MULTIDIMENSIONAL POVERTY MEASUREMENT AND PREFERENCES

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Multidimensional poverty measurement and preferences*

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Abstract

Poverty measurement based on income or consumption fails to be consistent with welfare: a higher utility (that is, preference satisfaction) of an individual may go together with an increase in the contribution of this individual to poverty. The equivalence approach, which consists of computing the money needed to maintain a given level of utility, is the way to adjust income poverty measurement so that it becomes consistent with welfare. We review four equivalence approaches, and we compare the properties that each approach satisfies or fails to satisfy. Poverty measurement based on deprivation measures, on the other hand, cannot be adjusted to become consistent with welfare. We discuss how weights and deprivation thresholds can be designed in order to decrease the discrepancy between poverty and welfare in deprivation measures.

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1 Introduction

The data that are used to measure poverty typically consist in quantities of consumed goods, prices of these goods, expenses in these goods, aggregate expenses or received

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incomes. The two simplest approaches to poverty measurement based on these data are the income approach and the deprivation approach. The income approach requires that either income is observed or that it can be computed by aggregating expenses. Then, an income poverty index is applied to the vector of incomes, and an individual is poor if her income is lower than what is needed to buy a reference consumption basket, which we call the poverty line. This approach has a long history and the theory of income poverty measurement traces back to Sen (1976) and Foster et al. (1984). In particular, Sen (1976) has initiated the axiomatic analysis of income poverty measures.

The deprivation approach was initiated by Townsend (1979). It requires data on consumption. An individual is deprived in one good if this individual's consumption of the good is below a given threshold. The vector of thresholds is the poverty line. A large part of the literature has focussed on how to identify the threshold (see, for instance, Nolan & Whelan, 1996, and Desai & Shah, 1988), but how to aggregate individual deprivation in different goods into one index is also complex. Alkire & Foster (2011) have proposed a family of indices of deprivation, according to what they called the counting approach, and they have given an interesting theoretical, including axiomatic, basis to it. The counting approach assigns weights to each commodity, the same weights for all individuals, and then it builds an individual deprivation index by summing up these weights and individual deprivation indices are compared with a threshold that is also common to all.

Both approaches, the income and deprivation approaches, face the same fundamental problem: their view of poverty is not compatible with preferences in the following sense: assuming that a higher satisfaction of preferences means a higher welfare, an individual may be better-off in a situation in which she is considered as poor than in a situation in which she is considered as non-poor. Indeed, both approaches fail to appropriately take account of heterogeneity in preferences and heterogeneity in the prices individuals face. This calls for introducing preferences as an argument of the poverty measure.

In this chapter, we assume that individual preferences are estimated, and we review the possibility to take them into account in the income and the deprivation approaches to poverty measurement. Several methods have been proposed in the literature to amend income poverty measurement so as to make it consistent with individual preferences and welfare, and we review them here in their ability to solve the different problems that make actual income a bad proxy for welfare: preferences and price heterogeneity. These methods are: equivalent incomes with a common reference price, the distance function, welfare ratios and equivalent incomes with individualized reference prices. All these methods consist in applying an income poverty index to incomes that are not the actual incomes of the individuals but are constructed to make them consistent with individual welfare and interpersonally comparable. In short, building equivalent incomes consists of computing the money needed to maintain a given level of utility.[1]

It is impossible, however, to reconcile the deprivation approach with welfare poverty: no weights, no deprivation thresholds can be defined to guarantee that an individual is poor if and only if she prefers the poverty line to her consumption basket. We deepen this point in the framework of the counting approach. We suggest to design weights and thresholds so as to minimize the number of individuals who are sorted by the counting approach in opposition to their welfare status. We discuss this possibility below, which leads us to argue that thresholds should depend on the prices faced by individuals as well as on preferences, and weights should also depend on preferences and should not be linear in commodities.

In Section 2, we compare the four methods that modify the income poverty measurement so as to make it compatible with preferences, under the assumption that observed consumption is the one that maximizes utility under a budget constraint. In Section 3, we turn to the deprivation approach, and we discuss the role that preferences may play in the design of weights and thresholds. In Section 4, we give some concluding comments.

2 Reconstructing incomes

The following notation and terminology will prove useful. There are L goods. The individual consumption set is denoted X and we assume that $X = \mathbb{R}^L_+$. Society is composed of a set N of n individuals. Each individual $i \in N$ consumes a bundle of goods $x_i \in X$. We assume that each good has a positive price, but prices may differ across individuals, because they live in different regions or they have been observed at different periods. We let $p_i \in \mathbb{R}^L_{++}$ denote the price vector faced by individual $i \in N$.

In the typical income poverty measurement, consumptions and prices are aggregated into an income $y_i = p_i x_i$, or income y_i is directly observed. We then obtain a vector of incomes, which we denote y_N. Each income is compared with an income threshold. Given the price heterogeneity, income thresholds may be different for different regions or different periods. The best way to take account of this deflation is to assume that there is a poverty line vector, $z \in X$, so that the relevant threshold for individual i is $p_i z$, the money value of the poverty line given the prices facing individual i.

[1] Why and how equivalent incomes have replaced the notions of equivalent and compensating variations in welfare analysis is explained in detail by Slesnick (1998).

The contribution of individual i to society's poverty depends on the ratio $\frac{p_i x_i}{p_i z}$. In case this ratio is above 1, the individual is not poor. If the ratio is equal to or smaller than 1, then the individual is poor, and individual poverty is aggregated into social poverty by a formula such as the FGT poverty indices (from Foster et al. 1984). What is relevant for our discussion in this chapter is the fact that an individual is poor if $p_i x_i < p_i z$. How this is translated into an individual contribution to poverty (for instance, whether it is relative or absolute) and which aggregator is used has no impact on the results we report.

The problem we study originates in the incompatibility between this way of measuring poverty and welfare. Let us assume that each individual has well-behaved (that is, continuous, monotonic and convex) preferences over consumption bundles, represented by a utility function $u_i : X \to \mathbb{R}$. We assume that these preferences are ethically compelling, so that a larger utility should be considered as a larger welfare for the individual.

There are two basic problems with income poverty measurement. The first problem is that, when individuals face different prices, income, even deflated, is not a welfare measure. This is illustrated in Fig. 1. An individual is represented in two different situations. In the first situation, she faces prices p_i, and, given her budget, her demand is x_i. In the second situation, prices are p_i' and her demand is x_i'. Indifference curves through x_i and x_i' are drawn in the figure. If we assume that prices have been adjusted so that $p_i z = p_i' z$, then we can see from the figure that

$$p_i x_i < p_i' x_i' \text{ whereas } u_i(x_i) > u_i(x_i').$$

where $p_i x_i < p_i' x_i'$ follows from the fact that the budget line through x_i crosses the ray to z below the budget line through x_i': income is lower whereas welfare is higher.

The second problem is that an individual may be described as poor even if she strictly prefers her consumption bundle over the poverty line. This is also illustrated in the figure when the individual faces prices p_i and her demand is x_i''. We see that

$$p_i x_i'' < p_i z \text{ whereas } u_i(x_i'') > u_i(z).$$

Not taking this possibility into account implies that poverty alleviation can be accompanied by a decrease in the welfare of the poor, a consequence that should be avoided.

This observation has led to a literature on price-independent welfare prescriptions, initiated by Roberts (1980), Slivinski (1983) and others. The key concept of this literature is that of equivalent income, a concept that was introduced earlier by Samuelson (1977) and Samuelson and Swamy (1974). It consists in fixing a price vector, the same for all individuals, and to measure income with respect to the demand that, at this price vector, leaves the individual indifferent to her current consumption.

This is illustrated in Fig. 1. The common reference price vector is $\overline{p}$. We can see that consumption bundle $\overline{x}_i$ has the two properties that $u_i(\overline{x}_i) = u_i(x_i)$ and $\overline{x}_i$ is the demand of this individual when prices are $\overline{p}$ for some income level. This income level is the equivalent income of this individual at $\overline{x}_i$. The figure also illustrates the construction of the equivalent income at $\overline{x}'_i$. The equivalent income comparison gives us $\overline{p}x_i > \overline{p}x'_i$ in line with the welfare comparison. The first problem is then solved.

Equivalent incomes with a common reference price, however, do not solve the second issue.[2] This is illustrated with bundle x''_i. The equivalent income at x''_i is equal to $\overline{p}x''_i$, smaller than $\overline{p}z$, whereas $u_i(x''_i) > u_i(z)$.

A second method has been proposed by Blackorby & Donaldson (1987), which solves the second problem but leaves the first one unsolved: welfare ratios. They consist in sticking to the actual demand and prices faced by the individual, but rather than comparing the actual income to the money value of the poverty line, they compute the income that would be necessary, at current prices, to reach the utility of the poverty line. This is illustrated in the figure with x_i. At prices p_i, bundle $\hat{x}_i$ is the demand at income $p_i\hat{x}_i$, and $u_i(\hat{x}_i) = u_i(z)$. Poverty for this individual would therefore be evaluated using the ratio $\frac{p_i x_i}{p_i \hat{x}_i}$. This does not guarantee, however, that these ratios are in line with welfare, because they depend on the actual price vector that individuals face as can be viewed from the figure as follows. If the actual prices were $\tilde{p}_i$ and actual demand $\tilde{x}_i$, then individual contribution to poverty, based on the observation that z is the demand at prices $\tilde{p}_i$ for income $\tilde{p}_i z$, would be $\frac{\tilde{p}_i \tilde{x}_i}{\tilde{p}_i z} \neq \frac{p_i x_i}{p_i \hat{x}_i}$ whereas $u_i(x_i) = u_i(\tilde{x}_i)$.

Two further methods have been proposed that solve both problems simultaneously. The main one is the distance function, and it was introduced by Deaton (1979). It consists in measuring the contribution of one individual to poverty by measuring the fraction of the poverty line that leaves this individual indifferent to her current consumption. This is illustrated with x'_i in the figure. Given that $u_i(x'_i) = u_i(\lambda' z)$, the contribution to poverty of this individual at x'_i is equal to λ'. The relationship to income poverty goes as follows: let p_i^d denote the supporting price of bundle $\lambda' z$. Then the contribution of this individual to poverty can be computed as an equivalent income with reference price p_i^d, as $\lambda' = \frac{p_i^d \lambda' z}{p_i^d z}$. Let us note that the reference price has become individual-specific, and, moreover, it may vary with the actual bundle of the individual, because the supporting price vector may

[2] The following extension of the equivalent income with a common price solves both problems at once: it consists in using the common price to estimate both the equivalent income at x_i and at z, that is fixing the income threshold at $\overline{pz_i}$ rather than $\overline{p}z$, where $\overline{z}_i$ is the demand at $\overline{pz_i}$ and $u_i(\overline{z}_i) = u_i(z)$. This concept has not been studied in the literature.

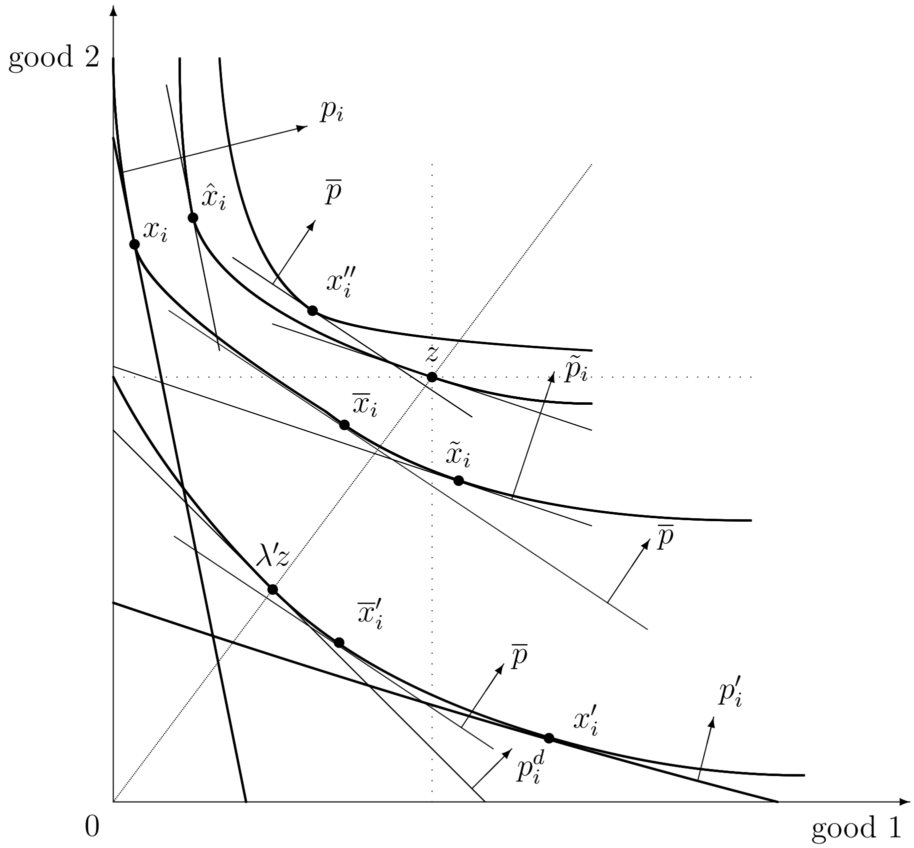

Figure 1: Illustration of the four methods to take preferences into account in the measurement of poverty.

not be constant along the ray through z.

The second method that solves both problems, contrary to the distance function, uses only one reference price vector per individual. It consists in computing the supporting price at the poverty line, and using it to compute equivalent incomes (see Dimri and Maniquet, 2019). It is illustrated in the figure, where the supporting price at z is $\tilde{p}_i$, and equivalent income at, for instance, x_i is computed at bundle $\tilde{x}_i$, which is the demand at prices $\tilde{p}_i$ for the equivalent income. The contribution to poverty of this individual at bundle $\tilde{x}_i$ is therefore $\frac{\tilde{p}_i \tilde{x}_i}{\tilde{p}_i z}$. The main drawback of this method compared to the distance function is that it requires the estimation of two indifference curves (the one through the actual bundle and the one through z), whereas the distance function only requires estimating the curve through the actual consumption.

We can summarize the discussion using the following table. It lists the different normative properties we have discussed, and, for each of them, it checks which method satisfies it. We have discussed five properties. The first one is to have a notion of income

	Equiv. inc. common prices	Distance function	Welfare ratios	Equiv. inc. ind. prices
less poor iff higher welfare	+	+		+
poor iff worse-off than at the PL		+	+	+
requires est. the IC through x_i	+	+	+	
requires est. the IC's through x_i and z	+	+	+	+
uses only one price per ind.	+			+

Table 1: Properties satisfied by different methods that take preferences into account

that is consistent with welfare. The second property is to describe an individual as poor only when her consumption has a lower utility than the poverty line. The third and fourth properties deal with the quantity of information that is needed to measure the corresponding income. The last property consists in having a single price vector to measure the equivalent income of an individual.

As suggested by Table 1, it is impossible to satisfy all properties simultaneously. It is rather intuitive: given an indifference curve through x_i and given a price vector p_i that depends only on this indifference curve and on z, it is always possible to construct preferences represented by u_i' and a bundle x_i' such that the indifference curve through x_i is unchanged, $u_i'(x_i') > u_i'(z)$ whereas $p_i x_i' < p_i z$ (see also the related characterization of poverty measures using equivalent income with individualized prices of Dimri and Maniquet, 2019, based on the axioms of the table). Poverty measures based on the distance function, on the other hand, have been axiomatized by Decancq et al. (2019) on the basis of the first three properties of the table, together with a property derived from the lattice structure of the set of indifference curves.[3]

In conclusion to this section, we can say that the income approach to poverty measurement is flexible enough, providing one uses equivalent rather than actual incomes, to meet the challenges raised by the objective of having poverty judgements in line with individuals' welfare. Two approaches seem to perform better than the others, though. Deaton's distance function approach satisfies the two basic requirements of consistency with welfare (the fist two lines in Table 2) and it is the most parsimonious in terms of needed information about preferences. Computing equivalent incomes with individualized

[3]A lattice is a partially ordered set in which every two elements have a unique supremum (in this case the lowest indifference curve that is above any two other indifference curves) and a unique infimum (the highest indifference curve that is below any two other indifference curves).

reference prices equal to the supporting prices at the poverty line is a way to satisfy the two basic requirements while using a single price vector for each individual, but it requires more information about individual preferences.

3 Preferences, deprivation, and the counting approach

Introducing preferences in the poverty measurement based on deprivation turns out to be much more difficult than in income poverty measurement. An individual $i \in N$ is said to be deprived in good $\ell \in \{1, \ldots, L\}$ if $x_{i\ell} < z_\ell$. The deprivation approach to poverty measurement begins by transforming the consumption vector x_i into a vector $d_i \in \{0,1\}^L$, in which $d_{i\ell} = 1$ if and only if $x_{i\ell} < z_\ell$. We use a function $d^z : X \to \{0,1\}^L$ to describe this transformation, that is

$$d_i = d^z(x_i) \text{ if and only if } d_{i\ell} = 1 \Leftrightarrow x_{i\ell} < z_\ell.$$

$$d^z_\ell(x_i) = 1 \Leftrightarrow x_{i\ell} < z_\ell.$$

and $d_i = (d_{i1}, \ldots, d_{i\ell}, \ldots, d_{iL}) = (d^z_1(x_i), \ldots, d^z_{i\ell}(x_i), \ldots, d^z_{iL}(x_i))$.

Then, there is a partition of the set of possible deprivation profiles $\{0,1\}^L$ into two classes, the class of profiles at which the individual is poor and the class of profiles at which she is non-poor. Formally, we can define a poverty function $P : \{0,1\}^L \to \{0,1\}$ for which $P(d^z(x_i)) = 1$ whenever the profile of deprivation of individual i consuming bundle x_i is such that i qualifies as poor. In the counting approach of Alkire & Foster (2011), this P function is defined with respect to a list of weights $w \in \mathbb{R}^L_+$, in which w_ℓ refers to the weight assigned to deprivation in good ℓ, and a threshold t, such that

$$P(d^z(x_i)) = 1 \Leftrightarrow wd^z(x_i) = \sum_{\ell \in \{1,\ldots,L\}} w_\ell d^z_\ell(x_i) > t,$$

that is the weighted sum of deprivation associated to x_i is above the threshold.

How consistent is this approach with poverty in welfare? There are two ways in which measuring deprivation contradicts welfare. The first, minor, way is that deprivation vectors force us to look at large sets of bundles as equivalent. This is exemplified in Fig. 1. Bundles x_i, $\hat{x}_i$ and x''_i, for instance, are all associated to deprivation in good 1 but not in good 2. They are, therefore, equivalent from the point of view of deprivation, but they do not correspond to the same welfare.

The second way in which measuring deprivation contradicts welfare is more problematic. It can be the case, indeed, that a bundle associated to a larger deprivation gives in fact a strictly larger welfare. This is illustrated in the figure when we compare $\overline{x}_i$ with x'_i. The former is associated to deprivation in all dimensions, whereas the latter is not deprived in good 1, but $u_i(\overline{x}_i) > u_i(x'_i)$. One may wonder how deep this problem is. That is, when comparing two deprivation vectors $d^z(x_i)$ and $d^z(x'_i)$, when can we be sure that $u_i(x_i) > u_i(x'_i)$. In the two dimensional case of the figure, it is easy to check that this is the case only when $d^z(x_i) = (0, 0)$ and $d^z(x'_i) = (1, 1)$, that is when there is no deprivation in one situation and full deprivation in the other. This is actually the case independently of the number of dimensions. The only deprivation judgement over any two bundles x_i and x'_i that is always in line with the corresponding welfare judgement is when $x_i \ll z \leq x'_i$ (we use $\ll$ to denote strict inequality in each dimension).

This observation should not be taken as a criticism towards the deprivation approach. In many situations, especially when prices are not observed, markets are imperfect and there is no way to estimate preferences, this is one of the only available approaches to poverty measurement. What this observation underlines, though, is that when some information about preferences becomes available, it is not clear at all how it can be used to refine the deprivation approach. We now deepen this point in the framework of the counting approach.

To the best of our knowledge, Bas (2018) offers the only attempt to use preferences to design weights in an application of the counting approach. The approach is empirical. Bas uses the following criterion: weights and threshold should be designed in such a way that the identification of the poor according to the counting approach is as close as possible to the identification according to welfare. More precisely, weights and threshold should be selected so as to maximize the sum of the number of individuals who qualify as poor in both approaches and the number of individuals who qualify as non-poor in both approaches.

Bas applies this idea to an evaluation of poverty in Turkey. Her main result is that weights should be different across groups of individuals. As a matter of fact, indeed, differences in preferences across groups makes it relevant to increase the weights of goods that a group of individuals with the same preferences like relatively more than other groups, so that being deprived in these goods can more easily be associated with being poor.

This suggests a more theoretical remark. The counting approach proposes to move from deprivation to poverty in an additively linear way, represented by the weight vector.

That looks unduly restrictive. To illustrate this point, let us assume that there are four goods. If profile $d = (0,1,0,1)$ is associated with poverty whereas $d' = (1,0,0,1)$ is not, we deduce that $w_1 < w_2$. Consequently, if the other profile $d'' = (1,0,1,0)$ is also associated with poverty, then we have no choice but to consider that profile $d''' = (0,1,1,0)$ is associated to poverty as well, because $wd''' - wd'' = w_2 - w_1 > 0$. This lack of choice about how to compare d'' and d''' prevents us from taking good complementarity appropriately into account. Indeed, if goods 2 and 4 are complements and goods 1 and 3 as well, then d'', in which there is no deprivation in 2 and 4 and d may perfectly well be associated to a larger welfare than d' and d'''. That is, the shape of function P should not necessarily be linear and preference estimation could be used to identify complementarities between the goods.

4 Concluding remarks

Neither income not consumption give us a well-behaved proxy for welfare. Poverty measures based on incomes or consumption, therefore, fail to deliver judgements that are in line with welfare. Several approaches based on the notion of equivalent incomes have been proposed in the literature to solve this problem in the framework of income poverty measurement. We have reviewed these approaches in this chapter and we have analyzed the different desirable properties that each of them satisfies or fails to satisfy.

The counting approach, which computes poverty based on a commodity-by-commodity comparison between consumption and poverty thresholds, on the other hand, is not easily adjusted to take preferences into account. More research is needed to determine how preferences can help design the deprivation weights and the poverty threshold so as to minimize the opposition between poverty and welfare judgements.

What we presented in this chapter raises a conceptual question and an empirical one. Conceptually, we may discuss the nature of the preferences that we claim should enter the measurement of poverty: are they the individuals' actual preferences or preferences that are laundered from aspects that are ethically irrelevant, such as time inconsistencies or other behavioral aspects related to constraints that poverty and the lack of resources impose on cognitive abilities (see, e. g., Mullainathan, & Shafir, 2013). Empirically, once we have solved the previous question and we know which preferences to use, which data and which method should be used to estimated them? To the best of our knowledge, there is no consensus in welfare economics about these two questions.

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