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Innovative Applications of O.R.

Measuring the effects of price controls using mixed complementarity models

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ABSTRACT

Government involvement in managing domestic prices of energy and other commodities is a major issue in emerging economies. We examine one aspect of the problem, price controls, when governments set or cap prices. We show how a Mixed Complementarity Problem (MCP) formulation can be used to model and assess the impacts of price controls in multi-sector economic-equilibrium models. Both the gains from deregulation and the consequences of imposing new or altering existing regulations can thus be measured. We present three distinct models that capture different price-control situations: firms have to meet demand and receive an implicit subsidy, demand rationing occurs due to an associated price control constraint, and subsidies limit demand rationing. We present an approach to measuring the effects on the equilibrium in the first case and the levels of disequilibrium induced by price controls in the other cases. We also show how to determine the most efficient allocation program when a government engages in rationing. We illustrate the cases described by these models using markets that have or had price controls.

1. Introduction

The impetus for this paper is the need to have tools that quantify the impacts of price controls and evaluate the consequences of alternative pricing regimes. The methodology presented here provides a means for countries to evaluate the costs of their price regulations and the trade-offs associated with alternative policies that either deregulate prices entirely or improve the economic efficiency of their price controls, reducing budgetary strains without compromising social objectives. We develop Mixed-Complementarity-Problem (MCP) models of multi-sector systems where prices of some or all goods are controlled, using examples from our experience in modeling energy markets. Although MCP models have been important in evaluating market power, (Gabriel, Kiet, & Zhuang, 2005; Hobbs & Pang, 2007; Masoumi, Yu, and Nagurney, 2012; Wogrin, Hobbs, Ralph, Centeno, & Barquín, 2013; Yu and Nagurney, 2013), we know of no models outside of Matar, Murphy, Pierru, and Rioux (2015) and Rioux, Galkin, Murphy, and Pierru (2017) that directly model prices controls of standard commodities and measure their impacts. Böhringer and Rutherford (2008) mention the rationing problem in an MCP framework but do not provide a mathematical formulation. Gouel (2013) presents an MCP in which a country uses an inventory policy to stabilize the price of a commodity, starting with the work of Miranda and Helmberger (1988). The complementarity constraints include violating the desired lower bound on the price when the inventory becomes too large and failing to cap the price when the inventory hits zero. Abrell and Rausch (2017) examine how to improve the efficiency of carbon policies when different political entities regulate different industries by providing relaxations of carbon prices and carbon constraints. They show how to minimize the costs of distortions by setting the bounds using a mathematical program subject to equilibrium constraints (MPEC).

This paper contributes to the literature in three different ways. First, it provides a general MCP framework and formalizes the approach for modeling controlled prices in multi-sector models. Second, it develops an approach to measuring and interpreting the levels of disequilibrium induced by price controls when rationing occurs and to measuring the levels of subsidies, including the complementarity structures that are added to the models. Third, it shows how to reduce the disequilibrium with a least-cost combination of subsidies. We also present an alternative formulation that determines the most economically efficient allocation plan under rationing.

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Murphy, Pierru, and Smeers (2016) discuss the limitations of linear programming in modeling government interventions and the advantages of using an MCP formulation. MCP is now the state-of-the-art method to model situations where the underlying problem is not an optimization model but an equilibrium model. MCPs facilitate the modeling of market imperfections and supplant the iterative techniques that use linear programming (Greenberg & Murphy, 1985). The imperfections can be roughly classified into three types, market power, regulation and taxes, and price controls. The first, market power, has captured much of the attention of the MCP literature, especially in energy (Gabriel, Conejo, Fuller, & Hobbs, 2012). Second, government policies on regulation and taxes lead to deviations from marginal-cost pricing. While linear programs can model flat taxes because a tax that is a fixed amount can be added as a cost in the objective function, MCP's are necessary for modeling valued-added and sales taxes because the tax is fraction of total costs instead of a fixed increment (Greenberg & Murphy, 1985). In an MCP a price that includes a value-added or sales tax is incorporated in a consumer's dual constraint by multiplying the original dual variable for the product by one plus the tax rate. Furthermore, in regulated industries, average-cost pricing, where customers are charged the average cost of delivering the good, is a standard approach to pricing that can be modeled directly in an MCP by setting the dual on the supply/demand balance equal to the average cost, as calculated in an added equation. See Gabriel et al. (2012), p. 521.

In this paper we focus on a third market distortion, price controls, which has not received sufficient attention in the literature. We examine a particular group of price controls encountered in many emerging economies and once common in developed countries, administered prices and price ceilings on different products in both intermediate and final consumption. We use the term "administered" when the prices are fixed by the government and the phrase "price ceiling" when the government sets a maximum price. Price controls are designed to achieve social objectives such as lowering costs for people with low-incomes. In oil rich countries, administered prices for fuels balance the budgets of power and water utilities that also have the prices of their products controlled. They are also used for economic development and diversification when lower energy costs incentivize the construction of new, industrial facilities such as petrochemical plants.

The standard economic prescription to reduce the inefficiencies associated with low controlled prices is to move to prices based on world markets, forcing higher prices on consumers. Most of the related literature deals with the impact of raising consumer prices on economic growth and household welfare, especially in countries that devote substantial proportions of their budgets to subsidies (e.g. Clements et al., 2007; Granado and Coady, 2012). This first best policy is justified on the basis of standard economics but may have social or industrial consequences that make implementation difficult. Thus, it is often useful to measure the extent of the economic distortion to show the costs as well as benefits of price controls and subsidies.

We distinguish economic curtailment from shortages as follows. Economic curtailment refers to situations where producers and consumers have substitutes, albeit at higher prices. The substitutes in inputs allow firms to produce market-clearing quantities of their products at higher prices, and likewise, consumers may have higher priced alternatives. With shortages of an input, firms have to cut back on production because of a curtailed supply and some end consumers find shelves that are empty when trying to purchase a product. That is, the economy cannot achieve an equilibrium with supply meeting demand.

The work presented in this paper expands on the modeling initiated in Matar et al. (2015), studying the effects of the administered energy prices in Saudi Arabia. We have not seen any literature that presents a general framework for modeling controlled prices in MCP's, the subject of this paper. We consider three possible cases involving controlled prices. The first is when prices are administered and the producer(s) must meet domestic demand at a loss or the prices are above average cost but below market prices, creating an implicit subsidy that does not appear as a line item in any budget. In Saudi Arabia for instance, Saudi Aramco sells fuels to utilities and other industrial sectors at prices set by the government that are below international market prices but above domestic production costs. Saudi Electricity Company and Saline Water Conversion Corporation sell their electricity and water at administered prices and receive subsidies to cover losses. In this case there are no shortages and no economic curtailment of activities.

The second involves situations where the level of production is an economic decision, and supply falls short of demand at controlled prices but substitutes exist. Matar et al. (2015) is an example of this case. They examine a situation in which Saudi Arabia's natural gas supply does not meet domestic demand at administered prices and natural gas is allocated to a range of customers. The industrial sectors and electric and water utilities then substitute other fuels for natural gas using processes that are represented in the model and other sectors do not consume natural gas. A first issue is to quantify the imbalances between supply and demand for the various sectors' products before applying an allocation rule. A second issue is to determine the most economically efficient allocation, which can serve as a benchmark for policy changes. Thus, the modeling issues center on the effects upon the market equilibrium and the allocation of supply, not the curtailing of economic activity or end-use energy services.

The third case consists of situations where the level of production is an economic decision, supply does not meet demand at the controlled price for some goods because the price is below marginal cost and no alternative technologies can substitute for the curtailed goods in meeting demand. This occurs most commonly with shortfalls in the production of goods used in final consumption where consumers do not have a ready alternative. Examples include the lack of household items in Venezuela beginning in 2014, and, in the 1970s in the USA, natural gas and gasoline shortages (Kalt, 1981; Lifset, 2014; Murphy, Sanders, Shaw, & Thrasher, 1981; Frum, 2000). Here one needs to reframe the model to understand the shortfall, rather than find the equilibrium. We also look at how prices of inputs or costs of outputs can be subsidized in order to reduce the amount of rationing (if any), or increase the efficiency of the sectors modeled.

The paper is organized as follows. Section 2 illustrates the pervasiveness of price controls. Section 3 gives a stylized approach to modeling markets with price controls and rationing using MCPs. Section 4 provides alternative MCP formulations of a multi-sector model with price controls and energy subsidies.

2. Price controls: a widespread practice

Although either price controls or government-imposed pricing rules exist in every economy (e.g. sugar in the United States and the Common Agricultural Policy in Europe), we limit ourselves to a few examples, mainly drawing on our experience in modeling the energy economies of Saudi Arabia and China. Matar et al. (2015) describe the consequences of currently administered energy prices in Saudi Arabia. Saudi power and water-desalination utilities buy crude oil from Saudi Aramco at a price lower than \$5/barrel, whereas the export price has ranged from \$30 to \$110 in the period from 2010 through 2017. In addition, Saudi Arabia varies the administered prices by consuming sector: prior to the recent price increases, cement companies bought heavy crude at \$6/barrel and electric utilities bought heavy crude at \$2.67/barrel.

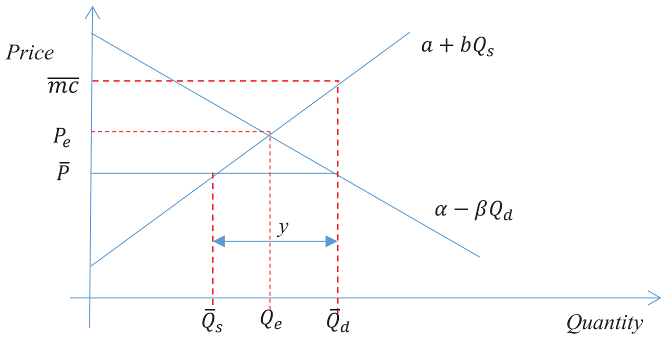

Fig. 1. Controlled price $\bar{P}$ lower than competitive equilibrium price P_e.

Domestic natural gas prices in the main gas-producing countries in the Middle East and North Africa are capped below both the international prices and the marginal cost of new production (Darbouche (2013)). Bangladesh sells natural gas and liquid fuels to its electricity sector at prices lower than the supply costs (Mujeri, Chowdhury, & Shahana, 2014). Commander (2012) estimates that between 2008 and 2010 over half of Asian countries passed on to consumers less than 75% of the increase in international prices for gasoline and diesel fuels.

In China's power sector, the National Development and Reform Commission (NDRC) caps the prices a utility can pay a generator for electricity, with the caps differentiated by technology and region (Rioux et al., 2017). China also imposes price ceilings in its domestic natural gas market (Rioux et al., 2018). Price ceilings are not restricted to developing countries. Most electricity markets, for example, have ceilings on peak electricity prices.

Price controls also apply to non-energy goods. For instance, Aljazira Capital (2013) reports that the Saudi government imposes a price ceiling on domestic sales of cement, while banning cement exports to ensure domestic demand is met. According to Al Rajhi Capital (2012), the Saudi government also caps the price of some agricultural and food products, such as fresh milk. Mulet (2009) reports that the Venezuelan government maintains and updates on a regular basis a list of various price-controlled products.

When implementing price controls, a government generally administers the price when there is only one supplier (often a state company) and no market mechanism, whereas the government caps the price when there is a market with partial or full competition.

3. Complementarity representations of price controls

In this section we develop graphical representations of price controls and the associated basic MCPs before presenting general MCPs of price controls in three cases.

3.1. Prices are not controlled

We begin with the deregulated case, followed by adding a limit on supply in a simple example and then provide more detailed MCP formulations. Assume an inverse supply curve with intercept a and slope b. Let Q_s be the supply quantity with supply curve $P = a + bQ_s$. Let Q_d be the quantity demanded with the inverse demand curve $P = \alpha - \beta Q_d$. Using Fig. 1, we illustrate the basic issues with controlled prices that we address in detail. Let (P_e, Q_e) be the equilibrium price and quantity with the deregulated equilibrium.

The complementarity conditions for the competitive model in the figure are as follows. Either the supply quantity is 0 or the marginal cost equals the price:

$$0 \le a + bQ_s - P \perp Q_s \ge 0 \quad (1)$$

where $\perp$ indicates the complementarity condition that we must have at least one of two equalities hold: $a + bQ_s - P = 0$ or $Q_s = 0$.

The equilibrium price is on the demand curve and is below the intercept on the demand curve or the quantity demanded is zero:

$$0 \le P - (\alpha - \beta Q_d) \perp Q_d \ge 0 \quad (2)$$

The quantity supplied equals the quantity demanded or the price is zero:

$$0 \le Q_s - Q_d \perp P \ge 0 \quad (3)$$

Note that there are three conditions and three variables. The system of equations is always square.

We now add an initial complication before introducing price controls. Say the supply curve is truncated at some capacity M. If this capacity is reached, the market-clearing price is above the supply curve and the marginal supplier captures a rent of R. The complementarity condition (1) becomes

$$0 \le a + bQ_s - (P - R) \perp Q_s \ge 0 \quad (4)$$

We add a complementarity condition on the capacity where supply equals capacity or the rent is zero:

$$0 \le M - Q_s \perp R \ge 0 \quad (5)$$

3.2. The price is capped and rationing occurs

Now assume the government imposes on the supplier a maximum sales price $\bar{P}$ in Fig. 1 that is below the equilibrium price. Demand increases to $\bar{Q}_d$, where the marginal cost of the supplier is $\overline{mc}$. We then consider several cases.

In the first case, the supplier decides to produce up to the point where its marginal cost equals the regulated price. Then supply is rationed with $\bar{Q}_s$ available for allocation. Let y be the amount of unmet demand. The relevant model is then obtained by replacing the market-clearing relation $0 \le Q_s - Q_d \perp P \ge 0$ by the complementarity condition

$$0 \le Q_s + y - Q_d \perp P \ge 0 \quad (6)$$

and imposing the additional condition:

$$0 \le \bar{P} - P \perp y \ge 0, \quad (7)$$

which states that demand is rationed only if the market price is equal to the controlled price. The demand and supply functions remain unchanged. An issue in modeling rationing is what happens to the unmet demand. For household consumers, a simple view could be that people buy on a "first come, first served" basis, rationing starts when shelves in stores are empty and the incremental utility beyond the regulated price is captured in the willingness of consumers to wait in line. When the lines from shortages become too long, formal rationing methods are required. For example, in response to inflation driven by the Vietnam War, in 1970 President Richard Nixon imposed wage and price controls on the United States economy. The combination of the rigid domestic prices and the jump in world oil prices during the 1973–1974 oil embargo led to shortages of gasoline. The form of rationing that the Nixon administration imposed was restricting the days when car owners could buy gasoline. See Cowan (1973) for a description of government actions as gasoline lines were lengthening in the United States. Another form of rationing, during WWII, was to issue ration coupons that had to be turned in with each purchase and rationed per person.

With industrial and commercial customers, the available supply has to be allocated among firms that then change their production processes to adapt to the available supply, if possible. In Saudi Arabia the natural gas price is controlled below the market clearing price, reducing potential production (Huppmann, 2013). At

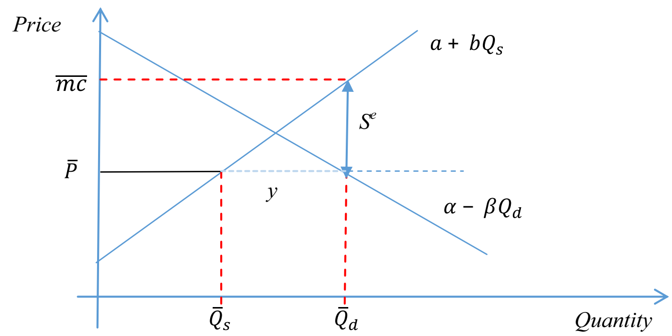

Fig. 2. Given a price cap $\bar{p}$, adding a per-unit subsidy increases supply along the original supply curve with S^e the equilibrium subsidy.

the same time the country allocates gas to industry so that higher-value customers receive gas before lower-value firms, consistent with the demand curve (Matar, Murphy, Pierru, & Rioux, 2013). In this case the shortage in Fig. 1 can be divided into two components. The first is the shortage relative to the market-clearing demand without price caps, which we term the actual shortage, $Q_e - \bar{Q}_s$. Here the customers without allocations would have positive demands at the market-clearing price. We term the remaining shortage at the regulated price the excess shortage, which is $\bar{Q}_d - Q_e$. Here the customers would consume the commodity at the regulated price but not the market-clearing price. This distinction is important because when customers with excess shortages receive allocations, they exacerbate the actual shortage for customers who would buy the gas at the market price. If no allocations meet customers' excess shortages and customers with actual shortages have alternatives, then the price regulations effectively shift rents from suppliers to customers. The benefit/cost to customers depends on the savings in the cost of the commodity versus the extra cost of the substitute. We address rationing in greater detail later in the paper.

For durable goods, rationing can take a form that involves maintenance costs, and the rationing induced by price controls can extend beyond just the quantity supplied. An example is rent control, enforced in New York (USA) in the past and Paris (France) currently. In Alston, Kearl, and Vaughan (1992), 76.3% of economists surveyed generally agree on the statement "a ceiling on rents reduces the quantity and quality of housing available."

3.3. The price is capped but the government provides subsidies

Subsidies can take different forms because they can bear on different cost elements. Subsidies can be lump sums independent of the amount produced or per unit payments to the producer. Because of the simplicity of this example model, we introduce a single subsidy that reduces the marginal cost of the supply sector. Fig. 2 illustrates the use of a per-unit subsidy $S = S^e$ that lowers the marginal cost to the controlled price, eliminating shortages.

The model is obtained by modifying the supply function to account for both rationing and a subsidy. This is done by replacing (6) and (7) with

$$0 \le a + bQ_s - S - P \perp Q_s \ge 0 \tag{8}$$

and

$$0 \le \bar{P} - P \perp S \ge 0.$$

3.4. The price is controlled and meeting demand is mandatory

Lastly, we assume that the government wants to regulate the sectors so that they satisfy the demand for their output at minimum cost while consumers pay exogenously given price controls. Because controlled prices are exogenous, demand is now fixed and the demand function reduces to a single point. This fixes the supply sector's production. Because prices and quantities are fixed, the supply sector no longer breaks even and the government must cover the deficit or collect the surplus. For an equilibrium to exist, the government must require that demand be met, and to balance the budget the government has to make a payment in the form of either a lump sum or per-unit subsidy, which is the same as a lump sum when demand is fixed. Typically, in these situations, there is only one supplier available to each customer and the supplier is a regulated utility.

We come back to these different situations in Section 4, using a multi-sector model.

4. Modeling controlled prices in multi-sector models

We now apply the different approaches to price controls introduced above in formulating a more realistic multi-sector energy model. We first introduce a generic multi-sector linear program. We then develop four versions of the model, the first under perfect competition and then three versions of price controls discussed below.

4.1. Building blocks

Our construction is based on two generic linear programs of production and the corresponding Karush–Kuhn–Tucker conditions. The first model is cost based and assumes that the producer minimizes the cost of supplying a given demand, taking the purchase prices of inputs as given. The second has price-taking profit-maximizing producers that choose both input and production quantities at market-clearing prices.

Noting that a sector can produce multiple products, let

I — the index set representing the energy sectors with $i, j \in I$

$p_{j,i}$ — the row vector of prices of goods produced by sector j and consumed by sector i and of dimension k_j

$q_{j,i}$ — the column vector of quantities of the goods produced by sector j and consumed by sector i and of dimension k_j

c_i — the row vector of unit costs for inputs coming from the sectors and agents not represented in the model and of dimension n_i.

x_i — the column vector of production levels of dimension n_i

z_i — the column vector of imports of sector i products

p_i^I — the row vector of import prices for sector i

$A_{j,i}$ — a matrix of the technology coefficients for providing the products of sector i from other sectors j, of dimension (k_i, k_j)

B_i — the matrix of technology coefficients for producing the products of sector i, of dimension (k_i, n_i)

e_i — the exogenous demand for goods produced by sector i and consumed outside the sectors j, a column vector of dimension k_i

$D_{j,i}$ — a matrix of coefficients for inputs from other sectors that add to constrained resources or help meet imposed requirements internal to sector i such as the level of capital stock, a matrix of dimension (m_i, k_j)

E_i — a matrix of technology coefficients of resources or requirements imposed internal to sector i, a matrix of dimension (m_i, n_i)

u_i — a column vector of dimension m_i that represents either requirements to be met such as petroleum product specifications, or levels of existing resources such as available equipment capacity.

The cost minimization model of sector i that produces k_i goods with e_i a fixed demand parameter is formulated as follows:

$$\underset{x_i,\ z_i,\ q_{j,i\ j\neq i}}{\text{Minimize}} \sum_{j\in I, j\neq i} p_{j,i}q_{j,i} + c_i x_i + p_i^I z_i \quad (9)$$

$$s.t.\ \textstyle\sum_{j\in I, j\neq i} A_{j,i}q_{j,i} + B_i x_i + z_i \geq \sum_{j\in I, j\neq i} q_{i,j} + e_i \qquad (\lambda_i) \quad (10)$$

$$\begin{aligned}&\textstyle\sum_{j\in I, j\neq i} D_{j,i}q_{j,i} + E_i x_i \geq u_i \quad (\mu_i)\\ &q_{j,i} \geq 0\ (j\in I,\ j\neq i), x_i \geq 0,\ z_i \geq 0\end{aligned} \quad (11)$$

where λ_i and μ_i are the vectors of dual variables associated with the sets of constraints (10) and (11), respectively.

Sector i sells $q_{i,j}$ to sector j and buys $q_{j,i}$ from sector j. The objective function consists of the costs for purchases from other sectors, $\sum_{j\in I, j\neq i} p_{j,i}q_{j,i}$, the within-sector production costs, $c_i x_i$, and the costs of imported goods, $p_i^I z_i$.

The set of Eq. (10) represents the domestic demands that sector i has to meet either through production within the sector, imports, or supplies from other sectors. The demands consist of demands from other sectors in the model, $\sum_{j\in I, j\neq i} q_{i,j}$, and demand e_i exogenous to the sectors represented, including end-use demands and demands from sectors not represented in the model. The set of Eq. (11) represents all other constraints contained in the sectoral model, including capacity limits on resources, limits on inputs, blending requirements, etc. The matrices $A_{j,i}$, B_i, $D_{j,i}$ and E_i are general statements of the technology coefficients for purchases from sector j and production and transportation in sector i. To illustrate what they can represent, let j be the crude oil sector and i be petroleum products. Typically, $A_{j,i}$ is all 0′s because crude is generally not considered a petroleum product. $D_{j,i}$ supplies the different crudes into the crude-oil material balances in the refining sector. E_i contains the rest of the crude-oil material balances, balances for intermediate product streams, unit capacity constraints, blending constraints, etc. B_i captures the delivery of refined products to the supply/consumption material balances. The variable x_i includes components for distillation, producing and using intermediate product streams, etc., as well as transporting the petroleum products to i. If j is expanded to include all hydrocarbons and the demands are for petroleum products and natural gas, $A_{j,i}$ directly transfers natural gas to sector i without going through a refinery and $D_{j,i}$ includes natural gas and gas liquids as feedstocks into refining.

Costs for inputs to the production processes from outside the sectors represented in the model are included in the cost coefficient, c_i. These costs include the costs of purchases from sectors not represented in the model as well as the cost of labor. Any equality constraints can be expressed using two inequalities and are, therefore, included in (11).

We now write the KKT conditions of sector i's cost-minimization problem (c_i' denotes the transpose of c_i):

$$0 \leq \sum_{j\in I, j\neq i} A_{j,i}q_{j,i} + B_i x_i + z_i - \sum_{j\in I, j\neq i} q_{i,j} - e_i \perp \lambda_i \geq 0 \quad (12a)$$

$$0 \leq \sum_{j\in I, j\neq i} D_{j,i}q_{j,i} + E_i x_i - u_i \perp \mu_i \geq 0 \quad (12b)$$

$$0 \leq \lambda_i' A_{j,i} + \mu_i' D_{j,i} - p_{j,i}' \perp q_{j,i} \geq 0\ j\in I,\ \ j\neq i \quad (12c)$$

$$0 \leq \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \geq 0 \quad (12d)$$

$$0 \leq p_i^I - \lambda_i \perp z_i \geq 0 \quad (12e)$$

The KKT conditions of the sectoral profit-maximization version of the problem are obtained from those of the cost model by simply adding:

$$0 \leq \lambda_i - p_{i,j} \perp q_{i,j} \geq 0\ j\in I,\ \ j\neq i \quad (13)$$

The multi-sector model is obtained by writing the KKT conditions of the different sector models and assembling them into a single complementarity model, as shown in the next sections.

4.2. Multi-sector model with no price controls

We assume that all sectors maximize profits. When concatenating the KKT conditions of all single-sector models, every price vector $p_{j,i}$ is removed, and the costs of purchases from other sectors are captured by the vectors λ_j of dual variables (i.e., the marginal cost of meeting domestic demand for sector j), which means that (13) is removed when constructing the multi-sector model. The multi-sector model, stated in the form of the equilibrium conditions, is then:

$$0 \leq \sum_{j\in I, j\neq i} A_{j,i}q_{j,i} + B_i x_i + z_i - \sum_{j\in I, j\neq i} q_{i,j} - e_i \perp \lambda_i \geq 0 \quad i\in I \quad (14a)$$

$$0 \leq \sum_{j\in I, j\neq i} D_{j,i}q_{j,i} + E_i x_i - u_i \perp \mu_i \geq 0 \quad i\in I \quad (14b)$$

$$0 \leq \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' \perp q_{j,i} \geq 0 \quad i\in I, j\in I \text{ and } j\neq i \quad (14c)$$

$$0 \leq \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \geq 0 \quad i\in I \quad (14d)$$

$$0 \leq -\lambda_i + p_i^I \perp z_i \geq 0 \quad i\in I \quad (14e)$$

It is easy to show that this is equivalent to the following TIMES-like optimization model (see for instance Loulou (2008) and Loulou and Labriet (2008) for a description of TIMES):

$$\begin{aligned}&\text{Minimize}_{x_i,\ z_i\ i\in I} \sum_{i\in I} (c_i x_i + p_i^I z_i)\\ &s.t.\ \sum_{j\in I, j\neq i} A_{j,i}q_{j,i} + B_i x_i + z_i \geq \sum_{j\in I, j\neq i} q_{i,j} + e_i \quad i\in I\\ &\sum_{j\in I, j\neq i} D_{j,i}q_{j,i} + E_i x_i \geq u_i \quad i\in I\\ &q_{j,i} \geq 0\ (j\in I,\ j\neq i), x_i \geq 0,\ z_i \geq 0 \quad i\in I\end{aligned} \quad (15)$$

The models (14) and (15) do not include a representation of a demand response. All of the standard demand representations can be added to (14), including demands represented using elasticities. Let $d_i(\lambda_i)$ be the demand for sector i's products not captured in the model, at the price λ_i. Let λ_i^s and $d_i^s(\lambda_i)$ be the components of λ_i and $d_i(\lambda_i)$, respectively. A constant elasticity demand curve with λ_i as the price is

$$d_i^s(\lambda_i) = d_i^{s,0} \prod_t \left(\lambda_i^t\right)^{\varepsilon_i^{s,t}} \quad (16)$$

Where $d_i^{s,0}$ is a constant and $\varepsilon_i^{s,t}$ are the cross-price elasticities for sector i's products.

This formula includes cross elasticities of sector i's products and can be extended to include the cross elasticities between the products grouped under i and the products grouped under j. However, the added notation complicates the discussion. After incorporating a demand curve (14a) becomes

$$0 \leq \sum_{j\in I, j\neq i} A_{j,i}q_{j,i} + B_i x_i + z_i - \sum_{j\in I, j\neq i} q_{i,j} - d_i(\lambda_i) \perp \lambda_i \geq 0 \quad i\in I \quad (14a)$$

If the elasticity matrix formed of the $\varepsilon_i^{s,t}$ is not symmetric, then the demand curve cannot be inserted into the linear program (15) (Hogan, 1975). However, asymmetric cross elasticities can be used in MCPs. To represent services, a subset of I can represent the demand for services and for this subset the technology matrices can represent the production of those services. For the remainder of the paper we leave the exogenous demands fixed to simplify the equations.

4.3. Prices are controlled and firms have to meet demand

Given prices are exogenously controlled, to maximize their profits or minimize their losses, firms must minimize costs while meeting demand. Because sectors are required to satisfy demand, the sectors with price controls are likely to lose money. We assume that these sectors receive fixed per-unit subsidies from the government. This is what happens with utility sectors in Saudi Arabia. In China some money-losing firms either get government subsidies that are either financial or in the form of subsidized inputs or are government-owned companies that are concerned only about breaking even across all of their businesses, profiting in some and running losses in others. These firms might also receive subsidies (Rioux et al., 2017). In one estimate 14% of the profits of nonfinancial firms that are listed on Chinese exchanges come from subsidies (Trivedi, 2016).

Let $C_{j,i}$ be the set of prices in j that are controlled when selling products j to sector i and $NC_{j,i}$ be the products j that face market prices when selling to i, that is, are not controlled. Let $\bar{p}_{j,i}$ be the exogenously given vector of price caps or administered prices at which sector j sells its goods to sector i. Let $\gamma_{j,i}$ represent an implicit per-unit subsidy on sector j's products guaranteeing that the demand for the products is met. When the subsidy is needed, sector j's marginal production cost less the subsidy is equal to the controlled price. The equilibrium conditions of the multi-sector model with price caps are the following:

$$0 \le \sum_{j\in I, j\ne i} A_{j,i} q_{j,i} + B_i x_i + z_i - \sum_{j\in I, j\ne i} q_{i,j} - e_i \perp \lambda_i \ge 0 \quad i \in I \tag{17a}$$

$$0 \le \sum_{j\in I, j\ne i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \ge 0 \quad i \in I \tag{17b}$$

$$0 \le \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' + \gamma_{j,i}' \perp q_{j,i} \ge 0 \quad i \in I, j \in I \text{ and } j \ne i \tag{17c}$$

$$0 \le \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \ge 0 \quad i \in I \tag{17d}$$

$$0 \le -\lambda_i + p_i^I \perp z_i \ge 0 \quad i \in I \tag{17e}$$

$$0 \le \bar{p}_{j,i} - \lambda_j + \gamma_{j,i} \perp \gamma_{j,i} \ge 0 \quad i, j \in C_{j,i} \tag{17f}$$

$$\gamma_{j,i} = 0 \quad i, j \in NC_{j,i} \tag{17g}$$

The term $\lambda_j - \gamma_{j,i}$ in (17c) represents the purchase cost of sector j's products by sector i, with $\gamma_{j,i}$ at 0 when the marginal cost is below the cap. This complementarity condition makes the model an MCP that cannot be represented as an optimization model. In (17a) supply must meet demand, and in (17f) the demand, $\sum_{j\in I, j\ne i} q_{i,j} + e_i$, is determined by the price caps when they are binding. When prices are not capped but administered at a fixed level, (17f) becomes the following equality:

$$\bar{p}_{j,i} = \lambda_j - \gamma_{j,i} \quad i, j \in C_{j,i} \tag{18}$$

To introduce rationing of j to i for goods with controlled prices, we introduce upper bounds $U_{j,i}$ on $q_{j,i}$. The components k of $U_{j,i}$ can be infinite when there is sufficient supply or the price is not controlled. In the case of Saudi Arabia, crude oil and petroleum products have no effective bound while natural gas does. The rationing constraint (19g) has a dual $\rho_{j,i}$ that is the difference between the controlled price and the cost of the lowest-cost alternative. This represents a rent provided by having access to the rationed good. In this model there are substitute technologies and no shortages. The formulation for price caps with rationing is:

$$0 \le \sum_{j\in I, j\ne i} A_{j,i} q_{j,i} + B_i x_i + z_i - \sum_{j\in I, j\ne i} q_{i,j} - e_i \perp \lambda_i \ge 0 \quad i \in I \tag{19a}$$

$$0 \le \sum_{j\in I, j\ne i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \ge 0 \quad i \in I \tag{19b}$$

$$0 \le \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' + \gamma_{j,i}' - \rho_{j,i}' \perp q_{j,i} \ge 0 \quad i, j \in C_{j,i} \text{ and } j \ne i \tag{19c}$$

$$0 \le \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' \perp q_{j,i} \ge 0 \quad i, j \in NC_{j,i} \text{ and } j \ne i \tag{19d}$$

$$0 \le \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \ge 0 \quad i \in I \tag{19e}$$

$$0 \le -\lambda_i + p_i^I \perp z_i \ge 0 \quad i \in I \tag{19f}$$

$$0 \le U_{j,i} - q_{j,i} \perp \rho_{j,i} \ge 0 \ i, j \in C_{j,i} \text{ and } j \ne i \tag{19g}$$

$$0 \le \bar{p}_{j,i} - \lambda_j + \gamma_{j,i} \perp \gamma_{j,i} \ge 0 \quad i, j \in C_{j,i} \tag{19h}$$

Interestingly, if $q_{j,i}$ is restricted to a quota, then we have two opposite effects: sector i benefits from a per-unit subsidy on the amount delivered, and at the same time, a per-unit scarcity cost is imposed because of the higher-cost technologies needed to meet demand. Note that the marginal value in the consuming sector of the controlled input is no longer the controlled price but becomes the marginal cost of using the substitute, or, equivalently, the marginal value of another unit of supply to the economy. With our formulation, for each product we can break down this marginal value into the marginal cost, the sectoral subsidy represented by the controlled price, and the sectoral scarcity cost when the controlled price leads to rationing. When the price cap on sector j's output is binding, the marginal value as an input into sector i is $\lambda_j' - \gamma_{j,i}' + \rho_{j,i}' = \bar{p}_{j,i}' + \rho_{j,i}'$ in (19c). Note that we assume that prices are capped. However, for some agricultural commodities governments set minimum prices to support farmers, leading to surpluses. The formulations here can be extended to that situation by adding a surplus variable that represents the amount the government must buy to support prices. A surplus variable, g_i, is introduced as an extra demand in (19a). The subsidy disappears from the formulation and (19h) changes to $0 \le \lambda_j - \underline{p_j} \perp g_j \ge 0$, where $\underline{p_j}$ is the minimum price for sector j's output.

This model captures the situation in which a good is imported, its price is controlled below the world price and the government subsidizes the costs of its importation. We model this by using the controlled price for imports and ex post calculating the expenditures on subsidies, the import volume times the difference between world and domestic prices. There have been times when Saudi Arabia has not had enough refining capacity to meet domestic demand and imported petroleum products were subsidized, keeping the domestic product prices fixed.

In Saudi Arabia, at the administered price of natural gas the potential domestic demand exceeds the domestic supply, which is physically constrained since the Saudi policy up to now is not to import gas. Some analysts have argued that this limited domestic supply results from the fact that the low administered price does not incentivize the development of more costly sources of natural gas, such as non-associated gas fields. In their model, Matar et al. (2015) impose quotas to replicate the allocation of the natural gas among consuming sectors (as in Eq. (16)) as decided by the government. Despite allocations lower than demand, there are no shortages because alternative fuels and technologies meet demand in the electric and water utilities and the major industrial sectors, and gas consumption in other sectors is negligible. In the case of Saudi Electricity Company (SEC), electricity prices are administered as well. In Matar et al. (2015) investment credits are paired with higher administered prices to improve the efficiency of resource allocation and not worsen the budget of SEC. The administered prices and investment credits are chosen to minimize government subsidies using a mathematical program subject to equilibrium constraints.

A different formulation for pricing with constrained supply was implemented in the United States in the 1970s when then President Nixon imposed price controls on domestic crude oil production while imported oil received the world price. A cap below the world price was placed on the domestic price of crude oil. If a price-adjustment mechanism to equalize the prices of imported and domestic crude oil had not been put in place, product prices would have been set to marginal cost, the cost of refining imported crude, giving refiners a windfall from the lower price of domestic crude. Effectively, what was done was importers were given a credit on the import cost and domestic producers paid a tax so that domestic and international crude costs were equalized below the world price. The tax varied so that revenues from the tax matched the expenditures on credits. The lower prices from crude-oil price controls were passed on to consumers through margin controls on the downstream portion of the oil industry. Let p_D be the controlled price of domestic crude and p_I be the price of imported crude, and q_D and q_I be the domestic and imported quantities. In our formulation above let I include the upstream and refining sectors and let s be crude oil. Through a tax on domestic crude and a credit on imports, the price refiners saw was

$$p_{refining}^{crude\ oil} = \frac{p_D q_D + p_I q_I}{q_D + q_I} \tag{20}$$

The formula (20) can be added to the MCP in (17) by substituting $p_{refining}^{crude\ oil}$ for the marginal cost of acquiring crude oil in the refining sector, whereas domestic crude oil producers produce until their marginal cost equals p_D. Note that the modified supply curve consists of the original supply curve for prices below the cap and is monotonically increasing with increasing imports, since the import price is higher than the domestic price cap.

During the oil price controls, domestic producers produced profitable oil only, reducing supply. Producers could theoretically withhold production, anticipating higher controlled or deregulated prices as the legislative debates evolved. This was not modeled and production from existing wells continued because production costs were well below the allowed price since exploration and development costs were sunk. To increase exploration and development, domestic oil was separated into two price categories, “old” oil from existing wells had a lower price than “new” oil from new wells. This prompted increased in-fill drilling in existing fields to move oil from “old” to “new” oil prices. Note that the MCP can readily capture multiple prices on a commodity as long as the supplies that get different prices are differentiated in separate supply curves, as (20) can include more than two sources of supply. Nevertheless, regulation-specific responses such as withholding in anticipation of raising price caps or in-fill drilling are virtually impossible to model because they are so hard to anticipate but the potential for novel behaviors should be considered when presenting model results.

Note that the ex post coverage of losses does not induce efficient outcomes because it does not incentivize agents to reduce their costs. We later explore a more powerful form of regulation after introducing a policy based on curtailment of demand.

4.4. Prices are capped and shortages occur

In contrast to the previous section, the government does not force firms to meet demand or there is no alternative source of supply to cover the gap between supply and demand. If, for the potential demand at the controlled price, the firm’s or sector’s marginal cost of production exceeds the controlled price, there is curtailment and the marginal cost of the firm’s production equals the controlled price. There is no curtailment if the marginal cost is below the controlled price. We first provide a formulation that allows for quantifying the unmet demand. We then propose an alternative formulation that yields the most efficient allocation of the available supply.

To quantify the imbalance between supply and demand created by the price caps, sector i’s equilibrium conditions (Eq. (21)) are derived from the MCP of the generic model (Eq. (14)) by introducing a curtailment variable, y_i, and its associated complementarity equation. The price ceilings are denoted by $\bar{p}_i^s$, with $s \in C_i$. For simplicity we apply the same price ceilings to all sectors j and we drop the index j from C and NC. Here C_i represents the set of sector i’s products with prices that are controlled and NC_i is the set of sector i’s products of which prices are deregulated (with $card(NC_i) + card(C_i) = k_i$). Let y_i^s be the components of y_i, with $y_i^s = 0$ when s belongs to NC_i. Unlike (17)–(19), in the next sets of complementarity conditions for sectors with controlled prices, when the marginal cost λ_i is equal the price cap, the price cap becomes the market price and shortages y_i^s can appear, as seen in the complementarity condition (21f).

Sector i’s equilibrium conditions are:

$$0 \leq \sum_{j \in I, j \neq i} A_{j,i} q_{j,i} + B_i x_i + z_i + y_i - \sum_{j \in I, j \neq i} q_{i,j} - e_i \perp \lambda_i \geq 0 \tag{21a}$$

$$0 \leq \sum_{j \in I, j \neq i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \geq 0 \tag{21b}$$

$$0 \leq \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' \perp q_{j,i} \geq 0 \quad j \in I \text{ and } j \neq i \tag{21c}$$

$$0 \leq \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \geq 0 \tag{21d}$$

$$0 \leq -\lambda_i + p_i^I \perp z_i \geq 0 \tag{21e}$$

$$0 \leq -\lambda_i^s + \bar{p}_i^s \perp y_i^s \geq 0 \quad s \in C_i \tag{21f}$$

$$y_i^s = 0 \quad s \in NC_i \tag{21g}$$

With this formulation, for a shortage to occur in a product, we need its import price to exceed its controlled price. Note that a mechanism to average prices like (20) can eliminate shortages when the import price exceeds the domestic price. The complementarity conditions (21) differ from the KKT conditions of the generic model by the appearance of the curtailment in the balance Eq. (21a) and a complementarity constraint that states that curtailment occurs only if the marginal value exceeds the price used to gauge the shortage (21f).

In (21), we take intermediate demand from other sectors and final demand as given, accepting that demand is not fully satisfied.

Eq. (21) represent the KKT conditions of the following optimization problem (22):

$$\begin{aligned}
&\text{Minimize}_{q_{j,i(j\in I, j\neq i)},\, x_i, z_i, y_i^{s(s\in C_i)}} \sum_{j\in I, j\neq i} p_{j,i} q_{j,i} + c_i x_i + p_i^I z_i + \bar{p}_i^s y_i^s \\
&s.t. \sum_{j\in I, j\neq i} A_{j,i} q_{j,i} + B_i x_i + z_i + y_i \geq \sum_{j\in I, j\neq i} q_{i,j} + e_i \quad (\lambda_i) \\
&\quad \sum_{j\in I, j\neq i} D_{j,i} q_{j,i} + E_i x_i \geq u_i \quad (\mu_i) \\
&\quad q_{j,i} \geq 0\ (j\in I, j\neq i), x_i \geq 0,\ y_i \geq 0,\ y_i^s = 0 \quad (s\in NC_i)
\end{aligned} \tag{22}$$

One can interpret (22) as customers in sectors j having access to an unlimited source, y_i, of product i at a cost equal to the controlled price (e.g., a backstop technology whose marginal cost is equal to the controlled price). The quantity y_i would be bought from this source. Another possible interpretation is that the firm i incurs an additional cost, which is the revenue loss for not being able to meet the entire domestic demand. Note that we defined e_i as a constant containing both final demand and demand from sectors not represented in the model. In the face of shortages, typically, a government would ban exports, as Saudi Arabia did with cement and e_i would be just domestic final demand.

The overall multi sector model is then stated as:

$$0 \leq \sum_{j\in I, j\neq i} A_{j,i} q_{j,i} + B_i x_i + z_i + y_i - \sum_{j\in I, j\neq i} q_{i,j} - e_i \perp \lambda_i \geq 0 \quad i\in I \tag{23a}$$

$$0 \leq \sum_{j\in I, j\neq i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \geq 0 \quad i\in I \tag{23b}$$

$$0 \leq \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' \perp q_{j,i} \geq 0 \quad i\in I, j\in I \text{ and } j\neq i \tag{23c}$$

$$0 \leq \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \geq 0 \quad i\in I \tag{23d}$$

$$0 \leq -\lambda_i + p_i^I \perp z_i \geq 0 \quad i\in I \tag{23e}$$

$$0 \leq -\lambda_i^s + \bar{p}_i^s \perp y_i^s \geq 0 \ s\in C_i,\ i\in I \tag{23f}$$

$$y_i^s = 0 \quad s\in NC_i,\ i\in I \tag{23g}$$

The model (23) represents a disequilibrium because some demand is unmet due to curtailments. Each sector curtails its own production because of the ceilings on the prices of its products, but still wants to buy all of the inputs it needs, as if there were no curtailments in the other sectors. The quantities demanded represent, however, what would actually be observed in an economy where businesses base their demands for inputs on the controlled prices they see. Introducing shortage variables, which measure the disequilibrium, makes the model feasible while representing the imbalances created by price controls. The size of the shortage is a measure of the scale of the disequilibrium and the need for government action to either loosen the price controls or engage in rationing. When rationing, governments intervene and prioritize which sectors and end users are curtailed. These priorities do not necessarily match the value the sectors place on the product and can leave some high-value uses unmet, which we term actual shortages.

Note that when shortages occur for good i, sectors that consume i can have shortages in their production that do not show up because of the artificial "supply." That is, the model measures only the initial shortages not the cascading effects throughout the economy. An equilibrium with rationing that balances supply with varying proportions of different sectors' demands cannot be computed directly using Eq. (23) because the shortages in one good can lead to shortages of other goods. These cascading shortages are hidden by the shortage variable y_i "meeting" demand in (23). In every sector that consumes the good, one has to impose government rationing of the goods that are short and iterate on the solution by updating the ration plan, until all curtailment variables and actual consumption match the government ration plan. The resulting equilibrium is not a market equilibrium but depends on the rules defining which sectors and final consumers are curtailed. Thus, the results have to be interpreted carefully to make clear that the shortages cascade beyond the shortage variables, are contingent on an allocation policy, and probably have major effects not captured in an equilibrium model. Venezuela has experienced this cascade and illustrates the extent that social costs exceed the losses measured by the shortage variables.

When y_i measures a substitute for good i, compared to the solution to Eq. (14) without price controls, the solution to (23) gives a lower bound on the system costs imposed by price ceilings, since any substitute not in the model that reduces y_i^s may have costs of using that input that do not appear in the model. This is especially true when meeting final demands in a model that does not have alternative technologies for meeting the demand for energy services.

To derive the most efficient allocation of products, one that reduces the actual, as opposed to excess shortages, we present an alternative formulation where the consuming sectors realize that their inputs will be curtailed. We fully understand that the scheme presented here is less efficient economically than deregulation, because there is economic curtailment. However, it allows us to measure the efficiency of an allocation plan versus the "optimal" allocation. In (23), consuming sectors do not realize that their inputs are rationed and base their decisions on the controlled prices only. By contrast, recognizing that there is rationing would lead them to attribute to every curtailed input a value that is higher than the price cap imposed on the sectors producing these inputs. This would be the case if, subsequent to any given government allocation, the products can be re-sold in a secondary market. The existence of this market ensures that the limited supply is consumed in the most efficient way, whatever the initial allocation rule might have been. The difference between the resulting allocation and the initial allocation represents the amount of trading that would have to occur in the secondary market for an economically efficient allocation. If reselling the allocated quotas on a secondary market is not officially permitted, then this market is often replaced by a black market, which has higher transaction costs.

Using a secondary market makes explicit that price controls are a form of rent reallocation. One virtue of this modeling approach is that the extent to which the beneficiaries of the allocations trade away their supply indicates how much the price can be raised without harming the target group, except for their loss of income from selling their rights to the good. In this model we remove the shortage variable y_i and add a variable r_i that represents the premium over the controlled price that clears the market when supply is constrained by the price cap. Sectors consuming sector i's products value these products at the price $\lambda_i + r_i$, i.e., the price at which they would sell or buy on the secondary market. Sector i's KKT conditions are stated as follows:

$$0 \leq \sum_{j\in I, j\neq i} A_{j,i} q_{j,i} + B_i x_i + z_i - \sum_{j\in I, j\neq i} q_{i,j} - e_i \perp \lambda_i \geq 0 \tag{24a}$$

$$0 \leq \sum_{j\in I, j\neq i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \geq 0 \tag{24b}$$

$$0 \leq \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' - r_j' \perp q_{j,i} \geq 0 \quad j\in I \text{ and } j\neq i \tag{24c}$$

$$0 \le \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \ge 0 \tag{24d}$$

$$0 \le -\lambda_i + p_i^l \perp z_i \ge 0 \tag{24e}$$

$$0 \le -\lambda_i^s + \bar{p}_i^s \perp r_i^s \ge 0 \qquad s \in C_i \tag{24f}$$

$$r_i^s = 0 \quad s \in NC_i \tag{24g}$$

Rewriting (24) for all i, the overall multi sector model is:

$$0 \le \sum_{j\in I, j\neq i} A_{j,i} q_{j,i} + B_i x_i + z_i - \sum_{j\in I, j\neq i} q_{i,j} - e_i \perp \lambda_i \ge 0 \quad i \in I \tag{25a}$$

$$0 \le \sum_{j\in I, j\neq i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \ge 0 \quad i \in I \tag{25b}$$

$$0 \le \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' - r_j' \perp q_{j,i} \ge 0 \quad i \in I, j \in I \text{ and } j \neq i \tag{25c}$$

$$0 \le \lambda_i' B_i + \mu_i' E_i - c_i' \perp x_i \ge 0 \quad i \in I \tag{25d}$$

$$0 \le -\lambda_i + p_i^l \perp z_i \ge 0 \qquad i \in I \tag{25e}$$

$$0 \le -\lambda_i^s + \bar{p}_i^s \perp r_i^s \ge 0 \; s \in C_i, \;\; i \in I \tag{25f}$$

$$r_i^s = 0 \quad s \in NC_i, \; i \in I \tag{25g}$$

Since e_i is fixed, we presume consumers do not sell into the secondary market.

In (25), if sector i's marginal cost of production reaches the price cap $\bar{p}_i^s$, the sectors j consuming sector i's product(s) see the cost $\bar{p}_i^s + r_i^s$ through (25c) and (25f). At the same time producers receive only $\bar{p}_i^s$ for their product. This cost is the product's price in the secondary market and adding r_i ensures domestic supply meets domestic demand. However, since producers do not get higher prices while their inputs increase in value, production is below what it would have been in deregulated markets, and the market clears at higher prices if only one commodity is controlled ($\overline{mc}$ in Fig. 1). Here the only benefit is to those consumers and producers who essentially get an income transfer because of their allocations at controlled prices. The income transfer is an ex post calculation: $r_i^s \times \left(\sum_{j\in I, j\neq i} q_{i,j} + e_i\right)$.

The model (25) directly finds the equilibrium without the need for iterating, with supply matching demand, since the secondary market removes the need for defining an allocation process. One could argue that this solution assumes that consuming sectors integrate into their optimizations the cost imposed on the system by curtailment, whereas this cost is not directly observable if there are no secondary markets for curtailed products. However, even if a secondary market does not exist, the sectoral consumption of inputs observed in the solution to (25) can be interpreted as the most economically efficient allocations under rationing. A political process, however, does not necessarily focus on economic efficiency and adds the dimension of equity, with the possible influence of lobbying by large stakeholders and stakeholder groups, implying a generally worse economic outcome. Furthermore, receiving a rationed good can lead to a situation in which the government seeks compensation or a cross-subsidy in return for the allocation.

The models presented here do not begin to capture the costs of the chaos surrounding shortages, including the disruption of home and work lives and the massive cost embodied in people waiting in lines, the signature feature of the Soviet Union. These costs extend well beyond the economic-surplus calculations presented here and include the disruption of lives, the increased violence and incivility, and the constant adjustments in response to revisions of the rules because of political pressures that are lessened only in special situations such as World War II. In Venezuela, price controls and currency controls that restrict imports have resulted in food shortages. Surveys (Sequera, 2018) show that this has contributed to a significant loss of weight in the general population. These social costs are very difficult to capture in an economic model.

As another example, see Cowan (1973) for a description of the disruptions that were occurring during the gasoline shortages due to the Nixon price controls. Here the government initially misallocated the gasoline supply to the different regions of the United States, causing shortages in some regions. This led to national panic buying. Because people saw gas lines, they started filling their tanks more frequently to avoid running out, and gasoline inventories shifted from gas stations to drivers' tanks, exacerbating the shortages resulted from the regional misallocations. Also, since gas stations had their margins controlled at low levels, there was no incentive for stations to keep evening hours and they closed early, which meant there was a shortage of pumping capacity, which became the main cause of the waiting lines, once the supply chain replenished inventories and gasoline became plentiful.

Despite not including all of the costs of shortages, a disequilibrium model of shortages is useful as a way to measure how distorted the economy is rather than an attempt to measure social welfare. Essentially, what the model provides is a rough measure of the level of shortages versus available supply and, not a fine-grained measure of the costs of price controls. The study of the consequences of shortages and their dynamics is a major subject in its own right as stressed by de la Grandville (2009), Chapter 14.

4.5. Introducing ex ante subsidies

Covering losses ex post is a costly regulatory policy because firms that expect subsidies to cover their losses have no incentive to be efficient. We now assume that the government wants to retain price controls while replacing ex post loss coverage by ex-ante subsidies. The case we examine is where the subsidy is proportional to the consumption of certain inputs, letting the companies keep the profits made after choosing their production and sales as a function of these price controls and subsidies. This is, for instance, the case in the food sector in Saudi Arabia, which is dominated by private firms, as explained by Rajhi Capital (2012). These firms, which rely on imports of raw materials, were facing an increase in crop prices while domestic food prices were capped by the government. To offset the rise in firms' costs, the government offered subsidies. For instance, animal feed (an important cost component in the dairy industry) and production equipment are subsidized. Providing a subsidy on an output to cover the difference between the marginal cost and the price cap, unlike the case considered here, can be formulated as an MCP.

The model presented here is not an MCP. Instead, it is a Mathematical Program subject to Equilibrium Constraints (MPEC) where the government optimizes its decisions subject to the equilibrium solution of an MCP. See Gabriel, Shim, Conejo, de la Torre, and García-Bertrand (2010) as an example of what we address here. In the MPEC below the government minimizes the cost of subsidies subject to the market being in equilibrium at a given level of curtailment. Consider the model based on the generic profit maximization problem after introducing subsidies on the operating costs of the companies and the possibility of some residual curtailment of final demand. The objective is to find the lowest-cost per-unit-of-input subsidies, s_i, to limit the curtailment y_i below a ceiling $\bar{y}_i$. The subsidies reduce the unit cost of x_i to $c_i - s_i$. The sector problem is

$$\underset{s_i}{\text{Minimize}}\, s_i x_i$$
$$\text{s.t. } 0 \le \bar{y}_i - y_i$$

and subject to the equilibrium conditions:

$$0 \le \sum_{j\in I, j\neq i} A_{j,i} q_{j,i} + B_i x_i + z_i + y_i - \sum_{j\in I, j\neq i} q_{i,j} - e_i \perp \lambda_i \ge 0 \qquad (26a)$$

$$0 \le \sum_{j\in I, j\neq i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \ge 0 \qquad (26b)$$

$$0 \le \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' \perp q_{j,i} \ge 0 \quad j \in I \text{ and } j \neq i \qquad (26c)$$

$$0 \le \lambda_i' B_i + \mu_i' E_i - \left(c_i' - s_i'\right) \perp x_i \ge 0 \qquad (26d)$$

$$0 \le -\lambda_i + p_i^I \perp z_i \ge 0 \qquad (26e)$$

$$0 \le -\lambda_i^s + \bar{p}_i^s \perp y_i^s \ge 0 \quad s \in C_i \qquad (26f)$$

$$y_i^s = 0 \quad s \in NC_i \qquad (26g)$$

This MPEC has as its constraints the KKT conditions (21) with condition (21d) modified.

The rationale for this MPEC is the following: when inputs from other sectors are bought at controlled prices, the economic trade-offs among all inputs in a sector is distorted. By subsidizing certain inputs, for instance energy-efficient equipment when energy prices are administered, it is possible to get closer to the correct relative prices for inputs and, therefore, to regain some economic efficiency lost to price controls, as was done in Matar et al. (2015). The efficiency gains are maximized when, for the desired level of curtailment, the total sum of subsidies is minimized.

This MPEC is highly nonconvex. One can resort to gridding the possible combinations of subsidies and solving the MCP for each grid point. The MCP solutions that do not satisfy the upper bounds on curtailments have to be discarded. Note that some components of s_i can be set to zero if subsidies target a subset of inputs, which is important if gridding is used to find the solution, as the number of grid points is multiplicative in the targeted components.

The objective function and equilibrium conditions of the multi-sector MPEC are the following.

$$\underset{s_{i\in I}}{\text{Minimize}} \sum_i s_i x_i$$
$$\text{s.t. } 0 \le \bar{y}_i - y_i \quad i \in I$$

and subject to the equilibrium conditions:

$$0 \le \sum_{j\in I, j\neq i} A_{j,i} q_{j,i} + B_i x_i + z_i + y_i - \sum_{j\in I, j\neq i} q_{i,j} - e_i \perp \lambda_i \ge 0 \quad i \in I \qquad (27a)$$

$$0 \le \sum_{j\in I, j\neq i} D_{j,i} q_{j,i} + E_i x_i - u_i \perp \mu_i \ge 0 \quad i \in I \qquad (27b)$$

$$0 \le \lambda_i' A_{j,i} + \mu_i' D_{j,i} - \lambda_j' \perp q_{j,i} \ge 0 \quad i \in I, j \in I \text{ and } j \neq i \qquad (27c)$$

$$0 \le \lambda_i' B_i + \mu_i' E_i - \left(c_i' - s_i'\right) \perp x_i \ge 0 \quad i \in I \qquad (27d)$$

$$0 \le -\lambda_i + p_i^I \perp z_i \ge 0 \quad i \in I \qquad (27e)$$

$$0 \le -\lambda_i^s + \bar{p}_i^s \perp y_i^s \ge 0 \quad s \in C_i,\ i \in I \qquad (27f)$$

$$y_i^s = 0 \quad s \in NC_i,\ i \in I \qquad (27g)$$

If the shortages are limited to $\bar{y}_i = 0$, then the subsidies amount to the marginal costs of producing the product less the controlled price of that product. If there are multiple firms and customers can buy from multiple firms, a subsidy based on marginal cost is the lowest subsidy that guarantees no shortages.

5. Conclusions

As is illustrated by the examples throughout this paper, governments have been very creative in imposing price controls. What is needed is to show the economic costs of those actions and evaluate less damaging alternatives. We have provided several situations where price controls can be readily represented in MCPs and doing this is simpler than using the older methods of developing algorithms based on iterating using the solutions of linear programs (e.g., Murphy et al. (1981) and Greenberg and Murphy (1985)). Thus, we can now better estimate the impacts of alternative policies on price regulations and subsidies and can compare these policies to deregulation.

We have developed three multi-sector models that represent the most common forms of regulation we have encountered, administered prices with no shortages allowed, price caps with shortages, price caps with subsidies limiting shortages. We present an approach for measuring the level of disequilibrium induced by price controls. We also show how to determine the most efficient allocation rule when the government has to engage in rationing. These three models do not cover the full range of possible forms of regulation. We expect to encounter many other situations with complicated regulations and view the modeling of regulations as a research opportunity given all of the forms these regulations can take.

The results from the models presented here should be tempered in any analysis because each imposition of price controls has marketplace consequences and social dynamics that go beyond a standard economic analysis. Many of the players become creative in working around the controls. For instance, Aljazira Capital (2013) reports that despite the price ceiling on cement, the Saudi market suffered from an ‘informal’ cement market throughout 2012. Traders were stockpiling cement to create artificial shortages, and cement was being sold at inflated prices. Black markets are a regular feature with controlled goods.

Nevertheless, having the ability to model price controls is the first step in measuring and making explicit the consequences of those controls, which is important for engaging in the discussion of whether they should be lifted or can be modified to retain social benefits and lessen their economic costs. Even if it is recognized that deregulated prices, with subsidies targeted to the poor are more efficient than controlled prices, transitioning to deregulated prices first requires understanding the distortions created by existing price controls.

All of the models developed here are partial equilibrium models. The virtue of a partial equilibrium model is that it has costs based on fixed prices from the non-modeled sectors and the price controls that are in place have values specified by the government. A general equilibrium framework adds two significant complications. One complication is that general equilibrium models have aggregate representations of sectors while the price controls are on specific goods. This problem can be solved by expanding the representation of the sector with the price controls, using the approach of Böhringer and Rutherford (2008). However, we have experienced the problem that the sectors modeled in detail can use different definitions for coefficients than the more aggregate sectors, requiring adjustments to link the models, a topic that deserves further research. The second is more conceptual and addresses how to specify a price cap when all prices are set relative to a numeraire. This problem essentially says that price controls really alter the relative prices of goods in a general-equilibrium context, while the controlled prices are stated in absolute terms. Again, this is another area for future research. Still another area of research is developing conditions under which a regulated equilibrium exists and is unique for the various types of regulations.

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