SEMI-MARKOV MODELING FOR CANCER INSURANCE

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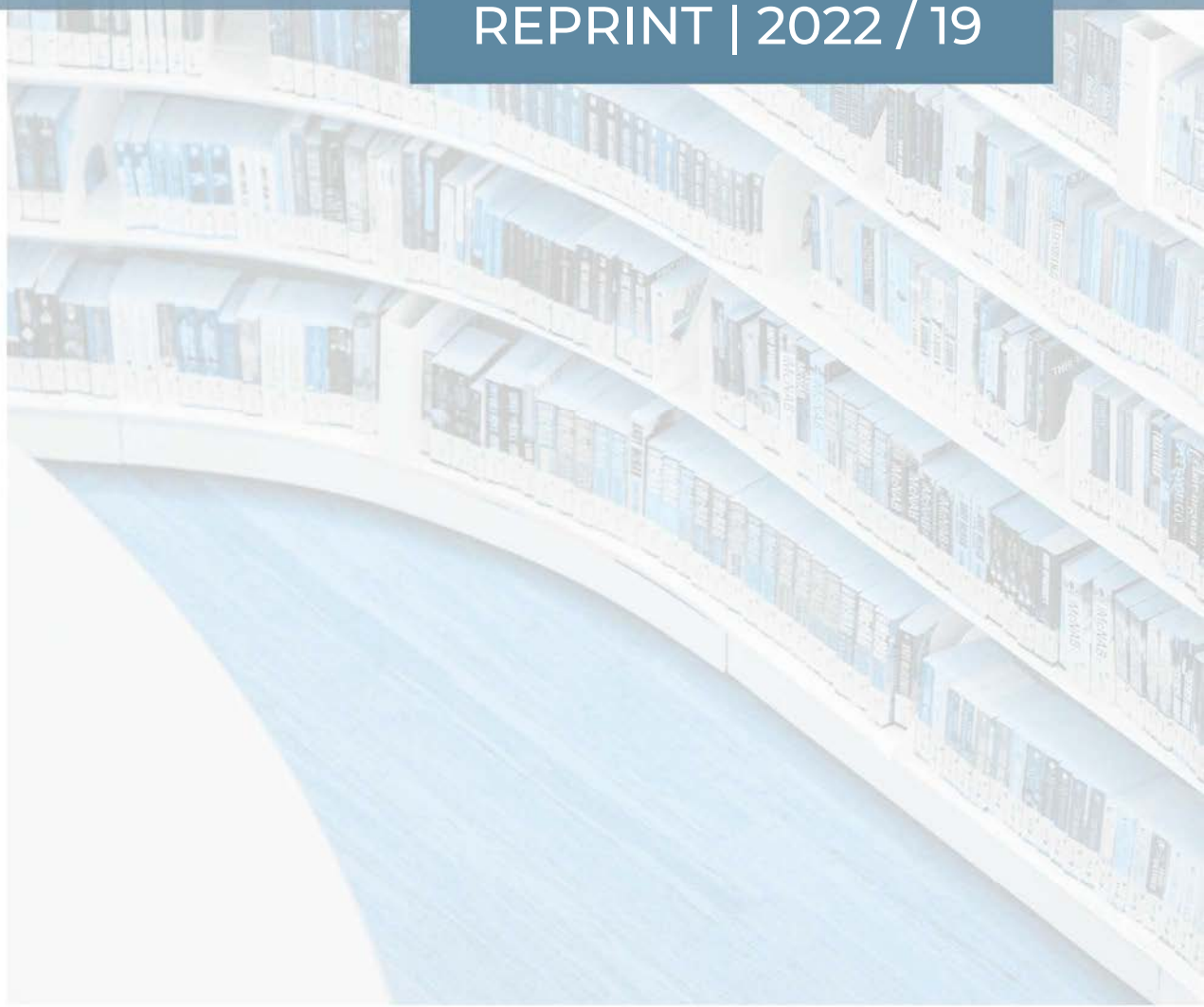

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ORIGINAL RESEARCH PAPER

Semi-markov modeling for cancer insurance

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Abstract
Advancements in medicine and biostatistics have already resulted in a better access to insurance for people diagnosed with cancer. This materializes into the "right to be forgotten" adopted in several EU member states, granting access to insurance after a waiting period of at most 10 years starting at the end of the successful therapeutic protocol. This paper concentrates on insurance covers on a market where such a right has been implemented. Stand-alone products are considered, as well as guarantees included as a rider in an existing package. The cost of offering standard premium rates to all applicants in mortgage insurance related to property loans is also evaluated. The 3-state (healthy–ill–dead) Semi-Markov hierarchical model developed in Denuit et al. [4] for long-term care insurance is adopted here for actuarial calculations. Semi-Markov transition intensities are estimated from cancer cases recorded by the Belgian Cancer Registry. The obtained results suggest that a new offer could develop, targeting the particular needs of cancer patients.

Keywords Critical illness · Medical insurance · Right to be forgotten · Multistate models

1 Introduction and motivation

Massart's [8] testimonial illustrates difficulties faced by cancer survivors to access life and health insurance products. See also Hendriks et al. [6] for the particular case of childhood cancer survivors. However, progress in medicine over the last 20 years greatly improved the prognosis of several types of cancer. In parallel, many tools have been developed in biostatistics and epidemiology to study mortality and morbidity associated with cancer. These advances have already resulted in a better

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access to insurance products for people diagnosed with cancer. For example, this led France and Belgium to establish a "*droit à l'oubli*" (translated literally as "right to be forgotten" in the remainder of this text) granting access to insurance after a waiting period of at most 10 years starting at the end of the successful therapeutic protocol, in absence of relapse within this period. The 10-year period has even been shortened for several types of cancer with a good prognosis. We refer the reader to Soetewey et al. [14] for an actuarial analysis of the "right to be forgotten" mechanism. Several initiatives purpose to extend this right throughout EU countries, by law or through a convention with insurance sector (as in Luxembourg). We refer the reader e.g. to Scocca and Meunier [12] as well as to "Survivorship challenge 3.4: Lack of knowledge of the stigma associated with cancer" listed in Lawler et al. [7] purposing to take advantage of the existing legal framework in four EU member countries (France, Belgium, Luxembourg and the Netherlands) to investigate a pan-European legal framework on access to financial services for cancer survivors.

Although the establishment of such a "right to be forgotten" is a clear progress for cancer survivors, there is still room for improvement by filling the coverage gap during the waiting period opening this right. Social security and supplemental health insurance cover most medical costs related to cancer, but non-medical costs are usually paid out of pocket. The latter include lost income (the part not covered by Social security or supplemental disability insurance), travel to and from hospital (especially for patients living in rural areas), travel and family lodging expenses, deductibles and co-payments, non-conventional comfort treatments, private nursing care costs, and non-nursing help with activities of daily living. They can rapidly become a financial hardship, even for patients who are treated on an outpatient basis.

Some insurance companies market supplemental cancer insurance policies that pay lump sum benefits upon diagnosis of cancer or a temporary life annuity to face these out-of-pocket expenditures. These products developed in Asia, North America and UK (Benett et al. [1], Nielsen and Mayer [9]). Using Taiwan National Health Insurance Database, Yue et al. [15] priced two types of whole-life insurance products: (i) a (lump-sum) benefit paid when the insured is diagnosed with cancer for the first time (and the contract terminates after the benefit is paid) and (ii) an annual benefit paid after the insured is diagnosed with cancer and as long as he or she survives. Shang [13] considered term life products and evaluated the extra cost related to cancer. Let us also mention that cancer is also typically incluced into the diseases covered by critical illness insurance policies.

The products considered in this paper are specifically related to the waiting period opening the "right to be forgotten", with temporary covers restricted to that period to fill the gap in coverage on a market where such a right has been implemented. First, stand-alone products are studied, including cancer insurance with lump sum payment at diagnosis, or temporary life annuity starting at diagnosis. In the latter case, periodic payments may correspond to insurance premiums of another product, or even to loan reimbursement. Then, riders included in a package are discussed. We consider term insurance with accelerated death benefit paid as a lump sum at diagnosis or as a temporary life annuity starting at diagnosis. The payment of rider reduces death benefit specified in term insurance. Finally, we discuss products granting access to some specific insurance cover (such as mortgage insurance) during the

waiting period opening the "right to be forgotten". This is especially important at young age, to guarantee access to property and home ownership (in case of house loan) and to entrepreneurship (in case of professional loan) to cancer patients whose health status has improved but who cannot benefit from the "right to be forgotten" because the waiting period is not exhausted. This guarantee can be bought by parents for their children (as it is commonly the case in medical insurance, where extra premiums ensure that children can continue their parents' medical cover when they leave the household, whatever their future health status) or by young adults starting their professional career (in supplement to individual health insurance, for instance). The cover may be subject to some deferred period after diagnosis in order to lower its cost (without real impact since it is very unlikely that cancer patients consider buying a house or developing professional activities right after diagnosis). The price of this cover also allows to evaluate the cost of offering standard premium rates to all applicants in mortgage insurance related to property loans. This is the approach followed by Crédit Mutuel in France, which announced in November 2021 the end of health questionnaire when applying for a home loan (under some conditions, like age below 62, amount borrowed less than 500,000 euros, having Crédit Mutuel as main bank since 7 years at least, this duration serving as an implicit waiting period as in the "right to be forgotten" mechanism, and electing domicile in the house to be bought). The bank even announces that this decision will have a retroactive effect for the ongoing loans. The loss in mortgage insurance premiums is evaluated to 70 millions euros per year for Crédit Mutuel. Our calculation help to assess the cost of this decision.

Cancer typically belongs to the set of critical illnesses and is thus also covered under several standard insurance policies. Long-term care or disability insurance policies also pay benefits in case of cancer. However, depending on the terms of the contract, a cancer patient may well have completed his or her treatment before the deferred period required to receive insurance benefits has been reached. Some patients even continue working while receiving chemotherapy or radiation therapy and thus do not qualify for loss of income. The cover comprised in the products considered in this paper is activated at diagnosis. These policies are targeted to help patients facing out-of-pocket expenditures while treatment occurs or to grant them access to other insurance products during the waiting period opening the "right to be forgotten". They thus constitute a new offer on a market where such a right has been implemented (as it is the case in Belgium or France, and may be generalized at EU level).

To illustrate our proposals, we consider the Belgian market where the "right to be forgotten" has been inserted in the Insurance Law in April 2019, with reduced waiting periods for some cancer types defined by Royal Decree in May 2019. Calculations are based on data available from the Belgian Cancer Registry (BCR), a national population-based cancer registry collecting data on all new cancer diagnoses in Belgium since the incidence year 2004. The paper considers three cancers with clear differences in terms of incidence, survival after diagnosis, and waiting periods defined by Royal Decree: melanoma (ICD-10 C43) with waiting period reduced up to 1 year after the end of the successful therapeutic protocol, thyroid (ICD-10 C73) with waiting period reduced up to 3 to 6 years after the end of the

successful therapeutic protocol, and female breast (ICD-10 C50) cancer subject to the standard 10-year waiting period. Melanoma and thyroid cancer patients are known to have limited excess mortality compared to the general population [14]. This has been recognized by reducing the waiting period opening the "right to be forgotten" in the Royal decree published in the Belgian Official Journal on June 14, 2019. The situation for female breast cancer patients is different with usually a high survival probability in the first years after diagnosis before it eventually decreases due to late cancer recurrences.

In this paper we perform our actuarial calculations in a 3-state Semi-Markov model assuming that a policyholder can be either "healthy", "ill" (diagnosed with cancer), or "dead". For the sake of easiness, only transitions from healthy to ill, healthy to dead and ill to dead are allowed so that cancer is assumed to be permanent (i.e., no recovery is possible). This non-reversibility greatly simplifies the computations (as the 3-state process is hierarchical) and appears to be reasonable for cancer insurance considered in this paper, with temporary cover restricted to the waiting period opening the "right to be forgotten" and benefits paid after the first diagnosis. For premium calculations, if mortality in cancer state reverts to the standard level after a sufficiently long period (like in cure models) then this is equivalent to a transition back to the initial healthy state. This model has been considered by Denuit et al. [4] for long-term care insurance. Let us mention that Debicka et al. [2] also considered multistate models to combine reverse annuity contracts with critical illness (in fact cancer) insurance for retired people. The cancer state is splitted into several stages to capture duration effects while remaining with a Markov structure. Here, we consider younger ages and focus on the waiting period opening the "right to be forgotten", performing calculations in the 3-state Semi-Markov model.

The remainder of the paper is organized as follows. Section 2 describes the 3-state Semi-Markov model used for premium calculation. Cancer insurance products are described in Sect. 3, where corresponding premium rates are computed in the model of Sect. 2. The final Sect. 4 discusses the results and concludes.

2 Semi-Markov 3-state model

2.1 State space and transitions

Multistate models offer a convenient representation for life and health insurance liabilities when benefits are associated to sojourns in, or transitions between different states [5, 10]. We consider an individual aged x at policy issue, taken as time 0. His or her history is described by the stochastic process $\{X_t, t \geq 0\}$ where X_t gives the state occupied at time t. Here, t corresponds to contract seniority. In this paper, we consider that $X_t \in \{a, i, d\}$ where state a stands for "active" (healthy), state i stands for "ill" (cancer) and state d stands for "dead" as represented in Fig. 1. At the time of diagnosis, individual moves from state a to state i. We do not allow for recovery (but mortality rates in state i may ultimately become similar to those applying in state a for cured individuals). Since benefits only relate to the first diagnosis, state i corresponds here to the first time a policyholder is diagnosed with cancer.

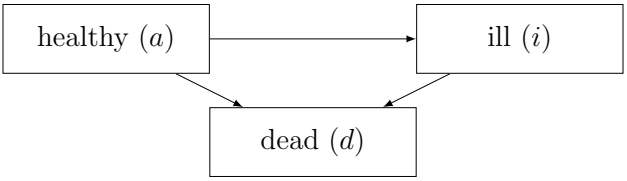


Fig. 1 Semi-Markov 3-state model for cancer insurance

The time spent in state i is known to influence mortality so that we introduce the random variable Z_t, defined as the time spent in the state occupied at time t. Formally,

$$Z_t = \max\{z \leq t | X_t = X_{t-h} \text{ for all } 0 \leq h \leq z\}.$$

For an individual in state i at time t, Z_t is the time since diagnosis. Henceforth, we work under the Semi-Markov assumption: only the current state X_t and the time Z_t spent in the current state influence future transitions. This means that stochastic process $\{(X_t, Z_t), t \geq 0\}$ is a Markov process.

2.2 Transition intensities

Transition intensities quantify the instantaneous risk of making a given transition, depending on the state currently occupied and sojourn time. Assuming that Z_t only matters in state i, transition intensities are defined by the following limits:

$$\mu_{x+t}^{ai} = \lim_{h \to 0} \frac{\mathrm{P}[X_{t+h} = i | X_t = a]}{h}$$
$$\mu_{x+t}^{ad} = \lim_{h \to 0} \frac{\mathrm{P}[X_{t+h} = d | X_t = a]}{h}$$
$$\mu_{x+t;z}^{id} = \lim_{h \to 0} \frac{\mathrm{P}[X_{t+h} = d | X_t = i, Z_t = z]}{h}, z < t.$$

State a remains Markovian so that transition intensities from that state do not depend on the time spent in the state, but only on attained age $x + t$. On the contrary, there is an influence of the duration of stay in state i so that transition intensities from state i depend on both attained age $x + t$ and time z since diagnosis.

2.3 Data

2.3.1 Belgian cancer registry (BCR)

We consider the data available from the Belgian Cancer Registry (BCR), a national population-based cancer registry collecting data on all new cancer diagnoses in Belgium since the incidence year 2004. For the execution of this main task, the BCR relies on its own specific legislation (more information can be found on the BCR website, at `kankerregister.org`).

To illustrate our work, we restrict our analyses to three cancer types: melanoma (ICD-10 C43), thyroid (ICD-10 C73) and female breast (ICD-10 C50) cancer. These three cancer sites have been selected to evaluate the proposed insurance products in different scenarios. Melanoma and thyroid cancer patients are known to have a limited excess mortality compared to the general population. The situation for female breast cancer patients is different with usually a high survival probability in the first years after the date of diagnosis before it eventually decreases due to late cancer recurrences. We consider only female breast cancer as there are very few registrations regarding to male breast cancer. We also limit our analyses to patients aged 20 to 69 at diagnosis since the products considered in this paper target young adults and active life.

A total of 24,325 persons were diagnosed with melanoma, 10,789 with thyroid and 105,127 with breast cancer between 2004 and 2018, and were followed-up until April 1, 2020. Follow-up thus ranged from 2 to 16 years. Only one record per patient (with the earliest incidence date) within each cancer site was kept for patients with multiple primary diagnoses. This is in accordance with the insurance products under consideration which are activated at the time of the first diagnosis. A minority of patients without national security number were excluded from the analysis. Patients lost to follow-up (mostly due to moving abroad) and patients still alive at the end of the follow-up period were treated as censored observations.

Table 1 summarizes the number of included cases, number and proportion of deaths and percentage of lost to follow-up before April 1, 2020 per type of cancer,

Table 1 Number of persons diagnosed with melanoma, thyroid and female breast cancer in Belgium between 2004 and 2018 (BCR data) by gender, site and age group, together with the percentage of lost to follow-up and the number of deaths

Gender	Cancer site	Age at diagnosis	Lost to follow-up	Number of included cases	Number of deaths
Men	Melanoma	20–34	3.38%	857	83
		35–49	2.24%	2812	354
		50–69	1.70%	6000	1268
	Thyroid	20–34	4.04%	322	6
		35–49	3.21%	841	58
		50–69	1.92%	1563	297
Women	Melanoma	20–34	3.22%	2174	70
		35–49	1.06%	5267	317
		50–69	1.12%	7215	874
	Thyroid	20–34	3.76%	1435	10
		35–49	2.51%	3029	78
		50–69	1.83%	3599	390
	Breast	20–34	2.87%	2685	423
		35–49	1.49%	29,007	3419
		50–69	1.07%	73,435	12,730
Total			1.40%	140,241	20,377

gender and age group. The fraction of patients lost to follow-up per subgroup varied from 1.06% for women with melanoma cancer aged 35–49 to 4.04% for male thyroid cancer patients aged 20–34. The total fraction of patients lost to follow-up cases, regardless of gender, site or age group was 1.4%.

2.3.2 General population

The products considered in this paper are sold to individuals before diagnosis (thus, in state a). This is in contrast with the study by Soetewey et al. [14] which considered individuals in state i. Therefore, we also need mortality in the general population. Belgian population life tables are available from Statbel (the Belgian statistical office) and can be freely downloaded from the website `statbel.fgov.be`.

2.4 Estimation

Transition intensities are often assumed to be piecewise constant in order to ease actuarial calculations. Starting from state a, this means that the identities

$$\mu^{ai}_{x+k+t} = \mu^{ai}_{x+k} \text{ and } \mu^{ad}_{x+k+t} = \mu^{ad}_{x+k} \tag{2.1}$$

hold for every integers x and k and fractional $0 \leq t < 1$. Transition intensity from state i is displayed as a function depending on the age at diagnosis and the time elapsed since diagnosis, i.e.

$$\mu^{id}_{x+\xi;z} = \widetilde{\mu}(x + \lfloor \xi - z \rfloor, \lfloor z \rfloor) \tag{2.2}$$

for some given function $\widetilde{\mu}$ with integer arguments, where $\lfloor \cdot \rfloor$ denotes rounding from below (i.e., the integer part). The arguments of $\widetilde{\mu}(\cdot, \cdot)$ represent, respectively, integer parts of age at entry in the ill state and time spent in that state. We thus work with age last birthday at diagnosis and the number of years since diagnosis, rounded from below. Of course, more accurate calculations can be performed by refining the time step, if needed.

When intensities are piecewise constant, they are easily estimated by the ratio of the observed number of transitions (diagnosis or death) to the corresponding exposure (in the state to be left). Precisely, consider a given integer age y and let N^{ai}_y be the number of transitions from state a to state i, that is, the number of diagnoses, among individuals aged y last birthday. Similarly, let N^{ad}_y be the number of transitions from state a to state d, that is, the number of deaths recorded among healthy individuals aged y last birthday. Let E^a_y denote the (central) exposure to risk in state a, that is, the time spent by all individuals aged y last birthday in state a. Because general population mortality statistics do not record exposures but only the number of individuals at the beginning of the period (or initial exposure to risk), we assume that transitions occur in the middle of the period. Under (2.1), the maximum likelihood estimators of μ^{ai}_y and μ^{ad}_y are respectively given by N^{ai}_y / E^a_y and N^{ad}_y / E^a_y and these values apply between ages y and $y + 1$. Similar formulas hold true under (2.2) for estimating $\widetilde{\mu}(y, z)$ for integer values y and z, classifying transitions and recording

exposures according to age last birthday y and integer number z of years since diagnosis.

Given that BCR covers the whole population, it would be possible to subtract from exposures and death counts available from Statbel the time lived and the number of deaths among cancer patients. In this way, the estimated μ_y^{ad} would only account for mortality not related to the cancer under consideration. However, in this paper, estimated intensities μ_y^{ad} have been obtained from the Belgian population life tables, so ignoring the influence of cancer mortality. The fact that population life tables include cancer mortality is not an issue as mortality for a given cancer represents only a small fraction of the overall mortality, and correcting for this over-representation of the cancer being studied has, in practice, an insignificant effect. The transition intensities from healthy to ill (cancer) and from ill to death can be estimated from BCR data (combined with general population exposures in the former case).

Even if the general shape of the mortality and incidence curves is generally clearly visible, erratic variations often remain. As long as these random departures do not reveal anything about the underlying mortality or morbidity pattern, they should be removed before entering actuarial calculations. This process is known as graduation in the actuarial literature. As there is no simple parametric model able to capture the structure of mortality and morbidity, actuaries generally use a generalized additive regression model with Poisson response distribution for transition counts, see e.g. Denuit and Legrand [3]. The method can be summarized as follows. Under (2.1), maximum likelihood inference can be equivalently conducted under the hypothesis that N_y^{ai} is Poisson distributed with mean $E_y^a \mu_y^{ai}$. The transition intensity μ_y^{ai} is then represented as $\ln \mu_y^{ai} = s(y)$ for some smooth function $s(\cdot)$ to be estimated from the data, under the assumption that N_y^{ai}, $y = 20, 21, \ldots, 69$ are mutually independent. The function $s(\cdot)$ is estimated with the help of a Poisson generalized additive model and the resulting estimate is used to produce the transition rates adopted to perform all actuarial calculations in the remainder of this paper. A similar procedure is followed to produce the other transition intensities entering the calculations. The resulting estimated transition intensities are visible in Figs. 2, 3, 4.

Figure 2 displays the estimated intensities μ_y^{ad} as functions of attained age y for males and females. Belgian regulatory life tables XR and XK are also displayed there: life table XK defines minimum premium amount for life insurance policies with a positive sum at risk (thus comprising mainly death benefits) whereas life table XR defines minimum premium amount for policies with a negative sum at risk (thus comprising mainly survival benefits). Life table XK is conservative and generates a relatively high safety loading. Dating back to the 1990s, life table XR does not comprise any safety loading anymore for women (but since it only defines minimal premium amounts, insurers remain free to charge higher premiums to remain solvent). We recognize on Fig. 2 the exponential increase in mortality at adult ages (the accident hump is not visible because actual values are displayed along the vertical axis, without log transform).

Figure 3 displays the estimated intensities μ_y^{ai} as functions of attained age y for the three types of cancer considered in this paper, separately for males and females. We can see there that incidence curves greatly differ among the three cancer types

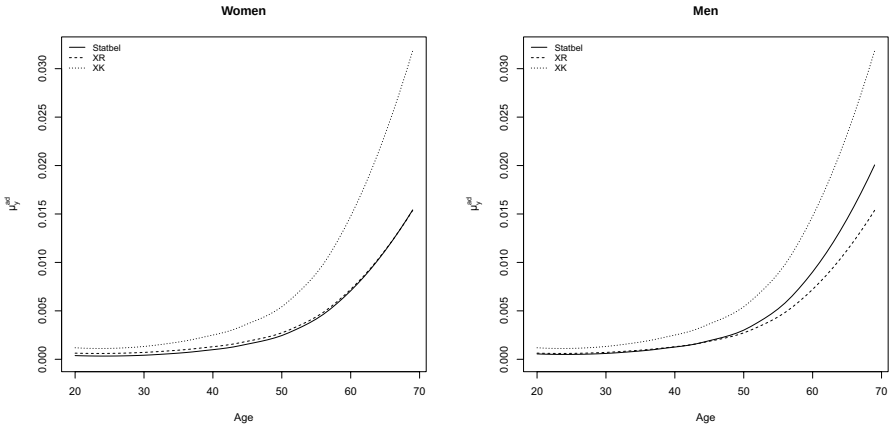


Fig. 2 Estimated transition intensities μ_y^{ad} as functions of attained age y. General population (Statbel, continuous line) and insurance regulatory life tables XR (broken line) and XK (dotted line)

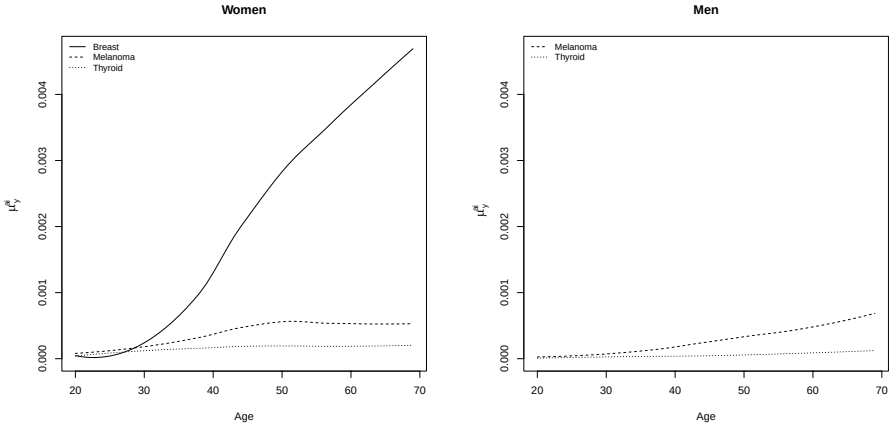


Fig. 3 Estimated transition intensities μ_y^{ai} as functions of attained age y, for different cancer types

under consideration. In particular, after age 30, incidence for women breast cancer largely exceeds the one for melanoma and thyroid. For males, incidence rates are closer at young ages but exhibit different age trends.

Estimated $\widetilde{\mu}(y, z)$ (with y and z corresponding to integer part of age at diagnosis and time since diagnosis, respectively) is displayed in Fig. 4. We can see there that mortality increases with age at entry and sojourn time for both genders and all three considered cancer sites. Mortality, however, increases less rapidly with sojourn time for young patients compared to old patients. We also see that, for patients below age 40, mortality remains low even after a long period after diagnosis (i.e., for large values of sojourn time). Note that since we computed these quantities considering all causes of death, they account for both mortality from the cancer of interest and mortality from

other causes. This is in line with the application to insurance since benefits do not vary according to the cause of death for the products considered in this paper.

2.5 Transition probabilities

The following probabilities are useful to perform actuarial calculations. Considering an individual who is healthy at age $x + t$, that is, who is in state a at time t, the probability of being in state i at time $t + u$ is denoted as

$$ {}_{u}p_{x+t}^{ai} = \mathrm{P}[X_{t+u} = i | X_t = a], $$

the probability of being in state d at time $t + u$ is denoted as

$$ {}_{u}p_{x+t}^{ad} = \mathrm{P}[X_{t+u} = d | X_t = a], $$

and the probability of being in state a at time $t + u$ is denoted as

$$ {}_{u}p_{x+t}^{aa} = \mathrm{P}[X_{t+u} = a | X_t = a]. $$

Since the time spent in state i influences future transitions, the random variable Z_t also enters the transition probabilities from that state. Precisely, considering an ill individual aged $x + t$ who has been diagnosed at time $t - z$, that is, who is in state i at time t since time $t - z$, the probability of being in state d at time $t + u$ is denoted as

$$ {}_{u}p_{x+t;z}^{id} = \mathrm{P}[X_{t+u} = d | X_t = i, Z_t = z] $$

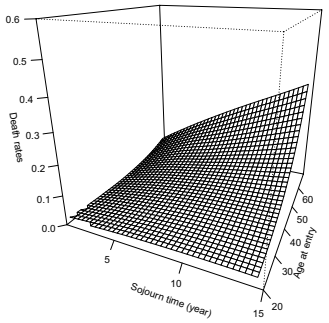
and the probability of being in state i at time $t + u$ is denoted as

$$ {}_{u}p_{x+t;z}^{ii} = \mathrm{P}[X_{t+u} = i | X_t = i, Z_t = z]. $$

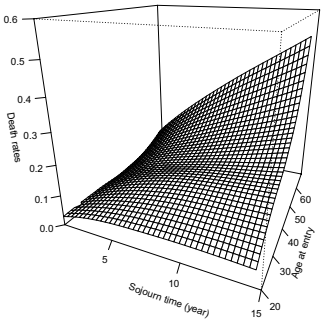
By assumption, recovery is not possible. Hence, transition probabilities ${}_{u}p_{x+t}^{aa}$ and ${}_{u}p_{x+t;z}^{ii}$ are in reality sojourn probabilities, i.e.

$$ \begin{aligned} {}_{u}p_{x+t}^{aa} &= \mathrm{P}[X_{t+h} = a \text{ for all } 0 < h \leq u | X_t = a] \\ {}_{u}p_{x+t;z}^{ii} &= \mathrm{P}[X_{t+h} = i \text{ for all } 0 < h \leq u | X_t = i, Z_t = z]. \end{aligned} $$

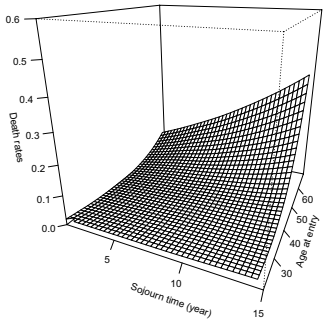
Sojourn probabilities are easy to compute when transition intensities are piecewise constant, as exponential functions of minus integrated exit rates (or cumulative hazards). This property will be used repeatedly in the next section.



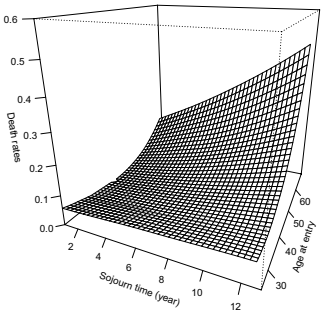
(a) Women with melanoma cancer



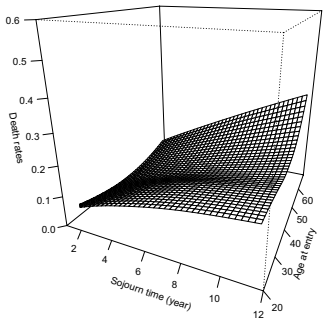
(b) Men with melanoma cancer



(c) Women with thyroid cancer



(d) Men with thyroid cancer



(e) Women with breast cancer

Fig. 4 Mortality intensities according to age at diagnosis and sojourn time in the cancer state

3 Cancer insurance products

3.1 Notation and specific policy conditions

Henceforth, $v(s, t)$ is the present value at time s of a unit payment made at time t (with $s < t$ and $v(s, s) = 1$). We assume that the technical interest rate used in actuarial calculation is constant and we denote as δ the corresponding instantaneous force of interest, that is, $v(s,t) = \exp\big(-\delta(t-s)\big)$. Also, we denote as $\mu^{a\bullet}_{x+t}$ the exit intensity from state a, that is, $\mu^{a\bullet}_{x+t} = \mu^{ad}_{x+t} + \mu^{ai}_{x+t}$.

In this paper, we consider temporary covers where diagnosis has to occur within the next n years to get the insurance benefit. The insured period is thus the time interval $[0, n]$, in the sense that a benefit is payable only if the time of diagnosis belongs to this interval. In principle, the insured period begins at policy issue and ends at policy termination, subject to the following specific policy conditions. The waiting period (or "elimination" period) w is the period following the policy issue during which the insurance cover is not yet operating. The waiting period aims at limiting the effects of adverse selection, in particular because of pre-existing insured's health conditions. The waiting period considered here is the one specified in the contract, not to be confused with the waiting period opening the "right to be forgotten" fixed by the law. Sometimes, the benefit is not payable at diagnosis but only if the policyholder has survived a certain minimum period after diagnosis, called the deferred period and denoted as f. This policy condition acts as a deductible and essentially purposes to reduce the cost and hence the premium of the insurance product; premium reduction can be particularly significant in case high mortality immediately follows diagnosis.

All premium calculations are performed on a unisex basis, in accordance with EU regulation which prohibits any difference in insurance cover or amount of premium by gender. Transitions observed for males and females have therefore been combined to produce a set of intensities independent of gender, which are used for all calculations performed in this section. For computations with respect to breast cancer, transitions for women only have been considered as only female breast cancer cases are included.

Let us comment on the particular case of the breast cancer cover. Even if this contract targets female policyholders, male breast cancer, though rare, does exist. To be consistent with the anti-discrimination EU directive, the coverage has to be offered to both males and females and priced under unisex basis. Notice that a unisex tariff may well be based on women's data exclusively (the requirement is that the premium cannot differ between male and female policyholders). Even if breast cancers are generally rare for males, they often have a very poor prognosis. This is because (i) these cancers are often detected at a later stage compared to women, (ii) there is not much research devoted to breast cancer affecting males and (iii) available treatments against female breast cancer are difficult to adapt to treat male patients because of the marked difference in hormonal status. According to the BCR, the 5-year prevalence from 2013 to 2017 in Belgium is 398 for men compared to 47,423 for women.

Since this cover may raise some concerns related to gender-based discrimination, let us consider the Femina cover sold in Belgium by AG Insurance (one of the market leaders). This is a sickness insurance product with lump sum benefits paid after diagnosis of some specific cancers affecting women, including breast cancer that may also affect men. As its name indicates, this product clearly targets women. The policy must be issued before the age of 60, offering a lifelong cover (subject to a severe underwriting conditions). Health Minister had to answer some specific queries by members of the Belgian Parliament about possible discriminatory issues related to Femina (see Question 7 by Deputy Karin Jiroflée to Minister Kris Peeters about Femina insurance, ref. P2321, plenary session of October 5, 2017). Minister Peeters asked the Financial Services and Markets Authority (FSMA, protecting consumers in the financial sector, including bank and insurance) to determine whether this product complies with the anti-discrimination EU Directive. The conclusion of the FSMA study was provided during the meeting of Parliament Economy Commission of March 28, 2018. Since AG Insurance indicated that Femina cover could also be bought by men, no discriminatory issue was found in relation to the product. This shows that even if products are marketed to address specific needs of the male or female population and are priced accordingly using data from the targeted population, this does not violate the EU Directive as long as both genders can access the cover at the same conditions (even if it is very unlikely that males will ever buy the Femina insurance cover).

3.2 Stand-alone covers

3.2.1 Lump sum

A first possibility is to pay a lump sum at diagnosis. The beneficiary can use this amount to face out-of-pocket expenditures related to treatment. The expected present value of a unit lump sum paid at diagnosis is given by

$$\overline{A}^{a;a\to i}_{x;n\rceil} = \int_0^n v(0,t)\,{}_tp_x^{aa}\,\mu_{x+t}^{ai}\,\mathrm{d}t.$$

In case the contract specifies a waiting period w, the integral is over (w, n) instead of $(0, n)$. Since diagnosis is recorded in the BCR database, the payment date t is easy to check.

When transition intensities are piecewise constant, we get

$$\begin{aligned}\overline{A}^{a;a\to i}_{x;n\rceil} =& \mu_x^{ai}\frac{1-\exp\left(-\delta-\mu_x^{a\bullet}\right)}{\delta+\mu_x^{a\bullet}} \\ &+\sum_{j=1}^{n-1}\mu_{x+j}^{ai}\exp\left(-\sum_{k=0}^{j-1}\mu_{x+k}^{a\bullet}-j\delta\right)\frac{1-\exp\left(-\delta-\mu_{x+j}^{a\bullet}\right)}{\mu_{x+j}^{a\bullet}+\delta}.\end{aligned} \tag{3.1}$$

In principle, the payment could be deferred. In many policies, the benefit is not payable until the need has lasted a certain minimum period called the deferred period [10]. Here, this would mean that the lump sum is not paid at diagnosis but the

payment is deferred later on. This may not be desired by the customers buying the product considered here so that we do not consider this possibility. Deferred periods may nevertheless be useful to lower premiums in case they are too expensive.

The values of $\overline{A}^{a;a\to i}_{x;n\rceil}$ obtained in the Semi-Markov 3-state model are displayed in Fig. 5 for ages $x \in \{20, 21, \ldots, 40\}$, coverage period $n = 20$ years, and yearly interest rate 1%, that is, $\delta = \ln 1.01$. Without discounting, that is, setting $\delta = 0$ or $v(s,t) = 1$ for all $s < t$, $\overline{A}^{a;a\to i}_{x;n\rceil}$ is the probability of being diagnosed with cancer for a healthy individual aged x over the next n years. Remember that for melanoma and thyroid cancers, it should be interpreted as the probability on the whole population while for breast cancer, it should be interpreted as the probability only among women. We can see on Fig. 5 that premium amounts remain rather low for melanoma and thyroid cancers, but considerably increase for breast cancer because of larger incidence within the Belgian population (culminating at 0.017 per unit of sum insured at age 40).

Remark 1 The single premium $\overline{A}^{a;a\to i}_{x;n\rceil}$ can be converted into a periodic one by dividing it with

$$\overline{a}^{aa}_{x;n\rceil} = \int_0^n v(0,t)\,{}_tp^{aa}_x \mathrm{d}t, \tag{3.2}$$

with the understanding that the premium is payable until diagnosis or death.

3.2.2 Temporary life annuities

Insured benefits can also consist in a temporary life annuity starting at diagnosis. This provides cancer patient with a periodic income to face out-of-pocket expenditures. The corresponding expected present value if payments are made continuously at a constant unit rate as long as cancer patient survives is given by

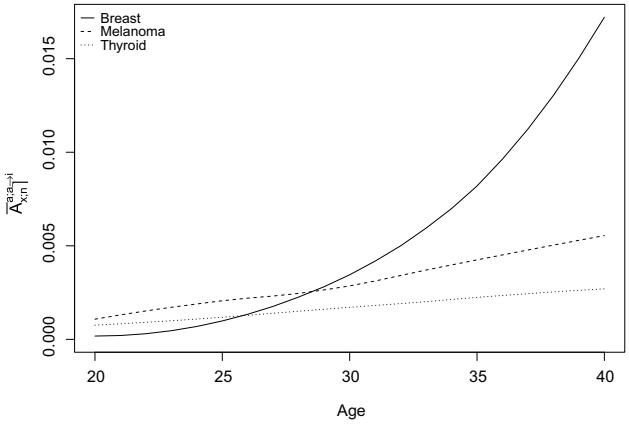


Fig. 5 Values of $\overline{A}^{a;a\to i}_{x;n\rceil}$ as function of age $x \in \{20, 21, \ldots, 40\}$ for different cancer types with $n = 20$ and yearly interest rate 1%

$$\overline{a}^{\mathrm{ai}}_{x;n\rceil} = \int_0^n {}_tp^{aa}_x \mu^{ai}_{x+t} v(0,t) \overline{a}^{ii}_{x+t;0} \mathrm{d}t \tag{3.3}$$

where

$$\overline{a}^{ii}_{x+t;0} = \int_0^m {}_sp^{ii}_{x+t;0} v(t,t+s) \mathrm{d}s \tag{3.4}$$

with m denoting the maximal payment duration. Here, m is given in policy conditions and may vary with the waiting period opening the "right to be forgotten" by the law, that is, it may depend on cancer type.

The duration m could be determined in two ways, at least. Either we adopt the reduced waiting periods specified by Royal decree but it would then be necessary to add the duration of treatment since the "right to be forgotten" in the law starts at the end of a successful treatment protocol. Or, we take the duration of the modified "right to be forgotten" since diagnosis as determined by Soetewey et al. [14]. While it could have been interesting to formally compare both approaches, individual data on the type and length of treatment for each case is not reported in the BCR and such information is not readily available. Even if it were available, the definition of the end of the treatment remains unclear (and this is precisely the reason why Soetewey et al. [14] suggested to let the waiting period start from diagnosis, to avoid endless disputes when a claim occurs). Moreover, durations of treatment are heterogeneous even within the same cancer type, usually unpredictable. Optimal durations are often still open to debates, see e.g. Schvartsman et al. [11], making it hard to include the duration of treatment in the actuarial computations. In any case, a reduction in treatment length due to the progress made in medical treatment of cancer would obviously lead to closer agreement between the two approaches. Since the date of diagnosis, as recorded in national registries, offers the great advantage of not being subject to any discussion and to allow the patient to know from the start when the waiting period will end, we think that all parties benefit from using the date of diagnosis instead of the end of treatment. For these reasons, we favor the second approach in the present paper.

When transition intensities are piecewise constant, we get

$$\begin{aligned} \overline{a}^{ai}_{x;n\rceil} &= \sum_{j=0}^{n-1} \int_j^{j+1} {}_tp^{aa}_x \mu^{ai}_{x+t} v(0,t) \overline{a}^{ii}_{x+t;0} \mathrm{d}t \\ &= \sum_{j=0}^{n-1} {}_jp^{aa}_x v(0,j) \int_0^1 {}_tp^{aa}_{x+j} \mu^{ai}_{x+j+t} v(j,j+t) \overline{a}^{ii}_{x+j+t;0} \mathrm{d}t \end{aligned} \tag{3.5}$$

for $j \in \{0, \ldots, n-1\}$ and $t \in [0,1)$, where

$$
\begin{aligned}
\overline{a}^{ii}_{x+j+t;0} &= \int_0^m {}_sp^{ii}_{x+j+t;0} v(j+t, j+t+s)\mathrm{d}s \\
&= \sum_{k=0}^{m-1} \int_k^{k+1} {}_sp^{ii}_{x+j+t;0} v(j+t, j+t+s)\mathrm{d}s \\
&= \sum_{k=0}^{m-1} {}_kp^{ii}_{x+j+t;0} v(j+t, j+t+k) \\
&\int_0^1 {}_sp^{ii}_{x+j+t+k;k} v(j+t+k, j+t+k+s)\mathrm{d}s \\
&= \sum_{k=0}^{m-1} \exp\left(-\sum_{l=0}^{k-1} \widetilde{\mu}(x+j, l) - k\delta \right) \int_0^1 \exp\left(-s\big(\widetilde{\mu}(x+j, k) + \delta\big)\right)\mathrm{d}s \\
&= \frac{1 - \exp(-\delta - \widetilde{\mu}(x+j, 0))}{\delta + \widetilde{\mu}(x+j, 0)} + \sum_{k=1}^{m-1} \exp\left(-\sum_{l=0}^{k-1} \widetilde{\mu}(x+j, l) - k\delta \right) \\
&\frac{1 - \exp(-\delta - \widetilde{\mu}(x+j, k))}{\delta + \widetilde{\mu}(x+j, k)} \\
&= \overline{a}^{ii}_{x+j;0}
\end{aligned}
\tag{3.6}
$$

which shows that $\overline{a}^{ii}_{x+j+t;0}$ can be taken out of the integral in the expression of $\overline{a}^{ai}_{x;n\rceil}$. Hence, we have

$$
\begin{aligned}
\overline{a}^{ai}_{x;n\rceil} &= \sum_{j=0}^{n-1} {}_jp^{aa}_x v(0, j) \overline{a}^{ii}_{x+j;0} \int_0^1 {}_tp^{aa}_{x+j} \mu^{ai}_{x+j+t} \exp(-t\delta)\mathrm{d}t \\
&= \mu^{ai}_x \overline{a}^{ii}_{x;0} \frac{1 - \exp(-\delta - \mu^{a\bullet}_x)}{\delta + \mu^{a\bullet}_x} \\
&+ \sum_{j=1}^{n-1} \mu^{ai}_{x+j} \overline{a}^{ii}_{x+j;0} \exp\left(-\sum_{k=0}^{j-1} \mu^{a\bullet}_{x+k} - j\delta \right) \frac{1 - \exp(-\delta - \mu^{a\bullet}_{x+j})}{\delta + \mu^{a\bullet}_{x+j}}.
\end{aligned}
\tag{3.7}
$$

Values of $\overline{a}^{\mathrm{ai}}_{x;n\rceil}$ are computed as a function of age at policy issue $x \in \{20, 21, \ldots, 40\}$ with $n = 20$ and a yearly technical interest rate of 1%, that is, $\delta = \ln 1.01$. The duration m is taken to be equal to 9 years for melanoma and 1 year for thyroid, in accordance with the reduced waiting periods fixed in the Belgian law for these cancers. For breast cancers, m is taken to be the standard waiting period of 10 years. The numerical values are displayed in Fig. 6. We can see there that the product is cheaper for melanoma and thyroid cancers from age 30, thanks to lower incidence rates and payment duration. Moreover, premiums increasing more rapidly with age are obtained for breast cancer, because of higher incidence rates and payment duration.

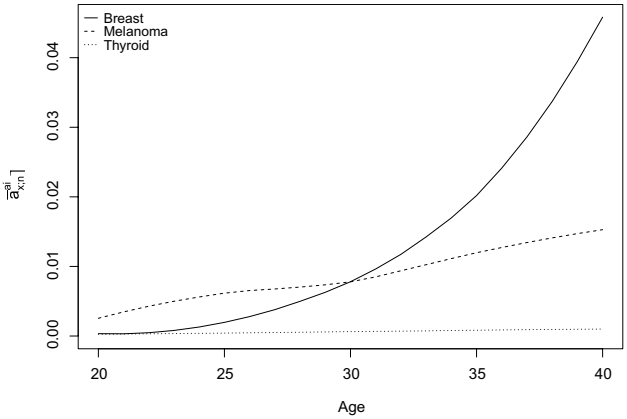


Fig. 6 Values of $\overline{a}^{\mathrm{ai}}_{x;n]}$ as function of age x for different cancer types with $n = 20$ and yearly interest rate 1%

3.3 Combined products

Combined products correspond to insurance packages where cancer insurance supplements a reference cover. Several examples are described hereafter.

3.3.1 Premium exemption

A first possibility is to use the temporary life annuity starting at diagnosis for paying the premiums of a reference cover, or even to reimburse a loan secured by mortgage insurance, for instance. Calculations are performed as explained before, with benefits matching amounts of premium or loan reimbursement.

3.3.2 Term-life insurance with cancer acceleration benefit

Cancer insurance can be added as a rider to a term-life insurance policy. In this case, the amount of death benefit is (totally or partially) converted into a lump sum paid at diagnosis. Specifically, let

$$\overline{A}^{a;a\to d}_{x;n]} = \int_0^n v(0,t)\,{}_tp^{aa}_x \mu^{ad}_{x+t}\mathrm{d}t \tag{3.8}$$

be the expected present value of a unit lump sum paid at death occurring in state a. In practice, this amount is replaced with the XK premium if the latter is higher. Let c_{ad} be the amount of death benefit for a policyholder in state a. A proportion $\alpha \in [0,1]$ of the death benefit may be paid at diagnosis and the remaining $1-\alpha$ at death in state i. For a unit death benefit, the expected present value of insurance benefits is then given by

$$\overline{A}^{(\alpha)}_{x:n\rceil} = \overline{A}^{a;a\to d}_{x:n\rceil} + \int_0^n {}_tp^{aa}_x \mu^{ai}_{x+t} \left(\alpha v(0,t) + \int_0^{n-t} {}_zp^{ii}_{x+t;0} \mu^{id}_{x+t+z;z} (1-\alpha) v(0,t+z) \mathrm{d}z \right) \mathrm{d}t. \tag{3.9}$$

When transition intensities are piecewise constant, we can compute the pure premium as follows. First, the part of the pure premium corresponding to death benefits for a healthy individual writes

$$\overline{A}^{a;a\to d}_{x:n\rceil} = \mu^{ad}_x \frac{1-\exp(-\delta-\mu^{a\bullet}_x)}{\delta+\mu^{a\bullet}_x} + \sum_{j=1}^{n-1} \mu^{ad}_{x+j} \exp\left(-\sum_{k=0}^{j-1} \mu^{a\bullet}_{x+k} - j\delta \right) \frac{1-\exp(-\delta-\mu^{a\bullet}_{x+j})}{\mu^{a\bullet}_{x+j}+\delta}. \tag{3.10}$$

Second, the accelerated death benefit payable at diagnosis is covered by $\alpha \overline{A}^{a;a\to i}_{x:n\rceil}$ where the expression for $\overline{A}^{a;a\to i}_{x:n\rceil}$ can be found in (3.1). The remaining part of death benefits involves the following integral

$$\int_0^n {}_tp^{aa}_x \mu^{ai}_{x+t} \int_0^{n-t} {}_zp^{ii}_{x+t;0} \mu^{id}_{x+t+z;z} v(0,t+z) \mathrm{d}z \mathrm{d}t = \int_0^n {}_tp^{aa}_x \mu^{ai}_{x+t} v(0,t) \overline{A}^{i;i\to d}_{x+t;\overline{n-t}\rceil} \mathrm{d}t \tag{3.11}$$

where

$$\overline{A}^{i;i\to d}_{x+t;\overline{n-t}\rceil} = \int_0^{n-t} {}_zp^{ii}_{x+t;0} \mu^{id}_{x+t+z;z} v(t,t+z) \mathrm{d}z \tag{3.12}$$

is the present value of a unit benefit payable at death before time n for an individual being diagnosed with cancer at time t. Then, (3.11) can be rewritten as

$$\sum_{j=0}^{n-1} \exp\left(-\sum_{l=0}^{j-1} \mu^{a\bullet}_{x+l} - j\delta \right) \int_0^1 \exp\left(-t\left(\mu^{a\bullet}_{x+j} + \delta\right) \right) \mu^{ai}_{x+j} \overline{A}^{i;i\to d}_{x+j+t;\overline{n-j-t}\rceil} \mathrm{d}t, \tag{3.13}$$

with the understanding that an empty sum is zero, where

$$\begin{aligned} \overline{A}^{i;i\to d}_{x+j+t;\overline{n-j-t}\rceil} = & \sum_{k=0}^{n-j-2} \exp\left(-\sum_{l=0}^{k-1} \widetilde{\mu}(x+j+l;l) \right) \widetilde{\mu}(x+j+k;k) v(j,j+k) \\ & \times \int_0^1 \exp\left(-z\left(\widetilde{\mu}(x+j+k;k)+\delta\right) \right) \mathrm{d}z \\ & + \exp\left(-\sum_{l=0}^{n-j-2} \widetilde{\mu}(x+j+l;l) \right) \widetilde{\mu}(x+n-1;n-j-1) v(j,n-1) \\ & \times \int_0^{1-t} \exp\left(-z\left(\widetilde{\mu}(x+n-1;n-j-1)+\delta\right) \right) \mathrm{d}z \end{aligned} \tag{3.14}$$

and

$$\begin{aligned}
&\int_0^1 \exp\Big(-z\big(\widetilde{\mu}(x+j+k;k)+\delta\big)\Big)\mathrm{d}z \\
&= \frac{1-\exp\Big(-\widetilde{\mu}(x+j+k;k)-\delta\Big)}{\widetilde{\mu}(x+j+k;k)+\delta} \\
&\int_0^{1-t} \exp\Big(-z\big(\widetilde{\mu}(x+n-1;n-j-1)+\delta\big)\Big)\mathrm{d}z \\
&= \frac{1-\exp\Big(-(1-t)\big(\widetilde{\mu}(x+n-1;n-j-1)+\delta\big)\Big)}{\widetilde{\mu}(x+n-1;n-j-1)+\delta}.
\end{aligned} \tag{3.15}$$

Premiums $\overline{A}^{(\alpha)}_{x:n\rceil}$ are computed as a function of age at policy issue $x \in \{20, 21, \ldots, 40\}$ with $n = 20$ and a yearly technical interest rate of 1%, that is, $\delta = \ln 1.01$. We take unit death benefit and $\alpha = 50\%$. The numerical values are displayed in Fig. 7. We can see there that the product is cheaper for melanoma and thyroid cancers for all considered ages, thanks to lower incidence rates. Furthermore, higher premiums increasing with age are obtained for breast cancer, because of higher incidence rates.

Remark 2 A temporary life annuity starting at diagnosis can also be financed by "accelerating" the payment of (part of) the death benefit. Specifically, the temporary life annuity starting at diagnosis (that is, at entry in state i) is payable continuously at rate b_i. Denoting as c_{ad} the amount of death benefit in case of transition from a to d, the residual amount of death benefit for an insured in state i dying after having spent a duration z in state i is given by

$$c_{id}(t,z) = \max\{c_{ad} - b_i z, 0\} = (c_{ad} - b_i z)_+. \tag{3.16}$$

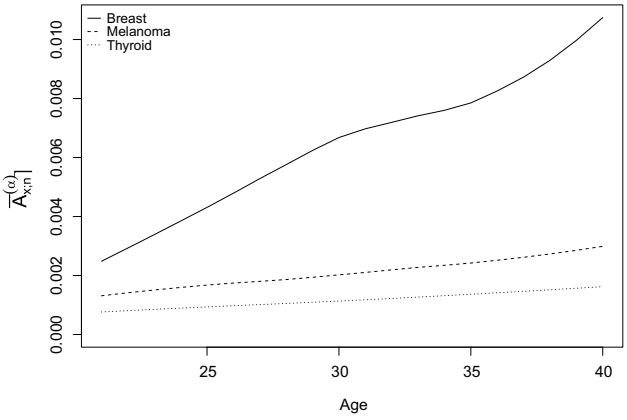


Fig. 7 Values of $\overline{A}^{(\alpha)}_{x:n\rceil}$ as function of age x for different cancer types with $n = 20$, yearly interest rate 1% and $\alpha = 50\%$

Setting the duration payment m equal to c_{ad}/b_i and converting the death benefit into a temporary life annuity starting at diagnosis, the expected present value of insurance benefits is given by

$$c_{ad}\overline{A}^{a;a\to d}_{x;n\rceil} + b_i\overline{a}^{ai}_{x;\overline{c_{ad}/b_i}\rceil} + \int_0^n {}_tp_x^{aa}\mu_{x+t}^{ai}\left(\int_0^{c_{ad}/b_i}(c_{ad}-b_iz)_zp_{x+t;0}^{ii}\mu_{x+t+z;z}^{id}v(0,t+z)\mathrm{d}z\right)\mathrm{d}t. \tag{3.17}$$

3.4 Cover option

Property loans are often accompanied with mortgage insurance that pays the balance of the loan if the mortgagor dies. Coverage is usually awarded in the form of term insurance with decreasing sum insured, with the amount of death benefit diminishing as the debt is reimbursed. This is common practice in France and Belgium. If the insurer refuses to cover the risk of premature death then the bank does not lend the money, resulting in a barrier to property and home ownership (in case of house loan) and to entrepreneurship (in case of professional loan).

The product considered in this section is issued in state a and offers the beneficiary the option to obtain mortgage insurance at standard conditions even if he or she has been diagnosed with cancer (subject to a deferred period f). The contract stipulates the characteristics of the loan (amount borrowed, amortization plan, maximal loan-to-value of the acquired building, etc.), or puts some limits. The sum insured is the difference between the actual single premium and the reference single premium computed from XK life table. This can also be seen as the expected cost on the Belgian market of a decision like the one taken by Crédit Mutuel in France. We restrict our analysis to single premiums to reduce risk for the insurer.

Let Π_{x+t}^{XK} be the reference single premium for mortgage insurance securing the loan described in the policy conditions, at age $x+t$. The actual premium, given the extra mortality related to cancer for a patient aged $x+t$ who has been diagnosed at time $t-z$ is denoted as $\Pi_{x+t;z}^{i}$. The sum insured is the difference between these two premiums. To avoid under-pricing, we consider that policyholders will exercise their option at the worst time for the insurer. This leads to the worst-case expected present value

$$\begin{aligned}&\mathrm{EPV}_{\mathrm{wc}}(x,n,f)\\&\quad=\max_s\int_0^n {}_tp_x^{aa}\mu_{x+tf+s}^{ai}p_{x+t;0}^{ii}\left(\Pi_{x+t+f+s;f+s}^{i}-\Pi_{x+t+f+s}^{XK}\right)v(0,t+s+f)\mathrm{d}t\end{aligned} \tag{3.18}$$

where n is the coverage period and f is the deferred period stipulated by the contract.

We consider here the same reference outstanding loan balance cover as in Soetewey et al. [14]. Specifically, we consider a home loan of duration 20 years (typical duration in Belgium). The mortgage insurance applicant aged x borrows an amount 100,000 at interest rate 2%. The technical interest rate for mortgage insurance is 1% and the insurance cover is over the full term of 20 years. These characteristics have been chosen as they represent a rather standard setting.

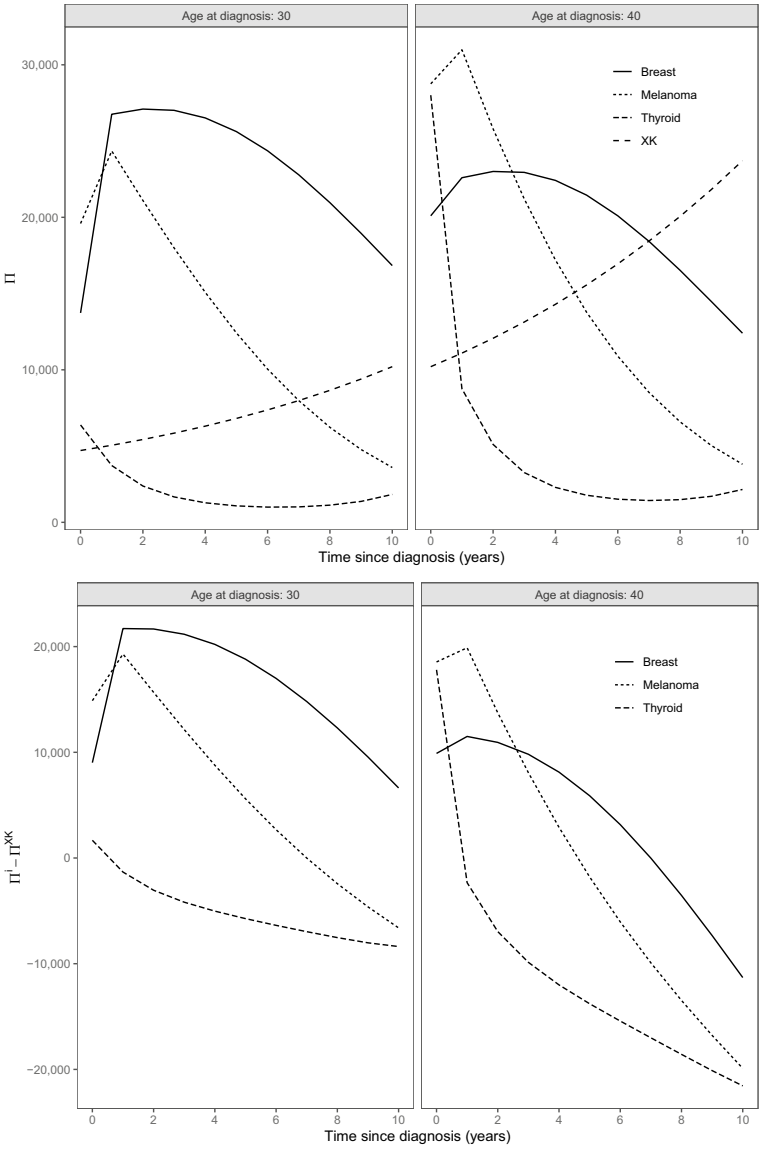


Fig. 8 Values of premiums $\Pi^{i}_{x+t+k;k}$ and Π^{XK}_{x+t+k} for $x+t=30$ and 40, and time k since diagnosis in $\{0,1,\ldots,10\}$, with $n=20$, yearly interest rate 1% and the reference outstanding balance cover

It is well documented that excess mortality generally decreases with time since diagnosis for most cancer types. Figure 6 in Soetewey et al. [14] shows that $\Pi^{i}_{x+t;z}$ peaks at $z=0$ for thyroid cancer at ages 30 and 50 and melanoma at age 50 or $z=1$ for melanoma at age 30 before decreasing for larger values of z. The upper panel in Fig. 8 displays premiums $\Pi^{i}_{x+t+k;k}$ and Π^{XK}_{x+t+k} for $x+t=30$ and 40, and time k since diagnosis in $\{0,1,\ldots,10\}$. The corresponding differences are shown in the lower

panel in Fig. 8. When the difference is negative, the cover described in this section is not needed since cancer patients can be covered at standard premium rates. We can see there that the maximum is generally attained at $s = 0$ when $f \geq 2$.

Let us take $f = 2$ so that access to mortgage insurance at standard rate is granted 2 years after diagnosis. This is expected to address patients' needs since it is very unlikely that they wish to buy a house right after diagnosis, the two-year deferred period being devoted to the acute phase of treatment. Notice that this can be offered at no cost for thyroid cancer since $\Pi^{i}_{x+t;t}$ falls below Π^{XK}_{x+t} two years after diagnosis in that case. The worst-case expected present value is then equal to

$$\begin{aligned}&\mathrm{EPV}_{\mathrm{wc}}(x,n,2)\\&\quad=\sum_{j=0}^{n} v(0,j)_j p_x^{aa}\int_0^1 {}_t p_{x+j}^{aa}\mu_{x+j;2}^{ai}p_{x+j+t;0}^{ii}v(j,j+t+2)\Big(\Pi^{i}_{x+j+2;2}-\Pi^{XK}_{x+j+2}\Big)\mathrm{d}t\end{aligned} \tag{3.19}$$

where we have assumed that insurance premiums only depend on integer age. When transition intensities are piecewise constant, we get

$$\begin{aligned}&\sum_{j=0}^{n-1}\exp\left(-\sum_{l=0}^{j-1}\mu_{x+l}^{a\bullet}-j\delta\right)\mu_{x+j}^{ai}\exp(-2\delta)\\&\int_0^1\exp\Big(-t\big(\mu_{x+j}^{a\bullet}+\delta\big)\Big)\exp\big(-\widetilde{\mu}(x+j;0)-\widetilde{\mu}(x+j+1;1)\big)\Big(\Pi^{i}_{x+j+2;2}-\Pi^{XK}_{x+j+2}\Big)\mathrm{d}t\\&=\sum_{j=0}^{n-1}\exp\left(-\sum_{l=0}^{j-1}\mu_{x+l}^{a\bullet}-j\delta\right)\mu_{x+j}^{ai}\exp(-2\delta)\\&\frac{1-\exp\big(-\mu_{x+j}^{a\bullet}-\delta\big)}{\mu_{x+j}^{a\bullet}+\delta}\exp\big(-\widetilde{\mu}(x+j;0)-\widetilde{\mu}(x+j+1;1)\big)\Big(\Pi^{i}_{x+j+2;2}-\Pi^{XK}_{x+j+2}\Big).\end{aligned} \tag{3.20}$$

The upper panel in Fig. 9 displays the sum insured $\Pi^{i}_{x+t+2;2}-\Pi^{XK}_{x+t+2}$ according to age $x+t \in \{20,21,\ldots,50\}$. We can see that the differences are negative for thyroid cancer so that the product is not needed in that case, as patients can be covered at standard rates after the deferred period. Amounts $\mathrm{EPV}_{\mathrm{wc}}(x,n,2)$ are displayed in the lower panel of Fig. 9 for $x \in \{20,21,\ldots,40\}$, with $n = 20$, yearly interest rate 1% and the reference outstanding balance cover. Only melanoma and breast cancer are considered since the cover is not relevant for thyroid cancer. The cost appears to be moderate for melanoma but much higher for breast cancer. This results again from the higher incidence of breast cancer compared to melanoma.

4 Discussion

In this paper, we have developed a Semi-Markov 3-state model for designing and pricing cancer insurance products on a market where the "right to be forgotten" has been implemented. Different covers are proposed and three cancer types are

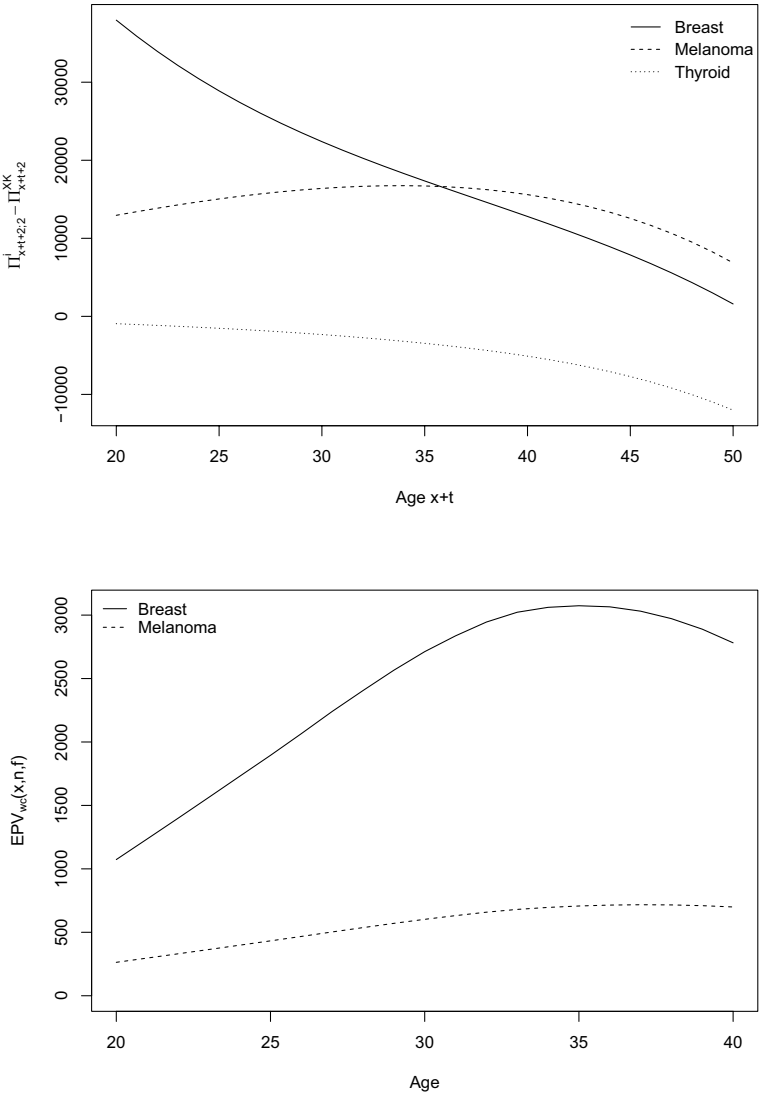


Fig. 9 Differences $\Pi^{i}_{x+t+2;2} - \Pi^{XK}_{x+t+2}$ according to age $x + t \in \{20, 21, \ldots, 50\}$ in the upper panel and $\text{EPV}_{\text{wc}}(x, n, 2)$ for $x \in \{20, 21, \ldots, 40\}$, with $n = 20$, yearly interest rate 1% in the lower panel

considered for illustration, with different incidence rates and survival prognosis. It is shown that insurance products can be developed to address the particular needs of patients during the waiting period opening the "right to be forgotten", but that costs greatly vary according to cancer type.

Insurance covers are typically limited to the first cancer occurrence. Several types of cancer can thus easily be combined into a single model to design products offering protection against more than just one cancer site. Considering the three cancer

types considered in this paper, this would result in a hierarchical Semi-Markov model with 5 states, with state i replaced with three states i_1, i_2 and i_3 corresponding respectively to thyroid, melanoma and breast cancer, with transitions from state a to states $\{i_1, i_2, i_3, d\}$ and from each i_1, i_2 and i_3 to state d. The extension to this more general setting is thus straightforward. Of course, distinguishing among cancer types is only relevant if coverage conditions vary with cancer site (for premium calculation, since distinguishing cancer types provides the actuary with a better understanding of cash flows, for instance to compute accurate reserves once a claim has been filed).

In Belgium, new diseases like, amongst others, HIV, some types of hepatitis and leukemia qualified for the "right to be forgotten". Even if the present paper restricts to cancer insurance, a more general critical illness approach would theoretically also be possible. The main difficulty however is the lack of nationwide registry for these diseases. An appropriate source of reliable and representative data must thus be identified to perform actuarial calculations assessing the actual costs of these extensions to the "right to be forgotten".

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