VARIETY, FERTILITY, AND LONG-TERM ECONOMIC GROWTH

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Abstract

This study develops a novel mechanism to explain the long-term economic and demographic evolution from the Malthusian stage to the modern stage. In the model, the progress in human history is characterized by not only technological advances but also the expansion of variety of goods. The technological progress, which enhances productivity, is in favor of population growth. Meanwhile, the growth of variety that expands consumption sets tends to reduce fertility. The change of fertility finally depends on the relative growth rate of these two kinds of innovations. With the help of some hypotheses that correspond to the stylized facts in the history of science and technology, the model predicts an evolutional pattern of technology and fertility that is consistent with unified growth theory.

Keywords Variety · Fertility · Economic growth · Innovations

JEL Classification J11 \cdot J13 \cdot N3

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1 Introduction

Technological progress in human history has resulted in either an expansion in population size or a sustained increase in per capita income. What is the gist? According to unified growth theory, it is the demographic transition starting in the second half of the 19th century that reversed the relationship between population growth and technological progress and finally led the economy out of the Malthusian trap and into the modern growth stage (Galor, 2005; Madsen, Islam, and Tang, 2020). Many theories have been developed to explore the cause of this demographic transition, the key of which is to find a mechanism that reverses the positive relationship between fertility and income growth. In literature, the human capital hypothesis based on "quantity-quality" tradeoff has received more attention from economists (Becker and Lewis, 1973; Galor and Weil, 2000; Galor and Moav, 2002; Lucas, 2002; de la Croix and Doepke, 2003). They argue that it is technological advances in the second half of the 19th century to raise the value of human capital, which turned the seesaw of 'quantity-quality' so that the 'quality' of children won the race and caused a permanent decline in fertility. In contrast to human capital theory, some studies have examined the historical changes in intergenerational income transfer and pointed to the reduction in the relative importance of children's labor wages caused by technological change or capital accumulation in the late 19th century as an economic reason for the demographic transition in the Western World (Blackburn and Cipriani, 2005; Brezis, 2001, 2010). Greenwood and Seshadri (2002) and Strulik and Weisdorf (2008) put more emphasis on the impact of economic structural change on fertility. In the theories, the faster development and expansion of the industrial sector led by technological progress resulting in a relative rise in child-rearing costs is viewed as the inducement for the demographic transition.

What happened in the second half of the 19th century to cause permanent changes in macro and micro demographical behaviors? Although the stylized facts and mechanisms emphasized in the above studies are different, they are identical in the assumption about the nature of technological progress in economy, that is, the progress in technology in human history is only referred to the advancement of production technology, which acts on households' behavior only through the production function or budget constraint. However, the motive of human beings engaging in creative activities is not only to increase productivity to make more quantity but also to seek new goods to consume more variety, which has been particularly evident in the last 150 years. In the history of science and technology, the middle of the 19th century was a watershed. Since then, while the production technology experienced continual changes, product innovation has also become increasingly active (see Fig. 1), and people living in the Western World witnessed a revolutionary change in consumption sets.



Fig. 1 Product innovation in history: 1700–1910. *Note*: Data collected from *An Encyclopedia of The History of Technology* (Ian McNeil, 2002).

The industrial revolution, which began again in the second half of the 19th century, brought not only steel, chemicals, and electricity but also bicycles, cameras, sewing machines, motion pictures, electric lights, running water, sodas, new vehicles, new transportation and travel, new houses and furniture, new kitchens, and many trivial but important inventions that cannot be enumerated. Perhaps the only one comparable to this industrial revolution in human history is the agricultural revolution 10,000 years ago. While promoting human survival and development capacity to a new level, that economic revolution also reshaped humans' material and spiritual world and brought human beings from barbarism to civilization. The second industrial revolution also has a similar significance in human history, which comprehensively rebuilt and expanded human consumption sets and brought human society from agricultural civilization to modern industrial civilization. As stated by McNeil (2002, p. 936), people in Western European countries in the early 19th century could not consume much more variety than those in ancient Rome, and it did not begin to change significantly until the middle of the 19th century.

¹ In contrast to economic historians who often focus on the innovations in production technology, social historians pay more attention to the influence of product diversification on social change. They note that the rush of new goods in the second half of the 19th century not only expanded people's consumption sets but also affected marriage, procreation, and power distribution between the sexes (Vago,

¹ The expansion of consumption sets, especially the feasible sets, is not solely caused by inventions. Some products, such as wallpapers and ceramic tiles, have been existing for a long time but were too expensive to be consumed by ordinary people due to inefficient production techniques. Until the technological innovations during the 19th century reduced the cost and improved the production capacity, these products entered the lives of ordinary people (McNeil, 2002, p. 903). In addition, some old products, such as clothing, have been transformed by new technologies. The advent of modern dyeing techniques and new manufacturing materials, including the rise of the clothing design industry, revolutionized the product attribute of clothes, shoes, and hats at the end of the 19th century. Since then, not only ladies but also women have become the target market for many manufacturing industries (Evans, 2006). The same is true in Pasteur's reinvention of an ancient product, that is, food.

2005).

In this study, we emphasize the role of the expansion of variety on social and economic change. Introducing the growth of variety into a dynamic model, we develop a new theory to explain the demographic transition and long-run economic development. In the model, technological and product innovations are incorporated. Technological innovation results in productivity improvement and directly increases output.² Product innovation generates new goods and directly expands consumption sets. Individuals choose the quantity of consumption, the number of children, and the investment for each child to maximize their lifetime utility subject to given disposal income and variety of goods. Giving productivity, the expansion of variety brings a redistribution of income among expenditures on all kinds of goods and children that tends to reduce the share of expenditure on children. Given variety, the improvement of productivity increases household income, which tends to raise the budget for children. Therefore, to some extent, the change of fertility depends on the relative speed of both types of innovation.

The model fully considers three hypotheses, which correspond to three stylized facts in the history of science and technology. First, different performances existed between technological and product innovations in the long history. Prior to the Scientific Revolution, there had been slight but continuous technological change, epitomized by the slow population growth. Compared with the achievement made in technology during that epoch, humans have made little progress in product innovation (McNeil, 2002, p. 936). Human society remained better at improving production techniques until the 19th century. However, by the second half of the century, product innovation finally came onto the fast track (see Fig. 1). This hypothesis allows the model to predict demographic transition following a period of technological growth.

Second, a change in the role of scientific knowledge on innovations occurred in history. Historians have established that prior to the Second Industrial Revolution, science was not indispensable to technological progress (McClellan III and Dorn, 2006, p. 289). However, assuming that science entered the function of technological innovation overnight is also inadvisable. The timing that science started to become involved in technological innovation can be dated to the Scientific Revolution. Mokyr (2002, p. 33) points out that scientific methods, mentality, and culture established in the Scientific Revolution paved the way for the application of science to technological development. The scientific method of precise measurements and controlled experiments has become the basic quality of skilled craftsmen. The scientific mentality not only breaks people's worship to the supernatural force but also makes knowledge production and technological development get rid of obedience to "authority." The scientific culture, in which science is supposed to serve business and manufacturing, became a common belief among scientists (Mokyr, 2002, p. 35). Famous scientists, such as Newton, had attempted to engage in some applied research, but unfortunately, the scientific knowledge at that time was insufficient to meet the practical technical needs (McClellan III and Dorn, 2006, p. 246). Therefore, in our model, knowledge enters the dynamic equations of technological and product innovations since the Scientific Revolution, but its contribution to innovations in the early stage is insignificant from the modern perspective.

² To prevent unnecessary confusion in definition, technology below is only referred to production technology.

Nevertheless, successive small advances in technology in the early stage are still important for economic development, which pushed west Europe gradually out of the Malthusian trap and diverged the trajectory of development between the East and the West (de la Croix, Doepke and Mokyr, 2018).

Finally, the Scientific Revolution changed the parameters of knowledge production (Mokyr, 2002, p. 31). In the model, society must produce knowledge beyond goods. The definition of knowledge in this paper is nearly close to the propositional knowledge defined by Mokyr (2002, p. 9). Mokyr (2002, p. 31) submits that the Scientific Revolution of the 17th century has epoch-making significance for the production of propositional knowledge. The most well-known is the establishment of the system of modern scientific methods. Before Galileo and Newton, empirical observation and logical reasoning still played dominant roles in knowledge production. However, since the Scientific Revolution, experimental methods and mathematics have become powerful tools for scientists to think about the world around us, which revolutionizes the scientific paradigm. The second one is the change in the way science is communicated. During the Scientific Revolution, scientific knowledge gradually got rid of the private property prevailing in the Middle Ages (Eamon, 1990). The discoverers no longer regarded knowledge as their own secret but recognized its public property and encouraged communication and sharing. Knowledge eventually became public goods, which had revolutionary significance for the improvement of knowledge production efficiency. In addition, the progress in access to knowledge during the scientific revolution cannot be ignored. Mokyr (2002, p. 43-52) discusses in particular the great importance of this achievement to knowledge production. Among his lists, the widespread use of mathematics and the publication of books such as the Encyclopaedia and the Descriptions des arts et metiers provide a common language and tools of visualization for the cross-cultural transfer of knowledge. Thus, the production function of knowledge in the model is discontinuous, and at least one jump is observed when the scientific revolution occurs. However, this kind of jump is the result of long-term evolution and accumulation of knowledge.

On the basis of the above three hypotheses, the model can predict an economic development process, which presents the evolutionary path characterized by unified growth theory (Galor, 2005). Before the Scientific Revolution, science and technology evolved in isolation. Technological advancement, following the path of empirical improvement, slowly crawled, whereas product innovation was in a state of stagnation for a long time. Technological progress increased output and ultimately resulted in an expansion in the population size, rather than a permanent rise in per capita income. The economy was in a typical Malthusian trap. The long-term accumulation of ancient scientific knowledge brought the revolutionary change of scientific paradigm, and science was increasingly involved in the process of technological innovation. Productivity and output continued to expand, but product innovation was still scarce. As a result, the new output was still distributed over the given consumption sets; population growth, which was strongly supported by the growth of output, increased unprecedentedly, whereas growth in per capita income had not been significant by modern standards during this period (Voth, 2003). A critical change began in the middle of the 19th century. On the one hand, science began to intervene in technological innovations like never before. On the other hand, the accumulation of modern science and technological progress provided knowledge and technical ways for continuous product innovation;

new goods, which were never seen in history, emerged endlessly and shocked households' consumption sets constantly. The growing income had finally found somewhere else to go, and all existing goods and children faced competition in the income distribution from newcomers. When income growth fails to keep pace with the expansion of variety, fertility falls while per capita income grows.

The model emphasizes the dynamics of science knowledge accumulation and innovations and their relationship. In the historical narrative, the Scientific Revolution and the Enlightenment have unique positions in the history of the Western World, but they have not been paid enough attention by growth economists. Modern growth theories simplify innovations and define technological advances as a continual function of population (Romer, 1990; Kremer, 1993), which makes the scale effect a key issue in the growth model (Jones, 1999). Based on the simple assumption, an innate scale effect is shown in Galor and Weil (2000), according to which, they would be China and India in the East, rather than the UK and the US in the West, to be the first echelon to face takeoff and achieve demographic transition. Therefore, Mokyr (2002, p. 28) sees that as a theory aimed at explaining when, rather than where economic revolution occurs. The emphasis on knowledge production offers a new approach for resolving the scale effect.

In this study, the unit direct cost of child-rearing is measured by the consumption of one child, which is assumed to be constant. It is distinct from Becker and Lewis (1973) and Galor and Weil (2000), where the input in raising a child is a fraction of parental time that is also taken as unchanged with technological progress. We incorporate the time cost into the model later, but the subsequent quantitative exercise shows that the introduction of time cost has little effect on the qualitative result. The model links the opportunity cost of child-rearing to the consumption opportunity of parents, which is essentially consistent with Galor and Weil (2000) and Strulik and Weisdorf (2008). However, the consumption opportunity in the model is an increasing function of variety. This insight is from Guzman and Weisdorf (2010) who examine the effect of variety on fertility. It captures a stylized feature of human economic development.

The rest of the paper continues as follows. Section 2 establishes a basic model to describe the configuration of the economy and the behavior of representative individuals. Section 3 analyzes the long-term growth path of the dynamic system and the evolution of the economy. Section 4 calibrates the model and takes a quantitative exercise. Section 5 provides some brief discussions about the timing and conditions of economic takeoff and demographic transition. Section 6 concludes the research.

2 Basic Model

2.1 Production of Knowledge and Goods

Two kinds of resources exist in the economy, namely, natural resource Z, such as land, and human resource H. The number of natural resources is fixed over time, but the quantity of human resources is determined by the amount of labor and level of human capital. Let L_t denote the amount of labor at period t and h_t the average human capital. Now, $H_t = h_t L_t$ is the total amount of human resources at period t. All the natural resources and the vast majority of human resources are used to produce a

variety of consumption goods; the rest of the human resources are invested in producing knowledge. Assuming $\kappa (0 \le \kappa < 100)$ percent of human resources are invested in knowledge production, the amount of human resources used to produce goods is $(100 - \kappa)H_t$.

Let K_t represent the stock of knowledge at time t, ΔK_{t+1} is the new knowledge generated during the period from t to t+1. ΔK_{t+1} is a function of existing knowledge stock K_t and the input of human resources κH_t . The return to scale of knowledge production is assumed constant, and the output elasticity of human resources is the same in different regimes. However, when the accumulation of knowledge reaches the critical value \overline{K} , a revolution in knowledge production is triggered. Under the new system, knowledge production becomes more efficient. The new knowledge generated during the period from t to t+1, ΔK_{t+1} , is

$$\Delta K_{t+1} = \begin{cases} \eta_1 K_t^{\xi} \left(\kappa H_t\right)^{1-\xi} & \text{for } K_t \leq \overline{K} \\ \eta_2 K_t^{\xi} \left(\kappa H_t\right)^{1-\xi} & \text{for } K_t > \overline{K} \end{cases}$$
(1)

where $\xi (0 < \xi < 1)$ is the output elasticity of human resources, η_1 and η_2 are the efficiency parameters in different regimes, $0 < \eta_1 < \eta_2$. Let g_K denote the growth rate of knowledge,

$$g_{K,t+1} \equiv \frac{K_{t+1} - K_t}{K_t} = \begin{cases} \eta_1 \left(\kappa H_t / K_t\right)^{1-\xi} & \text{for } K_t \le \overline{K} \\ \eta_2 \left(\kappa H_t / K_t\right)^{1-\xi} & \text{for } K_t > \overline{K} \end{cases}$$
(2)

The amount of variety at period t is I_t . To simplify the model, all variety of goods are assumed to be produced with the same technology A_t . The output of goods $i(i \in (0, I_t])$ at time t, $Y_{i,t}$, is

$$Y_{i,t} = \left(A_t Z_{it}\right)^{\gamma} H_{it}^{1-\gamma}$$
(3)

where $0 < \gamma < 1$, H_{ii} and Z_{ii} respectively represent the quantities of human resources and natural resources used to make goods *i* at time *t*. The total amount of natural resources, $Z \equiv \int_0^{l_i} Z_{ii} di$, is normalized as 1 over time. $A_i Z_{ii}$ refers to "effective resources." We can consider effective resources as intermediate capital goods produced by technology together with natural resources, implying the total amount of effective resources is equal to A_i and increases with time if a technological progress is observed. No property rights over natural resources and technology are found, and the output of each good is distributed equally by human resources. The price of goods *i* at time *t* is p_{ii} . Wage per unit of human resources is

$$w_{t} = \frac{1}{H_{t}} \int_{0}^{I_{t}} p_{it} Y_{it} di$$
(4)

2.2 Preference and Budget Constraints

Considering an overlapping-generation model, it assumes each individual lives for three periods, born in

period *t*-1, working in period *t*, and aging in period *t*+1. The consumption and investment in children are decided by parents. Therefore, each individual's utility is defined over the last two periods. Let $C_{1,t}$ and

 $C_{2,t+1}$ respectively denote the composite consumption in working and aging period. The lifetime utility function of an individual born at time *t*-1, U_t , is

$$U_{t} = \ln C_{1,t} + \beta \ln C_{2,t+1}$$
(5)

where β (0< β <1) is the discount rate. The composite consumption, $C_{1,t}$ or $C_{2,t+1}$, encompasses all kinds of goods on the market. The difference is that only the working individuals need to rear children and derive utility from children. Equations (6) and (7) give the composite consumption, $C_{1,t}$ and $C_{2,t+1}$,

$$C_{1,t} = \left(a \int_0^{l_t} x_{1,it}^{\alpha} di + (1-a) n_t^{\alpha}\right)^{1/\alpha}, 0 < \alpha < 1, 0 < a \le 1$$
(6)

$$C_{2,t+1} = \left(\int_0^{I_t} x_{2,it+1}^{\alpha} di\right)^{1/\alpha}$$
(7)

where $x_{1,it}$ is the consumption of goods *i* of a working individual and $x_{2,it}$ of an aging one. n_i is the number of children raised by a working individual at time *t*, and α is the parameter that determines the substitution elasticity among all variety of goods and children. Parameter *a* measures the relative importance of consumption of goods to the number of children. Given that having children is not necessary for parents, the model allows a = 1. However, considering the decreasing marginal utility of goods and children, the CES function makes sure that parents prefer to choose to have children even when having them is unnecessary, as long as *a* does not equal to 1.

Given that each kind of goods is introduced symmetrically into the utility function, and the production of each good takes the same technology and satisfies the constant return to scale, the consumption amount of each good must be the same, giving $x_{ji,t} = x_{j,t}$, j = 1, 2, for any $i \in (0, I_t]$. Then, the composite consumptions can be simplified as

$$C_{1,t} = \left(aI_{t}x_{1,t}^{\alpha} + (1-a)n_{t}^{\alpha}\right)^{1/\alpha}$$
(8)

$$C_{2,t+1} = I_t^{1/\alpha} x_{2,t+1} \tag{9}$$

and resources will be allocated equally among the production of all kinds of goods. Furthermore, relative price p_{ii} is always equal to 1 for any *i* at any *t*. That is, we can directly aggregate the output or income, consumption or expenditure, such as $Y_i \equiv \int_0^{I_i} Y_{ii} di$. The wage per unit of human resource is

rewritten as

$$w_{t} = \left(1 - \kappa\right)^{1 - \gamma} \left(\frac{A_{t}}{H_{t}}\right)^{\gamma} \tag{6}$$

In the second period of life, that is, the working period, individuals are endowed with one unit of labor, and h_t human capital results from the education investment by their parents in their childhood. They devote all their own human resources to work and receive returns. The income per capita is

$$y_t = w_t h_t = \left(1 - \kappa\right)^{1 - \gamma} \left(\frac{A_t}{H_t}\right)^{\gamma} h_t$$
(11)

The income is allocated to consumption, supporting old parents and raising young children. When individuals grow old, they retire and live on transfer from their children. The cost of raising a child is $\tau + e_r$, where τ represents the expenditure to raise a physically healthy child and is constant, and e_r is the investment in education for each child. In some literature, such as Galor and Weil (2000), τ is measured by the time parents devote to raising a child. The difference between the two settings has different meanings for the dynamic evolution of reproductive cost. If τ is measured by material input, the time needed to realize this part of the reproductive cost will decrease with technological progress, but if measured by time input, constant τ means that the change of technology has no effect on this part of the time input. Nevertheless, an extension of the model in Section 4 shows that this difference in cost setting has no essential effect on the qualitative result. Let *s* denote the intergenerational transfer ratio. The budget constraints of individuals are

$$I_{t}x_{1,t} + (\tau + e_{t})n_{t} = (1 - s)w_{t}h_{t}$$
(12)

$$I_{t+1}x_{2,t+1} = sw_{t+1}h_{t+1}n_t \tag{13}$$

$$(x_{1,t}, x_{2,t+1}, n_t, e_t) > 0, 0 < s < 1$$

Equations (12) and (13) respectively represent the budget of the second and third periods of life. Equation (13) offers one mechanism that associates the utility of an individual with the quantity and quality of their children. Of course, introducing the quantity and quality of children into the utility function of parents through the mechanism is unnecessary. A more intuitive hypothesis is altruism, similar to that in Galor and Weil (2000), which means parents care about the future welfare of children. In any case, parents' utility is a function of children's quantity and quality. Rewriting Equation (13) as $x_{2,t+1} = I_{t+1}^{-1} s w_{t+1} h_{t+1} n_t$ and inserting it into Equation (9), and then substituting Equation (8) and the new form of Equation (9) into utility function (5) yields

$$U_{t} = \frac{1}{\alpha} \ln\left(aI_{t}x_{1,t}^{\alpha} + (1-\alpha)n_{t}^{\alpha}\right) + \beta \ln\left(I_{t+1}^{1-\alpha/\alpha}sw_{t+1}h_{t+1}n_{t}\right)$$
(14)

Obviously, if we ignore variety I, then the mechanism presented in Equation (14) would be consistent with that in Galor and Weil (2000) and Galor and Mova (2002).

2.3 Dynamics of Resources

Individuals in the model are identical. They bear children in the second period of life and die in the last one. The evolution of the labor force is described by the following equation:

$$L_{t+1} = n_t L_t \tag{15}$$

where n_t is the number of children born by each working individual at time *t*. Thus, the growth rate of population at time t+1 is equal to $n_t - 1$. The human capital of workers results from the investment in education by their parents. The average human capital of labor forces at time t+1 is

$$h_{t+1} = e_t \tag{16}$$

And the dynamic of total human resource is

$$H_{t+1} = h_{t+1}L_{t+1} = e_t n_t L_t \tag{17}$$

2.4 Optimization of Individuals

In the framework, bearing and raising children bring parents positive utility; parents are also concerned about their children's future income. In light of Equations (14) and (16), individuals' optimization problem is to choose the consumption level of each good, number of children, and investment in educating children to maximize their lifetime utility, that is,

$$\max_{x_{t},n_{t},e_{t}} \frac{1}{\alpha} \ln \left(a I_{t} x_{1,t}^{\ \alpha} + (1-\alpha) n_{t}^{\ \alpha} \right) + \beta \ln \left(I_{t+1}^{\ 1-\alpha/\alpha} s w_{t+1} e_{t} n_{t} \right)$$
(18)

subject to Equation (12) and $(x_i, n_i, e_i) > 0$.

Solving the above optimization problem, we obtain the static optimal level of education, fertility, and consumption,

$$e_t = \beta \hat{I}_t \tag{19}$$

$$n_t = \frac{\left(1-s\right)y_t}{\left(1+\beta\right)\hat{I}_t} \tag{20}$$

$$x_{1,t} = \frac{b(1-s)y_t}{(1+\beta)\hat{I}_t}$$
(21)

where $b \equiv (a\tau/1-a)^{1/1-\alpha}$, and $\hat{I}_t \equiv \tau + bI_t$, which is defined as "effective variety." Equation (20) suggests that fertility increases with the growth of per capita income y but decreases with the expansion of effective variety \hat{I} . It shows the income effect and reveals the offsetting mechanism against the effect, which is the growth of variety. The reason why the expansion of variety tends to lower fertility is that the expansion of consumption sets increases the expenditure on consumption and causes the reduction of expenditure on birth. The share of expenditure on raising and educating children q_t is

$$q_{t} = \frac{(\tau + e_{t})n_{t}}{(1 - s)y_{t}} = \frac{\tau}{(1 + \beta)\hat{I}_{t}} + \frac{\beta}{1 + \beta}$$
(22)

Obviously, q_t decreases as the variety expands and ultimately converges to $\beta/1 + \beta$ when I_t approaches infinity. That is, individuals are willing to maintain a positive number of children even as the range of goods continues to grow. Similar to the effect on fertility, the expansion of variety has the same effect on any existing consumption of products. It is clear in Equation (21).

Considering the fact that the level of human resources at time t is determined at time t-1, Equation (11) implies that income per capita y_t , given the available human resources formed at the last period, is determined by technology A_t . The change of fertility depends on the relative growth speed of technology to variety. A more straightforward statement on the evolution of fertility is given by Proposition 1.

Proposition 1. The change of fertility depends on the relative growth speed of income per capita to the effective variety at time t. Fertility would rise (fall) if the growth rate of income per capita is greater (less) than that of the effective variety at time t.

In contrast to the effect on fertility, Equation (19) presents the positive effect of variety on education. In the model, the marginal cost of parents investing in children's education is n_t . Therefore, the decline of fertility reduces the marginal cost and thus motivates parents' investment in children's education. The positive effect of variety on education results from the negative effect of variety on fertility presented by Equation (20). Although the model shows an inverse relationship between "quantity and quality", the mechanism that generates this relationship is not entirely the Beckerian "quantity–quality" tradeoff. Note that the dominant force leading to such a substitution relationship is not the increase in the value of human capital brought about by technological progress, but the decline in fertility caused by the expansion of variety. This is quite different from the classical theory of "quantity–quality" tradeoff.

2.5 Technological and Product Innovations

In line with Kremer (1993), Jones and Romer (2010), we view an economy's innovations as functions of its total amount of human resources. The key in the model is the role of scientific knowledge in the long-run progress in human history. Following the insights of Mokyr (1990, 2002) and McClellan III and Dorn

(2006), the technological dynamics take the form with constant return of scale as

$$\Delta A_{t+1} = \begin{cases} \sigma_1 A_t^{\varepsilon} H_t^{1-\varepsilon} & \text{for } K_t \leq \tilde{K} \\ \sigma_2 A_t^{\varepsilon} K_t^{\phi} H_t^{1-\varepsilon-\phi} & \text{for } K_t > \tilde{K} \end{cases}$$
(23)

where $0 < \sigma_1 < \sigma_2$, $0 \le \varepsilon < 1$, $0 < \phi < 1$, ΔA_{r+1} is the new technology generated during the period from t to t-1, and A_t is the stock of technology at time t. $\varepsilon \ge 0$ means the existing technology has a nonnegative effect on producing ΔA_{t+1} , but $\varepsilon < 1$ suggests that it becomes difficult for innovations as technology increases without change in other inputs. This suggestion has been confirmed by the latest research from Bloom et al. (2020) and that is applicable to any regimes. The reason, as Mokyr (1990) points out in the distinction between macro-inventions and micro-inventions, is that creative activity aiming at seeking micro-inventions on the basis of existing macro-inventions may face diminishing returns in the absence of novel macro-inventions. \tilde{K} is the threshold value. When the stock of knowledge is less than the value, the technological progress is dominated by the experiential improvement mechanism. Once the stock of knowledge is greater than \tilde{K} , science begins to appear as a force to promote technological progress. $0 < \phi < 1$ yields a diminishing marginal output for knowledge. ϕ is strictly greater than 0, which suggests knowledge is necessary for innovations in the regime. $0 \le \varepsilon < 1$ and $0 < \phi < 1$ together guarantee that technological growth has a balanced path in any of two regimes. Furthermore, Equation (23) implies that there will be positive technological progress to maintain a slow population growth even if the economy is in the regime of $K_t \leq \tilde{K}$. The growth rate of technology is

$$g_{A,t+1} = \begin{cases} \sigma_1 A_t^{\varepsilon - 1} H_t^{1 - \varepsilon} & \text{for } K_t \le \tilde{K} \\ \sigma_2 A_t^{\varepsilon - 1} K_t^{\phi} H_t^{1 - \varepsilon - \phi} & \text{for } K_t > \tilde{K} \end{cases}$$
(24)

Similar to the dynamics of technology, the evolutionary paths of product innovation differ in the two regimes. Let ΔI_{t+1} denote the numbers of new varieties appearing during the period from t to t+1. In line with McNeil (2002), we directly set $\Delta I_{t+1} = 0$ in the regime of $K_t \leq \tilde{K}$. However, defining the product innovation in the regime of $K_t > \tilde{K}$ is a little roundabout. To ensure that a balanced economic growth path exists, and the equilibrium fertility is equal to 1 in the modern growth stage, the growth rate of effective variety must satisfy Theorem 1.

Theorem 1. In the regime of $K_i > \overline{K}$, there would be $\overline{g}_i^s = \overline{g}_A^s$ if income per capita, technology, and population grow on a balanced path and if $\overline{n} = 1$ in the economy.

PROOF:

See Appendix 1.

In Theorem 1, \overline{g}_i^s and \overline{g}_A^s respectively represent the steady-stable growth rates of effective variety and technology on the balanced path in the regime of $K_t > \overline{K}$, and \overline{n} is the equilibrium fertility. Theorem 1 stipulates that the dynamic of an effective variety must converge to a balanced growth path, and the growth rate on the balanced path must be equal to the steady-stable growth rate of technology. With that in mind, the dynamic equation of variety takes the form

$$\Delta I_{t+1} = \begin{cases} 0 & \text{for } K_t \leq \tilde{K} \\ \varsigma \hat{I}_t^{\psi} K_t^{\theta} H_t^{1-\psi-\theta} & \text{for } K_t > \tilde{K} \end{cases}, \varsigma > 0$$
(25)

Noting that $\hat{I}_t = \tau + bI_t$, Equation (25) gives

$$g_{\hat{i},t+1} = \begin{cases} 0 & \text{for } K_t \leq \tilde{K} \\ b\varsigma \hat{I}_t^{\psi-1} K_t^{\theta} H_t^{1-\psi-\theta} & \text{for } K_t > \tilde{K} \end{cases}$$
(26)

The meaning of parameters θ ($0 < \theta < 1$) and ψ ($0 \le \psi < 1$) in Equation (25) are similar with that of parameters ε and ϕ in Equation (23). Proving that the dynamic of effective variety \hat{I}_t has a balanced growth path, and its steady-stable growth rate is equal to the steady-stable growth rate of technology on the balanced path is easy (see proof of Theorem 3 in Appendix 3). The growth rate of variety, from Equation (25), is

$$g_{I,t+1} = \begin{cases} 0 & \text{for } K_t \leq \tilde{K} \\ \varsigma \hat{I}_t^{\psi} K_t^{\theta} H_t^{1-\psi-\theta} / I_t & \text{for } K_t > \tilde{K} \end{cases}$$
(27)

In the regime of $K_t > \tilde{K}$, the growth rate can be rewritten as $g_{I,t+1} = (1 + \tau/bI_t)g_{i,t+1}$, that is, $g_{I,t+1}$ converged to the growth rate of effective variety $g_{i,t+1}$ with $I_t \to \infty$. Ultimately, g_I converges to the steady-stable growth rate of effective variety \overline{g}_i^S . Therefore, the growth rate of variety defined by Equation (27) converges to a steady-state with time.

3 Dynamic System and Evolution of Economy

3.1 The Dynamic System

Defining $\omega \equiv \beta (1-s)(1-\kappa)^{1-\gamma}/(1+\beta)$, the dynamic equation of total human resources, that is, Equation (17) can be rewritten, in terms of the optimal education and fertility equations (19) and (20), as

$$H_{t+1} = \omega A_t^{\gamma} H_t^{1-\gamma} \tag{28}$$

The evolution of economy is governed by a dynamic system consisting of a series of dynamic equations, which are knowledge production function (1), technological innovation function (23), product innovation function (25), and dynamic equations of human resource (28). The dynamic system can be defined through a series of (K_t, A_t, I_t, H_t) .

Definition 1. Given the initial condition (K_0, A_0, I_0, H_0) , the dynamic system is defined by the sequences of (K_t, A_t, I_t, H_t) from t = 0 to the infinity, such that

a. Households choose consumption x_i , fertility n_i , and education of each child e_i to maximize their lifetime utility defined by Equation (18) subject to the constraint of Equation (12) and $(x_i, n_i, e_i) > 0$.

b. The evolution of total human resource H_t follows dynamic Equation (28).

c. Knowledge accumulation follows dynamic Equation (1).

d. Technology and variety respectively evolve according to dynamic Equations (23) and (25).

In the model, \tilde{K} is the critical value, around which knowledge, technology, and variety follow distinct dynamic paths. The growth paths of fertility and education, which are functions of technology and variety, also change because the stock of knowledge exceeds \tilde{K} , and the mechanism dominating the dynamic of human resources alters accordingly. Consequently, the dynamic system is characterized by two regimes.

In the regime of $K_t \leq \tilde{K}$, technological progress and the dynamic of variety are independent on knowledge. There exists a balanced growth path that characterizes the long-term growth rates of knowledge, technology, variety, population, average human capital, and income per capita. It is presented in Theorem 2.

Theorem 2. Given $K_t \leq \tilde{K}$, the variety is unchanged, and the average human capital is constant at a

given initial level in every period t. The growth rates of knowledge, technology, population, and income per capita converge to a balanced growth path such that the following are true:

a. Knowledge, technology, and population grow at a steady-stable speed, and on the balanced growth path, $\overline{g}_{K}^{M} = \overline{g}_{A}^{M} = \overline{n}^{M} - 1$.

b. Income per capita is constant at a steady state.

PROOF:

See Appendix 2.

where \overline{g}_{K}^{M} , \overline{g}_{A}^{M} , and $\overline{n}^{M} - 1$ respectively denote the steady-stable growth rates of knowledge, technology, and population in the regime of $K_{t} \leq \tilde{K}$, and $\overline{n}^{M} \geq 1$ is satisfied to prevent economic degradation.

Theorem 2 suggests that the dynamic system converges to its balanced growth path, given $K_t \leq \tilde{K}$ and the initial values K_0 , A_0 , I_0 , and H_0 . The optimal fertility of households on the balanced path is \bar{n}^M , and no additional investment is made in the education for children at any time. The growth of human resources results only from the growth of population. Theorem 2 also means that a positive growth in output and population in the regime is possible.

In the second regime of $K_t \leq \tilde{K}$, the evolution of knowledge takes a different path and gradually accounts for the technological progress and expansion of variety. Defining $\hat{H}_{t+1} \equiv H_{t+1}/A_{t+1}$, $\tilde{H}_{t+1} \equiv H_{t+1}/K_{t+1}$, and $\hat{H}_{t+1} \equiv H_{t+1}/\hat{I}_{t+1}$ as stationary variables, the growth rates of knowledge, technology, and variety can be respectively rewritten as

$$g_{K,t+1} = \eta_2 \left(\kappa \tilde{H}_t\right)^{1-\xi} \tag{29}$$

$$g_{A,t+1} = \sigma_2 \tilde{H}_t^{-\phi} \hat{H}_t^{1-\varepsilon}$$
(30)

$$g_{\hat{i},t+1} = \zeta \tilde{H}_t^{-\theta} \hat{H}_t^{1-\psi}$$
(31)

Correspondingly, in terms of Equation (28), the evolution of \hat{H}_{t+1} , \tilde{H}_{t+1} , and \hat{H}_{t+1} can be respectively given by

$$\hat{H}_{t+1} = \omega \frac{\hat{H}_{t}^{1-\gamma}}{1+g_{A,t+1}}$$
(32)

$$\tilde{H}_{t+1} = \omega \frac{\tilde{H}_t}{\left(1 + g_{K,t+1}\right) \hat{H}_t^{\gamma}}$$
(33)

$$\hat{H}_{t+1} = \omega \frac{\hat{H}_{t}}{\left(1 + g_{\hat{I},t+1}\right) \hat{H}_{t}^{\gamma}}$$
(34)

Now, the dynamic system consists of growth rate equations (29)-(31) and dynamics of stationary

variables (32)–(34). The equilibrium and balanced growth path of the dynamic system is described by Theorem 3.

Theorem 3. Given that $K_t > \tilde{K}$ and variety evolves following Equation (27), a balanced growth path exists, such that the following are true:

a. Knowledge, technology, effective variety, human capital, and income per capita grow at a steadystable speed \overline{g}_A^s , which is defined implicitly by the following equation

$$\overline{g}_{A}^{S} = \left(\eta_{2}^{\phi} \kappa^{\phi(1-\xi)} \sigma_{2}^{1-\xi} \left(\frac{\omega}{1+\overline{g}_{A}^{S}}\right)^{(1-\varepsilon)(1-\xi)/\gamma}\right)^{1/1+\phi-\xi}$$
(35)

b. Population growth rate converges to 0, that is, n_t converges to 1.

PROOF:

See Appendix 3.

Theorem 3 describes a world in which the economy continues to grow and the population stagnates. It implies that the total amount of human resources continually grows with the time on the balanced path, but the growth comes only from the continual enhancement of the average human capital. It is the difference between the two regimes. Furthermore, the rise of average human capital beyond technological progress in the regimes contributes to the growth of income per capita. Therefore, the difference between the two regimes is not only reflected in the growth rate of relevant variables but also in the mechanism.

3.2 Evolution of Economy

According to Galor and Weil (2000) and Galor (2005), economic development in human history has experienced three stages: Malthusian stage, post-Malthusian stage, and modern stage or Solow stage. The performances of technological progress, economic growth, and population growth are dissimilar among different stages. We incorporate knowledge and variety in the model. Economic development is jointly determined by the dynamics of knowledge, technology, variety, and human resources. Different from Galor and Weil (2000), the impetus driving economic change results from the evolution of knowledge in the model, and the continual expansion of population size cannot bring about an inevitably economic transition to modern growth.

With Definitions 2, 3, and 4, describing the economic evolution is easy. Initially, it starts out in an economy characterized by the Malthusian economy and eventually moves into the Solow growth stage through a transition characterized by the post-Malthusian economy.

Definition 2. Malthusian economy is an economy characterized by Theorem 2.

Definition 3. Solow economy is an economy characterized by Theorem 3.

Definition 4. Post-Malthusian economy is an economic transition from Malthusian economy to Solow

economy.

Specifically, in the beginning, the stock of knowledge was less than \tilde{K} , and the growth rate of knowledge was maintained at a relatively low level. In this stage, according to Formula (23), technological progress was independent of knowledge and was only a function of human resource. Meanwhile, no change was observed in the variety of goods available for humans. Technological progress converged, from Theorem 2, to a balanced growth path; the steady-stable growth speed was equal to population growth rate $\bar{n}^M - 1$, which means the Malthusian economy in the model allowed for positive technological and population growth; meanwhile, per capita income remained at a given level for long. It is in accordance with the result of one research on the economic performance of the Malthusian epoch from Ashraf and Galor (2011). Human resources grew slowly and were mainly led by population increasing because the average human capital was constant in the regime.

As knowledge accumulated and the stock of knowledge exceeded \tilde{K} , a revolution occurred in knowledge production that altered the way of knowledge production and made knowledge growth switch to a fast track. The link between knowledge and innovative activities gradually increased. Although population and output were still growing in tandem, for the first time in thousands of years, output was likely to win the race. Signs of sustained growth were found in per capita income. Moreover, the consumption sets, which had been tight for thousands of years, were beginning to loosen. This is a transitional phase, corresponding to the post-Malthusian stage described by Galor (2005).

After the transition, economic development entered the Solow stage in which the growth of knowledge, technology, variety, and per capita income all reached an unprecedented speed and finally converged toward a higher level of \overline{g}_{A}^{s} defined by Equation (35); meanwhile, the fertility rate continued to decline and finally stabilized at its replacement level. This is exactly the world described by Theorem 3.

The traits of Malthusian and Solow economies in the model have been presented respectively by Theorems 2 and 3, but the specifics of post-Malthusian economy in the model have not been shown directly. In the next section, a quantitative exercise will be performed for the purpose.

4 Quantitative Analysis

In this section, we simulate the evolution paths of some key variables. Specifically, we focus on the growth rates of technology, variety, and population. To simplify the simulation, we assume that the economy starts on the balanced growth path described by Theorem 2 and evolves on the path until

 $K_t > \tilde{K}$. The quantitative exercise below attempts to depict the whole evolution of the economy from the Malthusian stagnation to Solow growth.

4.1 Calibration

One period in the model is set to be 30 years. The economy is initialized on the Malthusian balanced path described in Theorem 2. The initial values of A, K, and I are normalized to 1. The model is calibrated so that (1) the initial Malthusian economy is consistent with the growth facts described by

Kremer (1993), (2) the Solow economy matches the growth facts describing post-World War II United States, (3) the evolutional patterns of technological growth and fertility correspond to unified growth theory (Galor, 2005), and (4) reaching the modern sustained growth stage from the Knowledge Revolution takes approximately 10 periods (300 years). These criteria give the values of some parameters shown in Table 1.

Parameter	Definition	Value	Comment
$\overline{n}^M - 1$	steady-stable growth rate of population in the Malthusian economy	0.015	Population doubles every 1,300 years, consistent with the evidence from Kremer (1993)
$\overline{n}^{s}-1$	steady-stable growth rate of population in the Solow economy	0	
\overline{g}^{s}	steady-stable growth rate of technology in the Solow economy	0.811	Annual growth rate of technology is equal to 0.02, consistent with de la Croix and Doepke (2003)

 Table 1
 Some parameter values calibrated to match the given criteria

The initial value of human resources H_0 must be calibrated to ensure that the economy starts out on the Malthusian equilibrium growth path, that is, $n_0 = \overline{n}^M$. Given that education is constant, $H_1/H_0 = n_0$ exists in terms of Equation (17). Combined with Equation (28), obtaining $H_0 = A_0 \left(\omega/\overline{n}^M\right)^{1/\gamma}$ is easy.

According to Table 1, \bar{n}^{M} is set to 1.015. To obtain the value of ω , we must calibrate some parameters first. Discount rate β is set to 0.299, which is from de la Croix and Doepke (2003) and implies an interest rate of 4.7% per year. For the relative importance of consumption of goods to children, we use a = 0.5 from Voigtlander and Voth (2013). According to Amaglobeli et al. (2019), the proportion of average pension spending to growth domestic product (GDP) per worker is approximately 30% between 1970 and 2010 in advanced countries. Transfer rate *s* is set to 0.3. We know little about the percentage of human resources invested in knowledge production in the early years, but the data from the Organization for Economic Co-operation and Development (OECD) database show the average percentage of researchers in the government and high education to the total labor force in OECD countries in 2016 is approximately 0.03. Hence, we take $\kappa = 0.03$. Parameter γ in goods production function is set to 0.3, equal to the share of land in Hansen and Prescott (2002). All of these four parameters together give $\omega = 0.806$. Given $A_0 = 1$, and $\bar{n}^M = 1.015$, it yields $H_0 = 0.464$.

According to Theorem 2, $\overline{g}_{\kappa}^{M}$ and \overline{g}_{A}^{M} are equal to 0.015. On the balanced path, $g_{\kappa,1} = g_{A,1} = 0.015$ exists. We can obtain $\eta_1 = 0.015/(\kappa H_0/K_0)^{1-\xi}$ and $\sigma_1 = 0.015/(H_0/A_0)^{1-\varepsilon}$.

Parameters ξ , ε , ϕ , θ , ψ , η_2 , σ_2 and ς jointly determine the transition dynamics and

steady-state values of g_K , g_A , and g_i . These parameters are calibrated to match the growth facts described above. The target steady-state growth rate of technology is equal to 0.811. Together with Theorem 2, we set to the target steady-state growth rate $\overline{g}_K^S = \overline{g}_i^S = \overline{g}_A^S = 0.811$.³ Another growth fact is connected with the transition from stagnation to modern growth. The western world took about 300 years (10 periods in model) to accomplish the transition since the beginning of the Scientific Revolution. The calibration of the parameters must guarantee that the economy accomplishes the transition in 10 periods and satisfies Equation (35).

Given that $\overline{g}_A = 0.811$ and subject to Equation (35), the target steady-state growth is achieved through 10 periods by choosing $\xi = 0.84$, $\varepsilon = 0.05$, $\phi = 0.75$, $\theta = 0.9$, $\psi = 0.05$, $\eta_2 = 3.838$, $\sigma_2 = 0.1$, and $\zeta = 0.005$. The choosing values of ξ , ϕ , and θ suggest whether the production of knowledge or the innovations of technology and variety rely much more on the existing knowledge than the scale of human resources in the past 300 years. The very small values of ε and ψ imply that the spillover effect of existing technology or variety on innovations is little pronounced once the role of knowledge on innovation is considered. It is consistent with the insight of Mokyr (1990) and the finding from Bloom et al. (2020). The transition time from Malthusian stagnation to Solow growth is sensitive to the values of these two parameters. That is, a little large value of ε or ψ , implying a slightly larger spillover effect, leads to a longer time to reach the steady state.

Given $\xi = 0.84$ and $\varepsilon = 0.05$, it yields $\eta_1 = 0.03$, $\sigma_1 = 0.03$. The remaining parameters are mainly related to the preference and budget of households. The value of α determines the magnitude of substitution elasticity $1/(1-\alpha)$. On the basis of the import data, Broda and Weinstein (2006) estimate that the substitution elasticity among commodities in the US is between 1.2 and 22.1. Considering the market shares, the expected elasticity is approximately 5.768, which gives $\alpha = 0.827$.

Haveman and Wolfe (1995) estimate the annual expenditures on children approximately 15% of GDP in the US in 1991. Given that the transfer rate is 0.3, it implies the whole cost of children is approximately 21.4% of the disposal income of a worker, that is, less than the share in Taiwan in 1985, which is one-third (Lai, 2012). In addition, Lino (2012) estimates the expenditures on children by families on the basis of a survey by the US Department of Agriculture and finds that the estimated expenditures on children

³ Exactly measuring and evaluating the advancement of knowledge and expansion of variety are not easy. However, the annual growth rate of 0.02 is in line with some documents. Larsen and von Ins (2010) report that between 1997 and 2016, the average annual growth rate of papers published in SCI journals that is used to measure the increase in scientific knowledge is 2.2%. Of course, scientometrics do some work to evaluate the advancement of science. De Solla Price (1965) estimates that the annual growth rate of scientific knowledge is 7%, which means scientific knowledge doubles every 10 years. Bornmann and Mutz (2015) report that doubling the stock of scientific knowledge only takes nine years and conclude that the growth of science is explosive. It obviously conflicts with the finding of Bloom et al. (2020). Bornmann and Mutz clearly know the defect of the research and already remind readers to pay more attention to the methodological meaning of their research. With respect to the expansion of variety, Broda and Weinstein (2006) calculate the number of varieties of imported goods by the US and find an average increase of 2.5% each year between 1972 and 2001.

ranges 21%–32% of households' total expenditures. Noting the lower limit of q_t given by $\beta/(1+\beta)$, we take the share of expenditure on raising and educating children as approximately 25% when economic growth is on the balanced path. In terms of Equation (22) and $\hat{I}_t = \tau + bI_t$, together with the values of parameter α and β , it yields $\tau = 1.137$, $\hat{I}_0 = 3.237$, and $e_0 = 0.968$. Finally, \tilde{K} is arbitrarily set to 1.061, which means the economy starts the transition from stagnation after four periods of development at the beginning. Table 2 in Appendix 4 reports all the calibrated values of parameters, initial values, and target steady-state values.

4.2 Results

We simulate the evolution of the economy for 20 periods. The economy starts on the Malthusian balanced growth path and experiences a transition to Solow growth. Fig. 2 illustrates the evolution of knowledge, technology, variety, fertility, education, and share of expenditure on raising and educating children. The point at which K_t approaches the threshold value \tilde{K} in the model corresponds to the time of the Scientific Revolution at the end of the 17th century.





In the model, the accumulation of knowledge is crucial for the evolution of the economic system. In the first few periods, knowledge grew slowly on the initial equilibrium path. After reaching the critical value \tilde{K} , a revolutionary breakthrough was made in knowledge production, which led to a jump in the growth rate of knowledge. The growth rate then began to slow down but eventually converged to a higher speed of balanced growth path.

The growth rate of technology began to go up since the stock of knowledge exceeded \tilde{K} . In the early stages after takeoff, technological progress was evident compared with historical periods, but still at a low level by modern standards. It is consistent with Mokyr's assessment of the achievement of technological innovation in Europe between 1500 and 1750 (Mokyr, 1990). Specifically, after four periods (120 years) of development since the Knowledge Revolution, the growth rate of technology achieved approximately 40% a period (approximately 1.13% annually). However, the subsequent technological advances started to accelerate. After a total of nine periods (270 years), the growth rate of technology approximated to its steady-stable level, that is, it reached the level of 0.811 per period or 2%

per year.

The evolution of variety follows a similar trajectory as technology evolves. However, the dynamic of variety goes through a much longer period of low-speed growth. It was not until nearly two centuries (six or seven periods) after the Knowledge Revolution that the growth rate of variety reached 0.4 a period. Since then, the expansion speed of variety has been significantly improved.

Change in fertility, furthermore in population growth $n_t - 1$, has undergone an inverted U-shaped process. Unlike technology and variety, demography responds more quickly to the Knowledge Revolution. In the first few decades after knowledge accumulation reaching the critical value, the population growth presented a trend distinct from that in history. Subsequently, with the continuous improvement of technology, the growth rate of population was rising and finally peaked 180 years (six periods) after the Knowledge Revolution. Then, fertility began to decline and fell to replacement level, that is, n = 1, for a century.

The change in education is quite different from that in previous variables. For nearly 200 years after the Knowledge Revolution, the increase in knowledge did not lead to a significant increase in education. The evident increase in investment in children's education is accompanied by the decline in fertility, just as predicted by the theories based on the mechanism of "quantity–quality" tradeoff. However, note that the model indicates that investment in education continues to rise rapidly even as fertility levels off. It means stabilization in fertility in the model is compatible with a continuous rise in education. This is where the model differs from Galor and Weil (2000) and de la Croix and Doepke (2003) in which education eventually converges to a stable level to maintain a stable fertility. It is most obvious in the quantitative exercise of Lagerlof (2006) for the model of Galor and Weil (2000). However, the dramatic rise in average human capital corresponds to the growth facts. Table 3 reports the change in average years of schooling of the population aged 16–64 from 1870 to 2010, which shows impressive increasing trends in educational attainment. The fact is summarized as one of "New Kaldor Facts" (Jones and Romer, 2010).

Year	Region (no. of countries)		
	World (111)	Advanced (24)	Developing (87)
1870	0.49	1.70	0.12
1910	1.37	3.94	0.47
1950	3.20	6.58	1.93
2010	8.40	11.94	7.72

 Table 3
 Average years of schooling of the population aged 15–64 for selected years

Source: Lee and Lee (2016), Table 5.

Fig. 2 displays the long-term change in the share of parents' spending on their children. The value of

 τ we chose is such that parents in the Malthusian economy might have spent 50% of their disposable income on their children, and the proportion would have lasted until approximately 100 years after the Scientific Revolution. Thereafter, the share might have begun to decline slowly and eventually converged to a level below 25%. The reduction of the share is the result of falling fertility and of technological progress, which has led to an increase in parental income without an increase in unit costs of children. It is quite different from the assumptions of Becker and Lewis (1973) and their successors, such as Galor and Weil (2000), where parents' investment in children is mainly time, the value of which rises with technological advances. A similar assumption will be introduced into the model in the following section. However, the result will suggest that such an extension does not substantially affect the conclusion.

In terms of the performances of technological progress (the main source of income growth in the model), fertility rates and education, the quantitative exercise perfectly simulates the past 600 years of economic change. Before the Scientific Revolution, the economy was dominated by the Malthusian mechanism. No change was observed in the growth rates of technology, fertility, and average human capital. For nearly 200 years after the Scientific Revolution, the growth rates of technology and population were increasing and went hand in hand, which is the typical fact of the post-Malthusian economy (Galor, 2005). However, given that product innovation accelerates, fertility continues to decline with rapid technological progress, whereas the investment in educating children from parents begins to increase significantly. The growth path of population and income begins to depart. Eventually, the economy enters the sustained growth regime.

4.3 Sensitivity Analysis

In the next two subsections, the robustness of the result of the basic model in Section 2 and benchmark calibration above is checked. First, we examine the sensitivity of different calibrations on some parameters to the result. Second, we introduce the time cost of child-rearing into the model to see what the extension means for the result.

In calibration, three modifications involving three parameters exist. First, the value of σ_2 is revised up by 20% to 0.12. Second, the elasticity of existing knowledge to technological advancement, ϕ , is adjusted to 0.8, which is a slight change from the baseline; thus, the population growth rate $n_t - 1$ at the peak period is no more than 0.35 (approximately 1% a year). Last, the efficiency parameter in variety dynamic function, ζ , has been reset to 0.006, a 25% improvement. In addition, the long-term influence of difference in initial values is considered. Specifically, we look at the effect of scale, which is whether an economy with more human resources will have a chance to takeoff earlier in the long run. Noting

 $H_0 = A_0 (\omega/n_0)^{1/\gamma}$, a higher initial level of technology (or effective resources, considering Z = 1) means a larger economy. We adjust A_0 to 1.5, 50% higher than the benchmark. We focus on the changes in the evolutions of technology, variety, and fertility. Fig. 3 shows the result.



Fig. 3 Evolution of the economy: sensitivity analysis. *Note*: The blue solid lines are the benchmark lines; all other parameters beyond those showed in the figure are given as the benchmark case.

Fig. 3 manifests that the behavior patterns of technology, variety, and fertility are scarcely affected by the adjustments in calibration. However, the new calibrations have some quantitative significance on the result in the regime of $K_i > \tilde{K}$. Specifically, the reset has little effect on the dynamic path of technology but an obvious effect on variety, resulting in a significant change in the dynamic of fertility. A greater value of σ_2 or ϕ , both are in favor of the former in the race between technological advancement and variety expansion, leads to a relatively lower growth speed of variety to benchmark case and brings about a higher level of fertility on the time path. Nevertheless, a slight improvement in efficiency in product innovation, which has no effect on technological evolution at all, generates a moderate increase in the growth rate of variety. It also causes a significant drop in fertility and 30 years ahead in the timing of demographic transition. It demonstrates the important role of variety in explaining the timing of demographic transition and economic change.

Fig. 3 illustrates no effect of scale on economic growth. A larger initial stock of human resources does nothing to help an economy escape from the Malthusian trap. By contrast, economies with larger populations in the Malthusian phase face a slower growth process than those with smaller populations after the Knowledge Revolution. The reason is that, in terms of Equation (24), a higher initial level of technology has a side effect on future technological growth. As a result, economies that start out with a larger population scale tend to grow at a lower rate once their economies takeoff. However, a larger initial human resource scale, in terms of Equation (24), is conducive to product innovation during the regime

of $K_t > \tilde{K}$. In the subgraph depicting variety, the line representing $A_0 = 1.5$ is slightly higher than the

others after the fifth period. Due to the combined effects on technological production and product innovation, economies with larger initial human resource sizes have a relatively lower population growth rate after economic takeoff and earlier demographic transition.

4.4 An Extension: Time Cost in Raising Children

In the majority of Q-Q models, the cost of raising and educating a child is measured by parental time. Folbre (2008, p. 113) estimates that the monetary value of parental time to care for children exceeds the cash expenditures. In this section, the time cost is introduced into the model. Let t be the fraction of time parents spend on raising a physically healthy child. Then, constraint Equations (12) and (13) are rewritten as

$$I_{t}x_{1,t} + (\tau + e_{t})n_{t} = (1 - s)(1 - \iota n_{t})w_{t}h_{t}$$
(36)

$$I_{t+1}x_{2,t+1} = s(1 - tn_{t+1})w_{t+1}h_{t+1}n_t$$
(37)

Now, the optimal education investment and fertility of households are

$$e_t = \beta \left(\hat{b}_t I_t + \hat{\tau}_t \right) \tag{38}$$

$$n_t = \frac{(1-s)y_t}{(1+\beta)(\hat{b}_t I_t + \hat{\tau}_t)}$$
(39)

where $\hat{b}_t = (a\hat{\tau}_t/1-a)^{1/1-\alpha}$, and $\hat{\tau}_t = t(1-s)y_t + \tau$ represents the total cost that parents spend to raise children beyond education, which is positively correlated with y_t . Although the incorporation of time cost into the model brings change in optimal education and fertility, the dynamic of total human resources cannot be affected in terms of $H_{t+1} = h_{t+1}L_{t+1} = e_tn_tL_t$. We redefine \hat{I}_t as

$$\hat{I}_t = \hat{b}_t I_t + \hat{\tau}_t \tag{40}$$

The growth of \hat{I}_t still follows the law ruled by Equation (26). The dynamic system is defined by the sequence of (K_t, A_t, I_t, H_t) in the same way. However, the growth rate of variety defined by (27) is changed because of the new definition of \hat{I}_t as Equation (40).

To simulate the dynamic path, we must calibrate ι . Folbre (2018, p. 113) shows parents devote a considerable amount of time to care for their children. For instance, a so-called *active care* takes one parent at least 24.4–38.3 hours a week, approximately 14.5%–22.8% of parental time, for one kid in a

two-child family. However, the care time here actually infers the whole time parents stay with their children, and how much work time is taken up is unclear. De la Croix and Doepke (2003) choose 0.075 as the ratio of care time to parents' endowed work time. It seems to be more acceptable. We take t = 0.075. The other parameters are calibrated as the benchmark case listed in Table 2 (see Appendix 4).



Fig. 4 Evolution of economy: extension case

The simulation result presented in Fig. 4 suggests that the introduction of time cost has no substantial effect on technological growth and fertility change but on variety dynamic. In terms of Equations (27), Equation (40), and the definition of $\hat{\tau}_t$, the time cost strengthens the role of effective variety on product innovation. Therefore, converging to the steady-stable growth rate of 0.811 for variety takes a long time. To approximately achieve the steady-stable growth rate in 10 periods (300 years) after the Scientific Revolution, a very small value (close to zero) of ψ is necessary.

5. Discussion

5.1 Timing and Condition of Takeoff and Great Divergence

Many economists and historians believe that economic takeoff is a long process spanning hundreds of years and the result of a long evolution of history. The rise of the Western world stems not only from its tireless and long-term pursuit of technological innovation (Mokyr, 1990) but also from its institutional advantages formed in history (North and Thomas, 1973). This viewpoint, however, has come in for several criticisms in the last two decades. The so-called "California School" represented by Pomeranz (2000), denies the inevitability of the rise of the Western world and views the economic takeoff as the result of some accidental element. They worked to find a common ground among different parts of the world and to seek support to date the Great Divergence between the East and the West beyond 1800.

However, accidental uncivilized factors do not determine the development difference between the East and the West. The rise of the West was a long creative process that produced elements, which were crucial to the subsequent industrial revolution, but not shared by the East. Recent studies, such as Broadbeery, Guan, and Li (2018) and de la Croix, Doepke, and Mokyr (2018), have re-stressed the long-run process of economic development. In their opinion, Europe, at least in the 17th century, laid the economic, intellectual, and cultural advantages for subsequent development.

This study emphasizes the role of the Scientific Revolution in the economic takeoff. In the model, the accumulation of knowledge drives the transformation of economic growth mechanism. Therefore, the permanent departure from Malthusian equilibrium relies on the occurrence of the Scientific Revolution. The model also shows that population size has nothing to do with the Scientific Revolution and economic takeoff. The reason for this is clear. While a large population is good for the early development of knowledge and technology, higher levels of knowledge and technology also make it more difficult to innovate later. Therefore, the impact of scale difference is mainly reflected in the time distribution of the speed during the process of knowledge production and technological innovation and has no substantial impact on the final results.

The Great Divergence between the East and the West was well established around the time of the Scientific Revolution. In the long race, with population advantage, China was once ahead, but the Chinese Empire did not make a breakthrough in innovating the knowledge production methods (Lin, 1995). In terms of knowledge production and technological innovation, Imperial China only approached the highest level it could achieve step by step and eventually fell into what is called the "high-level equilibrium trap" (Elvin, 1973). Confined in its traditional system of knowledge production, China soon lost out to Europe, which had experienced the Scientific Revolution, in the next season.

Emphasizing the importance of knowledge also lays the groundwork for explaining the economic rise of East Asia in the 20th century. The spread of modern scientific knowledge around the world has made it possible for backward countries to introduce knowledge. Ultimately, development performance depends on countries' willingness and ability to embrace and use new knowledge.

5.2 Difference in the Demographic Transition between France and England

France experienced an irreversible decline in fertility starting in the middle of the18th century (see Weir, 1994), although the fall of growth rate of population came later. The demographic transition in France predated that in England by nearly 80 years; meanwhile, the income per capita of the former had been lower than that of the latter (the long-run economic performance of France and UK see Bolt et al., 2018; the living standard of France vs. UK between the 18th and 19th centuries see Allen, 2001; Sharp and Weisdorf, 2012). Some studies have attempted to explain the phenomenon. An analysis from de la Croix and Perrin (2018) indicates that the rational-choice model based on the Beckerian approach relatively poorly explains the variation of fertility over time in France during the 19th century. Another research by Bree and de la Croix (2019) focuses on the fertility decline in Rouen, France between 1640 and 1792. It also argues that increasing materialism is one of the most important factors to account for the decline of fertility during the period. In the study, Bree and de la Croix clearly point out the role of expansion of

variety in fertility change. As the fashion leader and consumer center of Europe, the French have historically been more successful at innovating products than at innovating production technology. That is, the French have a relatively large ζ . The English, on the other hand, are better at applying scientific knowledge to production than the French (Mokyr, 2002, p. 52). It gives English a relatively large σ_2 . Perhaps the difference between the two kinds of innovation partly accounts for the difference in the fertility transition of French from England.

Moreover, France's population and economy size were larger than Britain's before and after the Scientific Revolution. By 1700, according to Maddison Project Database (version 2018), the population of France was more than 20 million, 2.5 times that of the UK, the GDP 2.1 times that of the UK. Nevertheless, after that time, Britain had a much faster growth. Take population growth as an example. During the 18th century, the population of England increased by 71%, almost twice that of France (Wrigley, 1985). Does France's initial scale contribute to its advantage in product innovation? We have no direct evidence for that. However, the historical relationship between initial size and population growth rate is consistent with the characteristic of effect of scale on the evolution of fertility shown in Fig. 3.

6 Conclusion

United growth theory, represented by Galor and Weil (2000), argues that the demographic transition that took place in the second half of the 19th century played a key role in the transition from stagnation to continual growth. It is the demographic transition, which reverses the positive relationship between population growth and technological progress, that makes the sustained growth of per capita income possible. On the basis of the insight of substitution between the quantity and quality of children from Becker, unified theory develops a model. This model argues that the rising value of human capital resulting from the long-term technological progress eventually motivates parents to make a choice preferring "quality" in the tradeoff, leading to an irreversible fall in fertility. It is the main reason for the demographic transition in western European countries in the 19th century.

In contrast to the unified growth model, some studies have stressed the turn of intergenerational income transfer flow, which is regarded as the force driving the decline of fertility. Some theories also give more attention to the role of change in economic structure brought by technological progress on the demographic transition. Although these theories focus on different mechanisms, the driving forces behind the progress of human society, in the models, are treated as identical ones that are only the improvement and innovation of production technology. The macro performance of these technological progress is the enhancement of the productivity or potential of human society, and the micro effect is to raise the budget income of individuals.

However, the progress in human history is referred to not only enhancement in productivity or production potential but also increase in the variety of goods. It is just that there has been no substantial progress in product innovation aimed at expanding common consumption sets for a long time in history. The advances in production technology mainly power the evolution of human history. The significant change appeared in the second half of the 19th century when the development of modern physics, chemistry, and biology provided enough knowledge for product innovation. During the second Industrial Revolution, the variety of goods expanded at an unprecedented speed, and the new products, which emerged in that era, have largely shaped the modern world.

This study develops a theory in which the progress in human society is manifested in technological and product innovations. The former improves productivity and moves up individuals' budget lines, whereas the latter increases the variety and expands individuals' consumption sets. In the model, technological progress enhances the income that is in favor of consumption of all goods, including "children." However, the expansion of variety tends to lower the spending on each good and child. Change in fertility therefore depends on the relative growth rate of income to variety. Corresponding to the second Industrial Revolution, as long as the growth speed of variety is fast enough (but not necessarily faster than technological innovation), fertility falls.

Although human capital is included in the model, the emphasis on the difference between human capital and physical capital is no sense in this research. The transition from Malthusian growth to Solow growth in the model does not depend on the change in the value of human capital. However, we do not deny the significance of human capital in modern growth.

The model incorporates more historical facts and responds positively to the findings of historians, particularly highlighting the importance of the Scientific Revolution in the growth history. This study explores a real and crucial achievement during the Second Industrial Revolution, that is, diversity of goods, which is barely discussed by economists. We state the relationship between product innovation and modern scientific knowledge in the 19th century. Responses to these historical facts enhance the explanatory power of the theory in the spatial heterogeneity of economic takeoff and eliminates the scale effect in unified growth theory. The theory predicts that although having the largest population and economy, the Chinese empire, which lacked sustained creativity and effective method in scientific discovery, would not have reached the knowledge threshold needed for a scientific revolution (Lin, 1995) and therefore would not have had an inevitable economic takeoff. Nevertheless, the theory leaves a door open for China. The spread of modern scientific knowledge around the world makes economic takeoff possible for backward countries, large or small.

The theory relies on the postulate that the speed of product innovation has been sustained at a relatively high level since the second half of the 19th century, which results in a permanent decline in fertility. Although reliable data to support this hypothesis are lacking, observations of progress in human society since the 19th century lend credence to the hypothesis. The theory also predicts that a faster increase in household income relative to variety of goods would lead to a rise in fertility if product innovation tends to slow down later while technological innovation remains at a high level. However, it is not seen as a flaw in the model; rather, it suggests that the model is more general.

Appendix 1

Proof of Theorem 1

In terms of Equations (11), (15), and (17), we obtain the growth rate of per capita income as follows:

$$g_{y,t+1} \equiv \frac{y_{t+1}}{y_t} - 1 = \left(\frac{e_t}{e_{t-1}}\right)^{1-\gamma} \left(\frac{1+g_{A,t+1}}{n_t}\right)^{\gamma} - 1 = \left(1+g_{\hat{I},t}\right)^{1-\gamma} \left(\frac{1+g_{A,t+1}}{n_t}\right)^{\gamma} - 1$$
(A1)

Obtaining the dynamic of fertility from Equation (20) is easy:

$$\frac{n_{t+1}}{n_t} = \frac{g_{y,t+1}+1}{g_{\hat{i},t+1}+1}$$
(A2)

Substituting Equation (A1) into Equation (A2), we obtain

$$\frac{n_{t+1}}{n_t} = \frac{1+g_{\hat{I},t}}{1+g_{\hat{I},t+1}} \left(\frac{1+g_{A,t+1}}{n_t \left(1+g_{\hat{I},t}\right)}\right)^{\gamma}$$
(A3)

Let the steady-state growth rates of A_i and \hat{I}_i be \overline{g}_A^S and \overline{g}_j^S . Given $\overline{n} = 1$, Equation (A3) yields $\overline{g}_A = \overline{g}_j$ on the balanced growth path.

Appendix 2

Proof of Theorem 2

Under the regime of $K_t \leq \tilde{K}$, effective variety and human capital are respectively constant at the initial level of \hat{I}_0 and h_0 because $g_T = 0$. In addition, knowledge has no effect on technological progress and economic growth. The evolution of economy is jointly governed just by the change in L_t and A_t . Substituting Equation (11) into Equation (20) and defining $z_t = A_t/L_t$, we obtain

$$n_t = \Omega z_t^{\gamma} \tag{A4}$$

where

$$\Omega = \frac{(1-s)((1-\kappa)h_0)^{1-\gamma}}{(1+\beta)\hat{I}_0}$$

Substituting Equation (A4) into Equation (15), it yields $L_{t+1} = \Omega z_t^{\gamma} L_t$. Using z_t and in terms of Equation (24), we obtain the growth rate of technology in the regime of $K_t \leq \tilde{K}$.

$$g_{A,t+1} = \ell z_t^{\varepsilon - 1} \tag{A5}$$

where $\ell = \sigma_1 h_0^{1-\varepsilon}$. In terms of the definition of z_t , the following is easy to obtain

$$g_{z,t+1} \equiv \frac{z_{t+1} - z_t}{z_t} = \frac{1 + g_{A,t+1}}{n_t} - 1$$
(A6)

Substituting Equations (A4) and (A5) into Equation (A6), we have

$$g_{z,t+1} = \frac{1 + \ell z_t^{s-1}}{\Omega z_t^{\gamma}} - 1$$
 (A7)

Taking the derivative of the above equation with respect z_t , we have

$$\frac{dg_{z,t+1}}{dz_t} = -\frac{\left(1 - \varepsilon + \gamma\right)\ell z_t^{\varepsilon - 1} + \gamma}{\Omega z_t^{\gamma + 1}} < 0 \tag{A8}$$

Equation (A8) suggests that $g_{z,t+1}$ is monotonically decreasing with z_t . We see $\lim_{z_t \to 0} g_{z,t+1} = +\infty$ and $\lim_{z_t \to \infty} g_{z,t+1} = -1$ easily. According to the intermediate value theorem, a value of $\overline{z}^M \in (0, +\infty)$ exists, such that $g_{z,t+1} = 0$. In addition, z_t would be increasing (decreasing) with time because $g_{z,t+1} > 0$ if $z_t < \overline{z}^M$ $(z_t > \overline{z}^M)$. Hence, z_t is globally stable and has a unique equilibrium defined by $z_t = \overline{z}^M$.

Correspondingly, in terms of Equations (A4) and (A5), the fertility and growth rate of technology have steady-states whose values are respectively denoted as \overline{n}^M and \overline{g}_A^M . Equation (A6) implies $\overline{g}_A^M = \overline{n}^M - 1$. Moreover, the income per capita would be constant if z_t is involved in the balanced growth path.

In terms of Equation (2), we get

$$g_{K,t+1} = \eta_1 \left(\kappa h_0\right)^{1-\xi} \left(\frac{L_t}{K_t}\right)^{1-\xi}$$
(A9)

Then, the evolution of $g_{K,t}$ is described by

$$\frac{g_{K,t+1}}{g_{K,t}} = \left(\frac{n_{t-1}}{1+g_{K,t}}\right)^{1-\xi}$$
(A10)

Given $n_t = \overline{n}^M$ on its balanced path, $g_{K,t}$ evolves as

$$\frac{g_{K,t+1}}{g_{K,t}} = \left(\frac{\bar{n}^M}{1+g_{K,t}}\right)^{1-\xi}$$
(A11)

Obviously, $g_{K,t}$ would be decreasing (increasing) with time if $g_{K,t} > \overline{n}^M - 1$ ($g_{K,t} < \overline{n}^M - 1$). That is, g_K has a unique steady state defined as \overline{g}_K^M , and $\overline{g}_K^M = \overline{n}^M - 1$ exists.

To sum up, a balanced growth path is found, thereby confirming Theorem 2.

Appendix 3

Proof of Theorem 3

Substituting growth rate equations (29), (30), and (31) into corresponding dynamic equations of stationary variables, namely, Equations (32), (33), and (34), we obtain

$$\hat{H}_{t+1} = \omega \frac{\hat{H}_{t}^{1-\gamma}}{1 + \sigma_2 \tilde{H}_{t}^{-\phi} \hat{H}_{t}^{1-\varepsilon}}$$
(A12)

$$\tilde{H}_{t+1} = \omega \frac{\tilde{H}_{t}}{\left(1 + \eta_2 \left(\kappa \tilde{H}_{t}\right)^{1-\xi}\right) \hat{H}_{t}^{\gamma}}$$
(A13)

$$\hat{H}_{t+1} = \omega \frac{\hat{H}_{t}}{\left(1 + \varsigma \tilde{H}_{t}^{-\theta} \hat{H}_{t}^{1-\psi}\right) \hat{H}_{t}^{\gamma}}$$
(A14)

Let $\hat{H}_{t+1} = \hat{H}_t = \hat{H}^*$ and $\tilde{H}_{t+1} = \tilde{H}_t = \tilde{H}^*$. In terms of Equation (A12), it yields

$$\tilde{H}^* = \left(\frac{\sigma_2 \left(\hat{H}^*\right)^{1-\varepsilon+\gamma}}{\omega - \left(\hat{H}^*\right)^{\gamma}}\right)^{1/\phi}$$
(A15)

In terms of Equation (A13), it yields

$$\tilde{H}^* = \frac{\eta_2^{1/\xi - 1}}{\kappa} \left(\frac{\omega}{\left(\hat{H}^*\right)^{\gamma}} - 1 \right)^{1/1 - \xi}$$
(A16)

Fig. 5 plots the phase diagram of system formed by Equations (A12) and (A13). It shows the transitional dynamics of the system and suggests that a global equilibrium defined by a set of (\hat{H}^E, \tilde{H}^E)

exists. In addition, \hat{H}^{E} is defined by the following implicit function (A17) by combining Equation (A15) with Equation (A16).

$$\begin{pmatrix} \sigma_{2}\left(\hat{H}^{E}\right)^{1-\varepsilon+\gamma} \\ \omega-\left(\hat{H}^{E}\right)^{\gamma} \end{pmatrix}^{1/\phi} = \frac{\eta_{2}^{1/\xi-1}}{\kappa} \left(\frac{\omega}{\left(\hat{H}^{E}\right)^{\gamma}} - 1\right)^{1/1-\xi} \quad (A17)$$

$$\overset{\tilde{H}_{i+1}}{\stackrel{\tilde{H}_{i+1}}{\longrightarrow}} = \tilde{H}_{i} = \tilde{H}^{i} \quad (A17)$$

$$\overset{\tilde{H}_{i+1}}{\stackrel{\tilde{H}_{i+1}}{\longrightarrow}} = \tilde{H}_{i} = \tilde{H}^{i} \quad (A17)$$

Fig. 5 Phase diagram of system formed by \hat{H}_{t+1} and \tilde{H}_{t+1} .

Given the equilibrium denoted by $(\hat{H}^{E}, \tilde{H}^{E})$, together with Equation (A14), it yields

$$\begin{aligned} \hat{H}_{t+1} &= \omega \frac{\hat{H}_{t}}{\left(\hat{H}^{E}\right)^{\gamma} \left(1 + \varsigma \left(\tilde{H}^{E}\right)^{-\theta} \hat{H}_{t}^{1-\psi}\right)} \\ &\frac{d\hat{H}_{t+1}}{d\hat{H}_{t}} = \frac{\omega}{\left(\hat{H}^{E}\right)^{\gamma}} \frac{1 + \psi\varsigma \left(\tilde{H}^{E}\right)^{-\theta} \hat{H}_{t}^{1-\psi}}{\left(1 + \varsigma \left(\tilde{H}^{E}\right)^{-\theta} \hat{H}_{t}^{1-\psi}\right)^{2}} > 0 , \\ &\frac{d^{2}\hat{H}_{t+1}}{d\hat{H}_{t}^{2}} = \frac{\omega\varsigma \left(1 - \psi\right) \hat{H}_{t}^{-\psi}}{\left(\hat{H}^{E}\right)^{\gamma+\theta}} \cdot \frac{\psi - 2 - \psi\varsigma \left(\tilde{H}^{E}\right)^{-\theta} \hat{H}_{t}^{1-\psi}}{\left(1 + \varsigma \left(\tilde{H}^{E}\right)^{-\theta} \hat{H}_{t}^{1-\psi}\right)^{3}} < 0 . \end{aligned}$$

For $d\hat{H}_{t+1}/d\hat{H}_t > 0$, $d^2\hat{H}_{t+1}/d\hat{H}_t^2 < 0$, \hat{H}_{t+1} evidently has a steady state that is denoted by \hat{H}^E (>0).

Together with Equations (A12), (A13), and (A14), at the equilibrium point, we have

$$\left(\tilde{H}^{E}\right)^{-\phi} = \frac{\omega\left(\hat{H}^{E}\right)^{-\gamma} - 1}{\sigma_{2}\left(\hat{H}^{E}\right)^{1-\varepsilon}}$$
(A19)

$$\eta_2 \left(\kappa \tilde{H}^E\right)^{1-\xi} = \frac{\omega}{\left(\hat{H}^E\right)^{\gamma}} - 1 \tag{A20}$$

$$\varsigma \left(\tilde{H}^{E}\right)^{-\theta} \left(\tilde{H}^{E}\right)^{1-\psi} = \frac{\omega}{\left(\hat{H}^{E}\right)^{\gamma}} - 1$$
(A21)

Substituting Equation (A19) into Equation (29), Equation (A20) into Equation (28), and Equation (A21) into Equation (30), it yields the equilibrium growth rates of technology, knowledge, and effective variety.

$$\overline{g}_{K}^{S} = \overline{g}_{\hat{i}}^{S} = \overline{g}_{A}^{S} = \omega \left(\hat{H}^{E}\right)^{-\gamma} - 1 \tag{A22}$$

Rewriting Equation (A22), we can obtain

$$\hat{H}^{E} = \left(\frac{\omega}{\bar{g}_{A}^{S} + 1}\right)^{1/\gamma} \tag{A23}$$

Substituting Equation (A23) into Equation (A17) and rearranging the expression yield

$$\overline{g}_{A}^{S} = \left(\eta_{2}^{\phi} \kappa^{\phi(1-\xi)} \sigma_{2}^{1-\xi} \left(\frac{\omega}{1+\overline{g}_{A}^{S}}\right)^{(1-\varepsilon)(1-\xi)/\gamma}\right)^{1/1+\phi-\xi}$$
(A24)

Noting $h_{t+1} = e_t = \beta \hat{I}_t$, human capital and income per capita evidently grow at the same speed of \overline{g}_A^S on the balanced growth path, and the fertility converges to 1.

Appendix 4

Table 2 Parameter values	Table 2	Parameter	values
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Parameter	Definition	Value
A_0	initial value of technology	1
K_0	initial value of knowledge	1
I_0	initial value of variety	1
n_0	initial value of fertility	1.015
${H_0}$	initial value of human resource	0.464
e_0	initial value of education	0.968
$g_{\scriptscriptstyle A,0}$	initial value of growth rate of technology	0.015
$g_{K,0}$	initial value of growth rate of knowledge	0.015

$g_{I,0}$	initial value of growth rate of variety	0
\overline{g}_A^S	target steady-state value of technology	0.811
\overline{g}_{K}^{S}	target steady-state value of knowledge	0.811
$\overline{g}_{\hat{i}}^{s}$	target steady-state value of effective variety	0.811
α	parameter determining substitution elasticity in utility function	0.827
β	discount factor	0.299
γ	share of natural resource	0.3
к	percentage of human resource investing in knowledge production	0.03
S	transfer rate	0.3
$\eta_{_1}$	efficiency parameter in knowledge production in the regime of $K_t \leq \tilde{K}$	0.03
η_2	efficiency parameter in knowledge production in the regime of $K_t > \tilde{K}$	3.838
$\sigma_{_{1}}$	efficiency parameter in technological dynamic function in the regime of $K_t \leq \tilde{K}$	0.03
$\sigma_{_2}$	efficiency parameter in technological dynamic function in the regime of $K_t > \tilde{K}$	0.1
ς	efficiency parameter in variety dynamic function in the regime of $K_{t} > \tilde{K}$	0.005
ξ	elasticity of existing knowledge in knowledge production	0.84
Е	elasticity of existing technology in technological dynamic function	0.05
ϕ	elasticity of existing knowledge in technological dynamic function	0.75
ψ	elasticity of existing effective variety in variety dynamic function	0.05
θ	elasticity of existing knowledge in variety dynamic function	0.9
τ	raising cost beyond education for each child	1.137
Ñ	threshold value of knowledge accumulation	1.061

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