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Hybrid Beamforming Design for Multiuser Massive MIMO-OFDM Systems

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***Abstract*—A multiuser hybrid beamforming system with orthogonal frequency division multiplexing (OFDM) is considered. We propose an alternating optimization algorithm based on manifold optimization to maximize the performance in terms of the system spectral efficiency. Moreover, two simplified methods with low complexity are also proposed which are of practical relevance for massive MIMO. The proposed methods are verified by simulation results to have smaller spectral efficiency gaps with the fully-digital beamforming than the scheme in another recent research. With the proposed designs, significant cost savings are obtained with only marginal loss in performance.**

I. Introduction

Multiple-input multiple-out (MIMO) is a well-known technology improving the capacity of a radio link using multiple transmit and receive antennas. In future communication systems, the base station (BS) will be equipped with hundreds or even thousands of antennas, i.e., an idea known under the name massive MIMO. Fortunately, massive MIMO could boost system capacity greatly thanks to its capability to achieve at the same time a much larger array gain and more aggressive spatial multiplexing than MIMO [1]–[3]. However, in conventional MIMO systems, fully-digital beamforming requires that each antenna be driven by its dedicated radio frequency (RF) chain which is not suitable for massive MIMO any more. The growing number of RF chains remarkably increases hardware cost, system complexity and power consumption which causes challenges in system implementation. To overcome this problem, the hybrid analog and digital beamforming architecture has been proposed which decreases the number of RF chains [4]–[7] by making many antennas share fewer RF chains. In detail, the hybrid beamforming consists of a high dimensional analog beamformer and a low-dimensional digital beamformer. Most popularly, a low-cost phase shifter network is applied to implement the analog beamforming to develop a tradeoff between degrees of freedom (DOF) and system complexity.

Among different application scenarios, hybrid beamforming designs have been considered for single-user MIMO systems [4] [5] and multiuser MIMO systems [6] [7] over single-carrier channels. However, hybrid beamforming designs over frequency selective channels are more challenging as the analog beamforming (that is implemented by analog components) can only be "frequency flat". Nonetheless, in [8] [9], codebooks and dynamic subarray architectures are designed in terms of system capacity for single user hybrid precoding systems. Moreover, for multiuser MIMO systems, a low-complexity beamforming design has been proposed in [10]. By applying the Jensen's inequality, a heuristic design is proposed for OFDM-based multiuser hybrid precoding in [11]. However, methods in [10] and [11] are based on the average channel information which reduces computational complexity but causes performance deterioration.

This paper considers a hybrid beamforming design in a multiuser MIMO downlink system operating over frequency selective channels. We aim to propose some novel hybrid beamforming methods to approach the performance of fully-digital beamforming. An alternating optimization algorithm is proposed in this paper to maximize the system spectral effeciency. Moreover, two methods with reduced computational complexity are also developed to cut much complexity without large performance loss. Specifically, a design only utilizing the channel information of one subcarrier for wide-band channels (WC-SSD) is proposed for popular mmWave system application, i.e., line-of-sight (LoS). It is still effective for multipath channels when the channels are sparse, i.e., the path number is not very large. Furthermore, we also develop a channel matrix decomposition design (CMDD) which only has a minor performance degradation in constrast with the alternating optimization algorithm.

Notations throughout this paper are as follows. Upper and lower case bold-face letters are matrices and vectors, respectively. Lower case normal letters are scalars. $\|\cdot\|_F$ and $(\cdot)^H$ represent the Frobenius norm and Hermitian of a matrix, respectively. $|\cdot|$ and $(\cdot)^*$ represent the absolute value and conjugation of a complex number, respectively. $\mathbb{C}^{m\times n}$ and $\mathbb{R}^{m\times n}$ respectively denote the ensemble of complex and real valued $m\times n$ matrices. $\mathbb{E}[\cdot]$ is used to denote expectation. $\Re[\cdot]$ represents the real part of a complex number. $\mathcal{CN}(\mu,\sigma^2)$ stands for the complex Gaussian distribution with mean μ and variance σ^2. $\mathcal{U}(a,b)$ denotes the uniform distribution between a and b. $\circ$ refers to the Hadamard product.

II. System and Channel Models

A. System Model

We consider an OFDM-based massive MIMO hybrid beamforming wireless communication system where the BS is equipped with U RF chains and M antennas where $U \ll M$ as shown in Fig. 1. For OFDM, K subcarriers are used for data

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transmission where U single-antenna users are simultaneously served on the entire band. At the BS, the signals at different subcarriers are first digitally-precoded respectively and then transformed to the time domain by using K-point inverse fast Fourier transforms (IFFTs). After that, the transformed signals are finally processed by an analog precoding matrix before transmission. Since the analog precoder is operated on signals after IFFT, it is the same for all subcarriers which indicates that it is flat in the frequency domain. This is the key challenge in designing the hybrid precoding over frequency selective wideband channels.

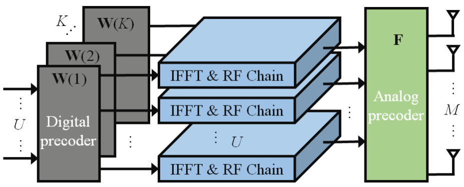

Fig. 1: OFDM-based Hybrid precoding architecture.

Assuming a block-fading channel model, the signal received at subcarrier k can be written by

$$\mathbf{y}[k] = \mathbf{H}[k]^H \mathbf{F} \mathbf{W}[k] \mathbf{s}[k] + \mathbf{n}[k] \tag{1}$$

where $\mathbf{s}[k] \in \mathbb{C}^{U \times 1}$ denotes the vector of transmitted data symbols at subcarrier k with $\mathbb{E}\{\mathbf{s}[k]\mathbf{s}^H[k]\} = P\mathbf{I}$ in which P is the transmit power for each user at each subcarrier, $\mathbf{W}[k] = [\mathbf{w}_1[k], \mathbf{w}_2[k], ..., \mathbf{w}_U[k]] \in \mathbb{C}^{U \times U}$ refers to the digital precoder at subcarrier k, $\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, ..., \mathbf{f}_U] \in \mathbb{C}^{M \times U}$ stands for the analog precoder, $\mathbf{H}[k] = [\mathbf{h}_1[k], \mathbf{h}_2[k], ..., \mathbf{h}_U[k]] \in \mathbb{C}^{M \times U}$ represents the channel matrix and $\mathbf{n}[k] \sim \mathcal{CN}(\mathbf{0}_K, P_n \mathbf{I}_K)$ refers to the additive white Gaussian noise for subcarrier k in which P_n is the noise power. It is notable that the analog part of the hybrid beamformer is typically implemented using simple analog components such as analog phase shifters which can only modify the angles of signals. Thus, every entry in $\mathbf{F}$ has the same constant amplitude. In this work, the fully-connected structure for hybrid precoding is considered in which each RF chain drives all antennas. Each RF chain and each antenna is connected through only one phase shifter. Therefore, the i-th element of $\mathbf{f}_u$ is normalized as

$$|f_{u,i}| = \frac{1}{\sqrt{M}}. \tag{2}$$

From (1), the received signal of the u-th user at subcarrier k is

$$y_u[k] = \mathbf{h}_u[k]^H \mathbf{F} \mathbf{W}[k] \mathbf{s}[k] + n_u[k] \tag{3}$$

where $y_u[k]$ and $n_u[k]$ respectively denote the u-th element of $\mathbf{y}[k]$ and $\mathbf{n}[k]$.

B. Channel Model

To characterize the practical scattering features of mmWave channels, we adopt the most widely used geometric channel model [12]. We assume that the channels between the BS and all users have the same number of paths, i.e., L. For the u-th user, each path has a time delay $\tau_{ul} \in \mathbb{R}$, angle of departure (AOD) $\theta_{ul} \in [0, 2\pi]$ and complex path gain α_{ul}. Assuming sampling with period T_s, in the frequency domain, we write the channel vector for the u-th user as

$$\mathbf{h}_u[k] = \sqrt{\frac{M}{L}} \sum_{n=1}^{N_c - 1} \left[\mathrm{e}^{-j\frac{2\pi kn}{K}} \sum_{l=1}^{L} \alpha_{ul} p(nT_s - \tau_{ul}) \mathbf{a}_{ul} \right] \tag{4}$$

where N_c refers to the number of delay taps, $p(\tau)$ is the pulse shaping filter and $\mathbf{a}_{ul}$ denotes the antenna array response vector of the base station. For simplicity, uniform linear arrays (ULAs) are utilized where

$$\mathbf{a}_{ul} = \frac{1}{\sqrt{M}} [1, \mathrm{e}^{-j\frac{2\pi}{\lambda} d \sin(\theta_{ul})}, ..., \mathrm{e}^{-j(M-1)\frac{2\pi}{\lambda} d \sin(\theta_{ul})}]^T \tag{5}$$

in which λ denotes the signal wavelength, and d represents the distance between antenna elements. Note that most of the results developed in this paper are general for massive MIMO channels, and not restricted to the channel model in this subsection. We describe the mmWave channel model here as it will be important for understanding the motivation of WC-SSD in Section IV-B. Furthermore, it will be adopted for the simulation in Section V.

C. Problem Formulation

The problem of interest for the multiuser MIMO case is to design the hybrid analog and digital beamformers in order to maximize the overall system spectral efficiency. In this case, the spectral efficiency of the u-th user at subcarrier k can be formulated as

$$R_u[k] = \log_2 \left(1 + \frac{\mathrm{SNR} |\mathbf{h}_u[k]^H \mathbf{F} \mathbf{w}_u[k]|^2}{\mathrm{SNR} \sum_{i \neq u} |\mathbf{h}_u[k]^H \mathbf{F} \mathbf{w}_i[k]|^2 + 1} \right) \tag{6}$$

where $\mathrm{SNR} = \frac{P}{P_n}$ is the signal-to-noise ratio of each user at each subcarrier. For the system, the problem is written as

$$\begin{aligned} \max_{\mathbf{F}, \mathbf{W}[k]_{k=1}^K} \quad & \sum_{k=1}^{K} \sum_{u=1}^{U} R_u[k] \\ s.t. \quad & |f_{ui}| = \frac{1}{\sqrt{M}}, \forall 1 \le i \le M, \ 1 \le u \le U, \\ & \|\mathbf{F}\mathbf{w}_u[k]\|_F = 1, \forall 1 \le u \le U, \ 1 \le k \le K. \end{aligned} \tag{7}$$

To tackle this problem, we would like to propose some design strategies in the following. Note that perfect channel state information (CSI) is assumed to be available in order to study the performance limits of the hybrid beamforming architecture.

III. Alternating Maximization Algorithm for Hybrid Precoding

As apparent from (7), the main difficulty of this problem is solving the maximization problem over two series of variables, i.e., $\mathbf{F}$ and $\{\mathbf{W}[k]\}_{k=1}^K$. However, alternating optimization algorithms could be used in such applications thanks to its iterative nature and simplicity. In this section, we propose an alternating maximization algorithm for hybrid precoding to maximize the system spectral efficiency which is based on an iterative analog precoder design. Instead of solving the original optimization problem over two series of variables, the proposed alternating maximization algorithm separates the problem into two problems where each one is concerned with only one series of variables. With the principle of alternating maximization, we will respectively maximize the system spectral efficiency with respect to $\mathbf{F}$ and $\{\mathbf{W}[k]\}_{k=1}^{K}$ while keeping the other one fixed. For hybrid precoding, the alternating optimization has been considered in [5]. The alternating minimization in [5] minimizes the difference between the optimal solution and hybrid beamforming design for single user case. Note that this method is not quite suitable for multiuser since the difference between the hybrid beamforming and optimal beamforming causes multiuser interference which further results in performance deterioration. In contrast, in this paper, the alternating maximization maximizes the system spectral efficiency which solves the original problem.

A. Analog RF Precoding Design

Since the analog precoding design is not trivial due to the constant amplitude constraints, we first consider designing the analog precoder $\mathbf{F}$ with fixed digital precoders $\{\mathbf{W}[k]\}_{k=1}^{K}$. Thus, problem in (7) can be restated as

$$\begin{aligned} \max_{\mathbf{F}} \quad & \sum_{k=1}^{K}\sum_{u=1}^{U} R_u[k] \\ s.t. \quad & |f_{ui}| = \frac{1}{\sqrt{M}}, \forall 1 \le i \le M,\ 1 \le u \le U. \end{aligned} \tag{8}$$

Unfortunately, we can not directly solve this problem in the Euclidean space dut to the constant amplitude constraints in (8). However, we could utilize a solution based on manifold optimization since the constant amplitude constraints define a Riemannian manifold. Due to the fact that the neighborhood of each point on a manifold resembles the Euclidean space, optimization algorithms in the Euclidean space could also be applied in manifolds. In this work, we would like to develop the conjugate gradient algorithm for the analog precoder design to maximize the spectral effeciency. Before explaining the exact algorithm, we first need to make some definitions of the Riemannian manifold as in [5].

We transfer $\mathbf{F}$ into a vector $\mathbf{x} = \mathrm{vec}[\mathbf{F}]$. Then, we can represent the manifold of the analog beamforming as

$$\mathcal{M} = \{\mathbf{x} \in \mathbb{C}^{MU} : |x_i| = \frac{1}{\sqrt{M}}, i = 1, 2, ..., MU\} \tag{9}$$

where x_i is the i-th element of $\mathbf{x}$. By treating $\mathbb{C}$ as $\mathbb{R}^2$ with the canonical inner product, we define the Euclidean metric in the complex $\mathbb{C}$ plane as

$$< x_1, x_2 >= \Re[x_1^* x_2] \tag{10}$$

which enables us to denote the tangent space of $\mathbf{x}$ as

$$\mathcal{T}_{\mathbf{x}}\mathcal{M} = \{\mathbf{z} \in \mathbb{C}^{MU} : \Re[\mathbf{z} \circ \mathbf{x}] = 0\}. \tag{11}$$

According to $\mathbf{x}$, we define the cost function as

$$f(\mathbf{x}) = R_{sum} = \sum_{k=1}^{K}\sum_{u=1}^{U} R_u[k]. \tag{12}$$

Moreover, since the inner product for the Riemannian manifold is defined on the tangent space of the given point $\mathbf{x}$, the optimization algorithm should also be conducted on the tangent space of $\mathbf{x}$. In other words, when calculating factors for the current point $\mathbf{x}$, we need to cancel the parts not in the tangent space of $\mathbf{x}$ by the orthogonal projection as

$$\mathrm{T}_{\mathbf{x}}(\mathbf{d}) = \mathbf{d} - \Re[\mathbf{d} \circ \mathbf{x}^*] \circ \mathbf{x}. \tag{13}$$

However, we use the algorithm on the tangent space of $\mathbf{x}$ which makes the new point be on its tangent space but not be guaranteed on the Riemannian manifold. Therefore, we should also map the new point onto the Riemannian manifold by retraction which can be stated as

$$\mathrm{R}(\mathbf{x}) = \frac{1}{\sqrt{M}} \mathrm{vec}\left[\frac{x_1}{|x_1|}, \frac{x_2}{|x_2|}, ..., \frac{x_{MU}}{|x_{MU}|}\right]. \tag{14}$$

Note that, in the proposed algorithm, the retraction restricts the constant amplitude contraints to the vector again.

Therefore, we develop the analog precoding as shown in Algorithm 1. After processed by Algorithm 1, $\mathbf{x}$ is transformed into the matrix $\mathbf{F}$ again.

Algorithm 1: Conjugate Gradient Algorithm for Analog Precoding Based on Manifold Optimization

Input: $\mathbf{W}[k]_{k=1}^{K}$,$\mathbf{x}_0$,s,ϵ_1
1 $\mathbf{d}_0 = \mathrm{T}_{\mathbf{x}_0}\left(\left.\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_0}\right)$, $t = 0$ and $f(\mathbf{x}_{-1}) = 0$;
2 **while** $f(\mathbf{x}_t) - f(\mathbf{x}_{t-1}) > \epsilon_1$ **do**
3 Choose the step size: $\alpha_t = \frac{s}{\|\mathbf{d}_t\|_F}$;
4 Update the position as $\mathbf{x}_{t+1} = \mathbf{x}_t + \alpha_t \mathbf{d}_t$;
5 Map the vector onto the manifold: $\mathbf{x}_{t+1} = \mathrm{R}(\mathbf{x}_{t+1})$;
6 Compute Riemannian gradient:
$\mathbf{g}_{t+1} = \mathrm{T}_{\mathbf{x}_{t+1}}\left(\left.\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}^*}\right|_{\mathbf{x}=\mathbf{x}_{t+1}}\right)$;
7 Calculate Polak-Ribiere parameter as
$\beta_{t+1} = \frac{\mathbf{g}_{t+1}^H(\mathbf{g}_{t+1} - \mathrm{T}_{\mathbf{x}_{t+1}}(\mathbf{g}_t))}{\|\mathrm{T}_{\mathbf{x}_{t+1}}(\mathbf{g}_t)\|_F^2}$;
8 Determine conjugate direction:
$\mathbf{d}_{t+1} = \mathbf{g}_{t+1} + \beta_{t+1}\mathrm{T}_{\mathbf{x}_{t+1}}(\mathbf{d}_t)$;
9 $t \leftarrow t + 1$.
10 **end**

It is notable that, in Step 6, $\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}^*}$ is the Euclidean gradient. Specifically, $\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}^*}$ is the vectorized Euclidean gradient of $\mathbf{F}^*$, i.e., $\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{x}^*} = \mathrm{vec}\left[\frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{F}^*}\right]$ where

$$\begin{aligned} \frac{\mathrm{d}f(\mathbf{x})}{\mathrm{d}\mathbf{F}^*} = & \sum_{k=1}^{K}\sum_{u=1}^{U} \frac{1}{\ln 2}\left[\frac{\mathrm{SNR}\mathbf{h}_u[k]\mathbf{h}_u[k]^H\mathbf{F}\mathbf{W}[k]\mathbf{W}[k]^H}{1 + \mathrm{SNR}\|\mathbf{h}_u[k]^H\mathbf{F}\mathbf{W}[k]\|_F^2}\right. \\ & \left. - \frac{\mathrm{SNR}\mathbf{h}_u[k]\mathbf{h}_u[k]^H\mathbf{F}\mathbf{W}_{\bar{u}}[k]\mathbf{W}_{\bar{u}}[k]^H}{1 + \mathrm{SNR}\|\mathbf{h}_u[k]^H\mathbf{F}\mathbf{W}_{\bar{u}}[k]\|_F^2}\right] \end{aligned} \tag{15}$$

in which $\mathbf{W}_{\bar{u}}[k]$ consists of all columns of $\mathbf{W}[k]$ except $\mathbf{w}_u[k]$.

B. Hybrid Precoding Design

With Algorithm 1, the hybrid precoding design via alternating maximization still requires digital precoding while fixing analog precoding. In this work, we directly follow minimum mean square error (MMSE) beamforming. In particular, MMSE beamforming computes the best beamforming vectors to maximize the system spectral effeciency by minimizing the error between the transmitted signal and received signal caused from both multiuser interference and noise. However, for hybrid beamforming, the digital beamforming is designed according to the effective channels but not the original channels. For each subcarrier, the effective channel matrix for all users is denoted by

$$\mathbf{G}[k]^H = \mathbf{H}[k]^H \mathbf{F}. \tag{16}$$

Then, the base station calculates MMSE precoder based on effective channels as

$$\mathbf{W}[k] = (\mathbf{G}[k]\mathbf{G}[k]^H + \frac{1}{\mathrm{SNR}})^{-1}\mathbf{G}[k]. \tag{17}$$

The digital precoder is finally normalized as $\mathbf{w}_u[k] = \frac{\mathbf{w}_u[k]}{\|\mathbf{F}\mathbf{w}_u[k]\|_F}$ to fulfill the power constraints.

Combining Algorithm 1 and MMSE digital precoding design, we finally characterize the alternating maximization algorithm for hybrid precoding in Algorithm 2.

Algorithm 2: Manifold Optimization Based Hybrid Precoding

Input: $\mathbf{F}^{(0)}$,$\mathbf{W}[k]^{(0)}$,ϵ_2
1. Set $t = 0$ and $R_{sum}^{(-1)} = 0$;
2. **while** $R_{sum}^{(t)} - R_{sum}^{(t-1)} > \epsilon_2$ **do**
3. Optimize $\mathbf{F}^{(t+1)}$ using Algorithm 1 when $\mathbf{W}[k]^{(t)}$ is fixed;
4. Calculate effective channels and MMSE digital precoding $\mathbf{W}[k]^{(t+1)}, k = 1, ..., K$ according to (16) and (17);
5. $t \leftarrow t + 1$.
6. **end**

IV. Low-Complexity Hybrid Precoding Design

In the last section, we have proposed an iterative solution to design OFDM-based multiuser MIMO hybrid beamformers. Although it is supposed to maximize the system performance, it requires high computational complexity which is not suitable for practical systems. Therefore, in this section, we would like to provide some simple designs with reduced complexity for multiuser MIMO hybrid beamforming over wideband channels.

A. Simplified Problem

Since the original problem (7) is complicated and non convex, to propose simple designs, we need to begin with simplifying the problem in this subsection. To simplify the problem, we would like learn some experience from the designs employed in single carrier systems. It can be learned from [6] [7] [13] that, for single carrier systems, the hybrid beamforming design for multiuser MIMO could be divided into two steps: (1) First maximize the signal power for each user utilizing the analog precoder ignoring the interference among users; (2) Second, design the digital beamformers according to the effective channels using the linear fully-digital beamforming schemes such as ZF or MMSE to cope with the multiuser interference. This strategy not only transfers the complicated multiuser analog precoder design into single-user case but also is proved to be asymptotically optimal in massive MIMO [13]. In the following, we simplify the spectral efficiency maximizing problem by extending this design into the OFDM system.

Similarly to the method in single carrier case, the OFDM-based hybrid precoding problem is split into two different domains, each with different constraints. The main idea of the proposed algorithm is to divide the calculation of the precoders into two stages. In the first stage, the analog precoder is designed to maximize the sum desired signal power on the entire band of each user and the interference among users is left to be addressed in the next stage. The analog precoder design is converted into a single-user problem which could be displayed as

$$\begin{aligned} \max_{\mathbf{f}_u} & \sum_{k=1}^{K} |\mathbf{h}_u[k]^H \mathbf{f}_u|^2, \ \forall 1 \le u \le U \\ s.t. \quad & |f_{ui}| = \frac{1}{\sqrt{M}}, i = 1, 2, \ldots, M. \end{aligned} \tag{18}$$

In the second stage, the digital precoder is designed according to MMSE to manage the multiuser interference which corresponds to (16) and (17).

For this scheme, the digital precoding design is so trivial that it does not require any explanation in this article. Therefore, in the remaining part of this section, we only focus on solving (18) to design the analog beamforming.

B. LoS Case

As mentioned in the last subsection, the key point of this simplified design is solving (18). Although the problem is simplified now, (18) is still non convex. Therefore, in the following, we will first study the performance of the proposed algorithm in one special case: with single-path channels. This case is of special interest as mmWave channels are likely to be sparse, i.e., only a few paths exist [12]. In this case, by letting $l = 1$ in the definition of the channel parameters, the model in (4) corresponds to single-path channels.

For single-user systems, the optimal design for a randomly selected subcarrier is also optimal for all subcarriers only if the number of channel paths is not larger than the numbers of antennas at both the transmitter and receiver sides [14]. In LoS case with single-antenna users in this work, the path number is equal to the number of antennas at each user side and obviously less than the numbers of the BS with large-scale antenna array. It implies that (18) is relaxed as

$$\begin{aligned} \max_{\mathbf{f}_u} \quad & |\mathbf{h}_u[\kappa]^H \mathbf{f}_u|^2 \\ s.t. \quad & |f_{ui}| = \frac{1}{\sqrt{M}}, i = 1, 2, \ldots, M \end{aligned} \tag{19}$$

where κ is randomly selected from $\{1, 2, ..., K\}$. As a consequence, the OFDM-based hybrid beamforming is transferred into a single-carrier problem. Simply, the solution for (19) is $f_{ui} = \frac{1}{\sqrt{M}} \frac{h_{ui}[\kappa]}{|h_{ui}[\kappa]|}$ where $h_{ui}[\kappa]$ is the i-th element of $\mathbf{h}_u[\kappa]$. Thus far, procedures of WC-SSD are clearly presented.

Since this scheme is simple enough, we would like to know whether it could be applied in all cases, even over multipath channels. However, this method is only spectral efficient over sparse channels. For sparse channels, the angle differences between channels at different subcarriers are small which indicates the channels have similar directions on frequency domain. Hence, the more sparse the channels are, the more spectral efficient this scheme becomes.

C. Channel Decomposition Among Subcarriers

Different from developing a scheme for sparse channels as proposed in the last subsection, we would like propose a general design strategy for multipath channels, i.e., CMDD, in this subsection. The difficulties of the design problem for multipath channels is due to the fact that the analog beamformer could only adjust phases but not amplitudes. Therefore, in this subsection, we first solve this problem as conventional beamformer design by dropping the constraint of the analog precoder and then restrict the analog beamformer matrix to satisfy the constant amplitude constraint. The steps of the proposed solution are described as follows.

Firstly, we apply eigenvalue decomposition to $\mathbf{S}_u = \mathbf{H}_u \mathbf{H}_u^H$ as

$$\mathbf{S}_u = \mathbf{P}\mathbf{M}\mathbf{P}^H \tag{20}$$

where $\mathbf{H}_u = [\mathbf{h}_u[1], \mathbf{h}_u[2], ..., \mathbf{h}_u[K]]$ stands for the channel matrix of u-th user at all subcarriers. Then, we find the eigenvector $\mathbf{p}_u^{opt}$ corresponding to the maximum eigenvalue λ_u^{opt} which indicates

$$\begin{aligned} \sum_{k=1}^{K} |\mathbf{h}_u[k]^H \mathbf{p}_u^{opt}|^2 =& \|\mathbf{H}_u^H \mathbf{p}_u^{opt}\|_F^2 \\ =& \mathrm{Tr}[(\mathbf{p}_u^{opt})^H \mathbf{H}_u \mathbf{H}_u^H \mathbf{p}_u^{opt}] \\ =& \lambda_u^{opt}. \end{aligned} \tag{21}$$

By this way, we obtain a suboptimal analog beamformer design for each user. However, the solution still does not fulfill the constant amplitude constraints of the analog beamforming, i.e., $|f_{ui}| = \frac{1}{\sqrt{M}}, i = 1, 2, \ldots, M$. Hence, the remaining step is to solve

$$\begin{aligned} \min_{\mathbf{f}_u} \quad & \|\mathbf{f}_u - \mathbf{p}_u^{opt}\|_F^2 \\ s.t. \quad & |f_{ui}| = \frac{1}{\sqrt{M}}, i = 1, 2, \ldots, M \end{aligned} \tag{22}$$

By applying the property of the Frobenius norm and trace, we can get

$$\|\mathbf{f}_u - \mathbf{p}_u^{opt}\|_F^2 = 2 - 2\mathrm{Tr}[\Re[\mathbf{f}_u^H \mathbf{p}_u^{opt}]]. \tag{23}$$

According to (23), the minimum value is achieved when $\mathbf{f}_u$ has the same phase components as $\mathbf{p}_u^{opt}$, i.e.,

$$f_{ui} = \frac{1}{\sqrt{M}} \frac{p_{ui}^{opt}}{|p_{ui}^{opt}|} \tag{24}$$

where p_{ui}^{opt} is the i-th element of $\mathbf{p}_u^{opt}$. Thus far, the analog beamforming design is completed.

V. Simulation Results

In this section, simulation results regarding the proposed beamforming methods are compared with another recently proposed design by [10] which is based on the spatial covariance matrix knowledge of all users. Regarding the channel model, the AODs and complex path gains are respectively distributed as $\theta_{ul} \sim \mathcal{U}(0, 2\pi)$ and $\alpha_{ul} \sim \mathcal{CN}(0, 1)$. Three cases are considered and their downlink average achievable sum spectral efficiencies per user are presented in Fig. 2 for different path numbers or different user numbers. At the BS, 64 antennas and 64 subcarriers are employed. In the figures, the results for the fully digital MMSE based on full CSI are displayed as reference. The alternating optimization algorithm based on manifold optimization proposed in Section III is denoted as Hybrid precoding-AO. Based on the results in Fig. 2, several observations can be made:

(1) First of all, in all three cases, the proposed alternating optimization algorithm has the best performance among all hybrid beamforming designs which has the smallest gap with the fully-digital beamforming system. Then, for the simplified schemes in this paper, CMDD outperforms WC-SSD. Finally, all designs except WC-SSD proposed in this article all have larger spectral efficiency than that of the design in [10]. WC-SSD is only beaten by the design in [10] in Fig. 2(b) at low SNRs but outperforms the design in [10] in other cases.

(2) Fig. 2 (a) and Fig. 2(b) show the performance differences between different beamforming designs for different path numbers. For WC-SSD, the gap with the alternating optimization algorithm is minor when $L = 4$ but becomes large when L increases to 6 which verifies that WC-SSD is more sufficient in sparse channels. However, CMDD still reduces complexity with only a small performance loss in constrast with the alternating optimization algorithm, i.e., about 1dB.

(3) Fig. 2 (a) and Fig. 2 (c) compare the performance of different designs for different user numbers. Although the performance gaps between fully-digital beamforming and the two simplified schemes become larger when the user number

grows, the gap between alternating optimization algorithm and the fully-digital beamforming is still not quite large.

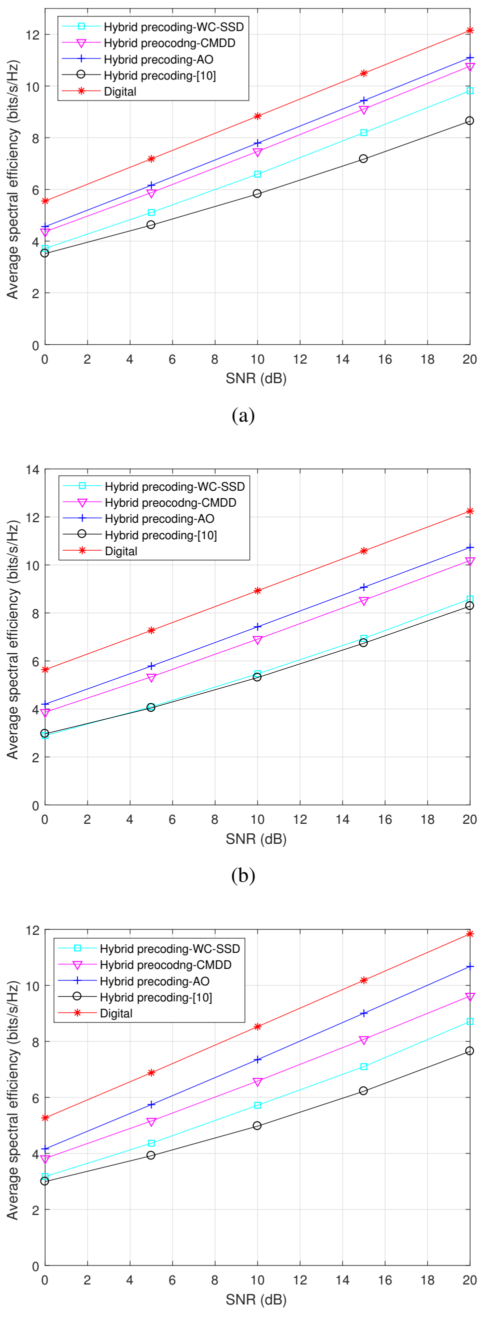

Fig. 2: Spectral efficiency per user for different SNRs with $U = 8$ and $L = 4$ in (a), $U = 8$ and $L = 6$ in (b) and $U = 16$ and $L = 4$ in (c).

VI. Conclusion

This paper considers hybrid analog and digital beamforming design for the OFDM-based multiuser massive MIMO system. Built on the principle of manifold optimization, we proposed an effective alternating algorithm design for hybrid beamforming. Specifically, analog beamforming is optimized by utilizing manifold optimization and digital beamforming is designed according to MMSE criterion. Moreover, two simplified designs are also proposed to reduce the computational complexity remarkably without large performance loss. In detail, WC-SSD designed for the popular mmWave scenario, i.e., LoS case, can also be applied into multipath case with few paths. A more general method proposed in this article, i.e., CMDD, is shown to have performance similar to that of the alternating optimization algorithm with far less complexity. Verified by the numerical results, all of the proposed methods outperform the design in the recent research [10].

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