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Adaptive local tracking of a temperature profile in tubular reactor with partial measurements

N. Beniich [a,*], A. El Bouhtouri [a], D. Dochain [b]

[a] LINMA, Département de Mathématique, Faculté des Sciences, BP 20, El Jadida, Morocco
[b] CESAME, Université catholique de Louvain, 4-6 avenue Georges Lemaître, B-1348 Louvain-LaNeuve, Belgium

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ABSTRACT

In this work, a local constrained adaptive output feedback is presented for a class of exothermic tubular reactors models described by a nonlinear partial differential equations. The considered output is the measured temperature in a fixed zone of the reactor to regulate the temperature throughout the reactor to a ball with radius λ (arbitrarily small) centered at the fixed temperature profile. For a given measurement zone with length given in terms of the desired profile and λ and for initial temperature in a fixed domain, it is shown that the tracking error through the reactor tends asymptotically to a ball of arbitrary prescribed radius $\lambda > 0$, centered at the given temperature profile. Numerical simulations have been performed to illustrate the performance of the proposed approach.

1. Introduction

In the last decades, an intensive research activity has been dedicated to the control design of linear and nonlinear infinite dimensional systems [9–11,17] and the references therein for linear systems and [8,15,18] for nonlinear systems, especially the nonlinear tubular reactor systems see [1–5].

A question of interest in the control of isothermal tubular reactor is the design of a controller for the temperature with a reference profile control, more specifically in the presence of additional practical considerations like the presence of input constraints typically encountered in the control of chemical processes. As in many control problems where variables need to satisfy constraints and where the output of the controlled process has to track a reference signal, one particular technique that addresses such problems is adaptive λ tracking controller see [13] that achieve the convergence to the desired profile taking into account the state and control constraints.

In this paper we propose an input constrained adaptive output feedback control with partial measurements for a non-isothermal tubular chemical reactor with axial dispersion. The dynamic of such reactors are described by nonlinear partial differential equations derived from mass and energy balances [23].

Our objective is to regulate the reactor temperature in a prespecified neighborhood of a given reference profile. Recently, a constrained adaptive output feedback has been developed for this class of models with same objective [4,5], by using a modified λ-tracking controllers originally developed in a finite dimensional context for a similar problem [2,12,13]. It has been shown that under a simple feasibility assumptions in terms of the reference temperature and the input constraints that the reactor temperature tends asymptotically to a ball of temperature profile with arbitrary prescribed radius $\lambda > 0$. The proposed control law has the shortcoming that it requires access to temperature measurements along the tubular reactor which present a practical limitation of these approaches. To overcome this limitation, we deal in this work with the possibility that the useful temperatures measurements for the regulation are only available via a finite number of zone sensors (without taking their emplacement into account). Installing all the necessary sensors may not be physically possible or the costs may become prohibitive. In other words, we consider only partial measurements obtained by a finite number of sensors.

* Corresponding author.
E-mail addresses: nadia_beniich@yahoo.fr (N. Beniich), abdelmoula_elb@yahoo.fr (A. El Bouhtouri), denis.dochain@uclouvain.be (D. Dochain).

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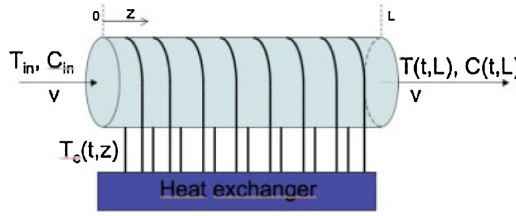

Fig. 1. Schematic view of an non-isothermal reactor with distributed heat exchange.

The paper is organized as follows. In Section 2 we present the basic dynamical model and we reformulate the problem within the framework of the semi-linear systems. Section 3 is dedicated to the development of our main result on adaptive and non-adaptive λ-tracking. An example is presented in Section 4 to illustrate our results via numerical simulation.

2. State space framework and statement of the control problem

Let us consider a non-isothermal reactor (Fig. 1) with the following chemical reaction:

$$A \longrightarrow bB \tag{1}$$

where $b > 0$, A and B are the stoichiometric coefficient of the reaction, the reactant and the product, respectively.

In the present study, along the lines of [15] as well as in line with practical chemical engineering considerations on the use of cascade non-isothermal reactors[1] (see [19]), we consider distributed heat exchange. The dynamics of the process in an exothermic tubular reactor with axial dispersion are readily obtained from energy and mass balances and are given by the following partial differential equations (e.g. [4,15]):

$$\begin{aligned} \frac{\partial T(t,z)}{\partial t} &= D_1\frac{\partial^2 T(t,z)}{\partial z^2} - v\frac{\partial T(t,z)}{\partial z} - \alpha f(T(t,z),C(t,z)) - k_0(T(t,z)-T_c(t,z)) \\ \frac{\partial C(t,z)}{\partial t} &= D_2\frac{\partial^2 C(t,z)}{\partial z^2} - v\frac{\partial C(t,z)}{\partial z} - f(T(t,z),C(t,z)) \end{aligned} \tag{2}$$

with the boundary conditions:

$$-D_1\frac{\partial T(t,0)}{\partial z} = v(T_{in}(t) - T(t,0)) \tag{3}$$

$$-D_2\frac{\partial C(t,0)}{\partial z} = v(C_{in}(t) - C(t,0)) \tag{4}$$

$$\frac{\partial T(t,L)}{\partial z} = 0 \tag{5}$$

$$\frac{\partial C(t,L)}{\partial z} = 0 \tag{6}$$

In the above equations, t (>0) and z ($\in [0, L]$ with $L = 1 > 0$) hold for the time and the reactor length, respectively. $k_0 = \frac{4h}{\rho C_p d}$ (>0), $\alpha = \frac{-\Delta H}{\rho C_p}$ (>0), T, C, $D_1 > 0$, $D_2 > 0$, $v > 0$, $\Delta H < 0$, ρ, C_p, h, d, T_c, T_{in} and C_{in} are the temperature reactor, the reactant concentration, the energy and mass dispersion coefficients, the superficial fluid velocity, the heat of the reaction, the density, the specific heat, the wall heat transfer coefficient, the reactor diameter, the coolant temperature, the inlet temperature, and the inlet concentration, respectively.

$f(T(t, z), C(t, z))$ represents the kinetics of reaction (1). It is a nonlinear, positive and locally Lipschitz function. The positivity of f is a direct consequence of the standard kinetics rules (e.g. [19]) for the irreversible reaction (1) in which the reactant A is consumed and the product B is synthesized. A typical example of $f(T(t, z), C(t, z))$ is the reaction rate of first-order kinetics with respect to the reactant concentration C and characterized by an Arrhenius type dependence with respect to the temperature T

$$f(T(t,z),C(t,z)) = kC(t,z)e^{-\frac{E}{RT(t,z)}} \tag{7}$$

with $k > 0$, $E > 0$ and $R > 0$ the kinetic constant, the activation energy and the ideal gas constant, respectively.

In order to write the model (2)–(6) in an abstract semigroup formulation in Hilbert space, we consider the Hilbert space $H = L^2[0, 1]$ endowed with the usual inner product:

$$\langle f, g\rangle_H = \langle f, g\rangle = \int_0^L f(z)g(z)\,dz \tag{8}$$

and the usual partial order defined by

$$f \le g \quad \text{if and only if} \quad f(z) \le g(z) \quad \text{for almost every} \quad z \in [0, L] \tag{9}$$

[1] The tubular reactor being then the limit case when the number of tanks in the cascade is large enough.

where f, g are in H. With respect to this order, the positive cone H^+ of H is defined by:

$$H^+ = \{y \in H, \quad s.t. \quad y \geq 0\} \tag{10}$$

We suppose that T_{in} and C_{in} are constant and consider the following definitions at time $t \geq 0$:

$$x_1(t) = T(t,.) - T_{in}, \quad x_2(t) = C_{in} - C(t,.), \quad u(t) = T_C(t,.) - T_{in} \tag{11}$$

and $\tilde{f} : H \times H \mapsto H$ so that

$$\tilde{f}(\psi, \phi)(z) = f(\psi(z) + T_{in}, C_{in} - \phi(z)) \tag{12}$$

for any ψ, ϕ in H and $z \in [0, L]$. The equivalent state space description of the model (2)–(6) is given by the following semi-linear differential equation on the Hilbert space $H \times H$:

$$\dot{x}_1(t) = A_1 x_1(t) + \alpha \tilde{f}(x_1(t), x_2(t)) + k_0 u(t) \tag{13}$$

$$\dot{x}_2(t) = A_2 x_2(t) - f(x_1(t), x_2(t)) \tag{14}$$

The operators A_1 and A_2 defined as follows:

$$A_1 x = D_1 \frac{\partial^2 x}{\partial z^2} - v \frac{\partial x}{\partial z} - k_0 x, \quad x \in \mathcal{D}(A_1) \tag{15}$$

$$A_2 x = D_2 \frac{\partial^2 x}{\partial z^2} - v \frac{\partial x}{\partial z}, \quad x \in \mathcal{D}(A_2) \tag{16}$$

defined on (for $i = 1, 2$):

$$\mathcal{D}(A_i) = \left\{ x \in H : x, \frac{dx}{dz} \text{ absolutely continuous}; \ \frac{d^2x}{dz^2} \in H \text{ and } D_i \frac{dx}{dz}(0) - vx(0) = \frac{dx}{dz}(1) = 0 \right\}$$

It is shown in [23] that A_1 and A_2 are Riesz spectral operators and infinitesimal generators of exponentially stable strongly continuous semigroups $(T_1(t))_{t \geq 0}$ and $(T_2(t))_{t \geq 0}$ of bounded operators in H. Besides, for all $x \in D(A_1)$ the inner product $\langle A_1 x, x \rangle \leq 0$, i.e. A_1 is dissipative.

The physical consideration leads us to assume that $u(t)$ is constrained so that there exist $\underline{u}$ and $\overline{u}$ with $\underline{u} < \overline{u}$ such that:

$$\underline{u} \leq u(t) \leq \overline{u} \tag{17}$$

The control objective is to regulate the temperature $x_1(\cdot)$ around a prespecified neighborhood of a given reference profile temperature x^* without violating the constraints on $u(\cdot)$ under (13) and (14).

The control error between x^* and $x_1(t)$ is denoted by: $e(t) = x^* - x_1(t)$, so that for all $z \in [0, L]$ $e(t)(z) = x^*(z) - x_1(t)(z) = x^*(z) - T(t, z)$, thus $e(t) \in H$.

In the present paper, the reactor temperature is assumed to be measured only in a zone $\Omega \subset [0, L]$. To this end, we define

$$\mathcal{C}(z) = \begin{cases} 1 & \text{if } z \in \Omega \\ 0 & \text{if } z \notin \Omega \end{cases} \tag{18}$$

The adaptive control law is defined by:

$$\hat{e}(t)(\cdot) = \mathcal{C}(\cdot)(x^*(\cdot) - x_1(t)(\cdot)) \quad \text{the tracking error in} \quad \Omega \tag{19}$$

$$u(t) = sat_{[\underline{u}, \overline{u}]}(\beta(t)\hat{e}(t) + u^*) \tag{20}$$

with

$$\dot{\beta}(t) = K \begin{cases} (||\hat{e}(t)|| - \lambda)^l & \text{if } ||\hat{e}(t)|| > \lambda \\ 0 & \text{if } ||\hat{e}(t)|| \leq \lambda \end{cases} \tag{21}$$

The constant $\lambda > 0$ is an upper bound for the asymptotic tracking error, $l \geq 1$, k, $\beta_0 > 0$ are chosen by the user, and $u^* \in (\underline{u}, \overline{u})$ is a constant offset.

The saturation function $sat_{[\underline{u}, \overline{u}]}$ is defined for all $y \in \mathbb{R}$ by:

$$sat_{[\underline{u}, \overline{u}]} y = \begin{cases} \overline{u} & \text{if } y \geq \overline{u} \\ y & \text{if } \underline{u} < y < \overline{u} \\ \underline{u} & \text{if } y \leq \underline{u} \end{cases} \tag{22}$$

Remark 2.1.

- The adaptive λ tracker consists of a proportional measured error feedback with saturation and a time-varying proportional gain $\beta(\cdot)$ determined adaptively by the measurement error only. The power l in the gain adaptation influences the speed of adaptation, a similar effect can be achieved by varying k or the initial gain β_0. The constant u^* is an input reference.
- $u(t)(z) = u^*(z)$ for all $z \notin \Omega$
- $u(t)(z) = sat_{[\underline{u}, \overline{u}]}(\beta(t)e(t) + u^*)$ for all $z \in \Omega$

The following assumptions will be considered to achieve our control objective:

(H_1) The positive cone $(H^+ \times H^+)$ is positively invariant under (13)–(14) for all nonnegative control $u(\cdot)$.
For a given $x^* \in H^+$ we assume:
(H_2) The desired profile $x^* \in H^+ \cap dom(A_1)$ and there exist $\rho > 0$, $\underline{x} \in H^+ \cap dom(A_1)$, $\overline{x} \in H^+ \cap dom(A_1)$ with $0 < \underline{x} < x^* < \overline{x}$ such that for all x_1, x_2 in H satisfying $\underline{x} \le x_1 \le \overline{x}$ and $0 \le x_2 \le x_2^{in}$:

$$\begin{cases} \underline{u} + \rho \le k_0 x_1 - \alpha f(x_1, x_2) - Ax^* \le \overline{u} - \rho \\ k_0 x_1 - \alpha f(x_1, x_2) - Ax^* - u^* \le -\rho \\ D_1 \dfrac{d^2(\overline{x} - x^*)}{d^2 z} - v \dfrac{d(\overline{x} - x^*)}{dz} \le 0 \end{cases} \tag{23}$$

where $A = A_1 + k_0 I$.
(H_3) $0 < \lambda < \overline{x} - x^*, \quad 0 < \underline{x} < x^* < \overline{x}$

Remark 2.2.

- Assumption (H_1) is natural for exothermic reactors. Indeed, the concentration and temperature should not become zero once they are positive, if the inlet temperature and the inlet concentration are strictly positive. The positivity of the trajectories is already studied in numerous investigations, in particular in [20–22].
- Assumption (H_2) is simply feasibility assumption given by relating the reference profile x^* and the positive input saturations $\underline{u}, \overline{u}$ to system data. Thus if x^* is the equilibrium profile of the open loop system, it implies that the associated input belongs to the interval of the input constraints.
- The upper bound $\overline{u}$ does not only depend on the feasibility condition H_2 but also on the physical limitations of the actuator when both conditions are compatible, i.e., the actuator limit is higher than the bound in H_2, $\underline{u}$ is chosen so that feasibility condition H_2 and saturation bound are checked.
- The additional assumption on the first and second derivatives of $\overline{x} - x^*$ guaranties the convergence of the tracking error through the reactor.
- We assume without loss of generality that $L = 1$. Note that if a continuous function $f \in H$ is such that $\|f\| > \lambda$ this implies that there exists $[a, b] \subset [0, 1]$ such that $f(z) > \lambda$ or $f(z) < \lambda$ for all $z \in [a, b]$.

Assumption H_3 requires to select a value of the design parameter λthat is small enough.

3. Main results

In this section we consider local λ-control in the sense that the initial temperature $x_1(0)$ is constrained to be in the set Δ_1 defined here below. We present two feedback strategies that force the temperature into a λ-neighborhood of the given reference profile. The first is non-adaptive while the second one is adaptive.

We define Δ_1 and Δ_2 as follows:

$$\Delta_1 = \{y_1 \in H \ \text{ such that } \ -x_1^{in} \le y_1(z) < \overline{x}(z) \ \ \forall z \in [0, 1)\},$$
$$\Delta_2 = \{y_2 \in H \ \text{ such that } \ 0 < y_2(z) \le x_2^{in} \ \ \forall z \in [0, 1)\}$$

3.1. Non-adaptive λ controller

In this section, we consider the non-adaptive version of the controller (19)–(21)

$$u(t) = sat_{[\underline{u}, \overline{u}]}(\beta(t)\hat{e}(t) + u^*) \tag{24}$$

$\beta : \mathbb{R}^+ \longrightarrow [\beta^*, \infty[$ some (a priori arbitrary) continuous function. Although the gain $\beta(t) \ge \beta^*$ might be conservatively too large, this non-adaptive controller is useful because it is even simpler than the already simple (19). We give explicit lower bounds for β^* in terms of weak conditions on the system data.

Let us now introduce the following notations:

$$C_1 = \max\{\|k_0 x_1 - \alpha \tilde{f}(x_1, x_2) - Ax^* - u^*\| / (x_1, x_2) \in (\Delta_1 \times \Delta_2)\}$$
$$C_2 = \max\{\|\overline{x} - x^*\|_\infty, \|x^*\|_\infty\}$$
$$\varepsilon = \frac{\rho \lambda^3}{4C_2^2 C_1}$$

Proposition 3.1. *Let the assumptions H_1, H_2, H_3 hold, $(x_1(0), x_2(0)) \in (\Delta_1 \times \Delta_2)$ and suppose:*

$$\beta^* \ge \max\left(\frac{u^* - \underline{u}}{\lambda}; \frac{\overline{u} - u^*}{\lambda}; \frac{u^* - \underline{u}}{\overline{x} - x^*}; \frac{2C_1}{\lambda}; \frac{\lambda C_1}{\varepsilon^2}\right) \tag{25}$$

then the following properties hold:

The controller (19) *applied to system* (13) and (14) *yields a unique solution:*

$$t \geq 0 (x_1, x_2) \in (\Delta_1 \times \Delta_2) \quad \text{for all} \quad t \geq 0$$

there exists $t_1 > 0$ *such that:*

$$\|\hat{e}(t)\| \leq \lambda \quad \text{for all} \quad t \geq t_1$$

Proof. The operators A_1 and A_2 are generators of C_0-semigroups and the nonlinearity $\tilde{f}$ is locally Lipschitz in the state variable, the existence of the mild solution over an maximal open interval $[0, w)$, $w \in [0, \infty]$ is assured by Theorem 4.1 ([19], pp. 185–186). In the following we show that for all $t \in [0, w)$, $(x_1(t), x_2(t)) \in (\Delta_1 \times \Delta_2)$. Therefore $x_1(t)$, $x_2(t)$ are bounded i.e. a finite time escape cannot occur. We apply Theorem 4.1 ([19], pp. 185–186) to conclude that $w = +\infty$.

H is an Hilbert space and $(x_1(0), x_2(0)) \in (dom(A_1) \times dom(A_2))$, Theorem 1.6 ([19], p. 189) yields that this mild solution is strong solution.

We note from (H_1) that we need only to show that $x_2(t) \leq x_2^{in}$ for all $t \in [0, w)$. This follows by integration of (14). The positivity of T_2 the C_0-semi group generated by A_2 and $\tilde{f}(x_1(s), x_2(s))$ implies that:

$$x_2(t) \geq T_2(t)x_2(0).$$

Thus if $x_2(0) \geq 0$ then $x_2(t) \geq 0 \quad \forall t \in [0, w)$.

From (H_1), if $x_1(0) \geq -x_1^{in}$ then $x_1(t) \geq -x_1^{in}$ for all $0 \leq t < w$.

Let us show that for all $x_1(0) \leq \bar{x}$, we have $x_1(t) \leq \bar{x}$ (to show the positive invariance of Δ_1):

$\tilde{f}(.,.)$ is a continuous function, then it is upper bounded on $(\Delta_1 \times \Delta_2)$ by some positive constant M. Then we have:

$$\begin{cases} x_1(t) = T_1(t - t_0)x_1(t_0) + \displaystyle\int_{t_0}^{t} T_{t-t_0-s}[\alpha\tilde{f}(x_1, x_2) + k_0 u]\, ds \\ \qquad \leq T_1(t - t_0)x_1(t_0) + \displaystyle\int_{t_0}^{t} T_{t-t_0-s}M\, ds. \end{cases}$$

We define the following system defined in Δ_1:

$$\begin{aligned} &\dot{y}(t) = A_1 y(t) + M \\ &y(t_0) = x_1(t_0). \end{aligned} \tag{26}$$

We use [15] and Theorem 5.1 ([18], p. 12) to show the positive invariance of Δ_1 with respect to (26).

Hence $x_1(0) \leq \bar{x}$ then $x_1(t) \leq \bar{x}$ for all $t \in [0, w)$.

We show now that there exists $t_1 > 0$ such that:

$$\|\hat{e}(t)\| \leq \lambda \quad \text{for all} \quad t \geq t_1$$

For $\lambda > 0$, we define the following distance function in H

$$d_\lambda(x) = \max\{\|x\| - \lambda, 0\} \quad \forall x \in H$$

for every solution of (13). Let us consider:

$$V_\lambda(t) = d_\lambda(\hat{e}(t))^2 \quad \forall t \geq 0$$

Suppose $\|\hat{e}(t)\| > \lambda$ and let $2\frac{\sqrt{V_\lambda(t)}}{\|\hat{e}(t)\|} = g(t)$, then:

$$\frac{d}{dt}V_\lambda(t) = 2\frac{\sqrt{V_\lambda(\hat{e}(t))}}{\|\hat{e}(t)\|}\langle \hat{e}(t), \dot{\hat{e}}(t)\rangle = g(t)\langle e(t), Ae(t) - k_0 e(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0 u(t) - A_1 x^*\rangle_\Omega$$

$$= g(t)\langle e(t), Ae(t) + k_0 x_1 - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0 u(t) - Ax^*\rangle_\Omega \leq g(t)\int_\Omega e(t)(z)[k_0 x_1 - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0 u(t, z) - Ax^*](z)\, dz.$$

Let $G_1 = \{z \in \Omega \backslash e(t)(z) > \lambda\}$;
$G_2 = \{z \in \Omega \backslash e(t)(z) < -\lambda\}$
and $G_3 = \{z \in \Omega \backslash -\lambda \leq e(t)(z) \leq \lambda\}$

If $z \in G_1$, then by (25):

$$\beta(t)e(t)(z) + u^* \geq \beta^*\lambda + u^* \geq \bar{u}$$

This implies that $u_2(t)(z) = \bar{u}$, hence:

$$I = \int_{G_1} e(t)(z)[k_0 x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0 u(t, z) - Ax^*](z)\, dz = \int_{G_1} e(t)(z)[k_0 x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0\bar{u} - Ax^*](z)\, dz$$

From the assumption (H_2), we obtain:

$$I < -\rho\lambda \text{mes}(G_1)$$

If $z \in G_2$, using (25) we find:

$$\beta(t)e(t)(z) + u^* \leq -\beta^*\lambda + u^* \leq \underline{u}$$

then

$$\beta(t)m(t)(z) + u_1^* \leq \underline{u}_1$$

This implies that $u(t)(z) = \underline{u}$
hence:

$$II = \int_{G_2} e(t)(z)[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0u(t, z) - Ax^*](z)\,dz = \int_{G_2} e(t)(z)[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0\underline{u} - Ax^*](z)\,dz$$

H_2 implies that:

$$II < -\rho\lambda \mathrm{mes}(G_2)$$

Now if $z \in G_3$, without loss of generality, we can suppose that $u(t, z) = \beta(t)e(t)(z) + u^*$. Let

$$III = \int_{G_3} e(t)(z)[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0u(t, z) - Ax^*](z)\,dz$$

To obtain an upper bound of III, we consider two case for $\mathrm{mes}(G_1) + \mathrm{mes}(G_2)$ namely:

$$\mathrm{mes}(G_1) + \mathrm{mes}(G_2) < \frac{\lambda^2}{2C_2^2}$$

$$\mathrm{mes}(G_1) + \mathrm{mes}(G_2) \geq \frac{\lambda^2}{2C_2^2}$$

In the first case, we obtain:

$$\begin{aligned} III &= \int_{G_3} e(t)(z)[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - Ax^* - k_0(\beta(t)e(t) - u^*)](z)\,dz \\ &\leq \int_{G_3} |e(t)(z)|\,|[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0u^* - Ax^*](z)|dz - \int_{G_3} k_0\beta(t)e^2(t)(z)\,dz \\ &\leq \lambda C_1 - \int_{G_3} k_0\beta(t)e^2(t)(z)\,dz \leq \lambda C_1 - k_0\beta^* \int_0^1 e^2(t)(z)\,dz + \beta^* \int_{G_1 \cup G_2} e^2(t)(z)\,dz \end{aligned}$$

We have $x^* - \bar{x} \leq e(t) \leq x^* + x_1^{in} \quad \forall t \geq 0$.
Thus we have for all $z \in \Omega$:
$|e(t)(z)| \leq \max\{\|\bar{x} - x^*\|_\infty, \|x^* + x_1^{in}\|_\infty\} = C_2$ Thus we have:

$$III \leq \lambda C_1 - k_0\beta^*\lambda^2 + k_0\beta^* C_2^2(\mathrm{mes}G_1 + \mathrm{mes}G_2) \leq \lambda C_1 - k_0\beta^*\lambda^2 + k_0\beta^* C_2^2 \frac{\lambda^2}{2C_2^2} \leq \lambda C_1 - k_0\beta^*\lambda^2/2 \leq \frac{\lambda}{2}(2C_1 - k_0\beta^*\lambda)$$

By condition (25), we obtain: $III \leq 0$.
So there exists a nonnegative C such that:

$$\frac{d}{dt}V_\lambda(t) < -C\frac{\sqrt{V_\lambda(\hat{e}(t))}}{\|\hat{e}(t)\|}$$

In the second case we suppose

$$\mathrm{mes}(G_1) + \mathrm{mes}(G_2) \geq \frac{\lambda^2}{2C_2^2}$$

Then:

$$\begin{aligned} III &= \int_{G_3} e(t)(z)[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - Ax^* - k_0\beta(t)e^2(t)](z)\,dz \\ &\leq \int_{\varepsilon \leq |\hat{e}(t)(z)| \leq \lambda} |\hat{e}(t)(z)||[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0u^* - Ax^*](z)| - k_0\beta(t)e^2(t, z)\,dz \\ &\quad + \int_{0 \leq |\hat{e}(t)(z)| \leq \varepsilon} |\hat{e}(t)(z)||[k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - k_0u^* - Ax^*](z)|\,dz - \int_{0 \leq |\hat{e}(t)(z)| \leq \varepsilon} k_0\beta(t)e^2(t)(z)\,dz \\ &\leq \int_{\varepsilon \leq |\hat{e}(t)(z)| \leq \lambda} (-k_0\beta^*\varepsilon^2 + \lambda C_1)\,dz + \int_{0 \leq |e(t)(z)| \leq \varepsilon} \varepsilon C_1\,dz \end{aligned}$$

By condition (25), we have:

$$\int_{\varepsilon\le|\hat{e}(t)(z)|\le\lambda}(-k_0\beta^*\varepsilon^2+\lambda C_1\,dz)<0$$

and

$$III\le\int_{0\le|e(t)(z)|\le\varepsilon}\varepsilon C_1\le\varepsilon C_1\,\mathrm{mes}(\{z/0\le|e(t)(z)|\le\varepsilon\})\le\varepsilon C_1$$

hence:

$$I+II+III\le\varepsilon C_1-\rho\lambda(\mathrm{mes}(G_1)+\mathrm{mes}(G_2))\le\frac{\rho\lambda^3}{4C_2^2C_1}C_1-\rho\lambda\frac{\lambda^2}{2C_2^2}\le\frac{\rho\lambda^3}{4C_2^2}-\frac{\rho\lambda^3}{2C_2^2}<0$$

This means that the time derivative of V_λ is bounded as follows:

$$\frac{d}{dt}V_\lambda(t)\le-\gamma\frac{\sqrt{V_\lambda(t)}}{\|\hat{e}(t)\|}\quad\text{if}\quad\|\hat{e}(t)\|\ge\lambda$$

where γ is a non-negative constant. This implies that

$$\frac{d}{dt}V_\lambda(t)\le-\theta\sqrt{V_\lambda(t)}\quad\text{if}\quad\|\hat{e}(t)\|>\lambda$$

$$\frac{d}{dt}V_\lambda(t)=0\qquad\text{if}\quad\|\hat{e}(t)\|\le\lambda$$

with $\theta=\gamma/\max(\|\hat{e}(t)\|)$.

Summarizing, we have for all $t\ge0$

$$\frac{d}{dt}V_\lambda(t)\le-\gamma\sqrt{V_\lambda(t)}$$

And therefore there exists $t_1>0$ such that:

$$\forall t>t_1\quad\|\hat{e}(t)\|\le\lambda$$

This completes the proof of the theorem. □

Remark 3.1.

1. Proposition 3.1 guarantees that a large value of β^* is a sufficient condition for ensuring the global existence of the solution of (13)–(14) and that the sets Δ_1 and Δ_2 remain positively invariant under the closed-loop system (13)–(14) and u given by (24).
2. Proposition 3.1 show that there exists some time t_1 such that $\|\hat{e}(t)\|\le\lambda\forall t\ge t_1$ the tracking error in Ω. It remains to show same property for the tracking error $e(t)$ on the whole interval [0, 1].

Theorem 3.2. *Let the assumptions H_1, H_2, H_3 hold and*

$$\beta^*\ge\max\left(\frac{u^*-\underline{u}}{\lambda};\frac{\overline{u}-u^*}{\lambda};\frac{u^*-\underline{u}}{\overline{x}-x^*};\frac{C_2C_1}{(1-k)\lambda^2};\frac{\lambda C_1}{\varepsilon^2}\right)\tag{27}$$

$$\omega\le\frac{k\lambda^2}{\|\overline{x}-x^*\|_\infty}\tag{28}$$

where $w=mes(\overline{\Omega})$ where $\overline{\Omega}=\{z\in[0,1]$ and $z\notin\Omega\}$ and $0<K<1$.

There exist t_2 such that:

$$\|e(t)\|\le\lambda\quad\text{for all}\quad t\ge t_2.$$

Proof. For $t\ge0$ and $z\in\overline{\Omega}$, we have:

$$\dot{e}(t)(z)=Ae(t)+k_0x_1(t)-\alpha\tilde{f}(x_1(t),x_2(t))-u^*-Ax^*$$

We use the exponential stability, the positivity of $(T(t))$ the C_0-semigroup of A and

$$k_0x_1-\alpha\tilde{f}(x_1,x_2)-u^*-Ax^*<-\rho$$

We obtain

$$e(t)(z)=T(t)e(0)(z)+\int_0^tT(t-s)(k_0x_1(s)-\alpha\tilde{f}(x_1(s),x_2(s))-u^*-Ax^*)(z)\,ds\le T(t)e(0)(z)$$

by passage to the limit:

$$\lim_{t\longrightarrow\infty}T(t)e(0)(z)=0$$

then there exist t_0 such that:

$$(\bar{x}-x^*)(z) \le e(t)(z) \le 0 \quad \text{for all} \quad t \ge t_0$$

The previous theorem show that there exist $t_1 > 0$ such that $\|\hat{e}(t)\| \le \lambda$. So we can assume that $u(t)(z) = \beta(t)e(t)(z) + u^*$ for all $z \in \Omega$ and $t > t_1$. We consider

$$V_\lambda(t) = d_\lambda(e(t))^2 \quad \text{for all} \quad t \ge \sup(t_0, t_1)$$

if $\|e(t)\| > \lambda$

$$\frac{d}{dt}V_\lambda(t) = 2\frac{\sqrt{V_\lambda(e(t))}}{\|e(t)\|}\langle e(t), \dot{e}(t)\rangle$$

Let $n(t) = 2\frac{\sqrt{V_\lambda(e(t))}}{\|e(t)\|}$.

$$\frac{d}{dt}V_\lambda(t) \le n(t) < e(t), k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - u(t) - Ax^* > \le n(t) < e(t), k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - \beta(t)e(t)(z) + u^* - Ax^* >_\Omega$$
$$+ \; n(t) < e(t), k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t)) - u^* - Ax^* >_{\overline{\Omega}} \le n(t)[< e(t), k_0x_1(t) - \alpha\tilde{f}(x_1(t), x_2(t))$$
$$- \; u^* - Ax^* > - \int_\Omega \beta(t)e^2(t)(z)dz] \le [C_2C_1 - \beta^*\lambda^2 + \beta^* \int_{\overline{\Omega}} e^2(t) \le 2\sqrt{V_\lambda(e(t))}[\frac{C_2C_1}{\lambda} - \beta^*\lambda - \beta^*\omega\|\bar{x}-x^*\|_\infty].$$

we have $w < \frac{k\lambda}{\|\bar{x}-x^*\|_\infty}$ and $\beta^* > \frac{C_2C_1}{(1-k)\lambda}$

Consequently:

$$\frac{d}{dt}V_\lambda(t) \le 2\sqrt{V_\lambda(e(t))}(C_2C_1 - \beta^*(1-k)\lambda)$$

We deduce that there is a $\gamma > 0$ such that

$$\begin{cases} \frac{d}{dt}V_\lambda(t) \le -\gamma\sqrt{V_\lambda(t)} & \text{si} \quad \|e(t)\| > \lambda \\ \frac{d}{dt}V_\lambda(t) = 0 & \text{si} \quad \|e(t)\| \le \lambda \end{cases} \tag{29}$$

Then there exist t_2 such that

$$\forall t > t_2 \quad \|e(t)\| \le \lambda$$

□

3.2. *Adaptive λ controller*

In this section we consider the adaptive version of the controller (19). The main results of this section show that a larger initial gain condition β_0 achieves the convergence of the feedback gain $\beta(t)$ and the error norm will approach the interval $[0, \lambda]$ as $t \longrightarrow +\infty$, i.e.

$$\limsup_{t\longrightarrow+\infty}\|e(t)\| \le \lambda$$

Theorem 3.3. *Assume that* (H_1), (H_2) *and* (H_3) *hold, and* $(x_1(0), x_2(0)) \in (\Delta_1 \times \Delta_2)$ *and suppose:*

$$\beta_0 \ge \frac{u^* - \underline{u}}{\bar{x} - x^*} \tag{30}$$

$$\omega \le \frac{k\lambda}{\|\bar{x}-x^*\|_\infty} \tag{31}$$

the closed loop system given by equations (13), (14) and (19) has the following properties:

(1) $x_1(\cdot), x_2(\cdot), \beta_(\cdot) : \mathbb{R}^+ \longrightarrow (\Delta_1 \times \Delta_2) \times \mathbb{R}^+$
(2) $\lim_{t\longrightarrow+\infty} \beta(t)$ *exists and is finite*
(3) $\limsup_{t\longrightarrow+\infty}\|e(t)\| \le \lambda$

Proof. The existence and uniqueness of the maximal solution in a maximal interval $[0, w)$ follow from Theorem ([7], pp. 56–57). From Proposition 3.1, the monotonicity of $t \longrightarrow \beta(t)$ and (30) yields that $\beta(t) \ge \beta_0$ with β_0 satisfying (25), so $(\Delta_1 \times \Delta_2)$ is positively invariant.

The adaptation gains in (21) are the integral of the error $\hat{e}(t)$ which are a bounded function then

$$\lim_{t\longrightarrow w}\beta < \infty$$

This yields that $\beta(\cdot)$ cannot exhibit a finite escape time on $[0, w)$ which implies that $w = +\infty$.

Now we show that $\beta(t)$ is bounded.

Suppose that β is unbounded. Then there exists $\hat{t}$ such that $\forall t \geq \hat{t}$:

$$\beta(t) \geq \max\left(\frac{u^* - \underline{u}}{\lambda}; \frac{\overline{u} - u^*}{\lambda}; \frac{u^* - \underline{u}}{\overline{x} - x^*}; \frac{2C_1}{\lambda}; \frac{\lambda C_1}{\varepsilon^2}\right)$$

For all $t \geq \hat{t}$, we define V_λ as in Proposition (3.1). We obtain:

$$\dot{V_\lambda}(t) = 0 \qquad \text{if} \quad \|e(t)\| \leq \lambda$$
$$\dot{V_\lambda}(t) \leq -\gamma\sqrt{V_\lambda} \quad \text{if} \quad \|e(t)\| > \lambda$$

where γ is a positive constant. In summary, we have for all $t \geq \hat{t}$, $\frac{d}{dt}V_\lambda(t) \leq -\theta\sqrt{V_\lambda(t)}$, and so there exists $t' \geq \hat{t}$ such that $\forall t \geq t'$ $\| e(t) \| \leq \lambda$, whence $\dot{\beta}(t) = 0 \quad \forall t \geq t'$, which contradicts the assumption of unboundedness of β. The result follows by the monotonicity of β.

Finally, $k_1 \int_0^t (d_\lambda^{l_1}(e(s))) = \beta(t) - \beta_0 \forall t \geq 0$, the continuity of $y \longrightarrow d_\lambda(y)$ and the boundedness and uniform continuity of $t \longrightarrow \| e(t)\|$ implies the uniform continuity of $t \longrightarrow d_\lambda^{l_1}(e(t))$. We apply Barbalat's Lemma [14] to conclude that

$$\lim_{t \longrightarrow +\infty} dist(\|e\|; [0, \lambda]) = 0$$

This completes the proof of the theorem. □

Remark 3.2. Assumption (28) on the length of the measurements zone is not restrictive. For a given zone Ω, we choose a convenable K and λ to ensure the convergence to the desired profile.

4. Simulation results

Numerical simulations have been carried out in order to illustrate the performance of the both versions of the controller. We have considered equations (2)–(6) with reaction kinetics modeled by first order with respect to the reactant concentration C and by an Arrhenius-type dependence for the temperature T, $f(x_1, x_2) = kx_2e^{-E/Rx_1}$. As in [5], we consider the following system parameters:

$$v = 1\,\text{m/s}, \quad D_1 = 0.25\,\text{m}^2/\text{s}, \quad D_2 = 0.25\,\text{m}^2/\text{s}$$
$$\frac{E}{R} = 15, \quad k = 0.83\,\text{s}^{-1}, \quad \alpha = 13\,\text{s}^{-1},$$
$$K_0 = 1.3, \quad T_{in} = 460\,\text{K}, \quad C_{in} = 0.07.$$

The objective is to regulate the temperature in a neighborhood of $T^* = 500\,\text{K}$ then

$$x^* = T^* - T_{in} = 40\,\text{K}$$

Our constraints for the input $u = T_C - T_{in}$ are chosen as follows:

$$\underline{u} = 10\,\text{K}, \quad u^* = 20\,\text{K}, \quad \overline{u} = 60\,\text{K}$$

It is easy to see that the feasibility assumption $(H2)$ is satisfied for the following constraints parameters:

$$\overline{x} = 44, \quad \underline{x} = 10$$

The initial condition:

$$T(0, z) = T_{in} = 460K, \quad C(0, z) = C_{in} = 0.07$$

and

$$\Omega = [0.2, 0.4] \cup [0.7, 0.9]$$

In the non-adaptive version of the controller (19) where $\beta(t) = \beta^*$, forall $t \geq 0$, in this case we have $C_1 = 599$, $C_2 = 10$, and $\varepsilon = 1125 \cdot 10^{-4}$.

Then, Assumption (27) is satisfied for $\beta^* = 16.\,10^6$.

We have considered a finite difference approximation for the space derivatives with 100 spatial discretization steps.

The result of the simulations is shown in Fig. 2.

For the adaptive version, the controller parameters have been set to the following values:

$$\lambda = 3, \quad \beta_0 = 3, \quad K = 0, 9, \quad l = 2\lambda = 3, \quad \rho = 1$$

The result of the simulations is shown in Fig. 3.

For both controllers, adaptive and non-adaptive, it is observed that the temperature in the reactor tends to the reference signal $T^* = 500\,\text{K}$ in a very short time, the concentration remains in $[0, C_{in}]$ and the convergence of the adaptation gain for the adaptive controller.

The simulation results are similar to those in [5], where the temperature measurements were taken along the reactor. The non-adaptive controller saturates in its upper bound on the measurements zone Ω, which is due to the choice of sufficient large β^* to achieve the tracking objective.

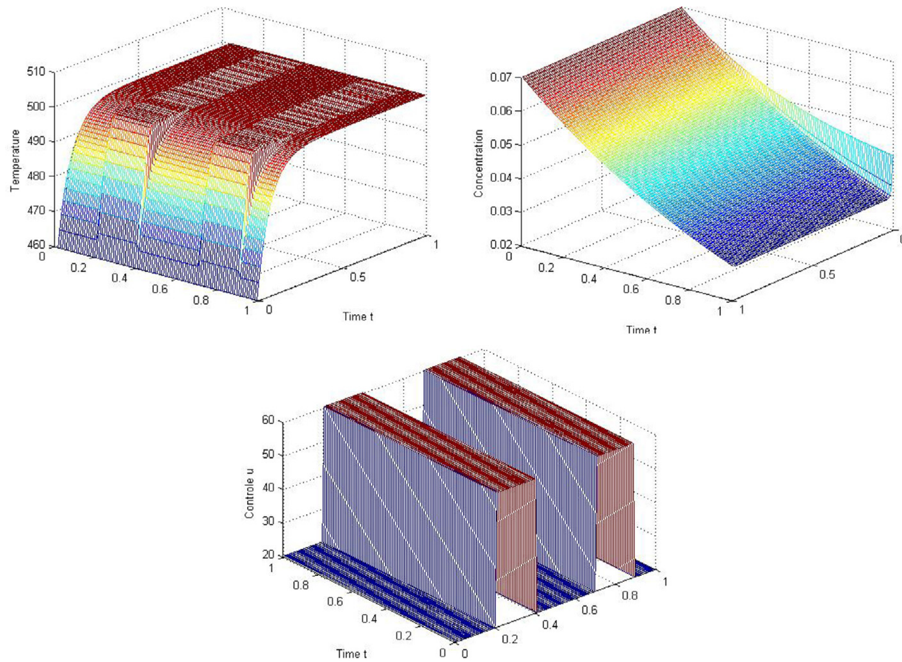

Fig. 2. Numerical simulation of the non-adaptive closed-loop system.

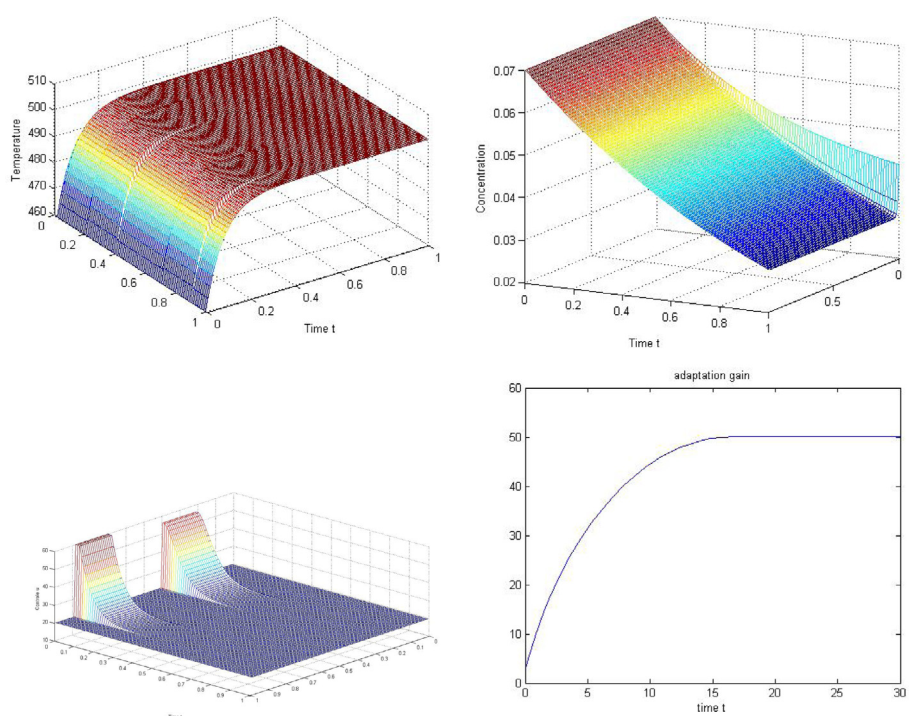

Fig. 3. Numerical simulation of the adaptive closed-loop system.

5. Conclusion

In this paper, the λ-tracking approach with partial measurements has been developed to the reference profile control of the temperature for an exothermic chemical tubular reactor with axial dispersion in the presence of input constraints. Under simple assumptions on the length of the measurements zone in terms of the reference temperature, input constraints and λ it has been shown that the λ-trackers (adaptive and non-adaptive) achieve the asymptotic convergence the temperature profile toward a ball of arbitrary prefixed radius $\lambda > 0$ centered at the given temperature profile. The performance of the controller is illustrated in numerical simulation.

The proposed control law is local in the sense that the initial temperature is forced to be in a fixed domain. The global convergence with partial measurements is under investigation.

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