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Managing Seller Conduct in Online Marketplaces and Platform Most-Favored Nation Clauses

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Abstract

This article investigates the incentive and ability of a platform to limit the extent of competition between the sellers it hosts. Absent contractual restrictions, a platform has an incentive to ensure competition between the sellers. This incentive can change with the introduction of so-called platform most-favored nation clauses (PMFN) that require the online sellers not to offer better conditions on other distribution channels. Such clauses can align the interests between sellers and platforms to restrict competition. I illustrate that a platform can stabilize seller collusion to its own benefit. These results offer a novel rationale to treat PMFNs with scrutiny.

JEL classification: L13, L40, L50

Keywords: platform MFN, digital economics, collusion in vertically-related markets, agency model

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1 Introduction

In an increasingly digitalized economy, consumers can purchase a wide range of goods and services via online platforms. Prominent examples are the Amazon Marketplace and online travel agencies such as Booking.com. These platform markets function well for consumers if there is a competitive environment between the sellers and a platform provider has an incentive to promote such an environment on its marketplace. Whereas the academic literature on this topic is relatively scarce, high-profile antitrust cases of illegal price fixing of sellers on such online platforms (discussed below) cast doubt on whether this premise is always fulfilled. These cases suggest that this form of collusive behavior is a concern for competition authorities more broadly.

The present article contributes to this debate with a specific focus on platform most-favored nation clauses (PMFN) by formally analyzing a platform's incentive and ability to encourage competition or collusion on its own marketplace in the presence of such clauses. A PMFN is a contractual requirement for sellers not to offer better prices and conditions on other distribution channels. Such clauses have triggered substantial antitrust scrutiny in several jurisdictions, and the recently enacted Digital Markets Act bans such clauses altogether for designated gatekeepers.[1] Moreover, PMFNs have played a role in cases of price fixing on online marketplaces.

This paper emphasizes that a platform's preferred seller conduct can change with the introduction of a PMFN. Table 1 depicts the main result schematically, which distinguishes whether a platform prefers seller competition or seller collusion. Initially, I take as given that sellers can coordinate on a collusive outcome and then focus below on how it can be sustained through tacit collusion in an infinitely-repeated game. I analyze a stylized model building on and extending Johansen and Vergé (2017) in which sellers have two distribution channels via which to sell to consumers. The first is a platform, which employs the *agency model*. This means the platform receives a commission for every intermediated transaction, and the sellers set the retail prices on the platform. The second distribution channel is a direct channel on which the online sellers do not incur per-transaction commission. I analyze both per-unit and revenue-sharing commissions on the platform and, for the sake of tractability, focus on a linear-demand specification.

	No PMFN	With PMFN
Seller competition	✓	
Seller collusion		✓

Table 1: Platform's preferred seller conduct.

The table shows schematically which form of seller conduct the platform prefers for the case of revenue-sharing commissions. For the case of per-unit commission, the result with PMFN relies on the qualifier that substitutability between sellers (interbrand competition) is sufficiently strong.

[1] EU Regulation 2022/1925, para 39.

Absent a PMFN, a platform realizes higher profits with seller competition than with seller collusion. To understand the result that collusion cannot be optimal in the case of per-unit commissions, note that at a given commission, seller collusion leads to a lower quantity sold on the platform, which *c.p.* decreases platform profits. Even at the optimal collusive rate, the platform would hence be better off inducing competition. Optimally adjusting the commission can only further increase profits. The same result is obtained in the case of revenue-sharing commissions. A seller's price strictly increases in the commission, and for a high enough commissions, exceeds the collusive price. Roughly speaking, I find that a platform prefers the combination of competition and a high commission to that of collusion and a lower commission – reinforcing that absent a PMFN a platform strictly prefers seller competition also with revenue-sharing commissions.

This result, however, changes if the platform introduces a PMFN. I show that the platform can charge higher commissions from colluding sellers than it can from competing ones. Importantly, this increase in the commission can render seller collusion more profitable for the platform. The result is driven by the fact that a PMFN induces sellers to charge uniform prices if they sell via the platform and the direct channel. If sellers compete and the platform charges high commissions, it is tempting for a seller to delist from the platform and to offer its product at a low price on the direct channel alone. This incentive to delist from the platform restricts the platform's commission. As delisting and competing aggressively on the direct channel alone is not a concern for the sellers if they coordinate their behavior, the platform can charge higher commissions from colluding sellers.

A PMFN therefore undermines a platform's incentive to ensure competition between sellers. Prior models instead (discussed in more detail in Section 2) emphasize that a PMFN has the potential to increase a platform's commission for a given degree of seller competition (Boik and Corts, 2016; Johnson, 2017). My findings provide a novel and complimentary theory of harm to treat PMFNs with scrutiny. Importantly, my findings suggest that a PMFN can be harmful even if commissions do not adjust after the introduction or removal of a PMFN, as seller collusion can be more sustainable with such clauses.

I continue to analyze the stability of collusion between online sellers in an infinitely-repeated game. This analysis is directly related to high-profile antitrust cases involving colluding sellers on digital platforms. A leading case is the famous e-book case that involved a PMFN, and in which five major publishers of e-books as well as the platform provider (Apple) were found guilty of engaging in illegal fixing of retail e-book prices.[2] In the year after the adoption of the PMFN, e-book prices for e.g., *New York Times* bestsellers increased by 40 percent as a result of this price fixing conspiracy (De los Santos and Wildenbeest, 2017). Interestingly, it was argued that an important role of the

[2] See Baker (2013) and Klein (2017) for comprehensive overviews of the antitrust case in the US, and Gaudin and White (2014) on the antitrust economics of this case. In 2011, the European Commission also opened an antitrust case against Apple and the e-book publishers with similar anticompetitive concerns (Case COMP/AT.39847-E-BOOKS).

PMFN was to ensure that the publishers have a credible threat to reduce the availability of their books on Amazon (Baker, 2013). As I demonstrate formally in this article, the credible threat of delisting due to a PMFN in fact has an important effect on the stability of seller collusion.

I determine to which extent the introduction of a PMFN allows a platform to affect the stability of tacit seller collusion. Sellers sustain collusion with grim-trigger strategies and coordinate their behavior in order to maximize their discounted stream of joint profits (i.e., coordinate on the best collusive outcome). In Appendix B, I focus on the case in which coordination on the joint profit maximum is not feasible and instead study constrained collusion. In the benchmark analysis, the platform charges time-constant commissions which appears to correspond most closely to real-world platform behavior. In line with the finding that a platform can benefit from seller collusion with a PMFN, I identify a range of commissions that the platform can choose in order to profitably stabilize collusion between sellers compared to the case without a PMFN. With a PMFN, the commission affects the sellers' distribution channel choices and thereby influences punishment and deviation behavior.

The remainder of the article is structured as follows: Section 2 discusses the related literature. In Section 3, I analyze the case of per-unit commissions in a static model in order to highlight that a PMFN can alter a platform's incentives regarding seller conduct. Section 4 focuses on the stability of seller collusion in an infinitely-repeated game. In Section 5.1, I show that the results are robust to the case of revenue-sharing commissions. Finally, in Section 5.2, I analyze the case in which the platform can change its commission rate during the infinitely repeated game complementing the main analysis with time-constant commissions. Section 6 concludes. All missing proofs can be found in Appendix A.

2 Related Literature

The present article contributes to three strands of the literature. First, it contributes to a nascent literature that links platform behavior to the interaction between sellers (e.g., their competitiveness) on the platform. Moreover, I focus on the so-called agency model, in which the sellers set the retail prices and the platform receives a commission payment, and which is predominantly used in digital markets. Second, this article fits into the analysis of collusion in vertically-related markets. This research analyzes how vertical relations and vertical restraints affect the stability of collusion at different stages of the vertical chain. Third, the present article relates to articles analyzing the competitive effects of the comparably new vertical restraint of platform most-favored nation clauses.

Platform Behavior and Seller Interaction. Given the platforms' importance as private rule-makers for the marketplaces that they have created, there is a surprisingly small related literature that relates strategic platform behavior to the interaction (e.g., competitiveness) between sellers on a platform. None of the existing literature investigates the impact of PMFN clauses on the incentives to limit competition between sellers. Teh (forthcoming) studies governance designs of a monopoly platform in order to affect on-platform competition. In particular, he studies governance decisions including seller entry, minimum quality standards, and on-platform search frictions. Karle et al. (2020) focus on the agglomeration and segmentation of sellers on different platforms and find that the competitive conditions between sellers shape the platform market structure. Relatedly, for a given market structure, Belleflamme and Peitz (2019) address the interaction of seller competition (i.e., negative within-group externalities on the platform) with platform pricing and product variety. Pavlov and Berman (2019) and Lefez (2020) study price recommendations that a platform sends to sellers which are active on the marketplace. Johnson et al. (2020) investigate a platform's ability to promote competition between sellers that use pricing algorithms with rules that reward firms with exposure to additional consumers if they cut prices.

Collusion in Vertically-Related Markets. The second strand of the literature studies the effects of vertical restraints on the stability of collusion. Closely related to the present analysis is Hino et al. (2019) who compare the stability of upstream collusion in the presence of either the traditional wholesale model (in which the retailer sets final consumer prices) or the agency model (in which sellers set these prices on the platform). I also focus on the agency model. Their main contribution is to analyze whether the distribution via wholesale contracts or agency contracts affects the stability of collusion between upstream sellers differently. They do not, however, analyze the use of platform most-favored nation clauses, which are common in markets that are operated via the agency model and have played an important role in multiple antitrust cases.

More broadly, the literature analyzes other forms of vertical restraints and their impact on collusion. The seminal articles by Nocke and White (2007) and Normann (2009) find that vertical integration can increase the stability of collusion between upstream firms. Relatedly, Biancini and Ettinger (2017) show that vertical integration generally also favors downstream collusion. The impact of resale price maintenance (RPM) on collusion on different levels at the vertical chain is analyzed by Jullien and Rey (2007), Overvest (2012), and Hunold and Muthers (2022). These articles demonstrate that the use of RPM can facilitate upstream collusion. Relatedly, I characterize the conditions under which a PMFN stabilizes seller collusion.

Further articles that study the effects of different contractual arrangements on collusion in vertically-related markets include Piccolo and Miklós-Thal (2012) and Gilo and Yehezkel (2020). They establish that contracts featuring slotting allowances and high wholesale prices during collusive periods can increase the stability of collusion between firms, as such a contract makes a deviation less profitable. Reisinger and Thomes (2017) study implications of the channel structure on seller collusion and find that seller collusion is easier to sustain if the sellers have independent retailers compared to the case in which they have a common retailer.

In non-vertical settings, contractual provisions have also been found to affect the stability of collusion between firms. Schnitzer (1994) analyzes the collusive potential of two forms of best-price clauses that guarantee consumers rebates on the purchase price if they find a better price for the purchased product. She finds that, in particular, contract clauses that promise consumers to meet price cuts from competing sellers have anticompetitive potential.

In the present paper, I emphasize that with a PMFN the platform's commission can affect punishment and deviation behavior differently and thereby stabilize seller collusion. Importantly, I demonstrate that the introduction of a PMFN can alter a platform's incentives to prevent collusion between online sellers. If a platform stabilizes seller collusion, a PMFN can therefore have a competition-weakening effect on the level of the sellers.

Competitive Effects of Platform Most-Favored Nation Clauses. The competitive effect of platform most-favored nation clauses have mostly been analyzed in static settings.[3] Recent articles such as Boik and Corts (2016), Johnson (2017), and Foros et al. (2017) support that such contract clauses have the potential to increase commissions and final consumer prices. In the presence of a PMFN, online sellers react less sensitively to changes in a platform's commission, which allows them to sustain higher rates in equilibrium than absent a PMFN. Moreover, these clauses may curtail entry in the platform market, as a new entrant in the platform market cannot win consumers by achieving lower retail prices on its own platform, and lead to excessive adoption of the platform's services as well as overinvestment in benefits to consumers (Edelman and Wright, 2015). Furthermore, Calzada et al. (2022) show that PMFNs can lead to segmentation of sellers across distribution channels in order to avoid interbrand competition.

In contrast, Johansen and Vergé (2017) show that accounting for the sellers' participation constraint can alleviate the anticompetitive price effects of a PMFN and can even lead to an increase in welfare if sellers have a direct channel through which to reach final consumers. Wang and Wright (2020) and Ronayne and Taylor (2022) analyze a setting in which a platform uses a PMFN in order to prevent sellers from engaging in *showrooming,* in which case consumers can search for products on the platform and buy on another channel in order to take advantage of lower prices. Both articles highlight that even in the presence of this efficiency defense for PMFNs, such clauses have the potential to harm consumers. Whereas the previous papers contrast the use of PMFNs with a complete ban of such clauses, Gomes and Mantovani (2021) study how a regulator should optimally cap a platform's commission.

These papers abstract from any effect of a PMFN on the competition between sellers on the platform and focus instead on the competition between the platform and other

[3]See Baker and Scott Morton (2017) and Fletcher and Hviid (2016) for comprehensive overviews of the competitive effects of PMFNs. They also informally discuss the effect of PMFN on the stability of upstream collusion but—in contrast to the present paper—neither the impact on the sellers' listing decisions nor the desirability of collusion for the platforms are considered in these discussions.

distribution channels. The present paper contributes to this literature by focusing on the competitive effects of PMFNs at the seller level, and their impact on the stability of seller collusion. This analysis, thus, offers a novel theory of harm regarding PMFNs that applies even in settings in which a platform does not adjust its commission after the introduction of a PMFN.

3 Static Model

3.1 Players and Environment

Consider an environment with two competing sellers $i \in \{1, 2\}$ producing differentiated products at constant symmetric marginal costs $c \geq 0$. Each seller offers a quantity q_{ij} of products to consumers through two distribution channels $j \in \{M, D\}$. The first distribution channel is a platform that provides a marketplace M and the second one is a direct channel D that sellers can use to reach consumers. For every intermediated transaction, the platform charges a commission from the sellers. Suppose that the marginal costs for an additional intermediated transaction between sellers and consumers on each distribution channel $j \in \{M, D\}$ is constant and normalized to zero.

3.2 Contracts and Timing

The platform uses the *agency model*, which implies that the sellers set retail prices on each distribution channel $j \in \{M, D\}$. Denote by p_{ij} the price that seller i sets on distribution channel j, and with $p_i = (p_{iM}, p_{iD})$ the vector of retail prices that seller i charges on both distribution channels. The vector $p = (p_1, p_2)$ is the vector of all retail prices. I analyze two forms of contracts between the platform and the sellers. For the main part of the analysis, I will focus on the case in which the platform receives a per-unit commission $w_M \geq 0$ from the sellers for every transaction that is intermediated on the platform.[4] The focus on simple per-unit commissions facilitates the analysis and allows for closed-form solutions.[5] Contract offers are observable.[6]

I compare the cases in which the platform can impose a platform most-favored nation clause (PMFN) in the contracts with the sellers and in which it cannot. A PMFN requires each seller to offer on the platform at least as favorable prices as on the direct channel, $p_{iM} \leq p_{iD}$.

[4] I obtain the same results when allowing for seller-specific commissions. In order to simplify the exposition, I impose symmetric commissions.

[5] In Section 5.1, I explain intuitively why the same economic forces are present when commissions are based on sellers' revenue, but formally the case is much less tractable. In line with the economic intuition, I numerically verify that the main economic results carry over to the case of revenue-sharing commissions.

[6] Platforms such as e.g., Amazon publicly announce the commission rates that apply for selling on their marketplace. See Johansen and Vergé (2017) for a related analysis with unobservable contract offers.

The timing of the game is as follows: First, the platform sets the commission. Second, sellers simultaneously decide whether to accept the platform's contract, and they set retail price p_{iB} on the direct channel as well as the retail price p_{iM} on the platform in case they accept the offer.[7] I solve this game for subgame perfect equilibria in pure strategies. If there is more than one equilibrium, I assume that the sellers coordinate on the Pareto-dominant equilibrium.[8] Below, I say that a seller is active on a distribution channel if it has accepted the contract offer (in the case of the platform), and sells a positive quantity to consumers via this channel.

3.3 Consumer Behavior

Building on Singh and Vives (1984) and Dobson and Waterson (1996), the demand function is derived from a representative consumer that maximizes a quadratic and strictly concave utility function of the following form

$$U(q) = \sum_{i=1,2} \sum_{j=M,D} q_{ij} - \frac{1}{2} q_{ij}^2 - \alpha \sum_{j=M,D} q_{1j} q_{2j} - \beta \sum_{i=1,2} q_{iM} q_{iD} - \alpha\beta \sum_{j \neq k = M,D} q_{1j} q_{2k},$$

where $q = (q_{1M}, q_{2M}, q_{1D}, q_{2D})$ is the vector of all quantities. The representative consumer considers both the sellers and the distribution channel to be imperfect symmetric substitutes. The parameter $\alpha \in (0,1)$ captures the degree of interbrand substitutability and $\beta \in (0,1)$ the degree of intrabrand substitutability. Utility maximization subject to a budget constraint yields the following inverse demand function

$$p_{ij}(q) = 1 - q_{ij} - \alpha q_{hj} - \beta q_{ik} - \alpha\beta q_{hk}, \tag{1}$$

with indices $i, h \in \{1,2\}$ for the sellers and $j, k \in \{M, D\}$ for the distribution channels. The representative consumer has demand for four differentiated seller-channel configurations. Inverting the system of inverse demand functions yields the direct demand function $q_{ij}(p)$. If both sellers offer products on both distribution channels, this demand function has the form

$$q_{ij}(p) \quad = \quad \frac{1}{(1-\alpha^2)(1-\beta^2)} \left(1 - \beta - p_{ij} + \beta p_{ik} - \alpha \left(1 - \beta - p_{hj} + \beta p_{hk}\right)\right). \tag{2}$$

[7] An alternative timing would be one in which sellers first take their listing decisions after the platform announced its commission and set prices in a subsequent stage. Calzada et al. (2022) analyze this timing in a model with two symmetric distribution channels. In the present setting with asymmetric distribution channels, the sequential timing can lead to multiple asymmetric equilibria in the static game, which renders the analysis considerably less tractable. The main results on the preferred seller conduct for the platform in Section 3 and the stability of seller collusion in Section 4, however, can also emerge in the case of sequential timing.

[8] For the case with PMFN, there is a small range of commission rates in which listing on both channels and listing on the direct channel alone is an equilibrium (see the proof of Lemma 2). This selection criterion picks the equilibrium in which both sellers list on both distribution channels. Further equilibrium refinements are not necessary to obtain a unique equilibrium.

If a seller delists from one of the distribution channels, demand for the other products is computed by replacing the price for that product-channel combination with a virtual price computed by equating q_{ij} to zero (Rey and Vergé, 2010).

This demand specifications has been widely employed to study collusion in vertically-related markets (Reisinger and Thomes, 2017; Hino et al., 2019) and PMFNs in the agency model (Boik and Corts, 2016; Johansen and Vergé, 2017; Calzada et al., 2022).[9] The demand function captures that a seller can reach more consumers if it is present on both distribution channels than if it is present only on one channel. Additionally, it implies that there is a substitution pattern for the product of a seller offered on different channels. This assumption is, for instance, supported by Cazaubiel et al. (2020) who document empirically that a hotel chain's direct channel is a credible alternative to an online travel agent such as Expedia. Similarly, estimates by Duch-Brown et al. (2017) show that there is considerable sales diversion between online and offline distribution channels for consumer electronics.

3.4 Analysis of the Static Model

In this section, I analyze how the introduction of a PMFN affects the profitability of seller competition for the platform. In order to do so, I characterize the static competitive market outcome and compare it to the outcome in which sellers can coordinate on the joint profit-maximizing behavior (i.e., the best collusive prices from the sellers' point of view). Throughout this section, I abstract from the exact mechanism supporting this monopolistic outcome in order to highlight the platform's incentive to restrict seller competition. In the following section, I analyze an infinitely-repeated game in order to study the stability of such collusive market outcomes when the platform can affect the stability of seller collusion by means of its commission.

For the analysis of per-unit commissions, I normalize the sellers' marginal costs to zero in order to simplify the exposition without affecting the qualitative results. For the analysis of revenue-sharing commissions below, I explicitly consider positive marginal costs.

The profit function of seller i that is present on both distribution channels is

$$\pi_i(p) \quad = \quad (p_{iM} - w_M)\, q_{iM}(p) + p_{iD} q_{iD}(p)\,. \tag{3}$$

The platform's profit is

$$\Pi_M(w_M) = w_M \sum_{i\in\{1,2\}} q_{iM}(p)\,, \tag{4}$$

[9] As noted by Johansen and Vergé (2017), this specification has the unusual feature $\partial q_{ij}/\partial p_{hk} < 0$. It is, however, important to keep in mind that the total quantity of one seller $q_{iM} + q_{iD}$ increases if the second seller increases its prices on either channel. Moreover, this specification is useful to keep the exposition simple and tractable and to avoid imposing restrictions on the demand parameters.

No Platform Most-Favored Nation Clause. Absent a PMFN, the presence of a positive commission w_M that sellers must pay to the platform leads to an incentive for the seller to charge different prices on each distribution channel. Given demand symmetry and the higher distribution costs on the platform, each seller charges lower prices on the direct channel if not restricted by a PMFN. Sellers' conduct leads either to competitive retail prices denoted by $\tilde{p}$ or collusive ones denoted by $\bar{p}$. The following lemma summarizes the seller behavior for both forms of conduct absent a PMFN.

Lemma 1. *For $w_M \in [0, 1-\beta]$ the sellers list on both distribution channels. Absent a PMFN (NP), seller i sets the retail prices*

$$\tilde{p}_{iM}^{NP}(w_M) = \frac{1-\alpha+w_M}{2-\alpha}, \text{ and } \tilde{p}_{iD}^{NP}(w_M) = \frac{1-\alpha}{2-\alpha}, \tag{5}$$

on distribution channel $j \in \{M, D\}$ if sellers compete, and

$$\bar{p}_{iM}^{NP}(w_M) = \frac{1+w_M}{2}, \text{ and } \bar{p}_{iD}^{NP}(w_M) = \frac{1}{2}, \tag{6}$$

in the monopolistic case.

The restriction on the commission $w_M \in [0, 1-\beta]$ ensures that—independent of their conduct—sellers prefer to be active on both distribution channels instead of listing on the direct channel only. As I verify below, the platform does not indeed find it profitable to charge higher commissions than $1-\beta$ because then sellers are not willing to list on the platform. The result of Lemma 1 shows that with collusion the sellers successfully eliminate the interbrand competition (as measured in α) on both distribution channels. This implies that retail prices are higher with collusion than they are with seller competition. Moreover in the linear demand specification, retail prices on distribution channel j are independent of the costs of distribution on the other channel $k \neq j$ and the degree of intrabrand substitutability β.[10]

Based on the seller behavior described in Lemma 1, the proposition below shows that the platform prefers seller competition to seller collusion. Intuitively, fixing the commission, the platform prefers competition over collusion whenever delisting is not a concern and lower competitive prices on both channels lead to an increase in the amount of sales on the platform. The latter seems to be a weak condition that holds for instance in the linear demand specification. Given that a platform benefits at any such fixed commission from seller competition, it also does so for the optimal commission. The following proposition summarizes the optimal platform behavior absent a PMFN and the resulting profits.

[10] An increase in β increases the sales diversion between the two channels (which c.p. increases prices) but also the demand sensitivity on each channel (which c.p. decreases prices). In equilibrium, both effects cancel each other out. An increase in the commission w_M makes selling via the direct channel more attractive, which c.p. decreases the direct channel price. A higher commission, however, also increases the prices on the platform, which c.p. also increases the prices on the direct channel. Again, both effects cancel each other out in equlibrium.

Proposition 1. *Without a PMFN, the optimal commission is*

$$w_M^{NP} = \frac{1-\beta}{2}, \tag{7}$$

which is independent of the seller conduct. The resulting platform profits depending on seller conduct are

$$\tilde{\Pi}_M^{NP}\left(w_M^{NP}\right) = \frac{1-\beta}{2\left(2-\alpha\right)\left(1+\alpha\right)\left(1+\beta\right)}, \tag{8}$$

$$\bar{\Pi}_M^{NP}\left(w_M^{NP}\right) = \frac{1-\beta}{4\left(1+\alpha\right)\left(1+\beta\right)}, \tag{9}$$

with $\tilde{\Pi}_M\left(w_M\right) > \bar{\Pi}_M\left(w_M\right)$ *for all* $w_M \in [0, 1-\beta]$.

Note that $w_M^{NP} < 1-\beta$, so that both sellers are active on both distribution channels. Importantly, since the platform's profit is greater when sellers compete than if they collude, a platform prefers to induce a competitive environment absent a PMFN and is not inclined to create a pro-collusive trading environment for the sellers.

The result that the optimal per-unit-commission is independent of the seller conduct is driven by the linear-demand specification. For non-linear demand specifications the commission in general changes for different seller conducts. As described above, however, the constant commission is not a necessary condition for the result that the platform prefers seller competition to hold. Intuitively, as long as seller collusion leads to a lower quantity sold on the platform for a given commission and delisting is not a concern, the platform is better off inducing competition.

Platform Most-Favored Nation Clause. Next, I turn to the analysis of the profitability of seller competition for the platform with a PMFN. In this case, it is important to take into account the sellers' listing decision on the platform as highlighted in Johansen and Vergé (2017). Due to the contractual restrictions of the PMFN, a seller is induced to charge higher than optimal prices on its direct channel if it is active on both distribution channels. It may therefore be more profitable for a seller to delist from the platform in order to charge more profitable prices on its direct channel and save the commission payments that accrue for every transaction via the platform. Hunold et al. (2018) and Cazaubiel et al. (2020) provide empirical evidence that listing decisions are economically important dimensions of adjustments in the hotel sector if online travel agents impose a PMFN. In the following, I characterize how a PMFN affects seller behavior in the case of competitive and monopolistic seller conduct.

Competitive Case. If present on both distribution channels, competing sellers maximize the profit function in Equation (3) subject to the constraint that $p_{iM} \leq p_{iD}$. If active on both channels, denote the resulting uniform retail price that seller i charges on both distribution channels by $\tilde{p}_i^P$. To show that these retail prices constitute an equilibrium, it is necessary to verify that no seller has an incentive to delist from the platform (explained below). In particular, taking as given that the rival seller h is active on both distribution channels and is anticipated to charge $\tilde{p}_h^P$, seller i can realize a profit of

$$\max_{p_{iD}} \ \pi_i \left(p_{iD}, \infty, \tilde{p}_h^P\right) = p_{iD} q_{iD} \left(p_{iD}, \infty, \tilde{p}_h^P\right), \tag{10}$$

from delisting from the platform, where ∞ indicates that seller i is not active on the platform. By delisting, a seller can avoid the contractual restrictions of a PMFN and charge more profitable prices on the direct channel.

If the profit on the direct channel alone (Equation (10)) exceeds the profit from being active on both channels, it cannot be an equilibrium for both sellers to be active on both channels. In the following lemma, I verify that this is the case if the platform's commission is too high and that in equilibrium both sellers are active on the direct channel only and offer no products via the platform in this case. Denote with $\tilde{\pi}_{i(D)}^P = \pi_i \left(\tilde{p}_D^P, \infty\right)$ seller i's equilibrium profit in case both sellers are only present on the direct channel. The following lemma summarizes the listing decision and prices of competing sellers as a function of the commission w_M if sellers compete.

Lemma 2. *Suppose that the platform imposes a PMFN* (P). *Both competing sellers are active on both distribution channels if*

$$w_M \quad \leq \quad \tilde{w}_{max} = \frac{4\left(1-\alpha\right)\left(2-\sigma\left(\beta\right)\right)}{4-\alpha\left(4-\sigma\left(\beta\right)\right)}, \tag{11}$$

with $\sigma\left(\beta\right) = \sqrt{2\left(1+\beta\right)}$, *and set the same retail price on both channels*

$$\tilde{p}_i^P\left(w_M\right) = \left(2-2\alpha+w_M\right)/\left(4-2\alpha\right). \tag{12}$$

Otherwise, both sellers are only active on the direct channel and set direct channel prices of $\tilde{p}_{iD}^P = \left(1-\alpha\right)/\left(2-\alpha\right)$ *as specified in Equation (5) in Lemma 1.*

The result of Lemma 2 provides a threshold value $\tilde{w}_{max}$ for the maximal commission on the platform for which sellers are active on both distribution channels (similar to Johansen and Vergé, 2017). If sellers are active on the platform, they optimally must set the same retail prices on both distribution channels (as they are contractually forced not to offer lower prices on the direct channel). In contrast to the case without a PMFN, the equilibrium retail price on distribution channel $j \in \{M, D\}$ therefore depends on the costs of distribution on both channels. In particular, the retail price on the direct channel is affected by the commission w_M that the platform charges for every intermediated transaction. A comparison of the equilibrium retail prices with and without a PMFN reported in Lemma 1 and Lemma 2 shows that the pass-through of the commission w_M on the retail price on the platform p_M is lower with a PMFN than without. Intuitively, a seller that wants to raise the retail price on the platform also needs to suboptimally increase it on the direct channel, which renders such adjustments less responsive than without a PMFN.

This property is at the core of the analyses that relate PMFNs to reduced competition on the platform level (see, for instance, Boik and Corts, 2016).

For commissions above the threshold $\tilde{w}_{max}$, it cannot be an equilibrium that both competing sellers are present on both channels as it is unilaterally profitable for a seller to delist from the platform if $w_M > \tilde{w}_{max}$. By delisting, a seller can charge more profitable prices on the direct channel and additionally benefits from the fact that the competing seller, which is anticipated to be present on both channels, is contractually induced to charge higher-than-optimal prices on the direct channel. Lemma 2 establishes that in this case both sellers are only active on the direct channel and optimally set the same retail prices as in the case without contractual restrictions specified in Lemma 1.

Joint Profit-Maximizing (Monopolistic) Case. The unilateral incentive to delist is not a concern for sellers if they can coordinate their listing decisions and retail prices in order to maximize their joint profits $\pi_{12} = \pi_1 + \pi_2$ because sellers internalize that delisting and competing aggressively on the direct channel alone cannibalizes the profits of the second seller. If present on both channels, the collusive maximization problem stipulates

$$\begin{aligned} \max_{p} \ \pi_{12}(p) &= \sum_{i \in \{1,2\}} (p_{iM} - w_M) \, q_{iM}(p) + p_{iD} q_{iD}(p), \\ & s.t. \ \ p_{iM} \leq p_{iD}. \end{aligned} \tag{13}$$

As in the case with seller competition, the constraint on the retail prices is binding in equilibrium. Denote the resulting collusive retail price on both distribution channels as $\bar{p}_i^P$. Sellers delist from the platform if the commission is such that their joint profits are larger on the direct channel alone than on both distribution channels. If only active on the direct channel, the sellers maximize

$$\max_{p_D} \ \pi_{12}(p_D, \infty) = \sum_{i \in \{1,2\}} p_{iD} q_{iD}(p_D, \infty), \tag{14}$$

where ∞ denotes that sellers are not active on the platform. Denote the monopolistic seller profit on the direct channel alone as $\bar{\pi}_{i(D)}^P$. In the following lemma, I characterize the behavior of colluding sellers.

Lemma 3. *Suppose that the platform imposes a PMFN* (P)*. Monopolistic sellers are active on both distribution channels if*

$$w_M \ \leq \ \bar{w}_{max} = 2 - \sqrt{2(1+\beta)} = 2 - \sigma(\beta), \tag{15}$$

with $\bar{w}_{max} > \tilde{w}_{max}$*, and set retail prices of*

$$\bar{p}_i^P(w_M) \ = \ (2 + w_M)/4. \tag{16}$$

Otherwise, sellers coordinate to be present on the direct channel only and set $\bar{p}_{iD}^{P} = 1/2$.

The threshold value $\bar{w}_{max} > \tilde{w}_{max}$ below which colluding sellers are willing to list on both distribution channels is larger than the threshold value $\tilde{w}_{max}$ for the competing sellers due to the fact that collusion allows sellers to overcome the unilateral incentive to delist from the platform. This implies that colluding sellers may be active on both distribution channels while such listing decisions cannot be sustained in equilibrium in the case of seller competition. Moreover, this result shows that a profit-maximizing platform, which imposes a PMFN, will never charge commissions above $w_M > \bar{w}_{max}$ as neither competing nor colluding sellers are willing to list on the platform and accept the contractual restrictions from a PMFN for such high commissions.

Platform Profits. As derived in Lemmas 2 and 3, the sellers' participation constraints restrict the platform's commission. In fact, the commissions that maximize the platform's profit are the same as the threshold values that make competing and colluding sellers indifferent to their outside option of being active on the direct channel only. Recall from the comparison of seller competition and seller collusion that this threshold value is smaller in the case of seller competition ($\bar{w}_{max} > \tilde{w}_{max}$). As a result, a platform can enforce a higher commission from colluding sellers than it can from competing sellers. This effect makes a platform more lenient toward seller collusion and can lead to the platform obtaining higher profits with seller collusion than with seller competition.

Proposition 2. *If seller conduct is competitive, the commission that maximizes the platform's profit with a PMFN is equal to the threshold value $\tilde{w}_M^P = \tilde{w}_{max}$ (Equation (11)). In the monopolistic case, this commission is equal to the threshold value $\bar{w}_M^P = \bar{w}_{max}$ (Equation (15)). The resulting platform profits depending on seller conduct are*

$$\tilde{\Pi}_M^P\left(\tilde{w}_M^P\right) = \frac{8(1-\alpha)(2-\sigma(\beta))\sigma(\beta)}{(1+\alpha)(1+\beta)(4-\alpha(4-\sigma(\beta)))^2}, \quad (17)$$

$$\bar{\Pi}_M^P\left(\bar{w}_M^P\right) = \frac{(2-\sigma(\beta))\sigma(\beta)}{2(1+\alpha)(1+\beta)}, \quad (18)$$

with $\sigma(\beta) = \sqrt{2(1+\beta)}$. The platform's profit with seller collusion is larger than with seller competition if interbrand substitutability α is sufficiently large. That is, $\bar{\Pi}_M\left(\bar{w}^P\right) > \tilde{\Pi}_M^P\left(\tilde{w}^P\right)$ if $\alpha > \bar{\alpha} = (16-8\sigma(\beta))/\left(16-8\sigma(\beta)+\sigma(\beta)^2\right)$.

Proposition 2 shows that there is more scrutiny warranted for marketplaces for a PMFN because the platform may adopt a pro-collusive trading environment for the sellers that it hosts. The result captures that monopolistic seller behavior has two diverging effects on the platform profits. First, joint profit-maximizing behavior of the sellers allows the platform to charge higher commissions without violating the sellers' participation constraint. This effect increases platform profits. Second, seller collusion leads sellers to charge higher retail prices at given commissions. This reduces demand and thereby decreases platform profits. The first effect dominates the second one if the degree of interbrand substitutability α is sufficiently large. If substitutability is large, the threat of a rival delisting and stealing market share is so severe that the platform has no incentive to discourage seller collusion ($\alpha > \bar{\alpha}$). The threshold value $\bar{\alpha}$ decreases in the degree of intrabrand competition β. This implies that the platform prefers seller collusion more the stronger the substitutability of the distribution channels is.

In Section 5.1, I analyze the case of revenue-sharing commissions. Importantly, the results reveal that the platform prefers monopolistic seller behavior for all degrees of interbrand substitutability α, and hence with this contract form a platform is even more prone to limit seller competition than with per-unit commissions.

Profitability of Platform Most-Favored Nation Clauses. In various digital markets, platform providers have revealed a strong interest in imposing a PMFN.[11] Comparing the platform's profit levels reported in Proposition 1 (for the case without a PMFN) and Proposition 2 (with a PMFN) allows to characterize the parameter space in which seller collusion and the use of a PMFN are the most profitable configuration for the platform. This result is summarized in

Proposition 3. *The use of a PMFN in combination with seller collusion is the most profitable configuration for the platform if*

$$\frac{16 - 8\sigma(\beta)}{16 - 8\sigma(\beta) + \sigma(\beta)^2} < \alpha < \frac{3\sigma(\beta) - 2}{2\sigma(\beta)}, \tag{19}$$

with $\sigma(\beta) = \sqrt{2(1+\beta)}$. *The intervall is nonempty for all* $\beta \in (0, 1)$.

Proof. Direct implication of comparing the platform profit levels provided in Propositions 1 and 2. □

This result shows that seller collusion in the combination with a PMFN is the most profitable configuration for the platform for intermediate values of α. The interval provided in Equation (19) allows for a wide range of degrees of interbrand competition α if the degree of interbrand competition β is large. For instance, if $\beta = 9/10$ the interval in which the platforms prefers seller collusion with a PMFN is $0.096 < \alpha < 0.99$. The lower bound is the threshold value provided in Proposition 2 that ensures that with a PMFN the platform prefers seller collusion over seller competition. The upper bound of the interval stems from the profit comparison of seller collusion and PMFN $\left(\bar{\Pi}_M^P\right)$ with seller competition and no PMFN $\left(\tilde{\Pi}_M^{NP}\right)$.

[11] See, for instance, the blog post Amazon Gets Bulk of Complaint in AAP Filing With US Trade Commission on www.publishingperspectives.com or Bundeskartellamt calls Booking.com's best-price clauses anticompetitive on www.triptease.com (last access, April 29, 2021). Another reason that makes a PMFN desirable for the platform that is outside of this model is the avoidance of *showrooming*, which means that consumers search on the platform for an online seller and purchase the product on the distribution channel that offers the product at the lowest price (see Wang and Wright (2020); Ronayne and Taylor (2022)).

Moreover, it is feasible to compare the profitability of a PMFN for the platform if the seller conduct is fixed: With seller competition (comparing the profits in Equations (8) and (17)), the comparison yields that a platform benefits from a PMFN only if the interbrand competition between the online sellers is not too strong, because otherwise the commission with a PMFN is too small to make a PMFN profitable.[12] In contrast, this case distinction on the intensity of the interbrand competition regarding the profitability of a PMFN does not apply in the case of colluding sellers. If sellers collude (comparing the profits in Equations (9) and (18)), the platform unambiguously prefers a PMFN. These results show that a platform can have a strong incentive to impose a PMFN if it faces monpolistic sellers and vice versa generally to foster seller collusion if it imposes a PMFN. A PMFN is therefore particularly profitable for a platform in the monopolistic case.

Online sellers typically complain about the use of PMFNs, suggesting that seller profits are higher absent a PMFN. For competing sellers this result is supported in the theoretical studies establishing the main theory of harm discussed in Section 2 (e.g., Foros et al., 2017).[13] Relatedly, I also find that monopolistic sellers realize lower profits with a PMFN than absent this clause. Moreover, comparing across different forms of seller conduct yields that sellers dislike a PMFN. In particular, seller competition absent a PMFN yields a higher profit than seller collusion with such a clause.

4 Dynamic Model

4.1 Infinitely-Repeated Game

In this section, I analyze the industry structure introduced above in an infinitely-repeated game in discrete time $t = 0, ..., \infty$. So far, I have imposed that sellers can coordinate on the joint profit-maximizing behavior without focusing on the exact stabilizing mechanism. The infinitely-repeated-game framework allows to study a possible mechanism with which such seller behavior can be supported. Moreover, this approach captures in a stylized fashion the collusive behavior documented and discussed in the e-book and other antitrust cases discussed in the Introduction. In Appendix B, I study the case in which coordination on the joint profit maximum is not feasible and sellers instead coordinate on constrained collusion (i.e., the highest prices that fulfill the incentive-compatibility constraint).

My focus is on the stability of collusion between the sellers under contracts with and without a PMFN. Sellers have a common discount factor $\delta \in (0, 1)$, and aim to maximize

[12] In particular, $\tilde{\Pi}_M^P \left(w_M^{NP}\right) > \tilde{\Pi}_M^{NP} \left(w_M^{NP}\right)$ if $\alpha < \left(8 - 2\sigma\left(\beta\right)\right) / \left(7 - \beta\right)$. See Johansen and Vergé (2017) for a similar condition.

[13] In contrast, Johansen and Vergé (2017) find that PMFNs can benefit all the actors (platforms, sellers, and consumers) in an industry. The result that profits of non-cooperative sellers can increase due to a PMFN is also supported in the present analysis for the case of large intrabrand substitution β (profits are reported in Appendix A). In this case, distribution channels are easily substitutable for the online seller, and the seller's participation constraint to be active on the platform commands a low commission.

present-discounted stream of profits

$$\sum_{t=0}^{\infty} \delta^t \pi_i \left(p_t\right), \tag{20}$$

where p_t is the vector of retail prices in period t, and π_i retailer i's stage profit at these retail prices.

Formally, the platform does not take part in the collusive agreement in the sense that it is involved in the joint-profit maximizing scheme of the sellers. As shown in Section 3 however, the platform generally has a preferred conduct and might choose its commission accordingly in order to influence seller conduct. I assume that the platform sets a constant and symmetric commission at the beginning of the first period that does not change in future periods.[14] In fact, this pricing behavior appears to be in line with actual platform behavior and this assumption therefore captures that pricing and listing decisions are generally more short term than commission changes. For instance, in the online hotel booking sector, a recent report by EU competition authorities indicates that there were little to no changes in the base and effective commissions paid by hotels to online travel agencies during the period 2014 to 2016.[15] This observation is perhaps surprising as the hotel sector is subject to strong seasonality effects and one could expect a platform to adjust its commission more frequently as reaction to changing market conditions (such as demand characteristics) change. Similarly, the commission that Apple negotiated with the major e-book publishers was set at 30 percent and did not change during and after the collusive period (Foros et al., 2017).[16]

I solve for a subgame-perfect Nash equilibrium of the infinitely-repeated game between the sellers based on this constant commission. On the path of play, the sellers coordinate to achieve in each period the joint profit maximum (i.e., the most collusive outcome) by coordinating their listing decisions and setting the collusive price $\bar{p}_{ij}$ on each active distribution channel j. For brevity, it is convenient to suppress that the retail prices depend on the constant commission.

I analyze the stability of collusion in an equilibrium sustained through grim-trigger strategies (Friedman, 1971). If a seller deviates from the collusive scheme, it makes its listing decision and sets $\hat{p}_{ij}$ such that its deviation profit is maximized.[17] After a deviation, all sellers revert to playing their static Nash equilibrium listing decision and prices $\tilde{p}_{ij}$ for all future periods. In Appendix B, I numerically analyze the case in which incentive-

[14] By offering asymmetric commissions, the platform would induce sellers with asymmetric costs of distribution. This cost asymmetry can affect collusive stability if sellers continue to collude on the joint profit-maximizing retail prices in this model. The sellers are, however, able to offset this effect on their critical discount factor by agreeing on a different distribution of profits or side payments. Both strategies render the effect of asymmetric costs of distribution on the stability of collusion negligibly small.

[15] See the Report on the Monitoring Exercise Carried out in the Online Hotel Booking Sector by EU Competition Authorities in 2016 (last access, April 29, 2021).

[16] This assumption implies that only the level of the commission but not changes in the commission can affect the stability of seller collusion.

[17] Note that the deviation can involve another listing decision than that of the seller who sticks to the collusive agreement.

compatibility prevents sellers from coordinating on profit-maximizing prices and sellers instead coordinate on the highest feasible (i.e., incentive-compatible) prices. The results are qualitatively comparable, and reinforce the finding that the platform can benefit from seller collusion if it imposes a PMFN.

Formally, in any period $t = 0, 1, ..., \infty$ in which sellers coordinate on collusion, seller i sets the collusive prices $\bar{p}_{ijt}$ on both distribution channels $j \in \{M, D\}$. For any future period t, it holds that

$$p_{ijt} = \begin{cases} \bar{p}_{ij} & \text{if } p_{hj\tau} = \bar{p}_{hj} \; \forall \; \tau < t, \;\; h \in \{1, 2\}\,, \;\; j \in \{M, D\}\,, \\ \tilde{p}_{ij} & \text{if otherwise.} \end{cases} \tag{21}$$

Denote the corresponding stage-game profits that are associated with the prices defined above by $\bar{\pi}_i$, $\tilde{\pi}_i$, and $\hat{\pi}_i$. The condition that there is no unilateral incentive to deviate from the collusive scheme is

$$\sum_{t=0}^{\infty} \delta^t \bar{\pi}_i \geq \hat{\pi}_i + \sum_{t=1}^{\infty} \delta^t \tilde{\pi}_i. \tag{22}$$

The discounted stream of profits from sticking to the collusive scheme needs to exceed the profit that an upstream firm can obtain from cheating and reverting afterwards to the static Nash equilibrium for all future periods. Rearranging yields that the common discount factor needs to exceed

$$\delta \geq \underline{\delta} = \frac{\hat{\pi}_i - \bar{\pi}_i}{\hat{\pi}_i - \tilde{\pi}_i} \;\; \in [0, 1]\,, \tag{23}$$

where $\underline{\delta}$ denotes the seller's critical discount factor for collusion to be sustainable.

In order to ensure that both sellers are active and sell positive quantities in all periods of the infinitely-repeated game, I assume that the degree of interbrand substitutability is not too large:

Assumption 1. $\alpha < \sqrt{3} - 1$.

In particular, this assumption ensures that a seller that charges collusive prices sells a positive quantity to the consumers even if the other seller deviates from the collusive scheme and charges lower prices in order to maximize the current-period profits (see also Ross, 1992).[18]

4.2 Analysis of the Dynamic Model

The aim of this section is to characterize how the introduction of a PMFN changes the stability of seller collusion by altering punishment and deviation behavior compared to the case without a PMFN.

[18]Recall that the profitability of seller collusion with PMFN (Proposition 2) depends on $\alpha > \bar{\alpha} = (16 - 8\sigma(\beta)) / \left(16 - 8\sigma(\beta) + \sigma(\beta)^2\right)$. Note that $\sqrt{3} - 1 > \bar{\alpha}$, $\forall \beta \in (0, 1)$ and hence this case is consistent with Assumption 1. Moreover, for the case of revenue-sharing commissions seller collusion is profitable for the platform for all degrees of interbrand substitutability α.

No Platform Most-favored Nation Clause. Given Lemma 1 specifies the static competitive and collusive profits, I next derive a seller's optimal deviation profits. The following lemma summarizes this for the case without a PMFN.

Lemma 4. *Absent a PMFN (NP), a deviating seller i is active on both distribution channels and optimally sets*

$$\hat{p}_{iM}^{NP}\left(w_M\right)=\frac{2-\alpha+\left(2+\alpha\right)w_M}{4},\ \text{and}\ \ \hat{p}_{iD}^{NP}\left(w_M\right)=\frac{2-\alpha}{4},\tag{24}$$

for all $w_M \in [0, 1-\beta]$.

If a seller deviates from the collusive agreement, it finds it profitable to be active on both distribution channels. The deviation prices that maximize the current-period profits of a seller in Equation (24) are below the collusive prices (Equation (6)) and above the competitive prices (Equation (5)). Independent of the conduct, the sellers prefer to set lower prices on the direct channel than on the platform as the costs of distribution on the direct channel are lower.

Based on the results in Lemmas 1 and 4 that characterize seller behavior in competitive, collusive, and deviation periods, the following proposition states the critical discount factor above which collusion is supported by a subgame-perfect equilibrium for the sellers.

Proposition 4. *Without a PMFN (NP), the critical discount factor is*

$$\underline{\delta}^{NP}=\frac{\left(2-\alpha\right)^2}{8-8\alpha+\alpha^2},\tag{25}$$

for both sellers $i \in \{1,2\}$. It increases in the degree of interbrand competition α, and is independent of the degree of intrabrand competition β and the commission w_M.

The result of Proposition 4 shows that the critical discount factor absent a PMFN is independent of the seller's cost level. This implies that a platform's per-unit commission does not affect the seller's incentive constraint for collusion to be stable in this setting. Relatedly, the degree of intrabrand substitutability between the distribution channels (as measured by β), which indirectly affects the per-unit commissions that the platform can impose, does not affect the sellers' critical discount factor either. The reason for this result is that all profit levels are proportional to the same function depending on w_M and β and, therefore, these parameters cancel out in the critical discount factor $\underline{\delta}^{NP}$. Moreover, with per-unit commissions, the critical discount factor $\underline{\delta}^{NP}$ depends on the degree of interbrand competition and increases in α, which shows that a higher degree of substitutability between the sellers decreases the stability of collusion.

Platform Most-Favored Nation Clause. Next, I turn to the analysis of the stability of collusion with a PMFN. Due to the contractual restrictions of the PMFN, a seller is induced to charge higher than optimal prices on its direct channel if it is active on both distribution channels. This can affect a seller's listing decision (Johansen and Vergé, 2017): with a PMFN, a seller may prefer to delist and charge the optimal price on the direct channel. This allows the seller to divert sales from the high-commission platform channel to the commission-free direct channel.[19]

In the following, I characterize how a PMFN affects the behavior of a deviating seller. As colluding and competing sellers, a deviating seller also needs to decide whether to be active on both channels or only on the direct channel. First, consider a deviating seller i that decides to be active on both channels and takes as given that the second seller h is also present on both channels and sets collusive prices $\bar{p}_h^P(w_M) = (2 + w_M)/4$ (derived in Lemma 3). The deviating seller then sets retail prices p_i in order to maximize

$$\max_{p_i} \ \pi_i\left(p_i, \bar{p}_h^P\right) = (p_{iM} - w_M)\, q_{iM}\left(p_i, \bar{p}_h^P(w_M)\right) + p_{iD} q_{iD}\left(p_i, \bar{p}_h^P(w_M)\right), \qquad (26)$$

subject to the constraint that $p_{iM} \leq p_{iD}$. Alternatively, the deviating seller may decide to delist from the platform, and to offer products only via the direct channel. In this case, such a seller sets the retail price p_{iD} in order to

$$\max_{p_{iD}} \ \pi_i\left(p_{iD}, \infty, \bar{p}_h^P(w_M)\right) = p_{iD} q_{iD}\left(p_{iD}, \infty, \bar{p}_h^P(w_M)\right). \qquad (27)$$

Denote the profit of a deviating seller that is present on the direct channel only as $\hat{\pi}_{i(D)}^P(w_M)$. The next lemma summarizes the optimal deviation behavior.

Lemma 5. *Suppose the platform imposes a PMFN (P). If seller i deviates from collusion, it is active on both distribution channels if*

$$w_M \ < \ \hat{w}_{max} = \frac{2(2-\alpha)(2-\sigma(\beta))}{4-\alpha(2-\sigma(\beta))}, \qquad (28)$$

and sets $\hat{p}_i^P(w_M) = (4 - 2\alpha + (2+\alpha) w_M)/8$. Otherwise, a deviating seller is only active on the direct channel and charges $\hat{p}_{iB}^P(w_M) = (4 - \alpha(2 - w_M))/8$ while the non-deviating seller stays active on both channels. One has

$$\tilde{w}_{max} < \hat{w}_{max} < \bar{w}_{max}.$$

The result of Lemma 5 shows that a deviating seller may be active on both distribution channels or on the direct channel only, depending on the commission on the platform. More specifically, if competing sellers are present on the platform $w_M < \tilde{w}_{max}$, it is also profitable for a deviating seller to do so. In contrast, at the other extreme, if the commission is very high such that colluding sellers are close to indifferent between listing

[19]If sellers do not delist and stay active on both distribution channels, the presence of a PMFN effectively undermines a seller's ability to price discriminate between distribution channels. For instance, Helfrich and Herweg (2016) show that the firms' ability to engage in preference-based price discrimination can destabilize collusion. As I derive below, the model based on per-unit commissions and the linear demand specification implies that the latter mechanism does not affect the stability of seller collusion. The analysis, therefore, highlights effects of altered punishment and deviation behavior due to a PMFN.

on both distribution channels or only the direct channel, a deviating seller prefers to delist from the platform and to sell only via the direct channel. In this case collusive prices are high due to the high costs of distribution on the platform and a deviating seller benefits strongly from avoiding contractual restrictions from a PMFN by delisting from the platform.

Based on the results in Lemma 2, 3, and 5, the following proposition characterizes the stability of collusion in the presence of a PMFN.

Proposition 5. *Suppose the platform imposes a PMFN (P). If $w_M \leq \tilde{w}_{max}$, the critical discount factor is*

$$\underline{\delta}^P = \underline{\delta}^{NP} = \frac{(2-\alpha)^2}{8 - 8\alpha + \alpha^2}, \tag{29}$$

as in the case without a PMFN (NP). At $w_M = \tilde{w}_{max}$, there is a discrete decrease in the critical discount factor such that $\underline{\delta}^P(\tilde{w}_{max}) < \underline{\delta}^{NP}$. Above this commission, the critical discount factor $\underline{\delta}^P(w_M)$ increases in $w_M \in (\tilde{w}_{max}, \bar{w}_{max})$, with a kink at $w_M = \hat{w}_{max}$. For a sufficiently large w_M in this range, it holds that $\underline{\delta}^P > \underline{\delta}^{NP}$.

The exact terms for the critical discount factor $\underline{\delta}^P$ for $w_M > \tilde{w}_{max}$ are provided in Equations (75) and (76) in Appendix A.

For small commissions $w_M \leq \tilde{w}_{max}$ the critical discount factor is equal to the case without a PMFN and independent of w_M. By conventional interpretation, it follows that the collusive stability between sellers is not affected by the introduction of a PMFN in this range of commissions. Moreover, this result emphasizes that the ability to engage in price discrimination between distribution channels itself, which is restricted due to a PMFN, does not affect the stability of collusion in this setting as long as the sellers list on both channels.

For higher commissions, Proposition 5 highlights that a PMFN has an effect on the stability of seller collusion due to the fact that it changes the sellers' listing decisions. In particular, at the threshold $w_M = \tilde{w}_{max}$, there is a discrete decrease in the critical discount factor due to the fact that competing sellers do not list on the platform. This effect renders punishment more severe in this range of commissions and stabilizes seller collusion. Importantly, sellers would realize higher profits if they were present on both distribution channels also for commissions $w_M > \tilde{w}_{max}$. But, as described above, in this range of commissions, each seller has a unilateral incentive to delist from the platform and to compete aggressively on the direct channel alone. The sellers therefore suffer from a Prisoner's Dilemma in their listing decisions and realize discretely lower competitive profits.

For commissions above $\tilde{w}_{max}$, the critical discount factor increases in w_M with a kink at $w_M = \hat{w}_{max}$ due to the fact that at this point the optimal deviation behavior (i.e., the listing decision) changes. This has the effect that $\underline{\delta}^P$ increases more strongly above this threshold because after delisting a deviating seller benefits if the second seller faces a higher commission. For the highest admissible commission of $w_M = \bar{w}_{max}$, the critical discount factor $\underline{\delta}^P$ is above the critical discount factor without a PMFN, $\underline{\delta}^{NP}$, indicating that collusion is harder to sustain at high commissions close to $\bar{w}_{max}$.

Figure 1 plots the critical discount factor $\underline{\delta}^P$ depending on the commission w_M on the platform as characterized in Proposition 5.

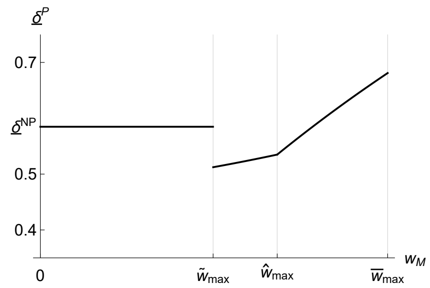

Figure 1: Critical discount factor with per-unit commissions.

The figure shows the critical discount $\underline{\delta}^P$ depending on the commission w_M for $\alpha = 7/10$ and $\beta = 1/2$.

The result of Proposition 5 allows to characterize the case in which the platform can at the same time (i) benefit from seller collusion compared to seller competition and (ii) increase the stability of seller collusion by choosing its per-unit commission appropriately. In particular, I solve for the commission w_M at which only colluding sellers list on the platform and the critical discount factor $\underline{\delta}^P$ equals the benchmark $\underline{\delta}^{NP} = (2-\alpha)^2 / (8 - 8\alpha + \alpha^2)$. This result is summarized in

Corollary 1. *The platform can profitably increase the stability of seller collusion if the degree of interbrand competition α is sufficiently large.*

The exact threshold value of α cannot be expressed in a closed-form solution but it depends only on the degree of intrabrand competition β and ranges from 0.62 for $\beta \to 1$ to 0.81 for $\beta \to 0$. Similar to the threshold value in Proposition 2 it is smaller for a strong degree of intrabrand competition β.

Novel Theory of Harm. The results presented in Sections 3 and 4 provide a novel theory of harm: A PMFN undermines a platform's incentive to ensure intense competition on its marketplace. Moreover, a PMFN gives the platform the ability to profitably stabilize seller collusion.

These results complement existing concerns regarding PMFNs. As described in Section 2, the main established theory of harm predicts that a PMFN leads to higher commissions and therefore higher consumer prices. As argued above (see, for instance, fn. 15), evidence from several markets, however, shows that there is little variation in commissions when platforms impose or waive a PMFN. Moreover, a platform may not want to increase its commission above a certain level in the shadow of regulation. An important aspect of my results is therefore that they do not necessarily require the platform to change its commission if it introduces a PMFN in order to lead to consumer harm. In particular, suppose that, absent a PMFN, the platform charges a commission that violates the participation constraint of competing sellers if it introduces such a clause. Absent adjustments in its commissions, the platform obviously has a strong incentive to alleviate the competitive pressure between the sellers in order to induce them to continue to sell via the platform. Moreover, in this range the introduction of a PMFN can also stabilize seller collusion without making it necessary for the platform to adapt its commission.

Discussion of the model framework applied to collusion in digital markets. In this section, I analyze the stability of collusion in online markets, taking the canonical approach of comparing the long-term benefits from collusion with the temptation of a one-time deviation from the collusive agreement. As already discussed in the Introduction, there are high-profile collusion cases on platform markets that motivate this analysis, and raise—among others—question about the stability of collusive agreements in online markets and, importantly, whether the stability changes with the introduction of a PMFN. Moreover, there are recent empirical studies from other industries lending support to the hypothesis that the stability constraint of collusive agreements is an economically relevant dimension to help understand the behavior of cartels (Igami and Sugaya, 2019; Miller et al., 2019).

A potential concern, however, may involve how, in principle, online markets can allow for timely responses to deviations. At an extreme of immediate reactions, this would render any deviation from collusion unprofitable and allow for stable collusion with any common discount factor (Ivaldi et al., 2003). I nevertheless take the view that this approach can offer fruitful insights for the study of collusion in online markets for the following reasons.

First, as derived above, the analysis links the stability of collusion to the listing decisions of the sellers on different distribution channels. Arguably, the channel choice is less flexible than an adjustment in the posted prices and takes more time to react to in case of a change in the market environment. Recent empirical studies such as Hunold et al. (2018) and Cazaubiel et al. (2020) provide evidence that the listing decision is an important dimension of adjustment in the hotel sector, particularly in the presence of a PMFN.

Second, there may be a fraction of online sellers that can react quickly to changes in the posted prices of other sellers, for instance, by using pricing algorithms in order to automate pricing decisions. In a recent paper, Chen et al. (2016) detect that 2.4 percent of online sellers use such algorithmic pricing on the Amazon Marketplace. For a large fraction of sellers, it is therefore still necessary to detect and react to a deviation without the help of algorithms, which may make them more comparable to other industries to which the approach is usually applied. Relatedly, deviations on other distribution channels than on the platform itself may be more difficult to monitor and also take more time to react to for the other sellers.

As a modeling choice, I abstract from information frictions. Arguably, a PMFN may improve the observability of secret price cuts and thereby stabilize collusion (see for informal discussions of this effect Fletcher and Hviid, 2016; Baker and Scott Morton, 2017). According to Stigler (1964), avoiding the threat of secret price cuts is the major obstacle for stable collusion, and this argument is reminiscent of the analysis by Jullien and Rey (2007) for the case with a resale price maintenance. This reasoning reinforces my findings that a PMFN stabilizes seller collusion. Importantly, however, when holding commissions fixed, prior arguments fail to establish that platforms that earn commissions from sales benefit from such collusion.

5 Extensions

5.1 Revenue-Sharing Commission

In this section, I verify that the main effects of a PMFN on the stability of seller collusion derived for the case of per-unit commissions also extend to the case with revenue-sharing commissions. I show that even small sellers' marginal costs are economically important in my setting and therefore allow for $c \geq 0$ in this section.

In contrast to existing contributions analyzing the agency model with revenue-sharing commissions such as Foros et al. (2017) or Hino et al. (2019), I allow for asymmetric distribution channels (one platform and one direct channel instead of two symmetric platforms), and online sellers facing (weakly) positive marginal costs $c \geq 0$. Both aspects prevent to fully analyze the model in closed-form solutions and hence I provide the results by means of numerical simulations.

I first provide results for the optimal symmetric revenue-sharing commission ϕ_M as a function of seller conduct and whether a PMFN is in place. Second, I establish that with this form of commissions the platform prefers seller competition absent a PMFN and seller collusion with a PMFN. Third, I analyze the stability of seller collusion.

Revenue-Sharing Commissions. If both sellers are active on the platform, the platform's profit is

$$\Pi_A(\phi_M) \quad = \quad \phi_M \sum_{i \in \{1,2\}} p_{iM} q_{iM}(p), \tag{30}$$

potentially subject to the constraint $p_{iM} \leq p_{iD}$ if the platform imposes a PMFN.

Depending on the seller conduct, Figure 2 plots the numerical results for the optimal revenue-sharing commissions that the platform sets for a representative parametrization. The left panel shows the case absent a PMFN and the right panel the one with a PMFN. If sellers compete, the optimal commission is depicted by the solid line, and if they collude by the dashed line.

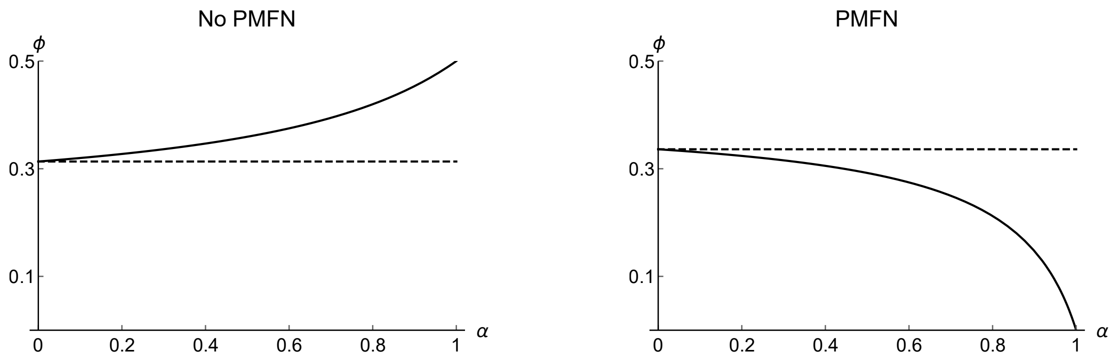

Figure 2: Revenue-sharing commissions.

The figure shows the revenue-sharing commissions for $\beta = 1/2$ and $c = 1/5$ depending on the degree of interbrand competition $\alpha \in (0, 1)$ for the case of seller competition (solid line) and seller collusion (dashed line). The left panel shows the case without a PMFN, the right panel the case with a PMFN.

Absent a PMFN, the optimal commission positively depends on the degree of interbrand competition α if the sellers compete. This finding is in contrast to the optimal per-unit commission w_M^{NP}, which is independent of α (see Proposition 1). The lowest commission that online sellers can obtain is at $\alpha \to 0$, which is exactly the commission that online sellers obtain if they collude (dashed line).

With a PMFN, this comparative static result is reversed (right panel of Figure 2). As in the case with per-unit commissions, the platform can charge higher commissions from colluding sellers than it can from competing ones. Again, the reason for this result is that competing sellers may have a unilateral incentive to delist from the platform, which also restricts revenue-sharing commissions on a low level.[20]

Preferred Seller Conduct. Next, I turn to the platform's preferred seller conduct. Absent a PMFN, the platform benefits from seller competition as in the case with per-unit commissions.[21] I illustrate this result numerically in the first panel of Figure 3.

[20]These comparative static results underline the robustness of the results of Johansen and Vergé (2017). Abstracting from the possibility of seller collusion, they derive qualitatively similar results on the basis of per-unit commissions and the assumption that contract offers are unobservable, but do not analyze its impact on seller collusion.

[21]With non-zero marginal costs $c > 0$ of the sellers and substitutability between two distribution channels, it is not a concern for the platform that competitive prices and realized revenue on the platform are too low. Each feature implies that the prices on the platform depend positively on the commission such that it can ensure a high revenue on the platform. If otherwise marginal costs $c = 0$ and no substitutable channel exists, this effect is not present and the platform would realize low profits if there is strong competition between the sellers. In this case, a platform may prefer weaker seller competition even absent a PMFN.

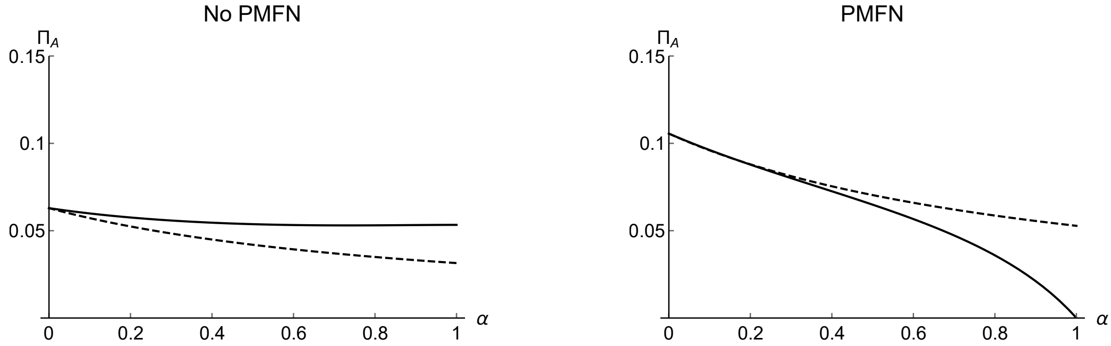

Figure 3: Platform profits with revenue-sharing commissions

The figure shows platform profits with the optimal revenue-sharing commissions for $\beta = 1/2$ and $c = 1/5$ depending on the degree of interbrand competition $\alpha \in (0, 1)$ for the case of seller competition (solid line) and seller collusion (dashed line). The left panel shows the case without a PMFN, the right panel the case with a PMFN.

This result is in contrast to Hino et al. (2019). They analyze the case without a PMFN and focus on two symmetric platforms as distribution channels in an extension. They conclude that for fixed commissions, platforms typically benefit from seller collusion. My analysis shows that if the platform charges optimal commissions based on seller conduct, it always benefits from seller competition absent a PMFN. Even for fixed commissions, I find that only for small degrees of intrabrand substitutability β and close to zero sellers' marginal costs c (as analyzed in Hino et al. (2019)) does it hold that the platform prefers seller collusion.[22] Economically, in Hino et al. (2019) firms may compete too fiercely and thereby destroy revenue. When marginal costs are positive, however, higher commission lead to higher price, so that the case of "excessive competition" for the platform becomes less relevant, and indeed cannot occur for optimal commissions.

The right panel of Figure 3 verifies that the preferred seller conduct changes if the platform can impose a PMFN. As described above, the platform optimally charges higher commissions from monopolistic sellers and this increase is sufficient to render seller collusion more profitable than seller competition. Importantly, I find that a platform prefers seller collusion for the whole parameter range $\alpha \in (0, 1)$. Hence, the anticompetitive potential of PMFNs is more pronounced in the model with revenue-sharing commissions than with per-unit commissions.

Stability of Seller Collusion. Recall that I restrict the range of interbrand competition to $\alpha \in \left(0, \sqrt{3} - 1\right)$ for the analysis of tacit collusion. With revenue-sharing commissions, I additionally restrict the commission to be lower than the threshold value $\hat{\phi}^{NP}_{max}$ (defined in Equation (94)) in order to ensure that a seller that charges collusive prices while the other seller deviates from the collusive agreement sells positive quantities on the platform. The following proposition summarizes the effect of PMFNs on the critical

[22] For instance, for $\phi_A = 3/10$ and $\beta = 2/10$, I find that seller collusion is not profitable for the platform for all $\alpha \in (0, 1)$ if $c \gtrsim 1/10$. For smaller degrees of marginal costs (e.g., $c = 3/100$) seller collusion is more profitable for the platform than seller competition for $\alpha \gtrsim 1/2$.

discount factor in the case in which the platform charges time-constant and symmetric revenue-sharing commission ϕ_M from the sellers.

Proposition 6. *Suppose that $\alpha \in \left(0, \sqrt{3}-1\right)$ and the commission is ϕ_M with $\phi_M \in \left(0, \hat{\phi}^{NP}_{max}\right)$. Without a PMFN, the critical discount factor is*

$$\underline{\delta}^{NP}\left(\phi_M\right) = \frac{\left(1-\phi_M\right)\left(2-\alpha\right)^2\left(1-\beta^2\right) - \left(1-\alpha\right)\beta^2\phi_M^2}{\left(1-\phi_M\right)\left(8-8\alpha+\alpha^2\right)\left(1-\beta^2\right) - 2\left(1-\alpha\right)\beta^2\phi_M^2}, \tag{31}$$

for both sellers $i \in \{1,2\}$. The critical discount factor increases in ϕ_M in the relevant parameter range.

The result of Proposition 6 characterizes the critical discount factor if the platform charges revenue-sharing commissions and does not impose a PMFN. Clearly, the case of $\phi_M = 0$ is formally equivalent to the case of a per-unit commission of zero, and, accordingly, the critical discount factor is the equal to $\left(2-\alpha\right)^2 / \left(8-8\alpha+\alpha^2\right)$ as in Proposition 4. In contrast to the case with per-unit commissions, for $\phi_M > 0$, the critical discount factor positively depends on the revenue-sharing commissions. This implies that a higher ϕ_M leads to less stable seller collusion. Quantifying the magnitude of the destabilizing effect of revenue-sharing commissions, however, reveals that there is only a minimal change in the critical discount factor if ϕ_M increases.[23]

For the case with a PMFN, the dependence of the critical discount factor on the commission ϕ_M is qualitatively the same as with per-unit commissions in Proposition 5. In particular, I also find threshold values on the commission for which competing $\left(\tilde{\phi}^P_{max}\right)$, deviating $\left(\hat{\phi}^P_{max}\right)$, and colluding sellers $\left(\bar{\phi}^P_{max}\right)$ prefer to be active on both distribution channels, and these threshold values exhibit the same ordering as for the case with per-unit commissions. The following proposition summarizes the result.

Proposition 7. *Suppose the platform imposes a PMFN (P). If sellers face a commission $\phi_M \leq \tilde{\phi}^P_{max}$, the critical discount factor is*

$$\underline{\delta}^P\left(\phi_M\right) = \frac{\left(2-\alpha\right)^2}{8-8\alpha+\alpha^2}. \tag{32}$$

At $\phi_M = \tilde{\phi}^P_{max}$, there is a discrete decrease in the critical discount factor. Above this commission, the critical discount factor $\underline{\delta}^P\left(\phi_M\right)$ increases in $\phi_M \in \left(\tilde{\phi}^P_{max}, \bar{\phi}^P_{max}\right)$, with a kink at $\phi_M = \hat{\phi}^P_{max}$. For a sufficiently large ϕ_M in this range, it holds that $\underline{\delta}^P\left(\phi_M\right) > \frac{(2-\alpha)^2}{8-8\alpha+\alpha^2}$. Over the complete parameter range, it holds that $\tilde{\phi}^P_{max} < \hat{\phi}^P_{max} < \bar{\phi}^P_{max}$.

The result of Proposition 7 is illustrated in Figure 5 in Appendix A. It highlights that the same pattern as with per-unit commissions emerges for the case with revenue-sharing commissions. This implies that both with per-unit as well as with revenue-sharing commissions, the platform can stabilize seller collusion.

[23]For the specification $\alpha = \beta = 1/2$ and $c = 0$, Figure 4 in Appendix A illustrates that the critical discount factor $\underline{\delta}^{NP}\left(\phi_M\right)$ maximally increases by 0.4%.

5.2 Responsive Commissions

Above, I analyze the case of time-constant commissions. In fact, sticky pricing behavior appears to employed by most online platforms nowadays, which implies that it is a natural benchmark for the analysis of seller collusion in these settings.

At the same time, a platform typically intermediates transactions for many different product markets with different characteristics (e.g., number of sellers, degree of substitutability, availability of additional distribution channels, etc.). In contrast to the observation that there is little change in commission rates over time, there is evidence that commissions do differ across product categories or markets. For instance, the referral fee on the Amazon Marketplace ranges from 8 percent (e.g., for personal computers) to 20 percent (e.g., for certain jewelries and gift cards).[24] It is therefore interesting to understand why there appears to be only limited variation in commission rates over time. In particular, it is clear that a commitment to time-constant commissions can be costly if market conditions change, and a platform generally can increase its current-period profit by adjusting its commission.

In this section, I analyze the effect on seller collusion if the platform can change its commission w_t every period. This analysis offers a complimentary view on a platform's incentive and ability to discourage seller collusion. In particular, I contrast two different platform strategies of how to manage seller collusion. The first strategy is to charge each period the commission that maximizes the platforms profits in each given period. It makes sense to adopt this strategy if the platform expects that it cannot influence the seller conduct. I refer to this strategy as spot-optimal commissions.

Second, I consider a platform strategy that aims at maximally destabilizing seller collusion. If the platform can influence the seller conduct and benefits more from seller competition than it does from seller collusion, it might decide to sacrifice current-period profits in order to attain a more favorable seller conduct and higher profits in the future. In order to analyze this inter-temporal trade-off for the platform, I characterize the platform's critical discount factor that describes the degree of patience of the platform that is necessary such that the strategy of destabilizing seller collusion is dynamically optimal for the platform. If the platform has to be more patient in order to destabilize seller collusion, I interpret this to mean that it has less incentive to organize a competitive marketplace.

Spot-Optimal Commissions As derived in Proposition 1 the spot-optimal commission is independent of the seller conduct absent a PMFN. This implies that there is no difference in the stability of seller collusion when comparing between time-constant and spot-optimal commissions.

With PMFN, the platform charges a lower commission of $\tilde{w}_M$ from competing sellers than it does from colluding ones. If the platform charges the collusive commission $\bar{w}_M$

[24] See the fee schedule for selling on Amazon on sellercentral.amazon.com (last access, November 29, 2022).

during collusive and deviation periods, the critical discount factor of the sellers can be expressed as follows

$$\underline{\delta}^{S}=\frac{\hat{\pi}_{i}\left(\bar{w}_{M}\right)-\bar{\pi}_{i}\left(\bar{w}_{M}\right)}{\hat{\pi}_{i}\left(\bar{w}_{M}\right)-\tilde{\pi}_{i}\left(\tilde{w}_{M}\right)}, \tag{33}$$

where S stands for spot-optimal commissions. Inserting all respective profit levels yields

$$\underline{\delta}^{S}=\begin{cases}\frac{\alpha(4-\alpha(4-\sigma))^{2}}{2\alpha^{3}(9+\beta-4\sigma)-8\alpha^{2}(13+\beta-5\sigma)+8\alpha(19+\beta-8\sigma)-32(2-\sigma)} & \text{if } \alpha\geq\frac{4(10-\beta(6+\sigma)-\sigma)}{(7-\beta)^{2}}\\ 1 & \text{if } \alpha<\frac{4(10-\beta(6+\sigma)-\sigma)}{(7-\beta)^{2}}.\end{cases} \tag{34}$$

There are two important results based on this critical discount factor. Fist, if the degree of interbrand competition α is sufficiently small, seller collusion is impossible to sustain under spot-optimal commissions. The reason for this result is that the commission under seller competition is so low that the competitive profits exceed the collusive profits at the substantially higher commission of $\bar{w}_M$. In other words, there is nothing to gain for the sellers to coordinate and, hence, collusion is infeasible for all discount factors δ.[25]

Second, even if collusion is sustainable for high discount factors, it is strictly more difficult to sustain than in the case absent a PMFN. Recall that without PMFN, the critical discount factor is $\underline{\delta}^{NP}=(2-\alpha)^{2}/\left(8-8\alpha+\alpha^{2}\right)$, which is strictly smaller than the first line of Equation (34) in the relevant parameter range.

Importantly, with a PMFN, a breakdown of collusion induces the platform to charge a lower commission in order to ensure that the sellers are willing to list on the platform. With responsive commissions, the sellers therefore realize a higher profit in punishment periods than with time-constant commissions and collusion is less stable. We summarize these findings in

Proposition 8. *Without a PMFN, there is no difference in the stability of seller collusion to the case of a time-constant commission. With a PMFN, spot-optimal commissions lead to a strictly higher critical discount factor than $\underline{\delta}^{NP}$. For*

$$\alpha<\frac{4\left(10-\beta\left(6+\sigma\right)-\sigma\right)}{\left(7-\beta\right)^{2}}$$

seller collusion is infeassible for all comon discount factors $\delta\in(0,1)$.

Proof. Direct implication of the derivations above. □

Whereas the platform is able to profitably stabilize seller collusion with a PMFN and a time-constant comission, it destabilizes collusion if it charges spot-optimal commissions. Hence, if the platform benefits from seller collusion, a commitment not to adjust the commission can help to stabilize seller collusion.

[25] For example, for $\beta=1/2$, collusion is infeasible for all discount factors if $\alpha\lesssim 0.42$.

Destabilizing Collusion As a final step, consider the case in which the platform aims at inducing a deviation from the seller collusion. This analysis is in the spirit of Snyder (1996) in which a buyer creates a backlog of unfilled orders in order to make a deviation more tempting. We consider the case in which the platform sets a commission $\hat{w}_M = 0$ for one period in order to maximally destabilize seller collusion. If collusion does not break down, the sellers expect the platform to return to the collusive commission $\bar{w}_M$ in future periods. In case it breaks down, they expect the platform to charge the competitive commission $\tilde{w}_M$. The sellers' critical discount factor in this case can be expressed as

$$\underline{\delta}^D = \frac{\hat{\pi}_i\left(\hat{w}_M\right) - \bar{\pi}_i\left(\bar{w}_M\right)}{\hat{\pi}_i\left(\hat{w}_M\right) - \tilde{\pi}_i\left(\tilde{w}_M\right)}, \tag{35}$$

where D stands for the regime in which the platform aims to destabilize seller collusion.

As can be expected, seller collusion is considerably less stable if the platform offers a one-time commission of zero, both with and without a PMFN. We summarize this result in

Proposition 9. *Both in comparison to time-constant as well as to spot-optimal commissions, the platform increase the critical discount factor if it offers a commission $\hat{w}_M = 0$ in one period.*

Proof. See Appendix A. □

The platform does not engage in this strategy if collusion remains stable. In this case it foregoes the current-period profit without affecting the seller conduct. Of course, the platform does not engage in this strategy either if it realizes a higher profit from seller collusion than it does from seller competition. Only if it can gain higher profits in the case of seller competition, it may decide to forego current-period profits in order to achieve higher profits in future periods.

In order to characterize this intertemporal trade-off for the platform, denote the platform's discount factor $\eta \in (0, 1)$. I derive the platform's critical discount factor $\underline{\eta}$ that renders the strategy of destabilizing seller collusion profitable for the platform. If the critical discount factor to engage in this destabilizing strategy is higher, the platform has to be more patient (i.e, has a lower incentive) to destabilize seller collusion.

Suppose that a one-time commission of $\hat{w}_M = 0$ leads to a breakdown of seller collusion. I establish

Proposition 10. *Without a PMFN, the platform prefers to destabilize seller collusion with $\hat{w}_M = 0$ if its discount factor exceeds*

$$\eta > \underline{\eta}^{NP} = 1 - \frac{\alpha}{2}.$$

With a PMFN, the platform prefers to destabilize seller collusion if

$$\eta > \underline{\eta}^{P} = \min\left\{\frac{\left(4 - \alpha\left(4 - \sigma\left(\beta\right)\right)\right)^2}{16\left(1 - \alpha\right)}, 1\right\},$$

with $\sigma(\beta) = \sqrt{2(1-\beta)}$*. The critical discount factor* $\underline{\eta}^{P}$ *is larger than* $\underline{\eta}^{NP}$*.*

Proof. See Appendix A. □

Importantly, the platform is more willing to destabilize seller collusion (as captured in the smaller critical discount factor $\underline{\eta}^{NP}$) than it is with a PMFN. The analytic solution can even exceed the value of 1 which implies that the platform cannot benefit from destabilizing seller collusion and, in fact, realizes higher profits if the sellers collude. The result of Proposition 10, therefore, highlights that a PMFN undermines a platform's incentive to organize a competitive marketplace.

6 Conclusion

This paper links the presence of a PMFN to reduced platform incentives to ensure seller competition on the platform. Absent contractual restrictions, a platform benefits from seller competition as it leads to more transactions on the platform and generally (weakly) higher commissions for the platform. In contrast, a PMFN can align the interests of sellers and platforms regarding seller collusion and, therefore, undermines a platform's incentive to organize a competitive marketplace. Intuitively, a reduction of competition between the sellers on both the platform and the direct channel enables a platform to collect higher commissions. Through this increase in the commission, the platform can benefit from seller collusion.

Moreover, in line with the incentive to reduce seller competition, the analysis highlights that a platform can profitably stabilize seller collusion if it imposes a PMFN. Recent antitrust cases (discussed in the Introduction) suggest that the conduct of price-fixing agreements between sellers is a concern for competition authorities more broadly. This concern can be especially pressing if platform providers have little interest in encouraging seller competition on their own marketplace, and my analysis reveals under which conditions this is the case in the presence of a PMFN.

In summary, my results offer a novel theory of harm, linking such clauses to potentially reduced competition at the seller level, and add to the vivid debate as regards their anticompetitive potential. Established concerns regarding PMFNs rely on the prediction that a PMFN leads to consumer harm via higher commissions and final prices (see e.g., Boik and Corts 2016). In several real-world cases (online hotel bookings, e-books, etc.) the prediction of increasing commissions, however, appears not to hold. Importantly, the results presented in this paper on a platform's incentive and ability to encourage seller competition also apply if a platform does not adjust its commission with the introduction of a PMFN. In particular, the result that collusive sellers are more likely to list on the platform for higher commissions than competing ones implies that seller collusion can be more sustainable than absent such a clause and that a platform may lose its incentive to fight seller collusion with the introduction of a PMFN even if it does not adjust its commission.

Future research should continue to analyze a platform's incentive and ability to affect the competitive interaction between sellers in different environments. Given that platforms are *private rule-makers* for the marketplace that they have created, it is important to identify situations in which a platform has little interest in ensuring a competitive environment between sellers, and to ensure that consumers can reap the full benefits of purchasing goods and services in the digital economy.

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Appendix A: Proofs

Proof of Lemma 1. Absent a PMFN and if sellers set prices non-cooperatively, each seller maximizes its profit function in Equation (3). Solving the corresponding first-order conditions yields the retail prices $\tilde{p}^{NP}$ reported in the lemma. It is straightforward to verify that the second-order conditions are fulfilled. Inserting these retail prices in the demand function in Equation (2), yields that the quantity that seller i sells via the platform is

$$q_{iM}\left(\tilde{p}^{NP}\right) = \frac{1-\beta-w_M}{(2-\alpha)(1+\alpha)(1-\beta^2)}, \tag{36}$$

which is non-negative if and only if $w_M \leq 1-\beta$.

In the monopolistic case, sellers set retail prices in order to maximize their joint profit

$$\pi_{12}(p) = \pi_1(p) + \pi_2(p) = \sum_{i\in\{1,2\}} (p_{iM} - w_M)\, q_{iM}(p) + p_{iD} q_{iD}(p). \tag{37}$$

The resulting retail prices $\bar{p}^{NP}$ are reported in the lemma, and the second-order conditions hold. The quantity that each seller i sells on the platform is

$$q_{iM}\left(\bar{p}^{NP}\right) = \frac{1-\beta-w_M}{2(1+\alpha)(1-\beta^2)}, \tag{38}$$

which is non-negative if and only if $w_M \leq 1-\beta$. This establishes the result.

For future reference, note that a seller's profit with competition is

$$\tilde{\pi}_i^{NP}(w_M) = \frac{(1-\alpha)\left(2-2\beta+w_M^2-2(1-\beta)\,w_M\right)}{(2-\alpha)^2(1+\alpha)(1-\beta^2)}, \tag{39}$$

where $\tilde{\pi}_i^{NP}(w_M) = \tilde{\pi}_i^{NP}\left(\tilde{p}^{NP}(w_M)\right)$. The resulting monopolistic seller profit is

$$\bar{\pi}_i^{NP}(w_M) = \frac{2-2\beta+w_M^2-2(1-\beta)\,w_M}{4(1+\alpha)(1-\beta^2)}, \tag{40}$$

where $\bar{\pi}_i^{NP}(w_M) = \bar{\pi}_i^{NP}\left(\bar{p}^{NP}(w_M)\right)$Note that $\tilde{\pi}_i^{NP}(w_M)$ and $\bar{\pi}_i^{NP}(w_M)$ decrease in $w_M \in [0, 1-\beta]$. $\square$

Proof of Proposition 1. Based on the seller behavior in the second stage of the static game (Lemma 1), the platform maximizes its profit in Equation (4) with respect to the per-unit commission w_M. The corresponding first-order condition $\partial \Pi_M / \partial w_M = 0$ can be written as

$$q_1(p(w_M)) + q_2(p(w_M)) + w_M \left(\frac{\partial q_1(p(w_M))}{\partial w_M} + \frac{\partial q_2(p(w_M))}{\partial w_M} \right) = 0. \tag{41}$$

Solving the first-order condition yields the optimal commission w_M^{NP} reported in the proposition at which the second-order conditions hold. Note that $w_M^{NP}=(1-\beta)/2 < 1-\beta$, which implies that sellers are willing to accept the platform's contract at this commission (Lemma 1). Based on w_M^{NP} , the platform realizes a profit of

$$\tilde{\Pi}_M^{NP}\left(w_M^{NP}\right) = \frac{1-\beta}{2(2-\alpha)(1+\alpha)(1+\beta)}, \tag{42}$$

if sellers compete, and it realizes

$$\bar{\Pi}_M^{NP}\left(w_M^{NP}\right) = \frac{1-\beta}{4(1+\alpha)(1+\beta)}, \tag{43}$$

in the monopolistic case. Calculations reveal that $\tilde{\Pi}_M(w_M) > \bar{\Pi}_M(w_M)$ for $w_M \in [0, 1-\beta]$. $\square$

Proof of Lemma 2. Suppose that the platform imposes a PMFN, which requires $p_{iM} \leq p_{iD}$. This proof characterizes the sellers' competitive price setting and listing behavior depending on the platform's commission.

Suppose that both sellers list on both distribution channels. In the competitive case, each seller faces the maximization problem

$$\begin{aligned} \max_{p_{iM}, p_{iD}} \pi_i(p_i, p_h) &= (p_{iM} - w_M) q_{iM}(p) + p_{iD} q_{iD}(p) \\ \text{s.t.} \quad & p_{iM} \leq p_{iD}. \end{aligned} \tag{44}$$

Based on the results of Lemma 1, the constraint is binding. Thus, sellers charge the same retail price on both distribution channels if active on the platform. Solving the corresponding first-order condition leads to the retail prices reported in the lemma leading to seller profits of

$$\tilde{\pi}_i^P(w_M) = \frac{(1-\alpha)(2-w_M)^2}{2(2-\alpha)^2(1+\alpha)(1+\beta)}. \tag{45}$$

Alternatively, a seller can deviate and list only on the direct channel and maximize the following profit function

$$\pi_i\left(p_{iD}, \infty, \tilde{p}_h^P\right) = p_{iD} q_{iD}\left(p_{iD}, \infty, \tilde{p}_h^P\right), \tag{46}$$

where ∞ indicates that seller i is not active on the platform while the rival seller h is present on both distribution channels and is expected to set $\tilde{p}_h^P$ on both distribution channels. Taking as given that seller h charges the competitive retail prices, seller i maximizes its profit by setting

$$\tilde{p}_{iD}^P(w_M) = \frac{4-\alpha(4-w_M)}{8-4\alpha}, \tag{47}$$

resulting in a profit of

$$\tilde{\pi}_i^P\left(\tilde{p}_{iD}^P(w_M), \infty, \tilde{p}_h^P\right) = \frac{(4-\alpha(4-w_M))^2}{16(\alpha-2)^2(1-\alpha^2)}. \tag{48}$$

In order to derive the threshold value $\tilde{w}_{max}$ reported in Lemma 2, equate the profit from being active on both channels in Equation (45) with the profit from being active on the direct channel in Equation (48), which yields

$$\begin{aligned} \tilde{\pi}_i^P(w_M) &= \pi_i\left(\tilde{p}_{iD}^P(w_M), \infty, \tilde{p}_i^P(w_M)\right) \\ \iff \quad \tfrac{4(1-\alpha)(2-w_M)}{2(2-\alpha)^2(1+\alpha)(1+\beta)} &= \frac{(4-\alpha(4-w_M))^2}{16(\alpha-2)^2(1-\alpha^2)} \\ \iff \quad \tfrac{(1-\alpha)(2-w_M)^2}{4-\alpha(4-w_M)} &= \sqrt{2(1+\beta)} \end{aligned} \tag{49}$$

The resulting threshold value is

$$\tilde{w}_{max} = \frac{4(1-\alpha)(2-\sigma(\beta))}{4-\alpha(4-\sigma(\beta))}, \tag{50}$$

with $\sigma(\beta) = \sqrt{2(1+\beta)}$.

Now suppose that both sellers are active on the direct channel only. In this case each seller maximizes $\pi_i(p_D, \infty) = p_{iD} q_{iD}(p_D, \infty)$. The resulting retail prices are $\tilde{p}_{iD}^P(w) = (1-\alpha)/(2-\alpha)$ as specified in Equation (5) in Lemma 1. The resulting profit is

$$\begin{aligned} \tilde{\pi}_{i(D)}^P(w_M) &= \pi_i\left(\tilde{p}_{iD}^P(w_M), \infty\right) \\ &= \frac{1-\alpha}{(2-\alpha)^2(1+\alpha)}. \end{aligned} \tag{51}$$

Alternatively, a seller can deviate and list on the direct channel and on the platform. In this case, it maximizes the following profit function

$$\pi_i\left(p_i, \infty, \tilde{p}_{hD}^P\right) = (p_{iM} - w_M)\, q_{iM}\left(p_i, \infty, \tilde{p}_{hD}^P\right) + p_{iD} q_{iD}\left(p_i, \infty, \tilde{p}_{hD}^P\right), \tag{52}$$

where ∞ indicates the seller h is not present on the platform. Taking as given the behavior of seller h, seller i maximizes its profit by setting

$$\tilde{p}_i^P(w_M) = \frac{(1-\alpha)(\alpha^2(1-\beta)-\alpha(1-\beta)+(\alpha^2-\alpha-2)w_M-4)}{2(\alpha-2)(2-\alpha^2(1-\beta))},$$

on both distribution channels. The resulting profit is

$$\frac{1}{8}\left(\frac{(2-w_M)^2}{1+\beta}+\frac{2(1-\alpha)^3}{(2-\alpha)^2(1+\alpha)}-\frac{(2+(2-\alpha)\alpha w_M)^2}{(\alpha-2)^2(2-\alpha^2(1-\beta))}\right) \tag{53}$$

Comparing the profit levels in Equation (51) and (53) shows that listing on the direct channel alone is an equilibrium if

$$w_M > \tilde{w}_{min} = \frac{4-\alpha(1+(2-\alpha)\alpha)(1-\beta)}{(2-\alpha)(1-\alpha^2)} - 2\sqrt{\frac{(1+\beta)(2-\alpha^2(1-\beta))}{(\alpha^3-2\alpha^2-\alpha+2)^2}}. \tag{54}$$

As a next step, I verify that there cannot be an equilibrium with asymmetric listing decisions. First note that it cannot be an equilibrium that one seller is active on the platform and the other seller is active on the direct channel. The seller that is active on the platform has an incentive to also list on the direct channel in order to increase its profit. Therefore, in the only candidate asymmetric equilibrium one seller is active on both distribution channels and one is active on the direct channel alone. For this outcome to be an equilibrium, two conditions have to hold. First, the seller on both channels cannot have an incentive to delist from the platform and, second, the seller on the direct channel cannot have an incentive to also list on the platform.

In order to show that there is no equilibrium with asymmetric listings call Seller 1 the seller that is active on both channels and Seller 2 the one that is active on the direct channel only. In the proposed candidate equilibrium, Seller 1 maximizes the profit

$$\begin{aligned}\max_{p_{iM},p_{iD}} \tilde{\pi}_1^P\left(p_1^P,\infty,p_{2D}^P\right) &= (p_{1M}-w_M)\,q_{1M}\left(p_1^P,\infty,p_{2D}^P\right)+p_{1D}q_{1D}\left(p_1^P,\infty,p_{2D}^P\right)\\ &\quad s.t.\;\; p_{1M}\le p_{1D}.\end{aligned}$$

and prefers to stay on both channels if the resulting profit exceeds the profit from deviating from this candidate equilibrium and being active on the direct channel alone. Anticipating that Seller 2 sets the price $\tilde{p}_{2D}^P$, Seller 1 faces the profit function

$$\tilde{\pi}_1^P\left(\infty,p_{1D}^P,\infty,p_{2D}^P\right)=p_{1D}q_{1D}\left(\infty,p_{1D}^P,\infty,p_{2D}^P\right).$$

Following the same procedure as above, I find that Seller 1 stays on both channels if the commission w_M is smaller than a threshold value $\tilde{w}_{max}^{asy}$. It is not tractable to report the threshold value and the corresponding profit levels, but I can show that at this (and lower) commissions, Seller 2 that is active on the direct channel only has a strict incentive to also list on the platform. Hence, there cannot be an equilibrium in asymmetric listing decisions.

As a last step, it is necessary to assess when both sellers list on both channels and when on the direct channel alone. Comparing the two relevant threshold values reported in Equation (50) and (54), it holds that $\tilde{w}_{min} < \tilde{w}_{max}$. This implies that there are three regions of commissions w_M to distinguish in order to characterize the equilibrium.

1. $w_M < \tilde{w}_{min}$: Both sellers list on both distribution channels.

2. $w_M \in [\tilde{w}_{min}, \tilde{w}_{max}]$: Listing on both distribution channels and listing on the direct channel only are a Nash equilibrium.

3. $w_M > \tilde{w}_{max}$: Both sellers list on the direct channel only.

In the first and third case, there is a unique Nash equilibrium in the seller industry. In the second case, the profit from being active on both channels $\tilde{\pi}_i^P(w_M)$ in Equation (45) is larger than the profit on the direct channel only $\tilde{\pi}_{i(D)}^P(w_M)$ in Equation (51) for $w_M \in (0, 2-\sigma(\beta))$, with $\tilde{w}_{max} < 2-\sigma(\beta)$. Equilibrium selection based on Pareto-dominance hence implies that both sellers list on the platform for $w_M \leq \tilde{w}_{max}$ and on the direct channel only for $w_M > \tilde{w}_{max}$. This establishes the result. □

Proof of Lemma 3. In the monopolistic case and if sellers are active on both distribution channels, the joint profit maximization of $\pi_{12} = \pi_1 + \pi_2$ is

$$\begin{aligned} \max_p \ \pi_{12}(p) &= \sum_{i\in\{1,2\}} (p_{iM} - w_M)\, q_{iM}(p) + p_{iD} q_{iD}(p) \\ &s.t. \ \ p_{iM} \leq p_{iD}. \end{aligned} \tag{55}$$

Solving the corresponding first-order conditions leads to the retail prices reported in the lemma, and the monopolistic profit of

$$\bar{\pi}_i^P(w_M) = \frac{(2-w_M)^2}{8(1+\alpha)(1+\beta)}. \tag{56}$$

Monopolistic sellers can also decide to only list on the direct channel in order to avoid the contractual restrictions of a PMFN. In this case, they set retail prices in order to maximize their profits on the direct channel

$$\max_{p_D} \ \pi_{12}(p_D, \infty) = \sum_{i\in\{1,2\}} p_{iD} q_{iD}(p_{1D}, \infty, p_{2D}, \infty). \tag{57}$$

The resulting retail prices are the same as the monopolistic direct channel prices for the case without a PMFN as reported in Lemma 1, $\bar{p}_{iD}^P = 1/2$, and the profit in this case for each seller i is

$$\bar{\pi}_i^P\left(\bar{p}_D^P, \infty\right) = \frac{1}{4+4\alpha}. \tag{58}$$

Monopolistic sellers prefer to be active on both distributions channels if the profit in Equation (56) exceeds the profit in Equation (58), which is equivalent to the commission w_M being sufficiently small:

$$w_M \ \ \leq \ \ \bar{w}_{max} = 2 - \sqrt{2(1+\beta)} = 2 - \sigma(\beta). \tag{59}$$

Finally, I show that the sellers do not coordinate on asymmetric listings. One possibility is that one seller is active on the platform and the second one is active on the direct channel. This configuration, however, does not maximize the joint profits as the first seller can also offer on the direct channel at a high retail price and thereby inrease the sellers combined profits. Hence, the only potential asymmetric listing decision is that one seller is active on both channels and seller is active on the direct channel only.

In this case the joint profit of the sellers is

$$\bar{\pi}_{12}^{P}\left(\bar{p}_{i}^{P},\infty,p_{hD}\right)=\frac{2\left(3+\alpha+\beta-\alpha\beta\right)+\left(1+\alpha\right)w_{M}^{2}-4\left(1+\alpha\right)w_{M}}{8\left(1+\alpha\right)\left(1+\beta\right)}. \tag{60}$$

However, for $w_M \leq \bar{w}_{max}$, the sellers' joint profit is higher if both of them list on both channels (Equation (56)), and if $w_M > \bar{w}_{max}$, their joint profit is higher from listing on the direct channel only (Equation (58)). This establishes the result. □

Proof of Proposition 2. First, I analyze the platform's unrestricted commission rate that it sets if the sellers cannot delist from the platform both for the case of seller competition and seller collusion. Inserting the the competitive downstream prices $\tilde{p}_i^P\left(w_M\right)$ reported in Equation (12) in the platform's profit function in Equation (4) yields

$$\tilde{\Pi}_{M}^{P}\left(w_{M}\right)=\frac{\left(2-w_{M}\right)w_{M}}{\left(2-\alpha\right)\left(1+\alpha\right)\left(1+\beta\right)}. \tag{61}$$

Solving the first-order condition yields that $w_M = 1$ is the profit-maximizing commission rate in this case. The second-order condition for a maximum is fulfilled. Note that at $w_M = 1$, the sellers optimally set retail prices of $\tilde{p}_i^P\left(w_M\right) = \left(3-2\alpha\right)/\left(4-2\alpha\right) < 1$, which implies that they incur losses on every unit sold via the platform and realize positive profits of selling on the direct channel. In sum, downstream firms realize positive profits for all $\alpha, \beta \in (0, 1)$.

Similarly, in the monopolistic case, inserting the downstream prices $\bar{p}_i^P\left(w_M\right)$ (Equation (16)) in the platform's profit function yields

$$\bar{\Pi}_{M}^{P}\left(w_{M}\right)=\frac{\left(2-w_{M}\right)w_{M}}{2\left(1+\alpha\right)\left(1+\beta\right)}, \tag{62}$$

which again is maximized at $w_M = 1$. This implies that both seller competition and monopolistic seller behavior, the unrestricted solution to the platform's maximization exceeds both threshold values $\tilde{w}_{max}$ and $\bar{w}_{max}$ derived in Lemma 2 and Lemma 3. The platform's profit increases in the per-unit commission up to $w_M = 1$ given that both sellers are willing to list on the platform, and hence the sellers' participation constraint binds at the optimal commission.

Based on the optimal commission $\tilde{w}_M^P = \tilde{w}_{max}$ (Equation (11)), the platform realizes a profit of

$$\tilde{\Pi}_{M}^{P}\left(\tilde{w}_{M}^{P}\right)=\frac{8\left(1-\alpha\right)\left(2-\sigma\left(\beta\right)\right)\sigma\left(\beta\right)}{\left(1+\alpha\right)\left(1+\beta\right)\left(4-\alpha\left(4-\sigma\left(\beta\right)\right)\right)^{2}}, \tag{63}$$

with seller competition, and based on the commission $\bar{w}_M^P = \bar{w}_{max}$ (Equation (15)), the platform realizes a period profit of

$$\bar{\Pi}_M^P\left(\bar{w}_M^P\right) = \frac{(2-\sigma(\beta))\,\sigma(\beta)}{2(1+\alpha)(1+\beta)}, \tag{64}$$

with seller collusion. Calculations show that $\bar{\Pi}_M\left(\bar{w}^P\right) > \tilde{\Pi}_M^P\left(\tilde{w}^P\right)$ if $\alpha > \bar{\alpha} = (16 - 8\sigma(\beta)) / \left(16 - 8\sigma(\beta) + \sigma(\beta)^2\right)$. □

Proof of Lemma 4. If a seller i deviates from the collusive agreement, it decides (i) whether it prefers to be active on both distribution channels or only the direct channel, and (ii) on retail prices on each active channel that maximize the seller's profits in the current period given the commission w_M, and given that the other seller h charges collusive retail prices $\bar{p}_h^{NP} = \left(\bar{p}_{hM}^{NP}, \bar{p}_{hD}^{NP}\right)$ as in Equation (6). If the deviating seller decides to be active on both distribution channels, this implies $\hat{p}_i^{NP} = \left(\hat{p}_{iM}^{NP}, \hat{p}_{iD}^{NP}\right) \in \arg\max_{p_i} \pi_i\left(p_i; \bar{p}_h^{NP}\right)$. The resulting retail prices of the deviating seller are

$$\begin{aligned}\hat{p}_{iM}^{NP}(w_M) &= \frac{2-\alpha}{4} + \frac{(2+\alpha)\,w_M}{4}, \\ \hat{p}_{iD}^{NP}(w_M) &= \frac{2-\alpha}{4},\end{aligned} \tag{65}$$

where the hat symbol indicates that seller i deviated from the collusive agreement. The deviating seller i receives a profit of

$$\hat{\pi}_i^{NP}(w_M) = \frac{(\alpha-2)^2\left(2 - 2\beta + w_M^2 - 2(1-\beta)\,w_M\right)}{16(1-\alpha^2)(1-\beta^2)}. \tag{66}$$

Denote the seller's profits in the case of being active only on the direct channel with $\pi_i\left(p_{iD}, \infty; \bar{p}_h^{NP}\right)$, where ∞ indicates that seller i is inactive on the platform. Note that the same direct channel price $\hat{p}_{iD}^{NP}$ as reported in Equation (65) maximizes the profit of the deviating seller in this case. The resulting profit in this case is $\hat{\pi}_i^{NP} = (2-\alpha)^2/(16(1-\alpha^2))$, which is strictly smaller than the profit from being active on both distribution channels reported in Equation (66) for all $w_M \in [0, 1-\beta]$.

Lastly, it is necessary to verify that the non-deviating seller h sells a positive quantity via the platform. That is, for w_M in the relevant range it has to hold that

$$\begin{aligned}\check{q}_{hM} = q_{hM}\left(\bar{p}_h^{NP}, \hat{p}_i^{NP}\right) &= \frac{(\alpha^2+2\alpha-2)(1-\beta-w_M)}{4(1-\alpha^2)(1-\beta^2)} > 0 \\ \iff \quad \alpha &< \sqrt{3}-1,\end{aligned} \tag{67}$$

where $\check{q}_{hM} = q_{hM}(\bar{p}_h, \hat{p}_i)$ indicates the quantity of the non-deviating seller h on the platform. The same inequality holds for the direct channel. This establishes the result. □

Proof of Proposition 4. Given that upstream firms sustain collusion by means of grim trigger strategies, inserting Equations (39), (40), and (66) into the formula for the critical discount factor in Equation (23) yields

$$\underline{\delta}^{NP} = \frac{(2-\alpha)^2}{8-8\alpha+\alpha^2}. \tag{68}$$

As reported in Proposition 4, the critical discount factor is independent of the degree of intrabrand competition β and the exact level of the symmetric commission w_M. Moreover, the critical discount factor $\underline{\delta}^{NP}$ is an increasing function of α in the relevant range $\alpha \in \left(0, \sqrt{3}-1\right)$. □

Proof of Lemma 5. If a seller decides to deviate from the collusive agreement characterized in Lemma 3, it has to decide whether to be active on both distribution channels or on the direct channel only. Consider that the commission is sufficiently small that $w_M \leq \bar{w}_{max}$ such that colluding sellers are active on the platform. First, consider that the seller is active on both channels. Restricted by the PMFN, the deviating seller maximizes

$$\begin{aligned} \max_{p_i} \; \pi_i\left(p_i, \bar{p}_h^P(w_M)\right) &= (p_{iM} - w_M)\, q_{iM}\left(p_i, \bar{p}_h^P(w_M)\right) + p_{iD} q_{iD}\left(p_i, \bar{p}_h^P(w_M)\right) \\ s.t. \quad & p_{iM} \leq p_{iD}, \end{aligned} \tag{69}$$

where the rival seller h sticks to the collusive agreement and charges $\bar{p}_h^P(w)$ as specified in Equation (16) on both distribution channels. The seller optimally charges

$$\hat{p}_i^P = \frac{1}{8}\left(4 - 2\alpha + (2+\alpha)\, w_M\right), \tag{70}$$

which results in a profit of

$$\hat{\pi}_i^P(w) = \frac{(2-\alpha)^2 (2-w_M)^2}{32\left(1-\alpha^2\right)(1+\beta)}. \tag{71}$$

Instead, the deviating seller can delist from the platform in order to maximize the profit function $\pi_i\left(p_{iD}, \infty; \bar{p}_h^P(w_M)\right)$, where ∞ indicates that the seller is not active on the platform. The seller is not restricted by the PMFN in this case and optimally charges $\hat{p}_{iD}(w) = \frac{1}{8}\left(4 - (2-w_M)\,\alpha\right)$. This price depends positively on the commission on the platform w_M due to the fact that it induces the collusive price of the other seller to be higher on both channels. The resulting profit is

$$\hat{\pi}_{i(D)}^P(w) = \hat{\pi}_i^P\left(\hat{p}_{iD}(w), \infty; \bar{p}_h^P(w)\right) = \frac{\left(4 - \alpha\,(2-w_M)\right)^2}{64\left(1-\alpha^2\right)}, \tag{72}$$

which is smaller than the profit from being active on both channels in Equation (71) only if the platform's commission is sufficiently small. By the same steps as above, the threshold value is

$$w_M \leq \hat{w}_{max} = \frac{2\,(2-\alpha)\,(2-\sigma(\beta))}{4-\alpha\,(2-\sigma(\beta))}, \tag{73}$$

with $\sigma(\beta) = \sqrt{2\,(1+\beta)}$. Otherwise, a deviating seller prefers to be present only on the direct channel $\left(\hat{\pi}_{i(D)}^{P}(w) > \hat{\pi}_{i}^{NP}(w),\ \forall w_M > \hat{w}_{max}\right)$, as the benefit from charging a more profitable direct channel price outweighs the forgone profit from the lost sales on the platform at high commissions. Comparing the threshold values given in Equations (50), (59), and (73) yields that $\tilde{w}_{max} \leq \hat{w}_{max} \leq \bar{w}_{max}$ over the complete parameter range. This establishes the result. □

Proof of Proposition 5. For the derivation of the critical discount factor with PMFN, I distinguish three cases: First, I consider the case for which the commission is sufficiently small such that sellers are active on the platform in all periods. In particular, this condition is fulfilled for $w_M \leq \tilde{w}_{max}$. In this case, I can insert the equilibrium profits of the stage games in which sellers are active on both channels (Equations (45), (56), and 71) in the formula for the critical discount factor derived in Equation (23). The resulting critical discount factor is

$$\underline{\delta}^{P} = \frac{\hat{\pi}_i^P - \bar{\pi}_i^P}{\hat{\pi}_i^P - \tilde{\pi}_i^P} = \frac{(2-\alpha)^2}{8-8\alpha+\alpha^2}, \tag{74}$$

as in the case without PMFN (see Equation (25) in Proposition 4).

Second, as derived in Lemma 2, for commissions $w_M > \tilde{w}_{max}$, competing sellers are only present on the direct channel and realize profits of $\tilde{\pi}_{i(D)}^{P}(w)$ derived in Equation (51) instead of $\tilde{\pi}_i^P(w)$. Due to the fact that, at $w_M = \tilde{w}_{max}$, $\tilde{\pi}_{i(D)}^{P}(\tilde{w}_{max})$ is strictly smaller than $\tilde{\pi}_i^P(\tilde{w}_{max})$ in Equation (45), and as the critical discount factor decreases in the punishment profit, there is a discrete decrease in $\underline{\delta}^P$ at $w_M = \tilde{w}_{max}$.

For the range $w_M \in (\tilde{w}_{max}, \hat{w}_{max}]$, the critical discount factor $\underline{\delta}^P$ is

$$\frac{(2-\alpha)^2\alpha^2(2-w_M)^2}{4\left(\alpha\left(\alpha\left((2-\alpha)^4-8\beta\right)-16(1-\beta)\right)-8\beta+8\right)+(2-\alpha)^4 w_A^2-4(2-\alpha)^4 w_M}, \tag{75}$$

which increases in $w_M \in (\tilde{w}_{max}, \hat{w}_{max}]$ for the complete parameter range.

Third, as derived in Lemma 5, a deviating seller is only present on the direct channel for $w_M > \hat{w}_{max}$. Compared to the critical discount factor for low commissions in Equation (74), the deviation profit is therefore $\hat{\pi}_{i(D)}^{P}(w)$ in Equation (72) instead of $\hat{\pi}_i^P(w)$ in Equation (71). As $\hat{\pi}_{i(D)}^{P}(\hat{w}_{max}) = \hat{\pi}_i^P(\hat{w}_{max})$ and as in the range $w_M \in (\hat{w}_{max}, \bar{w}_{max}]$, $\hat{\pi}_{i(D)}^{P}(w)$ increases more strongly in w_M, there is a kink in the critical discount factor $\underline{\delta}^P$ at $w_M = \hat{w}_{max}$. For the range $w_M \in (\hat{w}_{max}, \bar{w}_{max}]$, the critical discount factor $\underline{\delta}^P$ is

$$\frac{(2-\alpha)^2\left((4-\alpha(2-w_M))^2-\frac{8(1-\alpha)(2-w_M)^2}{1+\beta}\right)}{\alpha\left(4\alpha(8-(8-\alpha)\alpha)+\alpha(2-\alpha)^2 w_M^2+4(2-\alpha)^3 w_M\right)}, \tag{76}$$

which increases in w_M. At $w_M = \bar{w}_{max}$, it holds that the critical discount factor $\underline{\delta}^P$ is strictly larger than $\underline{\delta}^{NP}$. This establishes the result. □

Proof of Proposition 6. As in the case with per-unit commissions, I first analyze the case of no PMFN and afterwards analyze the case with a PMFN. Consider that the platform sets a symmetric commission ϕ_M. I restrict the platform's commission to

$$\phi_M \in \left[0, \frac{(\alpha(2+\alpha)-2)(1-\beta)(1-c)}{\alpha^2+2\alpha+(1-\alpha^2-\alpha)\beta(1-c)-2}\right], \tag{77}$$

in order to ensure that a seller that charges collusive prices remains active on the platform if the second sellers deviates from the collusive agreement. If a seller is present on both distribution channels, its profit is

$$\pi_i(p) = ((1-\phi_M)p_{iM}-c)\,q_{iM}(p) + (p_{iD}-c)\,q_{iD}(p). \tag{78}$$

Absent a PMFN, and with seller competition,each seller i maximizes the profit in Equation (78) taking as given the commissions and the rival seller's behavior. I verify below that a seller has no incentive to be active on the direct channel only. The resulting retail prices are

$$\begin{aligned}
\tilde{p}_{iM}^{NP}(\phi_M) &= \frac{(2-\alpha)\left(1-\beta^2\right)(1-\alpha+c)+(1-\alpha)(1-\beta)\phi_M(\alpha-\beta(1-\alpha+c)-2)}{(2-\alpha)^2(1-\beta^2)-(1-\alpha)\beta^2\phi_M^2-(2-\alpha)^2(1-\beta^2)\phi_M}, \\
\tilde{p}_{iD}^{NP}(\phi_M) &= \frac{(\beta-1)((1-\alpha)(1-\phi_M)(\beta\phi_M-(2-\alpha)(1+\beta))+c\phi_M(2-\alpha+\beta)-(2-\alpha)(1+\beta)c)}{(2-\alpha)^2(1-\beta^2)-(1-\alpha)\beta^2\phi_M^2-(2-\alpha)^2(1-\beta^2)\phi_M}.
\end{aligned} \tag{79}$$

Each seller $i \in \{1,2\}$ sets the same retail price on distribution channel $j \in \{M, D\}$ but the retail prices are strictly lower on the direct channel for $\phi_M > 0$. The price on the platform $\tilde{p}_{iM}^{NP}(\phi_M)$ positively depends on the commission ϕ_M for $c \geq 0$ and $\alpha, \beta \in (0,1)$ in the relevant range. The resulting seller profit is

$$\tilde{\pi}_i^{NP}(\phi_M) = \frac{(1-\alpha)\left({\phi_M}^2(1-\beta+\beta c)-(1-\beta)(3-c)(1-c)\phi_M+2(1-\beta)(1-c)^2\right)}{(1+\alpha)(2-\alpha)^2(1-\beta^2)-(1-\alpha^2)\beta^2{\phi_M}^2-(1+\alpha)(2-\alpha)^2(1-\beta^2)\phi_M}. \tag{80}$$

Suppose seller i does not accept the platform's contract offer, while the competing seller h is active on both distribution channels and charges retail prices as specified in Equation (79). In this case, seller i maximizes

$$\max_{p_{iD}} \; \pi_i\left(p_{iD}, \infty, \tilde{p}_h^{NP}(\phi_M)\right) = (p_{iD}-c)\,q_{iD}\left(p_{iD}, \infty, \tilde{p}_h^{NP}(\phi)\right). \tag{81}$$

The resulting retail price is

$$\begin{aligned}
\tilde{p}_{i(D)}^{NP}(\phi_M) &= \frac{1}{2}(1-\alpha+c) \\
&+ \frac{\alpha(\beta-1)((1-\alpha)(1-\phi_M)(\beta\phi_M-(2-\alpha)(1+\beta))+c\phi_M(2-\alpha+\beta)+(\alpha-2)(1+\beta)c)}{2\left((\alpha-1)\beta^2{\phi_M}^2+(2-\alpha)^2(\beta^2-1)\phi_M+(2-\alpha)^2(1-\beta^2)\right)},
\end{aligned} \tag{82}$$

where $\tilde{p}_{i(D)}^{NP}$ indicates that i is only active on the direct channel. The resulting profit for seller i is

$$\tilde{\pi}_{i(D)}^{NP}\left(\phi_M\right)=\frac{\left(1-\alpha\right)\left(2\left(2-\alpha\right)\left(1-\beta^2\right)\left(1-c\right)+\beta\phi_M{}^2\left(\alpha-\beta\left(1-c\right)\right)+\left(1-\beta\right)\left(1-c\right)\phi_M\left(\alpha\left(2+\beta\right)-4\left(1+\beta\right)\right)\right)^2}{4\left(1+\alpha\right)\left(\left(2-\alpha\right)^2\left(1-\beta^2\right)\left(1-\phi_M\right)-\left(1-\alpha\right)\beta^2\phi_M{}^2\right)^2}, \tag{83}$$

where $\tilde{\pi}_{i(D)}\left(\phi_M\right)=\tilde{\pi}_i^{NP}\left(\tilde{p}_{i(D)}^{NP}\left(\phi_M\right),\infty,\tilde{p}_h\left(\phi_M\right)\right)$. This deviation is not profitable if the profit in Equation (80) exceeds the profit in Equation (83), which is the case if

$$\phi_M\leq\tilde{\phi}_{max}^{NP}=\frac{\left(2-\alpha\right)\left(1-\beta\right)\left(1-c\right)}{2-\alpha-\beta(1-c)}. \tag{84}$$

Note that this restriction on the commission is weaker than the one imposed in Equation (77). Hence, competing sellers always prefer to be active on both distribution channels.

With collusion, sellers maximize joint profits $\pi_{12}=\pi_1+\pi_2$ and optimally set retail prices of

$$\begin{aligned}\bar{p}_{iM}^{NP}\left(\phi_M\right)&=\frac{\left(1-\beta\right)\phi_M\left(2+\beta+\beta c\right)-2\left(1-\beta^2\right)\left(1+c\right)}{\beta^2\left(2-\phi_M\right)^2-4\left(1-\phi_M\right)},\\ \bar{p}_{iD}^{NP}\left(\phi_M\right)&=\frac{\left(1-\beta\right)\left(\phi_M\left(2+3\beta+\left(2+\beta\right)c\right)-\beta\phi_M^2-2\left(1+\beta\right)\left(1+c\right)\right)}{\beta^2\left(2-\phi_M\right)^2-4\left(1-\phi_M\right)},\end{aligned} \tag{85}$$

with collusive profits of

$$\bar{\pi}_i^{NP}\left(\phi_M\right)=\frac{\left(1-\beta\right)\left(3-c\right)\left(1-c\right)\phi_M-\phi_A^2\left(1-\beta\left(1-c\right)\right)-2\left(1-\beta\right)\left(1-c\right)^2}{\left(1+\alpha\right)\left(\beta^2\left(\phi_M-2\right)^2+4\left(\phi_A-1\right)\right)}. \tag{86}$$

Alternatively, colluding sellers may decide to list on the direct channel only. In this case they maximize

$$\max_{p_D}\ \pi_{12}\left(p_D,\infty\right)=\sum_{i\in\{1,2\}}\left(p_{iD}-c\right)q_{iD}\left(p_D,\infty\right), \tag{87}$$

with resulting retail prices of $\bar{p}_{i(D)}^{NP}=\left(1+c\right)/2$ and a realized profit of $\bar{\pi}_{i(D)}^{NP}=\left(1-c\right)^2/\left(4\left(1+\alpha\right)\right)$. Colluding sellers prefer to be present on both distribution channels if

$$\phi_M\leq\bar{\phi}_{max}^{NP}=2-\frac{2\left(1+c\right)}{2-\beta\left(1-c\right)}. \tag{88}$$

This restriction on the commission is weaker than the one imposed in Equation (77), and colluding sellers are active on both distribution channels.

Consider that seller i deviates from the collusive agreement, while seller h is present on both distribution channels and charges collusive prices specified in Equation (85). The deviating seller sets retail prices p_i in order to maximize

$$\pi_i\left(p_i,\bar{p}_h^{NP}\left(\phi_M\right)\right)=\left(\left(1-\phi_M\right)p_{iM}-c\right)q_{iM}\left(p_i,\bar{p}_h^{NP}\left(\phi_M\right)\right)+\left(p_{iD}-c\right)q_{iD}\left(p_i,\bar{p}_h^{NP}\left(\phi_M\right)\right). \tag{89}$$

The resulting retail prices are

$$\hat{p}_{iM}^{NP}\left(\phi_M\right)=\frac{\left(1-\beta^2\right)\left(2-\alpha+\left(2+\alpha\right)c\right)+\left(1-\beta\right)\phi_M\left(2-\alpha+\beta\left(1-\alpha+c\right)\right)}{4\left(1-\phi_M\right)-\beta^2\left(2-\phi_M\right)^2}, \tag{90}$$

$$
\begin{aligned}
& \hat{p}_{iD}^{NP}\left(\phi_M\right) && (91) \\
= \;& \frac{(1-\beta)\left(1-\phi_M\right)\left((2-\alpha)(1+\beta)+\beta\phi_M\right)+(2+\alpha)\left(1-\beta^2\right)c-(1-\beta)c\phi_M(2+\alpha\beta+\alpha+\beta)}{4\left(1-\phi_M\right)-\beta^2\left(2-\phi_M\right)^2},
\end{aligned}
$$

yielding a deviation profit of

$$
\begin{aligned}
\hat{\pi}_i^{NP}\left(\phi_M\right) \;=\; & \frac{\left((2-\alpha)^2\left(1-\beta^2\right)-(1-\alpha)\beta^2\phi_M^2-(2-\alpha)^2\left(1-\beta^2\right)\phi_M\right)}{\left(1-\alpha^2\right)\left(\beta^2\left(2-\phi_M\right)^2-4\left(1-\phi_M\right)\right)^2} && (92) \\
& \frac{\left(\phi_M^2(1-\beta(1-c))-(1-\beta)(3-c)(1-c)\phi_M+2(1-\beta)(1-c)^2\right)}{\left(1-\alpha^2\right)\left(\beta^2\left(2-\phi_M\right)^2-4\left(1-\phi_M\right)\right)^2}
\end{aligned}
$$

The non-deviating seller h that sticks to the collusive agreement sells on the platform the quantity of

$$
\begin{aligned}
& q_{hM}\left(\bar{p}_h^{NP}\left(\phi_M\right),\hat{p}_i^{NP}\left(\phi_M\right)\right) && (93) \\
= \;& \frac{\phi_M\left(\alpha^2+2\alpha+\left(1-\alpha^2-\alpha\right)\beta(1-c)-2\right)-(\alpha(2+\alpha)-2)(1-\beta)(1-c)}{\left(1-\alpha^2\right)\left(4\left(1-\phi_M\right)-\beta^2\left(2-\phi_M\right)^2\right)},
\end{aligned}
$$

which is larger than zero if Assumption 1 is fulfilled $\left(\alpha<\sqrt{3}-1\right)$ and the commission ϕ_M is sufficiently small

$$
\phi_M \;\leq\; \hat{\phi}_{max}^{NP}=\frac{(\alpha(2+\alpha)-2)(1-\beta)(1-c)}{\alpha^2+2\alpha+\left(1-\alpha^2-\alpha\right)\beta(1-c)-2}, \tag{94}
$$

which is the restriction on the commission imposed in Equation (77). The critical discount factor is

$$
\underline{\delta}^{NP}\left(\phi_M\right)=\frac{\left(1-\phi_M\right)(2-\alpha)^2\left(1-\beta^2\right)-(1-\alpha)\beta^2\phi_M^2}{\left(1-\phi_M\right)\left(8-8\alpha+\alpha^2\right)\left(1-\beta^2\right)-2(1-\alpha)\beta^2\phi_M^2}. \tag{95}
$$

Note that the critical discount factor simplifies to $\underline{\delta}^{NP}(0)=\left((2-\alpha)^2\right)/\left(8-8\alpha+\alpha^2\right)$ for $\phi_M=0$, which is equal to the critical discount factor for the case without a PMFN and per-unit commissions reported in Equation (25) in Proposition 4. Moreover, the critical discount factor in Equation (95) increases in ϕ_M over the relevant range. This establishes the result. □

The following figure illustrates that the increase in $\underline{\delta}^{NP}\left(\phi_M\right)$ is small in the present setting. Note that the scaling of the y-axis ranges only from 0.529 to 0.532, and that the critical discount factor only increases by approximately 0.002 which translates to a relative increase from 0.4% over the admissible range of revenue-commissions ϕ_M.

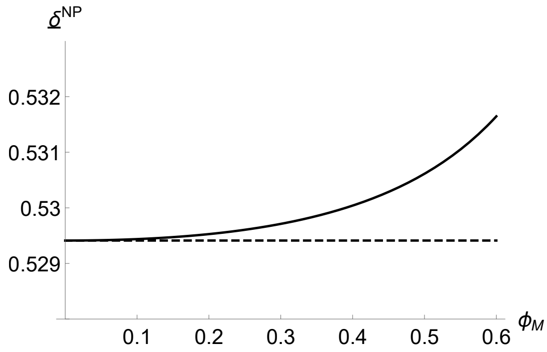

Figure 4: Critical discount factor with revenue-sharing commissions and without PMFN. The figure shows the critical discount $\underline{\delta}^{NP}(\phi_M)$ (solid line) and the critical discount factor for the case with per-unit commissions and without a PMFN (dashed line) depending on the exogenous commission ϕ_M for $\alpha = 1/2$, $\beta = 1/2$, and $c = 0$. As specified in Equation (77), the highest admissible commission for this specification is $\hat{\phi}^{NP}_{max} = 6/10$. For reference, the profit-maximizing commission that the platform charges from colluding sellers in this specification is $\bar{\phi}^{NP}_M \approx 0.465$.

Proof of Proposition 7. With a PMFN, competing sellers maximize their profit function in Equation (78) subject to the constraint that $p_{iM} \leq p_{iD}$. This constraint is binding for $\phi_M > 0$ and the retail price on both distribution channels is

$$\tilde{p}^P_i(\phi_M) = \frac{(1-\alpha)(2-\phi_M)+2c}{(2-\alpha)(2-\phi_M)}. \tag{96}$$

The resulting profit for each seller is

$$\tilde{\pi}^P_i(\phi_M) = \frac{(1-\alpha)(2c+\phi_M-2)^2}{(2-\alpha)^2(1+\alpha)(1+\beta)(2-\phi_M)}. \tag{97}$$

Alternatively, each seller can deviate and list on the direct channel only. As in the case without a PMFN in Equation (81), seller i maximizes in this case

$$\max_{p_{iD}} \ \pi_i\left(p_{iD},\infty,\tilde{p}^P_h(\phi_M)\right) = (p_{iD}-c)\,q_{iD}\left(p_{iD},\infty,\tilde{p}^P_h(\phi_M)\right), \tag{98}$$

with a resulting retail price on the direct channel of

$$\tilde{p}^P_{i(D)}(\phi_M) = \frac{2(1-\alpha)(2-\phi_M)+c(4-(2-\alpha)\phi_M)}{2(2-\alpha)(2-\phi_M)}, \tag{99}$$

and profits of

$$\tilde{\pi}^P_{i(D)}\left(\tilde{p}^P_{i(D)}(\phi_M),\infty,\tilde{p}^P_h(\phi_M)\right) = \frac{\left(c(4-\alpha(4-\phi_M)-2\phi_M)+2(1-\alpha)(2-\phi_M)\right)^2}{4(2-\alpha)^2(1-\alpha^2)(2-\phi_M)^2}. \tag{100}$$

The two sellers are active on both distribution channels if the profit in Equation (97) exceeds the profit in Equation (100). Define the threshold commission $\tilde{\phi}^P_{max}$ at which sellers are indifferent between being active on both channels and listing on the direct channel only. That is, $\tilde{\pi}_i^P\left(\tilde{\phi}_{max}^P\right) = \tilde{\pi}_{i(D)}^P\left(\tilde{\phi}_{max}^P\right)$. There is no closed-form solution for $\tilde{\phi}_{max}^P$ but it is possible to solve numerically for it. If the commission ϕ_M exceeds this threshold value, there is an equilibrium of the stage game in which both sellers are active on the direct channel. In this case they set $\tilde{p}_{iD}^P = (1-\alpha+c)/(2-\alpha)$ and realize an equilibrium profit of

$$\tilde{\pi}_{i(D)}^P\left(\tilde{p}_D^P, \infty\right) = \frac{(1-\alpha)(1-c)^2}{(2-\alpha)^2(1+\alpha)}. \tag{101}$$

Colluding sellers that are present on both distribution channels set optimal retail prices of

$$\bar{p}_i^P(\phi) = \frac{2+2c-\phi_M}{4-2\phi_M}, \tag{102}$$

and realize profits of

$$\bar{\pi}_i^P(\phi_M) = \frac{(2-2c-\phi_M)^2}{4(1+\alpha)(1+\beta)(2-\phi_M)}. \tag{103}$$

If sellers jointly decide to delist from the platform, face the same maximization problem as in Equation (87) and set the same retail prices of $\bar{p}_{i(D)}^{NP} = (1+c)/2$ in order to realize a profit of $\bar{\pi}_{i(D)}^{NP}(\phi_M) = (1-c)^2/(4(1+\alpha))$. Colluding sellers prefer to be present on both distribution channels if

$$\phi_M \leq \bar{\phi}_{max}^P = \frac{1}{2}(1-c)\left(3-\beta+(1+\beta)c-\sqrt{(1+\beta)(\beta+c(6-\beta(2-c)+c)+1)}\right). \tag{104}$$

Computations reveal that $\bar{\phi}_{max}^P > \tilde{\phi}_{max}^P$ over the complete parameter range. This implies that colluding sellers are willing to list on both distribution channels for higher commissions ϕ_M than competing sellers.

Consider that seller i deviates from the collusive agreement. If it is active on both distribution channels, it optimally charges

$$\hat{p}_i^P(\phi_M) = \frac{1}{4}\left(2-\alpha-\frac{2(2+\alpha)c}{\phi_M-2}\right), \tag{105}$$

and realizes a profit of

$$\hat{\pi}_i^P(\phi_M) = \frac{(2-\alpha)^2(2c+\phi_M-2)^2}{16(1-\alpha^2)(1+\beta)(2-\phi_M)}. \tag{106}$$

Alternatively, the deviating seller can decide to delist from the platform and only sell via the direct channel. In this case it optimally charges

$$\hat{p}_{i(D)}^P = \frac{1}{4}\left(2-\alpha+c\left(2+\frac{2\alpha}{2-\phi_M}\right)\right), \tag{107}$$

And realizes a profit of

$$\hat{\pi}_{i(D)}^P(\phi_M) = \frac{((2-\alpha)(2-\phi_M)-2c(2-\alpha-\phi_M))^2}{16(1-\alpha^2)(2-\phi_M)^2}. \tag{108}$$

Again, there exists a threshold commission $\hat{\phi}^P_{max}$ above which a deviating seller prefers to be active on the direct channel only. As in the case with seller competition, there is no closed-form solution for $\hat{\phi}^P_{max}$ but it can be characterized numerically. Simulations over the whole parameter range reveal that the same ordering of threshold values holds as in the case with per-unit commissions. That is, $\bar{\phi}^P_{max} > \hat{\phi}^P_{max} > \tilde{\phi}^P_{max}$.

Based on the threshold values and the seller profits for the different stage games, the critical discount factor is characterized for three intervals of commissions: The first interval is $\phi_M \in \left[0, \tilde{\phi}^P_{max}\right]$, the second one is $\phi_M \in \left(\tilde{\phi}^P_{max}, \hat{\phi}^P_{max}\right]$, and the third interval is $\phi_M \in \left(\hat{\phi}^P_{max}, \bar{\phi}^P_{max}\right]$.

In the first case, sellers are present on both distribution channels independent of seller conduct, and the critical discount factor is

$$\underline{\delta}^P(\phi) = \frac{(2-\alpha)^2}{\alpha^2 - 8\alpha + 8}, \ \phi_M \in \left[0, \tilde{\phi}^P_{max}\right]. \tag{109}$$

For the second interval, competing sellers prefer to be active on the direct channel only and realize the profit of $\tilde{\pi}^P_{i(D)}$ in Equation (101) instead of $\tilde{\pi}^P_i(\phi)$ in Equation (97), and the critical discount factor is characterized by

$$\begin{aligned}\underline{\delta}^P(\phi_M) \quad &= \quad \frac{\alpha^2 \left(2c + \phi_M - 2\right)^2}{16\left(1-\alpha\right)\left(1+\alpha\right)\left(1+\beta\right)\left(2-\phi_A\right)} \\ &\cdot \quad \frac{1}{\frac{(2-\alpha)^2(2-2c-\phi_M)^2}{16(1-\alpha^2)(1+\beta)(2-\phi_M)} - \frac{(1-\alpha)(1-c)^2}{(2-\alpha)^2(1+\alpha)}}, \ \phi_M \in \left(\tilde{\phi}^P_{max}, \hat{\phi}^P_{max}\right]\end{aligned} \tag{110}$$

Due to the fact that $\tilde{\pi}^P_{i(D)}\left(\tilde{\phi}^P_{max}\right) < \tilde{\pi}^P_i\left(\tilde{\phi}^P_{max}\right)$ at the threshold value $\tilde{\phi}^P_{max}$, and that the critical discount factor increases in the punishment profit $\tilde{\pi}_i$, there is a discrete decrease in $\underline{\delta}^P(\phi_M)$ at $\tilde{\phi}^P_{max}$. The critical discount factor $\underline{\delta}^P(\phi_M)$ increases in ϕ_M in the range $\left(\tilde{\phi}^P_{max}, \hat{\phi}^P_{max}\right]$.

In the third interval, not only competing sellers but also a deviating seller decides to be active on the direct channel only. Taking this listing decision into account, the critical discount factor in this range is

$$\underline{\delta}^P(\phi_M) = \frac{\frac{4(\phi_M-2)(2c+\phi_M-2)^2}{(1+\alpha)(1+\beta)} - \frac{((\alpha-2)(\phi_M-2)+2c(\alpha+\phi_M-2))^2}{\alpha^2-1}}{16\left(\phi_M-2\right)^2\left(\frac{(\alpha-1)(c-1)^2}{(\alpha-2)^2(\alpha+1)} - \frac{((\alpha-2)(\phi_M-2)+2c(\alpha+\phi_M-2))^2}{16(\alpha^2-1)(\phi_M-2)^2}\right)}, \ \phi_M \in \left(\hat{\phi}^P_{max}, \bar{\phi}^P_{max}\right], \tag{111}$$

which also increases in ϕ_M. This establishes the result. □

In the following figure, I illustrate that the effect of revenue-sharing commissions on the critical discount factor is qualitatively the same as with the per-unit commissions derived in Proposition 5 and depicted in Figure 1.

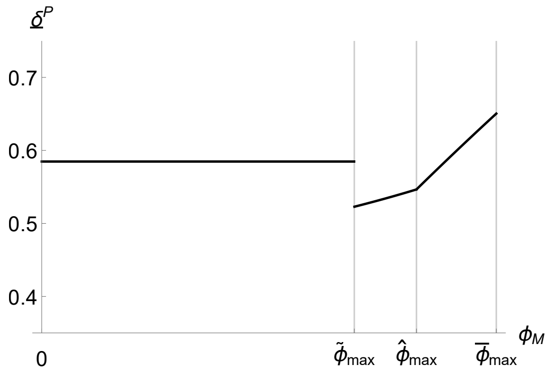

Figure 5: Critical discount factor with revenue-sharing commissions and PMFN.

The figure shows the critical discount $\underline{\delta}^P(\phi_M)$ depending on the exogenous commission ϕ_M for $\alpha = 7/10$, $\beta = 4/10$, and $c = 3/10$.

Proof of Proposition 9. Take the derivative of

$$\underline{\delta}^D = \frac{\hat{\pi}_i(\hat{w}_M) - \bar{\pi}_i(\bar{w}_M)}{\hat{\pi}_i(\hat{w}_M) - \tilde{\pi}_i(\tilde{w}_M)}, \tag{112}$$

with respect to $\hat{w}_M$, which is equal to

$$\frac{\partial \underline{\delta}^D}{\partial \hat{w}_M} = \frac{\bar{\pi}_i(\bar{w}_M) - \tilde{\pi}_i(\tilde{w}_M)}{\left(\hat{\pi}_i(\hat{w}_M) - \tilde{\pi}_i(\tilde{w}_M)\right)^2} \frac{\partial \hat{\pi}_i(\hat{w}_M)}{\partial \hat{w}_M} < 0. \tag{113}$$

Note that the first term is positive as long as the collusive profit $\bar{\pi}_i(\bar{w}_M)$ exceeds the competitive profit $\tilde{\pi}_i(\tilde{w}_M)$, which has to be the case if coordination on collusion can be a profitable alternative for the sellers. Moreover, the deviation profit $\hat{\pi}_i(\hat{w}_M)$ (derived in Equation (66) for the case absent PMFN and in Equations (71) and (72) for the case with PMFN) decreases in the commission $\hat{w}_M$. Hence, the critical discount factor increases if the platform charges a lower commission.

Note that this derivation is independent of the exact commissions $\bar{w}_M$ and $\hat{w}_M$ that the platform charges in collusive and competitive periods, respectively. Hence, charging a one-time commission $\hat{w}_M = 0$ destabilizes collusion in the case of time-constant commissions as well as in the case of spot-optimal commissions. This establishes the result. □

Proof of Proposition 10. Consider first the case without PMFN. In this case, the platform realizes the profit $\tilde{\Pi}_M^{NP}$ (Equation (8)) with seller competition, and the profit of $\bar{\Pi}_M^{NP}$ (Equation (9)) with seller collusion. The platform benefits to destabilize seller collusion if

$$0 + \sum_{t=1}^{\infty} \eta^t \tilde{\Pi}_M^{NP}\left(w_M^{NP}\right) > \sum_{t=0}^{\infty} \eta^t \bar{\Pi}_M^{NP}\left(w_M^{NP}\right), \tag{114}$$

which supposes that a one-time commission of 0 leads to a breakdown of seller collusion.

The inequality in Equation (114) simplifies to

$$\eta > \underline{\eta}^{NP} = \frac{\bar{\Pi}_M^{NP}}{\tilde{\Pi}_M^{NP}} = 1 - \frac{\alpha}{2}. \tag{115}$$

Only if the platform's discount factor η exceeds $\underline{\eta}^{NP}$ it prefers to destabilize seller collusion.

In the case with PMFN, the platform obtains the profit $\tilde{\Pi}_M^P$ (Equation (17)) if the sellers compete and the profit $\bar{\Pi}_M^P$ (Equation (18)) if they collude. By the same calculation, we find that the platform prefers to destabilize seller collusion if

$$\eta > \underline{\eta}^{P} = \frac{\bar{\Pi}_M^{P}}{\tilde{\Pi}_M^{P}} = \frac{\left(4 - \alpha\left(4 - \sigma\left(\beta\right)\right)\right)^2}{16\left(1 - \alpha\right)}, \tag{116}$$

with $\sigma\left(\beta\right) = \sqrt{2\left(1 - \beta\right)}$.

Note that this critical discount factor can exceed the value of 1 which indicates that the platform realizes a higher profit with colluding sellers than it does with competing ones. The platform therefore does not want for any discount factor η to destabilize seller collusion. Hence, define

$$\underline{\eta}^{P} = \min\left\{\frac{\left(4 - \alpha\left(4 - \sigma\left(\beta\right)\right)\right)^2}{16\left(1 - \alpha\right)}, 1\right\}. \tag{117}$$

Finally, it holds that $\underline{\eta}^{P} > \underline{\eta}^{NP}$ for all parameter values. This establishes the result. □

Appendix B: Constrained Collusion

The main analysis in Section 4 focuses on the sustainability of full collusion on the joint profit-maximizing retail prices. If the sellers' common discount factor is too small to sustain full collusion, the analysis assumes that sellers cannot coordinate at all and play competition in every period of the infinitely-repeated game.

It is possible, however, that sellers still coordinate on smaller than fully-collusive prices if this increases their joint profits (compared to the competitive level) and fulfills the incentive-compatibility constraint. I refer to this form of collusion as *constrained collusion.* I derive two important results from this analysis. First, a PMFN allows coordination on a higher retail price if the platform's commission is in an intermediate range. This result is the mirror image to the decrease in the critical discount factor for commissions above $\tilde{w}_{max}$ derived in Proposition 5. Second, I show that high commissions (which make a deviation more tempting in the model analyzed in Section 4), decrease the constrained collusive retail price that is necessary to keep the incentive-compatibility constraint binding. This reinforces the result that a platform may prefer seller collusion over seller competition with a PMFN, as this leads to higher commissions and potentially lower retail prices, and both aspects increase a platform's profit.

Again, I suppose that sellers sustain constrained collusion by means of grim trigger strategies. Denote punishment prices as $\tilde{p}$ and suppose that sellers cannot coordinate on fully-collusive prices $\bar{p}$. I consider instead that sellers coordinate on the highest feasible retail prices such that the incentive-compatibility constraint to be willing to stick to the collusive agreement is binding. Denote the constrained-collusive prices as $\bar{p}^{PC}$ and the deviation prices, which depends on the constrained-collusive prices, as $\hat{p}\left(\bar{p}^{PC}\right)$. The joint maximization problem is as follows:

$$\begin{aligned} \max_{p\in[\tilde{p},\bar{p}]} \ \pi_{12}(p) \quad &= \quad \sum_{i\in\{1,2\}} (p_{iM} - w_M)\, q_{iM}(p) + p_{iD} q_{iD}(p) \\ & \quad\ s.t. \quad \bar{\pi}_i(p) - (1-\delta)\,\hat{\pi}_i(\hat{p}(p)) - \delta\tilde{\pi}_i(\tilde{p}) \geq 0, \ \forall i, \end{aligned} \tag{118}$$

where the constraint in the second line ensures that the incentive-compatibility constraint in Equation (22) is fulfilled. With constrained collusion the sellers' common discount factor is sufficiently small such that the constraint needs to be binding with equality as otherwise sellers can coordinate on a higher constrained-collusive prices and realize higher joint profits on the equilibrium path. If the constraint is not binding at the fully-collusive price $\bar{p}$, sellers can sustain full collusion (the case analyzed in Section 4).

For the sake of exposition, I report a representative numerical result of the constrained-collusive prices. The findings are qualitatively the same for other parameter constellations for which coordination on constrained-collusive prices is the relevant case. The results for the retail prices absent and with a PMFN are depicted in Figure 6. The first panel shows the sellers' retail prices on the platform depending on the commission $w_M \in [0, 1-\beta]$ for three cases.[26] The dotted line is the competitive price $\tilde{p}_M$, the solid line is the fully-collusive price $\bar{p}_M$, and the dashed line shows the constrained-collusive price $\bar{p}_M^{CC}$. For $\delta = 3/10$, the incentive constraint is violated at the fully-collusive prices, but sellers can coordinate on constrained-collusive prices above the competitive level $\tilde{p}_M$. As the common discount factor δ increases, sellers are able to sustain higher constrained-collusive retail prices that approach the level of full collusion as δ approaches the critical discount factor reported in Equation (25) in Proposition 4.

[26] Recall that sellers are willing to list on the platform for commissions up to $1-\beta$.

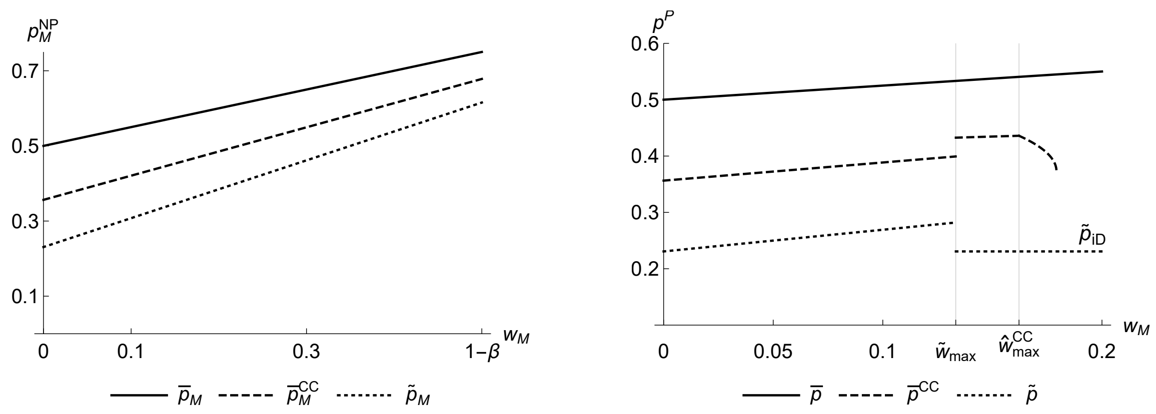

Figure 6: Retail prices with constrained collusion

The figure shows the highest feasible collusive retail price (i.e., constrained collusion) depending on the time-constant commission w_M for $\alpha = 7/10$, $\beta = 1/2$, and $\delta = 3/10$. The left panel shows the retail prices on the platform without a PMFN (NP) for the cases of competition (dotted), full collusion (solid) and constrained collusion (dashed). The right panel shows the retail prices for the case with a PMFN (P).

The second panel in Figure 6 depicts the case with a PMFN. The plot consists of three regions that are the analog to the three regions as in Figure 1 for the critical discount factor $\underline{\delta}^P$ necessary for full collusion to be stable. For a small $w_M \leq \tilde{w}_{max}$ (which is the same threshold value as in Proposition 5), sellers prefer to list on the platform for any conduct, and the plot exhibits the same features as the plot in the left panel: the constrained-collusive price lies between the competitive price level and the fully-collusive one and increases in the commission w_M.

For $w_M > \tilde{w}_{max}$, competing sellers are not willing to list on the platform, which has two consequences: First, the sellers are only active on the direct channel and optimally set the retail price $\tilde{p}_{iD}^P = (1-\alpha)/(2-\alpha)$ as derived in Lemma 2, and realize lower punishment profits compared to being present on both distribution channels. Second, this form of harsher punishment allows sellers to sustain higher constrained-collusive prices, which is apparent from the discrete increase in $\bar{p}^{CC}$ at $w_M = \tilde{w}_{max}$. This is the same mechanism that leads to the discrete decrease in the critical discount factor $\underline{\delta}^P$ characterized in Proposition 5 and depicted in Figure 1.

The third region in the plot is for commissions $w_M > \hat{w}_{max}^{CC}$, above which a seller that deviates from the constrained-collusive prices to be present on the direct channel only. In contrast to $\tilde{w}_{max}$, the threshold value $\hat{w}_{max}^{CC}$ is not the same as in the fully-collusive case ($\hat{w}_{max}$) and generally depends on the exact constrained-collusive price level. Again, above this level, deviation becomes more tempting for the sellers, which translates to lower constrained-collusive prices that can be sustained in equilibrium. Interestingly, in this range, an increase in the platform's commission leads to a decrease in the constrained-collusive price.

This result reinforces the finding that, with a PMFN, a platform may prefer seller coordination in contrast to seller competition on the platform: if sellers coordinate on constrained collusion, the platform can increase its commission above $\tilde{w}_{max}$, which is not profitable with seller competition as sellers would delist at higher commissions. Moreover, a commission above $\hat{w}^{CC}_{max}$ can lead to lower retail prices, and hence, the platform benefits from a higher commission payment than with seller competition, and, additionally, from the fact that sellers charge a low constrained-collusive retail price, which increases the quantity sold on the platform.