Optimization of the array element positions based on the analytical dependence of the mutual coupling in the GSM finite array formulation

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***Abstract*— Positions of array antenna elements are optimized taking advantage of the analytical dependence of the mutual coupling in the Generalized Scattering Matrix (GSM) finite array formulation. First, spatial derivatives for the GSM formulation are analytically computed. Then, they are applied to the synthesis of uniform amplitude finite arrays, by rotating the elements of the array or by changing their positions, through a gradient-based optimization method.**

Keywords—finite array synthesis; mutual coupling; generalized scattering matrix; gradient-based optimization

I. INTRODUCTION

A few years ago, a formulation for the analytical computation of the mutual coupling in finite arrays of antennas was proposed in [1]. This formulation is based on the translation and rotation theorems for spherical wave expansion. In the last years, it has been successfully applied to array synthesis optimizing the complex amplitudes of the array element excitations, due to its capability to deal with large arrays with a very low computational effort. Array Thinning has also been addressed by using Evolutionary and Greedy algorithms. However, up to now, synthesis of uniform amplitude arrays has not been carried out based on the proposed method.

In this work, spatial derivatives for the GSM finite array formulation given in [1] will be analytically computed, including derivatives of rotations and translations of spherical waves. They will allow to synthesize uniform amplitude finite arrays of rotated or non-equispaced elements, by using a gradient-based optimization method [2].

II. THEORY

A. Generalized Scattering Matrix Finite Array Formulation

As explained in [3], a hybrid 3-D Finite Element/Modal Analysis method can be used for a full wave and efficient analysis of a single antenna. This method provides the Generalized Scattering Matrix (GSM) of an isolated antenna i, which is given in terms of incident ($\mathbf{v}_i$) and reflected ($\mathbf{w}_i$) modes on the feeding port and incoming ($\mathbf{a}_i$) and scattering ($\mathbf{b}_i^{\mathrm{s}}$) modes on the spherical port as

$$\begin{bmatrix} \boldsymbol{\rho}_i & \mathbf{r}_i \\ \mathbf{t}_i & \mathbf{s}_i - \mathbf{I}_i \end{bmatrix} \begin{bmatrix} \mathbf{v}_i \\ \mathbf{a}_i \end{bmatrix} = \begin{bmatrix} \mathbf{w}_i \\ \mathbf{b}_i^{\mathrm{s}} \end{bmatrix} \tag{1}$$

where $\mathbf{v}_i$, $\mathbf{w}_i$, $\mathbf{a}_i$ and $\mathbf{b}_i^{\mathrm{s}}$ are column vectors and their elements are the complex amplitudes of waveguide or transmission line modes and spherical modes. $\boldsymbol{\rho}_i$, $\mathbf{r}_i$, $\mathbf{t}_i$, and $\mathbf{s}_i$ are respectively the antenna reflection, reception, transmission and scattering matrices.

Now, in order to analyze, in an efficient way and rigorously, an array of N antennas, the GSMs of every isolated antennas are connected by using the rotation and translation properties of spherical modes. For this purpose, we need to relate incoming spherical modes in one antenna j and scattering spherical modes in other different antenna k. These relations are given by means of matrices $\mathbf{G}_{jk}$,

$$\mathbf{a}_j = \mathbf{G}_{jk}\, \mathbf{b}_k^{\mathrm{s}} \tag{2}$$

where

$$\mathbf{G}_{jk} = \left[\mathbf{R}_k(-\varphi_{Rk})\, \mathbf{R}_k(\varphi_{kj})\, \mathbf{D}_k\!\left(\pi/2\right) \mathbf{C}\!\left(d/\lambda\right) \mathbf{D}_j\!\left(-\pi/2\right) \mathbf{R}_j(-\varphi_{kj})\, \mathbf{R}_j(\varphi_{Rj}) \right]^T \tag{3}$$

as explained in [1]. These matrices only depend on the electrical distance and the relative positions between the antennas. For the case where there are not rotated elements, (3) reduces to

$$\mathbf{G}_{jk}^{\mathrm{nr}} = \left[\mathbf{R}_k(\varphi_{kj})\, \mathbf{D}_k\!\left(\pi/2\right) \mathbf{C}\!\left(d/\lambda\right) \mathbf{D}_j\!\left(-\pi/2\right) \mathbf{R}_j(-\varphi_{kj}) \right]^T . \tag{4}$$

By using the GSM of each antenna and expression (2) for each pair of antennas, and after a few operations, we can obtain the formulation for the transmission matrix $\mathbf{T}_G$ of the array, taking the mutual coupling effects into account

This work was supported by the Spanish "Ministerio de Economía y Competitividad (MINECO)" and by FEDER (EU), under projects TEC2013-46282-C2-1-P and TEC2013-46282-C2-2-P, and by "Junta de Extremadura" and FEDER under contract GR15052.

978-1-5090-4837-3/17/$31.00@2017 IEEE

$$([\mathbf{I}-(\mathbf{S}-\mathbf{I})\mathbf{G}]^{-1}\mathbf{T})\,\mathbf{v}=\mathbf{T}_G\mathbf{v}=\mathbf{b}^{\mathbf{s}} \tag{5}$$

where **T** and **(S-I)** are the following diagonal block-matrices:

$$\begin{aligned}\mathbf{T}&=diag(\mathbf{t}_i)\\(\mathbf{S}-\mathbf{I})&=diag(\mathbf{s}_i-\mathbf{I}_i)\end{aligned} \tag{6}$$

and **G**, **v**, and $\mathbf{b}^{\mathbf{s}}$ are given by:

$$\mathbf{G}=\begin{bmatrix}\mathbf{0} & \mathbf{G}_{12} & \cdots & \cdots & \mathbf{G}_{1N}\\ \mathbf{G}_{21} & \mathbf{0} & \ddots & \mathbf{G}_{jk} & \vdots\\ \vdots & \ddots & \ddots & \ddots & \vdots\\ \vdots & \mathbf{G}_{kj} & \ddots & \mathbf{0} & \mathbf{G}_{N-1,N}\\ \mathbf{G}_{N1} & \cdots & \cdots & \mathbf{G}_{N,N-1} & \mathbf{0}\end{bmatrix};\ \mathbf{v}=\begin{Bmatrix}\mathbf{v}_1\\ \vdots\\ \mathbf{v}_i\\ \vdots\\ \mathbf{v}_N\end{Bmatrix};\ \mathbf{b}^{\mathbf{s}}=\begin{Bmatrix}\mathbf{b}_1^{\mathbf{s}}\\ \vdots\\ \mathbf{b}_i^{\mathbf{s}}\\ \vdots\\ \mathbf{b}_N^{\mathbf{s}}\end{Bmatrix}. \tag{7}$$

The radiated far-field of an array for a desired array excitation **v** will be obtained as an expansion of spherical modes weighted by complex coefficients $\mathbf{b}^{\mathbf{s}}$. In this way, considering a planar array of N antennas placed in the xy-plane, and M spherical modes for each antenna, and denoting by $\mathbf{e}_m(\theta,\varphi)$ the electric field corresponding to the m-th spherical mode on each antenna, the radiated field is expressed as

$$\vec{\mathrm{E}}(\theta,\varphi)=\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)\mathbf{b}^{\mathbf{s}} \tag{8}$$

with

$$\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)=\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}_1}\cdots\ \ \mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}_i}\cdots\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}_N}\right) \tag{9}$$

where $\mathbf{e}(\hat{u})$ is a row vector given by $\mathbf{e}(\hat{u})=$ ($\mathbf{e}_1(\theta,\varphi)$,..., $\mathbf{e}_m(\theta,\varphi)$, ..., $\mathbf{e}_M(\theta,\varphi)$), k is the wave number in free space, the vector $\hat{u}$ refers to the unitary vector in spherical coordinates and the vector $\mathbf{u}_i$ is the position vector of the antenna i.

For non-rotated elements (8) becomes

$$\vec{\mathrm{E}}(\theta,\varphi)=\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)\mathbf{T}_G\mathbf{v}\,. \tag{10}$$

However, if rotated elements are used, coefficients of $\mathbf{b}^{\mathbf{s}}$ must be referred to a local coordinate system with the same orientation for all the antennas to apply the array factor. In this case

$$\vec{\mathrm{E}}(\theta,\varphi)=\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)\mathbf{F}\mathbf{T}_G\mathbf{v} \tag{11}$$

where **F** is a diagonal matrix whose coefficients are given by the submatrices $\mathbf{R}_i(-\varphi_{Ri})$ related to the rotation of the spherical modes for each element i in (3). In this way, we can express **G** as

$$\mathbf{G}=\mathbf{F}^H\,\mathbf{G}^{\mathrm{nr}}\,\mathbf{F} \tag{12}$$

where the submatrices of $\mathbf{G}^{\mathrm{nr}}$ are given by (4).

B. Gradient-Based Optimization Method

The optimization method used in this work is based on a local and gradient-based method [2]. For this purpose, we calculate a cost function that involves the radiation intensity of the coupled array. The synthesis of the radiation pattern is carried out, taking mutual coupling effects into account, by minimizing the sidelobe level (SLL) while setting a mainbeam with a fixed width. The secondary lobes of the co-polar (CP) component are minimized through a weighted cost function that involves an average of the sidelobes. In the case of dealing with rotated elements, the cross-polar (XP) is also minimized with the same strategy. The excitation amplitudes are fixed a priori. Two cost functions are defined as

$$CF_\chi=\left(\int_u\left[W_\chi(\hat{u})\left|E_\chi(\hat{u})\right|^2\right]^p d\hat{u}\right)^{1/p} \tag{13}$$

where χ makes reference to the CP or the XP component, and W is the desired weighting function. The global cost function is obtained as the addition of both cost functions when the SLL and XP component are optimized simultaneously. A discussion about the weighting function W and the value of p can be found in [2].

The gradient of the cost function in terms of the position or the rotation of each element is computed as

$$\frac{\partial CF_\chi}{\partial c_i}=\left(CF_\chi\right)^{1-p}\int_u\left(W_\chi(\hat{u})\right)^p\left(\left|E_\chi(\hat{u})\right|^2\right)^{p-1}\frac{\partial\left|E_\chi(\hat{u})\right|^2}{\partial c_i}d\hat{u} \tag{14}$$

where c_i represents the coordinate (x_i or y_i), or the rotation angle (φ_{Ri}) for each antenna i.

C. Derivative of the Radiation Intensity in terms of the Position of Each Element

By using (10), the derivative of the Radiation Intensity w.r.t. the coordinates of every antenna is calculated as follows:

$$\frac{\partial\left|E_\chi(\hat{u})\right|^2}{\partial x_i}=\mathbf{v}^H\,\frac{\partial\left(\mathbf{T}_G{}^H\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)^H\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)\mathbf{T}_G\right)}{\partial x_i}\mathbf{v} \tag{15}$$

This expression is computed applying properties of the derivatives of products. Two derivatives are required to be computed separately. Thus, the term that relates the exponential function and the spherical modes is derived as

$$\frac{\partial\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)}{\partial x_i}=jk\left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)\frac{\partial(\hat{u}\cdot\mathbf{u})}{\partial x_i}, \tag{16}$$

and the derivative of the transmission matrix is calculated as

$$\frac{\partial \mathbf{T}_G}{\partial x_i} = \frac{\partial \mathbf{M}^{-1}}{\partial x_i}\mathbf{T} = -\mathbf{M}^{-1}\frac{\partial \mathbf{M}}{\partial x_i}\mathbf{M}^{-1}\mathbf{T} = -\mathbf{M}^{-1}\left[-(\mathbf{S}-\mathbf{I})\frac{\partial \mathbf{G}}{\partial x_i}\right]\mathbf{M}^{-1}\mathbf{T} \tag{17}$$

with $\mathbf{M}$ being

$$\mathbf{M} = [\mathbf{I} - (\mathbf{S}-\mathbf{I})\mathbf{G}] \tag{18}$$

The gradient of $\mathbf{G}$ is computed from (4) as

$$\begin{aligned}\frac{\partial \mathbf{G}_{jk}}{\partial x_k} = &\left[\frac{\partial \mathbf{R}_k(\varphi_{kj})}{\partial \varphi_{kj}}\frac{\partial \varphi_{kj}}{\partial x_k}\mathbf{D}_k\left(\pi/2\right)\mathbf{C}\left(d/\lambda\right)\mathbf{D}_j\left(-\pi/2\right)\mathbf{R}_j(-\varphi_{kj})\right]^T + \\ &\left[\mathbf{R}_k(\varphi_{kj})\mathbf{D}_k\left(\pi/2\right)\frac{\partial \mathbf{C}\left(d/\lambda\right)}{\partial d}\frac{\partial d}{\partial x_k}\mathbf{D}_j\left(-\pi/2\right)\mathbf{R}_j(-\varphi_{kj})\right]^T + \\ &\left[\mathbf{R}_k(-\varphi_{kj})\mathbf{D}_k\left(\pi/2\right)\mathbf{C}\left(d/\lambda\right)\mathbf{D}_j\left(-\pi/2\right)\frac{\partial \mathbf{R}_j(-\varphi_{kj})}{\partial \varphi_{kj}}\frac{\partial \varphi_{kj}}{\partial x_k}\right]^T\end{aligned} \tag{19}$$

The computation of the derivatives of $\mathbf{C}$ requires dealing with derivatives of Hankel functions. Details can be found in [4]. The computation of the derivatives of $\mathbf{R}$ is simple since they are composed of exponential functions.

D. Derivative of the Radiation Intensity in terms of the Rotation of Each Element

By using (11), the derivative of the Radiation Intensity w.r.t. the rotation of every antenna is calculated as follows:

$$\frac{\partial |E_\chi(\hat{u})|^2}{\partial \varphi_{Ri}} = \mathbf{v}^H \frac{\partial\left(\mathbf{F}^H \mathbf{T}_G{}^H \left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)^H \left(\mathbf{e}(\hat{u})e^{jk\hat{u}\mathbf{u}}\right)\mathbf{T}_G \mathbf{F}\right)}{\partial \varphi_{Ri}}\mathbf{v} \tag{20}$$

This expression is also computed applying properties of the derivatives of products. Dependence with the rotation angle is only due to matrix $\mathbf{F}$, directly, or indirectly through the transmission matrix. The computation of the derivatives of $\mathbf{F}$ is simple since they are composed of exponential functions. Derivatives of the transmission matrix are computed as

$$\frac{\partial \mathbf{T}_G}{\partial \varphi_{Ri}} = \frac{\partial \mathbf{M}^{-1}}{\partial \varphi_{Ri}}\mathbf{T} = \mathbf{M}^{-1}\left[(\mathbf{S}-\mathbf{I})\frac{\partial \mathbf{G}}{\partial \varphi_{Ri}}\right]\mathbf{M}^{-1}\mathbf{T} \tag{21}$$

where the gradient of $\mathbf{G}$ can be computed from (12) by using the chain rule

$$\frac{\partial \mathbf{G}}{\partial \varphi_{Ri}} = \frac{\partial \mathbf{F}^H}{\partial \varphi_{Ri}}\mathbf{G}^{\mathrm{nr}}\mathbf{F} + \mathbf{F}^H\mathbf{G}^{\mathrm{nr}}\frac{\partial \mathbf{F}}{\partial \varphi_{Ri}}. \tag{22}$$

III. Results

A. Linear array of rotated cavity-backed patch antennas with linear polarization

As an example, a linear array of 14 rectangular cavity-backed patch antennas is synthesized via element rotations. The aim is to reduce the maximum SLL of the CP, keeping a constant mainbeam width and controlling the cross-polar level at acceptable levels. The geometry of the radiating element can be found in [1]. The synthesized angles of rotation are shown in Fig. 1. The maximum CP level is reduced in the sidelobe region from −13.3 dB, for uniformly excited and distributed arrays, to −16.1 dB with optimized rotated elements, while the maximum XP component is kept lower than −18.9 dB in every direction of space.

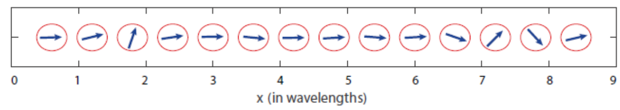

Fig. 1. Representation of the optimized rotation angles of the 14-element linear array of patches.

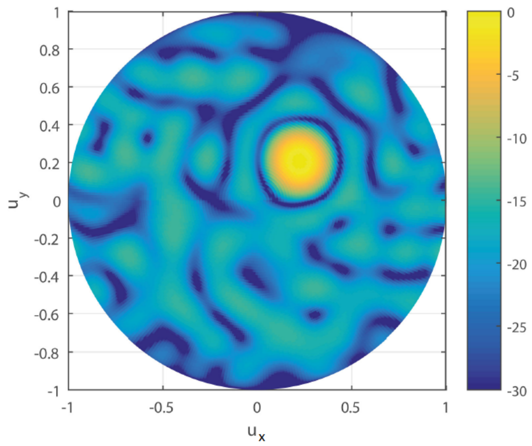

Fig. 2 . Synthesized field radiation pattern at 6.1 GHz of a 30-element array of cavity-backed patch antennas scanned at u_{x0} = 0.3 and u_{y0} = 0.2.

B. Aperiodic planar array of cavity-backed patch antennas

In this case, the same radiating element as in Section III.A is considered. A planar array, excited with a uniform amplitude distribution and a linear phase taper obtained with the classic theory in order to steer the beam towards a desired direction, is synthesized. It consists of 30 elements placed on a circle of radius 3 wavelengths at the resonant frequency of 6.1 GHz. The positions are optimized to achieved a fixed beam width while minimizing the average SLL. The field radiated by the array synthesized with the proposed method is shown in Fig. 2. A maximum SLL of -14.5 dB has been obtained.

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