Numerical Analysis of Modulated Metasurface Antennas using Fourier-Bessel Basis Functions

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***Abstract*—Metasurfaces are thin (2D) metamaterials designed for manipulating the dispersion properties of surface-waves (SWs) or the reflection properties of incident plane-waves. Thanks to the sub-wavelength sizes of the patches used in the implementation step, these surfaces can be described by a surface impedance boundary condition (IBC). In this paper, we investigate a "Method of Moments" (MoM) based analysis of such surface with a family of entire-domain basis functions named "Fourier-Bessel" functions. The orthogonality property of these functions on a disk allows us to represent any smooth current distribution in an effective manner and thereby to drastically reduce the size of the MoM matrix.**

I. Introduction

During the last decade, metasurfaces (MTSs) have received a great interest from the microwave and antennas community [1]. The possibility of controlling the propagation characteristics of surface-waves (SWs) at sub-wavelength level has enabled the design of low-profile, high-gain polarized leaky-wave (LW) antennas [2]. At microwave frequencies, metasurface antennas may be implemented as circular apertures with radius of several wavelengths and exited by a TM SW source (for example a dipole) placed at the center of the disk. The aperture is typically composed of a dense periodic texture of electrically small patches printed on a grounded dielectric slab. From a design point of view, these patches implement a spatially modulated periodic impedance boundary condition (IBC). By slowly varying the shape, dimension and/or orientation of the elements, we can assume a locally periodic surface. Under this assumption, the surface impedance can be locally interpreted as if the elements were placed in a periodically textured layer. This impedance modulation allows to locally modify the dispersion characteristics of the guiding surface waves and thereby gradually transform the surface-wave into a leaky-wave [3].
A classical analysis of this kind of surfaces requires the meshing of a dense array of sub-wavelength (around $\lambda/10$) printed elements (typically more than ten thousand) [2]. The resulting high number of degrees of freedom at a sub-wavelength scale leads to prohibitive computation times [4] and a possibly ill-posed formulation. However, the MTS can be homogenized and represented as an IBC within a good approximation. One possibility consists in using an opaque IBC [4], which assumes that the MTS is impenetrable (zero fields in the lower half-space) and relates the tangential electric and magnetic fields evaluated at the upper interface. Another possibility consists in modeling the surface as a transparent IBC, which relates the fields on both sides of the MTS and accounts separately for the contribution of the grounded slab and that of the sheet [5]. The sheet impedance relates the average tangential electric field at the surface level and the electric currents representing the jump-discontinuity of the average transverse magnetic field at the metasurface. It has been proven in [5] that the second configuration leads to a better conditioned formulation. Moreover, the first formulation does not take into account the spatial dispersion due to the thickness of the grounded slab.
In this paper, we will therefore use the second formulation.
Owing to the smoothness of the surface impedance, the current distribution on the surface will be smooth and therefore can be represented with much fewer basis functions than needed to mesh all the patches. The authors of [6] proposed the use of Gaussian Ring Basis Functions (GRBF) and they proved that using these basis functions, one can obtain a drastic reduction of the number of unknowns and an extremely short computation time (less than 2min). However, these basis functions are not rigorously orthogonal, which means that it should be possible to come up with a more general class of current distributions, which may be described more compactly.

The paper is outlined as follows. In section II, we present the family of Fourier-Bessel functions as well as their spectral behavior. Section III describes the MoM formulation using these functions and Section IV shows numerical results including comparisons with the results from [6].

II. Fourier-Bessel functions

One of the two known families of functions that are orthogonal on a disk is the Bessel family defined as follows [7]:

$$F_m^n(\rho) = J_n(\lambda_n^m \rho) \tag{1}$$

where λ_n^m is the m-th positive zero of $J_n(\rho)$. This obtained family of Bessel functions admits a closed-form Hankel transform:

$$\int_0^1 F_m^n(\rho) J_n(k_\rho \rho)\rho d\rho = -\frac{\lambda_n^m J_{n-1}(\lambda_n^m) J_n(k_\rho)}{(\lambda_n^m)^2 - k_\rho^2} \tag{2}$$

In order to account for the azimuthal dependence, we add the $e^{-jn\phi}$, where ϕ is the azimuthal coordinate. The Fourier-Bessel Basis Functions (FBBF) are then written as:

$$R_m^n(\rho,\phi) = F_m^n(\rho) e^{-jn\phi} \tag{3}$$

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The resulting family of Fourier-Bessel functions is rigorously orthogonal on a disk and admits a closed form Hankel transform on a disk of radius a:

$$R_m^n(k_\rho,\alpha) = -2\pi j^n e^{-jn\alpha}\left[\frac{a^2\lambda_n^m J_{n-1}(\lambda_n^m)J_n(k_\rho a)}{(\lambda_n^m)^2-(k_\rho a)^2}\right] \quad (4)$$

Where k_ρ and α are the spectral variables in cylindrical coordinates.
Finally the current distribution is decomposed into its x and y components. Each component is assumed to be expanded into a sum of Fourier-Bessel Basis Functions (FBBF), as follows:

$$\vec{J}(\vec{\rho}) = \sum_{n=-N}^{N}\sum_{m=1}^{M} i_{mn}^x R_m^{n,x}\left(\frac{\rho}{a},\phi\right)\hat{a}_x + i_{mn}^y R_m^{n,y}\left(\frac{\rho}{a},\phi\right)\hat{a}_y \quad (5)$$

where $\hat{a}_x$ and $\hat{a}_y$ are respectively the unit vectors along the x and y directions.

III. MoM formulation

By modeling the MTS as transparent IBC, one arrives to the following equation [5]:

$$\vec{n}\times\left[\int\int_{S'}\underline{\underline{G}}^{EJ}(\vec{\rho},\vec{\rho}\,')\vec{J}(\vec{\rho}\,')dS' - \underline{\underline{Z}}_S(\vec{\rho})\vec{J}(\vec{\rho})\right] = -\vec{n}\times\vec{E}_i \quad (6)$$

where $\vec{\rho}\,'$ and $\vec{\rho}$ are respectively the source and observation positions. $\underline{\underline{G}}^{EJ}$ is the dyadic Green's function of the grounded slab, $\underline{\underline{Z}}_S$ is the sheet impedance tensor and $\vec{E}_i$ is the excitation electric field.
We make the choice to test the fields with the complex conjugate of the FBBF. Equation (6) then leads to the following matrix equation:

$$\begin{pmatrix}[Z^{xx}] & [Z^{xy}]\\ [Z^{yx}] & [Z^{yy}]\end{pmatrix}\begin{pmatrix}[i^x]\\ [i^y]\end{pmatrix} = \begin{pmatrix}[v^x]\\ [v^y]\end{pmatrix} \quad (7)$$

The elements of the submatrix Z_{xx} are defined as:

$$Z^{xx}(m,n;m',n') = Z_G^{xx}(m,n;m',n') - Z_{IBC}^{xx}(m,n;m',n') \quad (8)$$

with:

$$Z_G^{xx} = \frac{1}{(2\pi)^2}\int_0^{2\pi}\int_0^{\infty} R_{m,n}^{x,*}(k_\rho,\alpha)G_{xx}^{EJ}(k_\rho,\alpha) R_{m',n'}^{x}(k_\rho,\alpha)\; k_\rho\; dk_\rho\; d\alpha \quad (9)$$

and:

$$Z_{IBC}^{xx} = \int_0^{2\pi}\int_0^{a} R_{m,n}^{x,*}\left(\frac{\rho}{a},\phi\right) Z_S^{xx}(\rho,\phi)\; R_{m',n'}^{x}\left(\frac{\rho}{a},\phi\right) \rho\; d\rho\; d\phi \quad (10)$$

Finally, the excitation vector v_x is given by

$$v_x(m,n) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{\infty} R_{m,n}^{x,*}(k_\rho,\alpha)\; G_{xz}^{EJ}(k_\rho,\alpha)\; k_\rho\; dk_\rho\; d\alpha \quad (11)$$

The elements in the other submatrices in (7) are given by definitions analogous to (9)-(10).

A. Grounded slab contribution

The G_{xx}^{EJ} component of the dyadic Green's function can be written as [8]

$$G_{xx}^{EJ}(k_\rho,\alpha) = G_A(k_\rho) + k_\rho^2\cos^2(\alpha)G_V(k_\rho) \quad (12)$$

where G_A and G_V are the scalar potentials (with respect to currents/charges), respectively. After substituting (12) in (9) and integrating along α, we obtain:

$$Z_G^{xx} = (-j)^n j^{n'}\int_0^{\infty} F_m^n(k_\rho a)F_{m'}^{n'}(k_\rho a) \left[2\pi G_A(k_\rho)\delta_{n,n'} + G_V(k_\rho)\epsilon_{n,n'}\right]\; k_\rho\; dk_\rho \quad (13)$$

with $\epsilon_{n,n'} = \frac{\pi}{2}(2\delta_{n,n'}+\delta_{n,n'+2}+\delta_{n,n'-2})$

This integral can be efficiently evaluated using a parabolic contour complex-deformation [8].
The same procedure can be used for Z_G^{xy}, Z_G^{yx} and Z_G^{yy}.

B. Sheet impedance contribution

The sheet impedance can be computed in space domain as follows:

$$Z_{IBC}^{xx} = \int_0^a F_m^n\left(\frac{\rho}{a}\right)F_{m'}^{n'}\left(\frac{\rho}{a}\right)\int_0^{2\pi} e^{j(n-n')\phi}\; Z_S^{xx}(\vec{\rho})\; d\phi\; \rho\; d\rho \quad (14)$$

After expanding Z_S^{xx} into Fourier series as proposed in [6], one has:

$$Z_S^{xx}(\vec{\rho}) = \sum_{r=-\infty}^{\infty} a_r(\rho)\; e^{jr\phi} \quad (15)$$

Then (14) reduces to:

$$Z_{IBC}^{xx} = 2\pi\int_0^a F_m^n\left(\frac{\rho}{a}\right)F_{m'}^{n'}\left(\frac{\rho}{a}\right)\; a_{n'-n}(\rho)\; \rho\; d\rho \quad (16)$$

C. Excitation contribution

The metasurface is excited at the origin ($\rho = 0$) with an elementary vertical electric dipole. The Green's function associated with the x-oriented E-Field can be written as $G_{xz}^{EJ}(\vec{k_\rho}) = \cos(\alpha)f(k_\rho)$. After inserting this expression in (11), we obtain:

$$v_x(m,n) = -\frac{(-j)^n}{2}a^2\lambda_n^m J_{n-1}(\lambda_n^m) \int_0^{\infty}\frac{J_n(k_\rho a)}{((\lambda_n^m)^2-(k_\rho a)^2)}f(k_\rho)k_\rho(\delta_{n,1}+\delta_{n,-1})dk_\rho \quad (17)$$

A similar expression is obtained for v_y.

IV. Results

The analyzed structure is a broadside circularly polarized metasurface antenna as in one of the examples described in [2]. The elements of the impenetrable MTS tensor are given by:

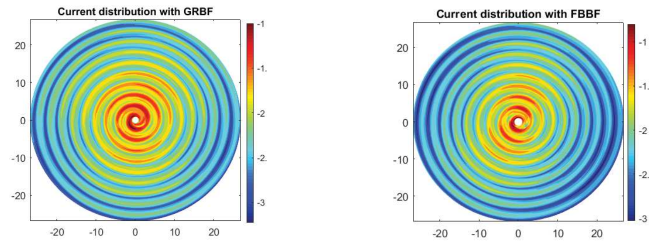
$$Z_+^{\rho,\rho}(\vec{\rho}) = jX_0[1+M_0 cos(2\pi\rho/d-\phi)] \quad (18)$$

$$Z_+^{\rho,\phi}(\vec{\rho}) = jX_0 M_0 sin(2\pi\rho/d-\phi) \quad (19)$$

$$Z_+^{\phi,\phi}(\vec{\rho}) = jX_0[1-M_0 cos(2\pi\rho/d-\phi)] \quad (20)$$

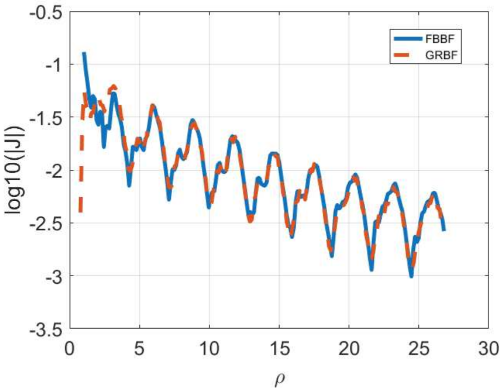
with: $X_0 = 279$, $M_0 = 0.4$, $\epsilon_r = 9.8$, h = 1.57 mm, f = 8.425 GHz, $d = \lambda_0/\sqrt{1+(X_0/\eta_0)^2}$ and $a = 7.6\lambda$.

The presence of the ρ and ϕ in the sine or cosine gives the different elements, a spiral shape.
Fig. 1 shows the obtained current distribution with GRBF and that obtained with FBBF for the same number of basis functions (N=16, M=96) and Fig. 2a shows a 2D plot of these two currents distributions on the axis $\phi = 0$.
We recall that the GRBF solution has been already validated in [6]. We will therefore consider it in this paper as a reference solution.

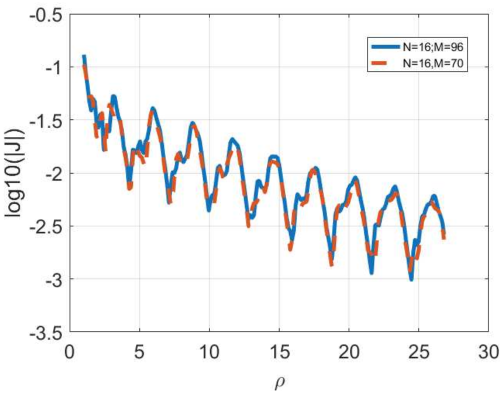


(a) Current distribution GRBF (b) Current distribution FBBF

Fig. 1: Current distribution



(a) GRBF vs FBBF



(b) FBBF (N16M70 vs N16M96)

Fig. 2: Current distribution on $\phi = 0$

We can observe a quite good agreement between the two results. The only discrepancy appears near the center because GRBF do not represent the current at the center, as in practical realizations.

Fig. 2b shows a convergence analysis with FBBF, comparing the obtained solution with (M=96, N=16) with that obtained with (M=70, N=16). We can observe a good agreement between the two curves, which means that the current distribution can be represent very efficiently thanks to the orthogonality property of FBBFs.

V. Conclusion

We have presented a method for the analysis of modulated metasurface antennas based on Fourier-Bessel decomposition. A very good correspondence has been obtained with respect to another set of basis functions. The main advantage of the FBBF lies in its orthogonality and its generality. Although the study has been focused on modulated metasurface antennas, those basis functions can be used in a more general application where one needs to represent a smooth current distribution on a disk.

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