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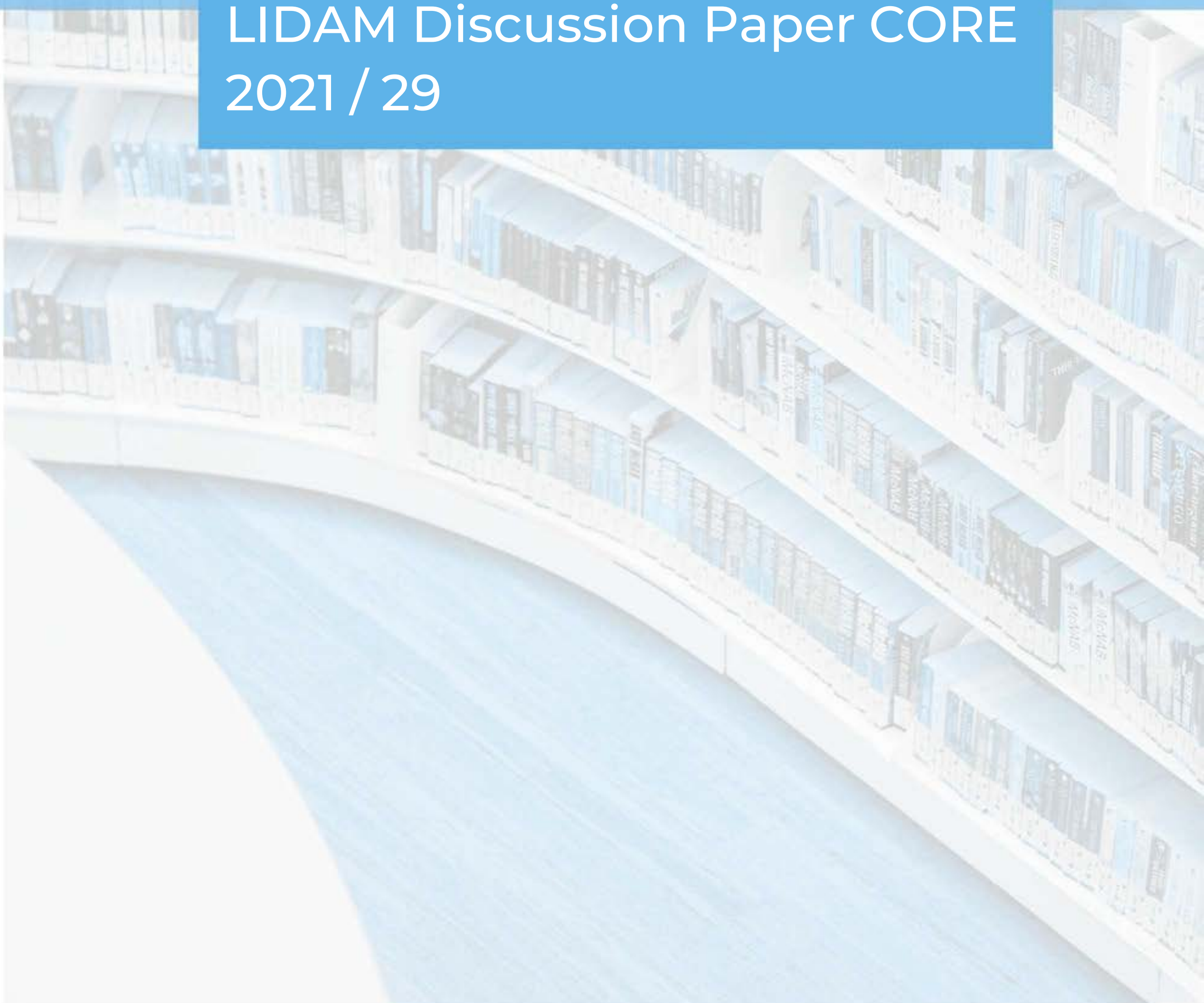

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# Income Inequality, Productivity, and International Trade


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**Abstract**

This paper discusses the effect of income inequality on selection and aggregate productivity in a general equilibrium model with non-homothetic preferences. It shows the existence of a negative relationship between the number and quantity of products consumed by an income group and the earnings of other income groups. It also highlights the negative effect of mean-preserving spread of income on aggregate productivity through the softening of firms' selection. This effect is however mitigated in the presence of international trade. In a quantitative analysis, it is shown that a too large mean-preserving spread of income may harm the rich as it raises firms' markups on her purchases. This is contrary to the general belief that income inequality benefits the rich.



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# 1 Introduction

Income inequality reappears as a hot social and economic issue in many developed countries (Atkinson, Piketty and Saez 2011, Piketty 2013). The majority of the economics literature has focused on studying the causes for income inequality, and technological progress and trade liberalization have been presented as two major driving forces.[1] In this paper, we ask a different question – how does income inequality affect aggregate economic performance and welfare in the context of an open economy? In particular, does there exist an equity-efficiency trade-off in the sense that an increase in income inequality (i.e., a decrease in equity) increases efficiency as measured by aggregate productivity? Or, could this be the other way around?

These questions has largely been ignored in the trade literature because of the usual premise of homothetic preferences (e.g. Krugman 1981, Melitz 2003[2]) or absence of income effects in the consumption of traded goods (e.g. Melitz and Ottaviano 2008). As those premises make most aggregate economic variables invariant to income redistribution there is no point to discuss its effect there. In contrast, the assumption of non-homothetic preferences allows to shed light on the effect of income inequality on aggregate productivity and welfare in the frameworks of the recent trade literature with firm heterogeneity and endogenous product variety *à la* Melitz (2003) and Melitz and Ottaviano (2008).

We first motivate our theoretical investigation by examining the conditional correlations between a country's TFP and its income inequality. Using a country-year panel data during 1996-2012, and using the Gini coefficient and top 10% income share of two measures of income inequality, we find significant and negative correlations of aggregate TFP with the two inequality measures, controlling for country and/or year fixed effects. Moreover, as the two major explanations for the cross-country differences in economic performance are institutions and geography (or market access), we also control for these two factors, and find that the negative correlation remains robust. In other words, even conditional on institution, geography, and history (the state of development of a country right before 1996 is subsumed into the country fixed-effect), income inequality provides an additional explanatory power on aggregate TFP of a country.

We propose a theoretical analysis in which income inequality and trade affect aggre-

[1] For skill-biased technical change, see, for example, Berman, Bound, and Machin (1998) and Acemoglu (2002). On the effect of globalization, see, for example, Grossman and Rossi-Hansberg (2008), Costinot and Vogel (2010), Helpman, Itskhoki, and Redding (2010), Behrens, Pokrovsky and Zhelobodko (2014), Grossman, Helpman, and Kircher (2017), Grossman and Helpman (2018), and Kim and Vogel (2018).

[2] In fact, this conclusion applies for all models in the model class characterized by Arkolakis, Costinot, and Rodriguez-Clare (2012).

gate productivity. We study a general equilibrium model in which firms have heterogeneous productivity and quality while individuals are endowed with different skills and same Stone-Geary non-homothetic preferences.[3] The presence of various skill groups results in income inequality and lead to demand patterns varying with individuals' incomes. We concentrate on an economy with two income groups (rich and poor) not only for the sake of analytical tractability but also because of the recent focus on top and bottom income groups. For tractability, we impose a Pareto productivity distribution in some parts of the analysis.

To clarify the basic properties of the model, we first analyze a closed economy where each firm enters and draws a differentiated variety with specific quality and then decides to exit or produce its variety according to its quality-adjusted unit production cost. Under the assumed preferences, the consumption choice of an individual is unambiguously represented by a *choke price* of her inverse demand function. A choke price is the maximum price at which she is willing to purchase a first unit of a variety. In contrast to Melitz and Ottaviano (2008) where there is no income effect due to quasi-linear preference, choke prices in our model differ across income groups. The choke prices of the rich and poor groups are then sufficient statistics of the demands for the whole set of varieties in the economy. Moreover, the price elasticity of individuals' demand also varies with choke price. In particular, ceteris paribus, the richer income group faces lower price elasticity of demand. For this reason, firms' pricing behavior hinges upon income groups and firms separate in two sets: the set of firms that have low quality-adjusted costs and target all consumers with low prices and the set of firms that have high quality-adjusted cost and target only the rich consumers with high prices. This is readily illustrated by the example of posters and art paintings: while both goods have the same decorative functionality, the latter is much more costly to make (especially in terms of per unit quality). At the equilibrium, only richer individuals are willing to purchase the two goods to decorate their houses. At the equilibrium, the price of each variety follows the movement of the rich and poor's choke prices.

Our analysis is mainly based on quality-adjusted productivity/cost, and our model predicts that the rich purchase relatively more the products with higher cost per unit quality. With rather reasonable assumptions, we also show that the rich purchase products with higher quality and those with higher prices, but our main results do not hinge on these assumptions.

Income inequality affects the average productivity across firms. It indeed alters the prices of varieties through its effect on the rich and poor's equilibrium choke prices. We

[3] The same preference is also used by Murata (2009) and Simonovska (2015).

show that an increase in the rich group's income raises this group's choke price, but there is a *cross effect* that such increase in the rich's income reduces the poor's choke price. The rich group is willing to consume a wider set of varieties and entices new firms producing more costly varieties to enter. At the same time prices augment and the poor reduce the basket and the quantity of her purchases. On average, firms uses more input to produce their goods, which decreases the average productivity. Similar effect emerges when the poor group becomes poorer because the cross effect implies that the rich's choke price becomes larger. As a result, a mean-preserving spread implies a lower average productivity because the rich's choke price unambiguously increases, and there are on average more costly firms in the economy.

We secondly study the effect of trade liberalization in an open economy. We find that the negative effect of a mean-preserving spread on aggregate productivity is mitigated by trade liberalization. The intuition is that lower trade costs expand variety and induce tougher selection. Smaller trade costs lead to a tougher selection of firms in favor of those with lower production costs. Compared with autarky, the number of unsold varieties is larger within the global economy. Hence, when the rich gets a higher income, she spreads her consumption towards the wider set of unsold goods in the whole world rather than concentrates her purchases on the narrower set of domestic unsold goods. In the end, consumed goods are produced with lower costs. In other words, it is the productivity gains of globalization that mitigate the negative impact of income inequality on average productivity.

We conduct a numerical analysis to further examine the properties that are difficult to obtain analytically. The above-mentioned analysis regarding average productivity is based on the unweighted average across firms. We examine how aggregate productivity (i.e., average productivity weighted by cost) reacts to mean-preserving spreads. In particular, when the poor become poorer, their consumption basket is more toward the varieties that are cheaper to produce. Can this force alter the previous result? The answer is no: we still find unambiguous decreases in aggregate productivity with mean-preserving spreads.

In the numerical analysis, we set the rich group to be the top $10\%$ income earners. In 2015, the income ratio between the two groups in the US is $7.9$. Using equivalent variation as a "real" measure of utility change, we find that an income reallocation from the income ratio of $7.9$ to $1$ is equivalent to a $69\%$ rise of the poor's *real income* and a $30\%$ fall of the rich's. However, this result suggests that for a given amount of additional income, the improvement in welfare in real terms would be larger if such additional income is given to the poor than to the rich. Similarly, even assuming Benthamite social welfare function, in which case the social planner does not actually value equality in utility, our result shows that income reallocation from the rich to the poor is welfare improving. Surprisingly, we also find that whereas mean-preserving spreads increase the rich's income, the effect on the rich's utility can actually fall when the income inequality is large. The reason behind this result is two-folds: increasing income inequality reduces aggregate productivity and increases markups when the rich/poor gain/lose presence in the market.

Our paper is closely related to the broad literature of heterogeneous firms and productivity that is pioneered by Melitz (2003) and Eaton and Kortum (2002). To our knowledge, our analysis is the first to offer new testable predictions about how income inequality affects firm selection and average productivity. In contrast to the traditional view of equity-efficiency tradeoff, Aghion et al. (1999) have highlighted reducing income inequality may promote economic growth through saving, investment and incentives; Murphy, Schleifer, and Vishny (1989) have found similar conclusion via a market-size effect. Matsuyama (2002) has studied the dynamic effect of income inequality on productivity in the context of homogeneous firms and learning by doing. Higher income inequality is detrimental to growth because it reduces the "mass of consumption" and therefore the dynamic productivity gains from learning by doing.[4] Through a different mechanism, our model shows that average productivity falls with inequality as it shuffles the mass of consumption from low-cost to high-cost goods.

This paper relates to the literature on the relationship between income heterogeneity and trade. Matsuyama (2000) and Fajgelbaum, Grossman and Helpman (2011) focus on the effect of income heterogeneity on the patterns of trade in contexts in which goods differ in some vertical attributes (quality or priority of consumption). Behrens and Murata (2012), Fajgelbaum and Khandelwal (2016), McCalman (2018), Hottman and Monarch (2018), and Borusyak and Jaravel (2018) make contributions on the welfare implications of trade liberalization for different income groups. Nevertheless, none of these studies discuss the effects of income heterogeneity on selection and productivity and how trade matters for these effects.

This paper is also related to the broad literature on the effect of nonhomothetic preference. It can be used to study pro-competitive effect and pricing to markets, such as in Simonovska (2015), Bertoletti, Etro, and Simonovska (2018), and Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2018), on optimality in monopolistic competition models, such as Parenti, Ushchev, and Thisse (2017) and Dhingra and Morrow (2019), on structural

[4] In a model with nonhomothetic preference, product innovation, and two income groups, Foellmi and Zweimüler (2006) show that more income inequality leads to faster growth as the new product is solely sold exclusively to the rich.

change, such as in Comin, Lashkari, and Mestieri (2018), or on trade flows and patterns of trade, such as in Fieler (2011) and Matsuyama (2015). Even though there are income effects in these models, there is no differentiation of income within a country.

The remainder of the paper is organized as follows. Section 2 provides an empirical motivation for our theoretical investigation. Section 3 lays out the model in the closed economy, and provides various comparative statics, with a focus on the effect of income. Section 4 extends the model to the open economy, and carries out similar analysis with a focus on the effect of trade liberalization. Section 5 provides a quantitative analysis of the effects on aggregate productivity and welfare. Section 6 concludes.

# 2 Empirical Motivation

To motivate our theory, this section provides suggestive evidence on the relationship between income inequality and productivity. This section presents essentially conditional correlations without attempting to establish a causal relation. As we are concerned with how productivity is related to income inequality, we control for two major factors affecting the level of development or technology of a country – institution and geography (i.e., market access).[5] Mainly, we ask the following question: conditional on institution and market access, is there a positive or negative correlation between income inequality and country-level productivity? We first describe our country-year panel data and empirical specification, and then presents the results.

Country-level productivity is measured by the total factor productivity (TFP) obtained from the Penn World Table (PWT) 9.0.[6] Note that since Version 8 of the PWT, quality-adjusted prices and quantities have been incorporated in the cross-country comparison; thus the following empirical investigation is consistent with our theoretical model in which we consider quality-adjusted prices, costs, and productivity.

A special feature of the PWT data is that there are one measure of TFP for cross-country comparison (CTFP), where the TFP level of the USA is set to $1$ for all years, and another by-country time-series measure (RTFPNA), where the TFP level is calculated relative to the country's 2011 level (hence TFP of each country at 2011 is set to $1$). To utilize the panel data nature of our regressors, we construct a panel of TFPs in the following way.

[5] There is a vast literature regarding these two factors. For the role of institution, see, for examples, Acemoglu, Johnson and Robinson (2005), Levchenko (2007), and Acemoglu and Robinson (2012). For the role of geography and market access, see, for examples, Krugman (1991), Diamond (1997), Redding and Venables (2004), Redding and Sturm (2008).

[6] For the detailed account for Penn World Table 8.0 and 9.0, see Feesntra, Inklaar, Timmer (2015).

We calculate a country $c$'s TFP at year $t$ relative to the US' level at 2011:

$$\text{TFP}_{c,t} \equiv \text{CTFP}_{c,t} \times \text{RTFPNA}_{\text{USA},t}.$$

A concern of such a panel of TFPs is that if year fixed-effects are controlled, then the panel is essentially reduced to a pool of cross-section TFPs because $\text{RTFPNA}_{\text{USA},t}$ is the same for all countries for each given year; thus, in this case, we basically rely on the cross-sectional variations of the regressors to explain the variation in the TFP. We include specifications where (1) only country fixed-effects are controlled and (2) both country and year fixed-effects are controlled.

We use two measures for income inequality for the period of 1996-2012: the *Gini Coefficient* and the share of total income by the top 10 percent (*Top* $10\%$ *Income Share*), both obtained from World Development Indicator.[7] Following the literature on institution, we use the rule of law as the measure for institutional quality of a country. The *Rule of Law* index is obtained from the Worldwide Governance Indicators by the World Bank.[8] Following the literature on economic geography, we define *Market Access* as trade–cost-discounted and price-deflated sum of market sizes around the world. We use the real market potential from CEPII's Market Potential database.[9]

We first peek at the simple correlation by plotting averages of log of TFP and averages of income inequality measures (the averages are taken over years). Panels (A) and (B) of Figure 1 plot the average *Gini Coefficient* and average *Top* $10\%$ *Income Share*, respectively. There is a clear negative correlation between income inequality and TFP.

[7] We interpolate (but not extrapolate) the missing values based on available years for each country. The number of countries in the overlap between the income-inequality measure and the TFP varies year by year, but the number of countries is significantly smaller than 66 before 1996 and after 2012. Hence, we restrict the sample to 1996-2012 to have a more inclusive set of countries.

[8] This index is calculated by including several indicators which measure the extent to which agents have confidence in and abide by the rules of society, including perceptions of the incidence of crime, the effectiveness and predictability of the judiciary, and the enforceability of contracts. During 1996-2012, the *Rule of Law* is missing in 1997, 1999, and 2001, and hence we also interpolate for the missing values for these years.

[9] This is computed using Head and Mayer's (2004) method, which adjusts for the impacts of national borders on trade flows.

Figure 1(A) Figure 1(B)

We will estimate the following equation:

$$\ln \text{TFP}_{it} = \beta_0 + \beta_1 \text{Inequality}_{it} + \beta_2 X_{it} + d_i + d_t + \varepsilon_{it}$$

where $\text{Inequality}_{it}$ is the measure of inequality for country $i$ in year $t$; $X_{it}$ denotes our set of covariates (*Rule of Law* and log of *Market Access*); $d_i$ and $d_t$ are country and year fixed effects. Note that country fixed-effect includes history, i.e., the state of development of a country right before 1996. To account for potential serial correlations and heteroscedasticity, standard errors are clustered at country level.

The regression results are reported in Table 1. Columns (1) - (6) show results based on the Gini coefficient, whereas Columns (7) - (12) use the top 10% income shares. For each income inequality measure, the first column estimates the case where both year and country fixed-effects are controlled, whereas the second one estimates the case where the year fixed-effects are dropped for the reason explained above. The third and fourth columns are similar to the first two, except that now the *Rule of Law* is included as a control variable. The sample includes 1297 observations with 100 countries.[10] In the fifth and sixth columns, we further include the *Market Access* as a control. One caveat is that the sample size is reduced by half when the *Market Access* is included because the available years for this measure are only up to 2003.

[Insert Table 1 here.]

Both measures of inequality exhibit significant and negative correlation with TFP across most columns, consistent with the observation from Figure 1. When we control

[10] Nevertheless, due to data constraint, it is an unbalanced panel.

for institution and geography, the coefficients on income inequality in Columns (5-6) and (11-12) are significant at $10\%$ level, whereas those in the other columns are significant at $1\%$ level. The difference in significance levels is mostly due to the smaller sample sizes in Columns (5-6) and (11-12) due to data constraint. Also, the TFP of a country is higher when the country has larger effective market size and better institution, confirming the rationales of including these controls.

Columns (6) and (12) are our most preferred specification, as it allows both the time-varying and cross-sectional variations of the regressors to explain the variation in TFP. Taking Column (12) as a benchmark, we can interpret the coefficient of $-0.807$ as the following: if the *Top* $10\%$ *Income Share* increases by 10 percentage points, the associated decline in TFP is about $7.7\%$, conditional on the same rule of law, market access, and year fixed-effects and country-specific time-invariant factors.

We next turn to our theory of how average productivity is affected by income inequality. We note here that our explanation is based on productivity selection, a mechanism that is distinct from institution or geography.

# 3 Closed Economy

We present a model where a mass $N$ of individuals are endowed with Stone-Geary preferences over a set of differentiated varieties $\omega \in \Omega$. Each individual $h$ belongs to either the high or low income group $h \in \{L, H\}$ with income $s_h > 0$ and probability $\alpha_h \in (0, 1)$, $h \in \{L, H\}$ ($s_H > s_L$ and $\alpha_H + \alpha_L = 1$). Each firm produces a distinct good $\omega$ with different quality level. Firms differ in marginal cost and face monopolistic competition.

## 3.1 Demand

An individual in the income group $h$ chooses the consumption profile $q\left(.\right)$ that maximizes her utility $\int_{\omega\in\Omega} \ln\left(1 + \beta(\omega) q\left(\omega\right)\right) \mathrm{d}\omega$[11] subject to her budget constraint $\int_{\omega\in\Omega} p\left(\omega\right) q\left(\omega\right) \mathrm{d}\omega = s_h$, where $\beta(\omega)$ is a quality shifter reflecting the number of quality units embedded in good

[11]This is an affine transformation of the original Stone-Geary utility function $\int_{\omega\in\Omega} \ln\left(q\left(\omega\right) + \overline{q}\right) \mathrm{d}\omega$.

$\omega$, and the price profile $p(\cdot)$ is taken as given.[12] Her demand is equal to

$$q_h(\omega) = \frac{\hat{p}_h}{p(\omega)} - \frac{1}{\beta(\omega)}, \tag{1}$$

where

$$\hat{p}_h = \frac{s_h + P_h}{|\Omega_h|}, \tag{2}$$

is a choke price, $\Omega_h$ is the set of goods that she consumes, $|\Omega_h| \equiv \int_{\omega \in \Omega_h} \mathrm{d}\omega$ is the measure of this set and

$$P_h \equiv \int_{\omega \in \Omega_h} \frac{p(\omega)}{\beta(\omega)} \mathrm{d}\omega \tag{3}$$

is her (personal) price index over her consumptions (see Appendix A). Note that the price index $P_h$ and therefore the choke price $\hat{p}_h$ are adjusted for quality. The intercept of the individual demand curve is given by the choke price $\hat{p}_h$ times the quality shifter $\beta(\omega)$; this is the willingness to pay for the first unit of a good $\omega$. In other words, $\hat{p}_h$ can be interpreted as the choke price for the first quality unit of a good $\omega$. At given exogenous set of prices and consumed goods, the choke price increases with larger income, larger price index and smaller set of consumed goods. Yet, for any endogenous set of consumed goods, it is readily shown that the choke price is larger for higher income individuals: $\hat{p}_H > \hat{p}_L$.

The aggregate demand for each good $\omega$ with price $p(\omega) = p$ is given by

$$Q(\omega, p) \equiv \begin{cases} \alpha_H N \left( \frac{\hat{p}_H}{p} - \frac{1}{\beta(\omega)} \right) & \text{if} \quad \frac{p}{\beta(\omega)} \in [\hat{p}_L, \hat{p}_H) \\ N \left( \frac{\hat{p}_{HL}}{p} - \frac{1}{\beta(\omega)} \right) & \text{if} \quad \frac{p}{\beta(\omega)} \in [0, \hat{p}_L) \end{cases}, \tag{4}$$

where $\hat{p}_{HL} \equiv \alpha_H \hat{p}_H + \alpha_L \hat{p}_L$ is the average of individual choke prices ($\hat{p}_H \geq \hat{p}_{HL} \geq \hat{p}_L$). Because of the presence of two income groups, it has a convex kink at $p = \hat{p}_L \beta(\omega)$. The model mixes the properties of Mussa and Rosen's (1978) unit-purchase model with two income groups of consumers who demand one unit of an indivisible good with continuous-purchase models where goods are infinitely divisible.

[12] We use an additive utility function that yields the Stone-Geary demand functions. Those are linear in income but do not exhibit expenditure proportionality (Pollak 1971). The linearity property is essential for the demand aggregation process below. The utility function belongs to the class of hierarichal preferences whereby the rich's basket of goods includes the poor's one, which has recieved good empirical support (Jackson 1984). Simonovska (2015) exploits this set-up to study international pricing-to-market under the assumption of homogenous income within a country.

The price elasticity is

$$\varepsilon(p) = -\frac{d\ln Q(\omega,p)}{d\ln p} = \begin{cases} \frac{\hat{p}_H}{\hat{p}_H - p/\beta(\omega)} & \text{if } \frac{p}{\beta(\omega)} \in [\hat{p}_L, \hat{p}_H) \\ \frac{\hat{p}_{HL}}{\hat{p}_{HL} - p/\beta(\omega)} & \text{if } \frac{p}{\beta(\omega)} \in [0, \hat{p}_L) \end{cases}.$$

Because $\hat{p}_H \geq \hat{p}_{HL}$, for a same price $p$, the elasticity is lower in the rich consumer segment.

## 3.2 Production

Labor is the only input. We consider two groups of individuals who differ only in the number of efficiency units they offer: a high (low) income individual is endowed with $s_H$ ($s_L$) efficiency units of labor. In other words, we can interpret this as a difference in human capital. We choose labor efficiency unit as the numéraire so that $s_H$ and $s_L$ also measure high and low incomes.

Each firm produces and sells a unique good $\omega$ under monopolistic competition. We assume the existence of a large pool of potential risk neutral entrants. By hiring $f$ units of labor, each entrant obtains a distinct good $\omega$ with quality shifter $\beta(\omega)$ and gets a feasible production defined by an idiosyncratic marginal input in labor efficiency units, $\beta(\omega)c$ where $c \in \mathbb{R}^+$ denotes the quality-adjusted marginal cost (i.e. the cost per unit of quality). Given the above choice of numéraire, $\beta(\omega)c$ also denotes the firm's marginal cost. The quality-adjusted cost parameter $c$ is drawn from a cumulative probability distribution $G : \mathbb{R}^+ \to [0,1]$. We denote the mass of entrants by $M$. Therefore, each measure of goods $\mathrm{d}\omega$ is identical to the measure $M\mathrm{d}G(c)$. We assume the natural condition that the marginal cost distribution have a bounded support and finite mean:

$$G : [0, c_M] \to [0,1] \text{ such that } \mathrm{E}(c) = \int_0^{c_M} c\mathrm{d}G(c) < \infty. \tag{A0}$$

Given the one-to-one mapping between goods, quality-adjusted costs and quality shifters, we can index the goods $\omega$ by their cost $c$ and their quality shifters by $\beta(c)$.

Each firm with quality shifter $\beta$ and quality-adjusted cost $c$ maximizes its profit $(p - \beta c)Q(c,p)$ taking the choke prices $\hat{p}_L$ and $\hat{p}_H$ as given. Under this specification, the profit turns out to be a function of the quality-adjusted price $p/\beta$ and quality-adjusted cost $c$ (but not the quality shifter $\beta$ alone). So, it is more convenient to discuss the firm's optimal choice in terms of its quality-adjusted price and cost. Because the demand $Q$ includes two segments, a firm can choose between targeting only the high income group or both income groups. The optimal quality-adjusted price is given by

$$\frac{p^*(c)}{\beta(c)} = \begin{cases} (\hat{p}_{HL}c)^{1/2} & \text{if} \quad c \leq \hat{c} \\ (\hat{p}_{H}c)^{1/2} & \text{if} \quad c > \hat{c} \end{cases} \tag{5}$$

and

$$\hat{c}^{1/2} \equiv \frac{(\hat{p}_{HL})^{1/2} - (\alpha_H \hat{p}_H)^{1/2}}{1 - \alpha_H^{1/2}}. \tag{6}$$

(see Appendix B). Except at $c = \hat{c}$, the optimal price is strictly concave increasing function of the quality-adjusted cost $c$. The concavity reflects that the presence of a pro-competitive effect whereby markups fall with higher cost $c$. Observe that because $\hat{p}_H > \hat{p}_{HL}$, the price jumps upward for the firm with cost $c$ just above $\hat{c}$, reflecting a switch towards targeting the high income consumers. Note that, in a partial equilibrium where we change one choke price and take the other as fixed, we have

$$\frac{\partial \hat{c}}{\partial \hat{p}_H} < 0 \quad \text{and} \quad \frac{\partial \hat{c}}{\partial \hat{p}_L} > 0. \tag{7}$$

This means that the cutoff $\hat{c}$ falls when the rich gets higher income and their choke price rises. This is because their willingness to pay improves and more firms find it profitable to target them. By contrast, the cutoff rises when the poor become richer and their choke price rises. Targeting the entire population becomes more profitable.

In the product market equilibrium, each income group purchases the goods that are targeted to them. In particular, the low income consumers buy only the goods produced at a quality-adjusted cost in the range $[0, \hat{c}]$ with $\hat{c} < \hat{p}_L$. High income individuals purchase goods produced at quality-adjusted costs in a range $[0, \hat{p}_H]$ with $\hat{p}_H > \hat{c}$. Their willingness to pay is too low to purchase the goods with quality-adjusted price $p/\beta$ higher than their choke price $\hat{p}_H$ (proof in Appendix B). Note that high income individuals purchase the goods with higher quality-adjusted prices, which are produced at higher costs per quality unit, $c \in [\hat{c}, \hat{p}_H]$.

The idea that richer people buy higher-quality goods can be easily incorporated in this model by assuming $\beta'(c) > 0$, i.e., goods with higher quality are more costly to produce per unit quality. Even if $\beta'(c) \leq 0$, richer people still buy the goods with higher product prices if the quality shifter $\beta(c)$ is not quickly decreasing in $c$ so that unit production cost $c\beta(c)$ is increasing in $c$ (or equivalently, $d \ln \beta(c)/d \ln c > -1$). We shall assume this to conform with the fact that richer people purchase more expensive varieties, but most of our results do not hinge on this assumption.

Note finally that firms' optimal markup is given by

$$m^* \equiv \frac{p^*(c)}{\beta(c)c} = \frac{\varepsilon(p^*(c))}{\varepsilon(p^*(c))-1} = \begin{cases} \frac{\hat{p}_H\beta(c)}{p^*(c)} & \text{if} \quad \frac{p^*(c)}{\beta(c)} \in [\hat{p}_L, \hat{p}_H) \\ \frac{\hat{p}_{HL}\beta(c)}{p^*(c)} & \text{if} \quad \frac{p^*(c)}{\beta(c)} \in [0, \beta(c)\hat{p}_L) \end{cases}.$$

Because $\hat{p}_{HL} \leq \hat{p}_H$, for a same price and same quality, markup is higher in the rich's market segment.

## 3.3 Equilibrium

Given the above analysis, equilibrium choke prices can be written as

$$\hat{p}_L = \frac{s_L + P_L}{MG(\hat{c})}, \quad \hat{p}_H = \frac{s_H + P_H}{MG(\hat{p}_H)} \quad \text{and} \quad \hat{p}_{HL} = \alpha_H\hat{p}_H + \alpha_L\hat{p}_L$$

and the price indices as

$$P_L = (\hat{p}_{HL})^{1/2} \int_0^{\hat{c}} c^{1/2} M \mathrm{d}G(c) \quad \text{and} \quad P_H = P_L + \hat{p}_H^{1/2} \int_{\hat{c}}^{\hat{p}_H} c^{1/2} M \mathrm{d}G(c).$$

Because choke prices and price indices are adjusted for quality, their equilibrium values depend only on the quality-adjusted costs $c$ (but not the quality shifters $\beta(c)$). Eliminating price indices, these equilibrium conditions can be expressed as

$$e_H(\hat{p}_H, \hat{p}_L) - \frac{s_H}{M} = 0, \tag{8}$$

$$e_L(\hat{p}_H, \hat{p}_L) - \frac{s_L}{M} = 0, \tag{9}$$

where

$$e_H(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_H - (\alpha_H\hat{p}_H + \alpha_L\hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c),$$

$$e_L(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_L - (\alpha_H\hat{p}_H + \alpha_L\hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c)$$

are the consumers' average expenditures per available good while $\hat{c}$ is given by its definition (6). After some algebraic manipulations, one can simplify the equilibrium conditions (8) and (9) as

$$M = \frac{s_H}{e_H(\hat{p}_H, \hat{p}_L)} = \frac{s_L}{e_L(\hat{p}_H, \hat{p}_L)}. \tag{10}$$

Thus, consumers' expenditures per unit of income are equal across income groups and equal to the equilibrium mass of entrants . The product market equilibrium is defined by the solution of those two equations for the choke prices $(\hat{p}_H, \hat{p}_L)$. For a given $M$, the equilibrium choke prices are sufficient statistics of product market equilibrium consumption and production choices.

In the long run firms enter the market. Before entry, each entrant expects to cover her entry cost so that

$$\int_0^\infty \max\{\pi(c), 0\} \mathrm{d}G(c) = f,$$

where the profit $\pi(c)$ is given by $N\left(\hat{p}_{HL}^{1/2} - c^{1/2}\right)^2$ if $c \leq \hat{c}$ and by $\alpha_H N \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2$ if $c > \hat{c}$. As stated earlier, profits are functions of quality-adjusted cost (but not quality shifter). Then, the entry condition writes as

$$\pi(\hat{p}_H, \hat{p}_L) = \frac{f}{N}, \tag{11}$$

where we define

$$\pi(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{p}_H} \max\left\{\left((\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} - c^{1/2}\right)^2, \alpha_H \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2\right\} \mathrm{d}G(c) \tag{12}$$

is the expected operational profit per capita and after entry.

The general equilibrium is defined by the variables $\hat{p}_H$, $\hat{p}_L$ and $M$ solving the conditions in (8), (9) and (11). Those conditions are expressed in terms of quality-adjusted cost $c$ (but not quality shifters $\beta(c)$). Since those equations include continuous expressions, the condition for general equilibrium existence requires that those expressions change signs on their supports. We show in Appendix C that this holds true under Assumption A0.

A condition for the uniqueness of the general equilibrium can be found as follows. First note that the expected operational profit $\pi(\hat{p}_H, \hat{p}_L)$ is an increasing function of both choke prices. So, the entry condition describes a decreasing relationship between the two choke prices. Second, it can be seen that the second equality in (10) describes an increasing relationship between the two choke prices if the conditions $\partial e_h / \partial \hat{p}_h > 0$ and $\partial e_h / \partial \hat{p}_l < 0$ hold for any $h \neq l \in \{H, L\}$. Under those conditions, it is clear that the two relationships cross in a single point $(\hat{p}_H, \hat{p}_L)$ that yields the unique equilibrium. The main question is to verify that those conditions are true.

Using (6), it is easy to verify that the poor's expenditure increases with own choke price and falls with the rich's choke price: $\partial e_L / \partial \hat{p}_L > 0$ and $\partial e_L / \partial \hat{p}_H < 0$. The symmetric condition holds for the rich provided that firms do not change consumer segment targets.

That is, if the cut-off cost $\hat{c}$ is fixed. However, by (6), the cut-off cost $\hat{c}$ falls ($\mathrm{d}\hat{c} < 0$) when $\hat{p}_H$ rises or $\hat{p}_L$ falls. Then a mass $-g(\hat{c})\mathrm{d}\hat{c} > 0$ of firms shift to the high income segment target, which reduces the rich's expenditure by the amount $\left(\hat{p}_H^{1/2} - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2}\right) \hat{c}^{1/2}$ $(-g(\hat{c})\mathrm{d}\hat{c})$. The change in firms' segment target therefore decreases the rich's expenditure and goes in the opposite direction of the effect of choke prices when $\hat{c}$ is fixed. Since this countervailing effect is proportional to the density $g(\hat{c})$, some smoothness property are required to guarantee that $G$ is not misbehaved about $c = \hat{c}$. Let

$$\partial e_H / \partial \hat{p}_H > 0 \quad \text{and} \quad \partial e_H / \partial \hat{p}_L < 0. \tag{A1}$$

We then have the following:

**Proposition 1.** *The equilibrium exists and is unique if the cost distribution $G$ satisfies (A0) and (A1).*

From now on, we assume that the cost distribution $G$ satisfies (A0) and (A1) so that a unique equilibrium exists.

## 3.4 Income Distribution

We are interested in understanding how demands and choke prices are affected by changes in income levels of the two groups. Intuitively, an increase in the income of one group raises its willingness to pay, choke price and product demands. Since demand elasticity falls with higher income, markups and prices increase. Facing higher prices, the other group is enticed to diminish its consumption, which should be reflected by lower choke prices. In Appendix D, we prove the following lemma:

**Lemma 1.** *Under (A0) and (A1), a rise in the rich (resp. poor) group's skill and income raises its choke price and demands whereas it reduces the poor's (resp. rich's). Formally,*

$$\frac{\mathrm{d}\ln \hat{p}_h}{\mathrm{d}\ln s_h} = -\frac{\mathrm{d}\ln \hat{p}_h}{\mathrm{d}\ln s_l} > 0, \quad h \in \{H, L\}, \ell \neq h. \tag{13}$$

This has implications about the effect of income distribution on the average productivity and set of consumption goods. First, in this model, the two group incomes can be written as $s_H = \overline{s} + \alpha_L v$ and $s_L = \overline{s} - \alpha_H v$ where $\overline{s} \equiv \alpha_L s_L + \alpha_H s_H$ is the average income and $v \equiv s_H - s_L$ the income differential. Using this definition, a mean-preserving spread of the income distribution is equivalent to a rise in $v$, holding $\overline{s}$, $\alpha_H$ and $\alpha_L$ constant. As a result, by (13), a mean-preserving spread increases the choke price of the high income group. It indeed increases the high income and decreases the low income so that

$$\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}v} = \frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H}\frac{\mathrm{d}\ln s_H}{\mathrm{d}v} + \frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_L}\frac{\mathrm{d}\ln s_L}{\mathrm{d}v} = \frac{\overline{s}}{s_H s_L}\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H} > 0. \tag{14}$$

Second, the average quality-adjusted productivity is negatively related to the (unweighted) average quality-adjusted cost in the economy, which is given by $\int_0^{\hat{p}_H} c\mathrm{d}G(c) / \int_0^{\hat{p}_H} \mathrm{d}G(c)$. The average cost moves in the same direction as the choke price $\hat{p}_H$ while the average productivity goes in the opposite direction. Hence, by (13), the average productivity falls with higher $s_H$. By (14), it also falls with a mean-preserving spread of the income distribution (higher $v$). The point is that when the high income group gets richer, it consumes more goods with high production cost, which raises the average cost and reduces the average productivity in the economy.[13]

Finally, we investigate how the baskets of goods is altered after income changes. The basket of the poor's individual is given by the cut-off cost $\hat{c}$. This cost falls with a higher income for the rich and decreases with a higher income for the poor. We indeed have

$$\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_H} = \frac{\partial\ln\hat{c}}{\partial\ln\hat{p}_H}\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H} + \frac{\partial\ln\hat{c}}{\partial\ln\hat{p}_L}\frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_H} < 0,$$

where the inequality stems from (7) and (13). The increase in the rich's income raises her demand so that more firms target her and raise their prices. Higher prices then decreases the poor's demand and raises further the incentives to target the rich. It is readily verified that the opposite effect holds with a change in the poor's income: $\mathrm{d}\ln\hat{c}/\mathrm{d}\ln s_L > 0$. As a consequence, a mean-preserving spread of the income distribution reduces the cut-off cost $\hat{c}$. Indeed one readily checks that

$$\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}v} = \frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_H}\frac{\mathrm{d}\ln s_H}{\mathrm{d}v} + \frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_L}\frac{\mathrm{d}\ln s_L}{\mathrm{d}v} < 0.$$

The mean-preserving spread therefore reduces the relative measure of goods consumed by the poor to the rich: that is, it reduces the ratio

$$\frac{MG(\hat{c})}{MG(\hat{p}_H)} = \frac{G(\hat{c})}{G(\hat{p}_H)}.$$

[13] Note that the same properties hold for the "unadjusted" average cost $AC(\hat{p}_H) \equiv \int_0^{\hat{p}_H} c\beta(c)\mathrm{d}G(c) / \int_0^{\hat{p}_H} \mathrm{d}G(c)$ if $c\beta(c)$ increases in $c$; i.e. if $d\ln\beta(c)/d\ln c > -1$, which holds under our earlier assumption. In this case, we have $AC' > 0$ so that average cost moves in the same direction as the choke price $\hat{p}_H$ while the average unadjusted productivity goes in the opposite direction. By (13) and (14), $AC$ also falls with a mean preserving spread of the income distribution (higher $v$).

**Proposition 2.** *A mean-preserving spread of the income distribution (i) increases the choke price of the high income group, (ii) reduces the (unweighted, quality-adjusted) average productivity in the economy and (iii) reduces the set of goods consumed by the poor relative to that by the rich.*

Income redistribution policies have the opposite effect of mean-preserving spreads: they lower the choke price of the high income group and raises average productivity. This model yields a clear-cut answer as to how mean-preserving spread of income distributions affect aggregate economic performances. Such a result does not show up in a model under homothetic preference or under a quasi-linear preference (Melitz 2003; Melitz and Ottaviano, 2008).[14]

Finally, the impact of a spread in the income distribution on the equilibrium utility is not clear at this stage. The equilibrium utility can be written as a function of optimal quality-adjusted prices, which are themselves functions of quality-adjusted cost and do not depend on the profile of quality shifters $\beta(\cdot)$. For instance, low income consumers have a utility successively given by $\int_0^{\widehat{c}} [1 + \beta(c)q_L^*(c)] \, \mathrm{d}G(c) = \int_0^{\widehat{c}} \widehat{p}_{HL}/ [p^*(c)/\beta(c)] \, \mathrm{d}G(c) = \int_0^{\widehat{c}} (\hat{p}_{HL}/c)^{1/2} \, \mathrm{d}G(c)$. Although we have seen above that a mean-preserving spread in the income distribution decreases $\widehat{c}$, its impact on the choke price $\hat{p}_{HL}$ cannot be signed. The same argument holds for the high income consumers' utility. For this result we now specify the distribution of productivity.

### 3.5 Pareto Productivity Distribution

To obtain more analytical results, we now assume Pareto productivity distribution. Since $c$ is the inverse of productivity, this implies that the c.d.f. of the cost distribution is given by $G(c) = (c/c_M)^\kappa$ for $c \in [0, c_M]$ and $\kappa \geq 1$. For the sake of conciseness, we further use $r = \hat{p}_H/\hat{p}_L$ to refer to the choke price ratio. The equilibrium quality-adjusted prices can be written as

$$\frac{p^*(c)}{\beta(c)} = \begin{cases} (\alpha_H r + \alpha_L)^{1/2} \hat{p}_L^{1/2} c^{1/2} & \text{if} \quad c \leq \hat{c} \\ r^{1/2} \hat{p}_L^{1/2} c^{1/2} & \text{if} \quad c > \hat{c} \end{cases}, \tag{15}$$

while the cutoff cost as

$$\hat{c}^{1/2} = \frac{(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1 - \alpha_H^{1/2}} \hat{p}_L^{1/2}. \tag{16}$$

[14]For example, in Melitz (2003), the homothetic preference implies that all that matters for selection is the mean (or total) income. In Melitz and Ottaviano (2008), the quasi-linear preference also implies the income elasticity of demand for differentiated goods is zero. That is, richer individuals spend the same amount on the differentiated products as the poor individuals, and they only spend more in the numeraire good.

This gives the following three equilibrium conditions

$$0 = \Phi\left(r; \kappa, \alpha_H, \frac{s_H}{s_L}\right), \tag{17}$$

$$\hat{p}_L = c_M^{\frac{\kappa}{\kappa+1}} \left(\frac{f}{N}\right)^{\frac{1}{\kappa+1}} \left[\Gamma_2\left(r; \kappa, \alpha_H\right)\right]^{-\frac{1}{\kappa+1}}, \tag{18}$$

$$M = \frac{N s_L}{f} \frac{\Gamma_2\left(r; \kappa, \alpha_H\right)}{\Gamma_1\left(r; \kappa, \alpha_H\right)}. \tag{19}$$

where $\Phi$, $\Gamma_1$ and $\Gamma_2$ are functions given in Appendix E. From (17) the value of the choke price ratio $r$ only depends on the exogenous parameters $\kappa, \alpha_H$, and the income ratio $s_H/s_L$. Given the value of $r$, one can determine the choke price $\hat{p}_L$ and the mass of entrants $M$ from (18) and (19). The Pareto cost distribution permits to separate the effects of some parameters and sufficient statistics such as $s_H/s_L$ and $f/N$.

It is shown in Appendix E that Assumptions (A0) and (A1) hold under Pareto cost distribution, so that the general equilibrium exists and is unique while Proposition 2 holds. Hence, a mean-preserving spread of the income distribution reduces the (unweighted) average productivity in the economy and reduces the set of goods consumed by the poor relative to that by the rich. Also, it is shown that the equilibrium choke price ratio $r$ strictly increases with income inequality (higher $s_H/s_L$). Income inequality therefore increases the discrepancy between the prices of the goods sold to the poor and to the rich.

Pareto distribution brings additional equilibrium features that are reminiscent of Melitz and Ottaviano's (2008) model in which entering firms draw their productivity parameters from a Pareto probability distribution. For instance, it is shown in Appendix E that the number of entrants $M$ is proportional to the population size $N$ and inversely proportional to entry cost $f$. In a similar spirit, the choke prices $(\hat{p}_L, \hat{p}_H)$, the cut-off cost $\hat{c}$, and each equilibrium price $p^*(c)$ decrease with larger population size $N$ and smaller entry cost $f$, which reflects the effect of increasing returns to scale. Finally, a proportional increase in incomes (i.e. higher $s_L$ and $s_H$ holding $s_H/s_L$ unchanged) raises the mass of entrants $M$, which reflects both increasing returns and love for variety.[15]

[15] With the Stone-Geary type preferences, proportional increases in income usually entail more complex effects than what we have here. For this reason, the literature on non-homotheticities along the balanced growth path has avoided the Stone-Geary; see, for examples, Boppart (2014) and Comin, Lashkari, and Mestieri (2018). The fact that the Pareto productivity distribution allows a tractable analysis under the Stone-Geary preferences may be used for other research topics.

Under Pareto productivity distribution, the equilibrium utility can be written as

$$U(s_H) = \frac{M\hat{p}_L^\kappa}{c_M^\kappa}\left[a^\kappa \ln\left(r^{1/2}(\alpha_H r + \alpha_L)^{-1/2}\right) + \frac{r^\kappa}{2\kappa}\right] \tag{20}$$

$$U(s_L) = \frac{M\hat{p}_L^\kappa}{c_M^\kappa}\left[\frac{a^\kappa}{2\kappa} - a^\kappa \ln\left[\left(1-\alpha_H^{1/2}\right)a + (\alpha_H a r)^{1/2}\right]\right] \tag{21}$$

where $a \equiv \hat{c}/\hat{p}_L = \left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right]^2 / \left(1-\alpha_H^{1/2}\right)^2$ with $1/r < a < 1 < r$. Using the equilibrium conditions, we get

$$\frac{M\hat{p}_L^\kappa}{c_M^\kappa} = s_L \left(\frac{N}{f}\right)^{\frac{1}{\kappa+1}} c_M^{-\frac{\kappa}{\kappa+1}} \frac{\left[\Gamma_2(r;\kappa,\alpha_H)\right]^{\frac{1}{\kappa+1}}}{\Gamma_1(r;\kappa,\alpha_H)}$$

So, utility rises with a larger population mass $N$ and lower fixed input $f$. Those parameters indeed increase labor supply and decrease labor demand so that more firms enter and generate more product diversity and more competition, which benefit consumers. Larger average productivity also raises utility as it can be easily shown that a decrease in $c_M$ increases both the intensive and extensive margins. The utility differential between high and low income consumers is equal to

$$U(s_H) - U(s_L) = \frac{M\hat{p}_L^\kappa}{2c_M^\kappa}\left\{a^\kappa \ln\left[\frac{ar\left[\left(1-\alpha_H^{1/2}\right)a^{1/2} + (\alpha_H r)^{1/2}\right]^2}{\alpha_H r + \alpha_L}\right] + \frac{r^\kappa - a^\kappa}{\kappa}\right\}.$$

As $ar > 1$, it can be easily shown that the logarithm term is positive. Hence, $U(s_H) > U(s_L)$ as $r > a$.

The effect income inequality on utility is not apparent from the above analytic, and we will further explore this in our quantitative analysis in Section 5.

# 4 Open Economy

We now study the implications of international trade and extend the above model to many trading countries and trade costs. We focus on the properties of income distribution and trade integration in the case of symmetric countries.

We consider $n$ countries each with the same population size $N$ and workers' skill distribution $s_H$ and $s_L$ with the probability $\alpha_H$ and $\alpha_L \in (0,1)$ ($\alpha_H + \alpha_L = 1$). Earnings in each country are respectively $ws_H$ and $ws_L$ where $w$ is the local wage. In each country, each firm produces a unique good $\omega$ with quality shifter $\beta(\omega)$ under monopolistic competition using an idiosyncratic quality-adjusted marginal cost $c$, which yields a variable cost $\beta(\omega)cw$. Firms now produce for the local and foreign locations and incur an iceberg trade cost $\tau - 1 > 0$ per unit of exported good. That is, every unit of exported good costs $\tau\beta(\omega)cw$. Firms incur no trade cost on their local sales. They pay a cost $wf$ to enter so that, in each country, each of the $M$ entrants obtains a distinct good $\omega$ and draw a quality-adjusted cost parameter $c$ from the cumulative probability distribution $G$. Again, the measure of goods produced in each country $\mathrm{d}\omega$ is equal to $M\mathrm{d}G(c)$. One can then replace the label of a good $\omega$ by its production cost $c$ and the quality shifter by $\beta(c)$. Given the symmetric setting, all economic variables are equal and we can normalize all local wages to one.

Because of the symmetry, the aggregate demand for imports or local goods is given by the expression $Q\left(c,p\right)$ in (4). A firm producing a good with quality $\beta$ and quality-adjusted cost $c$ makes a profit $\left(p-\beta c\right)Q\left(c,p\right)$ for its home sales and $\left(p-\tau\beta c\right)Q\left(c,p\right)$ for its exports. Under monopolistic competition, the firm chooses the price that maximizes its profit in each local market taking as given the equilibrium choke prices at each local market. Optimal domestic quality-adjusted prices are written as before as

$$\frac{p^{*}\left(c\right)}{\beta(c)}=\begin{cases}\left(\hat{p}_{HL}c\right)^{1/2} & \text{if} \quad c\leq\hat{c}\\ \left(\hat{p}_{H}c\right)^{1/2} & \text{if} \quad c>\hat{c}\end{cases},$$

where $\hat{c}$ is given by (6), while optimal export quality-adjusted prices are given

$$\frac{p^{x}\left(c\right)}{\beta(c)}=\begin{cases}\left(\hat{p}_{HL}\tau c\right)^{1/2} & \text{if} \quad \tau c\leq\hat{c}\\ \left(\hat{p}_{H}\tau c\right)^{1/2} & \text{if} \quad \tau c>\hat{c}\end{cases}.$$

The only difference is that the highest cost firm that sells to a foreign high (resp. low) income group has a cost equal to $\hat{p}_H/\tau$ (resp. $\hat{c}/\tau$). The equilibrium price levels and indices can be computed as before, and we have the following equilibrium condition:

$$M=\frac{s_H}{e_H\left(\hat{p}_H,\hat{p}_L\right)}=\frac{s_L}{e_L\left(\hat{p}_H,\hat{p}_L\right)}, \tag{22}$$

where

$$e_L(\hat{p}_H,\hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_L - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G(c) + (n-1)\int_0^{\hat{c}/\tau} \left(\hat{p}_L - \hat{p}_{HL}^{1/2}(\tau c)^{1/2}\right) \mathrm{d}G(c),$$

$$e_H(\hat{p}_H,\hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_H - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c)$$
$$+ (n-1)\left[\int_0^{\hat{c}/\tau} \left(\hat{p}_H - \hat{p}_{HL}^{1/2}(\tau c)^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}/\tau}^{\hat{p}_H/\tau} \left(\hat{p}_H - \hat{p}_H^{1/2}(\tau c)^{1/2}\right) \mathrm{d}G(c)\right]$$

express the consumers' average expenditure per available good.

A firm with cost $c$ and quality shifter $\beta$ gets the following profit from its home and foreign sales: $\pi(c) = (p^*(c) - \beta c)\, Q(c, p^*(c)) + (n-1)(p^x(c) - \tau\beta c)\, Q(c, p^x(c))$. Free entry implies that $\mathrm{E}[\pi(c)] = f$. We write this as

$$\pi(\hat{p}_H,\hat{p}_L) = \frac{f}{N}, \tag{23}$$

where $\pi(\hat{p}_H,\hat{p}_L) = \mathrm{E}[\pi(c)]/N$, or equivalently,

$$\pi(\hat{p}_H,\hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_{HL}^{1/2} - c^{1/2}\right)^2 \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \alpha_H \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2 \mathrm{d}G(c)$$
$$+ (n-1)\left[\int_0^{\hat{c}/\tau} \left(\hat{p}_{HL}^{1/2} - (\tau c)^{1/2}\right)^2 \mathrm{d}G(c) + \int_{\hat{c}/\tau}^{\hat{p}_H/\tau} \alpha_H \left(\hat{p}_H^{1/2} - (\tau c)^{1/2}\right)^2 \mathrm{d}G(c)\right].$$

The mass of surviving firms in a country is equal to $MG(\hat{p}_H)$.

As in the closed economy, the three market conditions in (22) and (23) determine the choke prices $(\hat{p}_H,\hat{p}_L)$ and mass of entrants $M$.

Using Pareto productivity distribution and a similar procedure for simplifying equilibrium conditions, we obtain

$$0 = \Phi\left(r;\kappa,\alpha_H,\frac{s_H}{s_L}\right), \tag{24}$$

$$\hat{p}_L = c_M^{\frac{\kappa}{\kappa+1}} \left(\frac{f}{N\left[1+(n-1)\tau^{-\kappa}\right]}\right)^{\frac{1}{\kappa+1}} \left[\Gamma_2(r;\kappa,\alpha_H)\right]^{-\frac{1}{\kappa+1}}, \tag{25}$$

$$M = \frac{s_L N}{f}\frac{\Gamma_2(r;\kappa,\alpha_H)}{\Gamma_1(r;\kappa,\alpha_H)}, \tag{26}$$

where we used the definition $\hat{p}_H = r\hat{p}_L$ and $\Phi$, $\Gamma_1$, and $\Gamma_2$ are the same functions as in the closed-economy model. The difference with the closed economy lies in the presence of the term $(n-1)\tau^{-\kappa}$, which accounts for the number of trade partners and trade costs. Using the same argument as in Proposition 1, we conclude that the symmetric open economy also entail a unique equilibrium. At the same time, Proposition 2 also applies in the open economy: a mean-preserving spread of the income distribution in every country reduces the country average productivity levels and reduces the set of goods consumed by the poor relative to that by the rich.

In the following we investigate the equilibrium properties under Pareto productivity distribution.

## 4.1 Income Distribution

We know by the structure of (24) to (26) that Proposition 2 hold in the open economy. That is, a mean-preserving spread of the income distribution increases the choke price of each country's high income group, reduces the country's (unweighted and quality-adjusted) average productivity and reduces the set of goods consumed by the poor relative to that by the rich in each country. The question then becomes whether lower trade costs amplify or attenuate the effect of a mean-preserving spread of the income distribution.

A mean-preserving spread of the income distribution (higher $v$ at constant $\overline{s}$) raises the highest income $s_H$ and reduces the lowest income $s_L$. In the open economy, each country (unweighted) average productivity is related to the opposite of its average cost $\int_0^{\hat{p}_H} cM\mathrm{d}G(c) \,/\, \int_0^{\hat{p}_H} M\mathrm{d}G(c)$, or equivalently, $\int_0^{\hat{p}_H} c\mathrm{d}G(c) \,/\, \int_0^{\hat{p}_H} \mathrm{d}G(c)$, which increases with a higher choke price $\hat{p}_H$ but is independent of the number of entrants, $M$. Hence, we need only to study the effect of a higher $\hat{p}_H$, irrespective of the numbers of entrants and firms. Because $\hat{p}_H = r\hat{p}_L$, we must discuss the effects of the mean-preserving spread on $r$ and $\hat{p}_L$ in equations (24) and (25). Let us consider the changes in income inequality $s_H/s_L$ from $s_H^a/s_L^a$ to $s_H^b/s_L^b$. By (24), the choke price ratio shifts from $r^a$ to $r^b$ where the superscripts $^a$ and $^b$ refer to the respective income ratios. By (25), we also have

$$\frac{\hat{p}_L^a}{\hat{p}_L^b} = \left(\frac{\Gamma_2\left(r^a;\kappa,\alpha_H\right)}{\Gamma_2\left(r^b;\kappa,\alpha_H\right)}\right)^{-\frac{1}{\kappa+1}},$$

which is independent of trade cost $\tau$. Multiplying all terms by $r_a/r_b$ we can write

$$\frac{\hat{p}_H^a}{\hat{p}_H^b} = \frac{r^a\hat{p}_L^a}{r^b\hat{p}_L^b} = \frac{r^a}{r^b}\left(\frac{\Gamma_2\left(r^a;\kappa,\alpha_H\right)}{\Gamma_2\left(r^b;\kappa,\alpha_H\right)}\right)^{-\frac{1}{\kappa+1}},$$

where the first equality stems from the definition of $r = \hat{p}_H/\hat{p}_L$. Hence the ratios of choke prices are also independent of trade costs $\tau$ and the number of trade partners $n-1$. However, (25) implies that a trade liberalization (in either an increase in $n$ or a decrease in $\tau$) lowers the level of $\hat{p}_L$ and hence $\hat{p}_H$.

As a result, we can infer that the difference between the choke prices, $\left|\hat{p}_H^b - \hat{p}_H^a\right|$, must also be smaller for smaller trade costs. As a result, the average cost $\int_0^{\hat{p}_H} c\mathrm{d}G(c) \,/\, \int_0^{\hat{p}_H} \mathrm{d}G(c)$ increases less with lower trade cost when income inequality is higher. Since the opposite holds for the average productivity, we can state the following:

**Proposition 3.** *A mean-preserving spread of income distribution reduces less each country's (unweighted, quality-adjusted) average productivity when trade costs are smaller.*

Stated differently, income redistribution from the rich to the poor improves each country's average productivity less under deeper trade integration. Also, the effects of income inequality on average productivity are the strongest under autarky ($\tau \to \infty$). The intuition is that trade expands product diversity and induces tougher firm selection. The cost cutoffs are smaller under trade so that the mass of surviving firms $MG(\hat{p}_H)$ is smaller but those firms are more cost effective. The number of unsold goods is larger within the global economy than $n$ autarkic economies. Hence, when the rich gets a higher income, she can spread her consumption towards the cheapest unsold goods in all countries rather than having to concentrate on the domestic unsold goods. In the end, consumed goods are produced at a lower cost.

## 4.2 Trade Integration and Welfare

The effects of trade integration/liberalization on consumptions and number of varieties are similar to those found in Melitz and Ottaviano (2008). Namely, trade integration induces both the rich and poor to consume more imported goods in both the quantity (intensive margin) and the number of variety (extensive margin). Moreover, the total set of goods expand for both groups. Due to pro-competitive effects of trade integration, product prices generally fall except for a few cases near $\hat{c}$ and $\hat{c}/\tau$.

However, the implications of trade on welfare are very different in our model from Melitz and Ottaviano because the presence of two income groups and the income effect. From Appendix G, equilibrium utility levels can be written as

$$U(s_H) = \frac{M\left(1+(n-1)\tau^{-\kappa}\right)\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[a^{\kappa}\ln\left(r^{1/2}\left(\alpha_H r+\alpha_L\right)^{-1/2}\right)+\frac{r^{\kappa}}{2\kappa}\right] \tag{27}$$

$$U(s_L) = \frac{M\left(1+(n-1)\tau^{-\kappa}\right)\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{a^{\kappa}}{2\kappa}-a^{\kappa}\ln\left[\left(1-\alpha_H^{1/2}\right)a+\left(\alpha_H a r\right)^{1/2}\right]\right], \tag{28}$$

where $a = \hat{c}/\hat{p}_L$. By using (25) and comparing (20-21) and (27-28), we have

$$U(s_h) = \left(1+(n-1)\tau^{-\kappa}\right)^{\frac{1}{\kappa+1}} U^A(s_h),$$

where $h = H, L$, and $U^A(s_h)$ denotes the utility level under autarky. Trade integration therefore raises equilibrium utility levels of high and low income groups in the same proportion. The utility difference between those groups therefore rises in that same proportion.

How does the above utility increase due to trade integration compared to an increase in income? To answer this question, we consider a proportional rise in income across groups so that $s_L/s_H$ and therefore $r$ and $a$ are fixed. A percentage decrease in trade cost yields the same change in utility resulting from a percentage increase in average income if it satisfies the following relationship:

$$\left[\frac{d\ln U(s_h)}{d\ln\tau}\right]_{s_L \text{ fixed}} d\ln\tau = -\left[\frac{d\ln U(s_h)}{d\ln s_L}\right]_{\tau \text{ fixed}} d\ln s_L.$$

We have the following proposition (See Appendix G for a detailed derivation).

**Proposition 4.** *A percentage decrease in trade cost is equivalent to a percentage increase in income by the following elasticity:*

$$\mu \equiv \frac{d\ln s_L}{d\ln\tau} = \frac{\kappa}{\kappa+1}\frac{(n-1)\,\tau^{-\kappa}}{1+(n-1)\,\tau^{-\kappa}}.$$

For example, with $n = 2$, $\tau = 1.7$ and $\kappa = 3.03$,[16] we have $\mu = 0.13$. That is, a $10\%$ fall in trade cost is equivalent to a $1.3\%$ percent rise in average income, would the income ratio ($s_H/s_L$) remain constant. The formula also shows that the benefit of trade liberalization becomes stronger for lower trade costs and larger trade networks.

# 5 Numerical Analysis

In this section, we conduct a numerical analysis to further examine the properties that are difficult to obtain analytically. Our numerical analysis focuses on the effects of income inequality based on a calibrated model.

## 5.1 Calibration

To calibrate the model, we attribute values to the nine parameters $(\alpha_H, s_L, s_H, c_M, f, \tau, \kappa, n, N)$. As we have shown above, the equilibrium consumption and utility are expressed in terms of quality-adjusted prices and costs and are independent of the quality shifter profile $\beta(c)$.

[16] These parameter values are the ones adopted in our quantitative analysis in Section 5.

Thus, we do not need to specify $\beta(c)$ in this exercise. We calibrate on a 2015 US-like baseline economy where top 10% income individuals earn $7.9$ more than the bottom 90% so that we set $\alpha_H^o = 0.10$ and $s_H^o/s_L^o = 7.9$ where the symbol $o$ denotes the baseline values.[17] Without loss of generality, we can normalize the population size and the cost to unity such that $N^o = 1$ and $c_M^o = 1$. We focus on two (blocks of) symmetric countries ($n^o = 2$) and set the iceberg trade cost to the value $\tau^o = 1.7$ as estimated in Novy (2013).[18] We identify the model on three additional empirical relationships about firms' markups, survival and employment rates. First, following empirical studies,[19] we impose an (unweighted) average markup on local sales $mrkup^o \equiv \int_0^{\widehat{p}_H} \frac{p}{\beta(c)c} dG$ of $115\%$. That is,

$$mrkup^o = \frac{\kappa}{\kappa - 1/2}\left[\frac{\left[(\alpha_H^o r + \alpha_L^o)^{1/2} - r^{1/2}\right]\left[(\alpha_H^o r + \alpha_L^o)^{1/2} - \alpha_H^{o1/2} r^{1/2}\right]^{2\kappa-1}}{\left(1 - \alpha_H^{o1/2}\right)^{2\kappa-1} r^{\kappa}} + 1\right],$$

This identity gives a relationship between $\kappa$ and $r$ as does the identity (24). Solving simultaneously those two identities allow us to pin down the values of $r^o$ and $\kappa^o$. In turn, we get the values $\Gamma_1^o \equiv \Gamma_1\left(r^o; \kappa^o, \alpha_H^o\right)$ and $\Gamma_2^o \equiv \Gamma_2\left(r^o; \kappa^o, \alpha_H^o\right)$. We finally make use the value of the firm's survival rate $surv^o = G\left(\hat{p}_H\right)$ of $90\%$[20] and average employment per firm $empl^o = N/(MG(\hat{p}_H))$ of 66 workers as reported in the 2015 US census data ($148 * 10^6$ workers in $2.22 * 10^6$ firms having more than 5 employees). Using (25) and (26), we compute

$$\begin{aligned} surv^o &= \left[\frac{1 + \tau_o^{-\kappa_o}}{f} r_o^{-(\kappa_o+1)} \Gamma_2^o\right]^{-\frac{\kappa_o}{\kappa_o+1}} \\ empl^o * surv^o &= \frac{f}{s_L}\frac{\Gamma_1^o}{\Gamma_2^o}, \end{aligned}$$

which allow us to pin down $f^o$ and $s_L^o$. This calibration process permits to recover the baseline economy parameter values $r^o = 1.539$, $\kappa^o = 3.03$, $f^o = 0.00887$ and $s_L^o = 0.00036$. In turn this yields $a^o = 0.860$ ($= \widehat{c}/\widehat{p}_L$). As an external validity check, we compute a $83\%$ share of domestic expenditure on domestic goods, which fits well the reality of the US

[17] The income ratio is calculated from top 10 percent income share data in 2014 for the US from the World Inequality Database.

[18] For examples, Novy (2013) estimated that the trade costs $\tau$ in 2000 between the US and Germany and between the US and the UK are $1.70$ and $1.63$, respectively. Using the same approach, the same set of estimates in 2014 reported by the World Bank's *International Trade Costs* data set are $1.723$ and $1.704$.

[19] For example, using Taiwanese manufacturing data and the markup-estimation approach by De Loecker and Warzynski (2012), Edmond, Midrigan, and Xu (2015) find an unweighted average markup of $1.13$.

[20] We take the average exit rate as $0.1$. See, for example, Klepper and Thompson (2006).

economy.[21]

## 5.2 Effects of Income Inequality

Table 2 presents the values of economic variables when workers incomes (or skills) increasingly spread about their mean. The first row presents the value of the income ratio that rises from $1$ (second column) to the baseline model $7.9$ (sixth column) and then to $3/2$ of this value (eighth column). The second row displays the respective values of the Gini coefficients. The next three lines show the value of skill endowment and their mean for the sake of completeness.

[INSERT TABLE 2 HERE]

In Section 2, our empirical analysis suggests a negative correlation between average productivity and income dispersion. The theoretical analysis in Section 3.4 shows that the average productivity falls with mean-preserving spreads. Because further analysis on productivity is analytically difficult, we resort to the quantitative analysis here. The seventh row in Table 2 shows reports the quantitative values for average productivity weighted by cost, given by

$$\frac{\int_0^{\hat{p}_H} (1/c)cQ\left(p^*\left(c\right)\right)dG(c)}{\int_0^{\hat{p}_H} cQ\left(p^*\left(c\right)\right)dG(c)},$$

which simplifies to the total output over total cost. The observation of this row confirms our previous analyses: average productivity falls with stronger income inequality. Ceteris paribus, as their income rises, rich consumers purchase larger quantity per good and add goods with higher quality-adjusted prices and costs. As mentioned, those goods also have higher prices if quality shifters are increasing in $c$ or not decreasing too quickly. As richer consumers buy more quantity and larger number of goods, their effect on total consumption dominates so that firms on average produce more costly goods.

The eighth and ninth lines of Table 2 compare the achieved utility levels compared to the baseline levels. The poor' utility monotonically falls with a mean-preserving spread of income distribution as they get lower incomes. Interestingly, the rich's utility first increases and then decreases with higher income dispersion. Too strong income inequality may thus turn out to be a disadvantage for the rich. The intuition balances the effects of

[21] Using information on domestic absorption and imports in Penn World Table 9.0, one can easily calculate the domestic expenditure share, and this share for the US in 2014 is $0.828$.

their larger purchasing power and larger number of expensive products. First, when income inequality strengthens, the rich get larger incomes and raise their demands so that they are willing to consume more in quantity and number of products. Second, income inequality reduces the poor' incomes and demands. This entices some firms with cost lower than $\hat{c}$ shift their consumer target from all individuals to only the rich ones. As a consequence, those firms raise their prices, which negatively affects the rich's consumption. In other words, for such products, the rich can no longer "hide behind the poor" and benefit from the low prices targeted to poorer people. Firms' price discrimination hits further the rich. Price hikes can be large because richer individuals have lower demand elasticity. One can then observe from the eighth row of Table 2 that the rich gain from larger income discrepancies only for income ratio $s_H/s_L$ lower than the baseline level $7.9$. Above that level, they are hurt by the above price hikes.

Finally, the last two rows of Table 2 display the *relative equivalent variations* as the percentage of additional income needed in the baseline model[22] to match the utility level obtained in another inequality configuration. To allow comparison, those measures take the baseline equilibrium price system and its product space as givens. Although they are a partial equilibrium measures, relative equivalent variations are better suited to express the magnitude of the impact of welfare inequality on the poor and rich. Hence, going from the sixth to the fifth column means to move from the baseline income ratio of $7.9$ to $5.41$. This implies a fall of $9$ points in the Gini coefficient, a rise of the poor's income from $0.36$ to $0.42$ and a fall in the rich's income from $2.82$ to $2.26$, which amounts respectively for about $16\%$ and $-20\%$ of their baseline incomes. However, the relative equivalent variations are $17\%$ and $-1\%$ respectively for the poor and rich. Hence, the negative impact on the rich is much lower than her actual income change. In the same vein, going from the sixth to the second column implies the most drastic move from the baseline model to full income redistribution (equal incomes). Poor's income moves up from $0.36$ to $0.60$, that is, for about $67\%$ of her baseline income. The rich's income moves down from $2.82$ to $0.60$, which is a $78\%$ income drop. However, in terms of equivalent variations, the rich lose only $30\%$ while the poor gain $69\%$ of their purchasing power. Finally, going from the sixth to the seventh or eighth column implies higher inequality compared to the baseline model. Yet, this move harms *both* the poor and rich in terms of utility level and equivalent variation. This is undesirable for either the social planner or each income group.

[22]That is, given the set of consumed goods and prices at the baseline. Detailed derivation for the formulas of relative equivalent variation is relegated to Appendix H.

# 6 Conclusion

In this paper, we propose a theory of how income inequality may affect aggregate productivity and welfare in a global economy via selection under a non-homothetic preference with pro-competitive effects. We find that there is a negative cross effect of one group's income on the other group's consumption. We also find that a mean-preserving spread of income reduces average productivity (both weighted and unweighted) through the softening of firms' selection and the shuffling of the mass of consumption from low-cost to high-cost goods.

In the quantitative analysis, it is shown that a too large mean-preserving spread of income may harm the rich. Moreover, when measuring welfare in real terms by equivalent variation, we find that a reallocation of nominal income increases the poor's real income more than the fall of the rich's real income. Taken together, regardless of whether efficiency is measured in aggregate productivity or welfare, we find the contrary to the equity-efficiency trade-off is true in our model.

Our result that the negative effect of income inequality on average productivity is mitigated by international trade is intriguing because most theoretical and empirical studies point to the negative effect of globalization on equity. Of course there is actually no conflict because the directions of the causal relationships are different. Not only our model is consistent with the general understanding that trade helps the poor in terms of consumption, but it also suggests another positive side of trade: with higher income, the rich can spread their extra consumption over goods from various countries, instead of having to concentrate their consumption domestically under autarky. Collectively, a more efficient part of each country's cost distribution is sampled in the presence of trade.

# Appendix A: Consumers' demands

Individuals are endowed with utility function $U = \int_{\omega\in\Omega} \ln\left(1+\beta(\omega)q\left(\omega\right)\right) \mathrm{d}\omega$ over the commodity space $\Omega \subset \mathbb{R}$. Note that, firm entry limits the mass of commodities that are offered. Let $\overline{\Omega}$ be the set of commodities that are actually offered and associated with a price $p\left(\omega\right)$, $\omega \in \overline{\Omega}$. Other commodities $\omega \in \Omega\backslash\overline{\Omega}$ are not offered and cannot be consumed so that $q(\omega) = 0$ for $\omega \in \Omega\backslash\overline{\Omega}$. An individual in the income group $h$ chooses the consumption $q\left(\omega\right)$, $\omega \in \overline{\Omega}$ that maximizes her utility $U$ subject to her budget constraint $\int_{\omega\in\overline{\Omega}} p\left(\omega\right) q\left(\omega\right) \mathrm{d}\omega = s_h$. The Lagrangian function of individual $h$ with income $s_h$ is therefore defined as

$$\mathcal{L}_h = \int_{\omega\in\overline{\Omega}} \ln\left(1+\beta\left(\omega\right) q\left(\omega\right)\right) \mathrm{d}\omega + \lambda_h \left(s_h - \int_{\omega\in\overline{\Omega}} p\left(\omega\right) q\left(\omega\right) \mathrm{d}\omega\right)$$

$\Omega_h \subseteq \mathbb{R}$. This is a concave function so that the following first order condition yields the consumer's best consumption choice:

$$\begin{array}{ll} \frac{\partial \mathcal{L}}{\partial q(\omega)} = \frac{1}{q(\omega)+1/\beta(\omega)} - \lambda_h p\left(\omega\right) = 0 & \text{if} \quad q\left(\omega\right) > 0 \\ \frac{\partial \mathcal{L}}{\partial q(\omega)} = \frac{1}{q(\omega)+1/\beta(\omega)} - \lambda_h p\left(\omega\right) < 0 & \text{if} \quad q\left(\omega\right) = 0 \end{array}$$

The set of consumed goods is given by $\Omega_h \equiv \{\omega : q\left(\omega\right) > 0\} = \{\omega : p\left(\omega\right) < 1/\lambda_h\}$. For $\omega \in \Omega_h$, the first-order condition entails

$$q_h\left(\omega\right) = \frac{1}{\lambda_h p\left(\omega\right)} - \frac{1}{\beta\left(\omega\right)},$$

and thus

$$\lambda_h = \frac{\int_{\omega\in\Omega_h} \mathrm{d}\omega}{s_h + \int_{\omega\in\Omega_h} \frac{p(\omega)}{\beta(\omega)} \mathrm{d}\omega}.$$

Plugging $\lambda_h$ back into the demand function, we obtain individual demand function

$$q_h\left(\omega\right) = \frac{\hat{p}_h}{p\left(\omega\right)} - \frac{1}{\beta\left(\omega\right)},$$

where

$$\hat{p}_h \equiv \frac{1}{\lambda_h} = \frac{s_h + P_h}{|\Omega_h|}$$

is the choke price of consumer with income $s_h$, $P_h \equiv \int_{\omega\in\Omega_h} \left[p\left(\omega\right)/\beta\left(\omega\right)\right] \mathrm{d}\omega$ is the aggregate price index for the goods consumed by $s$ and $|\Omega\left(s\right)| = \int_{\omega\in\Omega_h} \mathrm{d}\omega$ is the measure of the set of goods consumed by individual $h$. Combining the above results, we obtain (1) and (2).

Note that $\beta(\omega)\hat{p}_h$ is the highest price that $h$ is willing to pay to purchase any nonnegative amount of a good $\omega$. When $s_h$ increases, $\lambda_h$ falls and $\hat{p}_h$ rises so that $\Omega_h$ expands. As a result, one gets $s_H \geq s_L \iff \hat{p}_H \geq \hat{p}_L$.

Finally given that $q(\omega) = 0, \omega \notin \Omega_h$, the consumer's utility can successively be rewritten as

$$U_h = \int_{\omega\in\Omega_h} \ln(1 + \beta(\omega)q(\omega))\,\mathrm{d}\omega + \int_{\omega\in\Omega\setminus\Omega_h} \ln(1)\,\mathrm{d}\omega = \int_{\omega\in\Omega_h} \ln(1 + \beta(\omega)q(\omega))\,\mathrm{d}\omega$$

The indirect utility is thus equal to

$$V_h = \int_{\omega\in\Omega_h} \ln\left(\frac{s_h + P_h}{|\Omega_h|}\frac{\beta(\omega)}{p(\omega)}\right)\mathrm{d}\omega \tag{29}$$

# Appendix B: Firms' choices

Let us denote $\beta(c)$ by $\beta$. Note that it is often convenient to express conditions in terms of quality-adjusted prices $p/\beta$. The problem for a firm with cost $c$ is

$$\max_p \ \pi = (p - \beta c)\,Q(p)$$
$$= \begin{cases} (p-\beta c)\,\alpha_H N\left(\frac{\hat{p}_H}{p} - \frac{1}{\beta}\right) & \text{if} \quad p/\beta \in [\hat{p}_L, \hat{p}_H) \\ (p-\beta c)\,N\left(\frac{\hat{p}_{HL}}{p} - \frac{1}{\beta}\right) & \text{if} \quad p/\beta \in [0, \hat{p}_L) \end{cases}.$$

For $p/\beta \in [0, \hat{p}_L)$, the firm sells to both groups and choose the quality-adjusted price $p^*(c)/\beta = c^{1/2}(\hat{p}_{HL})^{1/2}$ and quality-adjusted markup $(\hat{p}_{HL}/c)^{1/2}$. The quality-adjusted price increases and the quality-adjusted markup decreases with higher marginal costs $c$, showing a pro-competitive effect. The firm gets a profit equal to $\pi^*_{HL}(c) = N\left[(\hat{p}_{HL})^{1/2} - c^{1/2}\right]^2$. For $p/\beta \in [\hat{p}_L, \hat{p}_H)$, a firm sells only to high income consumers and set a quality-adjusted prices $p^*(c)/\beta = c^{1/2}\hat{p}_H^{1/2}$ and quality-adjusted markup $(\hat{p}_H/c)^{1/2}$. Those prices increase and markups decrease in $c$. The firm gets a profit equal to $\pi^*_H(c) = \alpha_H N\left[(\hat{p}_H)^{1/2} - c^{1/2}\right]^2$. The firm chooses to charge $p^*(c)/\beta = c^{1/2}(\hat{p}_{HL})^{1/2}$ if and only if $\pi^*_{HL}(c) \geq \pi^*_H(c)$, which is equivalent to

$$c^{1/2} \leq \hat{c}^{1/2} \equiv \frac{(\hat{p}_{HL})^{1/2} - (\alpha_H\hat{p}_H)^{1/2}}{1 - \alpha_H^{1/2}}. \tag{30}$$

This argument yields (6) and (5). Observe that $p_H > p_{HL}$. So, there is upward jump of the price schedule $p^*(c)$ at $\hat{c}$.

In the product market equilibrium, it must be that each income group purchases the goods that are targeted to them. In particular, the low income consumers should buy only the goods produced at cost in the range $[0, \hat{c}]$. This means that their choke price $\hat{p}_L$ should satisfy $p^* (\hat{c} - 0) / \beta (\hat{c} - 0) < \hat{p}_L < p^* (\hat{c} + 0) / \beta (\hat{c} + 0)$. We show that this condition holds. Indeed, since $p^*(\hat{c} - 0)/\beta (\hat{c} - 0) = \hat{c}^{1/2} (\hat{p}_{HL})^{1/2}$ and $p^*(\hat{c} + 0)/\beta (\hat{c} + 0) = \hat{c}^{1/2}\hat{p}_H^{1/2}$ the previous condition becomes $\hat{c}^{1/2} (\hat{p}_{HL})^{1/2} < \hat{p}_L < \hat{c}^{1/2}\hat{p}_H^{1/2}$. Plugging the value of $\hat{c}$ and defining $r = \hat{p}_H/\hat{p}_L$ with $r > 1$ since $\hat{p}_H > \hat{p}_L$, we get the following inequalities:

$$(\alpha_H r + \alpha_L) - (\alpha_H r (\alpha_H r + \alpha_L))^{1/2} < 1 - \alpha_H^{1/2} < ((\alpha_H r + \alpha_L) r)^{1/2} - (\alpha_H)^{1/2} r$$

Because $\alpha_H + \alpha_L = 1$, we have that the left-hand side and right-hand side are equal to the middle term for $r = 1$. It can be shown that the left-hand side falls with higher $r$ while the right-hand side rises with it. Hence the inequalities are always satisfied.

For all goods to be supplied by firms with quality-adjusted cost $c$ to poor individuals, it must also that $c < \hat{p}_L$. This is obtained if $\hat{c} < \hat{p}_L$. Plugging the value of $\hat{c}$ and using $r = \hat{p}_H/\hat{p}_L$ we get the condition:

$$((\alpha_H r + \alpha_L))^{1/2} - (\alpha_H r)^{1/2} < 1 - \alpha_H^{1/2}$$

where the left-hand side decreases with larger $r$ and is equal to the right-hand side at $r = 1$. So the condition is always satisfied.

# Appendix C: Existence

The equilibrium is represented by the vector of variables $(\hat{p}_H, \hat{p}_L, M)$ with $\hat{p}_H \geq \hat{p}_L \geq 0$ and $M > 0$ that satisfy the market conditions (8) and (9) and entry conditions (12):

$$e_H (\hat{p}_H, \hat{p}_L) - \frac{s_H}{M} = 0 \tag{31}$$

$$e_L (\hat{p}_H, \hat{p}_L) - \frac{s_L}{M} = 0 \tag{32}$$

$$\pi (\hat{p}_H, \hat{p}_L) - \frac{f}{N} = 0 \tag{33}$$

where

$$e_H(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_H - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c)$$
$$e_L(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_L - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c)$$

are the consumers' average expenditures per available good and

$$\pi(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{p}_H} \max\left\{\left((\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} - c^{1/2}\right)^2, \alpha_H \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2\right\} \mathrm{d}G(c)$$

is the expected operational profit before entry. In those equations $\hat{c}$ is implicitly given by the solution of

$$\hat{c}^{1/2} = \frac{(\hat{p}_{HL})^{1/2} - (\alpha_H \hat{p}_H)^{1/2}}{1 - \alpha_H^{1/2}}$$

with $\partial\hat{c}/\partial\hat{p}_H < 0 < \partial\hat{c}/\partial\hat{p}_H$. It can readily be shown that $\pi_H > 0, \pi_L > 0$ and $e_{LL} > 0 > e_{LH}$ where $e_{hl} = \partial e_h/\partial \hat{p}_l$ and $\pi_l = \partial\pi/\partial\hat{p}_l$, $h, l \in \{H, L\}$.

Using (31) and (32), we can rewrite the equilibrium conditions as

$$H(\hat{p}_H, \hat{p}_L, M) \equiv M - \frac{\alpha_H s_H + \alpha_L s_L}{\alpha_H e_H + \alpha_L e_L} = 0 \tag{34}$$
$$F(\hat{p}_H, \hat{p}_L) \equiv \frac{e_H}{s_H} - \frac{e_L}{s_L} = 0 \tag{35}$$
$$\Pi(\hat{p}_H, \hat{p}_L) \equiv \pi(\hat{p}_H, \hat{p}_L) - \frac{f}{N} = 0 \tag{36}$$

The equilibrium is then given by the vector $(\hat{p}_H, \hat{p}_L, M)$ that solves (34), (35) and (36). Note that the choke prices are solutions of (35) and (36) while the mass of entrants is the solution of (34) at equilibrium choke prices.

To show the existence of the equilibrium, note that, since the $e_H$, $e_L$ and $\pi$ are continuous functions of $(\hat{p}_H, \hat{p}_L, M)$, the expressions in conditions (36), (34) and (35) are also continuous on $R^{+3}$. It then suffices to prove that each expression has opposite sign on two points in the support of $(\hat{p}_H, \hat{p}_L, M) \in R^{+3}$ with $\hat{p}_H \geq \hat{p}_L \geq 0$ and $M > 0$.

First, suppose that $(\hat{p}_H, \hat{p}_L, M) = (y, 0, M)$. Then, $\hat{c} = 0$ so that $e_H(y, 0) = \int_0^y \left(y - y^{1/2} c^{1/2}\right)$

$\mathrm{d}G\,(c) > 0$ and $e_L(y,0) = 0$. We compute

$$\Pi(y,0) = \alpha_H \int_0^y \left(y^{1/2} - c^{1/2}\right)^2 \mathrm{d}G\,(c) - \frac{f}{N}$$
$$H(y,0,M) = M - \frac{\alpha_H s_H + \alpha_L s_L}{\alpha_H \int_0^y \left(y - y^{1/2}c^{1/2}\right) \mathrm{d}G\,(c)}$$
$$F(y,0) = \frac{1}{s_H} \int_0^y \left(y - y^{1/2}c^{1/2}\right) \mathrm{d}G\,(c)$$

If $y$ is small enough, we have $\Pi(y,0) < 0$, $H(y,0,M) < 0$ and $F(y,0) > 0$.

Second, we consider that $G(c)$ has a bounded support and finite mean. That is, $G : [0, c_M] \to [0,1]$ such that $\mathrm{E}(c) = \int_0^{c_M} c\mathrm{d}G\,(c) < \infty$. We define $\hat{p}_L = x$, $\hat{p}_H = rx$, $\hat{p}_{HL} = (\alpha_H r + \alpha_L)\, x$ and $\hat{c} = ax$ where $1 \leq r < \infty$ and $a^{1/2} \equiv \left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right] / \left(1 - \alpha_H^{1/2}\right) \in (0,1]$. We further set $x$ such that $c_M < ax < x < rx$. This implies that $\int_0^{rx} \mathrm{d}G = \int_0^{c_M} \mathrm{d}G = 1$, $\int_0^{ax} c^{1/2}\mathrm{d}G = \int_0^{c_M} c^{1/2}\mathrm{d}G\,(c) = \mathrm{E}(c^{1/2})$, and $\int_{ax}^{rx} c^{1/2}\mathrm{d}G\,(c) = \int_{c_M}^{c_M} c^{1/2}\mathrm{d}G\,(c) = 0$. So, when $(\hat{p}_H, \hat{p}_L) = (rx, x)$, we have

$$e_H\,(rx,x) = rx - (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2} \mathrm{E}(c^{1/2})$$
$$e_L\,(rx,x) = x - (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2} \mathrm{E}(c^{1/2})$$

while

$$\Pi\,(rx,x) = \left[(\alpha_H r + \alpha_L)\, x - 2\,(\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2}) + \mathrm{E}(c)\right] - \frac{f}{N}$$
$$H(rx,x,M) = M - \frac{\alpha_H s_H + \alpha_L s_L}{(\alpha_H r + \alpha_L)\, x - (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2})}$$
$$F\,(rx,x) = \left(\frac{r}{s_H} - \frac{1}{s_L}\right) x + \left(\frac{1}{s_L} - \frac{1}{s_H}\right) (\alpha_H r + \alpha_L)^{1/2}\, x^{1/2}\mathrm{E}(c^{1/2})$$

For $x$ sufficiently large, it comes $\Pi\,(rx,x) > 0$ and $H(rx,x,M) > 0$ while $F\,(rx,x) < 0$ if $r < s_H/s_L$.

We can then choose five scalars, $x$ large enough, $y$ small enough, $r < s_H/s_L$, $M' > 0$ and $M'' > 0$, such that the functions $\Pi, H$ and $F$ have opposite signs at the points $(\hat{p}_H, \hat{p}_L, M) = (rx, x, M')$ and $(y, 0, M'')$. This proves the existence of an equilibrium.

# Appendix D: Income and Demand

In this appendix, we show how changes in income affect choke prices. Differentiating totally (36) and (35), we get

$$\left[\begin{array}{cc} e_{HH}s_H^{-1}-e_{LH}s_L^{-1} & e_{HL}s_H^{-1}-e_{LL}s_L^{-1} \\ \pi_H & \pi_L \end{array}\right]\cdot\left[\begin{array}{c} \mathrm{d}\hat{p}_H \\ \mathrm{d}\hat{p}_L \end{array}\right]=\left[\begin{array}{c} -e_H\mathrm{d}s_H^{-1}+e_L\mathrm{d}s_L^{-1} \\ 0 \end{array}\right]$$

where $e_{hl}\equiv\partial e_h/\partial\hat{p}_l$ and $\pi_l\equiv\partial\pi/\partial\hat{p}_l$, $h,l\in\{H,L\}$. In Appendix C, it has been shown that $\pi_H>0,\pi_L>0$ and $e_{LL}>0>e_{LH}$. Under the assumption $e_{HH}>0>e_{HL}$, the determinant of the matrix in the above LHS, $\Delta=\left(e_{HH}s_H^{-1}-e_{LH}s_L^{-1}\right)\pi_L-\left(e_{HL}s_H^{-1}-e_{LL}s_L^{-1}\right)\pi_H$ is strictly positive. We have

$$\left[\begin{array}{c} \mathrm{d}\hat{p}_H/\mathrm{d}s_H^{-1} \\ \mathrm{d}\hat{p}_L/\mathrm{d}s_H^{-1} \end{array}\right]=\frac{e_H}{\Delta}\left[\begin{array}{c} -\pi_L \\ \pi_H \end{array}\right]\quad\text{and}\quad\left[\begin{array}{c} \mathrm{d}\hat{p}_H/\mathrm{d}s_L^{-1} \\ \mathrm{d}\hat{p}_L/\mathrm{d}s_L^{-1} \end{array}\right]=\frac{e_L}{\Delta}\left[\begin{array}{c} \pi_L \\ -\pi_H \end{array}\right]$$

Noting that $e_h=s_h/(M)$ by (31) and (32) so that $\left(\mathrm{d}\hat{p}_h/\mathrm{d}s_h^{-1}\right)=e_h\hat{p}_hM\left(\mathrm{d}\ln\hat{p}_h/\mathrm{d}\ln s_h^{-1}\right)$, $h=H,L$, we can rewrite the above expression as

$$\left[\begin{array}{c} \mathrm{d}\ln\hat{p}_H/\mathrm{d}\ln s_H \\ \mathrm{d}\ln\hat{p}_L/\mathrm{d}\ln s_H \end{array}\right]=\frac{1}{\Delta}\left[\begin{array}{c} \pi_L \\ -\pi_H \end{array}\right]\quad\text{and}\quad\left[\begin{array}{c} \mathrm{d}\ln\hat{p}_H/\mathrm{d}\ln s_L \\ \mathrm{d}\ln\hat{p}_L/\mathrm{d}\ln s_L \end{array}\right]=\frac{1}{\Delta}\left[\begin{array}{c} -\pi_L \\ \pi_H \end{array}\right]$$

We then get

$$\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H}=-\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_L}=\frac{1}{\Delta}\frac{\pi_L}{\hat{p}_H}>0\quad\text{and}\quad\frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_L}=-\frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_H}=\frac{1}{\Delta}\frac{\pi_H}{\hat{p}_L}>0 \tag{37}$$

# Appendix E: Pareto productivity distribution

## Equilibrium Conditions

Assume Pareto productivity, which translate to cost distribution with the c.d.f given by $G(c)=\left(\frac{c}{c_M}\right)^{\kappa}$ for $c\in[0,c_M]$ and $\kappa\geq 1$. The equilibrium is the vector $(\hat{p}_H,\hat{p}_L,M)$ that solves (36), (34) and (35). With some algebraic manipulations, these conditions are translated to

$$0 = \Phi\left(r; \kappa, \alpha_H, \frac{s_H}{s_L}\right) \tag{38}$$

$$\hat{p}_L = c_M^{\frac{\kappa}{\kappa+1}} \left(\frac{N}{f}\Gamma_2\left(r; \kappa, \alpha_H\right)\right)^{-\frac{1}{\kappa+1}} \tag{39}$$

$$M = \frac{N s_L}{f} \frac{\Gamma_2\left(r; \kappa, \alpha_H\right)}{\Gamma_1\left(r; \kappa, \alpha_H\right)}, \tag{40}$$

where

$$\begin{aligned}
\Phi\left(r; \kappa, \alpha_H, \frac{s_H}{s_L}\right) \equiv{}& \frac{r^{\kappa+1}}{2\kappa+1} \frac{\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}}{\left[\left(\alpha_H r + \alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}\right]^{2\kappa}} - \frac{s_H}{s_L}\left(1-\alpha_H^{1/2}\right) \\
&+ \frac{2\kappa\left[r^{1/2} + \left(\frac{s_H}{s_L}-1\right)\left(\alpha_H r + \alpha_L\right)^{1/2}\right]\left[\left(\alpha_H r + \alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}\right]}{2\kappa+1}, \\
\Gamma_1\left(r; \kappa, \alpha_H\right) \equiv{}& \left(\frac{\left(\alpha_H r + \alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa} - \frac{2\kappa\left(\alpha_H r + \alpha_L\right)^{1/2}}{2\kappa+1}\left(\frac{\left(\alpha_H r + \alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa+1},
\end{aligned}$$

and

$$\begin{aligned}
\Gamma_2\left(r; \kappa, \alpha_H\right) ={}& \frac{\alpha_H r^{\kappa+1}}{\left(\kappa+1\right)\left(2\kappa+1\right)} + \alpha_L\left(\frac{\left(\alpha_H r + \alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa} \\
&+ \frac{4\kappa\left[\alpha_H r^{1/2} - \left(\alpha_H r + \alpha_L\right)^{1/2}\right]}{2\kappa+1}\left(\frac{\left(\alpha_H r + \alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa+1} \\
&+ \frac{\alpha_L \kappa}{\kappa+1}\left(\frac{\left(\alpha_H r + \alpha_L\right)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1-\alpha_H^{1/2}}\right)^{2\kappa+2},
\end{aligned}$$

The Pareto cost distribution permits to separate the effects of some parameters and sufficient statistics such as $s_H/s_L$ and $f/N$. In terms of the effect of income distribution, we show in Appendix E that $r^*$ strictly increases in $s_H/s_L$. In Appendix E, we show that using this fact and Lemma 1, $\Gamma_2$ and $\Gamma_2/\Gamma_1$ are both strictly increasing in $r^*$. From these, it is shown that (13) holds. We have the following proposition.

**Proposition 5.** *Suppose a Pareto productivity distribution. Then,*

1. *There exists a unique equilibrium.*

2. *Assumptions (A0) and (A1) hold so that Proposition 2 holds.*

3. *The equilibrium choke price ratio $r$ strictly increases with income inequality (higher $s_H/s_L$).*

4. *The number of entrants $M$ is proportional to the population size $N$ and inversely proportional to entry cost $f$.*

5. *The choke prices $(\hat{p}_L, \hat{p}_H)$, the cut-off cost $\hat{c}$, and the equilibrium price $p^*(c)$ of any firm with cost $c$ increase in the maximum cost $c_M$ and the entry cost $f$, and decrease with the population size $N$.*

6. *A proportional increase in incomes (i.e. higher $s_L$ holding $s_H/s_L$ unchanged) raises the mass of entrants $M$ proportional to the average income.*

## Points 1 and 2 in Proposition 5

For Points 1 and 2, it suffices to show that (A0) and (A1) holds under Pareto productivity, as Lemma 1 and Propositions 1 and 2 can be therefore applied. Under $G(c) = (c/c_M)^\kappa$, where $c \in [0, c_M]$ and $\kappa > 1$, it is immediate that $\mathrm{E}(c) = \frac{\kappa}{c_M^\kappa}\int_0^{c_M} c^\kappa \mathrm{d}c = \frac{\kappa c_M}{\kappa+1} < \infty$, and hence (A0) holds. The next task is to show that $\partial e_H/\partial \hat{p}_H > 0$ and $\partial e_H/\partial \hat{p}_L < 0$. Observe that we can rewrite $e_H(\hat{p}_H, \hat{p}_L)$ as

$$\begin{aligned} e_H(\hat{p}_H, \hat{p}_L) &= \frac{\kappa}{c_M^\kappa}\left[\int_0^{\hat{c}} \left(\hat{p}_H - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) c^{\kappa-1}\mathrm{d}c + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) c^{\kappa-1}\mathrm{d}c\right] \\ &\propto \hat{p}_H \int_0^{\hat{p}_H} c^{\kappa-1}\mathrm{d}c - \left[(\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} \int_0^{\hat{c}} c^{\kappa-1/2}\mathrm{d}c + \hat{p}_H^{1/2} \int_{\hat{c}}^{\hat{p}_H} c^{\kappa-1/2}\mathrm{d}c\right] \\ &= \frac{1/2}{\kappa(\kappa+1/2)} \hat{p}_H^{\kappa+1} - \frac{1}{\kappa+1/2} \frac{\left(\hat{p}_{HL}^{1/2} - \hat{p}_H^{1/2}\right)\left[(\hat{p}_{HL})^{1/2} - (\alpha_H \hat{p}_H)^{1/2}\right]^{2\kappa+1}}{\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}}. \end{aligned}$$

Thus,

$$\begin{aligned} &\partial e_H/\partial \hat{p}_L \\ &\propto -\frac{\alpha_L \hat{p}_{HL}^{-1/2}}{2}\left\{\left[(\hat{p}_{HL})^{1/2} - \alpha_H^{1/2}\hat{p}_H^{1/2}\right]^{2\kappa+1} + (2\kappa+1)\left(\hat{p}_{HL}^{1/2} - \hat{p}_H^{1/2}\right)\left[(\hat{p}_{HL})^{1/2} - \alpha_H^{1/2}\hat{p}_H^{1/2}\right]^{2\kappa}\right\} \\ &< 0. \end{aligned}$$

And,

$$\begin{aligned}&\partial e_H/\partial \hat{p}_H\\ &\propto \frac{1/2\,(\kappa+1)}{\kappa\,(\kappa+1/2)}\hat{p}_H^{\kappa}\\ &-\frac{\left(\frac{\alpha_H}{2}\hat{p}_{HL}^{-1/2}-\frac{1}{2}\hat{p}_H^{-1/2}\right)\times\left[\hat{p}_{HL}^{1/2}-(\alpha_H\hat{p}_H)^{1/2}\right]+\frac{\alpha_H(2\kappa+1)}{2}\left(\hat{p}_{HL}^{1/2}-\hat{p}_H^{1/2}\right)\left(\hat{p}_{HL}^{-1/2}-(\alpha_H\hat{p}_H)^{-1/2}\right)}{(\kappa+1/2)\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}\left[(\hat{p}_{HL})^{1/2}-(\alpha_H\hat{p}_H)^{1/2}\right]^{-2\kappa}}\end{aligned}$$

The above is positive if the second term is positive, that is, if

$$\left(\frac{\alpha_H}{2}\hat{p}_{HL}^{-1/2}-\frac{1}{2}\hat{p}_H^{-1/2}\right)\times\left[\hat{p}_{HL}^{1/2}-(\alpha_H\hat{p}_H)^{1/2}\right]+\frac{\alpha_H\,(2\kappa+1)}{2}\left(\hat{p}_{HL}^{1/2}-\hat{p}_H^{1/2}\right)\left(\hat{p}_{HL}^{-1/2}-(\alpha_H\hat{p}_H)^{-1/2}\right)<0.$$

The above is true iff

$$(2\kappa+2)\left(\alpha_H+\alpha_H^{1/2}\right)<\left[1+(2\kappa+1)\,\alpha_H^{1/2}\right]\left(\frac{\alpha_H\hat{p}_H+\alpha_L\hat{p}_L}{\hat{p}_H}\right)^{1/2}+\left[\alpha_H^{3/2}+\alpha_H\,(2\kappa+1)\right]\left(\frac{\alpha_H\hat{p}_H+\alpha_L\hat{p}_L}{\hat{p}_H}\right)^{-}$$

Let $y\equiv\left(\frac{\alpha_H\hat{p}_H+\alpha_L\hat{p}_L}{\hat{p}_H}\right)^{1/2}=\left(\alpha_H+\alpha_L\frac{\hat{p}_L}{\hat{p}_H}\right)^{1/2}\in(0,1)$. Thus, the above is true iff

$$\left[1+(2\kappa+1)\,\alpha_H^{1/2}\right]y^2-(2\kappa+2)\left(\alpha_H+\alpha_H^{1/2}\right)y+\left[\alpha_H^{3/2}+\alpha_H\,(2\kappa+1)\right]>0. \qquad (41)$$

As the determinant

$$\begin{aligned}\Delta&\equiv(2\kappa+2)\left(\alpha_H+\alpha_H^{1/2}\right)^2-4\left[1+(2\kappa+1)\,\alpha_H^{1/2}\right]\left[\alpha_H^{3/2}+\alpha_H\,(2\kappa+1)\right]\\ &=-2\alpha_H\left[\left(2+6\kappa+8\kappa^2\right)\sqrt{\alpha_H}+(1+3\kappa)\,\alpha_H+3\kappa+1\right]<0,\end{aligned}$$

and $\left[1+(2\kappa+1)\,\alpha_H^{1/2}\right]>0$, (41) is true.

## Comparative Statics of Income Distribution

**The effect of $s_H/s_L$ on $r^*$**

Observe that

$$\frac{\partial\Phi\,(r;\kappa,\alpha_H,x)}{\partial x}=-\left(1-\alpha_H^{1/2}\right)+\frac{2\kappa\,(\alpha_H r+\alpha_L)^{1/2}}{2\kappa+1}\left[(\alpha_H r+\alpha_L)^{1/2}-\alpha_H^{1/2}r^{1/2}\right].$$

The above is negative iff the following is negative

$$(\alpha_H r + \alpha_L) - (\alpha_H r + \alpha_L)^{1/2} \alpha_H^{1/2} r^{1/2} < \frac{(2\kappa + 1)}{2\kappa} \left(1 - \alpha_H^{1/2}\right). \tag{42}$$

Note that

$$\frac{d}{dr}\left[(\alpha_H r + \alpha_L) - (\alpha_H r + \alpha_L)^{1/2} \alpha_H^{1/2} r^{1/2}\right] = -\frac{2r\alpha_H^{\frac{3}{2}} + \sqrt{\alpha_H}\,(1-\alpha_H) - 2\sqrt{r}\alpha_H\sqrt{(1-\alpha_H) + r\alpha_H}}{2\sqrt{(1-\alpha_H)\,r + r^2\alpha_H}},$$

which is negative if and only if $1 - \alpha_H > 0$, which is true. Hence, the upper bound of $(\alpha_H r + \alpha_L) - (\alpha_H r + \alpha_L)^{1/2} \alpha_H^{1/2} r^{1/2}$ is its value at $r = 1$, $1 - \alpha_H^{1/2}$. Hence, (42) is true, and equilibrium $r^*$ strictly increases in $\frac{s_H}{s_L}$.

**$\Gamma_2$ and $\Gamma_2/\Gamma_1$ are both strictly increasing in $r^*$**

Next, we show that $\Gamma_2'(r^*) > 0$. Suppose $\Gamma_2'(r^*) \leq 0$, and consider an increase in $s_H$ with $s_L$ fixed. Then, $r^*$ increases. By $\Gamma_2'(r^*) \leq 0$, equilibrium $\hat{p}_L$ increases or stays the same, and this in turn implies that equilibrium $\hat{p}_H$ increases. Thus, $\frac{d\hat{p}_H}{ds_H} > 0$. By the lemmas proved in Appendix D, $\frac{d\hat{p}_L}{ds_L} > 0$, $\frac{d\hat{p}_H}{ds_L} < 0$, and $\frac{d\hat{p}_L}{ds_H} < 0$. But $\frac{d\hat{p}_L}{ds_H} < 0$ implies that $\hat{p}_L$ decreases, which reaches a contradiction. The result follows.

Next, we show that $(\Gamma_2/\Gamma_1)'(r^*) > 0$. Suppose $(\Gamma_2/\Gamma_1)'(r^*) \leq 0$, and again consider an increase in $s_H$ with $s_L$ fixed. Then, $r^*$ increases. By $\Gamma_2'(r^*) > 0$, equilibrium $\hat{p}_L$ decreases. Again, by the lemmas in Appendix D, $\hat{p}_H$ increases. As $e_{LL} > 0$ and $e_{LH} < 0$, $e_L$ decreases. As $(\Gamma_2/\Gamma_1)'(r^*) \leq 0$, equilibrium $M$ decreases or stays the same. Equilibrium condition $s_L/(M) = e_L$ is thus violated.

### Additional properties

Points 4-6 can be obtained by observing (17) and (19).

# Appendix F: Production in International Trade

Firms differ in their marginal cost $w_i c$. Given equilibrium $\hat{p}_L$ and $\hat{p}_H$, the problem for a firm located in $i$ with $c$ is

$$\max_{\{p_{ij}\}_{j=1}^n \geq c} \pi_i(c) = \sum_j [p_{ij} - \tau_{ij} w_i c]\, Q_{ij}(p_{ij}; c).$$

This is equivalent to solving, in each market $j$,

$$\max_{p_{ij}\geq c}\ \pi_{ij}(c)=\begin{cases}[p_{ij}-\tau_{ij}w_i c]\,\alpha_H N_j\left(\frac{\hat{p}_{H,j}}{p_{ij}}-1\right) & \text{if}\quad p_{ij}\in[\hat{p}_{L,j},\hat{p}_{H,j})\\ [p_{ij}-\tau_{ij}w_i c]\,N_j\left(\frac{\alpha_H\hat{p}_{H,j}+\alpha_L\hat{p}_{L,j}}{p_{ij}}-1\right) & \text{if}\quad p_{ij}\in[0,\hat{p}_{L,j})\end{cases}.$$

For $p_{ij}\in[0,\hat{p}_{L,j})$,

$$\pi_{HL,ij}(c)=\max_{p_{ij}}[p_{ij}-\tau_{ij}w_i c]\,N_j\left(\frac{\alpha_H\hat{p}_{H,j}+\alpha_L\hat{p}_{L,j}}{p_{ij}}-1\right),$$

which entails

$$\begin{aligned}p_{ij,HL}(c)&=\tau_{ij}^{1/2}w_i^{1/2}c^{1/2}\left(\alpha_H\hat{p}_{H,j}+\alpha_L\hat{p}_{L,j}\right)^{1/2}.\\ m_{ij,HL}(c)&\equiv\frac{p_{ijHL}(c)}{\tau_{ij}w_i c}=\left(\frac{\alpha_H\hat{p}_{H,j}+\alpha_L\hat{p}_{L,j}}{\tau_{ij}w_i c}\right)^{1/2}.\\ \pi_{ij,HL}(c)&=N_j\left[\left(\alpha_H\hat{p}_{H,j}+\alpha_L\hat{p}_{L,j}\right)^{1/2}-\left(\tau_{ij}w_i c\right)^{1/2}\right]^2.\end{aligned}$$

For $p_{ij}\in[\hat{p}_{L,j},\hat{p}_{H,j})$, the firms' problem is

$$\max_{p_{ij}}\pi_{H,ij}(c)=[p_{ij}-\tau_{ij}w_i c]\,\alpha_H N_j\left(\frac{\hat{p}_{H,j}}{p_{ij}}-1\right),$$

and the first-order condition entails

$$\begin{aligned}p_{H,ij}(c)&=\left(\tau_{ij}w_i c\right)^{1/2}\hat{p}_{H,j}^{1/2}.\\ m_{H,ij}(c)&\equiv\frac{p_{H,ij}(c)}{\tau_{ij}w_i c}=\left(\frac{\hat{p}_{H,j}}{\tau_{ij}w_i c}\right)^{1/2}.\\ \pi_{H,ij}(c)&=\alpha_H N_j\left[\hat{p}_{H,j}^{1/2}-\left(\tau_{ij}w_i c\right)^{1/2}\right]^2.\end{aligned}$$

The difference here from the closed economy model is that the existence of $\tau_{ij}$ raises prices but decreases markups, given $\hat{p}_{H,j}$ and $\hat{p}_{L,j}$.

Next, $\pi_{HL,ij}(c)-\pi_{H,ij}(c)>0$ if and only if

$$c^{1/2}<\hat{c}_{ij}^{1/2}\equiv\frac{\left(\alpha_H\hat{p}_{H,j}+\alpha_L\hat{p}_{L,j}\right)^{1/2}-\alpha_H^{1/2}\hat{p}_{H,j}^{1/2}}{\left(\tau_{ij}w_i\right)^{1/2}\left(1-\alpha_H^{1/2}\right)}.\tag{43}$$

To sum up, the optimal price is

$$p_{ij}^{*}(c)=\begin{cases} p_{HL,ij}(c)=\tau_{ij}^{1/2}w_i^{1/2}c^{1/2}\left(\alpha_H\hat{p}_{H,j}+\alpha_L\hat{p}_{L,j}\right)^{1/2} & \text{if} \quad c\leq\hat{c}_{ij} \\ p_{H,ij}(c)=\left(\tau_{ij}w_ic\right)^{1/2}\hat{p}_{H,j}^{1/2} & \text{if} \quad c>\hat{c}_{ij} \end{cases}.$$

Note that $p_{H,ij}(c)>p_{HL,ij}(c)$ for any $c$. So there is upward jump of the price schedule $p^*$ in terms of $c$ at $\hat{c}_{ij}$.

## Appendix G: Trade Integration and Welfare

Indirect utility is given by (29) or

$$U(s_h)=\int_{\omega\in\Omega_h}\ln\left(\frac{\hat{p}_h}{p^*(\omega)}\right)\mathrm{d}\omega.$$

The low income worker has a set of consumed goods $\Omega_L$ that includes the ranges $[0,M]\times[0,\hat{c}]$ and $[0,M]\times[0,\hat{c}/\tau]$ for local and imported goods. Using equilibrium prices $p^*$, $\hat{p}_L/p^*(\omega)=\hat{p}_L/(\hat{p}_{HL}c)^{1/2}$ and $\hat{p}_L/(\hat{p}_{HL}\tau c)^{1/2}$ for local and imported consumption, we get

$$U(s_L)=\int_0^{\hat{c}}\ln\left(\frac{\hat{p}_L}{(\hat{p}_{HL}c)^{1/2}}\right)M\mathrm{d}G(c)+(n-1)\int_0^{\hat{c}/\tau}\ln\left(\frac{\hat{p}_L}{(\hat{p}_{HL}\tau c)^{1/2}}\right)M\mathrm{d}G(c)$$

One can compute $\int\ln\left(Ac^{-1/2}\right)dG(c)=\frac{1}{2}\left(\frac{c}{c_M}\right)^{\kappa}\left[2\ln(A)+\frac{1}{\kappa}-\ln(c)\right]$ where $A$ is a positive constant. Applying this to the above expression and simplifying we get

$$U(s_L)=M\left[1+(n-1)\tau^{-\kappa}\right]\frac{\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{a^{\kappa}}{2\kappa}-a^{\kappa}\ln\left[\left(1-\alpha_H^{1/2}\right)a+\left(\alpha_H ar\right)^{1/2}\right]\right]$$

where $r=\hat{p}_H/\hat{p}_L$ and $a=\hat{c}/\hat{p}_L$.

The high income worker has a set of consumed goods $\Omega_H$ that includes the ranges $[0,M]\times[0,\hat{p}_H]$ and $[0,M]\times[0,\hat{p}_H/\tau]$ for local and imported goods. Using equilibrium prices,we get

$$\begin{aligned}U(s_H)=&\int_0^{\hat{c}}\ln\left(\frac{\hat{p}_H}{(\hat{p}_{HL}c)^{1/2}}\right)M\mathrm{d}G(c)+\int_{\hat{c}}^{\hat{p}_H}\ln\left(\frac{\hat{p}_H}{(\hat{p}_H c)^{1/2}}\right)M\mathrm{d}G(c)\\&+(n-1)\left[\int_0^{\hat{c}/\tau}\ln\left(\frac{\hat{p}_H}{(\hat{p}_{HL}\tau c)^{1/2}}\right)M\mathrm{d}G(c)+\int_{\hat{c}}^{\hat{p}_H/\tau}\ln\left(\frac{\hat{p}_H}{(\hat{p}_H\tau c)^{1/2}}\right)M\mathrm{d}G(c)\right]\end{aligned}$$

Using the same procedure as above, this simplifies to

$$U\left(s_H\right) = M\left[1+(n-1)\tau^{-\kappa}\right]\frac{\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[a^{\kappa}\ln\left(r^{1/2}\left(\alpha_H r+\alpha_L\right)^{-1/2}\right)+\frac{r^{\kappa}}{2\kappa}\right].$$

How does the above utility increase due to trade integration compared to an increase in income? Let us fix income ratio $s_H/s_L$ so that $r$ and $a$ remain constant. A percentage decrease in trade cost yields the same change in utility resulting from a percentage increase in average income if it satisfies the following relationship:

$$\left[\frac{d\ln U\left(s_h\right)}{d\ln\tau}\right]_{s_L\text{ fixed}} d\ln\tau = -\left[\frac{d\ln U\left(s_h\right)}{d\ln s_L}\right]_{\tau\text{ fixed}} d\ln s_L.$$

We have

$$\left[\frac{d\ln U\left(s_h\right)}{d\ln\tau}\right]_{s_L\text{ fixed}} = \frac{d\ln\left(1+(n-1)\tau^{-\kappa}\right)}{d\ln\tau}+\frac{d\ln\hat{p}_L^{\kappa}}{d\ln\tau} = -\frac{\kappa}{\kappa+1}\frac{(n-1)\,\tau^{-\kappa}}{1+(n-1)\,\tau^{-\kappa}}.$$

From (26) and (27), we observe that $U\left(s_h\right)$ is proportional to $M$, which, in turn, is proportional to $s_L$. Hence, we get

$$\left[\frac{d\ln U\left(s_h\right)}{d\ln s_L}\right]_{\tau\text{ fixed}} = 1.$$

Using the above results, we obtain

$$\mu \equiv \frac{d\ln s_L}{d\ln\tau} = \frac{\left[\frac{d\ln U(s_h)}{d\ln\tau}\right]_{s_L\text{ fixed}}}{-\left[\frac{d\ln U(s_h)}{d\ln s_L}\right]_{\tau\text{ fixed}}} = \frac{\kappa}{\kappa+1}\frac{(n-1)\,\tau^{-\kappa}}{1+(n-1)\,\tau^{-\kappa}}.$$

# Appendix H: Equivalent Variation

We define the *relative equivalent variation* to be the relative increase in income, $(\Delta s_h/s_h)^{\mathrm{eq}}$, that a worker $h$ must receive to raise her utility level from the equilibrium utility $U_h^*$ to the target utility level $U_h$ taking as given the equilibrium price system $p^*(\omega)$, $\omega\in\Omega^*$ and its product space $\Omega^*$. By (29), we can write worker $h$'s indirect utility as

$$V_h(s_h) = \int_{\omega\in\Omega_h^*}\ln\left(\frac{\widehat{p}_h^*(s_h)}{p^*\left(\omega\right)}\right)\mathrm{d}\omega$$

where $\widehat{p}_h^*(s_h) = \left(s_h+P_h^*\right)/\left|\Omega_h^*\right|$ is the workers $h$'s choke price expressed as a function of income $s_h$, $\Omega_h^*$ is her equilibrium set of purchased goods and $P_h^* = \int_{\omega\in\Omega_h^*}p^*\left(\omega\right)\mathrm{d}\omega$ her price index. Then, a relative increase in income $\Delta s_h / s_h$ implies an income change from $s_h$ to $s_h + \Delta s_h$, which yields a change in utility level such that

$$U_h^* - U_h = \int_{\omega \in \Omega_h^*} \ln \left( \frac{\widehat{p}_h^*(s_h)}{\widehat{p}_h^*(s_h + \Delta s_h)} \right) \mathrm{d}\omega = |\Omega_h^*| \;\; \ln \left( \frac{s_h + P_h^*}{s_h + \Delta s_h + P_h^*} \right)$$

Inverting this expression, we obtain

$$\frac{\Delta s_h}{s_h + P_h^*} = \exp \left( - \frac{U_h^* - U_h}{|\Omega_h^*|} \right) - 1$$

Using the definition of $\widehat{p}_h^*(s_h)$, the relative equivalent variation can be expressed as

$$\left( \frac{\Delta s_h}{s_h} \right)^{\text{eq}} = \frac{|\Omega_h^*| \, \widehat{p}_h^*(s_h)}{s_h} \left[ \exp \left( \frac{U_h - U_h^*}{|\Omega_h^*|} \right) - 1 \right] \tag{44}$$

The low income consumers have $\widehat{p}_L^*(s_L) = \widehat{p}_L$ while, under Pareto productivity distributions, the mass of goods is given by $|\Omega_L^*| = MG(\widehat{c}) \left[1 + (n-1)\tau^{-\kappa}\right]$. Then, their relative equivalent variation is given by the following formula:

$$\left( \frac{\Delta s_L}{s_L} \right)^{\text{eq}} = \frac{MG(\widehat{c})\widehat{p}_L}{s_L} \left[1 + (n-1)\tau^{-\kappa}\right] \left[ \exp \left( \frac{U_L - U_L^*}{MG(\widehat{c}) \left[1 + (n-1)\tau^{-\kappa}\right]} \right) - 1 \right]$$

The high income consumers with $\widehat{p}_H^*(s_H) = \widehat{p}_H$ and $|\Omega_H^*| = MG(\widehat{p}_H) \left[1 + (n-1)\tau^{-\kappa}\right]$ get

$$\left( \frac{\Delta s_H}{s_H} \right)^{\text{eq}} = \frac{MG(\widehat{p}_H)\widehat{p}_H}{s_H} \left[1 + (n-1)\tau^{-\kappa}\right] \left[ \exp \left( \frac{U_H - U_H^*}{MG(\widehat{p}_H) \left[1 + (n-1)\tau^{-\kappa}\right]} \right) - 1 \right]$$

Note that for small utility differences, we can approximate relative equivalent variations as

$$\left( \frac{\Delta s_h}{s_h} \right)^{\text{eq}} \simeq \frac{\widehat{p}_h}{s_h} \left( U_h - U_h^* \right), \quad h = L, H$$

Equivalent variations are proportional to utility differential and income group's choke price. Since $\widehat{p}_H > \widehat{p}_L$, a same rise in utility requires to give a larger increase in income to the higher income consumer because their marginal utility of consumption is lower. Finally, note that equivalent variations are partial equilibrium concepts because prices are fixed although they vary in our general equilibrium framework.

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