Analysis of Elliptical Aperture Metasurface Antennas

M. Bodehou[1], S. Hubert[1], H. Ali Kayani[1], C. Craeye[1], and I. Huynen[1]

[1]Universite Catholique de Louvain: ICTEAM, Place du Levant, 3, 1348, Louvain-la-Neuve, Belgium
husnain.kayani@uclouvain.be

Abstract – **A Method of Moments (MoM) based analysis is presented for elliptical aperture metasurface (MTS) antennas. The MTS is represented as a sheet transition impedance boundary condition incorporated in a transmission line which accounts for the substrate interaction with the patches layer. The algorithm uses the Fourier-Bessel basis functions (FBBFs) and is inspired from the circular domain analysis. The computation time is then comparable to that of an equivalent size circular aperture MTS. Numerical results obtained with the proposed method are compared with the full wave Contour-FFT (CFFT) based MoM simulation of a MTS implemented with sub-wavelength printed patches.**

I. Introduction

Metasurfaces are thin metamaterials designed to manipulate space or surface waves propagation [1]. These surfaces are used in many applications, covering the microwave [2], the TeraHertz [3] and the optical bands [4]. The present paper focuses on antenna applications. The flatness of MTS antennas makes them very attractive for many applications, since they can be easily incorporated on the platform [5]. MTS antennas are typically composed of dense sub-wavelength scatterers, which are excited by a surface wave launcher placed inside the substrate. The scatterers implement a designed surface impedance, which represents the ratio between the average electric and magnetic field in each unit cell of the surface [6]. The average field, stems for the fundamental mode of the local Floquet decomposition of the fields, with the local periodicity assumption. From a simulation point of view, it is simpler to analyze the MTS IBC (meaning the homogenized MTS), rather than the physical patches, since the current on the IBC is smooth and can then be described with much fewer basis functions. MTSs are usually designed over a circular aperture. This suggests the idea of analyzing the current distribution with entire-domain basis functions, namely defined on the whole circular domain [7]. Recently the Fourier-Bessel basis functions (FBBF) have been proposed, owing to their orthogonality on a disk and their non-admissible edge currents [8]. However, when the feed or the MTS illumination is not azimuthaly symmetric, the circular aperture can lead to a low aperture efficiency. An elliptical aperture may then be in some cases preferable [9]. The present paper extends the circular domain MTS analysis with Fourier-Bessel basis functions, to the elliptical domain MTSs analysis, based on the method detailed in [10]. We also provide a comparison based on the Contour-FFT (CFFT) analysis of an actual structure made of square patches implementing the MTS IBC.

II. MoM

The Fourier-Bessel basis functions (FBBFs) are traditionally defined over a circular domain as follows [8]:

$$R_{m,n}\left(\rho',\phi'\right)=F_{m,n}\left(\rho'\right)e^{-jn\phi'}=J_n(\lambda_n^m\rho')e^{-jn\phi'} \tag{1}$$

where ρ' and ϕ' are respectively the radial and azimuthal coordinates, and λ_n^m is the m-th positive zero of the n-th order Bessel function. The extension, to the elliptical domain of semi-major axis a and semi-minor axis b is obtained after stretching the FBBFs [10]. This leads to the mapping $\rho'=\sqrt{(x/a)^2+(yp/a)^2}$, $\tan(\phi')=yp/x$, where $p=a/b$ is the stretching factor, and (x,y) represents the Cartesian coordinates of a given point on the elliptical domain MTS.

The MTS is modeled as a sheet transition IBC since this formulation has been proven to be the most stable one [11]. The matrix impedance can be written as: $Z=Z_G-Z_{IBC}$, where Z_G is the substrate matrix contribution, and

978-1-5386-4702-8/18/$31.00 ©2018 IEEE

is the one obtained in a classical MoM for radiation problems. Z_{IBC} is the sheet transition matrix contribution. Since the FBBFs present a closed form Fourier transform and a small bandwidth (in practice, lower than $15k_0$, where k_0 is the free-space wavenumber), it is more convenient to compute the substrate contribution in the spectral domain. For circular domain analysis, each 2-D integral appearing in Z_G can be computed in closed form along the azimuthal spectral coordinates, which allows a fast computation of the integrals since the obtained matrix is also very sparse [8]. This is no longer true, after stretching the FBBFs. In this case, each integral involves a stretched testing function, a stretched basis function, and the spectral relevant Green's function. To compute these integrals, it is more convenient to report the stretching operation to the Green's function, in order to keep a "virtual no-stretched FBBFs". Then the integrals can be efficiently evaluated after decomposing the stretched Green's function into Fourier harmonics as follows [10]:

$$\tilde{G}\left(\sqrt{(k_x')^2 + (pk_y')^2}\right) \approx \sum_{q=-Q}^{Q} \tilde{G}_q(k_\rho^{'})e^{jq\alpha^{'}} \tag{2}$$

where $k_x^{'} = k_x$, $k_y^{'} = k_y/p$ and (k_x, k_y) are the spectral variables in a Cartesian system of coordinates. Each term of the sum involved in the integration can then be evaluated as in the case of circular domain analysis. It has been observed that for practical values of p, the sum converges very fast for the non-singular part of the Green's function ($Q = 5$ is sufficient), and requires about 15 harmonics for the singular part of the Green's function. The matrix is still sparse, but less sparser than the one obtained in the circular domain analysis. The same procedure can be used for the sheet transition matrix. In this case, it is more convenient to project the stretched sheet transition IBC into FBBFs since this projection can be computed very fast and leads to an integration of 3 Bessel functions defined over a unit disk. The latter can be tabulated, which accelerates the integration process. It is important to note that the stretched IBC requires in practice, the same number of basis functions projection than the no-stretched one. This means that the IBC matrix can be computed as fast as the one obtained for a circular domain with an equivalent radius. This is a great advantage since in an optimization process, the IBC matrix is the only one which needs to be calculated several times.

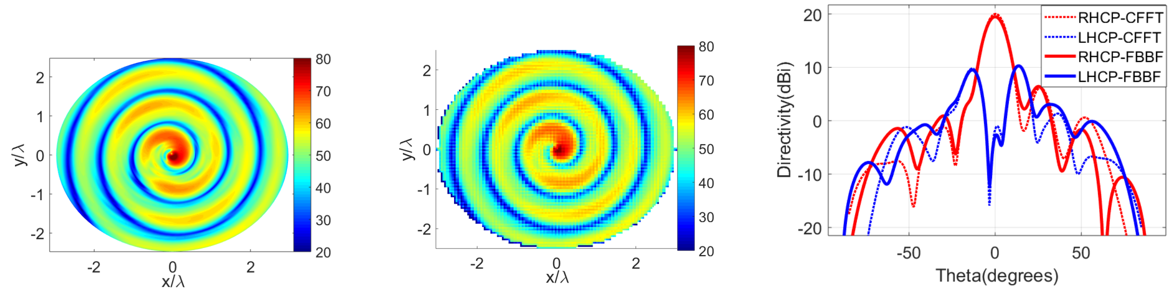
III. Results

As an example, we present the analysis of a scalar MTS designed to radiate a broadside circularly polarized beam. The impressed reactance modulation has been designed as proposed in [5]. The antenna aperture is an ellipse with 3λ semi-major axis and 2.5λ semi-minor axis. The MTS IBC has been implemented with square patches and is excited with a centered dipole. The obtained printed structure is analyzed using the MoM iterative based Contour-FFT (CFFT) method presented in [12], [13] with 128812 RWG basis functions. The CFFT [14] is a technique that allows the use of the Fast Fourier transform despite the presence of a singular factor, as appears when convolution between currents and Green's function is carried out directly in spectral domain. Contour deformation is reconciled with the use of the FFT through a series development of the real exponential that results from the contour deformation [14]. The impressed IBC is analyzed with 630 Fourier-Bessel basis functions. Fig. 1 compare the current distribution and the radiation pattern obtained with FBBF, with that of the CFFT. The CFFT iterative method takes several hours to complete, while the IBC-FBBF technique needs less than 2 min.

The lower resolution of the current distribution in Fig. 1(b) is due to the current projection on a Cartesian grid used in the iterative CFFT method [13]. The difference between the two simulated patterns (especially on the side lobes) may be explained by the IBC implementation process (with square patches) and the edge diffraction effects.

IV. Conclusion

Full-domain basis functions are well suited to the analysis and design of metasurfaces. Basis functions initially devoted to the analysis of circular apertures can be exploited on elliptical apertures, assuming a coordinate transform and an appropriate harmonic development of the Green's function. This quick analysis tool allows fast design and, in turn, translation into the actual patches, using a known link between surface impedance and patch shape. The performance of this final design has been verified through a recently developed iterative technique that makes use of the Contour-FFT. A good agreement between the radiation patterns has been found.



(a) FBBFs: Magnitude of the current distribution (dB).

(b) CFFT: Magnitude of the current distribution (dB).

(c) Right handed Circular polarization (RHCP) and Left handed Circular polarization (LHCP) radiation pattern comparison (CFFT vs FBBFs).

Fig. 1: Simulation results: the FBBFs based IBC method is compared with the iterative CFFT method.

Acknowledgement

This research has been supported by a FRIA grant from the Belgium FNRS fund.

References

[1] C. L. Holloway, E. F. Kuester, J. Gordon, J. O'Hara, J. Booth and D. Smith "An overview of the theory and applications of metasurfaces: The twodimensional equivalents of metamaterials," *IEEE Antennas Propag. Mag.*, vol. 54, no. 2, pp. 10-35, 2012.

[2] S. Maci, G. Minatti, M. Casaletti and M. Bosiljevac, "Metasurfing: Addressing waves on impenetrable metasurfaces," *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 1499-1502, Feb. 2011.

[3] D. González-Ovejero et al.,"Design, fabrication and testing of a modulated metasurface antenna at 300 GHz," *EUCAP.,* 2017.

[4] N. Yu et al.,"Flat optics: Controlling wavefronts with optical antenna metasurfaces," *IEEE J. Sel. Topics Quantum Electron.,* vol. 19, no. 3, pp. 4700423, May. 2013.

[5] G. Minatti, M. Faenzi, E. Martini, F. Caminita, P. De Vita, D. González-Ovejero, M. Sabbadini and S. Maci, "Modulated metasurface antennas for space: synthesis, analysis and realizations," *IEEE Trans. Antennas Propag.*, vol. 63, pp. 1288-1300, May. 2014.

[6] B. Stupfel, and D. Poget, "Sufficient uniqueness conditions for the solution of the time harmonic Maxwell's equations associated with surface impedance boundary conditions," *J. Comput. Phys.*, vol. 230, no. 12, pp. 4571-4587, Jun. 2011.

[7] D. González-Ovejero and S. Maci, "Gaussian ring basis functions for the analysis of modulated metasurface antennas," *IEEE Trans. Antennas Propag.*, vol. 63, pp. 3982-3993, Sep. 2015.

[8] M. Bodehou, D. González-Ovejero, C. Craeye and I. Huynen, "Method-of-Moment simulation of modulated metasurface antennas with a set of orthogonal entire-domain basis functions," submitted to *IEEE Trans. Antennas Propag.*, Dec. 2017.

[9] M. Casaletti, M. Smierzchalski, M. Ettorre, R. Sauleau and N. Capet, "Polarized beams using scalar metasurfaces," *IEEE Trans. Antennas Propag.*, vol. 64, no. 2, pp. 3391-3400, May. 2016.

[10] M. Bodehou, C. Craeye, H. Bui-Van, and I. Huynen, "Fourier-Bessel basis functions for the analysis of elliptical domain metasurface antennas," *IEEE Antennas Wireless Propag. Lett.*, vol. 17, pp. 675-678, April. 2018.

[11] M.A. Francavilla, E. Martini, S. Maci and G. Vecchi, On the numerical simulation of Metasurfaces with impedance boundary condition integral equations, *IEEE Trans. Antennas Propag.*, vol.63, pp. 2153-2161, May. 2015.

[12] S. Hubert, M. Bodehou, and C. Craeye, "Fast spectral iterative technique for the analysis of planar metasurface-type structures," *EUCAP 2018.*

[13] S. Hubert, S. Nath Jha, and C. Craeye, "Analysis of large non-regular printed scatterers using the Contour-FFT," submitted to *IEEE Trans. Antennas Propag.*, Nov. 2017.

[14] Shambhu Nath Jha, and C. Craeye,"Contour-FFT based spectral domain MBF analysis of large printed antenna Arrays," *IEEE Trans. Antennas Propag.*, vol. 62, no. 11, pp. 5752-5764, Nov. 2014.