AGE-RELATED TAXATION OF BEQUESTS IN THE PRESENCE OF A DEPENDENCY RISK

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Abstract

This paper studies the properties of the optimal taxes on bequests when individuals differ in wage and in their risks of mortality and old-age dependance. Survival is positively correlated to income but dependency is negatively correlated with it. The government cannot distinguish between bequests motives, that is whether bequests resulted from precautionary reasons or from pure joy of giving reasons. Instead, it observes the timing of bequests and the health status at death. Under the utilitarian social welfare criterion, we show that bequests taxation results from a combination of equity, insurance and public revenue motives. If redistribution concerns dominate insurance concerns, it is desirable to tax the most bequests of those individuals living long in good health and to tax the least bequests of those dying early. This is a direct consequence of the socio-demographic structure we assumed where richer agents live longer and in better health than poorer agents. To the opposite, if insurance concerns dominate redistributive concerns, early bequests should be the most taxed and, bequests under dependency the least taxed. Under the Rawlsian criterion, we find that early bequests should be the least taxed and bequests left by the healthy long-lived individuals should be the most taxed.

JEL-Codes: H210, H230, I140.

Keywords: bequest taxation, long term care, utilitarianism, Rawlsian welfare criterion, old-age dependency.

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1 Introduction

In the recent years, the taxation of wealth transfers (including inheritance and inter vivos gifts) has been going through a bad patch. Revenue raised through taxes on inherited wealth has shrunk significantly over the last decades. In OECD countries, the proportion of total government revenues raised by such taxes has fallen from 1.1% in the mid-60s to 0.4% in 2017 (OECD, 2018). Over the same period, some countries like Australia, Israel, Mexico, Sweden and Norway have abolished death duties. One of the reasons for this state of disgrace may be the poor design of those taxes (through exemptions, deductions, and avoidance opportunities). The purpose of this paper is then to look at the optimal design of wealth transfer taxation, from both an efficiency and a redistributive perspective, in a realistic framework where older individuals with different revenues face a double risk, the risk of death and that of dependence.

Following Cremer and Pestieau (2006), the optimal wealth transfer tax structure crucially depends on the assumed bequest motives and, thus, on the type of bequests left by the deceased. Some bequests are purely accidental. Since agents do not know how long they will live, they may save more money than they turn out to need. In that situation, the taxation of accidental bequests is quite efficient and can even be redistributive. However, savings can also be motivated by the direct utility obtained from the act of giving (i.e. a "warm glow" motivation). In that case, bequests are voluntary and reflect the preference of the deceased to leave some money to their heirs so that inheritance taxation would end up being distortionary. Yet, it is often quite difficult to disentangle those bequest motives. Instead, governments only have information on the timing of bequests, i.e. whether these are made early or late in life, which in turn, provides some information regarding the motives of wealth transmission and on which they can condition taxation.

Hence, savings comprise two main components. On the one hand, part of individuals' savings aims at benefiting their heirs, out of altruism or joy of giving. On the other hand, part of individuals' savings is precautionary and, aims at covering old-age needs that cannot be (fully) insured against. In case of a healthy retirement, these needs correspond to expenses not covered by pensions and due to annuity market imperfections. In case of old-age dependence, these needs correspond to the (extra) costs of long-term care (LTC hereafter) that are not insured. When agents die prematurely or when they live long but healthy, precautionary savings are transmitted to their heirs even though this transmission was not the main motivation. In most societies, the risk of a too long life is better insured (for instance, through public or private annuitization) than the risk of dependence.[1] In other words, one can expect that the precautionary savings related to the risk of dependence represent an important part of those unplanned bequests.[2]

[1] In most OECD countries, the market for LTC insurance is still largely under-developed. See, for instance, Brown and Finkelstein (2009), Pestieau and Ponthiere (2011), OECD (2011).

[2] See Lockwood (2018) who shows that in presence of bequest motives, individuals prefer to make pre-

So as to take into account these features, our paper models an economy where agents differ with respect to their labor productivity, their risk of mortality and their risk of dependence. Those who happen to die prematurely, bequeath all their savings (both the "warm glow" and the precautionary ones) to their children. Those who live long can be either healthy or dependent. It is most likely that those who are lucky enough to remain autonomous can afford to transfer also part of their precautionary savings to their children. To the opposite, dependent agents will consume the entirety of their precautionary savings and, will only be able to bequeath the "joy of giving" part of their savings. Hence, the question we ask is the following: considering that agents with different incomes, face differentiated risks of longevity and of old-age dependence, should early bequests be taxed more heavily than late bequests?

This implicitly amounts to ask whether the taxation of bequests should be adapted so as to take into account the fact that agents need to make extra precautionary savings so as to insure against the longevity and dependency risks but, that these risks are not shared equally in the population.

To answer these questions, we assume a two-period model. Agents obtain utility from consumption and from bequeathing their wealth to their kids. The first period is certain, while individuals survive to the second period with some probability. In the first period, agents supply labour and save for retirement and as well as for possible LTC expenditures. If they survive to the second period, they eventually become dependent, in which case they will need extra resources to finance LTC expenditures. Depending whether they die at the end of the first or of the second period, and on their health status at death, bequests will be more or less important depending on whether precautionary savings add up to the planned bequests.

As we show, the resulting bequest taxes closely depend on the distribution of the three individual characteristics (income, survival risk, dependency risk) as well as on their correlations. In that respect, we use some stylized facts regarding the risks of mortality and of disability taken from Lefebvre et al. (2018). These authors use the data from the Survey on Health and Retirement in Europe (SHARE) to establish that the risks of early death and of dependence are both negatively correlated with income. Even though low-income people have a relatively shorter life, they also face a higher risk of dependence. As a consequence, we have two opposing forces when deciding the amount of individual savings and of planned bequests. On the one hand, poorer individuals are induced to save less (and give lower planned bequests) because of both lower income and lower survival probability. But, on the other hand, they are pushed to save more (and give higher planned bequests) because of a higher probability to become dependent.

As already mentioned, the government does not observe the exact composition of bequests, i.e. the repartition between planned and unplanned bequests. Yet, he observes the

cautionary savings rather than buying LTC insurance.

timing of transmissions and the health status of the individual. Early transmissions always include both planned and unplanned bequests while late transmissions may or may not include unplanned bequests, depending on the health status of the deceased.

At the second-best optimum, the government only has access to a restricted number of fiscal instruments, which are a linear tax on labour, linear taxes on early bequests, on late bequests in case of good health and on late bequests in case of dependency, together with a demogrant. We show that the level of taxes on bequests always depend on three terms: an equity term, an insurance term and a public revenue term. The equity terms (which account for the redistribution to be made in this economy) depend on the specific relationship between income and demographic risks but they always push toward taxation of bequests. To the opposite, insurance terms (related to the fact that individuals would like to smooth consumption and bequests across periods and state of nature) push toward the taxation of early bequests but toward subsidization of late bequests and of bequests in case of dependence. Finally, the public revenue effect (which accounts for the impact the tax has on overall resources collected) is ambiguous in particular because bequests are composite goods. As a result, we obtain that, if insurance effects dominate equity effects (and revenue effects are negligible), that is, if individuals are very much eager to smooth bequests across the different states of nature, early bequests should be the most taxed and bequests left in case of dependency should be the least taxed. To the opposite, if equity effects dominate insurance effects, taxation of bequests in case of autonomy should be the most taxed and early bequests should be the least taxed. We find the same last ranking if the government were rawlsian and, aimed at maximizing the utility of the least well-off agent (that is, the individual with the lowest income, lowest survival probability and highest probability to become dependent).

Our paper can be related to at least three strands of the literature. First, it can be related to the vast literature on wealth transfer taxation (among others, Cremer et al. 2012; Pestieau and Sato, 2008; Brunner and Pech, 2012). This literature has shown how important it is to distinguish between the different motives that induce (heterogenous) agents to leave some bequests to their heirs and how it may affect the design of optimal wealth transfer taxation.[3] Recently, Piketty and Saez (2013) and Garcia-Miralles (2020) have underlined the importance of considering alternative social welfare criteria on the optimal design of inheritance taxation in settings where individuals are heterogeneous in terms of bequests tastes, labour productivities and wealth. We use some of these insights in our model, by considering agents with different income and demographic characteristics, as well as different bequest motives and different social welfare criteria. Second, our paper can be related to the developing theoretical literature on the optimal design of LTC public policies. Like us, these papers often assume an heterogenous population of individuals. While we do

[3]See for instance Blumkin and Sadka (2003), Farhi and Werning (2013) and, for a survey, Cremer and Pestieau (2006).

not directly study the optimal form of the LTC public insurance system, as in Cremer and Pestieau (2014), Leroux et al. (2021), Nishimura and Pestieau (2016) for example, we show how bequest taxation can be reformed so as to take into account the growing reality that an increasing number of older agents incur LTC expenses, for which they need to make precautionary savings.[4] Finally, it can be related to the (small) literature on the age-dependent taxation of bequests, initiated by Vickrey (1945). Some of the arguments in favour of a taxation of bequests varying with the age of the deceased are summarized in Pestieau and Ponthiere (2019). Fleurbaey et al. (2019) also study the optimal taxation of accidental bequests and show that the taxation of bequests should be increasing with the age of the deceased when the social objective is ex-post egalitarian, i.e. if it wants to neutralize any ex-post welfare inequality between agents with different lifespans. Nonetheless, none of these papers account for the possibility of old-age dependency, like we do.

Our paper is structured as follows. In the next section, we describe the model. Section 3 presents the first-best (unconstrained and constrained) optimum, while Section 4 gives the second-best results under successively the utilitarian and the rawlsian social welfare criterion. The last section concludes.

2 The model

2.1 Assumptions

We consider a society composed of N types of individuals, indexed by $i = \{1, ..., N\}$, characterized by a wage w_i. Each group is in proportion n_i with $\sum_i n_i = 1$. Agents live two periods. The first one, let say young adulthood, is certain, while the second one, the retirement period, is uncertain. Agents survive to that second period with probability $0 \leq \pi_i \leq 1$. In case they survive, agents also face a different (conditional) probability to become dependent, denoted by p_i, with $0 \leq p_i \leq 1$.

Relying on Lefebvre et al. (2018), we assume a positive correlation between wage and survival, so that higher wage individuals face a higher survival probability. We also assume that they have a lower conditional probability p_i to become dependent. Yet, combining those two effects, and relying again on Lefebvre et al. (2018), we assume that there exists a negative but small correlation between $\pi_i p_i$ and w_i.[5]

In the first period, agents supply labour for an amount ℓ_i, consume an amount c_i and save for their old age. They also determine how much income they would like to give to their

[4] Cremer al. (2016) also study the optimal design of public LTC insurance which includes a linear tax on bequests. Yet, these can only be voluntary and are used by parents as a way to induce more informal care from children.

[5] Equivalently, the unconditional probability to become dependent, i.e. $\pi_i p_i$, is higher for lower wage than for higher wage, showing that the dependency effect (through p_i) dominates the survival effect (through π_i), but the gap between the two is low.

offspring. If they survive, in the second period, they do not work anymore and, they consume an amount d_i in case of autonomy, but an amount m_i which includes LTC expenditures, in case of dependency.

Individuals preferences are additively separable in consumptions and the amount of bequests:

$$U_i = u(c_i - z(\ell_i)) + \pi_i p_i \left[H(m_i) + v(b_i^D)\right] + \pi_i(1-p_i)\left[u(d_i) + v\left(b_i^L\right)\right] + (1-\pi_i)v(b_i^E) \quad (1)$$

where $x_i = c_i - z(\ell_i)$ is first-period net consumption. We assume a quasi-linear form for the disutility of labour, $z(\ell_i)$ such that it is increasing, $z'(.) \geq 0$ and convex, $z''(.) \geq 0$ in labour. For simplicity, in the rest of the paper, we assume that it takes the following form: $z(\ell_i) = \ell_i^2/2$.

Utility of consumption in the first period and, in the second period under good health (equivalently autonomy) is denoted by $u(.)$. In the second period, under bad health (i.e. dependency), utility is denoted by $H(.)$. As usual, these utilities are increasing and concave, i.e. $u'(.) \geq 0$, $u''(.) \leq 0$ and $H'(.) \geq 0$, $H''(.) \leq 0$. For simplicity, we also assume that the utility in case of dependency takes the following form: $H(x) = u(x - \bar{L})$ where $\bar{L}$ is the monetary equivalent of the loss due to old-age dependency. This implies that marginal utility under dependence is higher than under autonomy: $H'(x) = u'(x - \bar{L}) > u'(x)$.

The utility obtained by parents from leaving bequests to their heirs is denoted by $v(b_i^j)$ where b_i^j is the amount of bequest received by the offspring of agent i. It depends on the timing $j = \{E, L, D\}$ of the bequests left by the agent, that is depending on whether he died early (E), late in good health (L) or late under dependence (D). We have $v'(.) \geq 0$ and $v''(.) \leq 0$. The modelling of the joy of giving utility is similar, for instance, to Fleurbaey et al. (2019), Glomm and Ravikumar (1992), and Piketty and Saez (2013) who also study bequest taxation. Like in these papers, we will assume that the deceased cares about the amount of bequests *received* by his heirs, i.e. net of taxation, and not about the gross amount (i.e. before taxation). Assuming a joy of giving motive instead of altruism enables to avoid the modelling of a more complicated (dynamic) setting and to account only for parents' preferences.[6]

[6] Under perfect altruism, parents care about the total well-being of their children while under imperfect altruism, parents value the bequests they leave to their children through their children's own preferences from consumption. Under joy of giving, parents value the bequests they leave to their children through their own preferences (which are possibly different from those of their children). See also Kopczuk and Lupton (2007) who prefer to model joy of giving rather than altruism on the grounds that "the evidence suggests motives other than the maximisation of a dynastic utility function" (page 210).

2.2 Laissez-faire

Assuming that the interest rate is zero and that individuals have no pure time preferences, the individual's problem can be written as follows:

$$\begin{aligned}\max_{\ell_i,s_i,b_i,g_i} U_i &= u(w_i\ell_i - s_i - b_i - \frac{\ell_i^2}{2}) + \pi_i p_i\left[H(s_i) + v(b_i)\right] + (1-\pi_i)v(b_i+s_i) \\ &+\pi_i(1-p_i)\left[u(s_i-g_i) + v\left(g_i+b_i\right)\right]\end{aligned}$$

where s_i is private saving, b_i is the amount of planned (or voluntary) bequests, and g_i is the extra amount the agent would leave to his offsprings in case the individual enjoys a healthy retirement period. By definition, $s_i \geq g_i$.

As this is clear from the above formulation of the laissez-faire problem, we assume away the existence of a private or public annuity market, as well as the existence of a private or public LTC insurance. This is close to what is observed in the real world where we only witness a partial annuitization of retirement savings through public or private defined benefit schemes. In addition, in most countries, the LTC insurance market is almost inexistent (see OECD, 2011).

As a consequence, in our model, individuals are forced to choose, in the first period of their life, a level of precautionary savings s_i higher than what would be needed if there was no risk to become dependent (i.e. if $p_i = 0$). In case of early death of the individual, this amount of unplanned (or equivalently, involuntary) bequests s_i is left to his children besides the amount of planned (or equivalently, voluntary) bequests b_i. In case the individual survives to the second period and remains autonomous, he will thus leave an additional transfer g_i to his children, which corresponds to some of the precautionary savings he had made to cover the risk of a long life under bad health. In the second period, if the parent is lucky enough to remain autonomous, there is thus some reallocation of the precautionary savings decided in the first period, to the benefit of the child. This corresponds to what is observed in reality where we may observe unintended bequests in the form of both accidental bequests s_i and transfers g_i.

These assumptions of no annuity market and of no LTC insurance are crucial for our model of differentiated bequest taxation. Indeed, if we had assumed instead the existence of an actuarially fair annuity market and of an actuarially fair LTC insurance, there would not have been any accidental bequest s_i nor any reallocation g_i of savings to the benefit of children. Hence, within such a setting, bequests would have been limited to voluntary ones (i.e. to b_i), and the issue of differentiated bequest taxation would have automatically vanished. It is true that we could still have assumed partial annuitization and LTC insurance schemes in the analysis. This would have evicted part of the precautionary savings and of the reallocation transfer. We decided not to make this assumption for at least two reasons. First, as long as they are taken as given, our results would be qualitatively unchanged.

We would still be left with different levels of bequests in the three states of the world.[7] Second, in this paper, we decided to focus on the direct redistributive and efficiency effects of introducing differentiated bequest taxation and to exclude any indirect redistributive and efficiency effects resulting from the introduction of public LTC or pension benefits. This is why we have assumed away any possibility of annuitisation and of LTC insurance.

This being said, the first-order conditions of the individual's problem are:

$$\frac{\partial U_i}{\partial s_i} = -u'(w_i\ell_i - s_i - b_i - \frac{\ell_i^2}{2})(w_i - \ell_i) = 0 \tag{2}$$

$$\frac{\partial U_i}{\partial s_i} = -u'(w_i\ell_i - s_i - b_i - \frac{\ell_i^2}{2}) + \pi_i p_i H'(s_i) + \pi_i(1-p_i)u'(s_i - g_i) + (1-\pi_i)v'(b_i + s_i) = 0 \tag{3}$$

$$\frac{\partial U_i}{\partial b_i} = -u'(w_i\ell_i - s_i - b_i - \frac{\ell_i^2}{2}) + \pi_i p_i v'(b_i) + (1-\pi_i)v'(b_i + s_i) + \pi_i(1-p_i)v'(g_i + b_i) = 0 \tag{4}$$

$$\frac{\partial U_i}{\partial g_i} = -u'(s_i - g_i) + v'(g_i + b_i) = 0 \tag{5}$$

The first condition yields that at the laissez-faire, individuals choose to supply an amount of labour equal to their wage rate, $\ell_i = w_i$.

From the last condition, we obtain that in case of good health at the old age, the extra amount of savings g_i left to offsprings should be set so as to equalize the marginal utility from consumption with the marginal utility from leaving bequests, i.e. $u'(s_i - g_i) = v'(g_i + b_i)$. Replacing for this equality in the FOCs (3) and (4), we also obtain that the marginal utility of consumption under dependence should be equalized to the marginal utility from leaving bequests when dependent at the old age, i.e. $H'(s_i) = v'(b_i)$.

Let us now make some comparative statics of s_i, b_i and g_i with respect to the wage w_i, the survival probability π_i and the conditional probability to become dependent, p_i. Considering the above first-order conditions, one can immediately see that the implicit function theorem cannot be used since each FOC includes the other endogenous variables. Instead, one needs to use the Cramer's rule which proves difficult with three equations and three unknowns and would certainly yield ambiguous results. Hence, in order to deal with this issue, we show in the Appendix, that if we assume that $u(.)$ takes an isoelastic form and that the joy of giving is modeled as $v(.) = \beta u(.)$ with $\beta \leq 1$, we can obtain unambiguous results. In such a case, we find that g_i only depends on the monetary equivalent of the loss due to dependency, $\bar{L}$, on the risk aversion parameter and on the intensity of the joy of giving preference, β, but not on w_i, π_i or p_i. Also, from condition $H'(s_i) = v'(b_i)$, we show that the variations of s_i and b_i with respect to w_i, π_i and p_i have the same sign. Finally, fully differentiating the FOC

[7] One difference would only be that, for some agents with low wages, private savings could have been totally evicted.

with respect to s_i, we prove that savings s_i and thus voluntary bequests b_i are increasing in w_i, π_i and p_i. Our results are summarized in the following proposition:

Proposition 1 *Suppose an economy where agents differ in income, survival probability and the probability to become dependent. Assuming isoelastic utility functions and a joy of giving utility function of the form $v(b) = \beta u(b)$ where $\beta \leq 1$, the laissez-faire allocation is such that*

- *Precautionary savings and planned bequests are increasing in income, survival probability, and in the probability to become dependent.*
- *The amount of income reallocated toward children in case of good health does not depend on income, survival probability, and the probability to become dependent.*

We will now study the first-best optimum and see how different it is from the laissez-faire problem.

3 The first-best optimum

3.1 The unconstrained optimum

In the main part of the paper, we will assume a utilitarian government.[8] Its problem therefore consists in maximizing the sum of individuals' expected utility defined in (1), subject to the resource constraint of the economy:

$$\begin{aligned}
\max_{\ell_i,c_i,d_i,m_i,b_i^j} \sum_i n_i U_i &= \sum_i n_i \{u(c_i - \frac{\ell_i^2}{2}) + \pi_i p_i \left[H(m_i) + v(b_i^D)\right] \\
&\quad + \pi_i(1-p_i)\left[u(d_i) + v\left(b_i^L\right)\right] + (1-\pi_i)v(b_i^E)\} \\
\text{s. to } \sum_i n_i w_i \ell_i &\geq \sum_i n_i \{c_i + \pi_i p_i (m_i + b_i^D) + \pi_i(1-p_i)(d_i + b_i^L) + (1-\pi_i)b_i^E\}
\end{aligned}$$

Rearranging the FOCs of this problem, we obtain the following trade-offs:

$$\ell_i = w_i \forall i \tag{6}$$

$$u'(x_i) = u'(d_i) = H'(m_i) = v'(b_i^j)\ \forall i\, \forall j = \{E, L, D\} \tag{7}$$

$$v'(b_i^D) = v'\left(b_i^L\right) = v'(b_i^E) \forall i \tag{8}$$

The above conditions show that it is optimal to equalize consumptions in good health in the first and the second periods, and to provide higher consumption in case of dependency.

[8] In Section 4.3, we assume instead a rawlsian government.

It is also optimal to set the level of consumptions identical across individuals with different wages, so that $m_i = \bar{m} > x_i = d_i = \bar{x}\,\forall i$.

Regarding the level of bequests, it is optimal to set them at the same level independently of whether the agent lives long or dies early, and whether he remains autonomous or dependent.[9] It should also be equalized across agents with different wages so that $b_i^D = b_i^L = b_i^E = \bar{b}$.

Our results are summarized in the following proposition.

Proposition 2 *Suppose an economy with agents differing in income, survival probability and probability to become dependent. At the first-best utilitarian optimum, we obtain that*

- *per period consumption is equalized across agents,*
- *consumption when dependent is higher than when autonomous,*
- *bequests are equalized across agents, and independent of either the timing or of the health state of the deceased.*

The implementation of this first-best unconstrained optimum would be made possible by introducing an actuarially fair system of annuities as well as an actuarially fair system of LTC insurance, together with individualized lump sum transfers across individuals with different wages and demographic characteristics.

3.2 The constrained optimum

In this paper, we exclude the possibility that agents have access to an actuarially fair system of annuities and of LTC insurance, as it is most often the case in reality. As we explain at the beginning of Section 2.2, this is not a strong assumption given the thinness of the annuity and of the LTC insurance markets in the real world.

In other words, we exclude the possibility of insuring perfectly against the risk of longevity and of dependence. This leads us to studying an alternative "constrained" first-best optimum where such a possibility is excluded:

$$\begin{aligned}
\max_{\ell_i, c_i, s_i, g_i, b_i} \sum_i n_i U_i &= \sum_i n_i \{ u(c_i - \frac{\ell_i^2}{2}) + \pi_i p_i \left[H(s_i) + v(b_i) \right] \\
&\quad + \pi_i (1 - p_i) \left[u(s_i - g_i) + v\left(b_i + g_i\right) \right] + (1 - \pi_i) v(s_i + b_i) \} \\
\text{s. to } \sum_i n_i w_i \ell_i &\geq \sum_i n_i \{ c_i + b_i + s_i \}
\end{aligned}$$

[9]This is due to the separability of preferences between consumption and bequests.

The FOCs of this problem lead to the following optimality conditions:

$$\begin{aligned} \ell_i^* &= w_i & (9)\\ u'(x_i) &= \mu \,\forall i & (10)\\ H'(s_i) - v'(b_i) &= 0 & (11)\\ u'(s_i - g_i) - v'(b_i + g_i) &= 0 & (12)\\ \pi_i p_i H'(s_i) + \pi_i(1-p_i)u'(s_i - g_i) + (1-\pi_i)v'(s_i + b_i) &= \mu \,\forall i & (13) \end{aligned}$$

where μ is the Lagrange multiplier associated with the resource constraint of the economy. Under the constrained optimum, agents supply labour for an amount equal to their wage rate, but they all obtain the same level of first-period consumption. Regarding the other variables, the first best constrained optimum leads to an equality between the marginal utility of consumption of the dependent and the marginal utility of the bequest left to his child, as well as between the marginal utility of consumption of the healthy long-lived individual and the marginal utility of the bequest left to his offspring. Combining equations (10) and (13), we also obtain that the marginal utility of first-period consumption should be equalized to the weighted sum of marginal utilities of consumption, the weights being the probabilities that the three states of nature realize (i.e. early death, healthy long life and long life under dependency). Note that using equations (11)-(13), we obtain that marginal utility of first-period consumption should also be set equal to the weighted sum of marginal utilities obtained from leaving bequests.

Comparing these FOCs (equations 9-13) with those obtained under the laissez-faire (equations 2-5), we immediately see that only a set of personalized lump sum transfers T_i are necessary to ensure the equality of $u'(x_i)$ across all agents. No other distortionary instruments, such as linear taxes on bequests are needed. Yet, as soon as such individualized lump-sum transfers are not available, we will need all these tax instruments as we will now study in the following section.

4 The second-best optimum

Up to now, we considered that the government had access to all possible policy instruments. In that case, only individualized lump-sum transfers at each period were needed to decentralize the constrained utilitarian optimum. Such a system of differentiated lump sum transfers being hardly available in real-world economies, this section considers a more realistic second-best setting, where the government has access only to a restricted number of uniform fiscal instruments, as in Sheshinski (1972).[10]

[10] The interest of resorting to linear taxation (instead of non linear taxation) is to have explicit formulations of the taxes.

We will therefore assume the following set of policy instruments: a uniform tax on labour θ as well as a uniform lump sum transfer T in the first period, and a system of taxes on bequests, $\{\tau_E, \tau_D, \tau_L\}$, which depend both on the health status and the age of the deceased. The tax τ_E corresponds to the tax on early bequests, that is on bequests left by the deceased if he dies at the end of the first period. If the agent survives to the second period, a tax τ_L will be paid on the bequests he leaves if he dies autonomous at the end of the second period. If the agent becomes dependent in the second period, a tax τ_D will be paid on bequests he leaves when he dies, i.e. on voluntary bequests. We assume that the health status at the old age is observable (through, for instance, medical check-ups, eligibility to public LTC programs, or the use of -public or private- LTC services).

The timing of the problem is the following one. In the first stage, the government announces the optimal policy. In a second stage, individuals make decisions regarding labour supply, private savings and bequests, for given policy instruments. As usual, we proceed backwards, by first deriving the individuals' problem and second, by deriving the optimal tax instruments, taking into account that they will affect individuals' decisions.

4.1 The individual's problem

The problem of an individual with type i, facing the public policy instruments $(T, \theta, \tau_E, \tau_L, \tau_D)$ is now modified in the following way:

$$\begin{aligned}\max_{s_i,b_i,g_i} U_i &= u(w_i(1-\theta)\ell_i - s_i - b_i + T - \frac{\ell_i^2}{2}) + \pi_i p_i \left[H(s_i) + v((1-\tau_D)b_i)\right] \\ &+ (1-\pi_i)v((1-\tau_E)(b_i+s_i)) + \pi_i(1-p_i)\left[u(s_i-g_i) + v((1-\tau_L)(g_i+b_i)\right]\end{aligned}$$

In the following, we will distinguish between three levels of bequests, net of taxation, received by the offspring of agent i and, denote them as follows: $b_i^E = (b_i + s_i)(1-\tau_E)$ in case of early death, $b_i^L = (b_i + g_i)(1-\tau_L)$ in case of a healthy long life, and $b_i^D = b_i(1-\tau_D)$ in case of dependency. Recall also that, because agents care about the amount *received* by their heirs, the levels of taxation τ_E, τ_L and τ_D appear in the function $v(.)$.

The first-order conditions of the above problem are:

$$\frac{\partial U_i}{\partial \ell_i} = -u'(x_i)(w_i(1-\theta) - \ell_i) = 0 \quad (14)$$

$$\frac{\partial U_i}{\partial s_i} = -u'(x_i) + \pi_i p_i H'(s_i) + \pi_i(1-p_i)u'(s_i - g_i) + (1-\pi_i)v'\left(b_i^E\right)(1-\tau_E) = 0 \quad (15)$$

$$\frac{\partial U_i}{\partial b_i} = -u'(x_i) + \pi_i p_i v'\left(b_i^D\right)(1-\tau_D) + (1-\pi_i)v'(b_i^E)(1-\tau_E) + \pi_i(1-p_i)v'(b_i^L)(1-\tau_L) = 0 \quad (16)$$

$$\frac{\partial U_i}{\partial g_i} = -u'(s_i - g_i) + v'\left(b_i^L\right)(1-\tau_L) = 0 \quad (17)$$

The first condition determines the level of labour supply as a function of the tax on labour: $\ell_i^* = w_i(1-\theta)$ and as usual, it is decreasing in the labour tax rate. Because of the assumption of quasi-linearity on labour, there is no impact of the taxes on bequests on labour supply. The above conditions jointly determine the levels of savings and bequests, as a function of the policy instruments. For the following, we denote these demand functions by $s_i^*(T, \tau_E, \tau_L, \tau_D, \theta)$, $b_i^*(T, \tau_E, \tau_L, \tau_D, \theta)$ and $g_i^*(T, \tau_E, \tau_L, \tau_D, \theta)$.

Like in the laissez-faire section, we can rearrange the last three conditions to obtain the following trade-offs:

$$v'\left(b_i^{L*}\right)(1-\tau_L) = u'(s_i^* - g_i^*) \tag{18}$$

$$v'\left(b_i^{D*}\right)(1-\tau_D) = H'(s_i^*) \tag{19}$$

In the appendix, we are able to show that when we assume that $u(x) = log(x)$ and $v(x) = \beta u(x)$ where $\beta \leq 1$, the optimal amount g_i^* reallocated to children in case of autonomy is equal to $\bar{g}$ and is independent of the policy instruments. It only depends on exogenous parameters such as the monetary equivalent of the loss due to old-age dependency, L and the joy of giving parameter β. In addition, we show that in this specific case, savings s_i^* and voluntary bequests b_i^* depend only on the lump-sum transfer, T, in the following way[11]

$$\frac{ds_i^*(T)}{dT} > 0 \text{ and } \frac{db_i^*(T)}{dT} > 0$$

As we will see in the next section, this will simplify a lot our computations so that we will consider the log utility case as a special case in order to obtain unambiguous results.

4.2 The utilitarian second-best policy.

The problem of the utilitarian government consists in choosing the level of policy instruments so as to solve the following problem:

$$\begin{aligned}
\max_{\theta,\tau_L,\tau_E,\tau_D,T} \quad & \sum_i n_i \quad \{u(\frac{w_i^2(1-\theta)^2}{2} - s_i^* - b_i^* + T) + \pi_i p_i \left[H(s_i^*) + v((1-\tau_D)b_i^*)\right] \\
& + \quad \pi_i(1-p_i)\left[u(s_i^* - g_i^*) + v((1-\tau_L)(g_i^* + b_i^*))\right] \\
& + \quad (1-\pi_i)v\left((1-\tau_E)(b_i^* + s_i^*)\right)\} \\
\text{s.to} \quad & \sum_i n_i \quad \{\theta(1-\theta)w_i^2 + \tau_D \pi_i p_i b_i^* + \tau_L \pi_i (1-p_i)(g_i^* + b_i^*) + \tau_E(1-\pi_i)(s_i^* + b_i^*)\} \geq T
\end{aligned}$$

where, for simplicity, we directly replaced for $\ell_i^*(\theta) = w_i(1-\theta)$. For ease of notation and when this does not introduce confusion, in the following, we will replace $\sum_i n_i$ by the expectation operator $E\{.\}$ and we will drop the arguments and indexes in the functions s_i^* and b_i^*. Denoting the Lagrange multiplier associated with the revenue constraint of the

[11] Note that assuming an isoelastic utility function yields ambiguous results.

government by μ, and using the envelope theorem for s_i^*, b_i^* and x_i^*, we obtain the following FOCs:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \theta} &= -(1-\theta)E\{u'(x^*)w^2\} + \mu[(1-2\theta)E\{w^2\} + \Phi_\theta] = 0 \\
\frac{\partial \mathcal{L}}{\partial \tau_L} &= -E\{\pi(1-p)(b^*+g^*)v'(b^{L*})\} + \mu[E\{\pi(1-p)(b^*+g^*)\} + \Phi_{\tau_L}] = 0 \\
\frac{\partial \mathcal{L}}{\partial \tau_E} &= -E\{(1-\pi)\,(b^*+s^*)\,v'(b^{E*})\} + \mu[E\,\{(1-\pi)\,(s^*+b^*)\} + \Phi_{\tau_E}] = 0 \\
\frac{\partial \mathcal{L}}{\partial \tau_D} &= -E\{\pi p b^* v'(b^{D*})\} + \mu[E\{\pi p b^*\} + \Phi_{\tau_D}] = 0 \\
\frac{\partial \mathcal{L}}{\partial T} &= E\{u'(x^*)\} - \mu\,[1-\Phi_T] = 0
\end{aligned}$$

where

$$\Phi_z \equiv \tau_D E\{\pi p \frac{\partial b^*}{\partial z}\} + \tau_L E\{\pi(1-p)\frac{\partial(b^*+g^*)}{\partial z}\} + \tau_E E\{(1-\pi)\frac{\partial\,(s^*+b^*)}{\partial z}\} \tag{20}$$

represents the indirect effects, through $s^*(T,\tau_E,\tau_L,\tau_D,\theta)$, $b^*(T,\tau_E,\tau_L,\tau_D,\theta)$, that the tax instrument $z = \{T,\tau_E,\tau_L,\tau_D,\theta\}$ has on public revenue.

Let us first compute the second-best level of the tax on labour, θ, in compensated terms, that is, compensated by a variation of the lump-sum transfer T such that it leaves the government's revenue unchanged. To do so, we combine the FOCs with respect to T and θ and write down the derivative of the Lagrangian with respect to the tax on labour in compensated terms as follows:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial \theta} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\theta} \tag{21}$$

where, from the budget constraint of the government, we obtain:

$$\frac{dT}{d\theta} = (1-2\theta)E\{w^2\}. \tag{22}$$

This leads to the following expression:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \theta} = -(1-\theta)E\{u'(x^*)w^2\} + \mu[(1-2\theta)E\{w^2\} + \Phi_\theta] + (1-2\theta)E\{w^2\}[E\{u'(x^*)\} - \mu\,[1-\Phi_T]]$$

After rearranging terms, we obtain that the optimal level of θ satisfies the following condition

$$\frac{\theta^{SB}}{1-\theta^{SB}} = \frac{-cov(u'(x^*),w^2) + \mu\frac{\tilde{\Phi}_\theta}{1-\theta^{SB}}}{E\{w^2\}E\{u'(x^*)\}} > 0$$

where SB stands for second-best and where

$$\begin{aligned}\tilde{\Phi}_z &\equiv \tau_D E\{\pi p \frac{\partial b^*}{\partial z} + \frac{\partial b^*}{\partial T}\frac{dT}{dz}\} + \tau_L E\{\pi(1-p)\frac{\partial(b^*+g^*)}{\partial z} + \frac{\partial(b^*+g^*)}{\partial T}\frac{dT}{dz}\} \\ &\quad + \tau_E E\{(1-\pi)\frac{\partial\left(s^*+b^*\right)}{\partial z} + \frac{\partial(s^*+b^*)}{\partial T}\frac{dT}{dz}\} \\ &\equiv \tau_D E\{\pi p \frac{\partial \widetilde{b^*}}{\partial z}\} + \tau_L E\{\pi(1-p)\frac{\partial \widetilde{b^*+g^*}}{\partial z}\} + \tau_E E\{(1-\pi)\frac{\partial \widetilde{s^*+b^*}}{\partial z}\}\end{aligned}$$

accounts for the indirect *compensated* effects a given tax instrument $z = \{\tau_E, \tau_L, \tau_D, \theta\}$ has on public revenues and where the tilde, on the second line, denotes the compensated effect of a variation of z on b^*, g^* and s^*.[12]

These results are in line with those obtained in optimal tax theory (see Atkinson and Stiglitz, 1980). As usual the denominator on the RHS shows the efficiency cost of increasing labour taxation on labour supply. The numerator comprises two terms. The first term accounts for the redistributive effect of increasing labour taxation. Since $cov(u'(x^*), w^2) < 0$, this term pushes toward a higher rate of labour taxation. Indeed, a high labour tax rate reduces inequalities between agents with different wage rate since agents with higher wage (and thus, income) pay more taxes, which are then redistributed through uniform lump-sum transfers. The second term accounts for the compensated effect of increasing labour taxation on government revenue, through its effect on b^* and s^*. On the one hand, an increase in θ is likely to decrease s_i^*, g_i^* and b_i^* $\forall i$, but, on the other hand, it enables to increase T, which in turn positively affects savings and bequests. The sign of this second term in the numerator is then ambiguous. Yet, if the public revenue effect is small compared to the redistribution effect, we then obtain a positive level of labour taxation, $\theta^{SB} > 0$.

Proceeding in the same way as for labour taxation, we now compute the derivative of the compensated Lagrangian with respect to τ_L:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_L} = \frac{\partial \mathcal{L}}{\partial \tau_L} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\tau_L} = 0 \tag{23}$$

where, from the budget constraint of the government, we obtain:

$$\frac{dT}{d\tau_L} = E\{\pi(1-p)\left(g^*+b^*\right)\}. \tag{24}$$

The above equation (23) enables to find the compensated effect of the tax on late bequests on aggregate welfare, that is, it enables to find whether a marginal change of τ_L would be beneficial to welfare, when that change is compensated by a variation of T in order to

[12] This compensated effect writes as follows: $\frac{\partial \tilde{X}}{\partial z} = \frac{\partial X}{\partial z} + \frac{\partial X}{\partial T}\frac{dT}{dz} = 0$ where $X = \{b^*, b^*+s^*, b^*+g^*\}$ and $z = \{\tau_L, \tau_E, \tau_D, \theta\}$.

maintain the government's budget balanced, as shown by equation (24). This yields:

$$\begin{aligned}\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_L} &= -E\{\pi(1-p)(b^*+g^*)v'(b^{L*}\} + \mu[E\{\pi(1-p)(b^*+g^*)\} + \Phi_{\tau_L}] \\ &+ E\{\pi(1-p)\,(g^*+b^*)\}[E\{u'(x^*)\} - \mu(1-\Phi_T)] \\ &= -cov(\pi(1-p)(g^*+b^*); v'(b^{L*})) + E\{\pi(1-p)(g^*+b^*)\}[E\{u'(x^*)\} - E\{v'(b^{L*})\}] + \mu\tilde{\Phi}_{\tau_L} = 0\end{aligned} \tag{25}$$

The first term in (25) accounts for the direct redistributive effect of the tax on late bequests. This covariance term, $cov(\pi(1-p)(g^*+b^*); v'(b^{L*}))$, is negative since higher wage leads to higher $\pi(1-p)$ and higher g^* and b^*. The equity term then pushes toward taxation of late bequests in order to redistribute more resources between agents with different incomes and demographic characteristics. The second term is the insurance effect of the tax and, it accounts for the fact that the individual would like to smooth bequests across the different states of the world (i.e. depending on whether he lives long or not, becomes dependent or not) as it is clear from condition (7) at the unconstrained first best. It is likely to be negative and would therefore push toward the subsidization of late bequests.[13] The last term represents the indirect effect (through the variations of aggregate s^*, b^* and g^*) of increasing the tax on late bequests on public revenue. The sign of this term is ambiguous since bequests are composite goods, which comprise a combination of either b^*, s^* or g^* and whose arguments may be affected differently by the tax on late bequests.

Rearranging equation (25), we obtain an explicit expression for the tax on late bequests:

$$\tau_L^{SB} = \frac{\begin{array}{l}[-cov(\pi(1-p)(g^*+b^*); v'(b^{L*})) + E\{\pi(1-p)(g^*+b^*)\}[E\{u'(x^*)\} - E\{v'(b^{L*})\}] \\ +\mu\left(\tau_D E\{\pi p \frac{\partial \tilde{b^*}}{\partial \tau_L}\} + \tau_E E\{(1-\pi)\frac{\partial \widetilde{b^*+s^*}}{\partial \tau_L}\}\right)]\end{array}}{-\mu E\{(1-p)\pi \frac{\partial \widetilde{b^*+g^*}}{\partial \tau_L}\}} \tag{26}$$

where the tilde denotes the compensated effect of a variation of τ_L on b^*, g^* and s^* (see footnote 12). This expression is again quite standard in optimal tax theory and in line with Atkinson and Stiglitz (1980). The denominator is the standard efficiency term, and accounts for the compensated effect of the tax τ_L^{SB} on late bequests (i.e. b^*+g^*).[14] The numerator comprises the same three terms we described above and its sign may well be positive or negative, depending on the magnitude of the different effects at play. All in all, as it is often the case in linear taxation problems, it is impossible to clearly establish whether the tax on late bequests should be positive or negative. It is only possible to decompose the different forces that plead for either taxation or subsidization of late bequests. Below, we take an example with log utilities so as to obtain unambiguous results.

[13]The sign of the insurance effect in τ_L^{SB} is proven at the end of Appendix 5.3.

[14]In Atkinson and Stiglitz (1980), the denominator is positive. Here, it is less clear as, in our problem, bequests are composite goods whose terms may vary differently depending on the tax considered.

In the Appendix, we also compute the derivative of the Lagrangian with respect to the tax on early bequests, in compensated terms:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_E} = -cov((1-\pi)(b^* + s^*); v'(b^{E*})) + E\{(1-\pi)(b^* + s^*)\}[E\{u'(x^*)\} - E\{v'(b^{E*})\}] + \mu \tilde{\Phi}_{\tau_E} = 0 \quad (27)$$

This formula is quite close to the one obtained for the tax on late bequests, and comprises the same three terms which account respectively for the redistributive, the insurance and the public revenue effects of the tax on early bequests, τ_E^{SB}. The sign of $cov((1-\pi)(b^* + s^*); v'(b^{E*}))$ is ambiguous a priori. On the one hand, higher wage individuals save more and give more to their children (i.e. $b^* + s^*$ increases), but, on the other hand, they are less likely to die at the end of the first period (i.e. $(1-\pi_i)$ is smaller for them). Yet, assuming reasonably that income effects are higher than survival effects, $cov((1-\pi)(b^* + s^*); v'(b^{E*}))$ is likely to be negative. Hence, this first equity term pushes toward the taxation of early bequests. In the Appendix, we also show that the second insurance effect is positive, which again pushes toward taxation of early bequests. Finally, the revenue effect is likely to be ambiguous.

In equation (27), we can isolate the optimal second best level of τ_E expressed in compensated terms:

$$\tau_E^{SB} = \frac{\begin{array}{l}[-cov((1-\pi)(b^* + s^*); v'(b^{E*})) + E\{(1-\pi)(b^* + s^*)\}[E\{u'(x^*)\} - E\{v'(b^{E*})\}] \\ +\mu\left(\tau_D E\{\pi p \frac{\partial \tilde{b}^*}{\partial \tau_E}\} + \tau_L E\{(1-p)\pi \frac{\partial \widetilde{b^* + g^*}}{\partial \tau_E}\}\right)]\end{array}}{-\mu E\{(1-\pi)\frac{\partial \widetilde{s^* + b^*}}{\partial \tau_E}\}} \quad (28)$$

As before, the denominator accounts for the efficiency impact of increasing τ_E. Again, the different effects at play in the numerator make it difficult to obtain a clear sign for the tax on early bequests, and to conclude whether early bequests should be taxed or subsidized. Nonetheless, if the revenue effect is small relative to the other two insurance and equity effects (and, if the denominator is positive as it would be the case in a problem *à la* Atkinson and Stiglitz), then early bequests should be unambiguously taxed, and $\tau_E^{SB} > 0$.

Finally, we compute the derivative of the Lagrangian with respect to the tax on bequests in case of dependency, in compensated terms:[15]

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_D} = -cov(\pi p b^*; v'(b^{D*})) + E\{\pi p b^*\}[E\{u'(x^*)\} - E\{v'(b^{D*})\}] + \mu \tilde{\Phi}_{\tau_D} = 0 \quad (29)$$

In the above expression, the first term is the equity term and, here again the sign of $cov(\pi p b^*; v'(b^{D*}))$ is ambiguous. On the one hand, a higher wage leads to higher b^* but

[15]See Appendix 5.3, for the details of the computations.

on the other hand, it decreases the probability to become dependent (i.e. w_i and $\pi_i p_i$ are negatively correlated). Relying on Lefebvre et al. (2018) which shows that the effect of higher wealth on the conditional probability to become dependent is negative but low, we can again reasonably assume that the effect of income on b^* dominates the effect of higher income on the probability to become dependent and set $cov(\pi p b^*; v'(b^{D*})) < 0$. The redistribution effect therefore pushes toward higher taxation of voluntary bequests, since richer agents leave higher voluntary bequests. In the Appendix, we show that the second term, which accounts for the insurance effect, is negative, which pushes toward subsidization of bequests in case of dependence. As before, the public revenue effect is ambiguous.

Rearranging terms in (29), we obtain an explicit formulation of the second best level of τ_D^{SB} expressed in compensated terms:

$$\tau_D^{SB} = \frac{\begin{array}{c}[-cov(\pi p b^*; v'(b^{D*})) + E\{\pi p b^*\}[E\{u'(x^*)\} - E\{v'(b^{D*})\}] \\ +\mu\left(\tau_L E\{\pi(1-p)\frac{\partial \widetilde{b^*+g^*}}{\partial \tau_D}\} + \tau_E E\{(1-\pi)\frac{\partial \widetilde{s^*+g^*}}{\partial \tau_D}\}\right)]\end{array}}{-\mu E\{\pi p \frac{\partial \widetilde{b^*}}{\partial \tau_D}\}} \tag{30}$$

All in all, depending on the magnitude of the different equity, insurance and revenue effects effects described above, we may well have a positive or negative tax on bequests left by the dependent individual.

Let us finally compare the taxes on bequests, τ_L^{SB}, τ_E^{SB} and τ_D^{SB}, with each other. They all comprise the three same equivalent terms in the numerator. As already explained, the first part of these three formulas reflects the redistributive effect of the taxes arising from the interplay between wages and probabilities of either dependence or survival. This term would vanish if there was no heterogeneity in wage and in survival probability or probability to become dependent. The second part reflects the insurance effect and is a consequence of the unavailability of actuarially fair annuities and LTC insurance. The third part reflects the compensated effect the tax in question has on total public revenue.

Let us start with the equity terms in these three equations. They rank as follows:

$$-cov(\pi(1-p)(g^*+b^*), v'(b^{L*})) > -cov(\pi p b^*; v'(b^{D*})) > -cov((1-\pi)(b^*+s^*); v'(b^{E*})) > 0.$$

The first part of the above inequality can be explained as follows. Both b^{L*} and b^{D*} increase with income but late bequests under autonomy concern the high-income individuals more than the low-income ones since their survival probability in good health is higher. In addition, Lefebvre et al. (2018) show that the correlation between survival (π) and income (implicit in $cov(\pi(1-p)(g^* + b^*), v'(b^{L*}))$) is much stronger than the correlation between the unconditional probability of dependency (πp) and income (implicit in $cov(\pi p b^*; v'(b^{D*}))$, the latter being close to zero. The last inequality above can be explained in the same way. Late bequests under dependency, b^{D*}, and early bequests b^{E*} increase with income. Yet, the correlation between the probability of dependence and income is low, while the probability of early death $(1-\pi)$ is clearly negatively correlated with income, resulting then in $-cov(\pi p b^*; v'(b^{D*})) > -cov((1-\pi)(b^*+s^*); v'(b^{E*}))$.

Considering only equity terms, it would then be optimal to tax more late bequests than bequests left in case of dependency and, early bequests would be the least taxed. Such a ranking reflects the fact that richer individuals live longer and in better health than poorer individuals who are more likely to die early, so that a way to redistribute resources across income groups is to tax more the bequests of those who live longer and in better health.[16]

We now turn to the insurance terms in our tax formula. Besides equity motives, without perfect insurance instruments (such as annuity markets and LTCI markets) as in our setting, inheritance taxes are also desirable in order to smooth bequests across the different states of nature. Starting from the case where the tax rates are the same, we clearly have $b^{E*} > b^{L*} > b^{D*}$. In addition, using the results regarding the signs of the insurance terms (see Appendix 5.3), we obtain that

$$E\{u'(x^*)\} - E\{v'(b^{D*})\} < E\{u'(x^*)\} - E\{v'(b^{L*})\} < 0 < E\{u'(x^*)\} - E\{v'(b^{E*})\}$$

Hence, the insurance effect pushes for a tax on early bequests, while it pushes toward subsidies on late bequests (independently of the health status at death). The subsidy on bequests in case of dependency would then be higher than in case of autonomy. The differences in magnitude of the insurance effects will depend on the concavity of $v(.)$. If concavity is high, the differences between the different insurance terms can be important.

Finally, the last term in the tax formulas accounts for the public revenue effect and, it quantifies the indirect impact a variation in the bequest tax has on government revenues through the variations of b^{E*}, b^{L*} and b^{D*}. But, as already mentioned, bequests are composite goods, which makes it difficult to infer what would be the precise impacts of the variation of a specific tax on those levels and to rank these revenue terms on prior ground.[17]

All in all, even if the differences in the revenue terms are negligible, with no further assumption, the ranking of taxes will be ambiguous and, will depend on the relative importance of both the insurance and the equity terms. If the insurance motive dominates the redistributive motive (that is, if individuals are very much eager to smooth bequests across the different states of nature), we therefore obtain that:

$$\tau_E^{SB} > \tau_L^{SB} > \tau_D^{SB}.$$

This seems quite intuitive since early bequests are higher than bequests in case of a long health life and, higher than bequests left in case of dependency and, the insurance motive

[16] Note that besides the strength of the correlations between π_i, p_i and w_i, the magnitude of the equity terms will depend on the concavity of $v(.)$ as well as on the distribution of wages in the economy.

[17] With log utilities (as we shall see below), or if we start from a zero taxation level, these revenue terms are null: $\tilde{\Phi}_z = 0$, $\forall z = \{\tau_L, \tau_E, \tau_D\}$.

pushes toward smoothing these. This ranking would be the one obtained if all individuals were identical or if individualized lump-sum taxes were available since, in these cases, the equity terms would disappear.[18]

If, on the contrary, the equity motive dominates the insurance motive, then the ranking would be

$$\tau_L^{SB} > \tau_D^{SB} > \tau_E^{SB}.$$

As a last example, we present here the level of taxes obtained when we assume log utility functions and a joy of giving function taking the following functional form, $v(x) = \beta u(x)$ with $\beta \leq 1$ (as in Section 4.1). Interestingly in this case, the public revenue effects of the taxes on bequests disappear, i.e. $\Phi_z = 0$, $\forall z = \{\tau_L, \tau_E, \tau_D\}$.[19] As we show in Appendix 5.4, we can obtain quite simple expressions:

$$\theta^{SB} = \frac{\mu E\{w^2\} - E\{u'(x^*)w^2\}}{2\mu E\{w^2\} - E\{u'(x^*)w^2\}} > 0 \tag{31}$$

$$\tau_L^{SB} = 1 - \frac{\beta}{\mu} \frac{E\{\pi(1-p)\}}{E\{\pi(1-p)(b^* + \bar{g})\}} \tag{32}$$

$$\tau_E^{SB} = 1 - \frac{\beta}{\mu} \frac{E\{1-\pi)\}}{E\{(1-\pi)(s^* + b^*)\}} \tag{33}$$

$$\tau_D^{SB} = 1 - \frac{\beta}{\mu} \frac{E\{\pi p\}}{E\{\pi p b^*\}} \tag{34}$$

As it is clear from the above formula, the comparison of the three taxes on bequests exclusively depends on the comparisons between the weighted average of late bequests, of the weighted average of early bequests, and of the weighted average of voluntary bequests, where the weights are the probability $\pi(1-p)$, $(1-\pi)$ and πp respectively. It is reasonable to assume that weighted average of early bequests is higher than the weighted average of late bequests, which itself is higher than the weighted average of bequests in case of dependence, yielding the following unambiguous ranking of the taxes on bequests:[20]

$$\tau_E^{SB} > \tau_L^{SB} > \tau_D^{SB}.$$

In that specific case with log utilities, we clearly obtain that insurance effects dominate redistributive effects and that early bequests should always be subject to higher taxation.

Our results for the utilitarian second-best optimum are summarized in the following proposition:

[18] In a model where there is no dependency at old age and individuals are identical, Fleurbaey et al. (2019) obtain a similar result regarding age-dependent bequest taxation.

[19] See Appendix 5.2.

[20] This result is even more obvious with uniform probabilities of dependence and of survival.

Proposition 3 *Assume that the utilitarian government can only use uniform lump-sum transfers as well as linear taxes on early bequests and on late bequests. The second-best optimal policy is such that*

- *The ranking of bequests taxes,* $\{\tau_E^{SB}, \tau_L^{SB}, \tau_D^{SB}\}$ *is in general ambiguous and depends on the magnitude of equity, insurance and public revenue terms.*
- *If insurance effects dominate equity effects (and revenue effects are negligible),* $\tau_E^{SB} > \tau_L^{SB} > \tau_D^{SB}$.
- *If equity effects dominate insurance effects (and revenue effects are negligible),* $\tau_L^{SB} > \tau_D^{SB} > \tau_E^{SB}$.
- *With log-utility functions, we always have* $\tau_E^{SB} > \tau_L^{SB} > \tau_D^{SB}$.

4.3 The rawlsian second-best solution

Let us now modify the above problem by considering instead a rawlsian government and see whether we can obtain more results. The problem of the government now consists in maximizing the welfare of the worst-off individual, subject to the revenue constraint of the government. Here, the worst-off individual corresponds to the individual with the lowest wage denoted by w_0, and thus, the lowest survival probability π_0 and the highest probability p_0 to become dependent. The problem now writes as follows:

$$\begin{aligned}\max_{\theta,\tau_L,\tau_E,\tau_D,T} \quad & u(\frac{w_0^2(1-\theta)^2}{2} - s_0^* - b_0^* + T) + \pi_0 p_0 \left[H(s_0^*) + v((1-\tau_D)b_0^*)\right] \\ + \quad & \pi_0(1-p_0)\left[u(s_0^* - g_0^*) + v((1-\tau_L)(g_0^* + b_0^*))\right] + (1-\pi_0)v\left((1-\tau_E)(b_0^* + s_0^*)\right) \\ \text{s. to} \quad & \sum_i n_i\{\theta(1-\theta)w_i^2 + \tau_D\pi_i p_i b_i^* + \tau_L\pi_i(1-p_i)(b_i^* + g_i^*) + \tau_E(1-\pi_i)(s_i^* + b_i^*)\} \geq T\end{aligned}$$

where we replaced for $\ell_0^* = w_0(1-\theta)$. Using the same procedure as in the previous section, we compute the derivatives of the Lagrangian with respect to the tax instruments in compensated terms (i.e. compensated by an increase in T), under the rawlsian criterion:

$$\frac{\partial\tilde{\mathcal{L}}}{\partial\theta} = -u'(x_0^*)[w_0^2(1-\theta) - (1-2\theta)E\{w^2\}] + \mu\tilde{\Phi}_\theta = 0 \tag{35}$$

$$\frac{\partial\tilde{\mathcal{L}}}{\partial\tau_L} = u'(x_0^*)E\{\pi(1-p)(g^* + b^*)\} - \pi_0(1-p_0)v'(b_0^{L*})(g_0^* + b_0^*) + \mu\tilde{\Phi}_{\tau_L} = 0 \tag{36}$$

$$\frac{\partial\tilde{\mathcal{L}}}{\partial\tau_E} = u'(x_0^*)E\{(1-\pi)(s^* + b^*)\} - (1-\pi_0)v'(b_0^{E*})(b_0^* + s_0^*) + \mu\tilde{\Phi}_{\tau_E} = 0 \tag{37}$$

$$\frac{\partial\tilde{\mathcal{L}}}{\partial\tau_D} = = u'(x_0^*)E\{\pi p b^*\} - \pi_0 p_0 v'(b_0^{D*})b_0^* + \mu\tilde{\Phi}_{\tau_D} = 0 \tag{38}$$

Let us concentrate on the conditions for τ_L^R, τ_E^R, τ_D^R, where R stands for Rawls.[21] The first term on the RHS of these expressions reflects the utility loss from an increase in the relevant inheritance tax. The second term represents the utility gain from the increase in the demogrant T generated by an increase in the tax. The last term is the compensated public revenue effect of increasing bequests taxation.

Let us note here that as before, both the concavity of $u(.)$ and $v(.)$ as well as the strength of the preference for leaving bequests (in comparison to consumption) will be important determinants of whether it is optimal to tax or subsidize bequests.

In order to interpret these equations and infer the signs of the taxes, let us directly use the log-utility simplification. In that situation, and as we showed in the previous sections, the public revenue effects vanish and, we obtain quite simple expressions for the tax rates under the rawlsian objective:[22]

$$\tau_L^R = 1 - \frac{\beta}{\mu} \frac{\pi_0(1-p_0)}{E\{\pi(1-p)(b^* + \bar{g})\}} \tag{39}$$

$$\tau_E^R = 1 - \frac{\beta}{\mu} \frac{1-\pi_0}{E\{(1-\pi)(s^* + b^*)\}} \tag{40}$$

$$\tau_D^R = 1 - \frac{\beta}{\mu} \frac{\pi_0 p_0}{E\{\pi p b^*\}} \tag{41}$$

Let us consider some extreme but realistic cases. For instance, let assume that $\pi_0 \to 0$ and $p_0 \to 1$ so that the poorest individual has almost no chance to survive and if so, he is almost certain to be dependent. In that situation, both τ_L^R and τ_D^R would be maximum (i.e. tend to 1), while $\tau_E^R < 1$. Basically, this example shows that if the poorest has a very low probability of survival and a very high probability of dependency, the taxes on late bequests and on bequests in case of dependency should be much higher than the tax on early bequests. The tax on early bequests could even turn out to be negative if the average probability to survive in the society is very high.

By continuity, for less extreme values of π_0 and p_0, we will obtain the following ranking of bequest taxes, $\tau_L^R > \tau_D^R > \tau_E^R$. This ranking is the same as the one obtained in the utilitarian section when the redistributive terms dominate the other terms, which is quite intuitive under the rawlsian objective.

[21] The details of our computations and the explicit tax formula have been relegated to Appendix 5.5.

[22] The tax on labour takes the following expression:

$$\theta^R = \frac{\mu E w^2 - u'(x_0^*) w_0^2}{2\mu E w^2 - u'(x_0^*) w_0^2} > 0.$$

5 Conclusion

As emphasized in Cremer and Pestieau (2006), the theoretical literature on wealth transfer taxation has been concerned by the distinction between planned and unplanned bequests with the result that the latter should be heavily taxed since it does not entail disincentive effects and can be quite redistributive. Unfortunately, in practice, it is quite difficult to sort out those two types of bequests. One way to overcome this issue would be to differentiate taxation rates according to the timing of bequests, as proposed by Vickrey (1945) and more recently, by Fleurbaey et al. (2019) and Pestieau and Ponthiere (2019). Nonetheless, this solution has not yet received a lot of attention both by researchers and by policy makers. Even though this differentiation is possible, the widespread practice around the world involves tax rates on bequests that do not depend on the age of the deceased.

In this paper, we have nonetheless shown that it is socially desirable to make the taxation of bequests depend explicitly not only on the age of the deceased, but also on his health status at death, a characteristic which the governments can observe. To do so, we have assumed away any public or private LTC insurance or pension annuities. It is obvious that if individuals could purchase actuarially fair LTC insurance or pensions, the difference between early and late bequests (under either dependence or autonomy) would disappear as there would not be anymore unplanned bequests. If those schemes are restricted because of loading costs, missing markets and public regulation as it is most often the case in reality, then the qualitative nature of our findings still holds and differentiated bequest taxation is desirable.

Under the utilitarian social welfare criterion, we show that bequests taxation results from a combination of equity, insurance and public revenue motives. If equity concerns dominate insurance concerns, it is then desirable to tax the most bequests of those individuals living long in good health and to tax the least bequests of those dying early. This is a direct consequence of the socio-demographic structure of the population we assumed where richer agents live longer in better health than poorer agents. To the opposite, if insurance concerns dominate redistributive concerns, that is if individuals are very much eager to smooth bequests across the different states of nature, we obtain that early bequests should be the most taxed and bequests under dependency the least taxed. Under the rawlsian criterion, which gives priority to the worst-off individual (i.e. the individual with the lowest income, lowest survival probability and highest probability to become dependent), early bequests should be the least taxed and bequests left by the healthy long-lived individuals should be the most taxed.

We believe that our theoretical model sheds light on important reasons as for why differentiated bequests taxation is justified and, which could help reforming the existing bequest taxation schemes. We hope our paper contributes to the still largely unexplored field of differentiated inheritance taxation. This topic is even more relevant that the uncertainty regarding autonomy at old age and the need for extra (LTC) resources is becoming more prominent.

References

[1] Atkinson A. and J. Stiglitz, 1980, Lectures on public economics, McGraw-Hill, New York.

[2] Blumkin, T. and E. Sadka, 2003, Estate taxation with intended and accidental bequests, Journal of Public Economics, 88, 1-21.

[3] Brown, J.R. and A. Finkelstein, 2009, The private market for long-term care insurance in the united states: a review of the evidence, The Journal of Risk and Insurance, 76 (1), 5-29.

[4] Brunner J. and S. Pech, 2012, Optimal Taxation of Bequests in a Model with Initial Wealth, 114 (4), 1368-1392.

[5] Cremer H. and P. Pestieau, 2006, Wealth transfer taxation: a survey of the theoretical literature, Chapter 16 in Handbook of the Economics of Giving, Altruism and Reciprocity, vol. 1, pp. 1107-1134 Elsevier North Holland, Amsterdam.

[6] Cremer, H., F. Gavahri and P. Pestieau, 2012, Accidental Bequests: A Curse for the Rich and a Boon for the Poor, Scandinavian Journal of Economics, 114, 1437-1459.

[7] Cremer H. and P. Pestieau, 2014, Social long-term care insurance and redistribution, International Tax and Public Finance, 21, 955-974.

[8] Cremer H., P. Pestieau and K. Roeder, 2016, Social long-term care insurance with two-sided altruism, Research in Economics, 70 (1), 101-109.

[9] García-Miralles E., 2020, The Crucial Role of Social Welfare Criteria and Individual Heterogeneity for Optimal Inheritance Taxation, The B.E. Journal of Economic Analysis & Policy, Vol. 20 (2), 20190274.

[10] G.Glomm and B. Ravikumar, 1992, Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality, Journal of Political Economy , 100(4), 818-834.

[11] Farhi, E. and I. Werning, 2013, Estate taxation with altruism heterogeneity. American Economic Review, Papers and Proceedings, 103 (3), 489-495.

[12] Fleurbaey, M., M-L. Leroux, P. Pestieau, G. Ponthiere and S. Zuber, 2019, Premature Deaths, Involuntary Bequests and Fairness, CESifo Working Paper No. 6802.

[13] Kopczuk, W. and J.P. Lupton, 2007, To Leave or Not to Leave: The Distribution of Bequest Motives. The Review of Economic Studies, 74(1), 207-235.

[14] Lefebvre M., Perelman S., Schoenmaeckers J. (2018). Inégalités face à la mort et au risque de dépendance. Revue Française d'Economie, 33(2), 75-112.

[15] Leroux, M-L., G. Ponthiere and P. Pestieau, 2021, Fair Long-Term Care Insurance, accepted Social Choice and Welfare.

[16] Lockwood, L., 2018, Incidental Bequests and the Choice to Self-Insure Late-Life Risks, American Economic Review, 108(9), 2513-2550.

[17] Nishimura Y. and P. Pestieau, 2016, Efficient taxation with differential risks of dependence and mortality, Economics Bulletin, 36 (1), 52-57.

[18] OECD, 2011, Help Wanted? Providing and Paying for Long-Term Care, OECD Publishing.

[19] OECD, 2018, The Role and Design of Net Wealth Taxes in the OECD, Book number 26. https://doi.org/10.1787/19900538

[20] Pestieau, P. and G. Ponthiere, 2011, The Long Term Care Insurance Puzzle. In Financing Long-Term Care in Europe - Institutions. J. Costa-Font, palgrave macmillan edition.

[21] Pestieau, P. and G. PonthiÃ?re, 2019, An age differentiated tax on bequests, in Age Policies - Normative Theory and Proposals, ed. by Greg Bognar and Axel Gosseries, forthcoming.

[22] Pestieau, P. and M. Sato, 2008, Estate Taxation With Both Accidental and Planned Bequests, Asia-Pacific Journal of Accounting & Economics, 15, 223-240.

[23] Piketty, T. and E. Saez, 2013, A Theory of Optimal Inheritance Taxation, Econometrica, Vol. 81 (5), 1851-1886.

[24] Salanié, B., 2003, The Economics of Taxation, MIT Press.

[25] Sheshinski, E., 1972, The Optimal Linear Income-tax, Review of Economic Studies, 39(3), 297-302.

[26] Vickrey, William, 1945, An integrated successions tax. Republished in: R. Arnott, K. Arrow, A. Atkinson and J. Drèze (eds.) (1994). Public Economics. Selected Papers by William Vickrey. Cambridge University Press.

Appendix

5.1 Comparative statics with respect to w_i, π_i and p_i.

Let assume the following functional forms for the utility functions:

$$\begin{aligned} u(x) &= \frac{x^\varepsilon}{\varepsilon} \text{ with } 0 \leq \varepsilon \leq 1 \\ v(x) &= \beta u(x) \text{ with } \beta \leq 1 \end{aligned}$$

At the laissez-faire, we have that $H'(s_i) = v'(b_i)$ and $u'(s_i - g_i) = v'(g_i + b_i)$. Replacing for the above functional forms and $H(x) = u(x - \bar{L})$, we obtain that:

$$\begin{aligned} (s_i - \bar{L})^{\varepsilon - 1} &= \beta b_i^{\varepsilon - 1} \\ (s_i - g_i)^{\varepsilon - 1} &= \beta (g_i + b_i)^{\varepsilon - 1} \end{aligned}$$

Solving this system, we obtain that

$$g_i = \frac{\bar{L}}{1 + \beta^{\frac{1}{\varepsilon - 1}}}$$

and it is constant in w_i, π_i and p_i. In the following, we denote it by $\bar{g}$. We then fully differentiate the laissez-faire FOC for s_i (equation 3 where we replaced for $\ell_i = w_i$) with respect to w_i and obtain after some rearrangements that:

$$\begin{aligned} -u''(x_i) w_i &+ \frac{ds_i}{dw_i} [u''(x_i) + \pi_i p_i H''(s_i) + \pi_i (1 - p_i) u''(s_i - \bar{g}) + (1 - \pi_i) v''(b_i + s_i)] \\ &+ \frac{db_i}{dw_i} [u''(x_i) + (1 - \pi_i) v''(b_i + s_i)] = 0 \end{aligned} \tag{42}$$

where $x_i = w_i^2/2 - s_i - b_i$ and where the expressions inside brackets are negative. Fully differentiating $H'(s_i) = v'(b_i)$ with respect to w_i, we obtain that

$$\frac{db_i}{dw_i} = \frac{H''(s_i)}{v''(b_i)} \frac{ds_i}{dw_i},$$

so that db_i/dw_i and ds_i/dw_i have the same sign. Replacing for that expression in (42), we obtain that

$$\begin{aligned} \frac{ds_i}{dw_i} \Big[& u''(x_i) + \pi_i p_i H''(s_i) + \pi_i (1 - p_i) u''(s_i - \bar{g}) + (1 - \pi_i) v''(b_i + s_i) \\ &+ \frac{H''(s_i)}{v''(b_i)} [u''(x_i) + (1 - \pi_i) v''(b_i + s_i)] \Big] = w_i u''(x_i) \end{aligned}$$

yielding that $\frac{ds_i}{dw_i} > 0$ and thus, $\frac{db_i}{dw_i} > 0$.

Using the same reasoning as above, we differentiate s_i with respect to π_i and p_i:

$$\begin{aligned}
\frac{ds_i}{dp_i} \quad & \Big[\quad u''(x_i) + \pi_i p_i H''(s_i) + \pi_i(1-p_i)u''(s_i - \bar{g}) + (1-\pi_i)v''(b_i + s_i) \\
+ \quad & \frac{H''(s_i)}{v''(b_i)}[u''(x_i) + (1-\pi_i)v''(b_i + s_i)]\Big] = p_i[u'(s_i - g_i) - H'(s_i)] + v'(b_i + s_i) - u'(s_i - g_i) \\
\frac{ds_i}{dp_i} \quad & \Big[\quad u''(x_i) + \pi_i p_i H''(s_i) + \pi_i(1-p_i)u''(s_i - \bar{g}) + (1-\pi_i)v''(b_i + s_i) \\
+ \quad & \frac{H''(s_i)}{v''(b_i)}[u''(x_i) + (1-\pi_i)v''(b_i + s_i)]\Big] = \pi_i[u'(s_i) - H'(s_i)]
\end{aligned}$$

and we obtain that $\frac{ds_i}{d\pi_i} > 0$ and $\frac{ds_i}{dp_i} > 0$, since $H'(s_i) = v'(b_i)$ and $u'(s_i - g_i) = v'(b_i + g_i)$. This in turn implies that $\frac{db_i}{d\pi_i} > 0$ and $\frac{db_i}{dp_i} > 0$.

5.2 Comparative statics with respect to the tax instruments.

From (14), we already know that $\ell_i^* = w_i(1-\theta_i)$, independently from any assumed functional form. We thus have that when θ_i increases, ℓ_i decreases.

We then assume that $u(x) = log(x)$ so that $v(x) = \beta log(x)$ and $H(x) = log(x - L)$. Replacing for these functional forms in (15)-(17), we obtain the following simplified expressions:

$$\begin{aligned}
\frac{\partial U_i}{\partial s_i} &= -\frac{1}{\frac{w_i^2(1-\theta_i)^2}{2} - s_i - b_i + T} + \frac{\pi_i p_i}{s_i - \bar{L}} + \frac{\pi_i(1-p_i)}{s_i - g_i} + \frac{(1-\pi_i)\beta}{s_i + b_i} = 0 \\
\frac{\partial U_i}{\partial b_i} &= -\frac{1}{\frac{w_i^2(1-\theta_i)^2}{2} - s_i - b_i + T} + \frac{\pi_i p_i \beta}{b_i} + \frac{(1-\pi_i)\beta}{b_i + s_i} + \frac{\pi_i(1-p_i)}{b_i + g_i} = 0 \\
\frac{\partial U_i}{\partial g_i} &= -\frac{1}{s_i - g_i} + \frac{\beta}{b_i + g_i} = 0
\end{aligned}$$

As it is clear from the above conditions, only the demogrant remains and may impact s_i^*, b_i^*, and g_i^*.

Solving this system of equations, one can show that:

$$g_i^* = \frac{\beta}{1+\beta}\bar{L} \ \forall i$$

Replacing for $g_i^* = \bar{g}$ in the last condition, $ds_i^*(T)/dT$ and $db_i^*(T)/dT$ have the same sign. Further differentiating fully the first two FOCs, one can show that

$$\frac{ds_i^*(T)}{dT} > 0 \text{ and } \frac{db_i^*(T)}{dT} > 0.$$

5.3 Second-best utilitarian optimum

So as to obtain the derivative of the Lagrangian with respect to the tax on early bequests in compensated terms, we proceed in the same way as for τ_L and combine the FOCs with respect to T and τ_E as follows:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_E} = \frac{\partial \mathcal{L}}{\partial \tau_E} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\tau_E} = 0 \tag{43}$$

where, from the budget constraint of the government, we obtain:

$$\frac{dT}{d\tau_E} = E(1-\pi)\left(s^* + b^*\right). \tag{44}$$

Rearranging terms, condition (43) yields (27).

Finally, we compute the derivative of the compensated Lagrangian with respect to τ_D:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_D} = \frac{\partial \mathcal{L}}{\partial \tau_D} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\tau_D} = 0 \tag{45}$$

where, from the budget constraint of the government, we obtain:

$$\frac{dT}{d\tau_D} = E\pi p b^*. \tag{46}$$

It yields expression (29).

We now determine the *signs of the insurance effects* described in Section 4.2. Reasonably, from the individual's problem, we have that $x_i^* > s_i^* > s_i^* - g_i^*$. Using equation (18) from the individual's problem with tax instruments, we can show that, starting from a tax level $\tau_L = 0$, $E\{u'(x^*)\} - E\{v'(b^{L*})\} < 0$.

In the same way, using the individual's FOC (19) together with $u'(x_i^*) < H'(s_i^*)$, one can show that, starting from a tax level $\tau_D = 0$, $E\{u'(x^*)\} - E\{v'(b^{D*})\} < 0$.

Finally, notice that condition (15) shows that $u'(x_i^*)$ is a linear combination of $H'(s_i^*)$, $u'(s_i^* - g_i^*)$ and $v'\left(b_i^{E*}\right)(1-\tau_E)$. Since $H'(s_i^*)$, $u'(s_i^* - g_i^*)$ are higher than $u'(x_i^*)$, we necessarily have that $v'\left(b_i^{E*}\right)(1-\tau_E) < u'(x_i^*)$. This implies that starting from a tax rate level $\tau_E = 0$, $E\{u'(x^*)\} - E\{v'(b^{E*})\} > 0$.

5.4 Second-best utilitarian optimum assuming log utilities

As we have shown in Section 4.1 and Appendix 5.2, if we assume log utilities, s^* and b^* only depend on T but not on the other fiscal instruments, so that $\Phi_z = 0$, $\forall z = \{\tau_L, \tau_E, \tau_D\}$. In addition, g is independent of fiscal instruments: $g_i^* = \bar{g}\, \forall i$.

The second-best FOCs can therefore be rewritten as follows

$$\frac{\partial \mathcal{L}}{\partial \theta} = -(1-\theta)E\{u'(x^*)w^2\} + \mu(1-2\theta)E\{w^2\} = 0 \tag{47}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_L} = -E\{\pi(1-p)(b^* + \bar{g})v'(b^{L*})\} + \mu E\{\pi(1-p)(b^* + \bar{g})\} = 0 \tag{48}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_E} = -E\{(1-\pi)v'(b^{E*})\,(b^* + s^*)\} + \mu E\,\{(1-\pi)\,(s^* + b^*)\} = 0 \tag{49}$$

$$\frac{\partial \mathcal{L}}{\partial \tau_D} = -E\{\pi p v'(b^{D*})b^*\} + \mu E\{\pi p b^*\} = 0 \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial T} = E\{u'(x^*)\} - \mu\,[1 - \Phi_T] = 0 \tag{51}$$

where $\Phi_T \equiv \tau_D E\pi p \frac{\partial b^*}{\partial T} + \tau_L E\pi(1-p)\frac{\partial b^*}{\partial T} + \tau_E E(1-\pi)\frac{\partial(s^*+b^*)}{\partial T} > 0$ represents the indirect effects, through $s^*(T)$, $b^*(T)$, that the demogrant T has on public revenue.

Rearranging equation (47), we can directly obtain expression (31). Under the reasonable assumption that $cov(u'(x^*), w^2) < 0$, we can also show that the numerator (and thus the denominator) are both positive. To see this, recognize that the numerator of (31) can be written as follows:

$$\frac{E\{u'(x^*)\}}{1-\Phi_T}E\{w^2\} - E\{u'(x^*)w^2\}$$

where we replaced for μ from eq. (51). Rearranging this expression yields

$$-cov(u'(x^*), w^2) - E\{u'(x^*)\}E\{w^2\}(1 - \frac{1}{1-\Phi_T}),$$

which is unambiguously positive.

Recognizing that with log utilities and under our specification of the joy of giving, $v'(x) = \beta/x$, we can rearrange equation (48) and obtain (32) defining τ_L^{SB}.

In the same way, we rearrange (49) and (50), and obtain equations (32) and (34) defining τ_E^{SB} and τ_D^{SB}.

5.5 Second-best rawlsian optimum

Using the envelope theorem for s_0^*, b_0^* and x_0^*, the FOCs are now:

$$\frac{\partial \mathcal{L}}{\partial \theta} = -u'(x_0^*)w_0^2(1-\theta) + \mu[(1-2\theta)E\{w^2\} + \Phi_\theta] = 0 \quad (52)$$

$$\frac{\partial \mathcal{L}}{\partial \tau_L} = -\pi_0(1-p_0)(b_0^* + g_0^*)v'(b_0^{L*}) + \mu[E\{\pi(1-p)(b^* + g^*)\} + \Phi_{\tau_L}] = 0 \quad (53)$$

$$\frac{\partial \mathcal{L}}{\partial \tau_E} = -(1-\pi_0)v'(b_0^{E*})\left(b_0^* + s_0^*\right) + \mu[E\left\{(1-\pi)\left(s^* + b^*\right)\right\} + \Phi_{\tau_E}] = 0 \quad (54)$$

$$\frac{\partial \mathcal{L}}{\partial \tau_D} = -\pi_0 p_0 v'(b_0^{D*})b_0^* + \mu[E\{\pi p b^*\} + \Phi_{\tau_D}] = 0 \quad (55)$$

$$\frac{\partial \mathcal{L}}{\partial T} = u'(x_0^*) - \mu\left[1 - \Phi_T\right] = 0 \quad (56)$$

where Φ_z is defined by (20) and, as before, μ is the Lagrange multiplier associated to the resource constraint.

We write the derivatives of the Lagrangian with respect to the tax instruments in compensated terms as follows :

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial \theta} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\theta} = 0$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_L} = \frac{\partial \mathcal{L}}{\partial \tau_L} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\tau_L} = 0$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_E} = \frac{\partial \mathcal{L}}{\partial \tau_E} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\tau_E} = 0$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau_D} = \frac{\partial \mathcal{L}}{\partial \tau_D} + \frac{\partial \mathcal{L}}{\partial T}\frac{dT}{d\tau_D} = 0$$

We use equations (52)-(56), equation (20) for the expression of Φ_z as well as

$$\frac{dT}{d\theta} = (1-2\theta)E(w^2) \quad (57)$$

$$\frac{dT}{d\tau_L} = E\pi(1-p)\left(g^* + b^*\right) \quad (58)$$

$$\frac{dT}{d\tau_E} = E(1-\pi)\left(s^* + b^*\right) \quad (59)$$

$$\frac{dT}{d\tau_D} = E\pi p b^*. \quad (60)$$

obtained from the budget constraint of the government. Rearranging terms, we obtain expressions (35)-(38). From these expressions we can isolate the taxes and obtain the following explicit expressions for the taxes on bequests,

$$\tau_L^R = \frac{u'(x_0^*)E\{\pi(1-p)(g^*+b^*)\} - \pi_0(1-p_0)v'(b_0^{L*})(g_0^*+b_0^*) + \mu\left(\tau_D E\pi p\frac{\partial\tilde{b}^*}{\partial\tau_L} + \tau_E E(1-\pi)\frac{\partial\widetilde{b^*+s^*}}{\partial\tau_L}\right)}{-\mu E(1-p)\pi\frac{\partial\widetilde{b^*+g^*}}{\partial\tau_L}}$$

$$\tau_E^R = \frac{u'(x_0^*)E\{(1-\pi)(s^*+b^*)\} - (1-\pi_0)v'(b_0^{E*})(b_0^*+s_0^*) + \mu\left(\tau_D E\pi p\frac{\partial\tilde{b}^*}{\partial\tau_E} + \tau_L E(1-p)\pi\frac{\partial\widetilde{b^*+g^*}}{\partial\tau_E}\right)}{-\mu E\{(1-\pi)\frac{\partial\widetilde{s^*+b^*}}{\partial\tau_E}\}}$$

$$\tau_D^R = \frac{u'(x_0^*)E\{\pi p b^*\} - \pi_0 p_0 v'(b_0^{D*})b_0^* + \mu\left(\tau_L E\pi(1-p)\frac{\partial\widetilde{b^*+g^*}}{\partial\tau_D} + \tau_E E(1-\pi)\frac{\partial\widetilde{s^*+g^*}}{\partial\tau_D}\right)}{-\mu E\pi p\frac{\partial\tilde{b}^*}{\partial\tau_D}}$$

We finally show how to obtain the optimal tax rates with log utilities and $v(x) = \beta u(x)$ where $\beta \leq 1$. With log utilities, $u'(x) = 1/x$ and $v'(x) = \beta/x$. As we showed in Section 4.1 and Appendix 5.2, in that case, s^* and b^* only depend on T but not on the other fiscal instruments. In addition, g is independent of fiscal instruments: $g_i^* = \bar{g}\,\forall i$. This implies that $\Phi_z = 0$, $\forall z = \{\tau_L, \tau_E, \tau_D\}$.

Replace for these simplifications in FOCs (53)-(56). After some rearrangements, one obtains eq. (39)-(41).