Resource Allocation and Subcarrier Pairing in Energy Efficient Relay-Assisted OFDMA Downlink Systems

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***Abstract*—The energy efficiency (EE) maximization problem in amplify-and-forward (AF) relay-assisted downlink channels with orthogonal frequency-division multiple access (OFDMA) is studied. The power allocation is optimized together with subcarrier allocation and pairing at the relay. As the base station and the relay have individual power constraints, an alternating optimization approach is adopted to solve the problem. Numerical results show that the proposed algorithm outperforms other benchmark schemes in terms of EE.**

I. Introduction

Energy efficiency (EE) for wireless communications is drawing increasing attention and corresponds to an alternative optimization criterion compared to rate maximization. EE is defined by the number of transmitted bits per Joule and leads to tradeoffs between higher transmission rates and reduced power consumption.

On another hand, coverage extension continues to deserve attention. Among various ways to counteract the large attenuation associated with long distances, relaying between base stations and users plays an important role.

A number of papers have already investigated EE in scenarios with assisting relays. Amplify-and-forward (AF) relaying was considered in papers [1]–[5]. Papers [1], [2] have studied EE maximization in multi-relay downlink systems. The authors of [3] maximized the worst EE among all users of an uplink under individual power constraints. EE maximization was also studied in [4] for two-way relay channels. In [5], EE was optimized by the proper selection of the modulation level out of a discrete set. While all these contributions assume multicarrier modulation, none of them has considered subcarrier pairing at the relay. Optimized subcarrier pairing leads to stronger link combinations and therefore a higher rate [6]. Paper [7] has studied the joint problem of subcarrier pairing and allocation for an AF downlink system, but with a total power constraint across the base station and the relay. Moreover, an approximated signal-to-noise ratio (SNR) expression was used in [7] for high SNR as in [2]. Paper [8] considered decode-and-forward (DF) relaying with subcarrier pairing but also with a total power constraint.

In this paper, we consider EE maximization in single-relay multi-user systems with orthogonal frequency-division multiple access (OFDMA) and the amplify-and-forward (AF) technique. We optimize the exact value of EE instead of an approximated value for high SNR. Individual power constraints at the base station and the relay node are considered which is of more practical significance. The problem is solved in an alternating way by means of an iterative algorithm and within only a few iterations. To the best of our knowledge, no work has studied this alternating subcarrier pairing method for maximizing EE. Thanks to the discontinuous quasi-concavity of the obtained EE function of each subproblem, the optimal EE value of each iteration is guaranteed and is obtained by a one-dimension search with bisection method. Different from the problem with total power constraint, where the rate function has to be approximated to be concave, the problem with individual power constraint can be solved by using the exact rate expression. Numerical results show that the proposed algorithm outperforms two benchmarks, especially at low SNR. A first one is based on the approximation of the rate function. Concerning the second one, for both hops the carriers are first sorted according to a gain decreasing order. Then sorted subcarrier k of the first hop is paired with sorted subcarrier k in the second hop.

Perfect channel state information (CSI) is assumed throughout the paper [7]. A robust extension considering imperfect CSI can be achieved by adding an additional term in the power of noise associated with the gap between real CSI and CSI estimation [9]. However, this is beyond the scope of this paper. The focus of this paper is on perfect CSI systems which provide an upper bound on the performance achievable in practice.

II. System Model and Problem Formulation

A. System model

Consider an OFDMA downlink system with a single cell equipped with one base station, one AF relay and a total of K subcarriers to be allocated to N users. The relay is assumed to be half-duplex and the transmission is divided to two time slots. The relay receives the signal from the base station in the

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first time slot and broadcasts an amplified signal to all users in the second time slot. Each subcarrier is allocated to one user at maximum in the second time slot. Therefore, there is no interference among different users at each subcarrier. We assume that perfect CSI is available at all transmission nodes and all nodes are equipped with a single antenna. During the transmission, slow fading is assumed so that within the two time slots the channel remains invariant. Direct link is ignored due to severe fading.

In cellular networks, the base station and the relay node have individual power constraints. We assume that their power constraints are P_s and P_r respectively in this paper.

In the first time slot, at the subcarrier i, the signal received at the relay is given by $r_i = h_i s_i + n_i^R$, where s_i is the source symbol with $\mathtt{E}\{s_i s_i^*\} = p_i$, h_i denotes the channel gain at the i-th subcarrier, and n_i^R is the noise with $\mathtt{E}\{n_i^R n_i^{R*}\} = \sigma^2$. In the second time slot, the relay transmits over carrier j the value r_i amplified to $a_{i,j} r_i$. It is subsequently received by user n as $y_{i,j,n} = g_{j,n} a_{i,j} r_i + n_{j,n}^D$, where $\mathtt{E}\{a_{i,j}^2 r_i r_i^*\} = q_{i,j}$ is the transmission power at subcarrier j in the second time slot if it is paired with subcarrier i in the first time slot, $g_{j,n}$ denotes the channel gain between the relay and user n at subcarrier j, and $n_{j,n}^D$ is the noise with $\mathtt{E}\{n_{j,n}^D {n_{j,n}^D}^*\} = \sigma^2$. Here we assume the power of noise at the relay and at each user is the same without loss of generality. Since each subcarrier in the second time slot is paired with unique subcarrier in the first time slot, we will remove the subscript i and denote $q_{i,j}$ as q_j hereafter. Thus, the signal-to-noise ratio (SNR) of the link from the subcarrier i at the first time slot to the subcarrier j which is allocated to user n at the second time slot is

$$\gamma_{i,j,n} = \frac{p_i |h_i|^2 q_j |g_{j,n}|^2}{\sigma^2 \left(\sigma^2 + p_i |h_i|^2 + q_j |g_{j,n}|^2\right)}. \quad (1)$$

B. EE model

The EE of the system is defined as the transmission rate normalized by the power consumption [10], which is

$$\epsilon = \frac{\frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{n=1}^{N} \alpha_{i,j,n} \cdot w(n) \cdot \log\left(1+\gamma_{i,j,n}\right)}{\phi_S \sum_{i=1}^{K} p_i + \phi_R \sum_{j=1}^{K} q_j + P_C}, \quad (2)$$

where $\alpha_{i,j,n} = 1$ if subcarrier j at the second time slot is paired with subcarrier i at the first time slot and is allocated to user n, otherwise $\alpha_{i,j,n} = 0$; ϕ_S and ϕ_R are respectively the inverse of the efficiency of the high power amplifier (HPA) of the base station and the relay, P_C denotes the sum of the constant circuit power of all nodes, $w(n)$ is the rate weight for user n, which provides different priorities for each user. The penalty term $\frac{1}{2}$ is due to the fact that the total transmission takes two time slots.

C. Problem formulation

The EE problem is formulated as maximizing the EE of the system with individual source and relay power constraints, that is,

$$\max_{\{p_i\},\{q_j\},\{\alpha_{i,j,n}\}\in\{0,1\}} \quad \epsilon \quad (3)$$

$$s.t. \quad \sum_{i=1}^{K} p_i \le P_s \quad (4)$$

$$\sum_{j=1}^{K} q_j \le P_r \quad (5)$$

$$\sum_{j=1}^{K}\sum_{n=1}^{N} \alpha_{i,j,n} = 1 \quad i \in \{1,...,K\} \quad (6)$$

$$\sum_{i=1}^{K}\sum_{n=1}^{N} \alpha_{i,j,n} = 1 \quad j \in \{1,...,K\}, \quad (7)$$

where (6) and (7) imply that there exists a one-to-one mapping between subcarriers in the first time slot and in the second time slot, and paired subcarriers are allocated to unique user.

III. Theoretical Analysis and Algorithm

In this section, we will first analyze the original problem and then solve it in an alternating manner. An algorithm will be provided following the analysis.

Even if there is no subcarrier pairing nor subcarrier allocation, i.e. $\{\alpha_{i,j,n}\}$ are fixed, ϵ in (2) is neither concave nor quasi-concave w.r.t $\mathbf{p} = [p_1, ..., p_K]^T$ and $\mathbf{q} = [q_1, ..., q_K]^T$ [11]. However, for fixed $\{\alpha_{i,j,n}\}$, ϵ in (2) is concave w.r.t. $\mathbf{p}$ for given $\mathbf{q}$ and concave w.r.t. $\mathbf{q}$ for given $\mathbf{p}$ [12]. Because problem (3) has two individual power constraints, it has been decided to solve it in an alternating manner in the sequel (see Algorithm 1).

One iteration is defined as the optimization over $\mathbf{q}, \{\alpha_{i,j,n}\}$ for given $\mathbf{p}$ or over $\mathbf{p}, \{\alpha_{i,j,n}\}$ for given $\mathbf{q}$. In every iteration, subcarrier pairing, subcarrier allocation, and power allocation are jointly optimized. It is notable that the optimizations over $\mathbf{q}, \{\alpha_{i,j,n}\}$ for given $\mathbf{p}$ or over $\mathbf{p}, \{\alpha_{i,j,n}\}$ for given $\mathbf{q}$ are symmetric. This is because, in the optimization of the first time slot, the subcarrier allocation can also be optimized. This is confirmed by observing that $\mathbf{p}$ and $\mathbf{q}$ are symmetric in the SNR function in (1). Therefore we only discuss the optimization over $\mathbf{q}, \{\alpha_{i,j,n}\}$ for a given $\mathbf{p}$, that is to say, the resource allocation and subcarrier pairing in the second time slot given a fixed power allocation at the source.

A. Optimize $\mathbf{q}, \{\alpha_{i,j,n}\}$ for given $\mathbf{p}$

In each odd numbered iteration, $\mathbf{q}$ and $\{\alpha_{i,j,n}\}$ are optimized for a given $\mathbf{p}$. Each sub-problem is a mixed-integer nonlinear programming (MINLP). The optimal solution is to exhaustively search all combinations of subcarrier allocation and pairing, which is not practical in real-time transmission when K is large (for example, there are 1200 subcarriers within a bandwidth of 20MHz in the Long Term Evolution (LTE) standard [13]).

Therefore we will first relax the integer constraints to obtain a criterion for subcarrier allocation. Different from [7], we take derivative to the relaxed time sharing variable, which gives a condition for subcarrier allocation that allows us to implement a one-dimension bisection search to maximize the obtained EE function. Then we will see that the Hungarian algorithm can be implemented to the result of subcarrier allocation for subcarrier pairing.

For time sharing, we introduce the duration $\rho_{i,j,n}$, which is the time fraction during which the subcarrier j in the second time slot is paired with the subcarrier i in the first time slot and allocated to user n. Denote $\hat{q}_{i,j,n}$ as the total transmit power at the relay during the time fraction $\rho_{i,j,n}$. Then the EE function in (2) becomes [14]:

$$\epsilon = \frac{\frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{n=1}^{N}\rho_{i,j,n}w(n)R_{i,j,n}}{\phi_S\sum_{i=1}^{K}p_i + \phi_R\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{n=1}^{N}\rho_{i,j,n}Q_{i,j,n} + P_C}, \tag{8}$$

where $Q_{i,j,n} = \frac{\hat{q}_{i,j,n}}{\rho_{i,j,n}}$ and

$$R_{i,j,n} = \log\left(1 + \frac{p_i|h_i|^2 Q_{i,j,n}|g_{j,n}|^2}{\sigma^2\left(\sigma^2 + p_i|h_i|^2 + Q_{i,j,n}|g_{j,n}|^2\right)}\right). \tag{9}$$

It can be shown that ϵ is concave w.r.t. $\rho_{i,j,n}$ and $\hat{q}_{i,j,n}$ [15]. For each pair (i,j), define $\sum_{n=1}^{N}\rho_{i,j,n} = \Sigma_{i,j}$. Then we have $0 \le \rho_{i,j,n} \le \Sigma_{i,j}$.

Similar with [14], checking the Karush-Kuhn-Tucker (KKT) condition of ϵ is equivalent to checking the KKT condition of its numerator. Thus, by checking the KKT condition w.r.t $\rho_{i^\star,j^\star,n^\star}$ for $0 \le \rho_{i^\star,j^\star,n^\star} \le \Sigma_{i,j}$, we have

$$\begin{aligned}
&\frac{1}{2}\frac{\partial\sum_{i,j=1}^{K}\sum_{n=1}^{N}\rho_{i,j,n}w(n)R_{i,j,n}}{\partial\rho_{i^\star,j^\star,n^\star}} \\
=&\frac{1}{2}w(n^\star)\left(R_{i^\star,j^\star,n^\star} - \rho_{i^\star,j^\star,n^\star}\frac{\hat{q}_{i^\star,j^\star,n^\star}}{\rho_{i^\star,j^\star,n^\star}^2}\frac{\partial R_{i^\star,j^\star,n^\star}}{\partial Q_{i^\star,j^\star,n^\star}}\right) \\
=&f_{i^\star,j^\star,n^\star}(Q_{i^\star,j^\star,n^\star}) - Q_{i^\star,j^\star,n^\star}f'_{i^\star,j^\star,n^\star}(Q_{i^\star,j^\star,n^\star}) \\
=&\beta_{i^\star,j^\star,n^\star} - \gamma_{i^\star,j^\star,n^\star},
\end{aligned} \tag{10}$$

where $\beta_{i^\star,j^\star,n^\star} > 0$ if $\rho_{i^\star,j^\star,n^\star} = \Sigma_{i,j}$ and $\beta_{i^\star,j^\star,n^\star} = 0$ if $\rho_{i^\star,j^\star,n^\star} < \Sigma_{i,j}$, $\gamma_{i^\star,j^\star,n^\star} > 0$ if $\rho_{i^\star,j^\star,n^\star} = 0$ and $\gamma_{i^\star,j^\star,n^\star} = 0$ if $\rho_{i^\star,j^\star,n^\star} > 0$, and

$$f_{i^\star,j^\star,n^\star}(x) \triangleq \frac{1}{2}w(n^\star)R_{i^\star,j^\star,n^\star}, \tag{11}$$

where its derivative is w.r.t $Q_{i^\star,j^\star,n^\star}$. By checking the KKT condition for $\hat{q}_{i^\star,j^\star,n^\star} > 0$, we have

$$\begin{aligned}
&\frac{\frac{1}{2}\partial\sum_{i,j=1}^{K}\sum_{n=1}^{N}\rho_{i,j,n}w(n)R_{i,j,n}}{\partial p_{i^\star,j^\star,n^\star}} \\
=&\frac{1}{2}w(n^\star)\rho_{i^\star,j^\star,n^\star}\frac{\partial R_{i^\star,j^\star,n^\star}}{\partial Q_{i^\star,j^\star,n^\star}}\frac{\partial Q_{i^\star,j^\star,n^\star}}{\partial p_{i^\star,j^\star,n^\star}} \\
=&f'_{i^\star,j^\star,n^\star}(Q_{i^\star,j^\star,n^\star}) \triangleq \lambda.
\end{aligned} \tag{12}$$

Similar with [14], [16], each potential subcarrier pair (i,j) is allocated to the user $n(i,j)$ such that

$$\begin{aligned}
n(i,j) =& \arg\max_n \Big[\Omega(i,j,n) \\
&= f_{i,j,n}(Q_{i,j,n}) - Q_{i,j,n}f'_{i,j,n}(Q_{i,j,n})\Big].
\end{aligned} \tag{13}$$

Let us define a cost matrix $\mathbf{A}$ by

$$\mathbf{A}_{(i,j)} = \max_n \Omega(i,j,n). \tag{14}$$

So far, we have allocated each subcarrier pair (i,j) to a certain user to maximize the EE (also the rate) for a given λ. That is, for a given λ, $f_{i,j,n}$ for each (i,j,n) can be calculated by using (12). For a given λ, we need to find the corresponding $Q_{i,j,n}$ such that $f'_{i,j,n}(Q_{i,j,n}) = \lambda$. This can be obtained thanks to the following formula.

Define $d(x) \triangleq w\log\left(1 + \frac{ax}{bx+c}\right)$. Then

$$d'(x) = \frac{1}{\ln 2}\frac{wac}{((a+b)x+c)(bx+c)}. \tag{15}$$

If $x \ge 0$, then we have $d'(x) \le d'(0) = \frac{wa}{c\ln 2}$ and

$$x = \frac{\sqrt{a^2c^2 + \frac{4wabc(a+b)}{\ln 2 \cdot d'(x)}} - c(a+2b)}{2b(a+b)}. \tag{16}$$

For each (i,j,n), define

$$\begin{aligned}
w(i,j,n) =& \frac{1}{2}w(n) && (17)\\
a(i,j,n) =& p_i|h_i|^2|g_{j,n}|^2 && (18)\\
b(i,j,n) =& |g_{j,n}|^2\sigma^2 && (19)\\
c(i,j,n) =& (p_i|h_i|^2 + \sigma^2)\sigma^2. && (20)
\end{aligned}$$

By substituting $w = w(i,j,n)$, $a = a(i,j,n)$, $b = b(i,j,n)$, $c = c(i,j,n)$, and $d'(x) = \lambda$ into (16), the values of $Q_{i,j,n}(= x)$ and $f_{i,j,n}(Q_{i,j,n})$ can be obtained.

Then $\Omega(i,j,n)$ are calculated and finally the matrix $\mathbf{A}$ is obtained by using (14).

Performing subcarrier pairing consists in selecting K elements from the cost matrix $\mathbf{A}$ such that every row or column has one and only one selected element. This can be realized by using the Hungarian algorithm [17].

After generating matrix A and selecting K elements, for each λ, we have maximized

$$\begin{aligned}
&\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{n=1}^{N}\left[f_{i,j,n}\left(Q_{i,j,n}(\lambda)\right) - \lambda Q_{i,j,n}(\lambda)\right] \\
&= R_{\text{sum}}(\lambda) - \lambda q_{\text{sum}}(\lambda),
\end{aligned} \tag{21}$$

where

$$q_{\text{sum}}(\lambda) \triangleq |\mathbf{q}(\lambda)|_1 = \sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{n=1}^{N}Q_{i,j,n}(\lambda), \tag{22}$$

and

$$R_{\text{sum}}(\lambda) = \sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{n=1}^{N}f_{i,j,n}\left(Q_{i,j,n}(\lambda)\right). \tag{23}$$

The values of $R_{\texttt{sum}}(\lambda)$ and $q_{\texttt{sum}}(\lambda)$ have been obtained. As analyzed in [14], the obtained rate function w.r.t. transmit power at the relay $\tilde{R}_{\texttt{sum}}(q_{\texttt{sum}}) = R_{\texttt{sum}}(q_{\texttt{sum}}^{-1}(\lambda))$ is a concave function of $q_{\texttt{sum}}$ (with discontinuity as discussed in [14]), and $\tilde{R}_{\texttt{sum}}(q_{\texttt{sum}})$ is maximized for each obtained $q_{\texttt{sum}}$. Hence the obtained EE function is a quasi-concave function of $q_{\texttt{sum}}$:

$$\texttt{EE}(q_{\texttt{sum}}) = \frac{\tilde{R}_{\texttt{sum}}(q_{\texttt{sum}})}{\phi_S \sum_{i=1}^{K} p_i + \phi_R q_{\texttt{sum}} + P_C}. \tag{24}$$

One-dimension bisection method can find the maximum of $\texttt{EE}(q_{\texttt{sum}})$ by checking $\texttt{EE}^{'}(q_{\texttt{sum}})$. The details of the algorithm in Algorithm 2 are provided in Subsection C.

B. Optimize $\boldsymbol{p}$, $\{\alpha_{i,j,n}\}$ for given $\boldsymbol{q}$

In each even numbered iteration, $\mathbf{p}$ and $\{\alpha_{i,j,n}\}$ are optimized for a given $\mathbf{q}$. It is exactly the same optimization as in the previous subsection; therefore we will omit the analysis and only give the necessary values to be used:

$$w^{'}(i,j,n) = \frac{1}{2} w(n) \tag{25}$$

$$a^{'}(i,j,n) = q_j |h_i|^2 |g_{j,n}|^2 \tag{26}$$

$$b^{'}(i,j,n) = |h_i|^2 \sigma^2 \tag{27}$$

$$c^{'}(i,j,n) = (q_j |g_{j,n}|^2 + \sigma^2)\sigma^2. \tag{28}$$

Algorithm 3 will show the details for this part.

C. Algorithm

In the algorithm, we first initialize the power at the base station by uniform allocation to the subcarriers. Then the algorithm operates iteratively until convergence. Given the power allocation at the base station, we jointly optimize the power allocation at the relay, the subcarrier allocation, and the subcarrier pairing. The power allocation at the base station, the subcarrier allocation, and subcarrier pairing will be optimized using the obtained power allocation at the relay. Therefore, Algorithm 1 operates in an alternating manner: the transmission time slot T_s changes between 1 and 2 alternatingly. Since the EE function is discontinuous quasi-concave at each iteration, EE is maximized by forcing the derivative to be zero. The EE value will converge since it increases at each iteration and it is upper bounded.

Algorithm 1 An alternating method for EE maximization

```
Initialization: Assume p = P_s/K × [1,...,1]^T, l = 0, ε^(0) = 0, Δε = 1, T_s = 2
while Δε > δ_ε do
    if T_s = 2 then
        l = l + 1; Go to Algorithm 2
        Δε = (ε^(l) − ε^(l−1)) / ε^(l); T_s = 1
    else
        l = l + 1; Go to Algorithm 3
        Δε = (ε^(l) − ε^(l−1)) / ε^(l); T_s = 2
    end if
end while
```

Algorithm 2 Optimize **q** for given **p**

```
1:  For each (i,j,n), calculate w(i,j,n), a(i,j,n), b(i,j,n),
    c(i,j,n) in the function f(x) using (17),(18),(19),(20)
2:  λ_max = max{ a(i,j,n) / (ln 2 · c(i,j,n)) }, λ_min = 0
3:  while λ_max − λ_min > Δλ do
4:      λ = (λ_min + λ_max) / 2
5:      For each (i,j,n), calculate Q(i,j,n) = x using (16)
        by substituting w(i,j,n), a(i,j,n), b(i,j,n), c(i,j,n)
        and λ into w, a, b, c and d'(x); obtain f_{i,j,n}(Q(i,j,n));
        calculate Ω(i,j,n) in (10)
6:      For each (i,j), calculate A(i,j)
7:      Find subcarrier pairing in A using Hungarian algorithm
8:      Calculate q_sum(λ) and R_sum(λ)
9:      if λ(φ_S|p|_1 + φ_R q_sum(λ) + P_C) − φ_R R_sum(λ) > 0 and
        q_sum(λ) ≤ P_r then
10:         λ_max = λ
11:     else
12:         λ_min = λ
13:     end if
14: end while
15: Output q* = q(λ)
```

Algorithm 3 Optimize **p** for given **q**

```
For each (i,j,n), calculate w'(i,j,n), a'(i,j,n),
b'(i,j,n), c'(i,j,n) in the function f(x) using
(25),(26),(27),(28)
λ_max = max{ a(i,j,n) / (ln 2 · c(i,j,n)) }, λ_min = 0
w(i,j,n) = w'(i,j,n); a(i,j,n) = a'(i,j,n); b(i,j,n) =
b'(i,j,n); c(i,j,n) = c'(i,j,n)
Implement line 3 to line 15 in Algorithm 2 until q* is
found
p* = q*
```

IV. COMPLEXITY COMPARISON

In this section, various schemes of resource allocation and subcarrier pairing are compared in terms of complexity. The EE performance of these schemes will be compared in the section dealing with numerical results. In Table I, the complexity of the proposed algorithm is compared with other algorithms which are explicitly described as follows.

- EEPA: the proposed method;
- RPA: rate maximization with both subcarrier pairing and allocation;
- EEA: EE maximization with only subcarrier allocation;
- EE: EE maximization without subcarrier pairing nor allocation;
- AEE (benchmark 1): the proposed method but using approximated rate function as in [7];
- FP (benchmark 2): a fixed pairing after per-hop decreasing gain ordering of the subcarriers.

In Table I, L is the number of iterations, which is mostly from 3 to 6 depending on system parameters such as SNR, power constraint, and constant circuit power. SP denotes subcarrier

TABLE I: Complexity comparison.

Operations	EEPA	RPA	EEA	EE	AEE	FP
SP	L	L	0	0	L	1
SA	L	L	L	0	L	L
PAl	L	L	L	L	L	L
PAd	L	0	L	L	L	L

pairing, SA denotes subcarrier allocation, PAl denotes power allocation, and PAd denotes power adaptation. Please note that power allocation is implemented in all schemes since the focus of this paper is to study the advantage of subcarrier pairing and allocation. Power adaptation means the total power changes with the instantaneous CSI for EE maximization, instead of keeping the same as for rate maximization. We take it into account in the table because power adaptation involves more manipulation for finding λ in Algorithms 2 and 3 to make the derivative of EE expression to be zero [14], [16]. In practical applications, a tradeoff can be considered by referring to Table I and the numerical results in the next section.

V. Numerical results

In this section we show the numerical results. The parameters are set as follows: $K = 16$, $N = 4$, $P_C = 150$mW, and the bandwidth for each subcarrier is $B = 10$KHz, $\phi_S = \phi_R = 2.5$, $w_1 = w_2 = w_3 = w_4 = 1$, $\delta_\epsilon = 0.01$, $\Delta\lambda = \lambda_{\max}/2^{20}$ for each $\lambda_{\max}$. The channels are assumed to be Rayleigh distributed and have an average channel gain $G_0 d_i^{-n}$ where the pass loss exponent $n = 4$, $G_0 = -(G_1 M_l) = -70$dB in which $G_1 = 30$dB is the gain factor at $d = 1$m. $M_l = 40$dB is the link margin compensating the hardware process variations and other noise and interference [18], the noise power $\sigma^2 = BN_0N_f$ where $N_0 = -170$dBm/Hz is the noise power spectral density and $N_f = 10$dB is the noise figure. d_1 is the distance between the base station and the relay and d_2 is the distance between the relay and the users. The results are averaged over 10000 channel realizations.

In all figures, the proposed algorithm is compared with other algorithms listed in Section IV.

In all figures, we plot the EE performances versus power constraints for various resource allocation strategies. Without loss of generality, we assume $P_s = P_r$. From Fig. 1 to Fig. 4, we set $d_1 = d_2 = 10, 15, 25, 50$m respectively. Therefore these figures correspond to the scenarios from high SNR to low SNR. We observe that EEPA always has the best performance. The EE of RPA even decreases at large power constraint because full available power is always used for rate maximization. AEE has a small gap with EEPA at high SNR but has a huge gap at low SNR. FP also has a small gap with EEPA at high SNR but has a huge gap at low SNR which is consistent with the conclusion in [12] that FP is optimal when the power is equally allocated to all subcarriers. In addition, an interesting finding is that the subcarrier pairing needs to be implemented only once to have almost the same performance as with multiple pairing operations. From the mathematical point of view, this is because the dominating entries of the matrix in the Hungarian algorithm always increase with almost the same gain at each iteration.

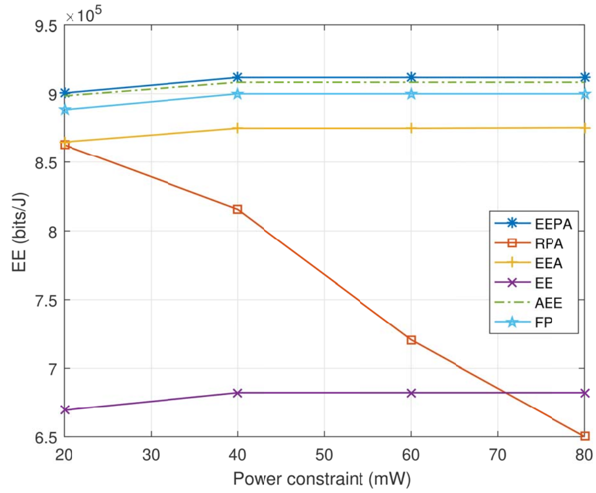

Fig. 1: EE performances versus power constraints. $d_1 = d_2 = 10$m.

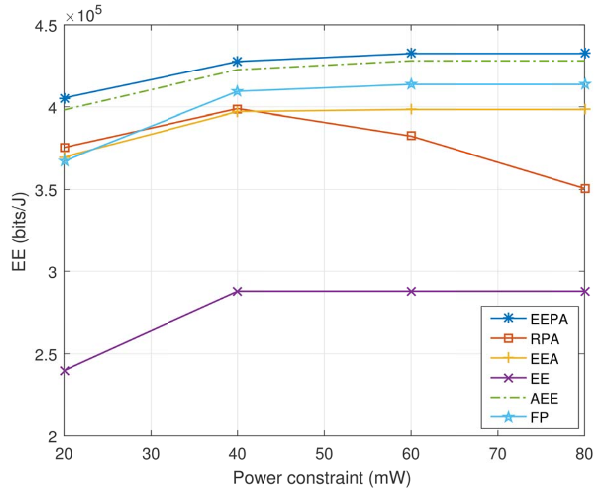

Fig. 2: EE performances versus power constraints. $d_1 = d_2 = 15$m.

VI. Conclusion

We have solved the EE maximization problem in relay-assisted OFDMA downlink channels using an alternating optimization approach. The resource allocation and subcarrier pairing are jointly optimized at each iteration under the power constraints of the source and the relay respectively. The method based on the approximated rate value and the one

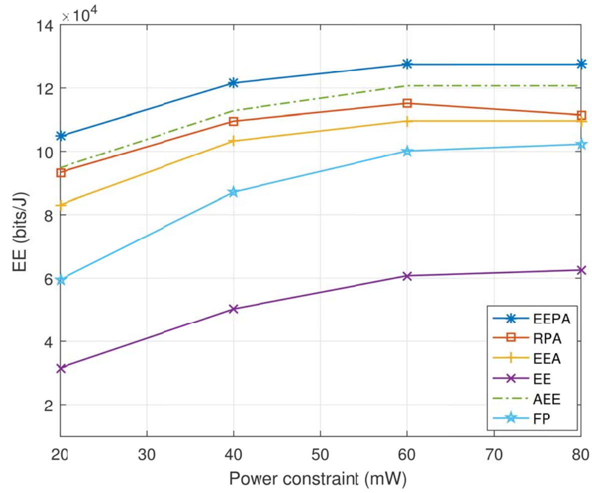

Fig. 3: EE performances versus power constraints. $d_1 = d_2 =$ 25m.

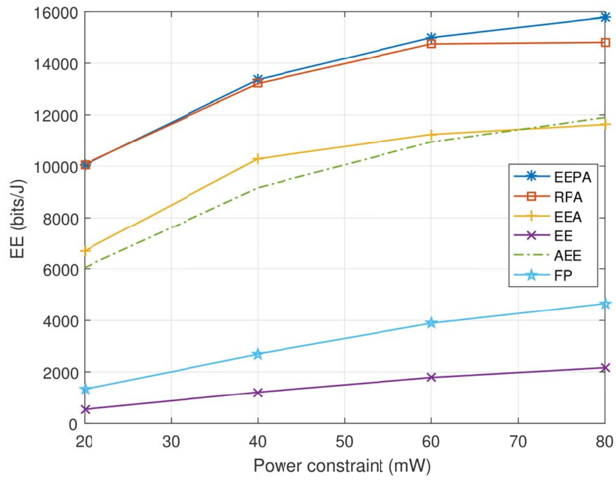

Fig. 4: EE performances versus power constraints. $d_1 = d_2 =$ 50m.

pairing subcarriers at identical positions after sorting, perform similarly to our proposed algorithm at high SNR, but they perform worse than ours at low SNR.

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