom and the Conditional Priority axiom.

Let us first examine the variant of the Conditional Priority axiom that will be used in this section. This new axiom, entitled Conditional Priority II, states that, whatever the level of lifetime, and independently of whether individuals have a life worth living or not worth living, individuals who care more about lifetime are worse off.

Conditional Priority II $\forall i, j \in N$, with $\widetilde{y_i} = \widetilde{y_j} = \widetilde{y}$, if $(y_i, L_i) = (y_j, L_j) = (y, L)$, with $y \neq \widetilde{y}$, and $IC(y, L, \succeq_i)$ is steeper than $IC(y, L, \succeq_j)$ at (y, L), then $M(y, L, \succeq_i) \geq M(y, L, \succeq_j)$.

The intuition behind that axiom goes as follows. When life is not worth living (i.e. $y_i < \widetilde{y_i}$), then lifetime is undesirable, and so it makes sense to suppose

that individuals who care more about their lifetime (to reduce it) are worse off. Conversely, when life is worth living (i.e. $y_i > \widetilde{y_i}$), then the least well off is the individual who cares more about increasing lifetime. Conditional Priority stated that the person who cares more about lifetime is the worst-off only in two cases: either when a life that is not worth living is not sufficiently short or when a life that is worth living is not sufficiently long, the terms "sufficiently short" and "sufficiently long" being defined by parameters $\overline{L^1}$ and $\overline{L^2}$. One may consider that these two cases are too restrictive. In particular, one may argue that *any* life not worth living is not sufficiently short, and that *any* life worth living is not sufficiently long, so that priority should always be assigned to the persons who care more, at the margin, about lifetime. Conditional Priority II does justice to that idea.

It may seem that Conditional Priority II is just a special case of Conditional Priority, but this is not the case. Conditional Priority assumes that the thresholds $\overline{L^1}$ and $\overline{L^2}$ are strictly positive, and this plays a key role in the proof of Theorem 1. In order to characterize an alternative well-being measure on the basis of Conditional Priority II, a strengthening of Sovereignty is needed, which extends it to comparisons involving different preferences but non-crossing indifference curves, as in the initial Sovereignty axiom. The new variant says that whenever individuals are on an identical indifference curve, they are equally well-off, and whenever one is on a "higher" indifference curve, this one is better off—independently of whether they have identical preferences or not.[13]

Sovereignty II $\forall i, j \in N : \widetilde{y_i} = \widetilde{y_j} = \widetilde{y}$, $\forall (y, L), (y', L')$, if $\mathbb{U}(y, L, \succeq_i) = \mathbb{U}(y', L', \succeq_j)$, then $M(y, L, \succeq_i) = M(y', L', \succeq_j)$; if $int\,[\mathbb{L}(y, L, \succeq_i) \cap \mathbb{U}(y', L', \succeq_j)] = \emptyset$, then $M(y, L, \succeq_i) \leq M(y', L', \succeq_j)$.

Let the equivalent income $\widehat{EI}(y_i, L_i, \succeq_i)$ be defined as follows:

- If $y_i < \widetilde{y_i}$, $(y_i, L_i) \sim_i (\widehat{EI}(y_i, L_i, \succeq_i), 0)$;
- If $y_i = \widetilde{y_i}$, $\widehat{EI}(y_i, L_i, \succeq_i) = \widetilde{y_i}$;
- If $y_i > \widetilde{y_i}$, $(y_i, L_i) \sim_i (\widehat{EI}(y_i, L_i, \succeq_i), \overline{L})$.

The following theorem provides the relevant variant of Theorem 1.

THEOREM 2 Consider the set of all $(y, L, \succeq_i)$ for which $\widehat{EI}(y, L, \succeq_i)$ is defined. On this domain, a well-being measure $M(y, L, \succeq_i)$ satisfies the

[13] This axiom is akin to Nested Contour Priority in Fleurbaey and Maniquet (2011).

axioms Sovereignty II, Conditional Priority II and Translation if and only if, up to an increasing transform, it takes the form:

$$M(y, L, \succeq_i) = \widehat{EI}(y, L, \succeq_i) - \widetilde{y_i}$$

PROOF: See the Appendix.

This well-being measure is simpler than the previous one since it does not require the ethical parameters $\overline{L^1}$ and $\overline{L^2}$, and this may make it appealing. Indeed, the literature so far has focused on lives worth living and has adopted the maximum lifetime as the natural reference. It is indeed natural to take the best possible value of lifetime, conditional on the level of income, as the reference, since this means that any individual who is not enjoying this best value is somehow suffering from this gap, and the more this individual cares about the gap, the worse off this individual is.

However, the measure proposed in the previous section may also have its appeal. Indeed, for lifetime, norms in social perceptions of what is desirable tend to stabilize not on the maximum possible, but on the upper level attained by a reasonable fraction of the population. For instance, those who live more than 100 years and care more about lifetime than others who live equally long may not necessarily be deemed worse off. On the contrary, they may be deemed lucky to have reached that old age. The previous measure is able to capture this intuition, whereas the variant introduced in this section exhibits much less flexibility to accommodate various intuitions about "normal" lifetime. Thus, not only do these two theorems involve different axioms and technical arguments, but they represent substantially different ethical views.

Concluding remarks

This paper started from a paradoxical result for standard equivalent income indexes in the (income, lifetime) space: when individuals differ in their conception of a life worth living, it is possible that equivalent income takes a higher level for individuals who regard their life as not worth living than for individuals who regard their life as worth living. This paradoxical result comes from the fact that the standard equivalent income abstracts from an important aspect of individual preferences—their conception of a life worth living. As a consequence, standard equivalent incomes could potentially lead to somewhat counterintuitive results

when making well-being comparisons between individuals with lives worth living or not worth living.

This paper proposed to provide foundations for an alternative equivalent income measure of well-being, which takes into account individual's conceptions of a life worth living. In a life-cycle model with heterogeneous preferences, including heterogeneous conceptions of a life worth living, we showed that simple axioms—the Sovereignty, Conditional Priority and Translation axioms—suffice to characterize two new indexes of well-being taking a similar form: an equivalent income net of the critical income making lifetime neutral, the equivalent income being defined for reference lifetime levels that differ depending on whether lives are worth living or not.

The two new well-being indexes are immunized against the counter-intuitive result highlighted at the outset of this paper, as they always take a lower value for a life not worth living than for a life worth living. This also provides a more promising account of how social priority should be assigned in the context of heterogeneous preferences in the (income, lifetime) space. When a life is not worth living, lifetime is an undesirable good, so that, when lifetime is sufficiently long, priority should be given to individuals who care more about their lifetime. On the contrary, when a life is worth living, lifetime is a desirable good, so that, when lifetime is not sufficiently long, priority should also be given to individuals who care more about their lifetime. Our paper explored two distinct ethical accounts of what "sufficiently long" means in those distinct contexts, and these account are directly translated into reference lifetime levels that depend on whether lives are worth living or not, and which have a clear ethical significance.

Thus, this analysis has implications that go beyond the measurement of well-being. The measures of well-being characterized in this paper can be applied for the reform of social security systems. Indeed, the design of social insurance systems often requires to deal with difficult trade-offs between allocating resources among groups differing in income and in survival conditions. In order to solve those policy trade-offs, the first stage is to assign priority to some groups, and this can only be done by measuring and comparing the well-being of different individuals. From this perspective, this paper suggests that one could hardly ignore, in that preliminary stage, how individuals conceive of a life worth living.

The present framework has also important implications for the design of other public interventions, such as humanitarian aid. Our analyses suggest that, when major crises - such as natural disasters or droughts - hit poor economies, international aid should not only focus on providing access to health care, but

should combine access to health care with massive transfers. Actually, the objective of "saving lives" requires both to provide health care (in order to "save lives" in the strict sense of the terms) *and* to redistribute substantial amounts of income to the victims of disasters (in order to avoid that their income falls below the critical level under which a life is not worth living). Humanitarian aid focusing only on the first aspect would have the perverse effect of reinforcing well-being deprivation. Humanitarian aid aimed at "saving lives" should thus combine access to health care with cash transfers.

More generally, if one considers the functions of the Welfare State in all economies (including advanced ones), the framework studied in this paper points to a new, original, motive for redistribution of resources by the State. In a laissez-faire world, some extremely poor individuals may find their life not worth living, and may thus not have incentives to invest in their health, which contributes to reducing their longevity. Redistribution could make their lives worth living, and, hence, favor the survival of those persons. The State could thus, thanks to transfer programs against poverty, have a key role in making lives worth living. This new motive for redistribution invites further research.

Declarations

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Appendix

0.2 Proof of Theorem 1

The proof proceeds in two steps. We first consider the proof of the statement that a well-being measure satisfying Sovereignty, Conditional Priority and Translation takes the form presented in Theorem 1. Then, in the second stage, we will prove that this measure of well-being satisfies indeed the three axioms.

First stage (Sufficiency).

The proof is organized in two stages.

We first focus on individuals whose preferences differ, but who have the same critical income level $\widetilde{y}_i$. We first show that if the indifference curves of two distinct individuals cross at the lifetime threshold $\overline{L^1}$ or $\overline{L^2}$, then the measure of well-being assigns the same well-being level to these two individuals.

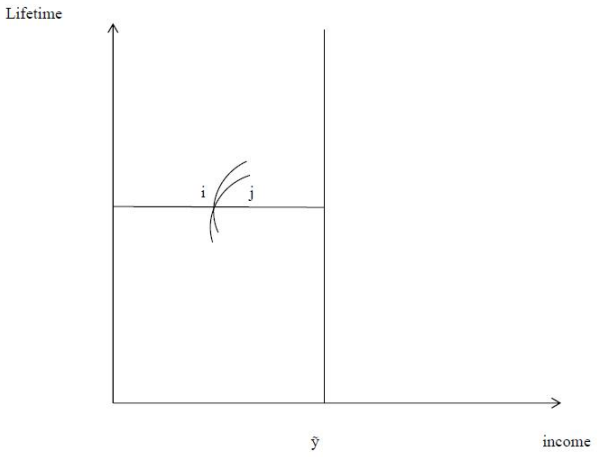

Figure 5: Well-being comparisons at $L = \overline{L^1}$

Then, in the second stage, we focus on individuals whose preferences do not share the same critical income level $\widetilde{y_i}$.

Consider first the case where two individuals i, j enjoy the same bundle $(y, \overline{L^1})$.

Suppose that their indifference curves of i, j intersect only once (single crossing), precisely at that bundle. Moreover, suppose, as on Figure 5, that the indifference curve of i is steeper, at $(y, \overline{L^1})$, than the indifference curve of j . We thus have that, at that bundle, individual i cares less about lifetime, and more about income, in comparison with individual j.

Then, by the Conditional Priority axiom, we obtain that, given $L_i = L_j \geq \overline{L^1}$, the well-being of individual j cannot exceed the well-being than individual i. However, since $L_i = L_j \leq \overline{L^1}$, we have also that the well-being of individual i cannot exceed the well-being than individual j. Hence it follows that the measured well-being must be equal for individuals i and j.

Consider now the case of no single crossing, that is, the indifference curves of individuals i, j intersecting at the bundle $(y, \overline{L^1})$ also intersect somewhere else in the space. That case can be dealt with as above. Indeed, in that case, it is possible to draw another indifference curve (let us say, for individual k) that intersects the indifference curves of i, j at the same bundle, but intersect these only once. Then, by the argument developed above, we have, given the single crossing, that individual k is exactly as well-off as individuals i and j. Hence, by transitivity of equality, the level of measured well-being must also be equal

for i and j in that case as well.

A similar argument can be developed for the case where indifference curves of i, j intersect only once (single crossing) at the bundle $(y, \overline{L^2})$. In that case, the same argument holds, and applying the Conditional Priority axiom implies that the same well-being level must be assigned to individuals i, j.

It follows from this that individuals with distinct preferences but same critical income can be ranked quite easily, in terms of well-being, whatever their bundle is. Clearly, for any bundle (y_i, L_i) on an indifference curve that crosses somewhere either the horizontal line at $L = \overline{L^1}$ in case of a life not worth living, or that crosses the horizontal line at $L = \overline{L^2}$ in case of a life worth living, we know, by the Sovereignty axiom, that the well-being measured at a bundle (y_i, L_i) is necessarily equal to the measured well-being of a hypothetical bundle located at the threshold lifetime, either $L = \overline{L^1}$ or $L = \overline{L^2}$, while remaining on the same indifference curve. For all these bundles, the measurement of well-being can thus be carried out by focusing on the equivalent income associated to the threshold lifetime $L = \overline{L^1}$ when the life is not worth living, or to the threshold $L = \overline{L^2}$ when life is worth living.

In other words, we have shown so far that individuals who have the same equivalent income have the same level of well-being, provided they share the same critical income making lifetime neutral.

Now, take the case of two individuals i, j with different preferences $\succeq_i, \succeq_j$, represented by different indifference maps, including different critical income levels $\widetilde{y_i} > \widetilde{y_j}$, and such that $EI(y_i, L_i, \succeq_i) - \widetilde{y_i} = EI(y_j, L_j, \succeq_j) - \widetilde{y_j}$. One can define a third indifference map $IM(\succeq_k)$, which is a translation of the indifference map of individual i, $IM(\succeq_i)$, and has the critical income level $\widetilde{y_j}$. This indifference map being a translation of $IM(\succeq_i)$, we can use the Translation axiom, which implies that $M(y_i, L_i, \succeq_i) = M(y_k, L_k, \succeq_k)$. But by construction of $\succeq_k$, $EI(y_j, L_j, \succeq_j) = EI(y_k, L_k, \succeq_k)$ and by the earlier stage of the proof, $M(y_j, L_j, \succeq_j) = M(y_k, L_k, \succeq_k)$. Thus, one has $M(y_i, L_i, \succeq_i) = M(y_j, L_j, \succeq_j)$.

By extension of the argument, if one assumed $EI(y_i, L_i, \succeq_i) - \widetilde{y_i} \gtrless EI(y_j, L_j, \succeq_j) - \widetilde{y_j}$, one would obtain $M(y_i, L_i, \succeq_i) \gtrless M(y_j, L_j, \succeq_j)$. This concludes the proof: $M(y_i, L_i, \succeq_i)$ is ordinally equivalent to $EI(y_i, L_i, \succeq_i) - \widetilde{y_i}$.

Second stage (Necessity).

It is straightforward to show that the measure $EI(y, L, \succeq_i) - \widetilde{y_i}$ satisfies the Sovereignty. Indeed, it ranks as equally good bundles that lie on the same indifference curve for a given individual. Moreover, it assigns a higher value to a bundle that lies on a lower indifference curve in case of a life not worth living,

and a higher value to a bundle that lies on a higher indifference curve in case of a life worth living.

One can also show that this index satisfies Conditional Priority. To see this, take the case of two individuals whose indifference curves cross above $\overline{L^1}$, and assume that the indifference curve of person i is steeper than the one of person j. When moving along those two indifference curves in the direction of $\overline{L^1}$, we see that the indifference curve of i will intersect the horizontal line drawn at $L = \overline{L^1}$ for a higher level of income, leading to a higher equivalent income, and a higher measured well-being level than the one of person j. This is clearly in line with what the Conditional Priority axiom requires.

Concerning the Translation Axiom, it is easy to see that, if two individuals i, j with translated indifference maps up to a distance $x > 0$ have the same lifetime, and lie at the same distance of their critical income level, then the associated measured well-being levels are equal, since the equivalent income of individual i is equal to the equivalent income of individual j $+x$. As a consequence, it follows that $EI(y_i, L_i, \succeq_i) - \widetilde{y_i} = EI(y_j, L_j, \succeq_j) + x - \widetilde{y_i} = EI(y_j, L_j, \succeq_j) + x - (\widetilde{y_j} + x) = EI(y_j, L_j, \succeq\ j) - \widetilde{y_j}$, so that the Translation axiom is satisfied.

Proof of Theorem 2

We only develop the parts of the proof that differ from that of Theorem 1.

First stage (Sufficiency).

We first focus on individuals whose preferences differ, but who have the same critical income level $\widetilde{y_i} = \widetilde{y_j} = \widetilde{y}$. We first show that if the indifference curves of two distinct individuals cross at the lifetime threshold 0 or $\overline{L}$, then the measure of well-being assigns the same well-being level to those two individuals.

Consider first the case where individuals have the same bundle $(y, 0)$ with $y < \widetilde{y}$. Suppose that the indifference curve of i is steeper, at $(y, 0)$, than the indifference curve of j . We thus have that, at that bundle, individual i cares less about lifetime, and more about income, in comparison with individual j. By Conditional Priority II, we know that individual i cannot be strictly worse off than individual j, that is: $M(y, 0, \succeq_i) \geq M(y, 0, \succeq_j)$. But notice that the indifference curve of i lies above the indifference curve of j in the (income, lifetime) space. I.e., one has $int\,[\mathbb{U}(y, 0, \succeq_j) \cap \mathbb{L}(y, 0, \succeq_i)] = \emptyset$. Hence, by Sovereignty II, $M(y, 0, \succeq_i) \leq M(y, 0, \succeq_j)$. As $M(y, 0, \succeq_i) \geq M(y, 0, \succeq_j)$ (by Conditional Priority II) and $M(y, 0, \succeq_i) \leq M(y, 0, \succeq_j)$ (by Sovereignty II), it follows that

$M(y,0,\succeq_i) = M(y,0,\succeq_j)$.

Note that, whereas the Conditional Priority II axiom presupposed single crossing of indifference curves, one can extend the above results to the case where indifference curves meeting at the bundle $(y,0)$ intersect more than once. To see this, take three indifference curves, denoted 1, 2, 3, that intersect only at $(y,0)$ with the indifference curve 1 being above the indifference curve 2, which is itself above the indifference curve 3. Then draw another indifference curve, called c, which also passes through $(y,0)$, but intersect indifference curve 2 also at another point, but without intersecting indifference curves 1 and 3 except at $(y,0)$. By Sovereignty II, we have that $M(y,0,\succeq_1) \leq M(y,0,\succeq_c) \leq M(y,0,\succeq_3)$. But by the argument developed in case of single-crossing (combination of Sovereignty II and Conditional Priority II), we have also that: $M(y,0,\succeq_1) = M(y,0,\succeq_3$). Moreover, by Sovereignty II, we have also: $M(y,0,\succeq_1) \leq M(y,0,\succeq_2) \leq M(y,0,\succeq_3)$. Hence it follows that: $M(y,0,\succeq_1) = M(y,0,\succeq_2) = M(y,0,\succeq_3)$. As a consequence, we obtain that: $M(y,0,\succeq_2) = M(y,0,\succeq_c)$.

A similar argument can be developed for the case where indifference curves of i,j intersect only once (single crossing) at the bundle $(y,\overline{L})$ for $y > \widetilde{y}$. In this case, the same argument holds, and applying the Conditional Priority II axiom with Sovereignty II implies that the same well-being level must be assigned to individuals i,j.

The final stage of the proof involving Translation is as in Theorem 1.

Second stage (Necessity).

It is straightforward to show that the measure $\widehat{EI}(y,L,\succeq_i) - \widetilde{y}_i$ satisfies Sovereignty II.

To check Conditional Priority II, take the case of two individuals whose indifference curves cross above $L = 0$, and assume that the indifference curve of person i is steeper than the one of person j. When moving along those two indifference curves in the direction of $L = 0$, we see that the indifference curve of i will intersect the horizontal line drawn at $L = 0$ for a higher level of income, leading to a higher equivalent income, and a higher measured well-being level than the one of person j. This is clearly in line with what the Conditional Priority II axiom requires.

Checking Translation is done as in Theorem 1.