Full-Wave Synthesis of Modulated Metasurface Antennas

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***Abstract*—A full-wave synthesis algorithm for modulated metasurface antennas is presented. It is able to provide arbitrary radiation patterns, with any polarization. The algorithm does not use the local periodicity approximation, but is directly based on the electric field integral equation (EFIE). Using Fourier-Bessel basis functions (FBBFs), one can efficiently discretize the surface currents. An inverse problem based on the EFIE is then formulated to derive the surface impedance from the knowledge of the currents. It has been observed that the FBBFs are also more suited than the Zernike basis for the surface impedance discretization. In the case of antenna applications, only the visible part of the surface currents spectrum is known from pattern specifications. This visible part can be combined with the near-field of the average reactance (SW contribution) to derive the required impedance boundary condition (IBC); this latter is constrained to be anti-Hermitian as required for implementation in the absence of losses. An example of shaped beam design is presented and numerically validated.**

***Index Terms*—beam shaping, metasurfaces (MTSs), integral equations, basis functions, impedance boundary condition.**

I. Introduction

Metasurfaces (MTSs) are the 2D version of metamaterials and can be used to manipulate surface waves (SW) or to control space waves transmission [1], [2]. The first type of application leads to a new class of transformation Electro-Magnetics devices [3], [4] and has attracted the interest of the antenna community. Indeed, the control of the surface wave dispersion also allows a suitable SW to leaky wave (LW) transformation mechanism [5]. Such an antenna possesses the advantage of being flat (low profile) by nature since the feeding role relies on the SW excitation, which is done through a coplanar feed [6], [7]. Additionally, the flexibility in achieving shaped beams (active elements are usually used to obtain reconfigurable patterns) makes the MTS antennas very attractive in many applications [8]. Those surfaces implement an Impedance Boundary Condition (IBC) on a circular [8] (possibly elliptical [9]) domain, and are usually fed from the center by a SW launcher. The IBC interacts with the excited SW and then generates the desired radiation characteristics. In the microwave regime, the designed IBC is usually implemented by means of sub-wavelength patches printed on a square lattice. The MTS synthesis problem then consists of finding the appropriate modulated IBC for a given excitation and pattern requirements. A lot of efforts have been made in this direction, thus leading to elegant analytical expressions of the IBC capable of generating high gain polarized pencil beams [8]. However, when one needs to arbitrarily shape the beam, the problem is much more complex. Recently, a LW Flat-Optics theory has been proposed to address this challenge [10]. The basic idea behind the Flat-Optics approximation consists of locally expanding the equivalent surfaces currents as well as the aperture fields in Floquet modes. The MTS synthesis can then be carried out by matching the desired aperture field with the -1 mode of the aperture electric field expansion. A variant of this technique has also been proposed in [11]. Once the IBC has been synthesized, its validation is carried out with the Method of Moments (MoM), by discretizing the relevant EFIE [12]-[15]. The present paper briefly describes a new synthesis method directly based on the solution of the EFIE, the latter being discretized with Fourier-Bessel basis functions (FBBFs) [14]. The proposed algorithm starts directly from the desired radiation pattern and follows a systematic procedure to derive the required surface impedance.

The paper is structured as follows. Section II briefly recalls the analysis formalism. Then, the MTS synthesis is described in section III. Section IV presents numerical results and section V concludes the paper.

II. Direct problem formulation

The metasurface can be efficiently described by modeling the metallization layer with a sheet transition impedance boundary condition incorporated in a transmission line network which accounts for the presence of the grounded substrate. The sheet impedance transition is defined as the tensor relating the electric field tangent to the surface to the jump-discontinuity of the magnetic field, corresponding to the equivalent surface current flowing on the MTS

$$\mathbf{E}_t = \underline{\underline{\mathbf{Z}}}_S \cdot \hat{\mathbf{z}} \times \left(\mathbf{H}_t|_{z=0^+} - \mathbf{H}_t|_{z=0^-} \right) = \underline{\underline{\mathbf{Z}}}_S \cdot \mathbf{J} \qquad (1)$$

where $\underline{\underline{\mathbf{Z}}}_S$ is the sheet impedance, $\mathbf{E}_t$ is the total tangential electric field on the MTS, and $\mathbf{J}$ is the equivalent surface current. Starting from equation (1), and identifying the total electric field as the sum of the one radiated by the surface currents and the one provided by the excitation, expression (1) leads to the following integral equation:

$$\hat{\mathbf{z}} \times \left[\iint_{S'} \underline{\underline{\mathbf{G}}}^{EJ}(\boldsymbol{\rho}, \boldsymbol{\rho}')\, \mathbf{J}(\boldsymbol{\rho}')\, dS' - \underline{\underline{\mathbf{Z}}}_S(\boldsymbol{\rho})\, \mathbf{J}(\boldsymbol{\rho}) \right] = -\hat{\mathbf{z}} \times \mathbf{E}_i \tag{2}$$

where $\hat{\mathbf{z}}$ is the unit vector normal to the MTS, $\underline{\underline{\mathbf{G}}}^{EJ}$ is the substrate dyadic Green's function, and $\boldsymbol{\rho}'$ and $\boldsymbol{\rho}$ are the source and observation coordinates, respectively. Finally, $\underline{\underline{\mathbf{Z}}}_S$ and $\mathbf{E}_i$ are the sheet impedance tensor and the excitation electric field, respectively.

For analysis purposes, the sheet impedance is known and the current distribution $\mathbf{J}$ in (2) is discrtized by means of basis functions, the unknowns being the coefficients of expansion. Expression (2) then leads to a system of equations, which can be written in compact form as:

$$(\underline{\underline{\mathbf{Z}}}_G - \underline{\underline{\mathbf{Z}}}_{IBC})\ \underline{\mathbf{I}} = \underline{\mathbf{V}} \tag{3}$$

where $\underline{\underline{\mathbf{Z}}}_G$ and $\underline{\underline{\mathbf{Z}}}_{IBC}$ are respectively the substrate and the sheet impedance matrix, and $\underline{\mathbf{V}}$ is a vector representing the discretized excitation.

Recently, the authors of [14] proposed the usage of Fourier-Bessel basis functions, and have proven that such a basis allows for an efficient discretization of the surface currents, which in turn translates into a well conditionned and relatively small MoM matrix size.

Next, we have shown that one can also numerically derive the surface impedance from the knowledge of the currents.

III. Inverse problem and MTS Synthesis

We assume a capacitive tensorial IBC of the general form:

$$\begin{aligned} Z_S^{\rho\rho}(\boldsymbol{\rho}) &= Z_0^{\rho\rho} + P^{\rho\rho}(\boldsymbol{\rho}) \\ Z_S^{\rho\phi}(\boldsymbol{\rho}) &= -(Z_S^{\phi\rho}(\boldsymbol{\rho}))^* = P^{\rho\phi}(\boldsymbol{\rho}) \\ Z_S^{\phi\phi}(\boldsymbol{\rho}) &= Z_0^{\phi\phi} - P^{\rho\rho}(\boldsymbol{\rho}) \end{aligned} \tag{4}$$

where $Z_0^{\rho\rho}$ and $Z_0^{\phi\phi}$ are the average reactances and are assumed to be purely imaginary, with negative imaginary parts. $P^{\rho\rho}$ and $P^{\rho\phi}$ are purely imaginary functions of $\boldsymbol{\rho}$ describing the IBC modulation and are assumed to be in absolute value lower than the average impedance. Note that, the average impedance $Z_0^{\rho\rho}$ and $Z_0^{\phi\phi}$ are fixed based on considerations relative to the IBC implementation with patches, the excitation efficiency [16], and the antenna bandwidth [17]. Such a tensorial IBC is anti-Hermitian and can therefore be implemented with lossless patches.

The surface impedance is now discretized into a Fourier-Bessel basis $R_{mn}(\boldsymbol{\rho})$ as follows:

$$Z_S^{\rho\rho} \approx \sum_{n_S, m_S} [K_S^{\rho\rho}(m_S, n_S) + X_S^{\rho\rho}(m_S, n_S)] R_{m_S, n_S} \tag{5}$$

$$Z_S^{\rho\phi} \approx \sum_{n_S, m_S} X_S^{\rho\phi}(m_S, n_S)\ R_{m_S, n_S} \tag{6}$$

$$Z_S^{\phi\phi} \approx \sum_{n_S, m_S} [K_S^{\phi\phi}(m_S, n_S) - X_S^{\rho\rho}(m_S, n_S)] R_{m_S, n_S} \tag{7}$$

Starting from (2) and (5), one can prove that the required surface impedance modulation is given by [18]:

$$\begin{bmatrix} \mathbf{X}_S^{\rho\rho} \\ \mathbf{X}_S^{\rho\phi} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^{S,\rho} & \mathbf{Z}^{S,\phi} \\ -\mathbf{Z}^{S,\phi} & \mathbf{Z}^{S,\rho} \end{bmatrix}^{-1} \cdot \underline{\mathbf{U}} \tag{8}$$

where

$$Z^{S,\rho}(m_S, n_S; m_t, n_t) = \sum_{n_b, m_b} i^{\rho}_{m_b n_b} Z(m_b, n_b; m_S, n_S; m_t, n_t) \tag{9}$$

and

$$Z^{S,\phi}(m_S, n_S; m_t, n_t) = \sum_{n_b, m_b} i^{\phi}_{m_b n_b} Z(m_b, n_b; m_S, n_S; m_t, n_t) \tag{10}$$

$\mathbf{Z}$ is a pre-tabulated matrix [18]. The vector $\underline{\mathbf{U}}$ in (8) is defined as:

$$\underline{\mathbf{U}} = \begin{bmatrix} \mathbf{U}^{\rho} \\ \mathbf{U}^{\phi} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_G^{\rho\rho} & \mathbf{Z}_G^{\rho\phi} \\ \mathbf{Z}_G^{\phi\rho} & \mathbf{Z}_G^{\phi\phi} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I}^{\rho} \\ \mathbf{I}^{\phi} \end{bmatrix} - \begin{bmatrix} \mathbf{V}^{\rho} \\ \mathbf{V}^{\phi} \end{bmatrix} - \begin{bmatrix} \mathbf{X}_K^{\rho} \\ \mathbf{X}_K^{\phi} \end{bmatrix} \tag{11}$$

Finally, $\mathbf{X}_K^{\rho}$ and $\mathbf{X}_K^{\phi}$ are respectively given by

$$\mathbf{X}_K^{\rho} = \mathbf{Z}^{S,\rho}\ \mathbf{K}_S^{\rho\rho} \tag{12}$$

$$\mathbf{X}_K^{\phi} = \mathbf{Z}^{S,\phi}\ \mathbf{K}_S^{\phi\phi} \tag{13}$$

Expression (8) establishes a full-wave link allowing the derivation of the surface impedance from the currents. Despite this expression does formally not require the usage of the same basis for the current and the IBC, we have observed in practice that better results are obtained when using FBBFs for both. Indeed, the usage of the Zernike basis [19] for example provided in our attempts a less stable solution, especially at the MTS border.

Now, let us consider the generation of a desired radiation pattern $\mathbf{F}(\theta, \alpha)$ in amplitude, phase and polarization. θ and α are respectively the elevation and azimuthal angles. This radiation pattern can be directly linked to the visible spectrum of the electric field on the MTS (see expression (44) in [18]). The desired visible spectrum of the currents is then obtained as:

$$\tilde{\mathbf{J}} = \left(\underline{\underline{\tilde{\mathbf{G}}}}^{EJ} \right)^{-1} (\tilde{\mathbf{f}} - \tilde{\mathbf{E}}_i) \tag{14}$$

where $\underline{\underline{\tilde{\mathbf{G}}}}^{EJ}$ is the spectral dyadic Green's function of the grounded substrate, $\tilde{\mathbf{E}}_i$ is the visible spectrum of the MTS excitation and $\tilde{\mathbf{f}}$ is the total tangential aperture field rescaled w.r.t. the desired radiated power. The radiated power can be calculated, for example, from the simulation of a pre-designed broadside pencil beam MTS [8] with the same average impedance and substrate.

The required surface impedance can now be computed after applying expression (8). However, a simple usage of this formula, despite giving a well radiating surface impedance, will not lead to an anti-hermitian IBC (property required in absence of loss), since the computed modulation exhibits a not negligible real part. This problem is fundamentally due to the fact the current $\tilde{\mathbf{J}}$ in (14) does not have a near-field (invisible) spectrum. Since MTS antennas are based on a SW to LW transformation, one cannot neglect the invisible spectrum of the currents. The latter, despite not being present in the far-field, is necessary to be consistent with the physics of the problem. For a given radiation pattern, one may try to find the appropriate invisible field spectrum that will produce the desired radiation pattern and ensure at the same time an anti-Hermitian surface impedance. This has been done in [8] for the particular case of a pencil beam. However, when dealing with an arbitrary shaped beam, one cannot make such an a priori prediction.

Here, we make a simple assumption, i.e the near-field current spectrum is estimated based on the one obtained in absence of modulation, namely that corresponding to the chosen average reactance. This simple estimate in combination with the visible part of the current spectrum (obtained from (14)) form the desired current distribution $\mathbf{J}$. Then we solve equation (8) while mathematically ensuring (in a least-squares sense) the absence of losses. This is carried out by imposing a sort of symmetry between the expansion coefficients of the impedance modulation [18]:

$$(X_S(m_S, -n_S))^* = -\frac{X_S(m_S, n_S)}{(-1)^n} \tag{15}$$

The advantages of the proposed method can be summarized as follows:

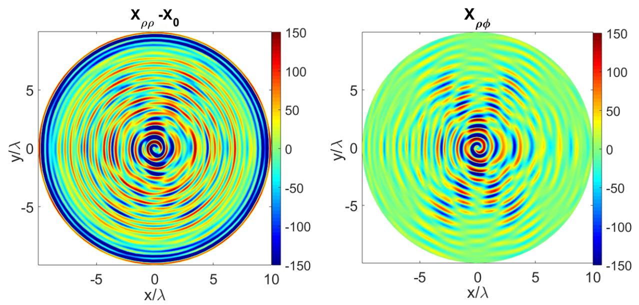
- First, the algorithm is systematic, i.e it starts from the desired radiation pattern $\mathbf{F}(\theta, \alpha)$ and follows a systematic procedure to directly compute the relevant surface impedance.
- Next, the extension into the invisible region is carried out in the least-squares sense, and can be applied to generate any radiation pattern with any polarization.
- Third, the proposed method can be used with any type of excitation as soon as the coupling between the excitation and the IBC modulation can be neglected.

IV. Results

Our intention in this section is to demonstrate the robustness of the proposed algorithm. To this end, we have designed a MTS radiating a multi-shaped polarized beam. The frequency of operation is 18 GHz. We used a substrate relative permittivity equal to 3.66, while the average impedance is $Z_0 = Z_0^{\rho\rho} = Z_0^{\phi\phi} = -461.5\ j$. The antenna is excited by a vertical elementary dipole placed at the MTS center ($\rho = 0$) and in the middle of the substrate.

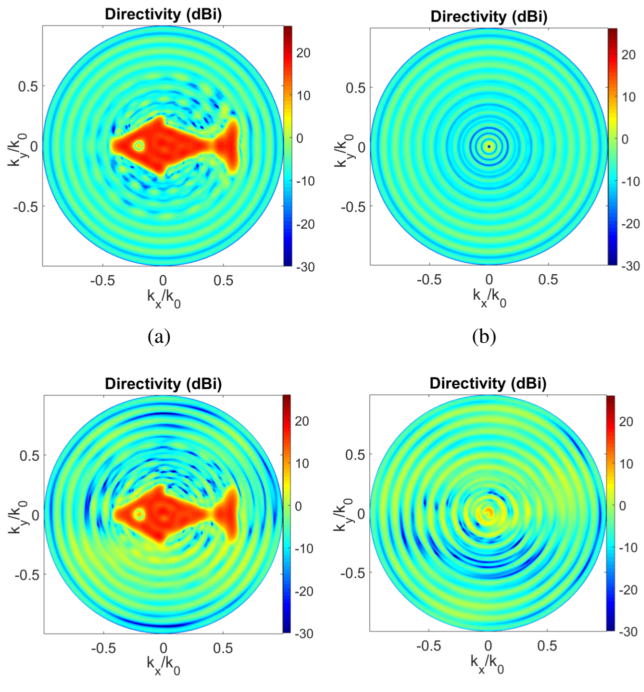
The objective pattern is a right-handed circularly polarized (RHCP) "fish like" pattern in the spectral $(k_x/k_0, k_y/k_0)$ plane. $k_x = k_0 \sin\theta\cos\alpha$ and $k_y = k_0 \sin\theta \sin\alpha$ are the spectral variables in Cartesian coordinates. The desired "fish like" radiation pattern $\mathbf{F}(k_x, k_y)$ is illustrated in Fig. 2(a). Basically, the main body of the fish is drawn with 2 triangles. The first triangle is perforated with a small ring to represent the eye. The tail is also drawn with a triangle. The radiation pattern in Fig. 2(a) is computed after filling the visible part of the current spectrum with the desired radiation pattern and the invisible part with the invisible spectrum corresponding to the average impedance. The obtained current is discretized in the spectral domain in the Fourier-Bessel basis.

The synthesis IBC tensor is illustrated in Fig. 1. The obtained IBC has been analyzed with the MoM code validated in [14]. Figs. 2(c), 2(d) and Fig. 3 show respectively the designed radiation pattern and its cut in the $\alpha = 0$ plane. One can observe a very good agreement of the Co-polar (RHCP) pattern w.r.t. the desired one. The main difference are in the cross-pol.



(a) (b)

Fig. 1. Modulated sheet transition IBC for a MTS radiating a "fish like" beam a) $X_{\rho\rho} - X_0$ and b) $X_{\rho\phi}$.



(a) (b) (c) (d)

Fig. 2. Circularly polarized "fish like" radiation pattern in the (k_x, k_y) plane. (a): Co-polar component for the ideal antenna (desired pattern) (b): Cross-polar component for the ideal antenna (c): Co-polar component for designed MTS (d): Cross-polar component for designed MTS.

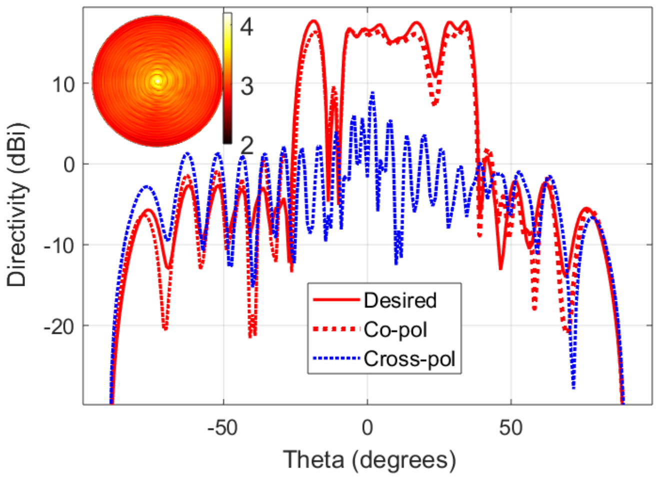


Fig. 3. Radiation pattern of the designed “fish-like” radiating MTS in the plane $\alpha = 0$. The co-polar and the cross-polar pattern corresponds respectively to the Right Handed Circular Polarization (RHCP) and the Left Handed Circular Polarization (LHCP) directivity. The inset disk corresponds to the absolute value of the current distribution on the designed MTS (on log_{10} scale) for a unit current excitation.

V. Conclusion

We proposed an EFIE based systematic formalism to design modulated metasurface antennas. An illustrative example has shown that a fine control of the antenna radiation pattern can be achieved. Concerning the surface impedance discretization, even if the formulation allows a free choice of this basis, we have observed that the Fourier-Bessel basis provides a much more stable solution than the Zernike basis, which is another orthogonal basis.

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