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Spatial segregation and urban structure

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Abstract

In this paper, we develop a model of spatial segregation mediated by competitive land prices. Agents of two groups consume city land and benefit from social interactions. Because of cultural or ethnic differences, intragroup interactions are more frequent than intergroup ones. When group sizes differ, population groups sort into distinct neighborhoods. We characterize two- and three-district urban structures. For high population ratios or strong intergroup interactions, only a three-district city exists. In other cases, multiplicity of equilibria arises. Both groups generally rank these equilibria differently. However, when group sizes are similar, all individuals agree on which spatial equilibrium is best.

KEYWORDS

segregation, social interaction, urban structure

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1 | INTRODUCTION

In many cities, different population groups tend to cluster in distinct neighborhoods. For example, many US metropolitan areas have a Chinatown, a little Italy, or other ethnic enclaves that host significantly high concentrations of particular ethnic or cultural groups. Such enclaves may range from a single block to a few square miles areas. The various explanations for such spatial segregation offered in the literature lie in the economic ties and social interactions that people maintain with their peers. The prevalence of such segregation is exacerbated by poverty, if poor people are more likely to see their economic prospects and social relationships improved within their own ethnic group. Spatial concentration also affects business and professional activities. Different industrial sectors often locate their business activities in separate areas. For instance, in Los Angeles, distinct neighborhoods host the movie, finance, fashion, and art industries. For urban economists, such industrial concentrations are partly explained by spillovers that benefit firms when locating close to other firms active in the same industry.

The sorting of both consumers and firms in a city leads to complex economic interactions. Yet, many cities around the world present a clear-cut spatial divide between two population groups on the basis of race, language, or ethnicity. For instance, the Island of Montreal in Canada displays an east–west division of its French- and English-speaking communities. A similar north-south divide is observed in the city of Brussels in Belgium between the Dutch- and the French-speaking communities. North American cities differ from European ones in several ways. For instance, most foreign ethnic groups live in the outskirts of Paris whereas US cities like New York or Detroit host several small ethnic and racial groups around their city center. The internal structure of a city also depends on the location of fixed and endogenous amenities. Spatial segregation can also have religious grounds. In the city of Belfast, the west and the east sides of the city are mostly inhabited by Catholics and Protestants, respectively. More generally, the spatial clustering of communities in cities may be influenced by many other personal attributes, ranging from professional activities to sexual orientation. The paper aims at a better understanding of the spatial sorting of two population groups differing in one such characteristic.

Unlike Schelling's (1971) seminal model, here spatial segregation is mediated by competitive land prices. We study a one-dimensional city where agents of two types engage in intra- and intergroup social interactions, choose their land consumption as well as their residential location. Due to greater affinity with members of one's own group than outsiders, intragroup interactions are more frequent than intergroup ones. Such preferential interactions reflect stronger relationships between individuals sharing a common culture, language, or ethnicity. They could also reflect more intense professional relationships between individuals sharing the same economic activity (e.g., bankers, lawyers, or designers) or economic status (e.g., employed or unemployed workers). We assume that populations are symmetric as to their benefit from intra- and intergroup interactions so that the intensities of social interactions are the same for both groups. Unlike existing urban models of segregation (e.g., Kanemoto, 1980; Yinger, 1976), our model does not rely on the presence of an exogenous city center (Alonso, 1964). Instead, urban districts emerge endogenously resulting from the interplay between a spatial externality due to social interactions and competition in the land market. Each agent travels along the city to visit other agents and derives a social benefit from face-to-face contacts. Each trip incurs a cost which is proportional to distance. In equilibrium, the benefit from social interactions balances residence and access costs. The focus of this paper is how these social interactions structure spatial neighborhoods.

Our results are as follows. First, we show that integration is never a spatial equilibrium when group sizes differ. Populations do not form an integrated city even if group sizes differ only slightly. This result comes from the fact that the net social benefit from intragroup interactions is larger than that from intergroup interactions. Because of this, agents always have an incentive to relocate closer to agents of their own population so as to save on trip costs.

Second, we analyze segregation patterns involving two or three urban districts. The two-district city configuration is a spatial equilibrium when group sizes are similar or when intergroup interactions are weak. In a three-district city, one population locates in the city center whereas the other resides in the two city edges. When the large population locates in the central district, the three-district configuration is always a spatial equilibrium. In contrast, when the small population locates in the central district, the three-district city is sustained in equilibrium only when population sizes are similar.

Third, we show that multiple equilibria may arise. Depending on the model parameters, various urban structures can coexist. The economy exhibits one, two, or three spatial equilibria. The more similar the population sizes and the weaker the intergroup interactions, the more likely it is for several equilibria to emerge. For high population ratios or strong intergroup interactions, only the three-district city with the large group occupying the central district exists. When several spatial configurations are possible, spatial equilibria can be ranked in terms of utility by each population group. A welfare analysis shows that when group sizes are similar, all individuals agree on which spatial equilibrium is best.

Our model helps to understand how city growth may affect urban structure. Given the multiple equilibria, city growth may induce spontaneous transitions from one urban structure to another. Thus, the spatial structure of cities depends on history. Over time, old cities like Paris have undergone several transitions and are now locked in a configuration with the large (native) population in the city center. In contrast, younger US cities like New York or Detroit may not have undergone such transitions and display an urban configuration with the minority group in the city center.

Our model also sheds light on the impact social integration programs may have on urban structure. Interpreting the frequency of intergroup interactions as an indicator of social integration, our model provides some interesting insights. In particular, spatial integration should not be considered as an indicator of efficiency of social integration programs. In many instances, social integration programs may be ineffective in reshaping the urban landscape. Moreover, social integration may even fragment spatially the minority group.

This paper is organized as follows. The next section discusses the contribution of our paper in light of the existing literature. Section 3 describes the model. The possibility of an integration equilibrium is studied in Section 4. Section 5 analyzes spatial segregation where population groups sort into two and three districts. Finally, Section 6 concludes.

2 | RELATED LITERATURE

The main contribution of the paper is to reconcile Schelling-like segregation patterns with a competitive land market and nonlocal interactions. In his seminal paper, Schelling (1971) presents a model where individuals' preferences for their local neighborhood composition lead to spatial segregation patterns.

Schelling's work has generated many studies extending the initial setup. Most of them rely on agent-based simulations of the two-dimensional checkerboard model. Grauwin, Goffette-Nagot, and Jensen (2012) propose an analytical approach that formulates Schelling model as a spatial game and relies on the existence of a potential function. Interestingly, when preferences for groups tend to be entirely deterministic, the stationary states resulting from Schelling dynamics turn out to maximize the potential function. Pancs and Vriend (2007) study the robustness of Schelling segregation patterns with respect to agents' preferences. Numerous simulations confirm the robustness of segregation even when agents have a strict preference for integration. Pancs and Vriend are also able to solve for Schelling segregation states when agents are distributed along a circle. This determines the sequences of White and Black individuals that constitute a Nash equilibrium of the spatial game along a ring. These two theoretical works have contributed to the understanding of Schelling model.

In our paper, segregation reflects the trade-off between the benefit of social interactions and the cost of land, and is mediated by the land price mechanism as in Becker and Murphy (2003). In constrast to the latter authors, the utility derived by agents does not depend only on the local neighborhood composition but also depends on population levels in other neighborhoods through the access cost because social interactions are nonlocal and take place across the whole city. This feature of the model is essential in understanding the impact of truly spatial urban interactions.

Zhang (2004) also extends Schelling model by incorporating the price of housing. To our knowledge, this study provides the first attempt to incoporate a housing market in Schelling's framework. There are several differences between Zhang's contribution and our paper. In Zhang, land consumption is still constant like in Schelling and the price of housing responds to the excess demand for housing resulting from the neighborhood vacancy rate. By contrast, the land price in our model clears the land market in all urban locations. Also, Zhang relies on evolutionary game theory to characterize stochastically stable patterns and presents agent-based simulation results of the checkerboard model. In contrast, we use the concept of spatial equilibrium, allowing us to address the issue of multiple equilibria of a one-dimensional city. In addition, Zhang studies segregation patterns that are driven by asymmetric preferences with White individuals having a preference for other White individuals. By contrast, in our model, social interactions are symmetric, in the sense that each group values equally interactions with the other group. In another interesting work, Zhang (2011) unifies both models initially introduced by Schelling: the checkerboard and the tipping models. This allows him to study how an integrated configuration tips into complete segregation. Zhang is able to determine multiple equilibria by relying on agent-based simulations. However, it is unknown how such a unifying framework could be handled in the presence of a housing market.

In contrast to Schelling and follow-on works, where segregation outcomes are obtained through computer simulations, our model admits simple closed-form solutions. This allows us to characterize and compare equilibrium configurations. This exercise would be hardly doable in Schelling's framework. In the particular case of groups of equal size, the integrated urban pattern coexists with the segregation ones. However, as soon as groups differ in size, even only slightly, the integration equilibrium ceases to exist giving rise to segregation only. This transition is reminiscent of Schelling's tipping point where Whites flee a neighborhood once some threshold of non-Whites is reached. However, here, tipping is from an integrated pattern to segregation. Moreover, further neighborhood transitions are possible here as spatial neighborhoods keep on restructuring as the model parameters change. When crossing such transition curves in the parameter space, some segregated patterns cease to exist or new ones emerge.

The paper also relates to the literature on urban market interactions and spatial segregation. Fujita and Ogawa (1982) and Lucas and Rossi-Hansberg (2002) have analyzed how market interactions between workers and firms can shape the internal structure of cities. Instead, here, the city structure results from nonmarket interactions. Our model builds on Beckmann (1976); Mossay and Picard (2011); and Blanchet, Mossay, and Santambrogio (2016) where social interactions are among homogeneous agents. To address segregation issues, we extend that single group framework to a two-group model, thereby allowing for both intra- and intergroup interactions among individuals. Kanemoto (1980) and Yinger (1976) also study the selection of spatial neighborhoods by two groups of households. While the former assumes that the poor group imposes a negative spatial externality on the rich group, the latter considers Whites and Blacks having biased preferences over their neighborhood composition. In contrast to those two works, we do not assume the pre-existence of a city center, to which residents are commuting, and we analyze the case of reciprocal segregation, where each population is affected by the location decisions made by individuals of the other group. Also, our model yields three-district configurations, which do not arise in either Kanemoto's or Yinger's work. Miyao (1978) provides an early model explaining how externalities between two groups of households can affect the segregation structure of a city. However, unlike our model or that by Kanemoto or Yinger, externalities in Miyao operate at city level and therefore do not decay over distance. As a result, segregation is induced by the inflow or outflow of agents of either group rather than by the sorting of households across urban locations.

Brock and Durlauf (2002) are known for their influential work on neighborhood effects. In their model, they study the impact of conformity where the individual's utility of a choice depends on the number of neighbors making the same choice. Nevertheless, in contrast to our framework, their approach focuses on a single group of individuals in a single neighborhood. In that sense, our work can be seen as an extension of social interaction modeling to the case of two population groups distributed over a set of neighborhoods, with social interactions represented by spatial externalities.

The literature on urban segregation has been highly influenced by Benabou's (1993, 1996) contributions on human capital and urban neighborhood composition. In his work, segregation stems from complementarities in education or production. Whereas the structure of urban equilibria shares some similarity with ours, the assumptions, the mechanisms at work, and urban policies are quite distinct from ours. Benabou's main focus is about how human capital decisions shape skill or income stratification and the city productivity, while our paper discusses the role of intra- and intergroup social interactions on the spatial structure of cities. Benabou relies on an endogenous formation of skill groups, highly localized educational interactions and global complementarity in production. By contrast, our paper does not include any specific educational or production features and rather builds on the presence of global social interactions.

Brueckner, Thisse, and Zenou (1999) study how endogenous amenities are affected by the income of individuals in the context of a monocentric city with two income groups. However, our analysis incorporates neither income heterogeneity nor commuting. Local neighborhood externalities are also influenced by the quality of housing which typically decreases with the age of the housing unit. Bond and Coulson (1989) are able to show how the filtering process and externality effects determine the neighborhood composition (high- vs. low-income households) as well as the neighborhood quality of housing. In our model, externalities are spatial in nature and spill over urban neighborhoods unlike in Bond and Coulson where externalities are confined to the neighborhood. Whether filtering issues could be studied in a model with spatial externalities remains unexplored.

In location models incorporating mobility across local jurisdictions and voting within jurisdictions, households are typically perfectly stratified in equilibrium. This result obtains when households differ along a single dimension (e.g., income) and preferences satisfy a single-crossing property. However, when agents differ across two dimensions (e.g., both preferences and income), incomplete income sorting may hold in equilibrium (see Epple & Platt, 1998). In our model, the households differ only in their intergroup interactions. The work by Epple and Platt could suggest the possibility of integration patterns if groups were to differ along multiple dimensions.

The importance of social interactions in the formation of cities has been stressed by Glaeser, Henderson, and Inman (2000). In this paper, we address social interactions in an explicit spatial framework and focus on intra- and intergroup social interactions between two groups of individuals. The intra- and intergroup interactions can be interpreted in terms of strong and weak ties in the sense of Granovetter (1973). They also reflect "homophily." This phenomenon has been studied in models of friendship formation (see Currarini, Jackson, & Pin, 2009, 2010). These authors have also identified preference and meeting bias in the formation of social networks. In particular, they have shown how friendship biases for the same population (White/Black/Hispanic) in American schools can be broken down into an intrinsic utility surplus and a better matching process with the members of the same group. Despite this, the network formation literature has remained silent about the role of distance in the cost of maintaining social interactions. The present paper attempts to bring the issue of segregation and homophily into a spatial and urban context. de Marti and Zenou (2017) study similar segregation issues arising in a social network. While they address various social aspects of the problem (e.g., assimilation or oppositional identities), we study a land market model with spatial interactions. Here, the access cost is assumed to be small enough so that each agent has an incentive to interact with all other agents distributed along the line segment. This means that according to the terminology of de Marti and Zenou, our economy always displays complete integration (e.g., each group is fully intraconnected and both groups are fully interconnected). However, in contrast to their work, the issue here is not about whether an individual will maintain a social link with other individuals, nor it is about the impact of the geometry of the social network. Rather, we are interested in how individual location choices affect the structure of spatial neighborhoods.[1]

3 | THE MODEL

We consider a linear city, with a unit land width in each location, that spreads over the interval $\mathscr{B} \equiv [-b, b]$ and hosts two populations of agents $P_1 \geq P_2$. The present framework extends the spatial model of social interactions by Mossay and Picard (2011) to the case of two interacting groups. The number of agents of population i residing at location x is denoted by the density $\lambda_i(x)$: $\mathscr{B} \to \mathbb{R}^{+}$, $i = 1, 2$. Each individual enjoys the same unitary benefit when interacting socially with another agent and incurs an access cost τ per unit of distance associated with the return trip to visit him. Because of cultural/ethnic differences or language barriers, social interactions are more frequent among individuals of a same group. While individuals meet each agent of their own population with a frequency normalized to one, they meet each agent of the other group with a lower frequency $0 < \alpha < 1$. The social utility derived by an agent of population i can be written as

[1]Note that the model by Helsley and Zenou (2014) addresses both location choices and endogenous network formation. However, it does not focus on segregation issues.

$$S_i(x) = \int_{\mathscr{B}} (1 - \tau|x - y|)\lambda_i(y)dy + \alpha \int_{\mathscr{B}} (1 - \tau|x - y|)\lambda_j(y)dy, \quad i \neq j,$$

where the first term (*resp*. the second term) reflects the net benefit from intragroup interactions (*resp*. intergroup interactions) accounting for the access cost with $|x - y|$ denoting the distance between locations x and y.

The above assumption $0 < \alpha < 1$ is not innocuous. According to us, it is a way of reflecting agents' preferences for their own group without precluding, a priori, the possibility of integration patterns. In the case where $\alpha > 1$, preferences are biased toward the other group, which would constitute such a strong integration force that segregation might not occur in equilibrium. In the case where $\alpha < 0$, agents do not benefit from interacting with the other group, which would result in repulsion forces leading the two groups to separate as much as possible from each other. Note that when $\alpha = 1$, agents meet each other agent irrespective of the group she belongs to, so that our model reduces to the single group model as studied by Mossay and Picard (2011).

The surplus $S_i(x)$ can also be interpreted in a context of uncertainty. In that case, it would correspond to the expected utility of an individual who plans to interact with a subset of agents whom location and identity are not known at the time of the residence choice. Such an interpretation applies to individuals moving to an urban area with no a priori acquaintances. This could also apply to the case of shopkeepers, sellers, as well as workers who expect to hold several jobs at different locations during their lifetime, or employers who do not have a precise idea about future workers' residences.[2]

Agents maximize the utility they derive from consumption and social interactions

$$U_i(s, z; x) = S_i(x) - \frac{\beta}{2s} + z,$$

subject to their budget constraint

$$z + R(x)s = Y,$$

where s and z are the consumption of land and of the composite good, $R(x)$ the land rent at location x, Y agents' income,[3] and β the preference parameter for land consumption.

For the sake of simplicity, we assume that land has no alternate use, so that $R(x) = 0$ in uninhabited locations. In the above functional form of utility, we consider an hyperbolic preference for land instead of the logarithmic preference used by Beckmann (1976) and Fujita and Thisse (2002, chapter 6). The present hyperbolic preference represents an intermediate case between Beckmann's demand and the inelastic demand for space that is regularly used in standard urban economics.[4]

Since Alonso (1964) and Fujita (1989), the urban economic literature has regularly relied on the bid rent approach to determine spatial equilibrium. Because agents are free to relocate anywhere along the geographical space, the absence of locational arbitrage requires that the utility level of agents of a same population remains constant across all locations they inhabit. The agent's bid rent ψ_i in location x is defined as the maximum rent that he is willing to pay for residing in x

$$\psi_i(x) = \max_s \frac{Y - z}{s} \text{ s.t. } U_i(s, z; x) \geq u_i \quad i = 1, 2,$$

for some given utility level u_i.

[2]In Currarini et al.'s (2010) terminology, the parameter α can be interpreted as a same-type bias.

[3]Y can also be interpreted as the valuation of the endowment in the composite good.

[4]The hyperbolic and logarithmic preferences for residential space are two particular instances of the class of preferences $s^{1-\rho}/(1 - \rho)$. They correspond to $\rho = 2$ and $\rho \to 1$ respectively, leading to iso-elastic demands for residential space with price elasticities equal to 1/2 and 1, respectively. The choice of hyperbolic preferences allow us to study our model analytically and to derive closed-form solutions for equilibrium spatial distributions.

Let $\hat{z}_i(x, u_i)$ and $\hat{s}_i(x, u_i)$ denote the bid-maximizing consumption of land and of the composite good for an individual of population i residing in x. By using the agent's budget constraint, the bid rent $\psi_i(x)$ can be written as

$$\psi_i(x) = \max_s \frac{Y - u_i + S_i - \beta/(2s)}{s} = \max_s \left(\frac{Y - u_i + S_i}{s} - \frac{\beta}{2s^2} \right).$$

The corresponding optimal consumption of space is given by

$$\hat{s}_i(x, u_i) = \frac{\beta}{Y - u_i + S_i}.$$

which yields the following bid rent:

$$\psi_i(x) = \frac{(Y - u_i + S_i)^2}{2\beta} = \frac{\beta}{2\hat{s}_i^2}. \tag{1}$$

A competitive spatial equilibrium is then defined by spatial distributions of consumption $\{z_i(x), s_i(x)\}$, land rent $R(x)$, agents $\lambda_i(x)$, and utility levels u_i which

- **(i)** maximize each population's bid rent ($z_i(x) = \hat{z}_i(x, u_i)$ and $s_i(x) = \hat{s}_i(x, u_i)$),
- **(ii)** allocate land to the highest bid ($R(x) = \max_i[\psi_i(x), 0]$ so that $R(x) = \psi_i(x)$ if $\lambda_i(x) > 0$, and $R(x) = 0$ if $\lambda_i(x) = 0, \forall\, i$),
- **(iii)** satisfy the land market equilibrium, $\sum_i \lambda_i(x)s_i(x) = 1$, and
- **(iv)** meet the total population constraint $\int_{\mathscr{B}} \lambda_i(x)dx = P_i, \forall\, i$.

4 | INTEGRATED DISTRICTS

In this section, we investigate the possible existence of integrated districts where both population groups live together. Integrated urban structures are often advocated in the urban planning literature. In the case of market interactions, integrated patterns of workers and firms may reflect the balance between dispersion and agglomeration forces in an urban economy (see, e.g., Fujita, 1989; Fujita & Ogawa, 1982; Lucas & Rossi-Hansberg, 2002). However, here our model of social interactions does not support spatial equilibria with integrated patterns when group sizes differ, even if only slightly. We show below that this is because agents always have an incentive to relocate closer to other agents of their own group so as to save on trip costs.

Suppose that both populations are integrated in some interval so that $\lambda_1(x) > 0$ and $\lambda_2(x) > 0$, for all x in that interval. For this configuration to constitute an equilibrium, land should be allocated to both populations. Hence, by equilibrium condition (ii), the bid rents of both populations must be equal: $\psi_1(x) = \psi_2(x)$. By expression (1), this implies that land consumption should be equal for both groups

$$s_1(x) = s_2(x) \equiv s(x).$$

All agents have an identical use of space because their benefit from social interactions and their preference for space are the same across groups. As the land market equilibrium (iii) implies that $s(x) = [\lambda_1(x) + \lambda_2(x)]^{-1}$, the bid rents (1) become

$$\psi_1(x) = \psi_2(x) = \frac{\beta}{2}[\lambda_1(x) + \lambda_2(x)]^2, \tag{2}$$

and the agent's utility $U_i = S_i(x) + Y - \beta[\lambda_1(x) + \lambda_2(x)]$. The spatial gradient of utility is then given by

$$U_i'(x) = \tau[P_i^+(x) - P_i^-(x)] + \alpha\tau[P_j^+(x) - P_j^-(x)] - \beta(\lambda_i'(x) + \lambda_j'(x)), \quad i \neq j = 1, 2, \tag{3}$$

where

$$P_i^+(x) = \int_x^b \lambda_i(y)dy \quad \text{and} \quad P_i^-(x) = \int_{-b}^x \lambda_i(y)dy = P_i - P_i^+(x)$$

denote the population i to the right and to the left of location x. Clearly, $P_i^+(x)$ (*resp.* $P_i^-(x)$) is a decreasing (*resp.* increasing) continuous function. A necessary condition for spatial equilibrium is that the utility of agents remains constant across inhabited areas ($U_1'(x) = U_2'(x) = 0$). By using expressions (3), we get

$$P_1^+(x) - P_1^-(x) = P_2^+(x) - P_2^-(x) = \frac{\beta}{\tau}\frac{\lambda_1'(x) + \lambda_2'(x)}{1 + \alpha}. \tag{4}$$

In equilibrium, both types of agents should have the same access to agents of their own group. Any difference in population access would reflect a change in the benefit from social interactions, which would translate into a change in the willingness to pay for land. If the bid rent gradients were to differ, then one population would be able to overbid the other one. A direct implication of this reasoning is that population densities should be identical, $\lambda_1(x) = \lambda_2(x)$, for any x in the integrated area. This imposes that the densities of both groups are equal within an integrated area. Hence, we can readily infer that a city can be supported by a single integrated area if the two groups have equal sizes ($P_1 = P_2$). It is far from obvious though that a combination of integrated and segregated areas could lead to a spatial equilibrium. The following Proposition shows that no integration area is actually possible when groups differ in size ($P_1 > P_2$).

Proposition 1. *When group sizes differ, there is no spatial equilibrium with integrated areas.*

Proof. See Appendix A . □

Proposition 1 results from the fact that intragroup interactions are more frequent than intergroup ones. At equilibrium, agents have an incentive to relocate closer to agents of their own population so as to save on trip costs.

Of course, when groups are the same size ($P_1 = P_2 = P$), integration is an equilibrium. In that case, differentiating relation (3) with respect to x gives

$$\beta\lambda_i''(x) + \tau(\alpha + 1)\lambda_i(x) = 0, \quad i = 1, 2.$$

Without loss of generality, the solution to this ordinary differential equation is given by

$$\lambda_1(x) = \lambda_2(x) = \lambda(x) = C\cos\delta_I x,$$

where $\delta_I^2 = \tau(\alpha + 1)/\beta$. The city border b and the amplitude C are determined by the zero opportunity cost of land at the city border b, $R(b) = 0$, and the population constraint $\int_{-b}^b \lambda(x)dx = P$

$$b = \frac{\pi}{2\delta_I}; \; C = \delta_I P.$$

5 | SEGREGATED CITIES

Given the analysis of the previous section, the economy has to organize into segregated districts, each of them hosting a single population. In each district, individuals favor intra- over intergroup interactions as they have a closer access to agents of their own group while having a more remote access to agents of the other group. As segregated patterns will reveal, a population may well spread into several districts, in which case, agents lose access even to agents of their own group.

The functional form of the population density within a segregated district is derived as follows. In a district hosting population i only, land market clears so that $\lambda_i(x)s_i(x) = 1$, for all locations x where $\lambda_i(x) > 0$ (condition (iii)). Given this, the utility and the population gradients are given by expressions (A4) and (A5) respectively. Differentiating once more the utility expression (A4) yields

$$\beta\lambda_i'' + 2\tau\lambda_i = 0.$$

This second order ordinary differential equation accepts the following class of solutions

$$\lambda_i(x) = C_i \cos\delta(x - \phi_i), \tag{5}$$

where $\delta^2 = 2\tau/\beta$ and the coefficients C_i and ϕ_i are constants still to be determined.

We now analyze the structure of cities with two and three segregated districts.

5.1 | The two-district city

Let the districts $[0, b_1]$ and $[-b_2, 0]$ host populations 1 and 2, respectively. Such an urban structure is best illustrated by the Island of Montreal or the city of Belfast where individuals segregate in two distinct areas based on language or religious grounds. Population densities are described by $\lambda_1(x) \geq 0$ for $x \in [0, b_1]$ and by $\lambda_2(x) \geq 0$ for $x \in [-b_2, 0]$. A spatial equilibrium is then defined by a set of scalars and functions (b_i, λ_i), $i = 1, 2$, which satisfy the following conditions:

- **(a)** the no-relocation arbitrage conditions within districts: $U'_1(x) = 0$, $\forall\, x \in [0, b_1]$ and $U'_2(x) = 0$, $\forall\, x \in [-b_2, 0]$,
- **(b)** the no-relocation arbitrage conditions across districts: $U_1(x) \leq U_1(0)$, $\forall\, x \in [-b_2, 0]$ and $U_2(x) \leq U_2(0)$, $\forall\, x \in [0, b_1]$,
- **(c)** the continuity of bid rents at district borders: $\psi_2(0) = \psi_1(0)$ and $\psi_1(b_1) = \psi_2(-b_2) = 0$, and
- **(d)** the total population constraint: $P_1 = \int_0^{b_1} \lambda_1(x)dx$ and $P_2 = \int_{-b_2}^{0} \lambda_2(x)dx$.

Conditions (a) and (b) ensure that agents have no incentive to relocate to another location regardless of which population inhabits it. Conditions (c) ensure that land is allocated to the highest bidder at district borders and that land is priced at its opportunity cost at the city edge. Conditions (d) guarantees that each district is occupied by its corresponding population. Note that the bid rent conditions (c) imply that $\lambda_2(0) = \lambda_1(0)$, $\lambda_1(b_1) = 0$, and $\lambda_2(-b_2) = 0$ as the bid rent $\psi_i(x)$ is inversely related to the use of space, $s_i(x)$, which is itself inversely related to the population density $\lambda_i(x)$.

Using conditions (a), (c), and (d), the spatial distributions (5) can be written as (see details provided in Appendix B)

$$\lambda_1 = C_1 \cos[\delta(x - \phi_1)] \quad \text{and} \quad \lambda_2 = C_2 \cos[\delta(x + \phi_2)], \tag{6}$$

where

$$C_1 = \frac{\delta}{2}(P_1 + \alpha P_2) \quad \text{and} \quad C_2 = \frac{\delta}{2}(\alpha P_1 + P_2), \tag{7}$$

$$\sin(\delta\phi_1) = \frac{P_1 - \alpha P_2}{P_1 + \alpha P_2} \quad \text{and} \quad \sin(\delta\phi_2) = \frac{P_2 - \alpha P_1}{P_2 + \alpha P_1}. \tag{8}$$

City borders are given by $b_i = \phi_i + \pi/(2\delta)$, $i = 1, 2$.

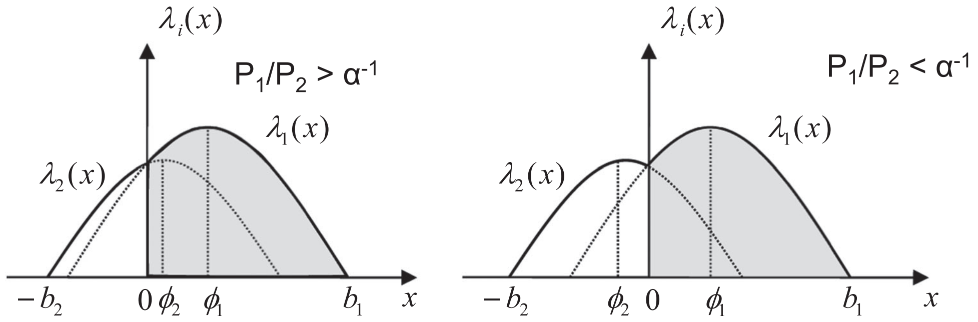

FIGURE 1 Two-district cities. Population distributions λ_1 and λ_2 are represented in terms of group sizes P_1 and P_2 and the intensity of intergroup interactions α. The shaded area corresponds to the large Group 1. The left panel (*resp*. the right panel) corresponds to an urban structure with a single subcenter (*resp*. two subcenters). Note that the land rent could be easily represented as it is proportional to the square of the population density: $R(x) = (\beta/2)\max[\lambda_1^2(x), \lambda_2^2(x)]$

Because $P_1 \geq P_2$, we have $C_1 \geq C_2$ so that *population* 1 *reaches higher densities than population* 2. From expression (8), it is readily checked that $\phi_1 > 0$, $\phi_1 > \phi_2$, and $b_1 > b_2$. The maximum density of population 1 is C_1 whereas that of population 2 may be less than C_2. Thus, population 1 is more concentrated and benefits from a better access to agents of its own group. Figure 1 depicts the population distributions λ_1 and λ_2 for an urban structure with two spatial districts. Note that the land rent $R(x)$ can be obtained easily by rescaling these population densities as $R(x) = \max[\psi_1(x), \psi_2(x)] = (\beta/2)\max[\lambda_1^2(x), \lambda_2^2(x)]$ given expression (1).

We still have to check whether this urban structure satisfies the no-relocation arbitrage conditions across districts (b). The first condition states that agents of population 1 have no incentive to relocate to population 2's district. This can be checked by using relation (A6)

$$U_1'(x) = \tau(1 - \alpha)\{P_1 - [P_2 - 2P_2^-(x)]\} > 0,$$

which implies that $U_1(x) \leq U_1(0)$, $x \in [-b_2, 0]$. This is because P_1 is larger than P_2 and $P_2^-(x)$ increases from 0 to P_2 in the interval $[-b_2, 0]$. No individual of the large population has an incentive to relocate to the small population area. In contrast, the second condition (b) does not always hold. By expression (A6), we have

$$U'_2(x) = \tau(1 - \alpha)\{-P_2 - [P_1 - 2P_1^-(x)]\},$$

where the curly bracket increases from $-(P_1 + P_2) < 0$ to $(P_1 - P_2) \geq 0$ in the interval $[0, b_1]$. Hence, the utility differential $U_2(x) - U_2(0) = \int_0^x U_2'(z)dz$ is a convex function that first falls under zero and then eventually increases above zero. Clearly, $U_2(0) \geq U_2(x), \forall\, x \in [0, b_1]$, if and only if $U_2(0) \geq U_2(b_1)$. Given that $U_2(0) = U_2(-b_2) = S_2(-b_2) + Y$ and $U_2(b_1) = S_2(b_1) + Y$, the no-relocation arbitrage condition (b) can be rewritten as $S_2(-b_2) \geq S_2(b_1)$. An individual of the small population area may well gain from moving to the large population area so as to benefit from a better access to the large group. In Appendix B, we show that the latter condition can be rewritten as

$$2\sqrt{\alpha P_1/P_2} \leq (1 + \alpha)\left[\pi - \arccos\left(\frac{P_1/P_2 - \alpha}{P_1/P_2 + \alpha}\right)\right]. \tag{9}$$

This can be summarized in the following Proposition.

Proposition 2. *The spatial configuration with two segregated districts is a spatial equilibrium if* $S_2(-b_2) \geq S_2(b_1)$, *that is, if condition (9) holds.*

When $P_1/P_2 \to \infty$, the inequality (9) is never satisfied. When $P_1/P_2 \to 1$, the condition becomes $2\sqrt{\alpha} \leq (1+\alpha)\{\pi - \arccos[(1-\alpha)/(\alpha+1)]\}$, which can be shown to be always satisfied. More generally, it can be shown that there exists a unique threshold P_1/P_2 below which this condition is satisfied. Thus, the two-district configuration is an equilibrium when population sizes are similar or when intergroup interactions are weak. In equilibrium, the large population occupies a larger share of the urban area repelling the small population toward the other city edge. The small population accommodates this situation because its interactions with the large population are weak and because land rents are too high in the other district.

We also investigate whether the spatial distribution of agents exhibits one or two subcenters. A subcenter is defined as a district interior location where the density $\lambda_i(x)$, and therefore, the land rent $R(x)$, are maximal. It readily comes from expression (8) that the city exhibits one center to the right of $x = 0$ if $P_1/P_2 > 1/\alpha$ (i.e., $\phi_1 > 0$, $\phi_2 < 0$), while it exhibits two of them, one on each side of $x = 0$, if $1 < P_1/P_2 < 1/\alpha$ (i.e., $\phi_1 > 0$, $\phi_2 > 0$).

Corollary 3. *The two-district city exhibits a single subcenter if* $\alpha > (P_1/P_2)^{-1}$ *and two subcenters otherwise.*

Both urban structures are depicted in the panels of Figure 1. Of course, when intra- and intergroup social interactions become equally frequent ($\alpha \to 1$), the location choices made by agents lead to the emergence of a single subcenter. In contrast, when intragroup social interactions dominate intergroup ones ($\alpha \to 0$), each population group locates around its own subcenter while still benefiting from intergroup interactions as both groups live in the same city. Relative group sizes also matter. When these are similar, each population group forms its own subcenter. This is because strong intragroup interactions create a separate basin of attraction for each group. In contrast, when one population is much larger than the other one, its density becomes so high that it also becomes a basin of attraction for the small population, which then ceases to have its own subcenter.

5.2 | The three-district city

Here we consider urban structures with three districts. In these configurations, the large population may locate at either the center or the edge of the city edge.

5.2.1 | The large population in the central district

We consider a symmetric spatial configuration where the large population 1 resides in the central district $[-b_1, b_1]$ and the small population 2 in the two edge districts $[-b_2, -b_1]$ and $[b_1, b_2]$.[5] Such an urban structure is reminiscent of some European cities like Paris where the native population concentrates around the city center and ethnic populations reside in the suburbs.

A spatial equilibrium is defined by a set of scalars b_i, $i = 1, 2$, and two even functions $\lambda_1 : [-b_1, b_1] \to \mathbb{R}^+$ and $\lambda_2 : [-b_2, -b_1] \cup [b_1, b_2] \to \mathbb{R}^+$ which satisfy:

(a) the no-relocation arbitrage conditions within districts: $U_1'(x) = 0$, $\forall\, x \in [0, b_1]$ and $U_2'(x) = 0$, $\forall\, x \in [b_1, b_2]$,

(b) the no-relocation arbitrage conditions across districts: $U_2(x) \leq U_2(b_1)$, $\forall\, x \in [0, b_1]$ and $U_1(x) \leq U_1(0)$, $\forall\, x \in [b_1, b_2]$,

(c) the continuity of bid rents at district borders: $\psi_2(b_1^-) = \psi_1(b_1^+)$ and $\psi_2(b_2) = 0$, and

[5] It can be shown that no asymmetric configuration with three districts can be a spatial equilibrium (see Appendix D).

(d) the total population constraint: $P_1 = \int_{-b_1}^{b_1} \lambda_1(x)dx$ and $P_2 = 2\int_{b_1}^{b_2} \lambda_2(x)dx$.

These conditions have an interpretation similar to that provided in the previous section. Conditions (a), (c), and (d) allow us to determine the spatial distributions as (see details provided in Appendix C)

$$\begin{aligned} \lambda_1(x) &= \lambda_1(-x) = C_1 \cos(\delta x), \text{ if } x \in [0, b_1] \\ \lambda_2(x) &= \lambda_2(-x) = C_2 \cos[\delta(x - \phi_2)], \text{ if } x \in [b_1, b_2]' \end{aligned} \tag{10}$$

where

$$C_1 = \frac{\delta}{2}\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2} \quad \text{and} \quad C_2 = \frac{\delta}{2}(P_2 + \alpha P_1), \tag{11}$$

while $\phi_2 = b_2 - \pi/(2\delta)$ and the district borders b_1 and b_2 are given by

$$\sin \delta b_1 = \frac{P_1}{\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2}}, \tag{12}$$

$$\cos \delta(b_2 - b_1) = \frac{\alpha P_1}{\alpha P_1 + P_2}. \tag{13}$$

The population distributions λ_1 and λ_2 corresponding to this urban structure is illustrated in the left panel of Figure 2. Like previously, the land rent $R(x)$ can be obtained easily by rescaling the population densities as $R(x) = (\beta/2)\max[\lambda_1^2(x), \lambda_2^2(x)]$

In the above urban structure, the no-relocation arbitrage conditions across districts (b) turn out to be always satisfied. This means that no individual has an incentive to relocate in the district hosting the other population. On the one hand, because of its size, the large population benefits from more numerous social interactions. It is better off locating around the city center where it gets a close access to agents of its own group. For $x \in [b_1, b_2]$, condition (A6) leads to the utility gradient $U_1'(x) = \tau(1-\alpha)[-P_1 - (P_2^+(x) - P_2^-(x))] \leq 0$ as $P_2 \leq P_1$. Hence, $U_1(x) \leq U_1(0)$ for $x \in [b_1, b_2]$. On the other hand, the small population has no incentive to relocate to the center. To show this, observe that by condition (A6), for $x \in [0, b_1]$, we get $P_2^+(x) = P_2^-(x) = P_2/2$ so that the utility gradient $U_2'(x) = \tau(1-\alpha)[-(P_1^+(x) - P_1^-(x))]$ increases from zero to $\tau(1-\alpha)P_1 > 0$ when x rises from 0 to b_1. This means that $U_2'(x) \geq 0$ and thus $U_2(x) \leq U_2(b_1)$, $\forall\, x \in [0, b_1]$. Intuitively, the higher density of the large population in the

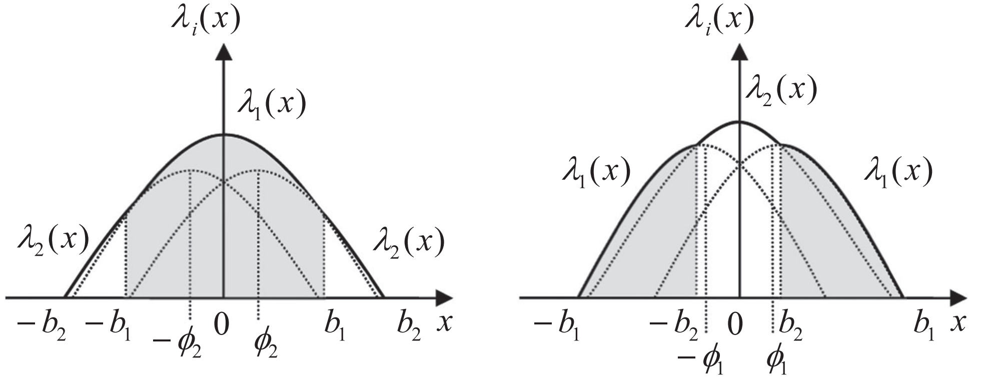

FIGURE 2 Two-district cities. Population distributions λ_1 and λ_2 are represented. The shaded area corresponds to the large population 1. In the left panel (*resp.* right panel), the larger population 1 is hosted in the central district (*resp.* the edge districts). Note that the land rent could easily be represented as it is proportional to the square of the population density: $R(x) = (\beta/2)\max[\lambda_1^2(x), \lambda_2^2(x)]$

city center also benefits the small population. Although it interacts less frequently with the large group, the small population gets a close access to a large number of agents of the other group.

These arguments can be summarized in the following Proposition.

Proposition 4. *The urban structure with three segregated districts and the large population in the central district is always a spatial equilibrium.*

In this three-district city, the large population occupies the city area where it benefits from a closer access to both populations at a high residence cost while the small population benefits from lower land rents in city edges at the expense of a higher access cost. Moreover, the city exhibits a single subcenter in $x = 0$. This is because the location $x = \phi_2$ is not a subcenter for population 2. If it were so, one should have $\phi_2 > b_1$, which contradicts the condition $\phi_2 = b_2 - \pi/(2\delta)$ and $\delta(b_2 - b_1) < \pi/2$ imposed by expression (13). Hence, in this urban structure, the large population constitutes a basin of attraction that is large enough to impede the creation of subcenters within the small population's districts.

5.2.2 | The large population in the edge district

We now consider a spatial configuration where the small population 2 resides in the central district $[-b_2, b_2]$ and the large population 1 in the two edge districts $[-b_1, -b_2]$ and $[b_2, b_1]$. This structure is reminiscent of some US cities like Detroit where the White population resides away from the city center while the Black (ethnic) population resides around the city center.

The equilibrium analysis performed in the previous subsection applies here by simply swapping subscripts 1 and 2. The corresponding urban structure is depicted in the right panel of Figure 2. Yet, an important change concerns the no-relocation arbitrage condition across districts (b). Here, the small population may have an incentive to relocate to a peripheral district. We have that, for $x \in [b_2, b_1]$, $U_2'(x) = \tau(1 - \alpha)[-P_2 - (P_1^+(x) - P_1^-(x))]$ which rises from $-\tau(1 - \alpha)P_2 < 0$ to $\tau(1 - \alpha)(P_1 - P_2) \geq 0$. Hence, $U_2(x)$ is a convex function on the interval $[b_2, b_1]$. Therefore, because $U_2(x)$ is constant for all $x \in [b_2, b_1]$, the condition $U_2(0) \geq U_2(x)$ is equivalent to $U_2(0) \geq U_2(b_1) = S_2(b_1)$. The utility differential $U_2(0) - U_2(b_1) \geq 0$ can be written as (see details provided in Appendix C)

$$\pi(1 + \alpha) - 2\sqrt{(2\alpha + P_1/P_2)P_1/P_2} \geq 2(1 + \alpha)\arcsin\left(\frac{\alpha}{\alpha + P_1/P_2}\right). \tag{14}$$

As in the analysis of two-district cities, only individuals of the small population may benefit from relocating to a district hosting the other group.

Proposition 5. *The urban structure with three segregated districts and the small population in the central district is a spatial equilibrium if $U_2(0) \geq U_2(b_1) = S_2(b_1)$, that is, if condition (14) holds.*

A numerical analysis of condition (14) shows that the above three-district city is a spatial equilibrium when population sizes are sufficiently similar. The explanation for this is as follows. Consider the situation where the small population in the city center shrinks and the large population in city edges grows. The growth of the large population increases the benefits of intra- and intergroup interactions while the decline of the small population diminishes these benefits. At city edges, stronger intragroup interactions entice the large population to increase their bid for land. This pressure on land rents in city edges transmits to the city center. At the same time, individuals of the small population benefit more from intergroup interactions than from intragroup ones, and have less incentive to stay close to each other. At some point, when the small population becomes small enough, its individuals find city edges more attractive than the city center and start relocating there. This three-district urban structure can then no longer be sustained as a spatial equilibrium. Note that the existence of this equilibrium pattern stems from a coordination problem. Although the large population would benefit from locating around the city center, no individual agent has an incentive to do so as he would face an excessive residence cost and lose access to individuals of his own group who are located in city edges.

By swapping subscripts 1 and 2 in expressions (11) and (12), we get the amplitudes C_i

$$C_1 = \frac{\delta}{2}(P_1 + \alpha P_2) \quad \text{and} \quad C_2 = \frac{\delta}{2}\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2},$$

and the district borders b_i

$$\begin{aligned} \sin \delta b_2 &= \frac{P_2}{\sqrt{P_1^2 + P_2^2 + 2\alpha P_1 P_2}}, \\ \cos \delta (b_1 - b_2) &= \frac{\alpha P_2}{\alpha P_2 + P_1} \end{aligned}$$

while $\phi_1 = b_1 - \pi/(2\delta)$.

The city exhibits a single subcenter at $x = 0$ as $b_2 - \phi_1 > 0$. This is because the above condition implies that $b_1 - b_2 < \pi/(2\delta)$, which yields $\phi_1 - b_2 < 0$ as $\phi_1 = b_1 - \pi/(2\delta)$. This result is similar to that found in the previous three-district configuration, where the large population locates in the central district.

Corollary 6. *Regardless of which population locates in the central district, the three-district city exhibits a single subcenter.*

In a three-district city, any population located around the city center creates a large basin of attraction for both populations, which impedes the creation of subcenters in the periphery (see Figure 2).

6 | DISCUSSION

In this section, we study the properties of the equilibrium structures obtained in Section 5, discuss the multiplicity of equilibria, and compare the utilities derived by each population group.

6.1 | Comparative statics

Table 1 summarizes the comparative statics analysis of the two- and three-district configurations denoted respectively by (21), (212), and (121) indicating the district occupied by each group (see Appendices A and B for mathematical expressions). So as to ease the comparison, we denote each district area of the two-district city and the central district of a three-district city by $B_i = b_i$, and the edge district area hosting population $i \neq j$ in a

TABLE 1 Comparative statics summary

City structure	$\frac{dB_i}{d\alpha}$	$\frac{dB_i}{d\delta}$	$\frac{dB_1}{d(P_1/P_2)}$	$\frac{dB_2}{d(P_1/P_2)}$	$\frac{dC_i}{d\alpha}$	$\frac{dC_i}{d\delta}$	$\frac{dC_i}{d(P_1/P_2)}$	Subcenter (s)
12	–	–	+	–	+	+	+	1 or 2
212	–	–	–	–	+	+	+	1
121	–	–	+	–	+	+	+	1

three-district city by $B_i = 2(b_i - b_j)$. Many results are identical to all city structures. For instance, *more frequent intergroup interactions* (a higher α), *weaker preferences for space or larger access costs* (a higher $\delta^2 = 2\tau/\beta$) *induce spatial concentration*: agents locate closer to individuals of their own group and reside in districts with smaller areas B_i and larger densities C_i (see columns 1, 2, 5, and 6). This result is intuitive: agents substitute the use of space for social interactions thus saving on trip costs.

Other results may differ across city structures. First, the emergence of a second subcenter may arise in a two-district city only (see column 8). This particular point has been commented in Section 5 already. Second, the impact of a rise in the population ratio P_1/P_2 on district areas depends on the city structure. On the one hand, for any urban structure, *a rise in the population ratio*P_1/P_2 induces the small population 2 *to live in a smaller district*B_2 so as to benefit more from the additional intergroup interactions (see column 4). This also increases the amplitude C_i for both populations (see column 7). This is because the larger size of group 1 raises their incentive to locate close to one another, thus raising C_1. The pressure on land rents exerted by population 1 increases and transmits to the district hosting group 2 who is then enticed to use less space, which raises the amplitude C_2. On the other hand, the effect of rise in the ratio P_1/P_2 on the district hosting the large population 1 depends on whether the large population locates in the city edge. When this is the case (i.e., configurations 21 and 121), population 1 can expand horizontally through an increase of the district area (i.e., a larger B_1 as reflected in rows 1 and 3 in column 3). When located at the city edge, population 1 can expand horizontally due to the availability of cheap land at city edges. In contrast, when the large group resides in the central district (i.e., configuration 212), land rents at the border of the central district are so high that any horizontal expansion is refrained. Instead, population 1 concentrates around the district center. Moreover, the rising share of population 1 increases the benefit from intragroup social interactions, which induces population 1 to concentrate even more. As a consequence, the central district B_1 shrinks (see row 2 in column 3).

6.2 | Multiplicity of equilibria

Here we analyze the conditions under which the urban structures studied in Section 5 exist. In particular, we highlight the possibility of multiple spatial equilibria. Figure 3 depicts the equilibrium urban structures with two or three segregated districts in terms of the population ratio P_1/P_2 and the intensity of intergroup interactions α. A population residing in a district exhibiting a subcenter (*resp.* no subcenter) is indicated by a bold number (*resp.* regular number). Note that the curves displayed in Figure 3 are independent of δ, and therefore of the preference for land β and the access cost τ. This means that Figure 3 accounts for all the relevant parameters of the model (α and P_1/P_2). The two-district city is an equilibrium provided that P_1/P_2 and α are not too large (see areas 2**1** and **21**). The three-district city with the large population 1 living in the central district is always an equilibrium regardless of parameter values (see area 2**1**2). In contrast, the three-district city with the small population 2 living in the central district is an equilibrium only for a low population ratio P_1/P_2 (see area 1**2**1). Figure 3 illustrates the existence of multiple equilibria. Depending on parameter values (P_1/P_2 and α), the economy exhibits one, two, or three spatial equilibria. The more similar the population sizes P_1 and P_2, and the weaker the intergroup interactions α, the more likely several equilibria to emerge.

When several spatial configurations coexist, spatial equilibria can be ranked in terms of the utility derived by each population group. Though this ranking could not be established analytically, it does not depend on the preference for land β nor on the access cost τ. Figure 4 displays each population's preferred urban configuration in terms of the population ratio P_1/P_2 and the intensity of intergroup interactions α.[6]

When population ratios are high and intergroup interactions strong, the urban structure (212) is the unique spatial equilibrium and leaves no other choice to individuals. In other cases, multiple equilibria exist. When

[6]Figure 4 summarizes the information contained in Tables E1 and E2. Note that unlike in Figure 3, the various configurations are indicated without making any explicit reference to the number of subcentres.

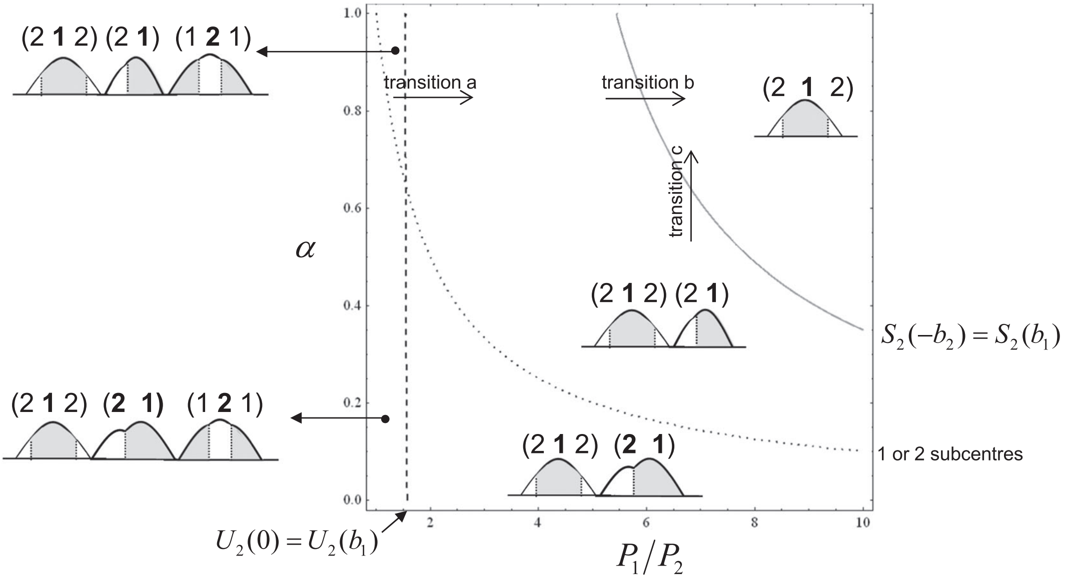

FIGURE 3 Urban structure equilibria. Two- and three-district equilibria in terms of the population ratio P_1/P_2 and the intensity of intergroup interactions α. The two-district city (21) (*resp*. the three-district city (121)) is a spatial equilibrium for parameter values at the left of the solid curve, representing condition (13) (*resp*. the dashed curve, representing condition (20)). Note that the three-district city (212) is a spatial equilibrium for all parameter values. The condition determining the number of subcentres in Corollary 4 is represented by the dotted curve so that a population residing in a district exhibiting a subcentre (*resp*. no subcentre) is indicated by a bold number (*resp*. a regular number)

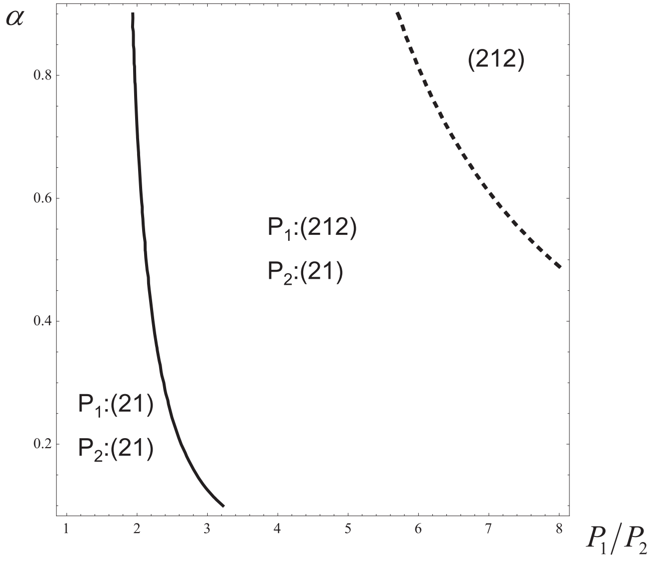

FIGURE 4 Urban configuration preferred by each population group. Preferred urban configuration in terms of the population ratio P_1/P_2 and the intensity of intergroup interactions α. Above the dashed curve, the only equilibrium is (212). Below the solid curve, both populations prefer the two-district city (21). In between these two curves, population 1 prefers the three-district city (212) while population 2 prefers the two-district structure (21)

population ratios or intergroup interactions are intermediate, both populations disagree about which urban structure to adopt, see Figure 4 where population 1 prefers the three-district structure (P_1:212) while population 2 prefers the two-district configuration (P_2:21). Finally, *when group sizes are similar*, *a* common agreement is reached so as to which spatial equilibrium is best as urban configurations are Pareto ranked (see Figure 4 where both populations prefer the two-district urban configuration [P_1:21 and P_2:21]).

A somewhat surprising result of our analysis is that the small population 2 always gets a higher utility in the two-district configuration. This configuration is preferred over that with three districts where it resides in the central district. The reason for this is the following. In the three-district urban configuration 121, the pressure on land prices exerted by population 1 in city edges transmits to the central district and outweighs the benefits of intragroup interactions of population 2 in the central district. Figure 4 also shows that population 1 displays a similar preference for the two-district configuration over that with three districts when populations sizes are similar. In this case, population 1 is worse off in configuration 212 as it faces too high land prices in the central district, which outweighs the benefit of intragroup interactions in the central district. It is the lack of access to city edges where land is cheaper that makes population 1 prefer configuration 21.

6.3 | City growth and social integration

Our model helps in understanding how city growth may affect urban structures. When city growth is not anticipated by agents, the urban structure depends only on population sizes. However, if both populations grow at the same rate, the urban structure remains unchanged. If some population grows at a faster rate than the other one, the city may incur a spontaneous restructuring process. To illustrate such a transition, suppose that the small population remains constant and has initially a size similar to that of the large group. In this case, it is possible that it resides in the central district surrounded by two edge districts hosting the large population (configuration 121). As the large population grows in size, it exerts a high pressure on land rents, which transmits from city edges to the city center through the land market. At some point, the small population find it beneficial to relocate to a city edge, replacing the former population which was living there. This corresponds to a transition from configuration 121 to configuration 21 (see transition *a* in Figure 3). The intuition is as follows. Rents have become too high in the central district so that some individuals of the small population have an incentive to move to a city edge so as to benefit from lower rents even though their intragroup interactions become more costly. As more of these individuals move to the edge district, their intragroup interactions become less costly, which makes the city edge more attractive. The restructuring process ends when all individuals of the small population 2 have relocated to the city edge. Of course, in our model this transition is instantaneous. Consequences of city growth do not end up here.

As the large population grows further in size, the city district hosting it expands horizontally, repelling the small population further away. At some point, the small population relocates to both city edges (configuration 212). This corresponds to a transition from configuration 21 to configuration 212 (see transition *b* in Figure 3). When the population ratio P_1/P_2 increases, configuration 21 has to restructure as it ceases to be an equilibrium. Intuitively, as the large population derives larger benefits from its intragroup social interactions, it can bid more for land. This pressure on land rents transmits to the small population's district through the land market so that this latter population find it beneficial to spread across city edges to face lower land rents. By relocating to both city edges, the small population ends up splitting into two subgroups. By doing so, it compensates more costly intragroup interactions by larger land plots.

Because of multiple equilibria, city growth may induce spontaneous transitions from one urban structure to another. So, urban structures depend on history. Whereas the growth of the large population can reshape the urban structure, a decline of this population has no effect on it. This is because any urban structure, which is an equilibrium for some initial population levels, remains so as the large population falls in size (see Figure 3). Interestingly, this suggests that over time, old cities like Paris have undergone several transitions and are now locked in configuration 212 with the large (native) population in the city center. In contrast, younger US cities like New York or Detroit may not have undergone such transitions and display an urban configuration with the minority group in the city center (configuration 121). Our model implies that over the long run, the minority group will be repelled to the city edge if the native population grows at a faster rate than the minority group.

Our model also sheds some light on the impact social integration programs may have on urban structures. Schooling and social programs aim at fostering social integration of immigrants with the native population. Urban planners and labor and urban economists often advocate a better social integration for efficiency and equity reasons. For instance, in Benabou (1993), under-investment in education is due to market imperfections arising from local human capital spillovers. Also, de Marti and Zenou (2017) point out that only substantial (vs. partial) lower intercommunity socialization costs can improve efficiency. Interpreting the frequency α of intergroup interactions as an indicator of social integration, our model provides some interesting insights (see Figure 3). First, segregation prevails as long as $\alpha < 1$. Therefore, the level of social integration should be very high (actually $\alpha = 1$) to eliminate spatial segregation and yield spatial integration. So, the lack of spatial integration should not be considered as an indicator of inefficiency of social integration programs. Second, when population sizes are similar, social integration programs that promote higher frequencies of intergroup interactions have no effect on the city structure. The population residing in the city center and city edges does not relocate as the frequency α of intergroup interactions increases. This means that social integration programs may be ineffective in reshaping the urban landscape. Third, when a population is significantly larger than the other one, social integration may even fragment spatially the minority group and split it into subgroups (see transition c from configuration 21 to configuration 212 in Figure 3).

7 | CONCLUSION

In this paper, we have studied how segregated districts emerge endogenously in a city and how multiple spatial equilibria arise. Our analysis derives Schelling-like segregation patterns mediated by competitive land prices. We have discussed various implications of our model regarding city growth and social integration programs. The paper also sets the stage for future research. Dynamic considerations, which are absent from our model, may be useful in understanding the evolution of spatial neighborhoods and how history may select spatial equilibria. The spatial segregation of several groups into urban districts is another issue to be examined. It would also be interesting to compare the equilibrium outcome with the socially optimal allocation of resources, as well as with the outcome of some spatial/social integration programs that would involve some specific social mix within urban districts. Finally, the present analysis might be usefully exploited to discuss issues related to urban labor markets, school segregation, and social capital. Indeed, part of the benefit of social interactions is the access to information about jobs (see Granovetter, 1973; Zenou, 2013). Pupil composition in schools may also shape the long run frequencies of intergroup social interactions and therefore affect urban segregation. Moreover, social interactions and the spatial distribution of agents contribute to the social capital that agents can build in urban areas.

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APPENDIX A

NO INTEGRATED AREAS

This appendix provides the proof of Proposition 1 stating that integration cannot be an equilibrium when population sizes differ ($P_1 > P_2$). The proof is divided into several steps.

First, we claim that at equilibrium, an integrated district cannot extend to city borders meaning that it must necessarily be interior to the city support. This is because at the city border $x = b$, the first equality in condition (4) would reduce to $-P_1^-(b) = -P_2^-(b)$ given that $P_1^+(b) = P_2^+(b) = 0$ as none lives beyond the city border. By the definition of the populations to the left of b, $P_i^-(b)$, this would imply that $-P_1 = -P_2$, which is impossible given that population sizes differ. Intuitively, as the densities of both groups are equal within an integrated area, integration at the city edge would imply equal population shares at the edge, and consequently a larger share of the large group in the rest of the city. This would inevitably induce individuals of the large group living at the city edge to relocate to areas where their peers are more numerous.

Second, at equilibrium, there exists only a single integrated district. This means that the city cannot include two or more integrated districts separated by single population areas or empty hinterlands. Imagine that such separated integrated districts would exist. On the one hand, interactions would be more valuable in single population districts because there agents would benefit from closer intragroup interactions. Other agents of the same group would therefore be enticed to move there from neighboring integrated districts. On the other hand, the presence of empty hinterlands would increase the access cost to agents. Since land is priced at its zero opportunity cost in hinterlands, agents would have an incentive to locate close to these empty areas as this would provide them with cheaper land while maintaining a good access to their own group. The formal argument is presented in the following Lemma.

Lemma 7. *When group sizes differ, integration can only arise in a single interior district.*

Proof. Our first argument in this appendix already shows that an integrated district should be interior to the city support. We now show formally that integrated areas cannot be separated by segregated or empty areas.

(i) Consider two integrated areas separated by a segregated district $[x_1, x_2]$ hosting a mass $m > 0$ of group– 1 agents. By the definition of the populations to the right and to the left of x, $P_i^+(.)$ and $P_i^-(.)$, we have

$$\begin{aligned} P_1^+(x_1) - P_1^+(x_2) &= m \text{ and } P_1^-(x_1) - P_1^-(x_2) = -m \\ P_2^+(x_1) - P_2^+(x_2) &= P_2^-(x_1) - P_2^-(x_2) = 0 \end{aligned}. \tag{A1}$$

Then taking the difference between conditions (4) evaluated at $x = x_1$ and $x = x_2$ yields $P_1^+(x_1) - P_1^-(x_1) - [P_1^+(x_2) - P_1^-(x_2)] = P_2^+(x_1) - P_2^-(x_1) - [P_2^+(x_2) - P_2^-(x_2)]$, which rewrites $2m = 0$ when accounting for relation (A1). This is a contradiction as $m > 0$. This proves the absence of segregated districts between integrated areas. Of course, our argument would also hold in the case of a segregated district $[x_1, x_2]$ hosting population 2.

(ii) Consider now two integrated districts separated by an uninhabited area (x_1, x_2). Given that land has a zero opportunity cost outside inhabited areas, bid rents are such that $\psi_i(x_1) = \psi_i(x_2) = 0$, $i = 1, 2$. By the bid rent expressions (2), population densities are equal to zero

$$\lambda_1(x_1) + \lambda_2(x_1) = \lambda_1(x_2) + \lambda_2(x_2) = 0. \tag{A2}$$

As area (x_1, x_2) is uninhabited, the population imbalances at x_1 and x_2 are identical, $P_i^+(x_1) - P_i^-(x_1) = P_i^+(x_2) - P_i^-(x_2)$, $i = 1, 2$. The latter condition and condition (4) imply that gradients of population densities sum equally at borders x_1 and x_2

$$\lambda_1'(x_1) + \lambda_2'(x_1) = \lambda_1'(x_2) + \lambda_2'(x_2). \tag{A3}$$

Relation (A2) suggests that the density $\lambda_1(x) + \lambda_2(x)$ falls to zero at x_1 in the first integrated area and rises from zero at x_2 in the second one. However, relation (A3) shows this cannot happen since the gradient $\lambda_1'(x) + \lambda_2'(x)$ should be the same in x_1 and x_2. This contradiction proves the absence of empty hinterlands between integrated areas. □

Third, Lemma 7 implies that an integrated neighborhood can only be surrounded by segregated districts. However, this last possibility will be shown to be impossible for the following reason: When populations differ in size, imbalances in population access provide the individuals of some group with an incentive to relocate away from the integrated neighborhood toward a segregated district.

So as to establish this result, we determine formally the incentive of an agent to relocate to a segregated area hosting the other group. Note that this argument is also of use in Section 5. To do this, let us consider a segregated district $[x_1, x_2]$ where $\lambda_i(x) > 0$ and $\lambda_j(x) = 0$, $i \neq j = 1, 2$. We derive the utility and the density gradient of group i living in this district, and only then the utility level that an agent of the other group j would obtain by relocating to this district.

Agents residing in the district have a utility level given by

$$U_i = S_i(x) + Y - \frac{\beta}{s_i} = S_i(x) + Y - \beta\lambda_i,$$

where the last term includes the density of population i only as the district is segregated. The equilibrium utility gradient is given by

$$U'_i(x) = \tau[P_i^+(x) - P_i^-(x)] + \alpha\tau[P_j^+(x) - P_j^-(x)] - \beta\lambda_i'(x) = 0, \tag{A4}$$

so that the population gradient can be written as

$$\lambda_i'(x) = \frac{\tau}{\beta}\{[P_i^+(x) + \alpha P_j^+(x)] - [P_i^-(x) + \alpha P_j^-(x)]\}. \tag{A5}$$

Population densities and land rents fall when less population can be accessed to. In equilibrium, the marginal residence cost $\beta\lambda_i'(x)$ equates the sum of the marginal access costs to individuals of her own group $\tau(P_i^+ - P_i^-)$ and to individuals of the other group $\alpha\tau(P_j^+ - P_j^-)$. The frequency of interaction α discounts intergroup interactions as they are less frequent than intragroup ones.

We now turn to an agent of the other group j, who does not reside in the segregated district $[x_1, x_2]$. When considering to relocate to some location $x \in [x_1, x_2]$, she will maximize her utility $U_j = S_j(x) - \beta/(2s_j) + z_j$ subject to her budget constraint $z_j + R(x)s_j = Y$, where the equilibrium land rent $R(x)$ is equal to the highest bid made by population i, $\psi_i(x) = \beta/[2s_i(x)^2]$. Given that $\lambda_i(x) = 1/s_i(x)$ and $s_j(x) = s_i(x)$, agent j's utility can be written as

$$U_j(x) = S_j(x) - \beta\lambda_i(x) + Y \quad x \in [x_1, x_2].$$

This expression reflects her social interactions (first term) and her use of space that diminishes with the land demand of population i (second term). Differentiating this expression and using the population gradient (A5) leads to

$$U_j'(x) = \tau(1 - \alpha)\{[P_j^+(x) - P_j^-(x)] - [P_i^+(x) - P_i^-(x)]\}. \tag{A6}$$

This expression reflects both groups' trade-off between population access and land prices. In the absence of intergroup interactions ($\alpha = 0$), the intuition is as follows: The first term in square brackets represents the population access to group j while the second term in square brackets corresponds to group i's willingness to pay for land, which is nothing but the population access to group i. In the presence of intergroup interactions ($\alpha > 0$), the interpretation remains similar but now population access accounts for these intergroup interactions too.

Last, the derivations obtained above allow us to show that the integrated district surrounded by segregated districts cannot be sustained in equilibrium. Consider some integrated district $[x_1, x_2]$ hosting both populations, as well as two neighboring segregated districts. Consider some location $x > x_2$ in the segregated right-district hosting say population 1. Let denote the mass of population 1 between locations x_2 and x by $n(x) \equiv \int_{x_2}^{x} \lambda_1(x) > 0$. For this configuration to constitute an equilibrium, it is necessary that

$$U_2(x_2) - U_2(x) \geq 0. \tag{A7}$$

By using the expression of the utility gradient for Group 2 (A6), we successively get

$$\begin{aligned} U_2'(x) &= \tau(1-\alpha)\{[P_2^+(x) - P_2^-(x)] - [P_1^+(x) - P_1^-(x)]\} \\ &= \tau(1-\alpha)\{[P_2^+(x_2) - P_2^-(x_2)] - [(P_1^+(x_2) - n(x)) - (P_1^-(x_2) + n(x))]\}. \end{aligned}$$

Given condition (4) in the integrated area (x_1, x_2), the above expression simplifies to $U_2'(x) = 2\tau(1-\alpha)n(x) > 0$. As a result, we obtain $U_2(x) - U_2(x_2) = \int_{x_2}^{x} U_2'(z)dz = 2\tau(1-\alpha)\int_{x_2}^{x} n(z)dz > 0$, which contradicts the equilibrium condition (A7). Note that an analogous argument applies when population 2 inhabits the segregated district.

The various results obtained in this appendix show Proposition 1.

APPENDIX B

The Two-district Urban Structure

By expression (5), we set $\lambda_1 = C_1 \cos[\delta(x - \phi_1)]$ and $\lambda_2 = C_2 \cos[\delta(x + \phi_2)]$.

First, $\psi_1(b_1) = \psi_2(-b_2) = 0$ implies

$$\lambda_1(b_1) = C_1 \cos[\delta(b_1 - \phi_1)] = C_2 \cos[\delta(-b_2 + \phi_2)] = \lambda_2(-b_2) = 0,$$

so that $b_1 = \phi_1 + (\pi/2\delta)$ and $b_2 = \phi_2 + (\pi/2\delta)$.

Second, $U_1'(b_1) = U_2'(-b_2) = 0$ leads to

$$-\tau P_2 - \alpha\tau P_1 + \beta C_2\delta \sin[\delta(-b_2 + \phi_2)] = 0; -\tau P_1 - \alpha\tau P_2 + \beta C_1\delta \sin[\delta(b_1 - \phi_1)] = 0.$$

Third, the population constraints $\int_0^{b_1} \lambda_1(x)dx = P_1$ and $\int_{-b_2}^{0} \lambda_2(x)dx = P_2$ imply

$$\frac{C_1}{\delta}(1 + \sin(\delta\phi_1)) = P_1; \frac{C_2}{\delta}(\sin(\delta\phi_2) + 1) = P_2,$$

which yields

$$\sin(\delta\phi_1) = \frac{P_1 - \alpha P_2}{P_1 + \alpha P_2} = -\cos(\delta b_1); \sin(\delta\phi_2) = \frac{P_2 - \alpha P_1}{P_2 + \alpha P_1} = -\cos(\delta b_2).$$

B.1 | Surplus differential

We now derive the equilibrium condition $S_2(-b_2) \geq S_2(b_1)$. For any $x \in [0, b_1]$, we can write

$$S_2(x) = \int_{-b_2}^{0} (1 - \tau|x - y|)\lambda_2(y)dy + \alpha \int_{0}^{b_1} (1 - \tau|x - y|)\lambda_1(y)dy.$$

Applying this expression to the locations $x = -b_2$ and $x = b_1$, we get

$$S_2(-b_2) - S_2(b_1) = -\tau \int_{-b_2}^{0} (2y - b_1 + b_2)\lambda_2(y)dy - \alpha\tau \int_{0}^{b_1} (2y - b_1 + b_2)\lambda_1(y)dy.$$

One computes

$$\begin{aligned} \int_{-b_2}^{0} (2y - b_1 + b_2)\lambda_2(y)dy &= (-b_1 + b_2)P_2 + \int_{-b_2}^{0} (2y)\lambda_2(y)dy \\ &= (-b_1 + b_2)P_2 + C_2 \int_{-b_2}^{0} (2y)\cos[\delta(y + \phi_2)]dy. \\ &= (-b_1 + b_2)P_2 + C_2 \frac{2}{\delta^2}(\cos\delta\phi_2 - \delta b_2) \end{aligned}$$

where the last equality is obtained by using

$$\begin{aligned} \int_{-b_2}^{0} (2y)\cos(\delta(y + \phi_2))dy &= \frac{2}{\delta^2} \int_{-\delta b_2}^{0} \delta y \cos(\delta y + \delta\phi_2)\, d\delta y \\ &= \frac{2}{\delta^2} \int_{-\delta b_2}^{0} z\cos(z + \delta\phi_2)dz \\ &= \frac{2}{\delta^2}(\cos\delta\phi_2 - \cos\delta(\phi_2 - b_2) + \delta b_2 \sin\delta(\phi_2 - b_2)) \\ &= \frac{2}{\delta^2}(\cos\delta\phi_2 - \delta b_2) \end{aligned}.$$

Also, one computes

$$\begin{aligned} \int_{0}^{b_1} (2y - b_1 + b_2)\lambda_1(y)dy &= (-b_1 + b_2)P_1 + \int_{0}^{b_1} (2y)\lambda_1(y)dy \\ &= (-b_1 + b_2)P_1 + 2C_1 \int_{0}^{b_1} y\cos[\delta(y - \phi_1)]dy, \\ &= (-b_1 + b_2)P_1 + 2C_1 \frac{1}{\delta^2}(-\cos\delta\phi_1 + \delta b_1) \end{aligned}$$

where the last equality is obtained by

$$\begin{aligned} \int_{0}^{b_1} y\cos[\delta(y - \phi_1)]dy &= \frac{1}{\delta^2} \int_{0}^{\delta b_1} \delta y \cos(\delta y - \delta\phi_1)d\delta y \\ &= \frac{1}{\delta^2} \int_{0}^{\delta b_1} z\cos(z - \delta\phi_1)dz \\ &= \frac{1}{\delta^2}(-\cos\delta\phi_1 + \delta b_1) \end{aligned}.$$

Therefore, the surplus differential is positive if $S_2(-b_2) - S_2(b_1) \geq 0$, that is, if

$$C_2(\cos\delta\phi_2 - \delta b_2) + \alpha C_1(-\cos\delta\phi_1 + \delta b_1) \leq \frac{\delta^2}{2}(b_1 - b_2)(P_2 + \alpha P_1),$$

or equivalently

$$-\delta b_1 P_2(-1+\alpha^2)+\alpha(P_1+\alpha P_2)\cos\delta\phi_1-(P_2+\alpha P_1)\cos\delta\phi_2\geq 0. \tag{B1}$$

Because

$$\begin{aligned}\cos^2\delta\phi_1 &= 1-\sin^2\delta\phi_1=1-(\frac{P_1-\alpha P_2}{P_1+\alpha P_2})^2=4\alpha\frac{P_1P_2}{(P_1+\alpha P_2)^2}\\ \cos^2\delta\phi_2 &= 1-\sin^2\delta\phi_2=1-(\frac{P_2-\alpha P_1}{P_2+\alpha P_1})^2=4\alpha\frac{P_1P_2}{(P_2+\alpha P_1)^2}\end{aligned},$$

we get

$$(\alpha P_1+P_2)\cos\delta\phi_2-\alpha(P_1+\alpha P_2)\cos\delta\phi_1=2(1-\alpha)\sqrt{\alpha P_1P_2}.$$

Thus, condition (B1) becomes

$$2\sqrt{\alpha P_1/P_2}\leq(1+\alpha)[\pi-\arccos(\frac{P_1/P_2-\alpha}{P_1/P_2+\alpha})].$$

B.2 | Comparative statics

Here is the comparative statics analysis of the two-district city equilibrium. First, population densities increase and district borders shrink as the access cost increases and the preference for space falls (a higher $\delta^2=2\tau/\beta$ raises C_i and reduces b_i; see relations (7) and (8)). The population density increases as population sizes grow in equal proportions (keeping P_1/P_2 constant, higher values of P_1 and P_2 raise C_i). A larger share of population 1 (P_1/P_2) leads district 1 to expand and district 2 to shrink. The city expands (with a larger b_i) if the frequency of intergroup interaction α falls (see relation (8)). Thus, more frequent intergroup interactions concentrate populations further as they are able to bid more for land.

B.3 | Utilities

Utilities can be computed as

$$\begin{aligned}U_1 &= P_1(1-\alpha^2)+\frac{\tau}{2}b_2\alpha(P_2+\alpha P_1)-\frac{\tau}{2}b_1(P_1+\alpha P_2)+\frac{\tau}{\delta}(\alpha-1)\sqrt{\alpha P_1P_2}+Y\\ U_2 &= \frac{\tau}{2}b_2(\alpha P_1+P_2)-\alpha\frac{\tau}{2}b_1(P_1+\alpha P_2)-\frac{\tau}{\delta}(1-\alpha)\sqrt{\alpha P_1P_2}+Y\end{aligned}.$$

APPENDIX C

THE Three-district Urban Structures

Here, we focus on the case where the large population 1 locates in the city center. The converse configuration can be obtained by swapping subscripts 1 and 2. By using the expressions $\lambda_1=C_1\cos(\delta x)$ if $x\in[0,b_1]$ and $\lambda_2=C_2\cos[\delta(x-\phi_2)]$ if $x\in[b_1,b_2]$, the conditions for population conservation and the land rent arbitrage at district borders become

$$\begin{cases} P_1 = \int_{-b_1}^{b_1} \cos(\delta x)dx = 2C_1\delta^{-1}\sin\delta b_1 \\ P_2 = 2C_2\delta^{-1}[\sin\delta(b_2-\phi_2) - \sin\delta(b_1-\phi_2)] \\ \psi_2(b_1) = \frac{\beta}{2}C_2^2\cos^2[\delta(b_1-\phi_2)] = \frac{\beta}{2}C_1^2\cos^2(\delta b_1) = \psi_1(b_1) \\ \lambda_2(b_2) = C_2\cos[\delta(b_2-\phi_2)] = 0 \end{cases}. \tag{C1}$$

The last line of (C1) implies that $b_2 - \phi_2 = \pi/(2\delta)$. In addition, note that $U_2'(b_2)= 0$ implies that by expression (A5), $\lambda_2'(b_2) = -(\tau/\beta)[P_2 + \alpha P_1]$, which yields $C_2 = \delta(P_2 + \alpha P_1)/2$ as $\lambda_2'(b_2) = C_2\sin\delta(b_2 - \phi_2) = C_2$.

The first three lines of (C1) become

$$\frac{\delta P_1}{2C_1} = \sin\delta b_1;\ 1 - \frac{\delta P_2}{2C_2} = \sin\delta(b_1-\phi_2);\ \frac{\cos[\delta(b_1-\phi_2)]}{\cos(\delta b_1)} = \frac{C_1}{C_2}. \tag{C2}$$

Squaring the first two expressions and using $\cos^2 x = 1 - \sin^2 x$ yields $C_1^2 = (\delta/2)^2(P_1^2 + P_2^2 + 2\alpha P_1P_2)$. Thus,

$$\sin\delta(b_1-\phi_2) = 1 - \frac{P_2}{(P_2+\alpha P_1)} = \frac{\alpha P_1}{\alpha P_1 + P_2};\ \sin^2\delta b_1 = \frac{P_1^2}{P_1^2+P_2^2+2\alpha P_1P_2}.$$

C.1 | Utility differential

We now derive the equilibrium condition $U_2(0) - U_2(b_1)$. For any $x \in [0, b_1]$, we can write

$$\begin{aligned} U_2(0) &= \alpha\int_{-b_1}^{-b2}(1+\tau y)\lambda_1(y)dy + 2\int_0^{b_2}(1-\tau y)\lambda_2(y)dy \\ &+ \alpha\int_{b_2}^{b_1}(1-\tau y)\lambda_1(y)dy - \beta\lambda_2(0) \\ U_2(b_1) &= \alpha\int_{-b_1}^{-b2}(1-\tau(b_1-y))\lambda_1(y)dy + \int_{-b_2}^{b_2}(1-\tau(b_1-y))\lambda_2(y)dy \\ &+ \alpha\int_{b_2}^{b_1}(1-\tau(b_1-y))\lambda_1(y)dy - \beta\lambda_1(b_1) \end{aligned}.$$

Note that $\lambda_1(b_1) = 0$ and $U_2(b_1) = S_2(b_1)$. By replacing the value of $(b_1, b_2, \phi_1, \phi_2, C_1, C_2)$, we get that the condition $U_2(0) - U_2(b_1) \geq 0$ is equivalent to

$$\pi(1+\alpha) - 2\sqrt{P(P+2\alpha)} \geq 2(1+\alpha)\arcsin\left(\frac{\alpha}{P+\alpha}\right),$$

where $P = P_1/P_2$.

C.2 | Comparative statics when the large population locates in the city center

First, population densities increase and district borders shrink as the access cost increases and the preference for space falls (a higher $\delta^2 = 2\tau/\beta$ raises both C_1 and C_2 by relation (11), while it reduces $b_2 - b_1$ and b_1 by relations (13) and (12)). Second, population densities increase as population sizes grow in equal proportion (keeping P_1/P_2 constant, larger populations P_1 and P_2 raise C_1 and C_2). Third, the city expands (with larger b_1 and $b_2 - b_1$) when the frequency of intergroup interactions α decreases. The lower returns from intergroup interactions induce lower bid rents, and thus the dispersion of agents. Fourth, a larger share of population 1 (P_1/P_2) leads the central district to shrink and the edge district to expand (a higher ratio P_1/P_2 raises $b_1 - b_2$ and decreases b_1 by relations (13) and (12)).

C.3 | Comparative statics when the large population locates in the city edge

The comparative statics analysis is derived in a way similar to that used in the previous three-district configuration. We simply need to swap subscripts 1 and 2. Hence, population densities increase and the district borders shrink as the access cost increases and the preference for space falls (a higher $\delta^2 = 2\tau/\beta$ raises C_1 and C_2 while it reduces $b_1 - b_2$ and b_2). Population densities increase as population sizes grow in equal proportions (keeping P_1/P_2 constant, higher values of P_1 and P_2 raise C_1 and C_2). The city expands (with larger b_2 and $b_1 - b_2$) when the frequency of intergroup interaction α decreases. A larger share of population 1 (P_1/P_2) decreases the area of the central district hosting population 2 and increases that of the edge district hosting population 1 (a higher ratio P_1/P_2 decreases b_2 and increases $b_1 - b_2$).

APPENDIX D

Here we show that no asymmetric configuration with three districts can be a spatial equilibrium. We consider the following equilibrium candidate

$$\begin{aligned}\lambda_1(x) &= C_1\cos(\delta x),\ y \in [-b_2, b_1]\\ \lambda_2(x) &= \begin{cases} C_2\cos(\delta(x-\phi_1)),\ x \in [b_1, b_3] \\ C_3\cos(\delta(x+\phi_2)),\ x\in[-b_4, -b_2]\end{cases},\end{aligned}$$

and we show that equilibrium conditions are not compatible with an asymmetric solution.

The conditions $U_2'(b_3) = U'_2(-b_4) = 0$ lead to $-\tau P_2 - \alpha\tau P_1 + C_2\sin(\delta(b_3 - \phi_1)) = \tau P_2 + \alpha\tau P_1 + C_3 \sin(\delta(-b_4 + \phi_2)) = 0$. By using the conditions $\psi_1(b_1) = \psi_2(b_1)$, $\psi_1(b_1) = \psi_2(b_1)$, and $\psi_2(-b_2) = \psi_1(-b_2)$, we get $\delta(b_3 - \phi_1) = -\delta(-b_4 + \phi_2) = \pi/2$ and

$$C_1^2 = C_2^2\cos^2(\delta(b_1 - \phi_1))/\cos^2(\delta b_1) = C_3^2\cos^2(\delta(-b_2 + \phi_2))/\cos^2(\delta b_2).$$

The conditions $U'_2(b_3) = U'_2(-b_4) = U'_1(0) = 0$ lead, respectively, to $C_2 = C_3 = \tau(P_2 + \alpha P_1)/(\beta\delta)$ and

$$\sin(\delta b_2) - \sin(\delta b_1) - \alpha(\sin(\delta(-b_2 + \phi_2)) + \sin(\delta(b_1 - \phi_1))) = 0.$$

The two above conditions can be written as

$$\begin{aligned} m_2 - m_1 - \alpha(m_4 + m_3) &= 0\\ (1 - m_3^2)(1 - m_2^2) - (1 - m_4^2)(1 - m_1^2) &= 0'\end{aligned}$$

where m_1, m_2, m_3, and m_4 denote $\sin(\delta b_1)$, $\sin(\delta b_2)$, $\sin(\delta(b_1 - \phi_1))$, and $\sin(\delta(-b_2 + \phi_2))$.

The solutions for m_2 and m_4 are

$$(m_1, -m_3) \quad \text{and}$$
$$\left(\frac{-m_1 + m_1^3 - 2\alpha m_3 + 2\alpha m_1^2 m_3 - \alpha^2 m_1 + \alpha^2 m_1 m_3^2}{-1 + m_1^2 + \alpha^2 - \alpha^2 m_3^2}, \frac{-m_3 + m_1^2 m_3 - 2\alpha m_1 + 2\alpha m_1 m_3^2 - \alpha^2 m_3 + \alpha^2 m_3^2}{-1 + m_1^2 + \alpha^2 - \alpha^2 m_3^2}\right).$$

The first solution corresponds to the symmetric equilibrium while the second one is our asymmetric candidate.

By plugging the second solution into the two constraints, $C_1/\delta(m_1 + m_2) = P_1$ and $C_2/\delta(m_4 - m_3 + 2) = P_2$, we get a system of equations for m_1 and m_3. The solutions are given by $(m_1, m_3) = (P_1(\alpha - 1)/(P_2 + \alpha P_1)$,

$P_1(\alpha - 1)/(P_2 + \alpha P_1))$ and $(m_1, m_3) = (-P_1(\alpha + 1)/(P_2 + P_1\alpha), P_1(1 + \alpha)(P_2 + P_1\alpha))$. Both solutions imply that $m_1 < 0$ meaning that the district border b_1 would be negative.

APPENDIX E

Table E1 (*resp.* Table E2) provides the ranking of urban configurations (21, 212, and 121) for individuals of population 1 (*resp.* for individuals of population 2).

TABLE E1 Urban configuration ranking for population 1

	P_1/P_2 = 1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
$\alpha = 0.9$	21 212 121	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212	212	212	212
0.8	21 212 121	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212	212	212
0.7	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212	212	212
0.6	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212	212
0.5	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212
0.4	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21
0.3	21 212 121	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21
0.2	21 212 121	21 212	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21
0.1	21 212 121	21 212	21 212	21 212	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21	212 21

Note: Each cell ranks urban configurations from most preferred to least preferred by population 1 in terms of the population ratio P_1/P_2 and the intensity of intergroup interactions α.

TABLE E2 Urban configuration ranking for population 2

	$P_1/P_2 = 1.5$	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
$\alpha = 0.9$	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212	212	212	212
0.8	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212	212	212
0.7	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212	212	212
0.6	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212	212
0.5	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	212
0.4	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212
0.3	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212
0.2	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212
0.1	21 121 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212	21 212

Note: Each cell ranks urban configurations from most preferred to least preferred by population 2 in terms of the population ratio P_1/P_2 and the intensity of intergroup interactions α.