



Efficient and full-wave electromagnetic analysis of MRI antennas using the Array Scanning Method

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D. Tihon

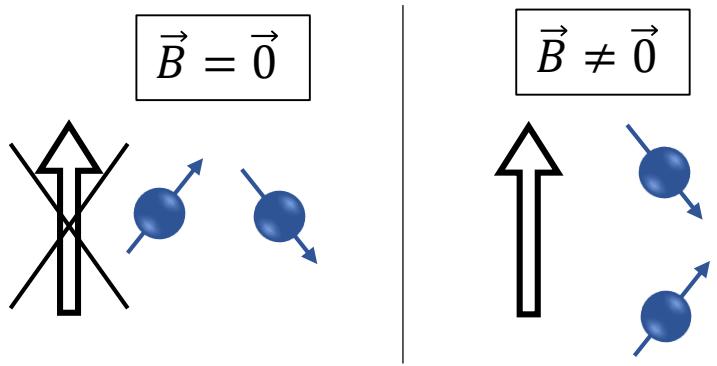
M. Dubois

R. Abdeddaim

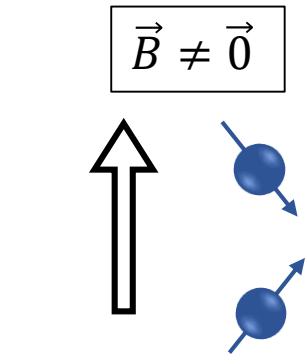
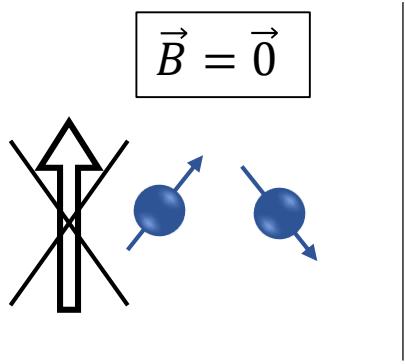
C. Craeye



Introduction



Introduction



Injected by antenna

$$\begin{aligned} & \text{+ Energy} = \text{Measured signal} \\ & = \text{Measured signal} + \text{Energy} \end{aligned}$$

Introduction

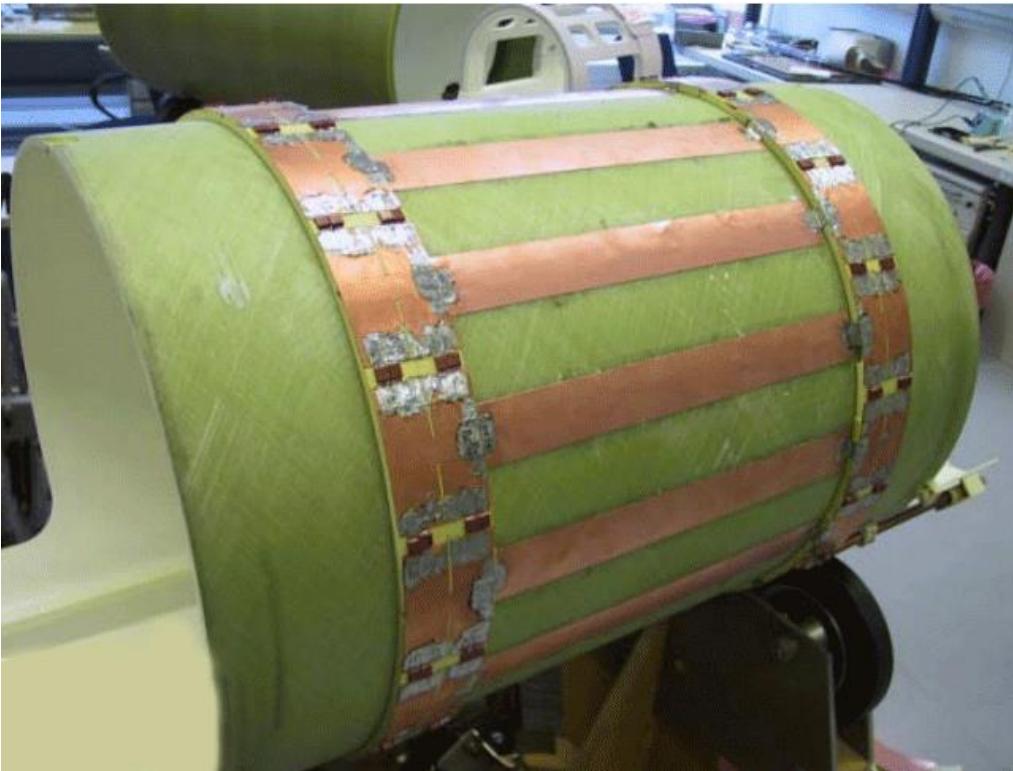


A « good » excitation:

- Narrow band (42,58 MHz/T)
- Homogeneous rotating magnetic field
- Low electric field (SAR)

Introduction

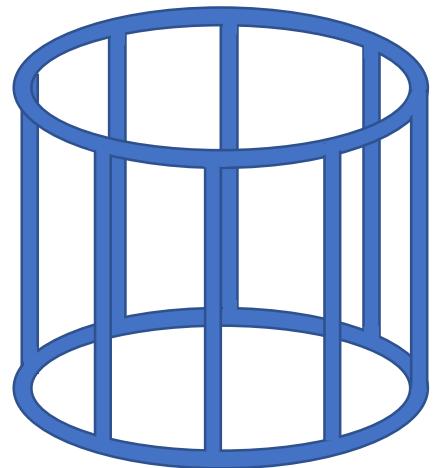
The Birdcage antenna



Picture from mriquestions.com/birdcage-coil.html

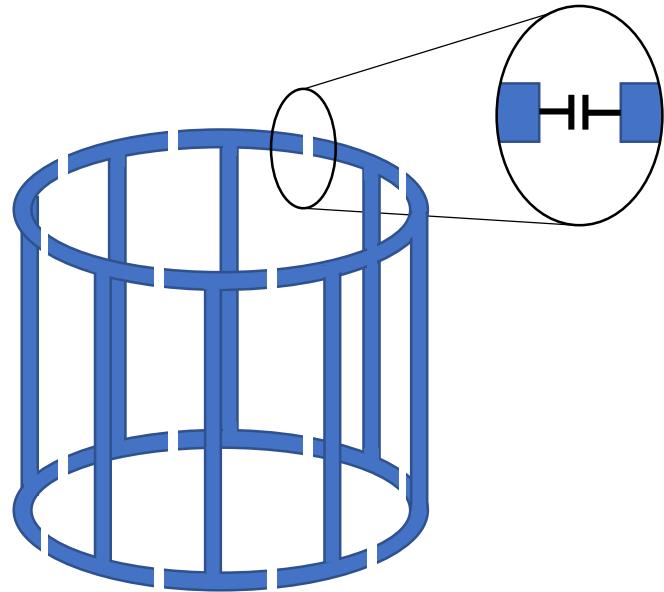
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The Birdcage antenna



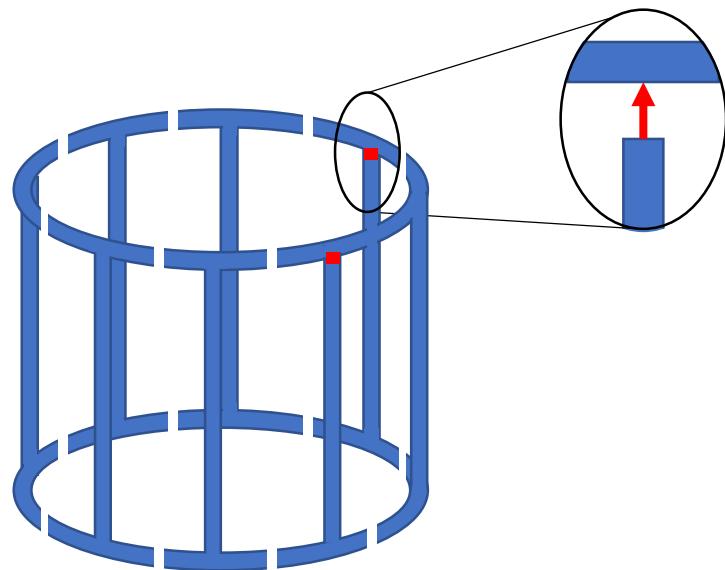
Introduction

The Birdcage antenna



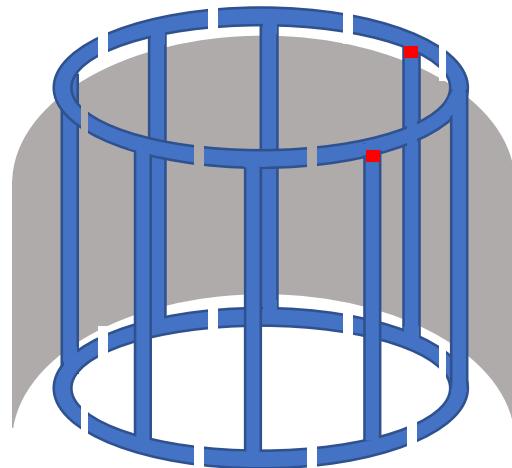
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The Birdcage antenna



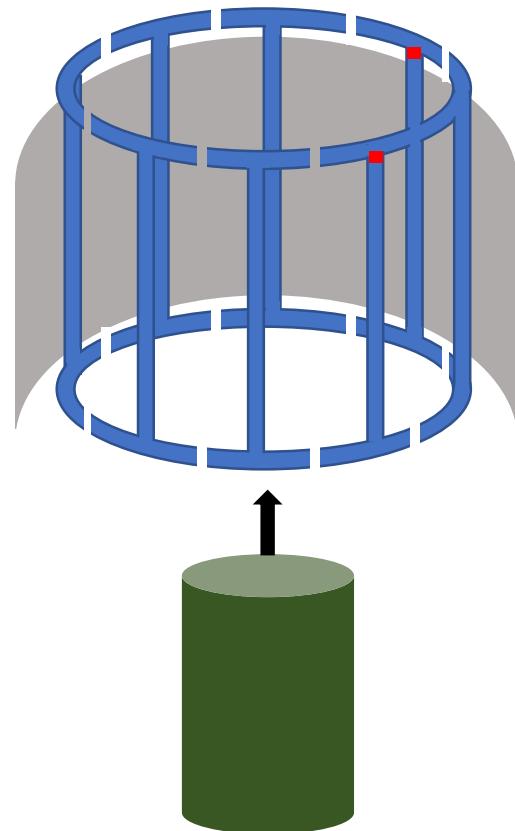
Introduction

The Birdcage antenna



Introduction

The Birdcage antenna



Salient features:

- Lossy dielectric and PEC
- Lumped elements and ports
- Nearly periodic
- Easy and fast design
(capacitors, frequency sweep)

Table of contents

I. The Method of Moments and Lumped elements

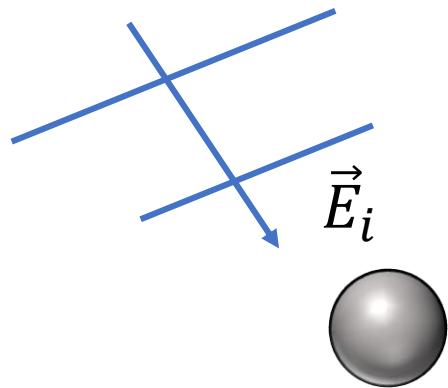
II. Fast frequency sweep

III. Periodicity and Schur complement

IV. Numerical validation

Method of Moments

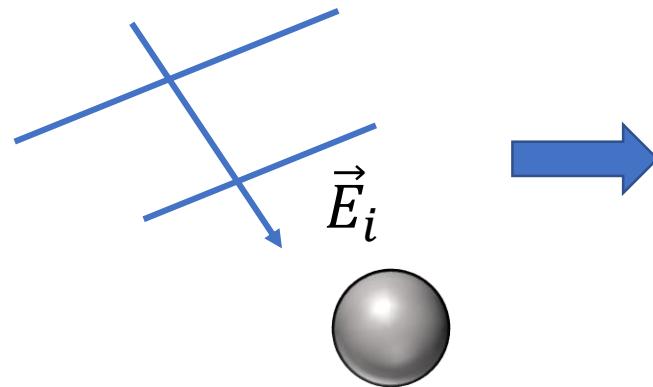
Penetrable body:



A. Poggio and C. Miller, 1973

Method of Moments

Penetrable body:



Outer problem

$$\vec{E}_{tot} = \vec{E}_i + \vec{E}_s$$

A diagram of a circle representing the body. Inside the circle, a red arrow labeled $\vec{E}_{in} = \vec{0}$ points to the right. Outside the circle, a red curved arrow labeled \vec{J} and \vec{M} indicates circulation around the body.

Inner problem

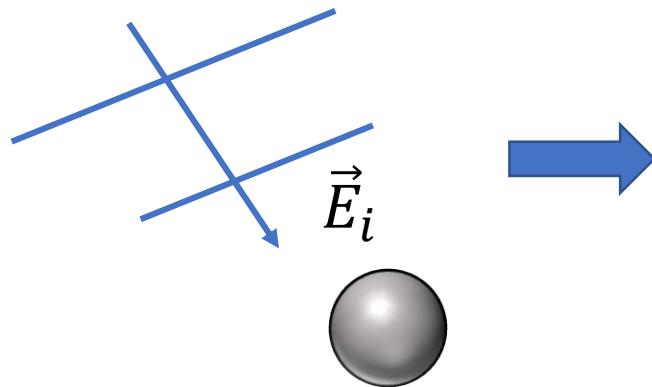
$$\vec{E}_{tot} = \vec{0}$$

A diagram of a circle representing the body. Inside the circle, a red arrow labeled \vec{E}_{in} points to the right. Outside the circle, a red curved arrow labeled $-\vec{J}$ and $-\vec{M}$ indicates circulation around the body.

A. Poggio and C. Miller, 1973

Method of Moments

Penetrable body:



$$\vec{E}_{tot} = \vec{E}_i + \vec{E}_s$$

The diagram shows a circle representing the body. Inside the circle, $\vec{E}_{in} = \vec{0}$ is written in red. On the right side of the circle, there is a red curved arrow labeled \vec{J} and \vec{M} , also in red. The entire equation $\vec{E}_{tot} = \vec{E}_i + \vec{E}_s$ is in black.

Outer problem

$$\vec{E}_{tot} = \vec{0}$$

The diagram shows a circle representing the body. Inside the circle, \vec{E}_{in} is written in black. On the right side of the circle, there is a red curved arrow labeled $-\vec{J}$ and $-\vec{M}$, also in red. The entire equation $\vec{E}_{tot} = \vec{0}$ is in red.

Inner problem

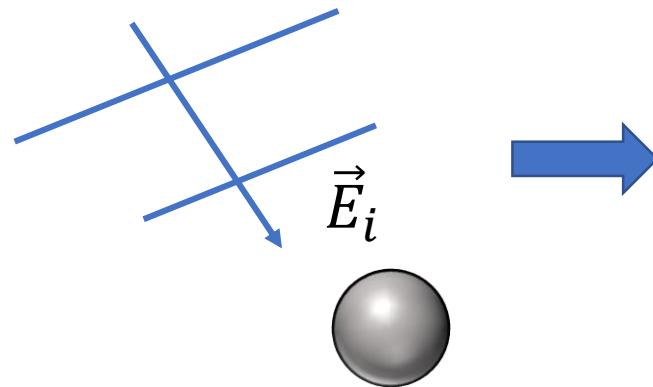
J, M?

→ Continuity of
tangential fields

A. Poggio and C. Miller, 1973

Method of Moments

Penetrable body:



Outer problem

$$\vec{E}_{tot} = \vec{E}_i + \vec{E}_s$$

A diagram of a circle representing the boundary of the penetrable body. Inside the circle, a red arrow labeled $\vec{E}_{in} = \vec{0}$ points to the left. Outside the circle, a red arrow labeled \vec{J}, \vec{M} points clockwise around the boundary.

Inner problem

$$\vec{E}_{tot} = \vec{0}$$

A diagram of a circle representing the boundary of the penetrable body. Inside the circle, a red arrow labeled \vec{E}_{in} points to the left. Outside the circle, a red arrow labeled $-\vec{J}, -\vec{M}$ points clockwise around the boundary.

$J, M?$

→ Continuity of tangential fields

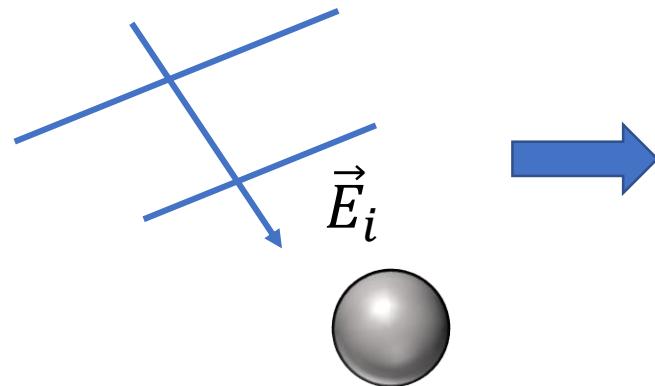
PEC: Just the outer problem $\rightarrow \vec{M} = \vec{0}$

$J?$ → Vanishing tangential electric field

A. Poggio and C. Miller, 1973

Method of Moments

Penetrable body:



$$\vec{E}_{tot} = \vec{E}_i + \vec{E}_s$$

A diagram of a circular boundary representing the outer problem. Inside the circle, the electric field is zero ($\vec{E}_{in} = \vec{0}$). Outside the circle, the total electric field is the sum of the incident field (\vec{E}_i) and the scattered field (\vec{E}_s). A red arrow labeled \vec{J} and a red vector labeled \vec{M} are shown on the boundary, representing tangential current density and magnetization.

Outer problem

$$\vec{E}_{tot} = \vec{0}$$

A diagram of a circular boundary representing the inner problem. Inside the circle, the electric field is zero ($\vec{E}_{in} = \vec{0}$). On the boundary, the total electric field is zero ($\vec{E}_{tot} = \vec{0}$). A red arrow labeled $-\vec{J}$ and a red vector labeled $-\vec{M}$ are shown on the boundary, representing tangential current density and magnetization.

Inner problem

J, M?

→ Continuity of
tangential fields

PEC: Just the outer problem $\rightarrow \vec{M} = \vec{0}$

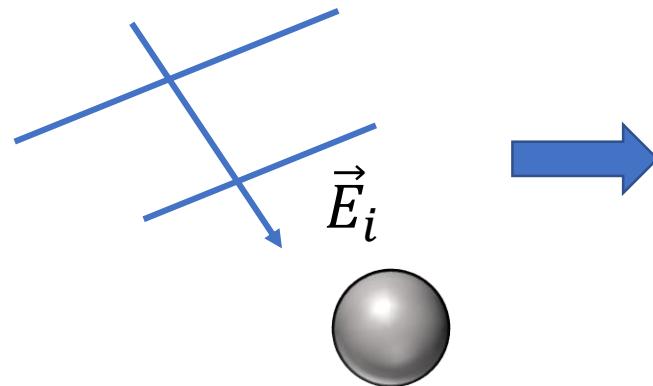
J? → Vanishing tangential electric field

$$(Z_{in} + Z_{out}) \mathbf{x} = -\mathbf{b}$$

A. Poggio and C. Miller, 1973

Method of Moments

Penetrable body:



$$\vec{E}_{tot} = \vec{E}_i + \vec{E}_s$$

A diagram of a circle representing the outer boundary of the penetrable body. Inside the circle, the total electric field $\vec{E}_{in} = \vec{0}$ is shown in red. On the boundary, there is a clockwise current \vec{J} and a clockwise magnetic moment \vec{M} , both also shown in red.

Outer problem

$$\vec{E}_{tot} = \vec{0}$$

A diagram of a circle representing the inner boundary of the penetrable body. Inside the circle, the electric field \vec{E}_{in} is shown in red. On the boundary, there is a counter-clockwise current $-\vec{J}$ and a counter-clockwise magnetic moment $-\vec{M}$, both also shown in red.

Inner problem

J, M?

→ Continuity of tangential fields

PEC: Just the outer problem $\rightarrow \vec{M} = \vec{0}$

J? → Vanishing tangential electric field

$$(Z_{in} + Z_{out}) \mathbf{x} = -\mathbf{b}$$

Fill Z $\rightarrow O(N^2)$

Invert Z $\rightarrow O(N^3)$

A. Poggio and C. Miller, 1973

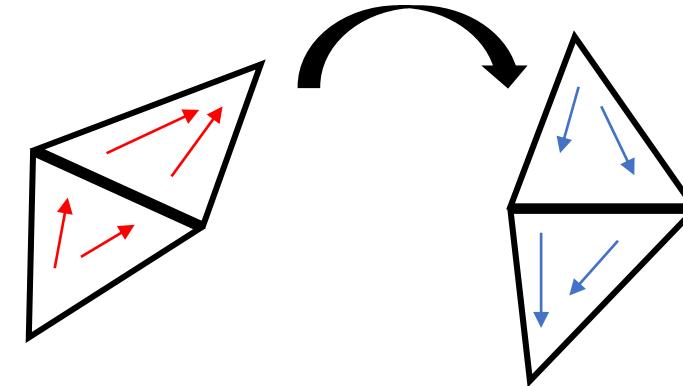
Method of Moments

The Z matrix:

$$(Z_{in} + Z_{out}) \mathbf{x} = -\mathbf{b}$$

4D integral:

2D for the source, 2D for the test

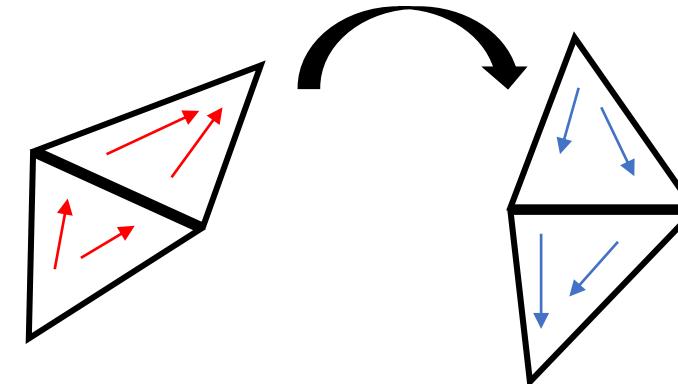


Method of Moments

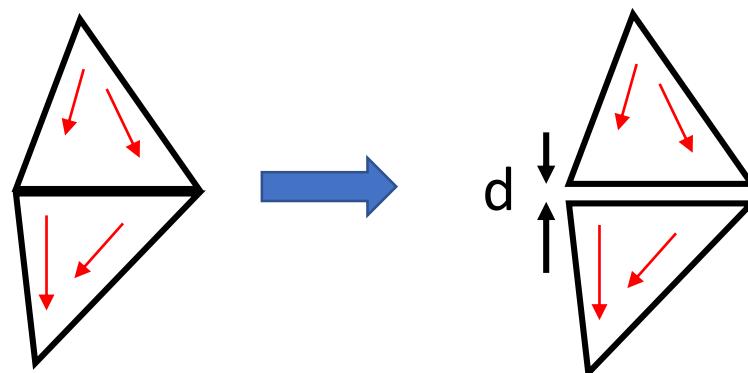
The Z matrix:

$$(Z_{in} + Z_{out}) \mathbf{x} = -\mathbf{b}$$

4D integral:
2D for the source, 2D for the test



Lumped elements: delta-gap model

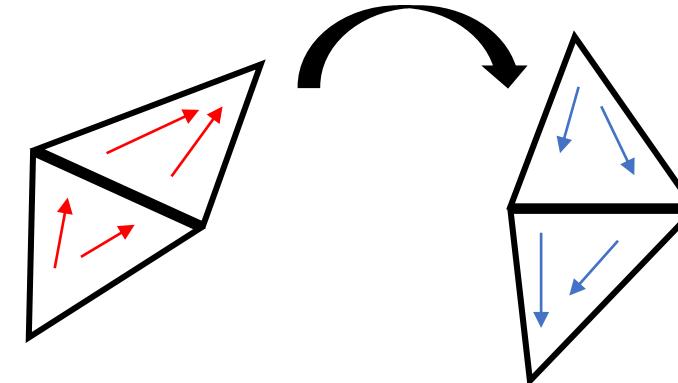


Method of Moments

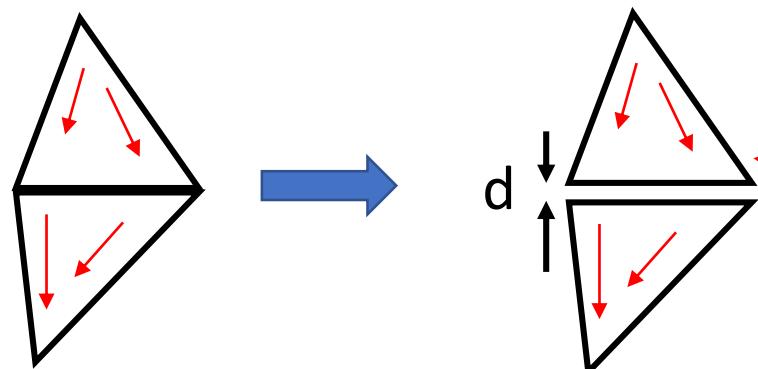
The Z matrix:

$$(Z_{in} + Z_{out}) \mathbf{x} = -\mathbf{b}$$

4D integral:
2D for the source, 2D for the test



Lumped elements: delta-gap model



Extra \vec{E} generated for the self-term
 $|\vec{E}| = V(x)/d$

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Frequency interpolation

The Z matrix:

4D integral of the type

$$Z(\omega) = f(\omega) \iiint g(\mathbf{r}, \mathbf{r}') \exp(-j k(\omega) R) dS' dS$$



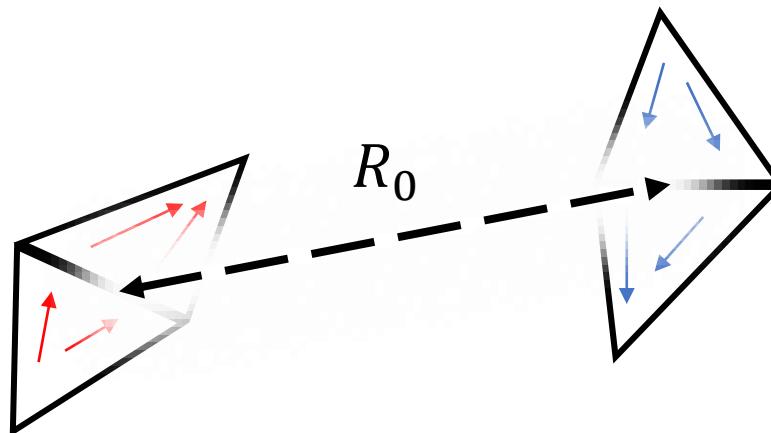
Need a small R

Frequency interpolation

The Z matrix:

4D integral of the type

$$Z(\omega) = f(\omega) \iiint g(\mathbf{r}, \mathbf{r}') \exp(-j k(\omega) R) dS' dS$$



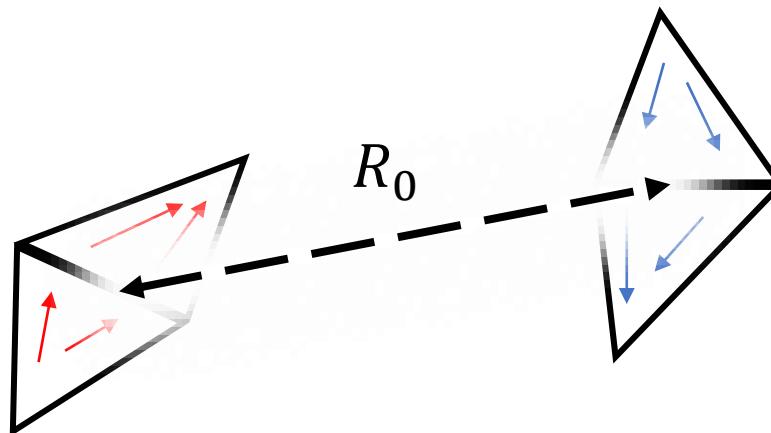
Need a small R

Frequency interpolation

The Z matrix:

4D integral of the type

$$Z(\omega) = f(\omega) \iiint g(\mathbf{r}, \mathbf{r}') \exp(-j k(\omega) R) dS' dS$$



Need a small R

$$Z(\omega) = f(\omega) \exp(-j k(\omega) R_0) \times \boxed{\iiint g(\mathbf{r}, \mathbf{r}') \exp(-j k(\omega) (R - R_0)) dS' dS}$$

Smooth with respect to ω

Interpolation of $\frac{Z(\omega)}{f(\omega)} \exp(j k(\omega) R_0)$

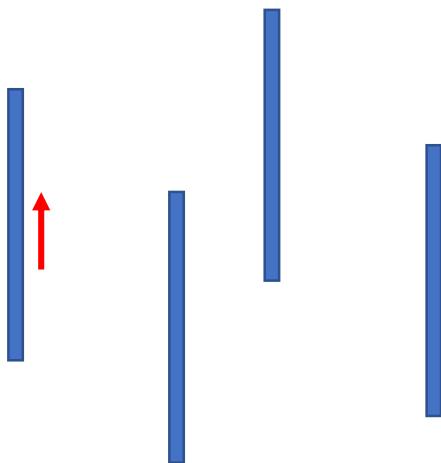
E. H. Newman, 1988

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The Array Scanning Method

Non-periodic excitation:



The Array Scanning Method

Non-periodic excitation:

$$\times \exp\left(j \frac{2\pi}{m}\right) = \frac{1}{M} \sum_m$$

The Array Scanning Method

Non-periodic excitation:

$$\times \exp\left(j \frac{2\pi}{m}\right)$$
$$= \frac{1}{M} \sum_m$$

m=0	1	1	1	1	1
m=1	1	j	-1	-j	-j
m=2	1	-1	1	-1	-1
m=3	1	-j	-1	-1	j
total	4	0	0	0	0

The Array Scanning Method

Non-periodic excitation:

$$\times \exp\left(j \frac{2\pi}{m}\right) = \frac{1}{M} \sum_m$$

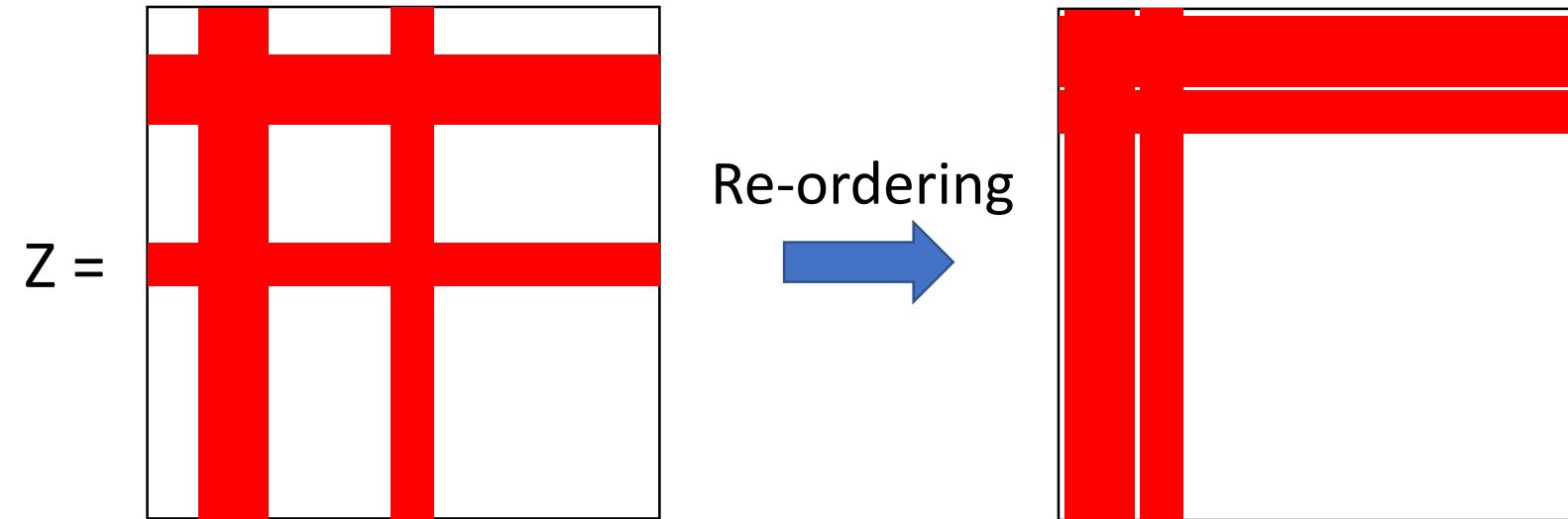
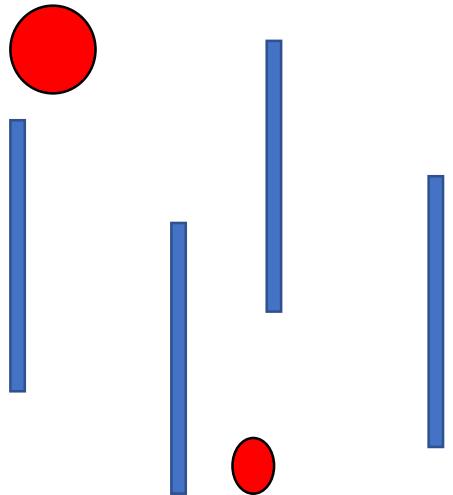
m=0	1	1	1	1	1
m=1	1	j	-1	-j	-j
m=2	1	-1	1	-1	-1
m=3	1	-j	-1	-1	j
total	4	0	0	0	0

Advantages: ➤ Filling time: $O((MN)^2) \rightarrow O(M(N)^2)$

➤ Solution time: $O((MN)^3) \rightarrow O(M(N)^3)$

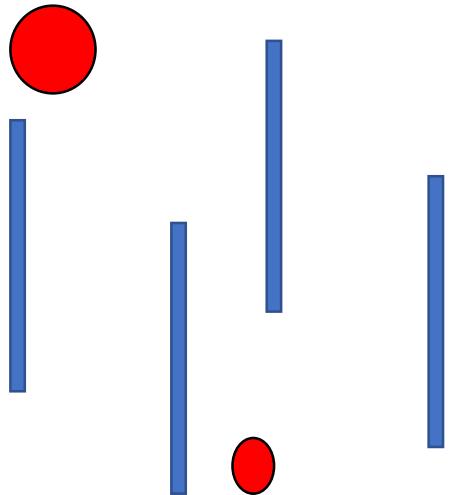
Schur complement

Nearly periodic structure:



Schur complement

Nearly periodic structure:



$$Z = \begin{matrix} & & \\ & \textcolor{red}{\#} & \\ & & \end{matrix}$$

Re-ordering

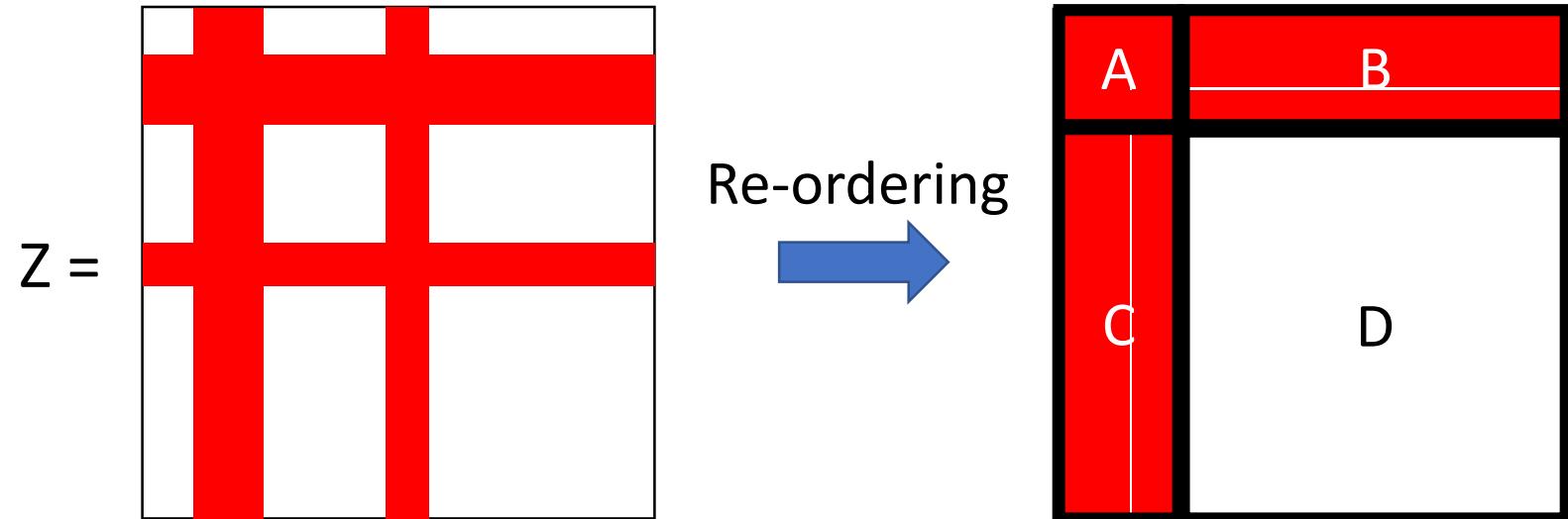
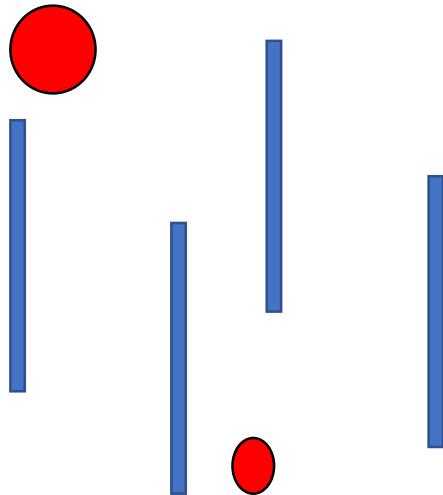
$$\begin{matrix} A & B \\ C & D \end{matrix}$$

Schur complement

$$\tilde{A}$$

Schur complement

Nearly periodic structure:



Schur complement



$$(A - C \cdot D^{-1} \cdot B) x_{red} = b_{red}$$

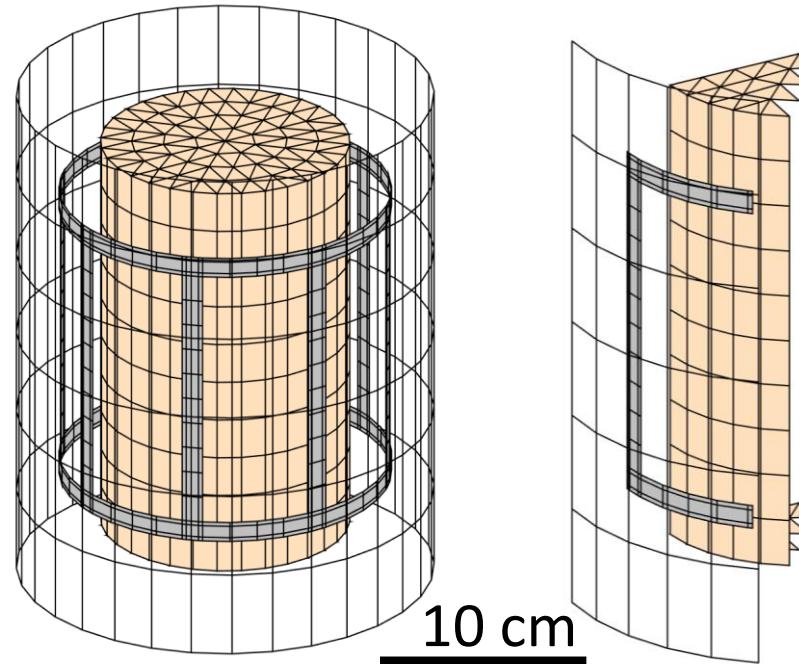
Periodic part \rightarrow rapid inversion

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Numerical validation

Simulation of birdcage antenna:

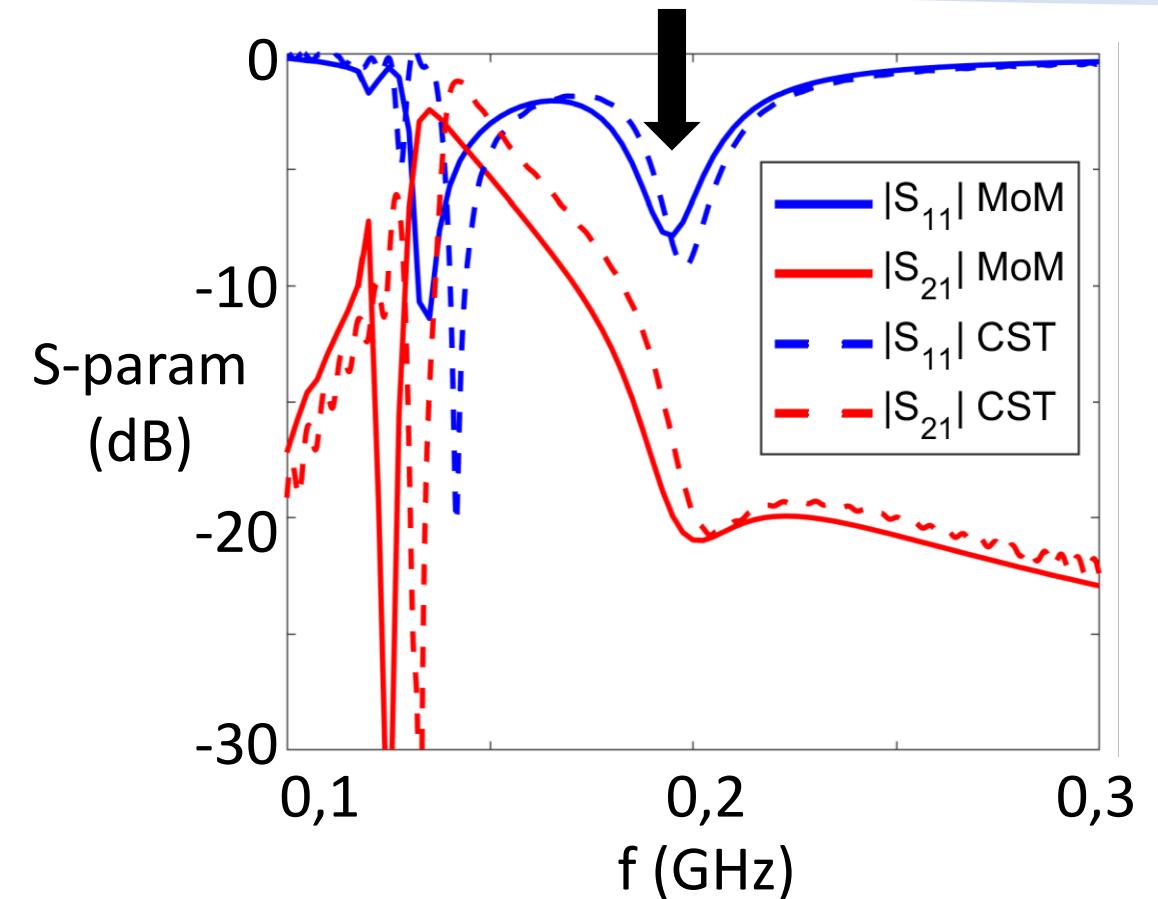


Frequency : 100-300 MHz (100 points)

Capacitance : 11 pF

Phantom : $\epsilon_r = 61, \sigma = 0,8 \left[\frac{S}{m} \right]$

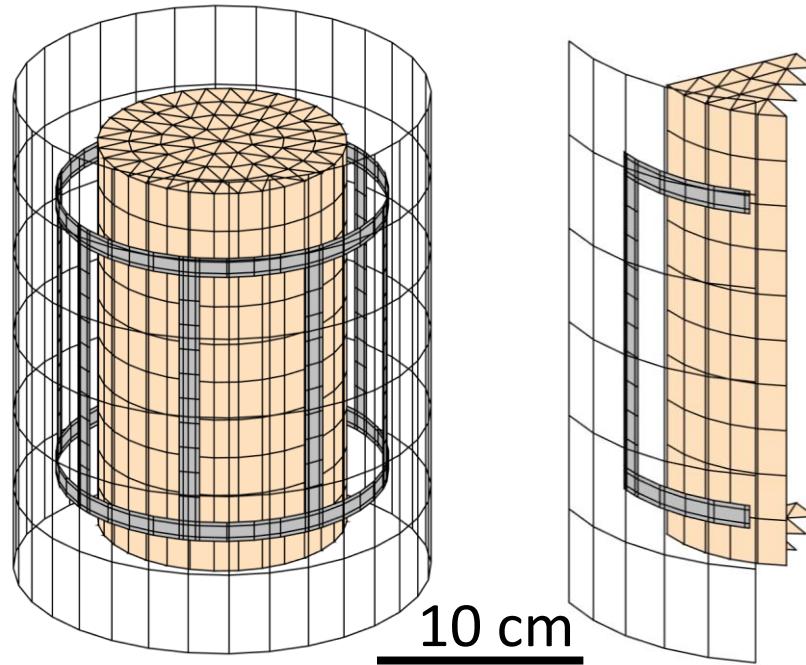
Frequency interpolation : 3 points



Time required: MoM : 12 minutes
CST : 40 minutes

Numerical validation

Simulation of birdcage antenna:



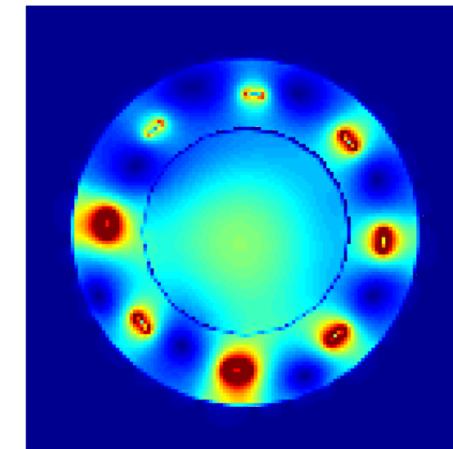
Frequency : 200 MHz

Capacitance : 11 pF

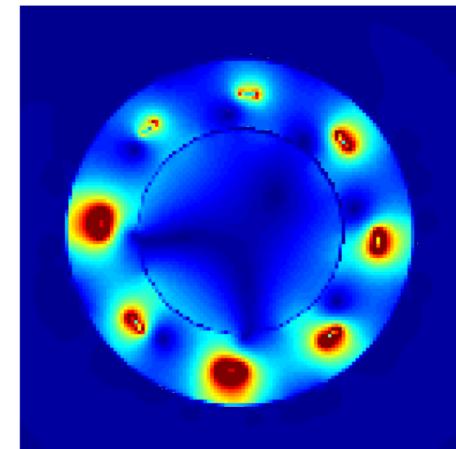
Phantom : $\epsilon_r = 61, \sigma = 0,8 \left[\frac{S}{m} \right]$

MoM

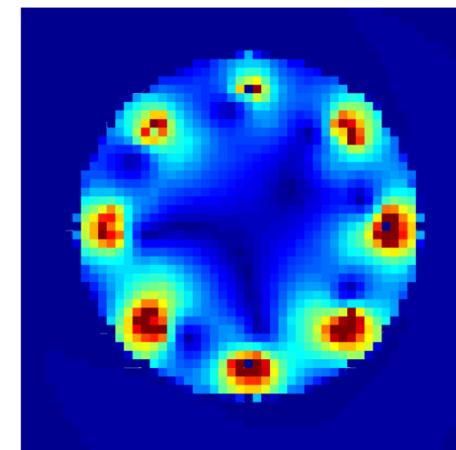
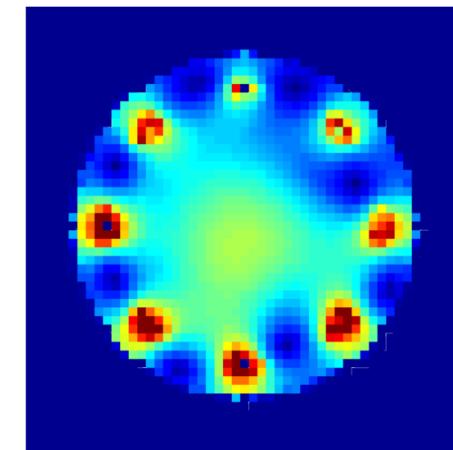
B_1^+



B_1^-



CST



Acceleration techniques:

- Frequency sweep: fast interpolation of Z
- Solution: use of the partial periodicity
- Design: rapid sweep over value of the lumped elements

Results:

- Acceleration by a factor of 3 (10 if parallelized)
- Computation for different capacitances done in negligible time

Question time

Thank you for your attention