Upright and Grand Piano Actions Dynamic Performances Assessments using a Multibody Approach

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Abstract

The behavior of the highly dynamic mechanical transmission between the key and the strings of pianos remains insufficiently understood. Called action, this mechanism is essential for the instrument playability and touch. Upright and grand piano actions, although based on similar principles, present quite different behaviors. This work outlines two models, one for each action, that have been carried out using a similar multibody approach with equivalent modeling hypotheses. The models take all the moving bodies into account as well as the intermittent contacts geometry and specific force laws. In addition, experimental validation with high-speed camera have been successfully achieved. Simulations of the models allow, among others, to estimate the maximal playing frequency, to discover the bridle strap and butt spring usefulness in the upright piano, to illustrate the fast repetition capability at halfway keystroke in the grand piano action and to virtually adjust its settings. These results help in understanding the actions functioning and capabilities, and should contribute to a useful tool for piano makers, showing the interest of the multibody modeling approach for demystifying piano actions behavior and performances.

Keywords: Upright and Grand Action, Multibody Modeling, Piano Regulation, Experimental Validation

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1. Introduction

While being a very common instrument, the piano and its mechanical functioning remain poorly understood by most professional pianists. In pianos, the sound is produced by the impact between the hammer and the strings. The resulting string vibrations are amplified by the sounding board and the frame that give the piano its typical sound amplitude. A sophisticated mechanism – called piano *action* – propels the hammer, as a result of the key motion which is controlled by the pianist's finger.

In the upright piano actions, the hammer travels mainly horizontally to hit vertical strings. On the contrary, the grand piano hammer principally moves upwards to hit horizontal strings, and contains, in its present form, an additional body named the repetition lever. Both complex and with very dynamic behavior, these two actions can be suitably modelled through a multibody dynamic approach to understand their functionings and analyze their behaviors.

This paper presents the development, validation and various analyses of two multibody models, one for each action. Compared to the existing literature, our approach: (i) follows the same modeling technique for both actions while being in good agreement with experimental validations; (ii) offers the advantage of matching each real action component with a corresponding physical parameter of the model; (iii) includes the regulation parameters, which allows to directly predict their impact on the actions behavior; (iv) accepts any force or displacement input to predict the resulting action dynamics.

The models parameters have been identified through experimental characterizations with the equivalent Renner® demonstrators of Fig. 1. These two have also been used for high-speed camera validations. Additional simulations aim at demystifying the actions functioning to a certain extent, in particular the bridle strap and butt spring usefulness in the upright piano and the keystroke repetition at halway key stroke in the grand piano. Furthermore, the hammer blow distance, intentionally modified during the simulation, reveals the importance of the action tuning on both mechanisms.

Figure 1: Upright (left) and grand (right) piano action Renner® demonstrators, with their equivalent Computed Aided Design (CAD) replicas. [UCLouvain2020]

The paper structure is the following. First, a *state-of-the-art* of both piano actions models is presented in section 2. Section 3 details the broad lines of the so-called multibody approach and the numerical implementation. Sections 4 and 5 bring forward the description of the upright and grand piano models, along with the experimental characterizations of the main parameters. Section 6 deals with validations via a high-speed camera that have been achieved for the global actions motion and for the hammers rotations. Then, section 7 goes deeper in the understanding and analysis of the actions functionings through specific simulated situations. Finally, section 8 discusses the results and section 9 concludes the paper.

2. Piano action modeling: state-of-the-art

In spite of many acoustical studies on the piano, the scientific community started only recently to tackle a complete dynamic model of its action. Whereas many publications have concentrated on the grand piano, only few have focused on the upright one.

2.1. Upright piano

A first noticeable attempt by Oledzki (1972) [1] presented a dynamic model containing only two masses and a nonlinear spring, based on experimental observations. After that, several publications of Ramin Masoudi, John McPhee and Stephen Birkett (2009) [2] (2014) [3] (2015) [4] led to the most complete vertical action dynamic model so far. Using the graph theory, they modelled the physical interactions between the five main bodies of the action [2]. Their simulations already allowed to explore the model behavior under different key impulses. They also developed a simplified model of felt compression, which takes its hysteretic behavior into account. Additional features (bridle strap and butt spring, flexibility of some parts, new felt contact models) were introduced in [3] to enhance the model fidelity and an experimental validation was presented later on [4].

2.2. Grand piano

Starting in the early 20^{th} century, most studies have concerned the grand piano equipped with the so-called *double escapement*. After the first works on extremely simplified models of Matveev (1937) [5], Dijksterhuis (1965) [6], Pfeiffer (1967) [7] and Topper (1987) [8], VandenBerghe (1995) [9] used conditional states to take the behavior of the five bodies of the action into account. Gillespie (1992) [10] developed a four rigid bodies model: key, whippen, escapement and hammer, linked with kinematic constraints. Afterwards, he added (1996) [11] the repetition lever to get the complete morphology.

Hayashi (1999) [12] improved Matveev's model by adding a third mass that represents the hammer free flight. In 2004, Hirschkorn [13] published his thesis in which the action dynamic behavior is thoroughly analyzed. For the first time, every component of the mechanism is considered independently and with its own physical properties. Following his recommendations, Izadbakhsh (2006) [14] introduces flexibility to the hammer shank using Rayleighs method.

More recently, Thorin (2013) [15] proposed a single degree of freedom model whose simulation is driven with a displacement whilst, until then, only force driven models have been studied in the literature.

In 2017, Bokiau [16] showed the interest of the multibody approach for explaining the dynamic behavior of piano actions, focusing on the modern *double escapement* and two 18^{th} century striking mechanisms [17].

2.3. Research aims

In line with the upper modeling works, we propose to expand the understanding and comparison of the grand and upright piano actions behavior, via the multibody approach. Following the work of Hirschkorn [13], each part of both the upright and grand piano actions is characterized independently, with its own physical parameters (length, mass, inertia, equivalent stiffness, ...), while identifying the parameters via well-targeted experimentation.

First, a novel multibody model of the upright piano action is presented, which complements the studies conducted by Masoudi *et al.* [4]. Simulations on the model and experiments on the Renner® demonstrator of Fig. 1 are investigated. In particular, the influence of the bridle strap and butt spring for a double key strike is analyzed.

Secondly, the grand piano model based on Bokiau's work at UCLouvain [16] is noticeably improved, via the experimental characterization of the internal contacts. The enhanced model shows consistent results with the experiments, in particular for the maximum playing frequency and for the fast repetition at halfway key stroke.

In this paper, we propose two models based on the same approach and validated experimentally. This constitutes a first step to compare different playing characteristics of vertical actions and horizontal grand piano actions, as suggested in [4]. The maximum playing frequency is estimated by simulation and compared to experiments for both actions.

In addition to enhancing the understanding of these two mechanisms, an obvious application would be to exploit these models as a dynamic tuning tool for piano makers. Indeed, the regulation parameters are crucial for the action dynamic behavior. In this paper, the multibody model approach allows showing results of a continuous simulated deregulation of one action setting parameter, both for the upright and grand piano.

Last but not least, thanks to the real-time capabilities of our symbolic models, the latter can be inserted inside a haptic keyboard to reproduce the touch of a piano action, as we proposed in [18]. This would allow, among other things, to virtually switch between different actions on the same haptic keyboard.

3. Multibody dynamic approach

3.1. Model formulation

Multibody dynamic approach enables to predict the motion of any kind of polyarticulated system composed of bodies connected by joints. The equations of motion can be generated via various formalisms like the virtual principle, the Lagrange equations, or directly from the Newton/Euler laws. In our case, by using our Robotran software [19], the multibody model is obtained from a Newton/Euler recursive formalism generated symbolically, involving a relative joint coordinates approach.

More specifically, the *direct dynamic* of a multibody system is the computation of the generalized joint accelerations $\ddot{q}$ for a given configuration $(q,\ \dot{q})$ of the system on which forces and torques are applied. The multibody equations of motion read:

$$M(q,\ \delta)\,\ddot{q} + c(q,\ \dot{q},\ \delta,\ frc,\ trq,\ g) = Q(q,\ \dot{q}) \tag{1}$$

where, for a system containing n joints, $q\,[n*1]$ (resp $\dot{q}\,[n*1]$ and $\ddot{q}\,[n*1]$) are the relative generalized position (resp. velocity and acceleration) coordinates; $M\,[n*\ n]$ is the symmetric generalized mass matrix; $c\,[n\ *\ 1]$ is the nonlinear dynamic vector which contains the gyroscopic, centripetal, Coriolis and gravity terms as well as the contribution of external forces frc and torques trq; $\delta\,[10\,n\ *\ 1]$ gathers the body masses, centers of mass and inertia matrices. The right-hand side $Q\,[n\ *\ 1]$ represents the generalized joint forces and torques, i.e. the contributive effort components in the joints.

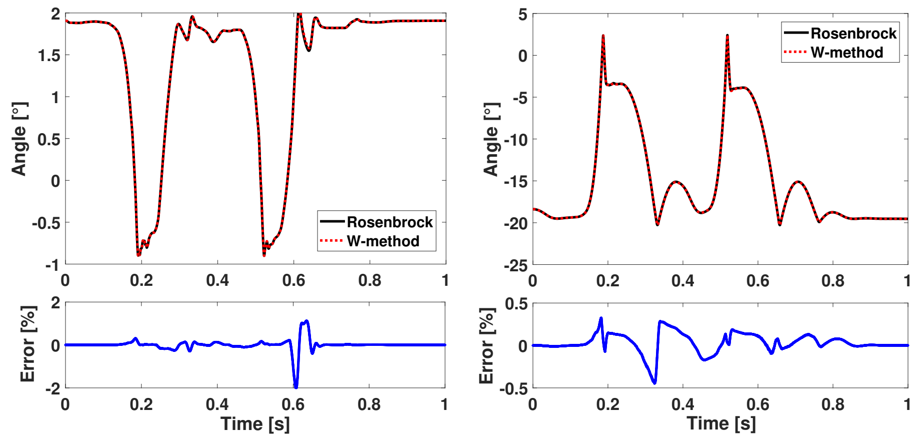
In short, the resolution of the linear system (1) provides the joint accelerations $\ddot{q}$ for each system configuration. Velocities and positions are computed by time integration to find the motion of the system, at the root of the subsequent analyses.

In the same way as the two actions presented in the following sections, models of any piano action morphology can be modeled by a multibody approach.

3.2. Time integration

Regarding the time integrator, among the various candidates, a two stages W-method [20] has been used in this paper for those specific mechanisms, in combination with a stabilization method applied on the acceleration level [21]. This choice results from a compromise between accuracy and time efficiency, the latter being required by the above-mentioned haptic perspective of the work.

The system being very stiff and containing intermittent contacts, a time step of $1.0\mathrm{e}^{-5}$ s turns out to be the best compromise between calculation time and precision. Due to the fact that W-method is an implicit scheme, the numerical jacobian is needed. It is approximated by numerical finite differences and is frozen for 4 time steps. To illustrate the relevance of this choice, a *double blow* is applied on the key (system input) and the hammer motion (output) is compared in Fig. 2.



(a) Key. (b) Hammer.

Figure 2: Numerical time integrators comparison for a double blow on the grand piano action model.

The reference curve is obtained with an error-controlled time integrator, that is based on Rosenbrock methods [22]. W-method with time integrator stabilization gives the same solution as the reference integrator for that highly dynamic motion, while having a fixed step size and thus a constant computing time. The maximum of the error between the integrators is less than 2 %, while the computing time is similar, see Table 1. All the simulations presented in this work use this combined W-method time integration scheme.

Grand piano action	Rosenbrock	W-method
Execution time [s]	**0.72**	**0.87**

Table 1: Computing times of the grand piano action model for one second of simulation.

3.3. Modeling general hypotheses

The first hypothesis of the proposed approach is the rigidity of all wooden bodies, the goal being to deal with models that are both sufficiently faithful to the reality and computationally efficient. This may appear as a strong assumption, as the hammer is flexible and may influence the sound production [23]. However, even without flexibility, results are encouraging for the piano actions at hand, see sections 6 and 7. Furthermore, it has been shown [14] that the hammer shank deformation during its motion does not impact significantly the other components behavior.

A second general assumption claims that the motion is planar and that rotation joints are perfectly revolute without backlash, which appears reasonable for the envisaged study.

4. Upright piano model

A multibody model of the upright piano Renner® action (Fig. 1, left) is developed, following the works of Masoudi [4] and based on the same multibody approach that of the grand piano model of section 5.

4.1. Model components

The model consists of five articulated wooden bodies, moving in a vertical plane, plus several stops and springs as shown in Fig. 3. The five Degrees of Freedom (DoFs) correspond to the rotations of these bodies, i.e. the key (A), the whippen (B), the jack (C), the hammer (D) and the damper (E) in Fig. 3.

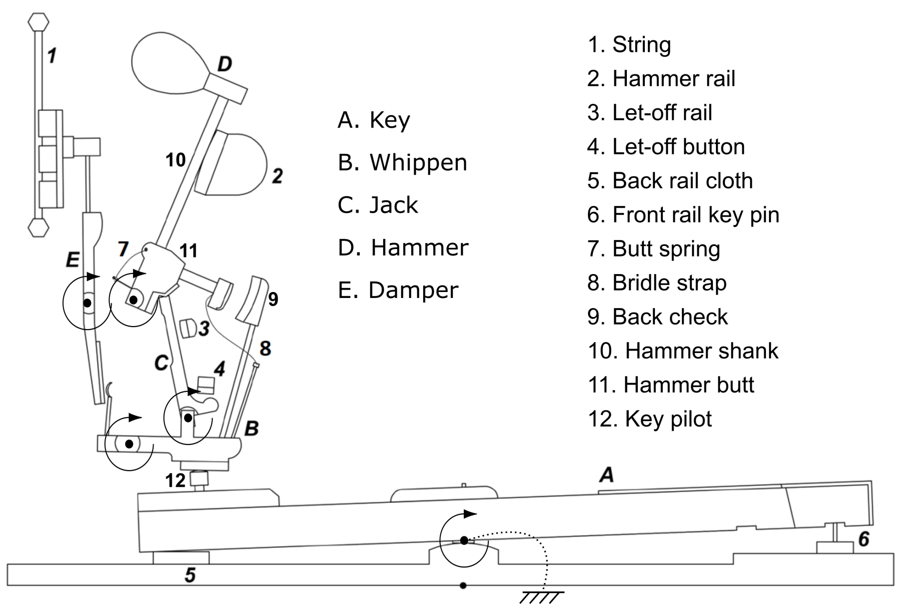


Figure 3: Upright piano action model main components: mobile bodies (resp. other elements) are indicated by letters (resp. numbers). Circle arrows represent the DoFs. [UCLouvain2020]

When the key (A) is pressed, it pushes the action parts upwards. At some point, the jack (C) touches the let-off button (4), rotates and loses contact with the hammer butt (11). This is called the escapement. The hammer (D) will then freely hit the string (1), rebound and finally be caught by the back check (9). A more detailed explanation of the functioning can be found in [2].

4.2. Intermittent contact modeling

The model considers suitable geometries to compute the contact forces between bodies. Their shapes are approximated by circular arcs or straight line segments to find the contact patch location and to compute the penetration x between bodies at the patch center point, as in [3] and [16]. As a piano action is made of wood, leather and felt, each contact possesses its own force-penetration and force-sliding laws characteristics. For the normal contact involving felt, the reaction force F follows the parametrized exponential law [3]:

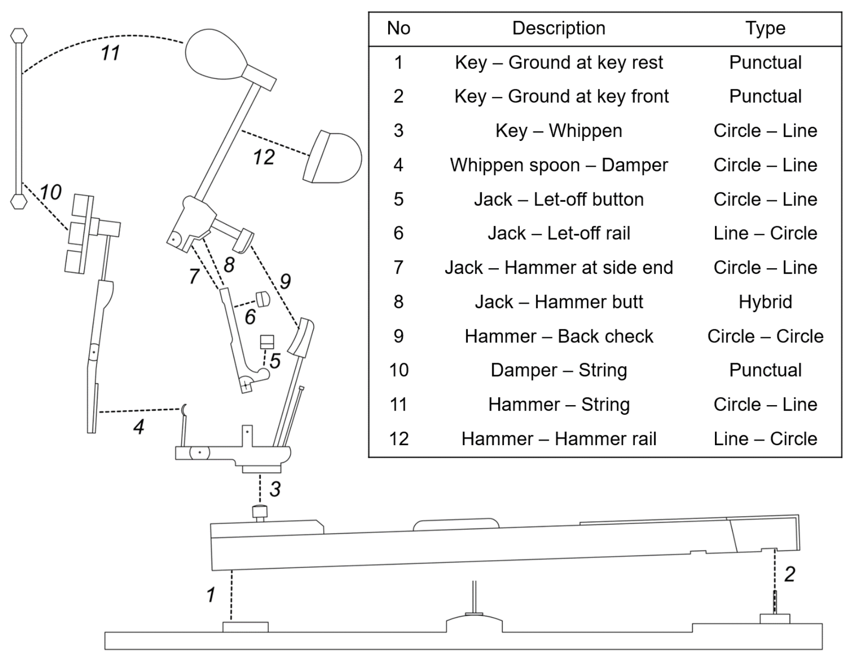
$$F(x) = a\ x\ e^{b\ x} + c\ x \tag{2}$$

where x stands for the normal penetration; a, b and c are contact-specific coefficients. A hysteretic behavior is also taken into account, depending on the penetration velocity $\dot{x}$:

$$F_n(x, \dot{x}) = \begin{cases} F_L(x) & \text{if } \dot{x} \geq 0 \\ F_L(x) + [F_L(x) - F_U(x)] \tanh(\alpha\ \dot{x}) & \text{if } \dot{x} < 0 \end{cases} \tag{3}$$

where F_n is the resulting normal force between the two bodies, F_L (resp. F_U) is the loading (resp. unloading) law that follows (2) and α is a parameter that adjusts the transition between the loading and unloading cases.

The 12 contact points and their type are depicted in Fig. 4.

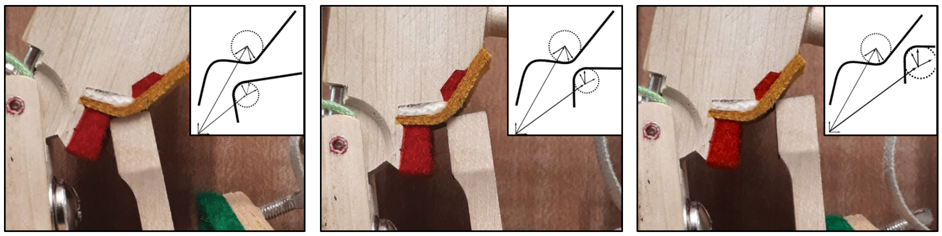


No	Description	Type
1	Key – Ground at key rest	Punctual
2	Key – Ground at key front	Punctual
3	Key – Whippen	Circle – Line
4	Whippen spoon – Damper	Circle – Line
5	Jack – Let-off button	Circle – Line
6	Jack – Let-off rail	Line – Circle
7	Jack – Hammer at side end	Circle – Line
8	Jack – Hammer butt	Hybrid
9	Hammer – Back check	Circle – Circle
10	Damper – String	Punctual
11	Hammer – String	Circle – Line
12	Hammer – Hammer rail	Line – Circle

Figure 4: Upright piano action contacts location and type (inspired by [2]).

The punctual type corresponds to a point-to-point acting force, for which the point of application is fixed on the two involved bodies. For all contacts, a normal force is computed according to (3). Friction laws with saturation effect w.r.t the tangent slip, adapted from Cull [24], have been used for the tangential forces of contacts no. 3, 8, 9 in Fig. 4.

Fig. 5 illustrates the so-called hybrid case of the hammer-jack contact, in which the geometry changes according to the hammer jack relative position. It combines circle-line and circle-circle contact shapes.



(a) Circle-line (b) Circle-circle (c) Line-circle

Figure 5: Hybrid contact of the hammer-jack geometry. [UCLouvain2020]

Regarding the torques appearing in the rotational joints, the constitutive law has been adapted from [13] in which this problem is studied in detail, given the difficulty to take into account the presence of the compressed felt around the turning axle. The torque law reads:

$$T = A\left(\tanh(\omega/\omega_t) + \frac{B_1(\omega/\omega_t)}{1 + B_2(\omega/\omega_t)^4}\right) \tag{4}$$

where T is the resulting torque, A, B_1, and B_2 are constants determined for each joint, ω is the rotational speed, and ω_t is the threshold rotational speed [13].

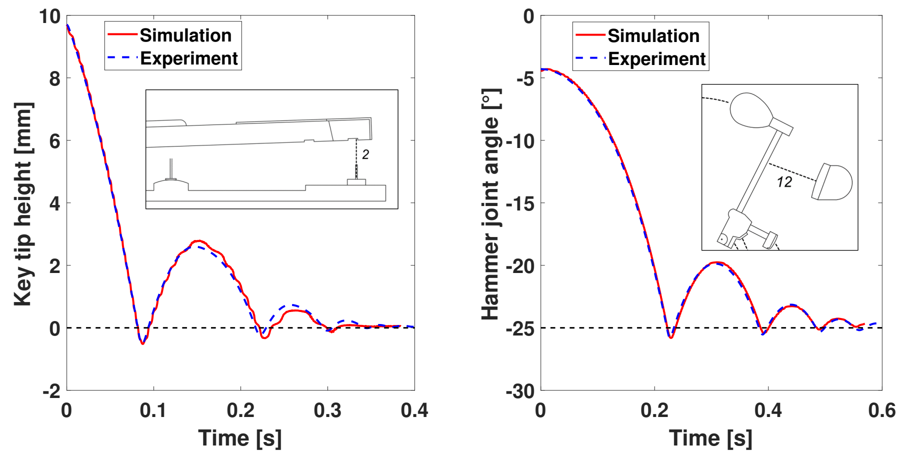
4.3. Experimental characterization of the parameters

In addition to precisely measuring the component parameters (mass, inertia, length, ...) via standard technique, experiments have been conducted to refine the coefficients of the model laws and to provide the model with better-tuned parameters.

The behavior of the key alone has been investigated by discarding the rest of the mechanism. Released from the classic resting position, the key rotates on its pivot due to gravity. It allows to characterize the friction in the joint as well as the contact at the front rail key pin (no. 2 in Fig. 4).

Another experiment has dealt with the contact between the hammer shank and its rail – no. 12 in Fig. 4. Its main purpose is to damp the hammer movement after it has hit the string and it allows a better catching phase, see section 6.

As shown in Fig. 6a for the key free rotation and Fig. 6b for the hammer shank contact with the rail, the experimental tuning has allowed to characterize the model as well as possible with the real Renner® action on Fig. 1.



(a) Key free rotation around its pivot

(b) Hammer rail contact

Figure 6: Upright piano action parameters experimental characterization.

5. Grand piano model

Invented by Erard in the early 19$^{\text{th}}$ century, the *double escapement* action equips most modern grand pianos. The multibody model presented in this paper is based on a previous work of our lab led by Bokiau [16] and follows the modeling approach presented in the previous sections.

5.1. Model description

In Fig. 7, the six DoFs of the grand piano action are represented by circled arrows. Four bodies out of six are articulated on the keyboard frame: the key, the whippen, the hammer and the damper; on the contrary, the jack and the repetition lever rotate with respect to the whippen. Two return springs – no. 7 in Fig. 7 – and several stops condition these bodies motion. The detailed functioning of the action can be easily found in the literature, see [16] and [25] for example.

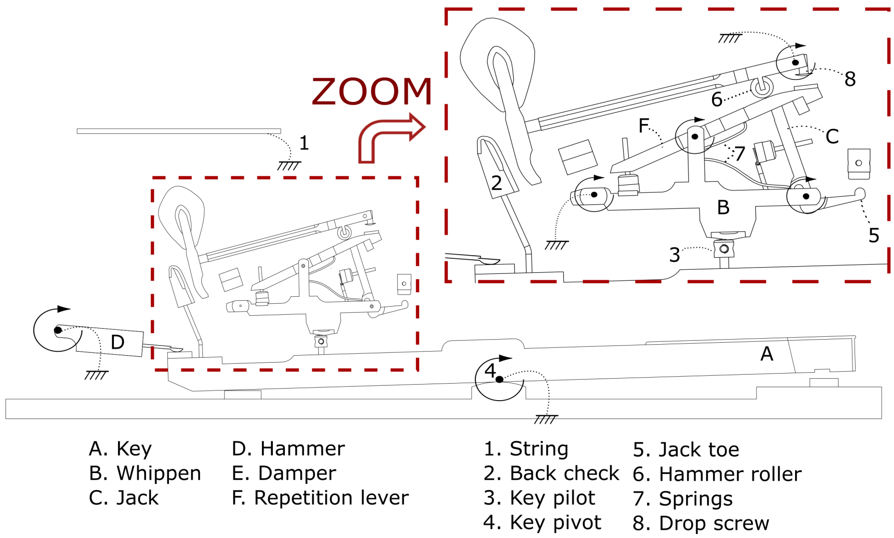
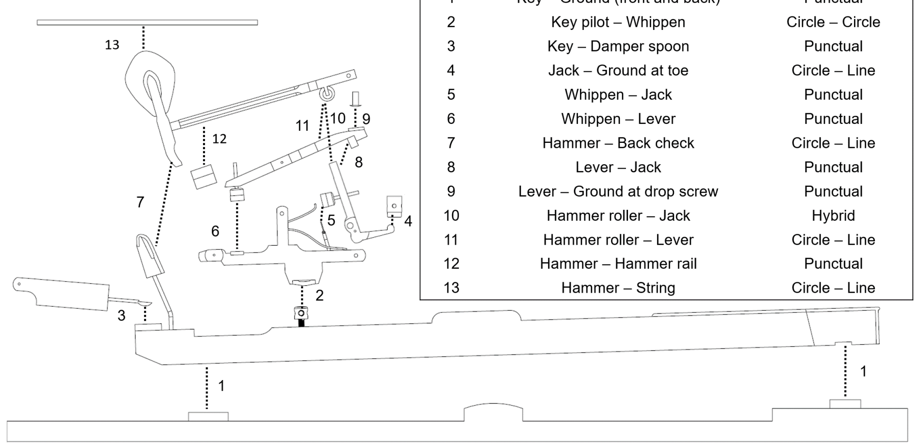


Figure 7: Grand piano action multibody model main components: mobile bodies (resp. other elements) are indicated by letters (resp. numbers). Circle arrows represent the DoFs. [UCLouvain2020]

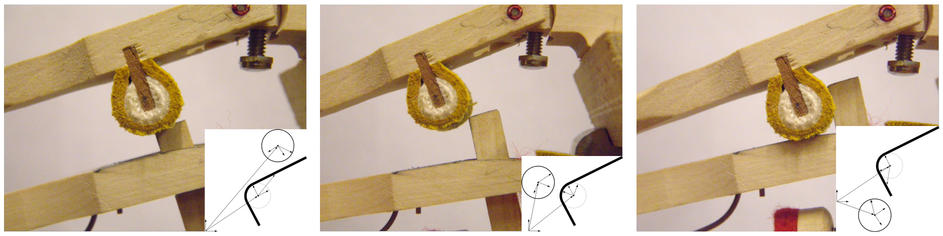
Fig. 8 shows the intermittent contacts occurring in the action. These are computed with the same approach as for the upright piano action of section 4: punctual, circle or line geometry.



No	Description	Type
1	Key – Ground (front and back)	Punctual
2	Key pilot – Whippen	Circle – Circle
3	Key – Damper spoon	Punctual
4	Jack – Ground at toe	Circle – Line
5	Whippen – Jack	Punctual
6	Whippen – Lever	Punctual
7	Hammer – Back check	Circle – Line
8	Lever – Jack	Punctual
9	Lever – Ground at drop screw	Punctual
10	Hammer roller – Jack	Hybrid
11	Hammer roller – Lever	Circle – Line
12	Hammer – Hammer rail	Punctual
13	Hammer – String	Circle – Line

Figure 8: Grand piano contact locations and types. [UCLouvain2020]

The hybrid contact – no. 10 in Fig. 8 – between the hammer roller and the jack mixes circle-circle and circle-line, as illustrated on the pictures Fig. 9.



(a) Circle-line (b) Circle-circle (c) Circle-line

Figure 9: Hammer-jack hybrid contact in grand piano action. [UCLouvain2012] [26]

5.2. *Experimental characterization of the contacts*

Starting from the existing Bokiau's model [27], the morphology is preserved but the physical parameters (masses, inertia, ...) have been experimentally measured to correspond to the Renner® action of Fig. 1.

In addition, 12 contacts out of the 13 of Fig. 8 have been characterized individually with a Universal Mechanical Tester (UMT) Tribolab from Brucker, see Fig. 10. Both normal and tangent forces have been measured with the this tool. Then the model force laws coefficients were fitted to the experimental curves.

Figure 10: UMT from Brucker used for characterization. The contact analyzed here is the Jack-Ground at toe, no. 4 in Fig. 8. [UCLouvain2020]

For illustrating purpose, Fig. 11 shows the normal force for the whippen-key pilot contact, with the fitted curve based on equation (2).

Normal force was measured on the UMT for a penetration up to 1 mm. It is noticeable that the curve of Bokiau's former model differs from the experimental data. In the literature, no *double escapement* piano action model have been characterized so precisely with experimentation for each felt compression.

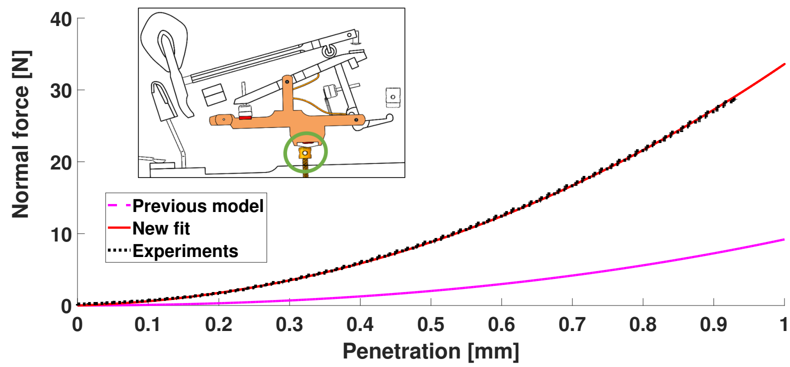


Figure 11: Experimental fitting according to equation (3) for the normal force of the key-whippen contact, no. 2 in Fig. 8.

Furthermore, each contact profile is different, which highlights the need to characterize each contact separately.

Fig. 12 presents the ratio between normal and tangent force – i.e. the friction coefficient – during a relative displacement of the hammer nose and the felt back check, for three different tangent velocities.

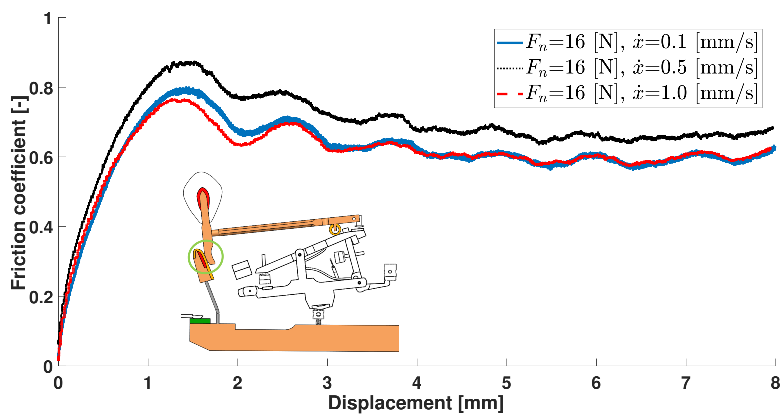


Figure 12: Experimental results for the tangential force of the hammer-back check contact, no. 7 in Fig. 8.

After the movement start at [0;2] mm, the friction coefficient reaches an almost constant value around 0.65. The oscillations between 2 and 8 mm are due to the presence of several striae on the hammer nose.

The velocity does not seem to consistently influence the friction coefficient value. However, the velocity amplitudes are small compared to the ones occurring during the hammer-back check contact in real situations. More experiments are needed to assess precisely the velocity influence on the contact forces, which is out of scope of our current investigations.

5.3. Action regulation

In addition to tuning the string tone pitch, professional piano tuners also regulate the actions from a geometrical point of view. Tuning an action consists in making small adjustments on several elements so that the action operates properly. It is worth noting that a minor deregulation may result in significant differences on the action behavior and on the pianist's touch feeling.

Seven parameters are adjustable on the double escapement mechanism, as pointed out in Fig. 13.

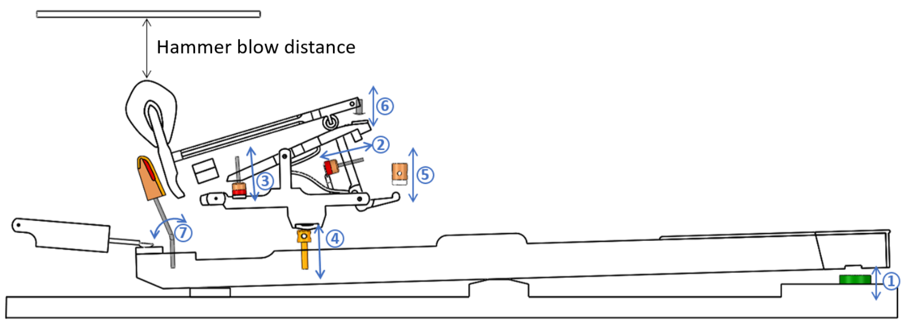


Figure 13: Regulation steps for the double escapement action. [UCLouvain2020]

Moreover, the regulation must follow a rigorous sequential procedure, as proposed by Reblitz [28]:

1. **Key dip:** The keystroke must be 10 mm. This is adjusted by adding tiny sheets of paper under the felt.
2. **Jack engagement:** By screwing its stop, the left vertical edge of the jack must be aligned with the vertical wooden frame of the hammer roller.
3. **Lever height:** Its stop must be positioned in such a way that the distance between the hammer roller and the jack is 0.1 mm at rest.
4. **Hammer blow:** The distance between the hammer top and the string is adjustable by screwing the key pilot. The resting hammer blow distance, see Fig. 13, should be 45 mm.
5. **Let-off:** The jack toe is adjusted so that the hammer escapes when its top is 1.5 mm apart from the string.
6. **Hammer fall:** When the key is pressed down softly, the hammer is not caught by the back check and rests on the repetition lever. Its position is adjustable via the drop screw and must be positioned so that the hammer-string distance is 3 mm after pressing.
7. **Back check:** For a *forte* blow, the hammer must be caught when its top is 16 mm distant from the string. The tilt angle of the back check can be changed by twisting the metal rod.

These steps are implemented in both upright and grand piano Robotran models. A bisection method allows to find precisely and automatically the setting values. Although only the double escapement is presented here, the detailed tuning procedure for the upright action can be found in [2] or in [28].

Using the multibody approach to create a model of the piano action allows to easily tune or deregulate the actions. Results of an automatic tuning is presented in section 7.2 for both upright and grand piano actions.

The first main interest of this model-based approach is to explain and demonstrate the causes and consequences of the action regulation, for example for teaching piano tuners or makers.

A second aspect refers to the dynamical effects occurring in the action that could be precisely captured by the model. For example, during the tuning of the no. 5. **Let-off**, one needs to precisely know the escapement height at the escapment time, which is difficult to estimate in practice while being straightforward in the model, as the escapement event can be precisely determined.

6. Experimental validations

Experimentation conducted on the Renner® demonstrators of Fig. 1 allows to validate the models, using a high-speed camera tracking at 2000 fps. Fig. 14 presents the set-up, while Fig. 15 shows the location of the markers on the double escapement action.

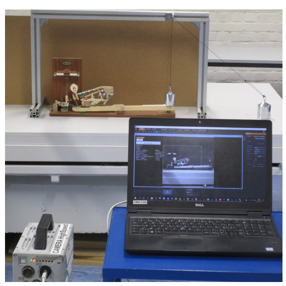


Figure 14: Experiments set-up. [UCLouvain2020]

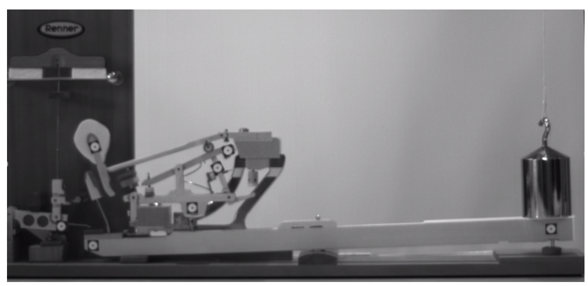
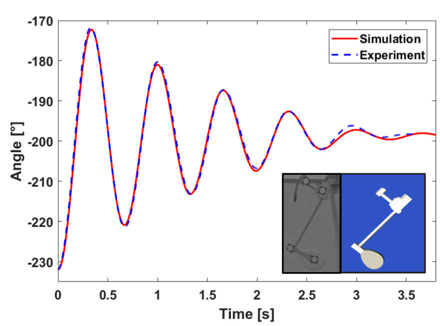
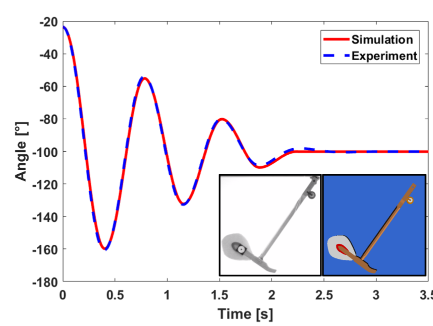


Figure 15: Double escapement action with markers. [UCLouvain2020]

The friction within the hammer rotating joint has been characterized by identifying multibody simulations and experiments, as depicted in Fig. 16a and Fig. 16b for both hammers.



(a) Upright piano action.



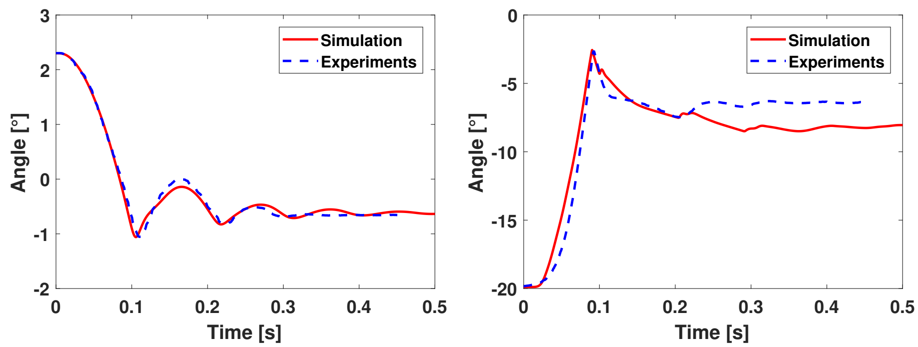
(b) Double escapement action.

Figure 16: Hammer joint identification.

It is worth noting that piano tuners check this joint friction by manually achieving exactly the same experiment. The hammer should make about two round trips before stopping [28]. For instance, in the present cases, the upright hammer joint lacks some friction while the grand one oscillates almost two times.

To validate the whole action behavior, a 1 kg mass positioned at the key tip is used as a known input (Fig. 15 on the right).

Fig. 17 displays the upright action behavior for this experiment.

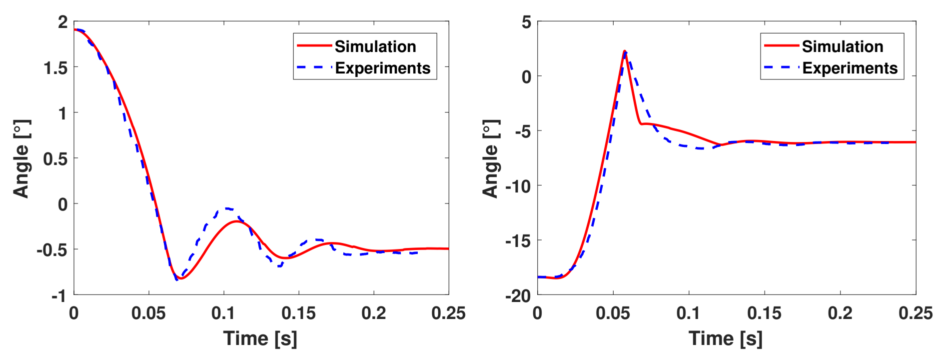


(a) Key. (b) Hammer.

Figure 17: Upright piano action validation for a 1 kg input.

For very close key angles in Fig. 17a, the hammer motion differs slightly in Fig. 17b. During the pre-impact phase [0;0.1] s, the hammer starts moving earlier but hits the string at the same time. After that, the catch phase by the back check results with an improper hammer resting position, but in a quite satisfactory manner given the fact that this catch position is very sensitive in practice and in simulations.

Fig. 18 shows a similar behavior for the double escapement action.



(a) Key. (b) Hammer.

Figure 18: Double escapement action validation for a 1 kg input.

The key angles are a bit different but result in very close hammer motions. Again, the hammer capture by the back check is not perfectly caught between [0.06;0.12] s, but this can be understood, given the remaining model parameters uncertainties, for a mechanism mainly made of wood and felt.

7. Actions advanced simulations

Now that the models have been validated, one can observe their responses to various inputs. In what follows, the maximal playing frequency and an automatic online regulation are performed. Besides, the role of the bridle strap and butt spring of the upright piano is clearly shown for a double blow, while the fast repetition at halway key stroke is illustrated for the grand piano action. All investigations are compared with experiments.

7.1. Maximal frequency

Exploration of the upright piano action model response to a key position input at an increasing frequency reveals that the hammer is unable to hit the string from around 11 Hz, see Fig. 19. Therefore, for the action at hand, this value can be considered as the maximal playability frequency of the model.

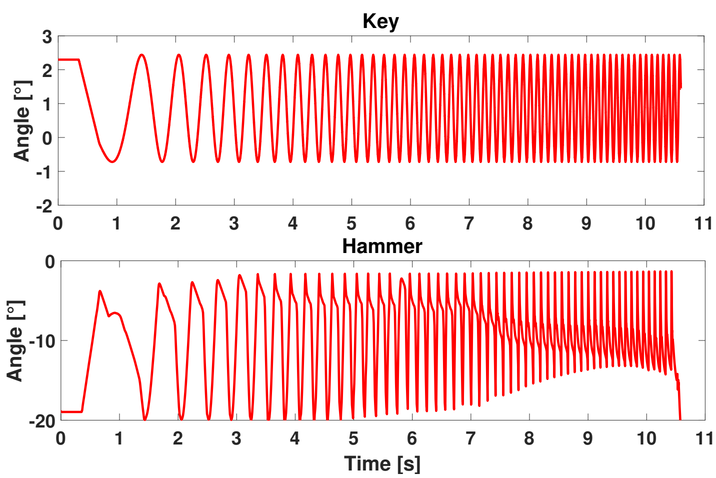


Figure 19: Upright piano action submitted to an increasing frequency sinusoidal input. The frequency increases linearly with the time (at time 10 s, the frequency is 10 Hz).

To validate this result, an external linear actuator (Faulhaber LM1247) applies a sinusoidal position input to the key at a known frequency (experimental set-up shown in Fig. 20). A high-speed camera captures the motion (see Fig. 14).

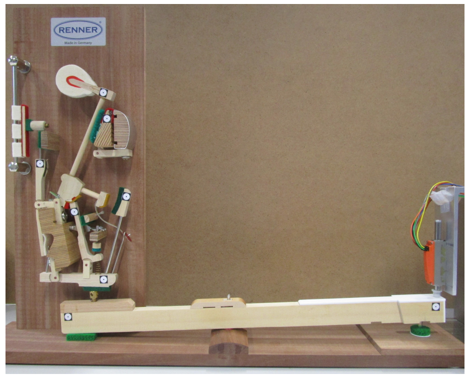


Figure 20: Upright piano action demonstrator submitted to an external actuation (on the right) and whose motion is recorded with a high-speed camera via visual markers, as in Fig. 14.

In Fig. 21, the hammer motion is shown for three different frequencies of key input. At 10 Hz, the action is able to repeat the note. At 13 Hz, the hammer motion becomes uneven but still follows the required frequency. At 14 Hz, the hammer is clearly not repeating correctly to follow the input.

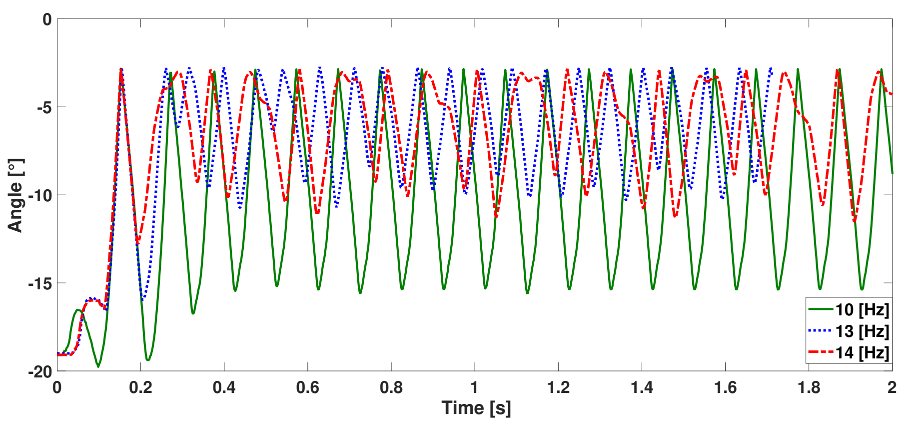


Figure 21: Hammer displacement of the upright piano action demonstrator under external actuation at three sinusoidal inputs (constant frequency).

The same approach has been followed for the grand piano action, as presented in Table 2.

Piano action	**Simulation**	**Experiments**
Upright	11	13
Grand	13	14
Grand : at halfway key stroke	19	$\geqslant 20$

Table 2: Maximum playing Frequencies.

On the grand piano, the first repetition problems appear approximately at the same frequencies as for the upright piano – both in simulation and experiments – if we consider the same input motion, i.e. when a full up-and-down sinusoidal key stroke is used for the input motion.

However, when trying to repeat the key strike from its fully pressed configuration to a halfway stroke position corresponding to the escapement reegagement, the repetition frequency is much higher as espected, around 20 Hz. This value is less – but more realistic – than the one obtained by Bokiau in [26], with its preliminary model. Note that, in experiments, the external actuator is not able to produce a motion above 20 Hz, which explains why the experimental value in Table 2 is above or equal to 20 Hz.

In any case, all other things being equal, the results of Table 2 confirm that the double escapement action can repeat faster than the upright piano one thanks to the repetition lever, which has been a well established fact for pianists since the invention of Érard at the beginning of the 19$^{\text{th}}$ century.

7.2. Automatic online regulation

Action regulation is paramount to ensure its proper functioning and to satisfy the pianist's touch requirements. Thanks to the models, the effects of a *deregulation* can be illustrated by simulation. Although every of the 7 steps of section 5.3 can be performed, only step no. 4 **Hammer blow** is described here for the sake of conciseness. This setting can be continuously adjusted during a simulation with an input key motion sinusoidally moving at a constant frequency of 3 Hz. The simulation ends when the imposed key movement produces too extreme forces due to non-physical penetrations between bodies.

The key pilot is virtually screwed up during the simulation, so that the hammer blow becomes smaller and smaller over time. Fig. 22 presents the results for the upright piano. The whippen is clearly moved upwards, as the hammer blow distance becomes smaller.

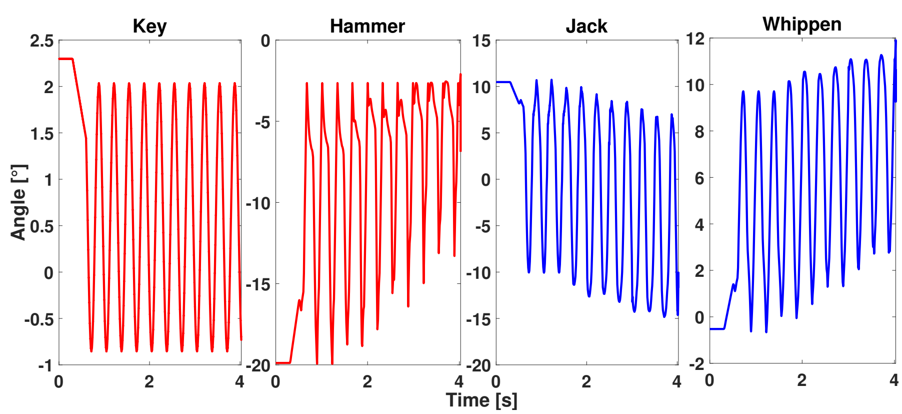


Figure 22: Upright action deregulation for a stable sinusoidal key input. The hammer blow is modified by virtually screwing upwards the key pilot – no. 12 in Fig. 3 – (0.5 mm each 0.5 s).

The same operation on the double escapement action exhibits similar results. Fig. 23 shows the hammer angle hitting the string normally, with a more and more reduced hammer blow.

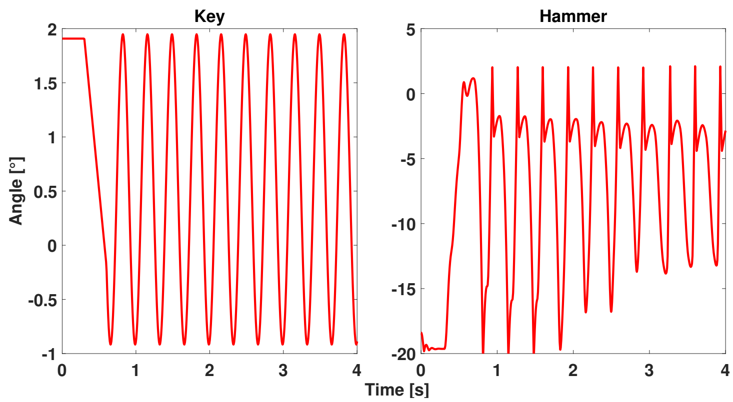


Figure 23: Grand piano action deregulation for a stable sinusoidal key input. The hammer blow – no. 4 in Fig. 13 – is modified by virtually screwing upwards the key pilot – no. 3 in Fig. 7 – (0.5 mm each 0.5 s).

At one point, the jack begins – at around 4 s in Fig. 24 – to penetrate in the lever through the contact no. 8 in Fig. 8 and the resulting forces rise sharply. The simulation ends when this penetration becomes unrealistic.

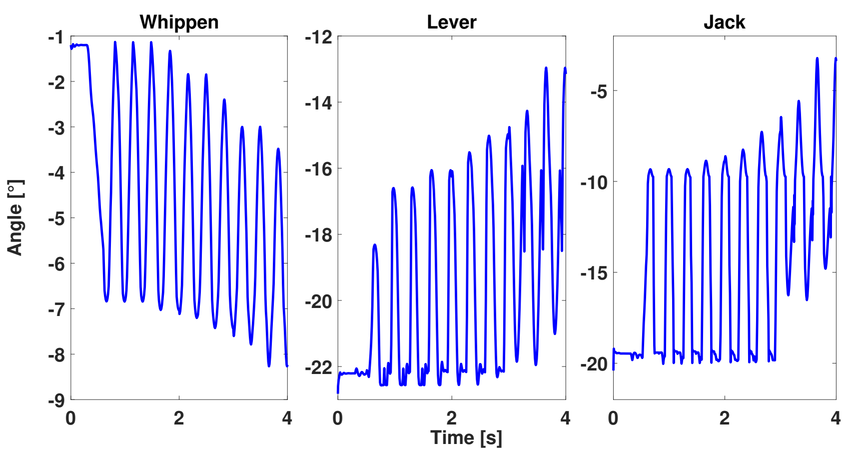


Figure 24: Grand piano action deregulation: motions of the whippen, lever and jack, related to Fig. 23.

7.3. Upright action: bridle strap and butt spring

To clearly highlight the potential of action multibody models, let us illustrate a specific behavior related to the importance of the bridle strap and butt spring of the upright action (parts 7 and 8 in Fig. 3). It is analyzed with a double blow force input, both in simulation (Fig. 25) and in experiments (Fig. 26).

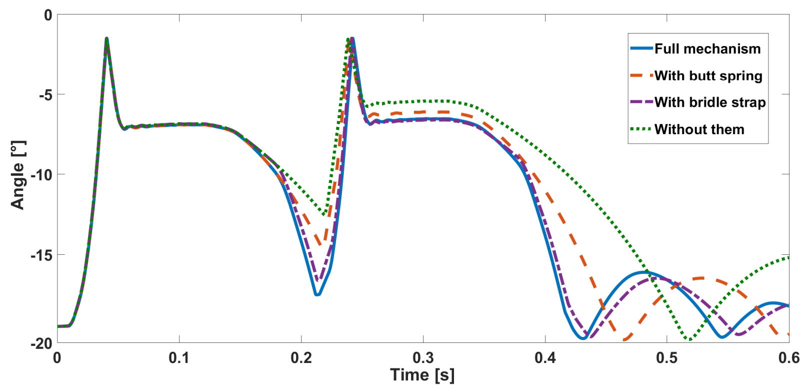


Figure 25: Simulation - Effect of the bridle strap and butt spring on the hammer angle for a double blow applied to the upright piano action.

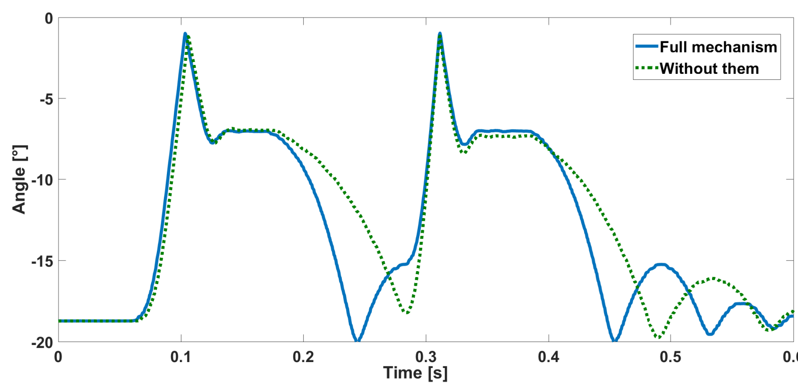


Figure 26: Experiments - Effect of the bridle strap and butt spring on the hammer angle for a double blow applied to the upright piano Renner® demonstrator of Fig. 1.

In Fig. 25, the bridle strap and the butt spring both help the hammer to travel back faster to its rest position so that keystrokes can be repeated faster, as explained in Reblitz [28]. However, the butt spring influence seems to be less than the bridle strap. These results are very similar to those of Masoudi [4], indicating that the models are consistent and are able to highlight and quantify the function of these components which appear difficult to master in practice.

Experimental results, shown in Fig. 26, are similar and demonstrate that the bridle strap and butt spring both help the hammer to travel back to its rest position.

7.4. Double escapement action: repetition at halfway key stroke

As explained in [16], the *double escapement* denomination is actually erroneous as the hammer escapes only once from the jack. In fact, the main difference between the upright and the double escapement action is due to the presence of the repetition lever that was introduced by Erard in the 19$^{\text{th}}$ century. The repetition lever, by pushing the hammer upwards, allows the jack to re-position itself under the hammer roller, even when the key is only slightly released from its fully pressed position. In this way, the note can be repeated faster as the finger has to achieve a shorter stroke. This constitued a breakthrough invention, as it broadened the possibilities to create new compositions with faster note repetitions, especially for the composers of the romantic period.

Using the experimental set-up of Fig. 14, a manually applied repetition at full key stroke is done for the double escapement action demonstrator. The real key motion captured by experimental tracking, visible in Fig. 27, is fitted to serve as input in the multibody model.

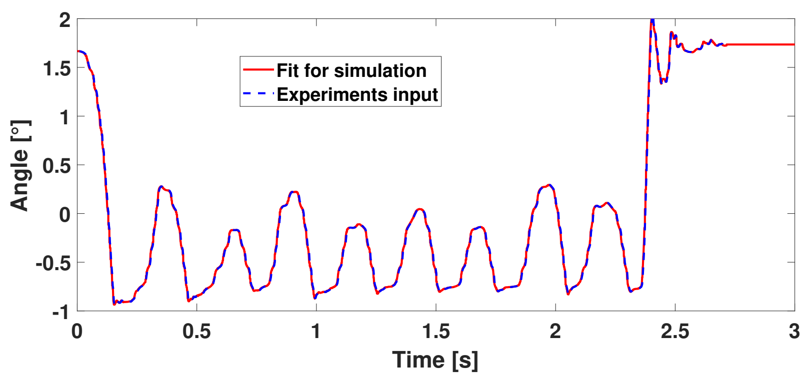


Figure 27: Key angular position for a fast repetition with the double escapement action.

For this particular movement, the hammer behaves in simulation in the same way as the real one by hitting the string at the same times, see Fig. 28.

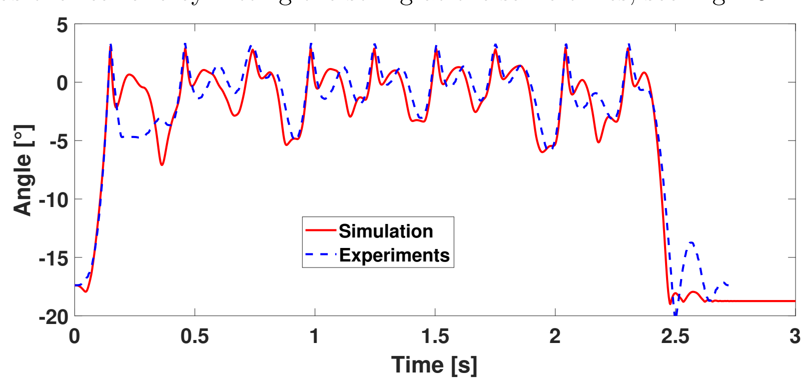


Figure 28: Validation of the double escapement action hammer angular position for the manually applied fast repetition of Fig. 27.

One notices that the post-impact behavior between 0.3 s and 2.4 s differs as the hammer is maintained closer to the string in the experiments. This difference probably comes from the difficulty to capture very precisely the lever spring – no. 7 in Fig. 7 – characteristics. Indeed, the action behavior is very sensitive to that spring stiffness. As explained in [28], in practice, it is hand-adjusted by slightly bending the spring.

Despite theses differences, the results show that the model is able to reproduce the double escapement fast repetition capabilities.

8. Discussion

In the light of the above, both upright and grand piano action models give quite satisfactory and encouraging results, in terms of experimental validation and analysis potentiality. However, despite numerous experimental characterizations and validations, some discrepancies still exist in the observed behaviors.

First, the impact of the flexibility of the wooden action parts – mainly the hammer shank and back check rod – is difficult to encompass, leading us to consider rigid bodies, as a first step. Enhancing the approach by introducing those flexibilities would probably allow a better capture of the action dynamic behavior. This could for instance be achieved as in [27], using finite segments methods or by coupling with other modeling techniques, as already explored in [29] with a much simpler action model.

In this connection, through the developed models, the action behavior has been proven to be very sensitive to its settings, which is hopefully consistent with the piano tuning practice. For example, the repetition lever spring greatly influences the repetition capabilities of the double escapement action. This deformable element is approximated by an equivalent spiral spring in our model and its effect may not be sufficiently well captured, for instance w.r.t. the tangent friction taking place at its end that slides in a wooden slider. However, we have shown that the model-based tuning works well for the setting parameters (section 7.2).

According to the experience of a professional pianist, being able to automatically regulate but also more suprizingly to deregulate the action is of prime interest for the pianists. Indeed, this allows them to pratice on virutally baldy tuned piano to help to improve their technics.

As discussed by Masoudi [4], the type of input provided to the model influences the simulation output, especially in terms of timing accuracy. With our multibody modeling approach, both motion-driven or force profiles input can be applied to the key so that we could easily investigate the effect of using one or another.

9. Conclusions

The action plays a crucial role in a piano as it is responsible for making the link between the pianist's finger and the hammer-string impact that produces the sound. As discussed in Masoudi's conclusion [4], the playing characteristics of the grand and upright pianos show significant differences that have never been tackled with a common dynamic approach in the literature until now.

In this work, two distinct multibody models of upright and grand piano actions have been carried out following the same modeling approach and based on a series of experimental identifications on two Renner® demonstrators. Both actions have shown a clear consistency with high-speed camera validations. Furthermore, the main models parameters have been identified through experimental characterization. Simulations have shown the dynamic potentialities of the multibody models, in particular for assessing the fast key strike repetition for the grand piano and, for instance, the bridle strap and butt spring role in the upright action.

An automatic procedure for the action regulation followed by technicians is proposed, showing the importance of the settings on the action behavior. In this connection, it appears that the behavior of some flexible parts should be modeled in a more refined way in the future.

Having such models at our disposal offers various perspectives. First, from a strictly educational point of view, they could help the piano players to better understand the differences between upright and grand piano actions, as it appears that most of them are not well acquainted with this mechanical transmission between their finger and the strings. Secondly, a multibody modeling tool may be used by piano makers to visualize in real-time and observe the *dynamic* influence of the adjustment of such or such parameter.

Last but not least, as far as we are concerned, we are planning to insert these models inside a haptic keyboard – thanks to the real-time capabilities of our symbolic approach – to reproduce the touch of a piano action, as we already proposed in [30]. This would allow, among other things, to virtually switch between different actions on the same haptic keyboard.

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