DIRECT ADAPTIVE CONTROL OF A LINEAR PARABOLIC SYSTEM

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Abstract: This paper presents an adaptive model reference algorithm for a linear distributed parameter system with input boundary control and output boundary reference. The control and adaptation laws are based a model reference adaptive control approach. . This controller is applied to a tubular reactor model with unknown kinetic parameters. Simulation results are shown for set-point changes, variation of kinetic parameters and input perturbation.

Keywords: distributed-parameter systems, Lyapunov methods, adaptive control, process control, boundary conditions, parameter estimation

1. INTRODUCTION

Several chemical reactors are essentially distributed processes and their dynamics can be appropriately represented by partial differential equations (PDE). For instance, a tubular reactor modeled by mass balances leads to parabolic PDE which account for convection, dispersion and reaction phenomena occurring in the reactor. Traditional process control uses a transfer function representation obtained by input/output identification for those systems. Feedback controllers designed with these models often include adaptive or predictive strategies to account for process nonlinearities and model mismatch (Ogunnaike and Ray, 1994). Using a PDE model represents an interesting approach since it gives a more accurate representation of reality and then more information for the process engineer. Moreover, it gives an interesting framework for the analysis of sensor and actuator location.

Various approaches have been considered to use the PDE phenomenological model directly. Ray (1981) proposed to divide control approaches on PDEs in two groups . The first group is composed of early lumping methods. These approaches use a preliminary discretization of the PDE model to obtain a set of ODEs. That lumping is often realized by numerical techniques such as finite difference, orthogonal collocation or finite elements. Regarding the numerous equations obtained, model reduction technique may also be used (Christofides, 1996). Finally, lumped control design methods could be applied on those models. The other group is based on late lumping methods where the controller design problem is solved directly with the PDE model. When necessary, some lumping may be applied for controller implementation.

The problem addressed here is the control of a tubular reactor by a late lumping approach. The dynamics of the reactor are defined by two parabolic equations representing mass balances of each species. It is assumed that the reaction kinetic is not well known and could vary with time, thus an adaptive approach have been considered.

The control of parabolic PDE have been addressed previously by Hong and Bentsman (1994) and more recently in a more theoretical framework by Böhm et al (1998). They provide a design solution for systems in which the control action appears in explicitly in the PDE system. In this tubular reactor problem, the controller action is the concentration of one of the reactants at the inlet. The problem to be solve is then a boundary control problem. Bourrel et al. (1996) have addressed this problem for bioreactor control and they have proposed a feedback control law based on exact linearization in the case of hyperbolic systems of PDEs .

We propose in this paper an adaptive controller using a reference model based on a parabolic PDE system. The first part of this paper shows the development of the model base adaptive controller using a Lyapunov approach. In the second part, simulation results are shown for step-point changes and for perturbation in the kinetic parameters.

2. MODEL REFERENCE CONTROLLER DESIGN

A tubular isothermal chemical reactor can be modeled using mass balances on each reactant. This leads to the well-known dispersive model. A reactor with two species is modeled here with two mass balances. The first one is on reactant L for which a set point is specified at the end of the reactor but it may be variable at the inlet. The second reactant C, is used as control variable at the inlet. This leads to the following distributed parameter system (DPS) described by two parabolic linear equations and their boundary conditions:

$$\frac{\partial C(z,t)}{\partial t} = -v\frac{\partial C(z,t)}{\partial z} + D\frac{\partial^2 C(z,t)}{\partial z^2} - k_1 C(z,t) - k_2 L(z,t) \quad (1)$$

$$\frac{\partial L(z,t)}{\partial t} = -v\frac{\partial L(z,t)}{\partial z} + D\frac{\partial^2 L(z,t)}{\partial z^2} - k_3 C(z,t) - k_4 L(z,t) \quad (2)$$

$$\left.\frac{\partial L(z,t)}{\partial z}\right|_0 = \frac{v}{D}(L(0,t) - L_{in}(t)) \quad (3)$$

$$\left.\frac{\partial C(z,t)}{\partial z}\right|_0 = \frac{v}{D}(C(0,t) - C_{in}(t)) \quad (4)$$

$$\left.\frac{\partial C(z,t)}{\partial z}\right|_1 = \left.\frac{\partial C_{out}}{\partial z}\right|_1 = 0 \quad (5)$$

$$\left.\frac{\partial L(z,t)}{\partial z}\right|_1 = \left.\frac{\partial L_{out}}{\partial z}\right|_1 = 0 \quad (6)$$

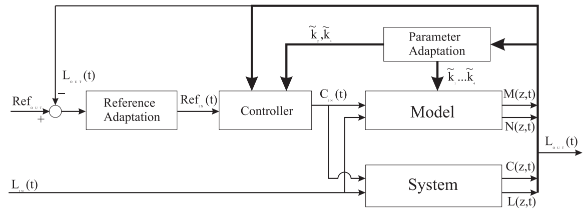

Fig. 1. Controller structure

In these equations, the parameters v and D can be determined from hydrodynamic experimentation on the process but parameters $k_1 - k_4$ are considered unknown. The following PDEs are then used as a reference model with the same boundary conditions as the system:

$$\frac{\partial M(z,t)}{\partial t} = -v\frac{\partial M(z,t)}{\partial z} + D\frac{\partial^2 M(z,t)}{\partial z^2} - \tilde{k}_1 M(z,t) - \tilde{k}_2 N(z,t) \quad (7)$$

$$\frac{\partial N(z,t)}{\partial t} = -v\frac{\partial N(z,t)}{\partial z} + D\frac{\partial^2 N(z,t)}{\partial z^2} - \tilde{k}_3 M(z,t) - \tilde{k}_4 N(z,t) \quad (8)$$

In this approach we consider that one input, C_{in}, in used as the control variable and the other variable input, L_{in}, is free. The controlled output will be L_{out} while C_{out} is free. The proposed controller structure is illustrated in figure (1). A Lyapunov approach is used to design the adaptation mechanism and the control law. Let us first define the error equations of the system:

$$e_C(z,t) = C(z,t) - M(z,t) \quad (9)$$

$$e_L(z,t) = L(z,t) - N(z,t) \quad (10)$$

and the parameter estimation errors:

$$\psi_1(t) = \tilde{k}_1(t) - k_1 \quad (11)$$

$$\psi_2(t) = \tilde{k}_2(t) - k_2 \quad (12)$$

$$\psi_3(t) = \tilde{k}_3(t) - k_3 \quad (13)$$

$$\psi_4(t) = \tilde{k}_4(t) - k_4 \quad (14)$$

Time differentiation of equations (9)-(14) those errors leads to:

$$\dot{e_C} = -v\frac{\partial e_C}{\partial z} + D\frac{\partial^2 e_C}{\partial z^2} - k_1 e_C M - k_2 e_L + \psi_1 M + \psi_2 N \quad (15)$$

$$\dot{e_L} = -v\frac{\partial e_L}{\partial z} + D\frac{\partial^2 e_L}{\partial z^2} - k_3 e_C M - k_4 e_L + \psi_3 M + \psi_4 N \quad (16)$$

$$\dot{\psi_1} = \dot{\tilde{k}}_1 \quad (17)$$

$$\dot{\psi_2} = \dot{\tilde{k}}_2 \quad (18)$$

$$\dot{\psi_3} = \dot{\tilde{k}}_3 \quad (19)$$

$$\dot{\psi_4} = \dot{\tilde{k}}_4 \quad (20)$$

The above equations can be used to design control and adaptation laws based on the Lyapunov second method. The objective is then to find a positive definite function with continuous first partial derivative $V(x,t)$ such that $V(0,t) = 0$. If the time derivative of this function $\dot{V}(x,t)$ is definite negative then the system is asymptotically stable. If $V(x,t)$ is radially unbounded, this property is global. Consider the following Lyapunov function:

$$\begin{aligned} V(z,t) = & \frac{1}{2}\langle e_C, e_C\rangle + \frac{1}{2}\langle e_L, e_L\rangle \\ & + \frac{1}{2\epsilon}(C_{in} - Ref_{in})^2 + \frac{1}{2\gamma}(C_{out} - Ref_{out})^2 \\ & + \frac{1}{2a}\psi_1^2 + \frac{1}{2b}\psi_2^2 + \frac{1}{2c}\psi_4^2 + \frac{1}{2d}\psi_3^2 \end{aligned} \tag{21}$$

The time derivative of this function is equal to:

$$\begin{aligned} \dot{V} = & \langle e_C, \dot{e}_C\rangle + \langle e_L, \dot{e}_L\rangle + \frac{1}{\gamma}(C_{out} - Ref_{out})\dot{C}_{out} \\ & + \frac{1}{\epsilon}(C_{in} - Ref_{in})(\dot{C}_{in} - \dot{Ref}_{in}) \\ & + \frac{1}{a}\psi_1\dot{\psi}_1 + \frac{1}{b}\psi_2\dot{\psi}_2 + \frac{1}{c}\psi_3\dot{\psi}_3 + \frac{1}{d}\psi_4\dot{\psi}_4 \end{aligned} \tag{22}$$

Introducing equation (15) into (21) leads to:

$$\begin{aligned} \dot{V} = & \langle e_C, -v\frac{\partial e_C}{\partial z}\rangle + \langle e_C, D\frac{\partial^2 e_C}{\partial z^2}\rangle + \langle e_C, -k_1 e_C\rangle \\ & + \langle e_C, -k_2 e_L\rangle + \langle e_C, \psi_1 M\rangle + \langle e_C, \psi_2 N\rangle \\ & + \langle e_L, -v\frac{\partial e_L}{\partial z}\rangle + \langle e_L, D\frac{\partial^2 e_L}{\partial z^2}\rangle + \langle e_L, -k_3 e_C\rangle \\ & + \langle e_L, -k_4 e_L\rangle + \langle e_L, \psi_3 M\rangle + \langle e_L, \psi_4 N\rangle \\ & + \frac{1}{\epsilon}(C_{in} - Ref_{in})(\dot{C}_{in} - \dot{Ref}_{in}) \\ & + \frac{1}{a}\psi_1\dot{\psi}_1 + \frac{1}{b}\psi_2\dot{\psi}_2 + \frac{1}{c}\psi_3\dot{\psi}_3 + \frac{1}{d}\psi_4\dot{\psi}_4 \\ & + \frac{1}{\gamma}(C_{out} - Ref_{out})\dot{C}_{out} \end{aligned} \tag{23}$$

Integration by part gives:

$$\begin{aligned} \dot{V} = & \langle e_C, -v\frac{\partial e_C}{\partial z}\rangle + \langle \frac{\partial e_C}{\partial z}, -D\frac{\partial e_C}{\partial z}\rangle \\ & + e_C D\frac{\partial e_C}{\partial z}\Big|_{out} - e_C D\frac{\partial e_C}{\partial z}\Big|_{in} + \langle e_C, -k_1 e_C\rangle \\ & + \langle e_C, -k_2 e_L\rangle + \langle e_C, \psi_1 M\rangle + \langle e_C, \psi_2 N\rangle \\ & + \langle e_L, -v\frac{\partial e_L}{\partial z}\rangle + \langle \frac{\partial e_L}{\partial z}, -D\frac{\partial e_L}{\partial z}\rangle \\ & + e_L D\frac{\partial e_L}{\partial z}\Big|_{out} - e_L D\frac{\partial e_L}{\partial z}\Big|_{in} + \langle e_L, -k_3 e_C\rangle \\ & + \langle e_L, -k_4 e_L\rangle + \langle e_L, \psi_3 M\rangle + \langle e_L, \psi_4 N\rangle \\ & + \frac{1}{\epsilon}(C_{in} - Ref_{in})(\dot{C}_{in} - \dot{Ref}_{in}) \\ & + \frac{1}{a}\psi_1\dot{\psi}_1 + \frac{1}{b}\psi_2\dot{\psi}_2 + \frac{1}{c}\psi_3\dot{\psi}_3 + \frac{1}{d}\psi_4\dot{\psi}_4 \\ & + \frac{1}{\gamma}(C_{out} - Ref_{out})\dot{C}_{out} \end{aligned} \tag{24}$$

Using the following control law:

$$\dot{Ref}_{in} = \theta(L_{out} - Ref_{out}) \tag{25}$$

$$\begin{aligned} \dot{C}_{in} = & -\frac{\epsilon}{(C_{in} - Ref_{in})}(\langle e_C, -v\frac{\partial e_C}{\partial z}\rangle \\ & + e_C D\frac{\partial e_C}{\partial z}\Big|_{out} - e_C D\frac{\partial e_C}{\partial z}\Big|_{in} \\ & + \langle e_L, -v\frac{\partial e_L}{\partial z}\rangle + e_L D\frac{\partial e_L}{\partial z}\Big|_{out} \\ & - e_L D\frac{\partial e_L}{\partial z}\Big|_{in} + \frac{1}{\gamma}(C_{out} - Ref_{out})\dot{C}_{out} \\ & - \theta(L_{out} - Ref_{out}) \\ & - \tilde{k}_2\langle e_C, e_L\rangle - \tilde{k}_3\langle e_C, e_L\rangle) \end{aligned} \tag{26}$$

the derivative of the Lyapunov function becomes:

$$\begin{aligned} \dot{V} = & \langle \frac{\partial e_C}{\partial z}, -D\frac{\partial e_C}{\partial z}\rangle + \langle \frac{\partial e_L}{\partial z}, -D\frac{\partial e_L}{\partial z}\rangle \\ & + \langle e_C, -k_1 e_C\rangle + \langle e_L, -k_4 e_L\rangle \\ & + \langle e_L, \psi_1 M\rangle + \langle e_C, \psi_2 N\rangle \\ & + \langle e_C, \psi_2 e_L\rangle + \langle e_L, \psi_3 e_C\rangle \\ & + \langle e_L, \psi_3 M\rangle + \langle e_L, \psi_4 N\rangle \\ & + \frac{1}{a}\psi_1\dot{\psi}_1 + \frac{1}{b}\psi_2\dot{\psi}_2 + \frac{1}{c}\psi_3\dot{\psi}_3 + \frac{1}{d}\psi_4\dot{\psi}_4 \end{aligned} \tag{27}$$

Finally, the following adaptation laws are used:

$$\dot{\psi}_1 = -a\langle e_C, M\rangle \tag{28}$$

$$\dot{\psi}_2 = -b(\langle e_C, N\rangle + \langle e_C, e_L\rangle) \tag{29}$$

$$\dot{\psi}_3 = -c(\langle e_L, M\rangle + \langle e_L, e_C\rangle) \tag{30}$$

$$\dot{\psi}_4 = -d\langle e_L, N\rangle \tag{31}$$

The final expression of the derivative of the Lyapunov function is then:

$$\begin{aligned} \dot{V} = & \langle \frac{\partial e_C}{\partial z}, -D\frac{\partial e_C}{\partial z}\rangle + \langle e_C, -k_1 e_C\rangle \\ & + \langle \frac{\partial e_L}{\partial z}, -D\frac{\partial e_L}{\partial z}\rangle + \langle e_L, -k_4 e_L\rangle \end{aligned} \tag{32}$$

This expression is negative definite, ensuring the asymptotic stability of the adaptation and control laws. To avoid division by zero in equation 26, the following modified control law is considered:

$$\begin{aligned} \dot{\tilde{C}}_{in} = & \frac{(C_{in} - Ref_{in})^2}{(w + C_{in} - Ref_{in})^2}\tilde{C} \\ & + \frac{(C_{in} - Ref_{in})}{(w + C_{in} - Ref_{in})^2}f_c \end{aligned} \tag{33}$$

$$\dot{C}_{in} = \tilde{C}_{in} \tag{34}$$

$$\begin{aligned} f_c = & (\langle e_C, -v\frac{\partial e_C}{\partial z}\rangle + \langle e_L, -v\frac{\partial e_L}{\partial z}\rangle \\ & + e_C D\frac{\partial e_C}{\partial z}\Big|_{out} - e_C D\frac{\partial e_C}{\partial z}\Big|_{in} \end{aligned} \tag{35}$$

$$
\begin{aligned}
&+ e_L D \frac{\partial e_L}{\partial z}\bigg|_{out} - e_L D \frac{\partial e_L}{\partial z}\bigg|_{in} \\
&+ \frac{1}{\gamma}(C_{out} - Ref_{out})\dot{C}_{out} \\
&- \theta(L_{out} - Ref_{out}) \\
&- \tilde{k}_2\langle e_C, e_L\rangle - \tilde{k}_3\langle e_C, e_L\rangle) \qquad (36)
\end{aligned}
$$

were w is a tuning parameter with a small positive value. Note that both version of the control law are identical for $\dot{\tilde{C}}_{in} = 0$.

3. RESULTS AND DISCUSSION

Numerical simulation of the control algorithm applied to the system has been performed by using a sequencing algorithm with a 100 node mesh for a 1 meter reactor (Renou *et al.*, 2000). In this algorithm, convection, dispersion and reaction phenomena are considered successively for each time step. Initial parameters of the system are:

$$
\begin{aligned}
v &= 0.05\ m/s & k_2 &= 0.03\ 1/s \\
D &= 0.0005\ m^2/s & k_3 &= 0.04\ 1/s \\
k_1 &= 0.02\ 1/s & k_4 &= 0.03\ 1/s
\end{aligned}
$$

Controller parameters have been chosen to minimize overshoot and oscillatory response. The objective here is to reject perturbations and model parameters variation rather then reference tracking. Simulations are started at steady state for an input of L and C equals to 1. The tuning parameters are equal to:

$$
\begin{aligned}
\epsilon &= .1 \;\; \theta = .05 \;\; \gamma = 100 \\
a &= 2 \;\; b = 2 \;\; c = 2 \;\; d = 2
\end{aligned} \qquad (37)
$$

$$
Ref_{out} = 0.1745 \qquad (38)
$$

Figure (2) show the response of the system to a L set-point step from 0.17 to 0.20 g/L. The output curves of L and C are smooth and do not present any overshoot as required. A perfect match between the model and system is obtained and no variation of adaptation parameters is observed.

Figure (3) shows the response of the controlled system for a system variation. The rate constants have been modified to the following values:

$$
\begin{aligned}
k_1 &= 0.02\ 1/s \;\; k_2 = 0.03\ 1/s \\
k_3 &= 0.04\ 1/s \;\; k_4 = 0.03\ 1/s
\end{aligned}
$$

The adaptation mechanism is oscillatory but rapidly converges to the system values as shown in figure (4). Model and system simulation curves show differences for a shorter time than the adaptation mechanism. This situation suggests some oscillation between multiple possible choices for parameter convergence. Output L admits a important deviation from desired set point but returns to set point without steady state error. This behavior could be explained by the slow output reference adaptation mechanism used. But in practice, rate constants are not changing so aggressively, thus the simulated case can be considered as a worst case situation.

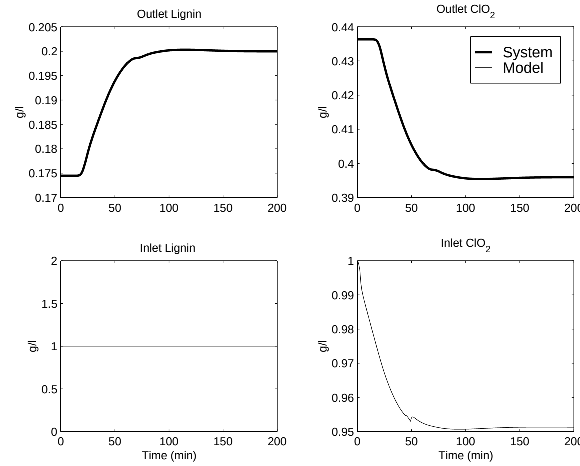

Fig. 2. Set point change on Ref_{out}

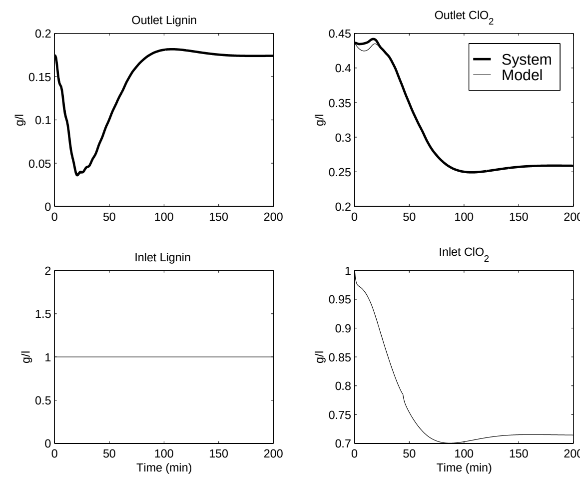

Fig. 3. Model perturbation

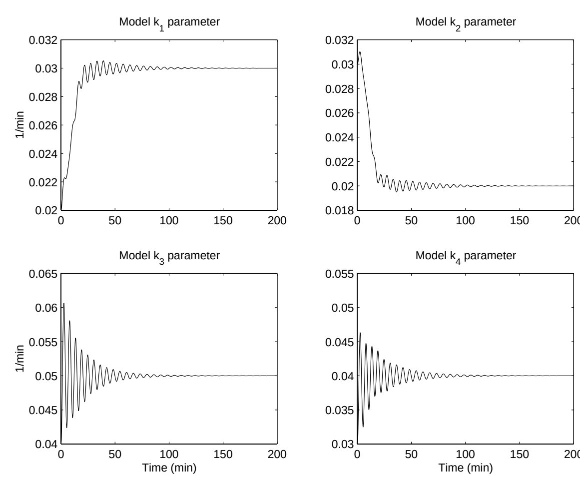

Fig. 4. Parameters adaptation

Figure (5) shows the response of the system to a step response combined to system parameter variation. The parameter adaptation curves are the same as observe in figure (4). This shows that the adaptation mechanism is a function or the error between the model and the system instead of the characteristics of the input. Figure (6) shows the effect of adding a noisy L input to the condition of the preceding simulation. The convergence around the set point and adaptation mechanism still works similarly but the variation of L affects directly the output and is not rejected efficiently.

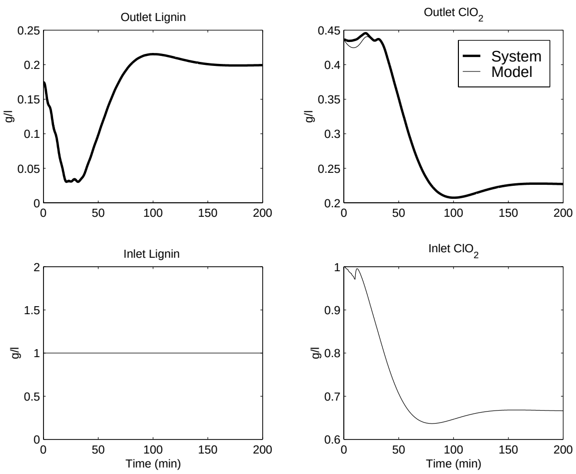

Fig. 5. Set point change and model perturbation

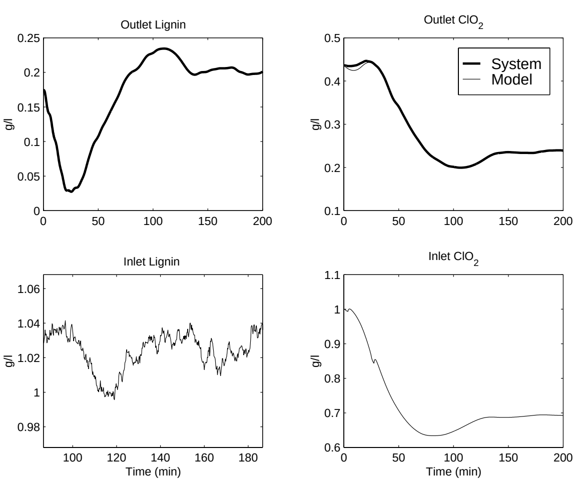

Fig. 6. Set point change, model perturbation and L_{in} variation

4. CONCLUSION

An adaptive model reference controller has been presented for a distributed parameter system with input boundary control and output boundary reference. This algorithm includes a PDE model reference, an adaptation law and a reference modification law. Simulations have shown a good set-point step response and efficient parameter tracking. Response to noisy input of the uncontrolled reactant L needs to be improved. This could probably be done by using a feedforward strategy in the reference modification law. Moreover, the nonlinear kinetics case has to be studied to cover a wider class of applications in reactor control field.

5. ACKNOWLEDGEMENTS

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