

# Emergent Quasiparticles. Or How to Get a Rich Physics from a Sober Metaphysics.

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## 1 Introduction

Among the very architects of the recent re-emergence of emergentism in the physical sciences, Robert B. Laughlin certainly occupies a prominent place. Through a series of works beginning as early as his Nobel lecture in 1998, a lecture given after having been awarded, together with Störmer and Tsui, the Nobel prize in physics for its contribution in the elucidation of the fractional quantum Hall effect, Laughlin openly and relentlessly advocated a strongly anti-reductionistic view of physics – and, more particularly, of the interface between condensed matter and particles physics – which culminated in what can be considered his emergentist manifesto: *A Different Universe. Reinventing Physics from the Bottom Down* (2005). In spite of this prominent role in the vindication of emergentism, rare are the philosophers, among whom even those sympathetic to the idea of emergence, who have paid serious attention to Laughlin’s insights. The subtleties of his view – it is true, often concealed in many technicalities – have accordingly, and somewhat unfortunately, mainly passed unnoticed.<sup>1</sup>

The starting point of this paper is a willingness to somehow remedy this situation by taking Laughlin’s emergentism seriously. More specifically, reflecting on Laughlin’s insight according to which “one of the things an emergent phenomenon can do is create new particles” (1999, 863), we propose an exploration of the way in which emergence can shed light on the ontological status of some quantum entities – more particularly, so-called “quasiparticles” – that would come into being on the occasion of certain

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\*This work is fully collaborative. Authors are listed in alphabetical order.

<sup>1</sup>Two recent notable exceptions are Lancaster & Pexton 2015, and Lederer 2015.

physical transformations. On the occasion of such an exploration, we will be in a position to claim that a precise and metaphysically kosher way of capturing Laughlin’s insightful emergentism exists, which allows vindicating the existence of a rich physical world – made of diverse, ontologically irreducible, yet connected physical domains – within the boundaries of a metaphysically sober worldview. Incidentally, we will show that a natural interpretation of quantum electrodynamics turns out to be, perhaps surprisingly, rather hospitable to (a version of) ontological emergence, in favor of the non-reductive physicalists’ greed, *pace* Kim (2005, 22), for “having the cake and eating it too”.

Here is our battle plan. In section 2, we open the discussion by unpacking Laughlin’s intuition that pertains to the emergence of quasiparticles. More particularly, after having explicated what we think is the philosophical core of Laughlin’s emergentism (subsection 2.1), we describe what constitutes his most convincing and well-articulated exemplification of an emergent phenomenon – the one that has actually driven him towards emergentism in the first place –, namely the fractional quantum Hall effect (subsection 2.2), on the occasion of which anyons are claimed to emerge. In section 3, we turn to an exposition of one of the rare, explicit philosophical reconstructions of Laughlin’s emergentism that is available on the current philosophical market, namely Carl Gillett’s “mutualism”, and contend that such a reconstruction, though perhaps successful, involves an unnecessary excess of metaphysics. In support of such a claim, we show in section 4 that a metaphysically cheaper alternative way of understanding emergence, embedded in Paul Humphreys’ recent “transformational” framework, is available, which better captures Laughlin’s emergentist insights. Finally, in section 5, we build on our previous reflexion in order to explicate, and subsequently evaluate the plausibility of, the claim according to which anyons are to be considered as emergent quasiparticles.

## 2 Laughlin on emergent quasiparticles

### 2.1 Laughlin on emergence

In a nutshell, there is emergence in Laughlin’s view as soon as physical entities are organized together in a very specific, “emergence-engendering” way, to the effect that the transition between these entities – the “parts” –, and the specifically organized collective configuration they give rise to, – the “whole” – is governed by new physical “principles of organization” (hereafter [POS]; see e.g. Laughlin 2005, xiv). Because these [POS] are at the core of

Laughlin’s emergentism, let us proceed to unpack them.

First, [POs] are claimed to be “physical” in the *via negativa* sense that they are not tied to anything – forces, powers or entities – that one would be ready to qualify as non-physical (e.g. mental or social). In spite of the fact that, as we will explicate later on, [POs] “transcend” the laws and rules of microphysics – microphysics being understood here as the physics that pertains to the “ultimate parts”, viz. the most basic constituents of natural things, whatever they are –, [POs] still “grow out of the microscopic rules” (Laughlin et al. 2000, 29-32), thereby guaranteeing some minimal nomological continuity or self-sufficiency for the physical realm broadly construed. It is in this last sense that Laughlin’s emergentism opposes both the view that physics simply reduces to a set of “master rules”, and the opposite view according to which it is rather an “open frontier” (Laughlin 2005, 6).

[POs] are also inherently “organizational”, to the extent that they appear upon the organization of entities into systems of entities, and regulate the passage from the former to the latter. Accordingly, Laughlin’s emergence primarily has a collective, holistic or hierarchical flavor, in the way that it is mainly about a transition between “levels” – the (lower or microphysical) level of the parts and the (higher or macrophysical) level of the whole – in the spirit of the traditional slogans: “The whole is more than the sum of its parts” or “More is different” (Laughlin & Pines, 2000, 29).

Finally, [POs] are “novel”. As with any emergentist view, dealing with the crucial tenet of novelty turns out to be very delicate. In order not to let too much ambiguity bear on this tenet, it can prove helpful to first stress the possible senses in which Laughlin’s [POs] are *not* (necessarily) supposed to be novel. To begin with, the type of novelty considered here is to be contrasted with what can be referred to as *degenerate* novelty, that is to say, [POs] are not merely *different* from other physical principles or rules that govern the evolution of emergent systems or their composing parts (and, ultimately, their most basic constituents). Second and more interestingly, the novelty of [POs] does *not* also reduce to mere *epistemic* novelty, that is, unexpectedness relative to a given state of knowledge or cognitive/technological access at a given moment in time. Third and finally – let’s call this *cosmological* novelty –, Laughlin’s [POs] are not (necessarily) novel in the sense that their advent and action is unprecedented in the history of the whole evolving universe.

Actually, it turns out that [POs] are novel in a sense that is at the same time more radical than mere degenerate or epistemic novelty, and that doesn’t have the unique and contingent character of cosmological novelty. It essentially amounts to the idea – let’s refer to it as *ontological* novelty – that

[POs] are *sui generis* or fundamental, in the sense that they appear as brute empirical laws that aren't already in existence, even potentially or implicitly, outside the specific empirical context that gave rise to them.<sup>2</sup> Correlatively, their appearance and action could not have been predicted theoretically before the appropriate experimental configuration was reached (spontaneously or not), nor could they have been deduced from "first principles", namely and ultimately, the laws of microphysics (Laughlin & Pines 2000, 30).

Now, it seems to be the case that, in Laughlin's implicit ontology where laws or principles govern the behavior of entities, the emergent status of [POs] entails the emergence of entities that [POs] are about, along the exact same lines. Accordingly, upon emergence, new entities arise that are (i) physical (for they are made up of the physical entities that constitute their emergence basis), (ii) high-level (for they only exist at the level of an organized collection of their basal entities) and (iii) ontologically new (for they are as fundamental as their ultimate basal entities are). This being said, there is a similar sense in which, in Laughlin's view, properties, processes, behaviors or forces can also be said to be emergent as soon as some [POs] are at play in the relevant way.

Elizabeth Barnes recently proposed a way to characterize emergence that nicely fits with Laughlin's implicit construal of the concept, and can accordingly be used to put some philosophical flesh on Laughlin's emergentist bone. According to Barnes (2012), emergents are entities that are fundamental and yet dependent. As we will see later on, emergent entities like anyons are indeed to be conceived of, in Laughlin's view, as as fundamental as electrons – both are parts of the building blocks of reality –, and yet the former depend on the latter (and not the other way around, a fact that justifies the idea that anyons are "higher-level" than electrons).<sup>3</sup> It should be clear from this that, as it is the case in Barnes' account, Laughlin's emer-

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<sup>2</sup>That Laughlin actually endorses this last point is of course a matter of exegesis. According to us, it is the only way to make sense of the radical epistemic cut-off that Laughlin's emergence inexorably entails (see below). Furthermore, ontological novelty seems a reasonable interpretation of expressions like: emergent states are "*fundamentally* different" from basal states (Laughlin 1999, 864, our emphasis), emergent principles are "transcendent", and their existence shows us "that for at least some *fundamental* things in nature the Theory of Everything is irrelevant" (Laughlin & Pines 2000, 28-29, our emphasis), or "collective organizing principles [...] are *in a real sense* independent of them [the microscopic rules]" (Laughlin et al. 2000, 32, our emphasis).

<sup>3</sup>Should it turn out that electrons themselves are not among the basic building blocks of reality, this would not modify Laughlin's view on emergence in any significant way. What he would probably say in this context is that electrons are themselves to be considered as emergent on the whatever-fundamental-entity-there-is-below in the exact same sense as anyons are to be considered emergent on electrons.

gence is primarily an ontological matter – emergence ascriptions are to be taken realistically as saying something about the way the world is – even if, concomitantly, they are systematically accompanied by principled epistemic effects, like unpredictability or non-deducibility from “first principles”.

## 2.2 Emergence *par excellence*: the fractional quantum Hall effect

Among the phenomena that Laughlin is ready to call “emergent” in the sense just described, the fractional quantum Hall effect (hereafter FQHE) certainly occupies a prominent place. If we take seriously Laughlin’s claim that the discovery of this effect was the “opening movement of the age of emergence” (Laughlin 2005, 76), then it is rather natural to focus on this peculiar quantum phenomenon in order to provide some empirical concreteness to Laughlin’s emergence ascriptions.<sup>4</sup>

To understand this effect and, more importantly, its idiosyncrasies, it is worthwhile to briefly describe its underlying physics.<sup>5</sup> To begin with, the Hall effect was first discovered by Edwin H. Hall in 1879. In an experimental set-up consisting of a conductor lying in a magnetic field and in which flows an electric current orthogonal to the field, the effect consists in the advent of a Hall potential  $V_H$  and a corresponding Hall resistance  $R_H$  perpendicular to both the magnetic field and the current. Such a phenomenon is due to the accumulation of the electrons on one side of the conductor, given the fact that the electrons are deflected by the magnetic field. In this purely classical effect, the Hall resistance is proportional to the intensity of the magnetic field and inversely proportional to the electronic density (as can be seen through the straight dotted line on figure 2).

In 1980, Klaus von Klitzing discovered that the Hall resistance doesn’t actually vary in linear fashion with the intensity of the field, but rather exhibits plateaus as a function of the field (see figure 1). Furthermore, these plateaus occur at some values of the Hall resistance that are insensitive to the nature of the material involved in the experimental set-up, but are a function

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<sup>4</sup>Especially in his 2005 book *A Different Universe*, Laughlin generalizes his emergentism to other domains of physics and, by extension, of science, to the effect that his emergence becomes rather mundane, if not almost universal. By sticking to the specific case of the FQHE, we wish not to commit ourselves to such generalizations. We indeed think that some idiosyncrasies of the FQHE are responsible for it being emergent in an interesting sense (see section 4.2), but we don’t know whether other phenomena, especially in less formalized sciences like biology or psychology, share enough of these idiosyncrasies to also be plausibly qualified as emergent.

<sup>5</sup>For a more complete treatment and some philosophical insights, see Lederer 2015.

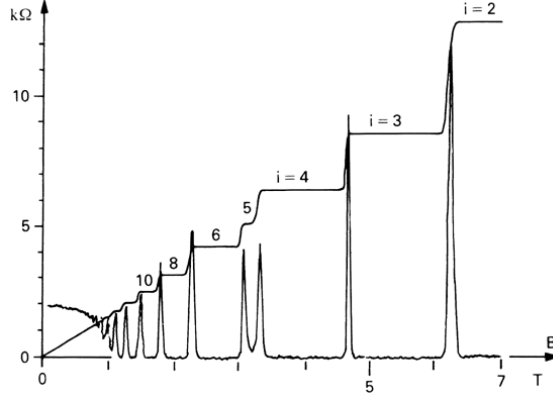


Figure 1: The integer quantum Hall effect [retrieved from the 1998 “Nobel Press Release” (2015 October 22), <http://www.nobelprize.org>]. The hall resistivity shows different plateaus as a function of the magnetic field. The Ohmic resistance, represented by the lower curve, drops to zero for each plateau.

of some constants – namely the electric charge  $e$  and Planck constant  $h$  – divided by an integer (called the filling factor; see below). This phenomenon, known as the integer quantum Hall effect (hereafter IQHE), can be explained through the machinery of the quantization of cyclotron orbits of electrons in magnetic fields (called the Landau quantization). Three facts are relevant here for providing such an explanation. First, the energy levels (called the “Landau levels”) are discrete and equidistant, with a spacing proportional to the intensity of the field. Second, these levels are highly degenerate (that is, a lot of electrons can occupy each level) and such a degeneracy is also itself proportional to the intensity of the magnetic field. Third, electrons are fermions. Accordingly, following the exclusion principle, they cannot occupy the same states as other electrons. It follows from these facts that, starting from an experimental situation where a given Landau level is completely filled by electrons (let’s say, with a field of approximately  $5T$ , when the third Landau level is fully occupied; see figure 1), decreasing the intensity of the field, and consequently decreasing the amount of available states in each Landau levels, will lead electrons, of which there is a fixed density, to begin to fill the next Landau level (here the fourth), leading to a transition to a next plateau of Hall resistance. Conversely, by raising the intensity of the field and, concomitantly, the number of possible occupation states in each Landau level, there will be a point (here around  $\pm 6T$ ) when the lower Landau level

(here the second) will have enough states available to house all the electrons, hence leading to a transition to an adjacent plateau of Hall resistance. It is of common use to ascribe each Landau level, and correspondingly each of the Hall plateaus, with a filling factor  $\nu$  that can only take integer values. Given this, the IQHE is an indirect way of making manifest a purely quantum phenomenon, *viz.* the quantized nature of electronic orbitals in a magnetic field.

In 1982, through experiments involving lower temperatures and a stronger magnetic field, Störmer, Tsui and their co-workers discovered other steps in the Hall resistance than the ones involved in the IQHE. In particular, these new steps can also be expressed through the same constants ( $e$  and  $h$ ), but it turns out that – in sharp contrast with the IQHE – they occur at values of the filling factor that are fractions over an odd integer (like, *e.g.*,  $1/3$ ,  $2/5$ ,  $3/7$ ,  $2/3$ , *etc.*; see figure 2). In order to account for such an effect, naturally referred to as the “fractional” quantum Hall effect, the Landau machinery proves inadequate. A new approach had to be envisioned that would take into account something that had so far been neglected, namely the electron-electron interactions inside the experimental set-up. It was in this context that Laughlin made his insightful proposal that led him to be awarded the 1998 Nobel prize in physics.

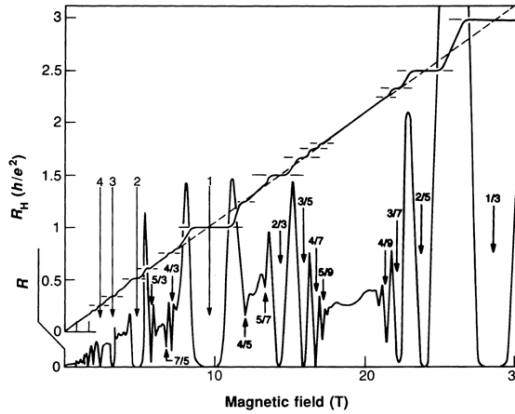


Figure 2: The fractional quantum Hall effect [retrieved from the 1998 “Nobel Press Release” (2015 October 22), <http://www.nobelprize.org>]. The straight dotted line captures the dependence relation between the Hall resistance and the field in the classical limit. Beside plateaus of integer filling factors (for values of the field below  $\pm 10T$ ), plateaus with fractional filling factors can be seen.

It is not the place here to enter into the details of Laughlin’s approach, which involves many technicalities that turn out to be irrelevant for our present purpose. Moreover, we will lay down some aspects of the theoretical underpinnings of Laughlin’s work in a slightly different perspective (namely a quantum field theoretical perspective rather than a non-relativistic quantum mechanical perspective) in due time (see section 4.2). For the time being, let us content ourselves with stressing Laughlin’s main insight: the fractional occupation of the Landau levels corresponds to fractionally charged excitations obeying fractional statistics, which correspond, at least according to Laughlin himself, to new (types of) particles, namely *quasiparticles* (famously referred to by Wilczek as “anyons”). These new particles are “the elementary excitations of a distinct state of matter” (Laughlin 1999, 863), called a topological state of matter, and exert a new type of force, namely “a long-range velocity-dependent force [...] – a gauge force – which is unique in the physics literature in having neither a progenitor in the underlying equations of motion nor an associated continuous broken symmetry” (Laughlin 1999, 871).

It is noteworthy that what makes the FQHE a case of emergence, in contrast with the IQHE, is the fact that in the case of the former, but not (presumably) of the later, electron-electron interactions occur, leaving room for the advent of collective effects of the electron gaz, *viz.* anyons. What matters for us here is that anyons are to be considered emergent in Laughlin’s sense (Laughlin & Pines 2000, 29), for they satisfy the three basic ingredients identified above, namely dependence (anyons are sustained by underlying electrons), hierarchy (anyons are in some sense collective effects of electrons) and fundamentality or ontological novelty (anyons are fundamental particles in their own right, not simply “second-class electrons”).<sup>6</sup> This last point is worth emphasizing, for it constitutes the core of Laughlin’s intuition about emergent quasiparticles, namely that a physical process involving emergence is able to create new particles out of “old” ones. Such an intuition is doubly

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<sup>6</sup>The corresponding [PO] that is responsible for such an emergence is “Anderson’s localization” (Laughlin 1999; see also Anderson 1958), which governs the fact that electrons removed from, or added to, a filled Landau level are localized by the impurities in the conductor, and hence do not contribute to electric transport (a fact that accounts for the constant conductivity that is typical of the Hall plateaus). As an anonymous referee rightly pointed out to us, considering localization as the only relevant [PO] at stake here cannot possibly be the whole story for getting at emergent anyons, notably because the very same principle is also supposed to explain the IQHE. Complementing Laughlin’s proposal in this respect is beyond the scope of this paper, especially given the fact that the reconstruction of Laughlin’s insights that we propose in section 4 doesn’t need the invocation of any [PO].



vindicated in Laughlin’s work. First, from an empirical point of view, anyons in the experimental set-up and electrons in the vacuum are on a par, for they behave in qualitatively similar ways: “They carry momentum, energy, spin, and charge, scatter off one another according to simple rules, obey Fermi or Bose statistics depending on their nature” (Laughlin & Pines 2000, 29). Second, they are also theoretically indistinguishable, in the sense that “there is no logical way to distinguish a real particle from an excited state of the system that behaves like one” (Laughlin 1999, 863). This being said, the only thing that could significantly differentiate anyons from electrons in terms of ontological status is that the conditions of existence of the former are more restrictive than the ones of the latter – hence the former, and not the latter, are called *quasiparticles*.<sup>7</sup>

Considering anyons as emergent quasiparticles is philosophically significant on three different levels. First, from the *ontological* point of view we’ve just stressed, it makes the FQHE the place for the advent of new entities that are to be considered as as fundamental or ultimate as the other fundamental entities they are emerging from.<sup>8</sup> Put differently, with the FQHE, a new physical domain is made accessible. Second, as an *epistemological* correlate of this, this new physical domain is representationally broken off the “old” one, in the sense that the epistemic resources of the latter are irrelevant or inadequate to account for the former. Accordingly, as a case of Laughlin’s emergence, the FQHE marks a dividing line between particles physics and condensed matter physics. This obviously bears on the debates relating to the possible unity of science. In Laughlin’s own words, rather than a “Theory of Everything” from which every phenomenon could be deduced – including the advent of anyons –, science is made of “a hierarchy of theories of things” that are epistemologically autonomous from one another (Laughlin & Pines 2000, 30).<sup>9</sup> Beside these ontological and epistemological stakes, Laughlin’s emergence also suggests a *methodological* prescription

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<sup>7</sup>Gelfert (2003) considers quasiparticles as unreal, for, according to his intuition of “entity realism”, derivative or composite things cannot be real. Of course, such an intuition merely amounts to a denial of the very possibility of ontological emergence, something that can prove to be empirically falsified, as we claim it is in section 4.2. It is also somewhat ironic that the Nobel press release relative to Laughlin’s shared prize refers to anyons as *not* being particles, but entities that merely *result from* a “dance of electrons”.

<sup>8</sup>It is noteworthy that Laughlin’s implicit construal of fundamentality certainly doesn’t amount to something like “being non-composite”. A plausible interpretation is then to consider Laughlin’s fundamentality as meaning something like “being subject to fundamental laws”, such laws being those deemed fundamental by practicing physicists.

<sup>9</sup>In Laughlin’s layered view, this brings about autonomous theories at different “levels”. In the perspective that we embrace in section 4, such a picture will be reconsidered. The possible irreducibility of theories will be construed in a diachronic sense, with respect to

for physics. Pure theoretical physics, characterized by a high weight given to deductive knowledge, should be abandoned in favor of a form of experimentalism that grants preference to inductive knowledge. According to Laughlin, “the central task of theoretical physics in our time is no longer to write down the ultimate equations but rather to catalogue and understand emergent behavior in its many guises [...]” (Laughlin & Pines 2000, 30).

Now that what constitutes the core of Laughlin’s implicit emergentism has been described and exemplified, with a special focus on the ontological status of emergent quasiparticles, let us turn to one of the rare philosophical reconstructions of Laughlin’s emergence that has been attempted in the recent literature pertaining to the metaphysics of emergence.

### 3 Gillett’s philosophical reconstruction

In 2010, Carl Gillett put forward a complete reconstruction of the metaphysical underpinnings of a view he himself dubbed “scientific emergentism”, of which he took scientists like Anderson and Laughlin to be the prominent figures. According to Gillett, such an emergentism is committed to a strong version of emergence that is incompatible with “compositional reductionism”, a view according to which, when it comes to part-whole configurations, high-level property instances are causally inert, all of their potency being systematically and completely preempted by their lower-level, realizer property instances.

Gillett’s main contention is that scientific emergentists have recently provided us with some tools to oppose such a reductionism without having to embrace a dualistic stance, by pointing towards a “missed option” in the way parts and whole can come to be related in some empirical situations. Such an option, which has been overlooked by philosophers in the recent debates, can be delineated on the basis of two interconnected ingredients, namely a new view of aggregation – the “Conditioned View” – and a new determinative relationship – the so-called “*machresis*”.

Unpacking these two notions requires some preliminaries, among which are (any variant of) the causal theory of properties together with a notion of natural hierarchy built in terms of levels of mechanism. On such a basis, one can capture the relationship between higher-level and lower-level property instances through Gillett’s own notion of “dimensioned realization”, on the following model (Gillett 2010, 29, with slightly modified notations): A set of

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which the theories at play are to be considered as being about domains of fundamentality that are temporally, rather than hierarchically, separated.

property instances  $B$  realizes an instance of a property  $X$ , in an individual  $s$  under condition  $\$$ , if and only if  $s$  has powers that are individuated by an instance of  $X$  in virtue of the powers contributed, under  $\$$ , by  $B$  to  $s$  or  $s$ 's constituents, but not vice versa.

This being said, the conditioned view of aggregation, in contrast with the “simple view” that inexorably leads to compositional reductionism, allows that lower-level realizer property instances only contribute certain powers on the condition they are realizing a certain higher-level property instance. What is relevant here for emergence is that this view of aggregation, unlike the simple one, allows for higher-level properties, though realized, to make a genuine difference in the world, for it allows for the existence of a peculiar species of downward determination – the so-called *machresis* – holding between the higher-level realized property instances and their lower-level realizer property instances. *Machresis*, which is distinct from the more usual, dualistically-inclined determinative relation that is downward causation, is a relation through which a realized property instance can be efficacious by determining synchronically, non-causally and non-compositionally, the contributions of powers by property instances in its own realizers (and not by contributing powers itself).

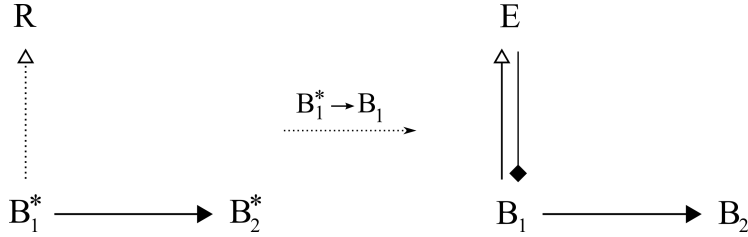


Figure 3: Gillett’s “mutualism” or strong emergentism as defended by scientific emergentists like Laughlin. On the left diagram: the dynamics of the simple view of aggregation. A realized property instance  $R$  is merely resultant from, or compositionally reducible to, its basis  $B_1^*$ , insofar as its defining powers are preempted by the powers contributed by  $B_1^*$ .  $B_1^*$  causes  $B_2^*$  at  $t_2$  regardless of it realizing  $R$  at  $t_1$ . On the right diagram: the dynamics of the conditioned view, which is reached from that of the simple view as soon as the relevant basis  $B_1$  obtains. In this case, the higher-level realized property instance  $E$  is strongly emergent, for it *machretically* determines the powers contributed by  $B_1$ . Put differently,  $B_1$  would never causes  $B_2$  at  $t_2$  if not realizing  $E$  at  $t_1$ .

According to Gillett, this “mutualist” determinative dynamics (see figure 3) adequately captures the metaphysical picture that scientists like Laughlin

have implicitly in mind when making emergence ascriptions. Should this be true, considering the emergence of anyons in the peculiar case of the FQHE would lead Laughlin to be committed to at least these three facts: (1) anyons (or their properties) are realized by an underlying, simultaneous basis (probably made of electrons or their properties), to the effect that there is a sense in which anyons are higher-level than such a basis (in a mechanistic sense of level); (2) realizing anyons (or their properties) is the appropriate conditions for the basis to contribute new powers that would never have appeared otherwise; and (3) anyons are then causally efficacious by machretically determining the powers contributed by their underlying realization basis. It is noteworthy that, together, these three facts cover the three ingredients defining of Laughlin’s emergence as presented in section 2, namely dependence (here cashed out in terms of realization), hierarchy (here cashed out in terms of levels of mechanism) and ontological novelty (here cashed out in terms of higher-level causal efficacy).

This being said, it is important to note that applying Gillett’s reconstruction of Laughlin’s emergentism to the case of the FQHE is to be done with some caution. Indeed, Gillett’s analysis turns out to be mainly inspired from emergence ascriptions that are tied to a different [PO] than the one at play in the FQHE, namely spontaneous symmetry breaking (instead of localization; see e.g. Gillett 2010, 32), and nothing *a priori* requires that the determinative dynamics at play in these different cases should be the same. This leaves us with three possible options. First, Gillett’s reconstruction does apply *mutatis mutandis* to the case of the FQHE, as it is suggested by Gillett himself (2016, 346, footnote 14).<sup>10</sup> Second, Gillett’s reconstruction does *not* apply to the FQHE, and therefore Gillett is mistaken in arguing that his mutualism captures the implicit thought of scientific emergentists like Laughlin (remember that, according to Laughlin, the FQHE is the paradigmatic example of an emergent phenomenon). Third, Gillett’s reconstruction does *not* apply to the FQHE, but it applies adequately to other emergence ascriptions made by Laughlin (like the one relative to, say, superconductivity), to the effect that Gillett’s analysis is merely incomplete. Laughlin’s emergentism turns out to be richer than it first seemed, in the sense that it allows a single template to accommodate very different kinds of emergence.

Which one of these options is really the case doesn’t actually matter to us, for knowing exactly what Gillett and Laughlin have in mind is irrelevant

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<sup>10</sup> A possible way to accomplish this extension is to adopt a model of the FQHE where symmetry breaking plays a role. In this class of models, the FQHE is explained by the fact that a gas of composite-bosons condenses spontaneously (Rao 2001).

to what we want to claim in what follows, namely, that it is possible to recover Laughlin’s important insights that pertain to the idea of emergent quasiparticles without the metaphysical inflation put forward by Gillett. To put it more bluntly, we contend that it is possible to get Laughlin’s rich physics without Gillett’s convoluted metaphysics.

## 4 A metaphysically cheaper alternative

In this section, we concisely describe an alternative account of emergence, called “transformational emergence”, which we have developed more extensively elsewhere (Guay & Sartenaer 2016) on the basis of an idea first proposed by Paul Humphreys (2016). By exemplifying such an emergence in the case of the FQHE, we claim to recover Laughlin’s strong intuition about emergent quasiparticles without having to embrace Gillett’s heavy metaphysical machinery. In the next section, we draw some conclusions with respect to the idea of emergent particles.

### 4.1 On transformational emergence

As with any other account of emergence, transformational emergence (hereafter [TE]) is devised in order to make sense of two ideas that are *prima facie* in tension, namely that a putatively emergent entity *depends on* its emergence basis and that, at the same time, the former is *novel* with regard to the latter. More precisely, considering the case of a physical system  $S$  at two successive stages of its evolution ( $S_1$  at  $t_1$ ;  $S_2$  at  $t_2$ ), one will say that  $S_2$  transformationally emerges from  $S_1$  when the ideas of dependence and novelty are met in the following sense:

- (DEP)  $S_2$  is the product of a spatiotemporally continuous process going from  $S_1$  (for example causal, and possibly fully deterministic). In particular, the “realm”  $R$  to which  $S_1$  and  $S_2$  commonly belong (e.g. the physical realm) is closed, to the effect that nothing outside of  $R$  participates in  $S_1$  bringing about  $S_2$ . And yet:
- (NOV)  $S_2$  exhibits new entities, properties or powers that do not exist in  $S_1$ , and that are furthermore *forbidden* to exist in  $S_1$  according to the laws governing  $S_1$ . Accordingly, different laws govern  $S_2$ .

It is noteworthy that the “forbidden” clause in (NOV) introduces a modality aspect to [TE], according to which, upon emergence, a physical domain

that was previously barred – including entities and their properties, subject to specific laws – becomes accessible. Put differently, a [TE] ascription is essentially a modal claim. It is about impossible phenomena (according to the pre-emergence laws) that, upon emergence, become possible, and even actual (due to the advent of new laws that reconfigure the space of possibilities).<sup>11</sup>

[TE] shares some commonalities with Gillett’s emergence, among which the fact that the advent of emergent entities makes a genuine causal difference in the world without breaking causal closure or entailing a rejection of physicalism<sup>12</sup>. What sharply distinguishes [TE] from Gillett’s proposal, though, is that the former, in contrast with the latter, is a *diachronic* variety of the concept, that is, there always necessarily is a temporal delay between a putative transformational emergent and its emergence basis. Another contrast that is worth stressing is that [TE] is essentially *non-holistic*, in the sense that it can perfectly tolerate a thorough egalitarianism when it comes to the idea of a discrete hierarchy of levels in nature. It is on the basis of these two distinguishing ingredients – diachrony and “level egalitarianism” – that [TE] actually turns out to be immune to exclusion-style worries (see figure 4) and that the account captures, instead of the traditional emergentist slogans, the ideas that “*after* is different” or “the whole is the sum of the *transformed* parts”.<sup>13</sup>

## 4.2 Transformational emergence and the FQHE

Because no one can claim to have a direct and privileged access to the ontology of natural systems, exemplifying [TE] in a concrete phenomenon, and in particular in the FQHE, requires one to identify what would be the

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<sup>11</sup>It should be noted that transformational emergence does not rely on a specific metaphysical conception of natural laws. However, laws have to be able to circumscribe the domain of physically possible states.

<sup>12</sup>For these reasons, we are inclined to consider [TE] as a form of “weakly ontological” emergence, while Gillett prefers to refer to his account as “strong emergence”, “ontological emergence” being reserved for anti-physicalistic versions of the notion. This difference is only terminological.

<sup>13</sup>It is in this last respect that [TE] makes sense of an idea that Gillett’s emergence also captures – though differently –, namely that “parts behave differently in wholes”. For the reader who is perhaps not well acquainted with Kim’s debated “causal exclusion argument” (see e.g. 2005, chapter 2), “exclusion-style worries” refer to the fact that not all combinations of (DEP) and (NOV) yield acceptable forms of ontological emergence. Typically, versions of emergence that combine a construal of (DEP) in terms of supervenience with a construal of (NOV) in terms of downward causation lead to a vexing causal overdetermination (when the basal level is supposed to be causally closed).

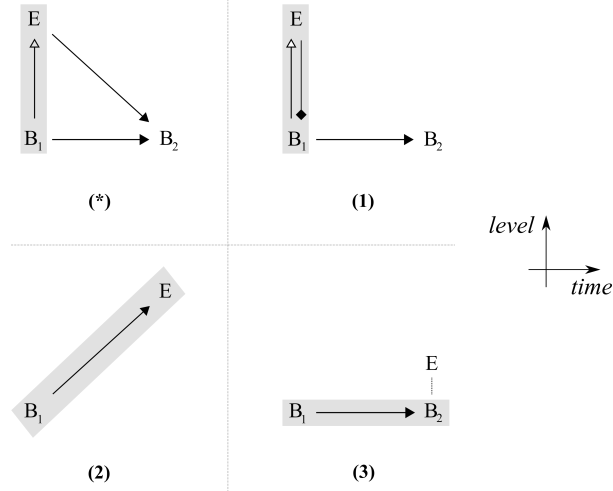


Figure 4: Some possible ways (1-3) to define emergence without facing exclusion worries. Diagram (\*) represents Kim (1999)’s emergence that is subject to exclusion, as it combines some synchronic upward determination with downward causation. Diagram (1) captures Gillett (2010)’s strong emergence, which avoids exclusion by conceiving of downward determination as non causal and synchronic. Diagram (2) is a representation of Humphreys (1997)’s fusion emergence, which avoids exclusion by replacing synchronic upward determination by a diachronic, inter-level fusion operation. Diagram (3) captures [TE], which avoids exclusion by combining diachrony and “level egalitarianism” (*i.e.*  $E$  can be identical to  $B_2$ , though it is of course not necessary).

traces that [TE] is supposed to leave in the formal constructs we use to investigate these natural systems. The two clues we are considering are the following:

- (C<sub>1</sub>)  $M_1$  and  $M_2$ , which both describe the same system  $S$  at two successive stages  $S_1$  and  $S_2$  of its evolution, are models of one and the same non-trivial theory  $T$ . And yet:
- (C<sub>2</sub>)  $M_2$  is not derivable from  $M_1$  as a matter of principle, for  $M_2$  contains features that are forbidden in  $M_1$  according to theory  $T$ . Accordingly,  $S_2$ ’s dynamics as described by  $M_2$  is not continuously deformable into  $S_1$ ’s dynamics as described by  $M_1$ .

In a nutshell, what this operational formulation of [TE] entails is that, although transformationally emergent states are the products of their bases,

and can accordingly be accounted for in the light of one and the same theoretical framework, the former are epistemologically broken off the latter, in the sense that there is an unbridgeable representational gap between them. Such a gap actually constitutes the best – and even also the only – evidence we could ever have to ground [TE] ascriptions.

This being said, one can make the case that there is [TE] at play in the FQHE. More particularly, we argue that the experimental production of the FQHE is an instance of a physical transformation between two states – a pre-transformation state  $S_1$  (4-dimensional fermions) and a post-transformation state  $S_2$  (3-dimensional particles) – to the effect that the clues ( $C_1$ ) and ( $C_2$ ) obtain, and consequently (DEP) and (NOV) hold.<sup>14</sup> It is not the place here to enter into the details of how to justify such a claim (for some details, see Guay & Sartenaer 2016). Suffice it to mention the key elements of such a justification, in the following way (see also figure 5):

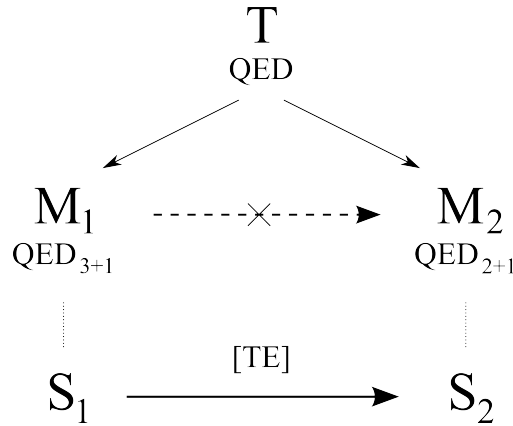


Figure 5: [TE] in the FQHE.

- (1) It can be shown that 4-dimensional quantum electrodynamics ( $\text{QED}_{3+1}$ ) and 3-dimensional quantum electrodynamics ( $\text{QED}_{2+1}$ ) are both models of one and the same non-trivial theory, namely quantum electrodynamics (QED).

<sup>14</sup>Note that we do not claim the the initial state is a 3-dimensional conductor in a classical regime. Contrary to the FQHE, there is no necessity to model the conductor before the effect as to be in four dimensions. In consequence, we prefer to be careful about the dimensionality of the model in the pre-transformation phase.



- (2)  $\text{QED}_{3+1}$  and  $\text{QED}_{2+1}$  are the adequate models for describing the universal properties of the dynamics of  $S_1$  and  $S_2$ , respectively.
- (3) The experimental production of the FQHE can be seen as a transformation of  $S$  going from a state  $S_1$  described by  $\text{QED}_{3+1}$  to a later state  $S_2$  described by  $\text{QED}_{2+1}$ .<sup>15</sup>

As such, (1), (2) and (3) entail that  $(C_1)$  obtains, to the effect that we have good reasons to believe that (DEP) holds. Already at this stage, the FQHE is very surprising, for it involves the production, in our 4-dimensional world, of a system that can be best described by  $\text{QED}_{2+1}$  (and, as we will see, that *cannot*, even in principle, be described by  $\text{QED}_{3+1}$ ).

- (4)  $\text{QED}_{2+1}$  contains features that are forbidden in  $\text{QED}_{3+1}$  according to QED. In particular, the respective Lagrangian densities are analogous (*modulo* a difference in dimensionality), with the notable exception that  $\mathcal{L}_{\text{QED}_{2+1}}$  contains an additional term – a so-called Chern-Simons term – that cannot exist in  $\mathcal{L}_{\text{QED}_{3+1}}$ . The presence of this term radically changes the accessible quantum states of matter.<sup>16</sup>
- (5) Accordingly,  $\text{QED}_{2+1}$  is non derivable from  $\text{QED}_{3+1}$ , though both are models of one and the same general theory, namely QED. Derivability should be understood here as the fact that it is not possible to continuously deform one model into the other, because there is no analog to the Chern-Simons term in four dimensions.

From (3) and (5), it can be seen that  $(C_2)$  obtains for the transformation involved in the FQHE.<sup>17</sup> Of course, what matters from a physical point of view, and what concomitantly justifies why we think thesis (NOV) also holds in this case, is the set of empirical consequences of the presence of the Chern-Simons term that captures the very distinguishing features of the dynamics of the Hall states. Some notable empirical facts, which the Chern-Simons term codes for, are that “photons” (more precisely, spin 1 excitation states of the gauge field) acquire a topological mass and that “charged particles”

<sup>15</sup>We are not claiming anything about the actual dimensionality of the physical system. This is a claim about how to best describe the dynamics.

<sup>16</sup>The main new aspect is not the presence of parastatistics, since fractional statistics could in principle exist in a 4-dimensional world. The real novelty is the access to new topological quantum states (Wen 2004, chap. 8).

<sup>17</sup>In Laughlin’s own phrasing: “The fractional quantum Hall state is *not* adiabatically deformable to any noninteracting electron state” (Laughlin 1999, 869; emphasis in the original).

are endowed with magnetic fluxes and have fractional statistics. Such facts are *forbidden* to happen in a 4-dimensional world described by  $\text{QED}_{3+1}$ , and yet they are produced in a continuous way from a 4-dimensional system adequately described by  $\text{QED}_{3+1}$ . This modal claim captures the very essence of what is *really* surprising about the FQHE, namely, to borrow the words of Laughlin himself, that the effect “was unanticipated by any theory and not analogous to anything previously known in nature [...] [The discoverers of the effect found something] which should have been impossible” (Laughlin 2005, 77).

## 5 Transformationally emergent anyons?

Transformational emergence implies that the transformationally emergent state is impossible according to the very laws that govern the pre-transformation state. In a certain manner, all entities in such an emergent state are themselves emergent since they participate in an emergent state. This trivial fact is not what we are aiming at in this section. The question we will answer is rather do we encounter a new kind of individual in the context of the FQHE?

As early as 1984, the presence of anyons has been invoked to explain the surprising empirical behavior of the FQHE (Arovas et al. 1984). Anyons are 3-dimensional “particles” that exhibit very unusual statistics, interpolating between fermions and bosons (Wilczek 1982). They apparently can explain the FQHE. They are very probably no anyonic solutions in  $\text{QED}_{3+1}$  (Guay & Sartenaer 2016). In consequence, they seem to be good candidates as emergent entities.

### 5.1 The problem with quantum individuals in general

Quantum physics is generally considered hostile to the notion of the individual. In fact, since Schrödinger the doxa is that if there are particles, they cannot be individuals. Since the defense of this claim, almost every position has been argued, from the fact that the metaphysics is undetermined by physics (French & Krause 2006) to the case that individuals are an emergent phenomenon in nonrelativistic quantum mechanics (Saunders 2016). In all cases, the notion of an individual used in classical physics has to be deeply revised. In this paper, we will put aside this important debate and focus on the possibility of emergent particles (individuals or not). However, this simplification is not a panacea. The concept of “particle” is not always well defined in quantum physics. Moreover, we have good reasons to believe

that there is no such a thing as a coherent particle concept applicable to all quantum models (Ruetsche 2011). At best, only a local concept could be defined. This said, in this section we will try to answer the following question: are emergent particles (understood loosely as relatively stable field quanta) involved in the FQHE?

## 5.2 Some relevant theoretical results

In this subsection, we will expose some theoretical results that are relevant to the discussion of the status of anyons. These results, we believe, strongly limit the number of ontological options available.

Do we have good reasons to believe that anyons are field quanta that are sufficiently stable to be understood as particles? It seems unlikely that we could obtain a simple relativistic quantum field theory where fundamental field quanta are themselves anyons (Khare 1997). However, this does not exclude that anyons are quanta of another kind. For example, Fröhlich and Marchetti (1989) proposed a model where the FQHE could be represented by a relativistic quantum field theory, with a Chern-Simons term, where topological solitons (vortices) are quantized. According to them, such solitons (anyons) should be considered as particles, since they behave as such. It seems that anyons are good candidates for being considered as emergent particles since they appear in the emergent context of the FQHE.

As already mentioned in a previous work (Guay & Sartenaer 2016), in the context of the FQHE, the dependence between electrons and anyons could go both ways. As discussed by Zee (2010, 326-27), it is possible to conceive anyons as a collective behavior of electrons. Nevertheless, it is also possible to model electrons as bond objects composed of anyons. This may be surprising but not unexpected in a quantum field theory where particles and quasiparticles are only types of field configurations. This result is a very strong challenge to the position understanding anyons as being at a higher ontological level than electrons, because, in this metaphysical context, the dependence relation – e.g. supervenience or realization – does not allow for such a symmetry between putative higher entities (anyons) and lower ones (electrons). For that reason, we will claim that anyons and electrons are at the same ontological level. They still can ontologically differ in a significant way.

Mund has demonstrated (1998) that, under mild assumptions, no free model of relativistic anyons is possible. What Mund means by “free” is that the basic fields create one-particle state out of vacuum. It does not exclude the other possible meaning of “free”, namely that the anyons’ S-matrix leads

to a trivial cross-section. Apparently, according to quantum field models, electrons and anyons are different kinds of beasts. The autonomy of electrons is stronger since they can in principle exist alone. Anyons do not. From this we should conclude that even if they occupy the same ontological level, anyons are not on a par with electrons. Anyons are dependent entities, in the sense that, to be produced, appropriate conditions must be fulfilled. Anyons are therefore context-dependent entities, hence they are *quasiparticles*. It is important to notice that this conclusion relies on the fact that free electrons and the electrons involved in the FQHE are essentially the same. This is not obvious. If the topology change is more than a mere idealization, then the electrons after the transformation are to a great extent different than the ones before. In consequence, the electrons interacting with anyons could also be context-dependent and therefore quasiparticles. Even if you do not take seriously the topological change of the dynamics, you should worry about the ontological continuity between electrons before and after the transformation. According to Haag’s theorem (Earman & Fraser 2006), the interacting field theory is unitarily inequivalent to a free theory.<sup>18</sup> If our conception of an electron as a particle comes basically from a theory without interaction, electrons in interaction in the FQHE cannot be the same (Ruetsche 2011). Moreover, electrons in  $\text{QED}_{3+1}$  cannot be the same as free electrons. The simple possibility of interaction changes the ontology. Apparently, all particles seem, in a certain way, context-dependent.<sup>19</sup>

### 5.3 Anyons = emergent particles?

Let us begin this last subsection with two recapitulative remarks:

- Are anyons collective behaviors of electrons? This position, quite natural if one works in a nonrelativistic quantum model of the FQHE, is not convincing in quantum field theory. Even if anyons, which are solitons, are an apparently different kind of quanta than electrons, there is no reason to believe that the former are subordinated to the latter. They are both quantas of the fundamental fields. They are both equal candidates to be particles.<sup>20</sup>

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<sup>18</sup>To avoid this concern, we could limit ourselves to non-relativistic models, since they fall outside the theorem’s domain of application. Indeed, the FQHE could be represented by an effective non-relativistic quantum field model. Nevertheless, in this paper we want to aim at the general case and anticipate a broader set of objections.

<sup>19</sup>For a similar idea, see for instance Falkenburg 2015; or Wallace 2012. Wallace actually also believes that electrons, like other particles, are themselves to be considered emergent (see, in particular, Wallace 2009).

<sup>20</sup>More on this question is available in Manton & Sutcliffe (2004).

- Do anyons supervene on electrons? As mentioned above, Zee provided a strong reason for believing that, at least in the context of the FQHE, electrons and anyons are on the same ontological level.

These remarks exclude the possibility that anyons would synchronically emerge from an electron basis after the transformation occurred. Anyons are unmediated products of the transformation. However, this is not the end of the story. The status of anyons is strongly dependent on the interpretation of the topological change between the representations of the states before and after the transformation. The dynamics of the system are best described by a 3-dimensional quantum field model but, from this fact, should we infer that 3-dimensional “particles”, namely anyons, must be involved?

The first possibility is that there is some kind of topological change involved in the FQHE. In this case, since anyons are well-defined quanta of  $\text{QED}_{2+1}$ , they are emergent quasiparticles. However they are not the only ones. The 3-dimensional analog of the electron, a fermion, is possibly an emergent quasiparticle. This conclusion could push us to entertain the hypothesis that 4-dimensional electrons are also contextual particles depending on the dynamics’ topology.

The second possibility is that the topological change in the representations is an idealization. In this context, because they could well be the result of an artefact, we have to be careful about any ontological claim. To guide us we will adopt a principle attributed to John Earman and Laura Ruetsche and reformulated by Shech (2015, 1065):

Earman–Ruetsche sound principle—If a scientific account (theory, model) uses an idealization to predict an effect which disappears when the idealization is removed then either the effect is an artifact of the idealization or else (if experiment confirms the effect) the theory is inadequate.

The existence of the FQHE is a proof that a model of  $\text{QED}_{2+1}$ , with Chern-Simons terms, can be successfully applied. But what is the status of this so-called idealization? We agree with Shech that the topological change understood as an idealization is problematic. Any mathematical series of models starting from  $\text{QED}_{3+1}$  and going towards  $\text{QED}_{2+1}$  is inhospitable to the Chern-Simons terms. In consequence, anyonic solutions seem to exist only in  $2+1$ . Since we claimed that this kind of transformation is emergent in a modal sense, this result was expected. To recover anyons, Shech’s proposition is to describe the system using a geometrical approach, in the spirit of Arovas et al. (1984), rather than a topological one. This idealization seems, at least in the context of nonrelativistic quantum mechanics,

to recover anyons appropriately. We do not know how to develop a similar strategy in the context of a quantum field theory, but the recovery of anyons in nonrelativistic quantum mechanics suggests that the pathological idealization can be avoided.

Do we have a reason to choose among these possibilities? Does the FQHE involve a genuine topological change of some sort? We do not have a definite answer to this question. However emergence seems to imply that something quite radical must happen between the states before and after the transformation. A topological change of the dynamics would be such a change.

## 6 Conclusion

We are now in a position to capture Laughlin’s intuition about emergent quasiparticles in a metaphysically more sober way than what has been suggested by Gillett. By making a case that anyons can be considered transformationally emergent on 4-dimensional electrons (represented by  $\text{QED}_{3+1}$ ) – and not, as in Gillett’s case, strongly emergent on an underlying, simultaneous basis of 3-dimensional “electrons” –, we recover Laughlin’s insights as they have been exposed in section 2.

Within the transformational framework, anyons are indeed fundamental entities of a new physical domain, governed by new laws that do not exist, and that cannot exist, outside of this domain. Hence anyons are fundamental particles in their own right, provided that there is a sense in which electrons are themselves particles in their own rights. Put differently, anyons and electrons are ontologically on a par. Emergence is seen here as some kind of mechanism that can produce (ontologically) new particles out of “old” ones.

Of course, through [TE], Laughlin’s idea that what emerges has to be “higher-level” or “composite” is lost, though it can be kept as a parasitic feature (in the sense that it can be entertained, but has no role to play in emergence ascriptions). As a result, thinking about (ontologically) emergent anyons doesn’t necessarily require one to be committed to the existence of (at least) two facts that are problematic when it comes to (micro)physics, namely “levels of mechanism” and downward determination (under the form of *machresis* or anything else).

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