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Bargaining in endogenous trading networks

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HIGHLIGHTS

- We study the formation of trading networks when link costs are infinitesimally small.
- Sellers and buyers are randomly selected to bargain through a chain of intermediaries.
- We determine both the trading path and the allocation of the surplus at equilibrium.
- With patient and impatient agents, core–periphery networks with all impatient agents in the core are stable.
- Once there is private information, core–periphery networks may not be stable.

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ABSTRACT

We study a model in which heterogeneous agents first form a trading network where linking costs are positive but infinitesimally small. Then, a seller and a buyer are randomly selected among the agents to bargain through a chain of intermediaries. We determine both the trading path and the allocation of the surplus among the seller, the buyer and the intermediaries at equilibrium. We show that, under the initiator bargaining protocol, a trading network is pairwise stable if it is a core–periphery network where the core consists of all impatient agents who are linked to each other and the periphery consists of all patient agents who have a single link towards an impatient agent. Once agents do not know the impatience of other agents, each bilateral bargaining session may involve delay. Then, core–periphery networks may not be pairwise stable because agents may prefer to add links for reducing the length of trading paths and so avoiding costly delays in reaching a global agreement.

1. Introduction

We are interested in markets where trades between a buyer and a seller can occur through intermediaries and where each agent can be some day the buyer, some other day the seller, and the day after acting as an intermediary.[1] In such cases it is natural to model the market using a network where only pairs of connected agents may engage in trade. Which trading networks are likely to emerge when agents can be either patient or impatient and the division of the surplus between the seller, the buyer and the intermediaries is determined through a series of bilateral bargaining sessions?

We develop a model where agents having different discount rates first form a trading network with link costs being infinitesimally small. Second, a seller and a buyer are randomly selected among the agents. The seller owns an indivisible good and the buyer has a valuation normalized to one for the good. The buyer can obtain the good from the seller if and only if they are connected to each other. Agents on a given path between the seller and the buyer can act as intermediaries if trade occurs along this path. Third, the trading path and the allocation of the surplus among the seller, the buyer and the intermediaries are determined following the so-called *initiator procedure*. The buyer first chooses one of her predecessors, say the first intermediary, on a path from the seller to the buyer to negotiate bilaterally a partial agreement. Each bilateral negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model. Once a partial agreement is reached, the buyer exits the game and the first intermediary chooses one of her predecessors, say the second intermediary, on a path from the seller to the first intermediary. Once a partial agreement is reached between the first intermediary and the second intermediary, the

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1 See Goyal (2007), Jackson (2008), Easley and Kleinberg (2010) for a comprehensive introduction to the theory of social and economic networks.

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first intermediary exits the game; and so on until a partial agreement is reached between the last intermediary and the seller. Each agent receives her share of the surplus once all partial agreements have been reached.

In each bilateral session the agent who can exit the bargaining is the one who selects her partner to negotiate her share to exit with a partial agreement and makes the first proposal. One could object that this bargaining protocol is extreme giving much more power to the agent who can exit. In addition, the buyer and the intermediaries close to her get a share of the surplus substantially larger than the one obtained by the seller and the intermediaries close to him. However, Suh and Wen (2009) have shown that the initiator procedure can emerge endogenously from a multi-agent bilateral bargaining model where, in each period of a bilateral bargaining session, the proposer can choose between demanding a share to exit the bargaining or offering the responder a share to exit the bargaining. Moreover, each agent decides the links she wants to form with other agents, and she has ex-ante an equal likelihood of being the seller or the buyer.

Suppose that the population of agents is partitioned in two types of agents: patient agents and impatient agents. Our main result is that a trading network is pairwise stable if it is a core–periphery network where the core consists of all impatient agents who are linked to each other and the periphery consists of all patient agents who have a single link towards an impatient agent. Intuitively, agents have first incentives to create and share some surplus, and so any pairwise stable trading network consists of only one component connecting all agents. Agents have also incentives to occupy a position in the trading network that enables them to extract more rents from intermediation. In addition, agents have incentives to negotiate a partial agreement with an impatient agent to exit with a larger share of the surplus.[2] Hence, in any pairwise stable trading network each agent is linked to an impatient agent. This result is not immediate. Indeed, when an agent links to an impatient agent she loses from trades (for instance, when she is the seller) that before were going more straightforwardly to her and now go through a longer chain of intermediaries including the impatient player she is linked to. However, those losses are compensated by the gains she makes when she is the buyer.

Each impatient agent will also try to circumvent intermediaries to obtain more of the surplus for her. It follows that in any pairwise stable trading network all impatient agents are linked to each other. Each patient agent will then destroy links to other patient agents because those links are never used or are harmful (more intermediaries lie on the trading path when she is the seller). Finally, in any pairwise stable trading network, each patient agent is linked to exactly one impatient agent to avoid sharing the surplus with more intermediaries when she is the seller. Thus, core–periphery networks are the unique pairwise stable trading networks.[3]

Our paper introduces heterogeneous agents in Goyal and Vega-Redondo (2007) model of trading networks. In Goyal and Vega-Redondo (2007), agents are homogeneous and the surplus is shared equally among the buyer, the seller and the essential intermediaries. An intermediary is essential if she lies on all paths between the seller and the buyer. This way of dividing the surplus implicitly assumes that bargaining is multilateral rather than consisting of a series of bilateral bargaining sessions. In addition, some intermediaries will get no surplus because they are not essential even though they may become essential once trade and exchange reach some intermediary on a path between the seller and the buyer. They find that, if the formation of links is costly, a star network where a single agent acts as an intermediary for all transactions and enjoys significantly higher payoffs is the unique non-empty equilibrium architecture. We go further their analysis by considering heterogeneous agents and allowing them to hold private information about their bargaining strength. In addition, we endogenize the trading path and we show that, with two types of agents (patient and impatient), a core–periphery architecture can emerge even when link formation is infinitesimally costly. In our model, the bargaining with the initiator procedure consists of a finite series of bilateral negotiations along the trading path and the identity of the player who can exit in each bilateral sessions is determined by the trading path, two features which are more realistic for the markets we are interested in (e.g. real estate, antiques, drugs, over-the-counter, etc.).

We also explore the limits of our main result and we find that relaxing the main conditions (two types of agents/complete information/initiator procedure) can destabilize the core–periphery networks. For instance, once agents become homogeneous, there is a unique pairwise stable architecture, namely the complete network. In addition, once agents do not know the discount rate of the other agents, each bilateral bargaining session may involve delay, but not perpetual disagreement, in equilibrium. We find that the maximum delay time in reaching an agreement can be substantial and is increasing with the amount of private information. Hence, when an agent chooses another agent on a path from the seller to the buyer to negotiate bilaterally a partial agreement, her choice now depends both on the type of this other agent and on how much time the preceding agents will need to reach their partial agreements. Therefore, core–periphery networks are likely to be pairwise stable only if impatient agents are quite less patient than patient agents. Otherwise, agents may prefer to add links for reducing the length of trading paths and so avoiding costly delays in reaching a global agreement.

Most of the literature on decentralized trade in networks has focused on the exchange of goods in networks with no intermediation. See, among others, Abreu and Manea (2012), Calvo-Armengol (2003), Condorelli and Galeotti (2012), Condorelli et al. (2015), Corominas-Bosch (2004), Elliott (2015), Kranton and Minehart (2001), Manea (2011), Mauleon et al. (2011), Polanski (2007) and Wang and Watts (2006). There are a number of papers where intermediation is present. Blume et al. (2009) have analyzed a complete information model where buyers and sellers are connected through intermediaries who strategically choose bid and ask prices to offer to the sellers and buyers they are connected to, and where the exogenously given network structure determines the amount of competition among intermediaries.[4] Siedlarek (2015) has studied a stochastic model of bargaining and exchange with common discount factor and intermediation on an exogenously given network. In Siedlarek (2015), bargaining is multilateral instead of having a series of bilateral negotiations

2 Xiao (2015) has shown that it is optimal for a real estate developer to bargain first with requisite landowners having smaller opportunity cost for their land. In Manea (2015), the manufacturer prefers dealing with the costliest suppliers in the last stages.

3 The labels of buyers and sellers can be reversed without consequence for our main results. In fact, agents are traders who can exchange goods. This exchange creates a surplus of 1 and it can be carried out only if both traders know each other personally (i.e. they are linked to each other) or there is a sequence of personal connections (i.e. there is a path which indirectly links the two traders). If both traders have equal probability of initiating the negotiation, core–periphery networks are the unique pairwise stable trading networks.

4 Gale and Kariv (2009) have done an experimental study of trading networks where each trader can only exchange assets with a limited number of other traders and intermediation is used to transfer the assets between initial and final owners. Gofman (2014) has studied a reduced-form model of bargaining in over-the-counter markets where intermediaries receive an exogenous share of the surplus.

along the trade route. It is close as if in each period a coalition of agents is drawn and has to divide some surplus and if they do not reach an agreement, then a new coalition is drawn to bargain over the division of some surplus, and so forth until an agreement is reached. We rather adopt a finite series of bilateral negotiations along the trade route and we make endogenous the trade route, two features which are more realistic for the markets we are interested in. Babus and Hu (2015) have studied over-the-counter markets where bargaining along a given trading path consists of a finite series of bilateral bargaining sessions with a common discount factor.[5] If links are costly and agents are forward-looking, then a star network that connects all agents is an absorbing state of a dynamic network formation process. Recently, Manea (2015) has investigated how competing paths of intermediation determine the terms of trade between buyers and sellers in exogenously given networks. He has found that trade does not always proceed along the shortest path between the seller and buyers, and that the sharing of the surplus depends on the complete collection of competing path available to each intermediary.

The paper is organized as follows. In Section 2 we consider the series of bilateral bargaining sessions with complete information and we determine both the equilibrium trading path and the equilibrium shares of the surplus to be divided. In Section 3 we characterize the pairwise stable trading networks. In Section 4 we discuss the limits of our main result by relaxing some main conditions.

2. Multi-agent bilateral bargaining in networks

Let $N = \{1, 2, \ldots, n\}$ denote the set of players. A network g is a list of which pairs of players are linked to each other and $ij \in g$ indicates that i and j are linked under g. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \exists j \text{ such that } ij \in g\}$ be the set of players who have at least one link in the network g. A path in a network g between i and j is a sequence of players $i_1, i_2, \ldots, i_{K-1}, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$, and such that each player in the sequence $i_1, \ldots, i_K$ is distinct. We say that player i is connected in g to j if there is a path between i and j in g. A subnetwork $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$.[6] We denote by $C(g)$ the set of components of g.

Players participate in the market and can be active either as a seller or as a buyer or as an intermediary. A pair of players is randomly selected. The probability that the pair (s, b) is selected, where s is the seller and b is the buyer, is $1/\left(n(n-1)\right)$. The seller owns an indivisible good and the buyer has a valuation $v = 1$ for the good. The buyer can obtain the good from the seller if and only if the two are connected. In other words, the buyer and the seller can trade the good if and only if they belong to the same component. Players on a path between the seller and the buyer can act as intermediaries if trade occurs along the path. When there is no path between the randomly selected pair, no surplus will be realized and both players receive 0. One central question is how the surplus is shared among the buyer, the seller and the intermediaries when trade is feasible.

Suppose that (s, b) is a pair randomly matched with s being the seller and b being the buyer and s and b are connected in the trading network g. Since for a given network g there may exist more than one path connecting s and b, we need to determine which sequence $(i_1, i_2, \ldots, i_k)$ of intermediaries between s and b is going to emerge at equilibrium as well as how the surplus is shared.

We assume that the negotiation starts with the buyer b who first chooses one of her predecessors, say intermediary i_k, on a path from s to b to negotiate bilaterally a partial agreement.[7] In the bilateral bargaining session (i_k, b), the negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model where players make alternate offers, with i_k making offers in even-numbered periods and b making offers in odd-numbered periods. The length of each period is Δ. The negotiation starts in period 0 and ends when one of the players accepts an offer and leads to a partial agreement. No limit is placed on the time that may be expended in bargaining and perpetual disagreement is a possible outcome. In case of perpetual disagreement, all players get 0.

Under the initiator procedure, only the player who selects her partner to negotiate and makes the first proposal in the bilateral bargaining session can exit. Hence, a partial agreement specifies the share of the surplus, $0 \le x_b \le 1$, for b to exit the game. Players have time preferences with constant discount rates, $r_i > 0$. Once a partial agreement is reached, b exits the game and i_k chooses one of her predecessors, say intermediary i_{k-1}, on a path from s to i_k such that b does not lie on the path. In the bilateral bargaining session (i_{k-1}, i_k), the negotiation proceeds as in Rubinstein's alternating-offer bargaining model to specify the share of the surplus, $0 \le x_{i_k} \le 1$, for i_k to exit the game. Once a partial agreement is reached between i_k and i_{k-1}, i_k exits the game. Then, i_{k-1} chooses one of her predecessors, say intermediary i_{k-2}, on a path from s to i_{k-1} such that i_k and b do not lie on the path. Once a partial agreement is reached between i_{k-1} and i_{k-2}, i_{k-1} exits the game; and so on until a partial agreement is reached between i_1 and s.

An outcome consists of a sequence $(i_1, i_2, \ldots, i_k)$ of intermediaries between s and b and $(k + 1)$ partial agreements that specify player i's share of the surplus, $0 \le x_i \le 1$, for $i \in \{s, i_1, i_2, \ldots, i_k, b\}$, such that $x_s + x_{i_1} + \cdots x_{i_k} + x_b = 1$. Each player only receives her share once all $(k + 1)$ partial agreements have been reached. Under complete information it does not matter in our model whether a player can exit and obtains her share immediately or only at the end of the process. Players anticipate that an agreement will be reached immediately in all subsequent bilateral negotiations. Proposition 1 provides the unique subgame perfect equilibrium (SPE) outcome under the initiator procedure (see Appendix A for details).

Proposition 1. *As the interval between offers and counteroffers shortens and shrinks to zero, there is a unique limiting subgame perfect equilibrium outcome under the initiator procedure given by*

$$x_b^* = \frac{r_{i_k}}{r_{i_k} + r_b},$$

$$x_{i_k}^* = \frac{r_{i_{k-1}}}{r_{i_{k-1}} + r_{i_k}} \left(1 - \frac{r_{i_k}}{r_{i_k} + r_b}\right),$$

[5] Recent empirical evidence suggests that financial networks exhibit a core–periphery network structure (see e.g. Craig and von Peter, 2014). in't Veld et al. (2014) have found that for sufficiently large differences between large and small banks, a core–periphery network with large banks in the core becomes stable. Our main result seems consistent with such findings. Financial institutions resell assets over the counter and big traders often occupy a central role in the network but have less time to devote to each financial transaction. Core–periphery networks can also emerge in models of communication network formation (see Hojman and Szeidl, 2008).

[6] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, # will refer to the notion of cardinality.

[7] In our model, each player has the same probability of being the seller or the buyer. Hence, all the results we obtain are robust to the alternative order where first the seller would negotiate with an intermediary i_1.

$$x^*_{i_{k-1}} = \frac{r_{i_{k-2}}}{r_{i_{k-2}} + r_{i_{k-1}}}\left(1 - \frac{r_{i_{k-1}}}{r_{i_{k-1}} + r_{i_k}}\right)\left(1 - \frac{r_{i_k}}{r_{i_k} + r_b}\right), \quad (1)$$

$$\vdots$$

$$\vdots$$

$$x^*_{i_1} = \frac{r_s}{r_s + r_{i_1}}\left(1 - \frac{r_{i_1}}{r_{i_1} + r_{i_2}}\right)\left(1 - \frac{r_{i_2}}{r_{i_2} + r_{i_3}}\right)\cdots \left(1 - \frac{r_{i_{k-1}}}{r_{i_{k-1}} + r_{i_k}}\right)\left(1 - \frac{r_{i_k}}{r_{i_k} + r_b}\right),$$

$$x^*_s = \left(1 - \frac{r_s}{r_s + r_{i_1}}\right)\left(1 - \frac{r_{i_1}}{r_{i_1} + r_{i_2}}\right)\left(1 - \frac{r_{i_2}}{r_{i_2} + r_{i_3}}\right)\cdots \left(1 - \frac{r_{i_{k-1}}}{r_{i_{k-1}} + r_{i_k}}\right)\left(1 - \frac{r_{i_k}}{r_{i_k} + r_b}\right),$$

where *s is the seller, b is the buyer and $(i_1, i_2, \ldots, i_k)$ is the equilibrium sequence of intermediaries that facilitate the transaction in this order.*[8] *All $(k+1)$ partial agreements are reached immediately so that delay cannot occur in equilibrium.*

Notice that once a partial agreement is reached between i_{k-l} and i_{k-l-1}, i_{k-l} exits the game and i_{k-l-1} chooses one of her predecessors, say intermediary i_{k-l-2}, on a path from s to i_{k-l-1} such that $i_{k-l}, i_{k-l+1}, i_{k-l+2}, \ldots, i_k$ and b do not lie on the path. Then, i_{k-l-1} and i_{k-l-2} negotiate bilaterally a partial agreement for i_{k-l-1} given that $(1 - x_b - x_k - \cdots - x_{k-l})$ is the surplus left to be shared after intermediaries $i_{k-l}, \ldots, i_{k-1}, i_k$ and buyer b have taken their shares. The bilateral negotiation leads to a share $x_{i_{k-l-1}} = (1 - x_b - x_k - \cdots - x_{k-l}) r_{i_{k-l-2}} (r_{i_{k-l-2}} + r_{i_{k-l-1}})^{-1}$ for i_{k-l-1}, and this share only depends on i_{k-l-1}'s own discount rate, the discount rate of her predecessor i_{k-l-2}, and the discount rates of the players who have already exited the game with a partial agreement. Therefore, when i_{k-l-1} chooses her predecessor on a path from the seller to her for a bilateral negotiation, she chooses her most impatient predecessor (i.e. the one with the highest discount rate). In case a player is indifferent between two or more predecessors, she chooses to negotiate with the predecessor leading to the shortest path between the seller and herself because of a positive but infinitesimally small probability that a link fails to deliver the good once an agreement is reached.[9]

Lemma 1. *The path $(s, i_1, i_2, \ldots, i_k, b)$ in g is an equilibrium trading path under the initiator procedure if and only if*

(i) *for each player $j \neq s$ in $(s, i_1, i_2, \ldots, i_k, b)$ the discount rate of her predecessor in $(s, i_1, i_2, \ldots, i_k, b)$ is greater than or equal to the discount rate of her predecessor in any other paths in g between s and j such that her successors in $(s, i_1, i_2, \ldots, i_k, b)$ do not lie on those paths, and*
(ii) *there is no strictly shorter path in g connecting s and b than $(s, i_1, i_2, \ldots, i_k, b)$ that satisfies* (i).

The trading network depicted in Fig. 1 illustrates the lemma. Suppose that $r_4 > r_5$ and $r_3 > r_1$. After having negotiated with player 7 the share of 7 to exit the bargaining, player 6 can choose between players 4 and 5 to negotiate her share to exit. From players 4 and 5 there is some path leading to the seller and neither 6 nor 7 lie on the path. Player 6 chooses 4 instead of 5 because 4 is less patient than 5. Similarly, player 2 chooses 3 instead of 1 because 3 is less patient than 1. Finally, player 3 can only negotiate with the seller (player 1) and $(1, 3, 2, 4, 6, 7)$ is the unique equilibrium trading path. The driving force for obtaining such equilibrium trading path is the initiator procedure where in each bilateral session the player who can exit the bargaining is the one who is selecting her partner to negotiate her share to exit.

Suppose now that $r_4 = r_5$ and $r_3 > r_1 = r_2$. Then, $(1, 3, 5, 6, 7)$ is the unique equilibrium trading path. Player 6 is indifferent between players 4 and 5. Player 6 chooses 5 as her predecessor, because of the shortest path assumption for breaking ties, anticipating perfectly that player 3 will choose to negotiate with player 1. The trading path $(1, 3, 5, 6, 7)$ involves five players. If player 6 had chosen 4 as her predecessor then the trading path would have been $(1, 3, 2, 4, 6, 7)$ and would have involved six players.

3. Pairwise stable trading networks

Players form a trading network before knowing which pair of players will be randomly selected to become the seller and the buyer. Let $u_i(g, (s, b))$ be player i's SPE payoff (or share) in the trading network g with player s being the seller and player b being the buyer, and let $U_i(g)$ be player i's SPE expected payoff in the trading network g before knowing which pair of players will be randomly selected. For instance, suppose that the trading network is a star network $\{12, 13\}$ where player 1 is the center. Player 1's expected payoff will be equal to

$$\begin{aligned} U_1(\{12, 13\}) = {} & \frac{1}{6}u_1(\{12, 13\}, (2, 1)) + \frac{1}{6}u_1(\{12, 13\}, (3, 1)) \\ & + \frac{1}{6}u_1(\{12, 13\}, (1, 2)) \\ & + \frac{1}{6}u_1(\{12, 13\}, (1, 3)) + \frac{1}{6}u_1(\{12, 13\}, (2, 3)) \\ & + \frac{1}{6}u_1(\{12, 13\}, (3, 2)). \end{aligned}$$

That is,

$$\begin{aligned} U_1(\{12, 13\}) = {} & \frac{1}{6}\frac{r_2}{r_2 + r_1} + \frac{1}{6}\frac{r_3}{r_3 + r_1} + \frac{1}{6}\left(1 - \frac{r_1}{r_1 + r_2}\right) \\ & + \frac{1}{6}\left(1 - \frac{r_1}{r_1 + r_3}\right) + \frac{1}{6}\frac{r_2}{r_2 + r_1}\left(1 - \frac{r_1}{r_1 + r_3}\right) \\ & + \frac{1}{6}\frac{r_3}{r_3 + r_1}\left(1 - \frac{r_1}{r_1 + r_2}\right). \end{aligned}$$

As our interest is in understanding which networks are likely to arise in trading networks when bargaining is with complete information and players are heterogeneous, we need to define a notion which captures the stability of a network. We suppose that there are positive but infinitesimally small costs to forming links. Hence, the cost of any link is always less important than any effect it may have on the expected payoff from the bargaining, and so, as in Goyal and Joshi (2003) or Polanski and Vega-Redondo (2014) we use a strict version of Jackson and Wolinsky's (1996) notion of pairwise stability. A network is pairwise stable if no player does not lose from severing one of her links and no other two players strictly benefit from adding a link between them.[10]

8 The subgame perfect equilibrium outcome under the initiator procedure converges to $((1/2)^{k+1}, (1/2)^{k+1}, \ldots, 1/8, 1/4, 1/2)$ for $(s, i_1, \ldots, i_k, b)$ when players have the same discount rate and the equilibrium path involves k intermediaries.

9 In case there are more than one predecessor leading to the shortest path between the seller and herself, then she chooses them with equal probability.

10 Players are not farsighted in the sense that they do not forecast how others might react to their actions. Dutta et al. (2005), Herings et al. (2009) and Page and Wooders (2009) have recently developed notions to predict which networks are likely to be formed among farsighted players.

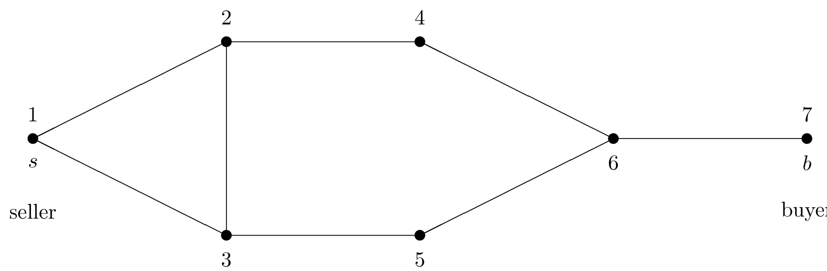

Fig. 1. Equilibrium trading paths.

Definition 1. A network g is pairwise stable if

(i) for all $ij \in g$, $U_i(g) > U_i(g - ij)$ and $U_j(g) > U_j(g - ij)$, and
(ii) for all $ij \notin g$, if $U_i(g) < U_i(g + ij)$ then $U_j(g) \geq U_j(g + ij)$.

Part (i) reflects that if any player cuts one of her links, she incurs a loss from bargaining that could not be compensated by the saving of the infinitesimally small linking cost. Part (ii) reflects that no pair of players benefit from creating a link between them. If one of them would like to add the link, then the other one would incur a loss. Even if her expected payoff from the bargaining would not change, her linking cost would infinitesimally increase.[11]

Our first result is that, in the presence of infinitesimally small costs for forming links, any pairwise stable trading network will consist of only one component connecting all players in N. Indeed, linking two players belonging to two different components (or being isolated) would not affect the outcome of any previously feasible trade but would allow for new possible trades.

Lemma 2. *A network g such that $\#C(g) > 1$ or $N(g) \subsetneq N$ is never pairwise stable.*

Suppose now that we have two types of players: impatient players and patient players. Let $I = \{1, 2, \dots, m\}$ be the set of impatient players and r_I be the discount rate of impatient players. Let $P = \{m + 1, m + 2, \dots, n\}$ be the set of patient players and r_P be the discount rate of patient players. Obviously, $r_I > r_P$. Let g^T be the collection of all subsets of $T \subseteq N$ with cardinality 2. Then, g^I is the complete network among the impatient players. The degree of i is the number of players that i is linked to. That is, $d_i(g) = \#\{j \mid ij \in g\}$. Which trading networks are pairwise stable?

Our main result is that pairwise stable networks are core–periphery networks where the core only consists of impatient players who are linked to each other and the periphery only consists of patient players who are linked to one impatient player. Fig. 2 illustrates a core–periphery network where $I = \{1, 2, 3\}$ and $P = \{4, 5, \dots, 11\}$.

Proposition 2. *Suppose that $r_i = r_I > 0$ for $i \in I = \{1, \dots, m\}$ and $r_j = r_P$ for $j \in P = \{m + 1, \dots, n\}$ and $r_P < r_I$. Under the initiator procedure, a network g is pairwise stable if and only if*

(i) $g^I \subseteq g$;
(ii) $d_i(g) = 1$ *for all* $i \in P$;
(iii) $\#C(g) = 1$.

Proposition 2 tells us that, under the initiator procedure, a trading network g is pairwise stable if and only if (i) all impatient players are linked to each other, (ii) each patient player has exactly one link (and so, each patient player is only linked to one impatient player), and (iii) g consists of only one component connecting all players in N. Part (iii) follows from Lemma 2. The proof of part (i) and part (ii) of Proposition 2 proceeds in five steps. First, we show that networks that can be pairwise stable are such that each patient player is linked to at least one impatient player.

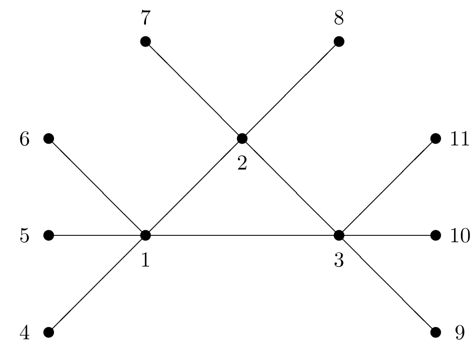

Fig. 2. A core–periphery network.

Lemma 3. *Suppose that $r_i = r_I > 0$ for $i \in I = \{1, \dots, m\}$ and $r_j = r_P$ for $j \in P = \{m + 1, \dots, n\}$ and $r_P < r_I$. Under the initiator procedure, a network g cannot be pairwise stable if there is some patient player that is not linked to at least one impatient player.*

All the proofs not in the main text can be found in Appendix B. Suppose that g consists of one component connecting all players and that there is some patient player $i \in P$ that is not linked to at least one impatient player $j \in I$ in g. The patient player i has incentives to add the link ij because, as a buyer or intermediary or seller, she will get a larger share of the surplus when bargaining with the impatient player j rather than having to bargain with another patient player. Moreover, she will endorse more often the role of intermediary in $g + ij$. Precisely, in $g + ij$ player i is winning when she is matched as a buyer to the impatient player j or to one of the patient players on the geodesic between i and j or to any other player l such that player j lies on a path between i and l in g.[12] Indeed, player i will choose to negotiate with the impatient player j to obtain a larger share than the one she would get when bargaining with patient players. Player j is indifferent between g and $g + ij$ when he is the buyer. In $g + ij$ player i is winning when she is matched as a seller to a player l such that player j was lying on the trading path in g since the trading path in $g + ij$ will be shorter and j will end the sequence of bilateral bargaining sessions negotiating with i. In addition, in $g + ij$ player i is winning when she is matched as a seller to a player l such that player j was not lying on the trading path in g and the length of the geodesic between l and j is shorter than the length of the geodesic between l and i. When j is the seller, he is either better off or equal off depending if

[11] This definition of pairwise stability incorporates the fact that if player i is indifferent between g and $g - ij$ she will delete the link ij because of infinitesimally small linking costs. Hence, links which are never used at equilibrium in trading networks are going to be deleted.

[12] The distance between two nodes is the length of (number of links in) the shortest path or geodesic between them.

the length of the equilibrium trading path becomes shorter or not in $g + ij$. When i was an intermediary in g for some match then she is still an intermediary for the same match in $g + ij$ and she is either better off or equal off. Finally, it may happen that i was not an intermediary in g for some match and now becomes in $g + ij$ an intermediary for the same match. Similarly, for player j. Thus, both players i and j have incentives to add the link ij.

Lemma 4. *Suppose that $r_i = r_I > 0$ for $i \in I = \{1, \dots, m\}$ and $r_j = r_P$ for $j \in P = \{m + 1, \dots, n\}$ and $r_P < r_I$. Under the initiator procedure, a network g cannot be pairwise stable if $g^I \not\subseteq g$.*

Lemma 4 follows from two observations. Firstly, two impatient players $i, j \in I$ having a common impatient player $l \in I$ as neighbor (i.e. $il, jl \in g$ but $ij \notin g$) have incentives to link to each other in g to form $g + ij$. Both i and j never make losses by adding the link ij. When i is the buyer, her payoff does not change since she is already linked to another impatient player l (that is linked to j) with whom she can negotiate first. When i is the seller or an intermediary, her payoff increases for all trades such that player j is either the buyer or a preceding intermediary in g since the new equilibrium trading path in $g + ij$ will be shorter than the one in g avoiding one intermediary, namely player l. Similarly for player j. Next, we proceed from $g + ij$ by adding a link between any two impatient players having a common impatient player as neighbor until we cannot add such links and we end up with the new network g' where the set of impatient players can be partitioned into coalitions such that all impatient players within each coalition are linked to each other and no impatient player from a given coalition is linked to an impatient player from another coalition. Secondly, two impatient players i and j of different coalitions have incentives to add the link ij to form the network $g' + ij$. When i is the seller she is winning for all trades where j or one of his coalition partner is the buyer or an intermediary since the new equilibrium trading path in $g' + ij$ will be shorter than the one in g' avoiding one patient intermediary. When i is the buyer she is indifferent. When i is an intermediary in g' she is also an intermediary in $g' + ij$ and she is either equal off or better off. Similarly for player j. Next, we repeat the process until we end up with a network where all impatient players are linked to each other and all patient players have exactly the same links as in g.

Lemma 5. *Suppose that $r_i = r_I > 0$ for $i \in I = \{1, \dots, m\}$ and $r_j = r_P$ for $j \in P = \{m + 1, \dots, n\}$ and $r_P < r_I$. Under the initiator procedure, a network g cannot be pairwise stable if there is some link between two patient players that are linked to the same impatient player.*

Here, the main point is that, when in g there is a link between two patient players $i, k \in P$ that are linked to the same impatient player $j \in I$, either the link ik is never used or one of the patient players is better off in $g - ik$. For instance, suppose that i is only linked to one impatient player j. If k is only linked to i and j then the link ik will never be used. If k is only linked to i and j and to another impatient player then player i has incentives to delete the link ik because when the match is (i, j) player j will choose to negotiate first with the other impatient player instead of negotiating directly with i.

Lemma 6. *Suppose that $r_i = r_I > 0$ for $i \in I = \{1, \dots, m\}$ and $r_j = r_P$ for $j \in P = \{m + 1, \dots, n\}$ and $r_P < r_I$. Under the initiator procedure, a network g cannot be pairwise stable if there is some link between two patient players that are not linked to the same impatient player.*

Suppose that in g there is a link between two patient players that are not linked to the same impatient player: $ik \in g, jl \in g$ and $ij \in g$ with $i, j \in P$ and $k, l \in I$. Depending on the other links in g, either player i (or j) has incentives to delete the link ij or player i (or j) has incentives to add a link with another impatient player ($\neq k, l$). For instance, if i and j do not have other links then i has incentives to delete the link ij. By deleting ij she is only losing the payoff she obtains as an intermediary for the match (j, l). This loss is largely compensated by the gains she makes by shortening the trading path for the match (i, k) in $g - ij$. If j is linked to another impatient player (say $m \in I$) then i would have even more incentives to delete ij since otherwise she would earn less from the match (i, k) and she would get nothing from the matches (j, l) and (j, m).

Lemma 7. *Suppose that $r_i = r_I > 0$ for $i \in I = \{1, \dots, m\}$ and $r_j = r_P$ for $j \in P = \{m + 1, \dots, n\}$ and $r_P < r_I$. Under the initiator procedure, a network g cannot be pairwise stable if some patient player is linked to more than one impatient player.*

We already know that the candidates for being pairwise stable are networks g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^I \subseteq g$, (iii) $ij \notin g$ if $i \in P$ and $j \in P$. Suppose that in g player $i \in P$ is linked to two impatient players $k, l \in I$. Clearly, player i is indifferent when she is the buyer and she is never an intermediary. When she is matched to an impatient player ($\neq k$) or to a patient player that is not linked to player k she is better off by deleting the link ik since the equilibrium trading path is shortened of one link. Hence, we obtain our main result that a network g is pairwise stable if and only if $g^I \subseteq g$, $d_i(g) = 1$ for all $i \in P$, and $\#C(g) = 1$.[13]

Core–periphery networks, where the core consists of impatient players who are linked to each other and the periphery consists of patient players who are only linked to the same impatient player, give to the patient players (and to player $i \in I$ for which $d_i(g) = n - 1$) their best payoffs among pairwise stable networks. In those core–periphery networks the payoff of a patient player may be greater or smaller than the payoff of the impatient player $i \in I$ for which $d_i(g) = n - 1$ depending on the discount rates, the number of impatient players (m) and the number of patient players ($n - m$). In fact, the SPE expected payoff for a patient player $i \in P$ in such core–periphery trading network g is equal to

$$U_i(g) = \frac{1}{2(n-1)} \frac{r_I}{r_I + r_P} \left(n + \frac{m-1}{2} + (n - m - 1) \frac{r_P}{r_I + r_P} \right),$$

and the SPE expected payoff for the impatient player $j \in I$ who is linked to all patient players is equal to

$$U_j(g) = \frac{1}{2(n-1)} \left(m - 1 + \frac{r_P}{r_I + r_P} (n - m) \times \left(m + 1 + 2(n - m - 1) \frac{r_P}{r_I + r_P} \right) \right).$$

For instance, suppose $m = 1$. If $2(n - 1)r_P > nr_I$ then the SPE expected payoff for the impatient player $j \in I$ who is linked to all patient players is greater than the SPE expected payoff for a patient player $i \in P$.

4. Discussion

4.1. Ranked or homogeneous players and imperfect reliability

Consider the case where there can be more than one player of each type and more than two types of players. Core–periphery

[13] Linking costs are assumed to be positive but infinitesimally small. Increasing costs would destabilize the core–periphery networks up to disconnecting the trading networks if costs become very large.

networks where the core only consists of the most impatient players $i \in I$ who are linked to each other and the periphery consists of all other players who are linked to one player in I are pairwise stable.

Proposition 3. *Suppose that $r_i = r_1 > 0$ for $i \in I = \{1, 2, \dots, m\}$, $r_i = r_2 > 0$ for $i \in \{m+1, \dots, l\}$, $r_i = r_3 > 0$ for $i \in \{l+1, \dots, k\}, \dots$, with $r_1 > r_2 > r_3 > \cdots$. Under the initiator procedure, the network g such that $g^I \subseteq g$, $d_i(g) = 1$ for $i \in \{m+1, \dots, n\}$ and $\#C(g) = 1$ is pairwise stable.*[14]

However, there might be other pairwise stable networks. For instance, take $N = \{1, 2, 3\}$ and $r_1 > r_2 > r_3$. The star network $\{12, 23\}$ with the second most impatient player being the center is pairwise stable when r_2 is close to r_1. In fact, player 3 has no incentive to link to player 1 because the gains player 3 would obtain when he is the buyer (3 would negotiate with 1 instead of 2) do not compensate the losses he would incur when player 2 is the buyer and player 3 is the seller.[15]

Suppose now that players are homogeneous in terms of their discount rates: $r_1 = r_2 = \cdots = r_n$. From Lemmas 2 and 4 we have that there is a unique pairwise stable architecture, namely the complete network.[16]

Corollary 1. *Suppose that $r_i = r$ for all $i \in N$. Under the initiator procedure, the complete network g^N is the unique pairwise stable network.*

Suppose now that players have the same discount rate ($r_i = r$ for all $i \in N$) but there is a positive probability that a link fails to deliver the good/money once an agreement has been reached.[17] The delivering reliability of a link is measured by a parameter $\mu_{ij} \in [0, 1)$. Here, μ_{ij} is the probability that an established link between i and j fails in delivering the good, while $1 - \mu_{ij}$ is the probability that it succeeds. Link reliability across different pairs of agents is assumed to be independent. For instance, let $\mu_{ij} < \mu_{ik} = \underline{\mu} < \mu_{kl} = \overline{\mu}$ for $i, j \in L = \{1, 2, \dots, m\}$ and $k, l \in H = \{m+1, \dots, n\}$. Then, the network g such that $g^L \subseteq g$, $d_i(g) = n - 1$ for $i \in L$ and $d_k(g) = m$ for $k \in H$ is pairwise stable if and only if $(1 - \overline{\mu})/2 < (1 - \underline{\mu})^2/2$. In such a core–periphery network, the most reliable players are linked to all players but the less reliable ones are not linked to each other. Thus, once some nodes/links are more reliable than others to deliver the good/money, core–periphery networks can emerge in the long run even if all players are equally patient. However, when players can be patient or impatient, a trade-off is likely to occur between linking to either impatient players or reliable ones.

4.2. Private information

Under complete information, agreement is reached immediately in each bilateral bargaining session. We now suppose that players do not know the impatience of the other players. It is common knowledge that player i's discount rate lies in the range $[\underline{r}_i, \overline{r}_i]$, where $0 < \underline{r}_i \le \overline{r}_i$ and $i \in N$, and that types are independently drawn from the set $[\underline{r}_i, \overline{r}_i]$ according to some probability distribution p_i. Each bilateral bargaining session may involve delay, but not perpetual disagreement, in equilibrium. In fact, delay is positively related to the distance between the discount rates of the most and least patient types of the players. If the range of types is reduced, then this leads to a smaller range of possible payoffs and less delay. Delay can occur even when the game is close to one of complete information (as the type distributions converge to point mass distributions).

We propose to analyze the maximum delay time in reaching an agreement. Only on average is this measure a good proxy for actual delay.[18] In each bilateral bargaining session (i, j), the maximum real time player j would spend bargaining is the time $D(i, j)$ such that player j is indifferent between getting her lower bound perfect Bayesian equilibrium (PBE) payoff at time 0 and getting her upper bound PBE payoff at time $D(i, j)$. In Appendix C we derive the expression for the maximum delay in equilibrium which shows that an agreement is reached in finite time and that delay time equals zero as incomplete information vanishes (in that $\underline{r}_i$ and $\underline{r}_j$ converge to $\overline{r}_i$ and $\overline{r}_j$, respectively).

Proposition 4. *Under the initiator procedure, the maximum real delay time in reaching a partial agreement in each bilateral bargaining session (i, j) is given by*

$$D(i, j) = -\frac{1}{\underline{r}_j} \cdot \log\left[\frac{\underline{r}_i}{\overline{r}_i} \cdot \frac{\overline{r}_i + \underline{r}_j}{\underline{r}_i + \overline{r}_j}\right].$$

In fact, $D(i, j)$ is the maximum real time player j would spend negotiating if she were of the most patient type. We have $\partial D(i, j)/\partial \underline{r}_j < 0$, $\partial D(i, j)/\partial \overline{r}_j > 0$, $\partial D(i, j)/\partial \underline{r}_i < 0$ and $\partial D(i, j)/\partial \overline{r}_i > 0$. Given the trading path $(s, i_1, i_2, \dots, i_k, b)$, the maximum real delay time in reaching $k + 1$ partial agreements is equal to $D(s, i_1, i_2, \dots, i_k, b) = D(s, i_1) + D(i_1, i_2) + \cdots + D(i_{k-1}, i_k) + D(i_k, b)$.

We now provide an example of the maximum delay. Suppose that (s, i_1, i_2, i_3, b) is the trading path and let $\underline{r}_i = \underline{r}$, $\overline{r}_i = \overline{r}$, $\overline{r} = 0.33 - \underline{r}$ with $\underline{r} \in [0.04, 0.17]$, $i \in \{s, i_1, i_2, \dots, i_k, b\}$. Table 1 gives the integer part of the maximum delays for $\Delta = 1/365$. So, we can interpret r as the annual discount rate and the numbers in Table 1 as the maximum number of days needed to reach an agreement. We observe that many bargaining rounds may be needed in equilibrium before an agreement is reached and this number is increasing with the amount of private information $\left|\overline{r} - \underline{r}\right|$.

When a player chooses one of her predecessors on a path from the seller s to the buyer b to negotiate bilaterally a partial agreement once there is private information about the impatience of the players, her choice still does not depend on how the predecessors are going to share the rest of the surplus but now depends on how much time the predecessors will need to reach their partial agreements. Suppose that $N = \{1, 2, 3\}$, $g = \{12, 23, 13\}$ and that $\overline{r}_2 = \underline{r}_2 = \overline{r}_3 = \underline{r}_3 < \underline{r}_1 < \overline{r}_1$. That is, it is common knowledge that player 1 is an impatient player

[14] Hence, if $r_1 > r_2 > \cdots > r_{n-1} > r_n$ then the star network with the most impatient player, namely player 1, being the center is pairwise stable.

[15] An alternative protocol is Suh and Wen (2006) multi-agent bilateral bargaining model with the demand procedure, where the proposer demands a share to exit the bargaining game. Under such procedure, the discount rates of all players on the trading path between the seller and her predecessor now matter for her share to exit the bargaining game, and so, any player always prefers to negotiate with a patient player instead of negotiating with a less patient player and adding at least one more intermediary on the trading path between the seller and the buyer. Therefore, the complete network g^N is the unique pairwise stable network under the demand procedure. Other bargaining protocols leading to similar outcomes can be found in Chae and Yang (1994), Krishna and Serrano (1996), Vannetelbosch (1999), Huang (2002), and Li (2010) among others.

[16] Babus and Hu (2015) have shown that, if the formation of links is costly, players are homogeneous but forward-looking, and players incur monitoring costs for each transaction along the trading path, then a star network that connects all players is an absorbing state of Dutta et al. (2005) dynamic network formation process.

[17] Bala and Goyal (2000) have analyzed the impact of imperfect link reliability in endogenous communication networks.

[18] It is not uncommon in the literature on bargaining to analyze the maximum delay before reaching an agreement. See, for instance, Cramton (1992) and Cai (2003).

Table 1
Maximum delay in reaching an agreement.

$\underline{r}$	0.17	0.16	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.08	0.07	0.06	0.05	0.04
"Partial delay"	0	1	2	3	4	5	7	9	12	15	20	26	36	51
"Total delay"	0	4	8	12	16	20	28	36	48	60	80	104	144	204

and players 2 and 3 are patient players. Suppose first that player 1 is the seller and player 3 is the buyer. Since player 3's worst PBE payoff when she negotiates directly with player 1, $\underline{r}_1/(\underline{r}_1+\overline{r}_3)$, is greater than her best PBE payoff when she negotiates first with intermediary 2, $\overline{r}_2/(\overline{r}_2+\underline{r}_3)=1/2$, she will choose to negotiate directly with player 1. Suppose now player 2 is the seller and player 3 is the buyer. Since player 3's PBE payoff when she negotiates directly with player 2, $\overline{r}_2/(\overline{r}_2+\underline{r}_3)=1/2$, can be greater or smaller than her worst PBE payoff when she negotiates first with intermediary 1, $\underline{r}_1/(\underline{r}_1+\overline{r}_3)\exp(-\overline{r}_3 D(1,2))$ where $D(1,2)$ is the maximum real delay time in reaching a partial agreement between 1 and 2, it is not excluded that player 3 would choose to negotiate directly with player 2 instead of going through the impatient player 1. Player 3 will choose to bargain with player 2 instead of player 1 if the expected delay for reaching an agreement in a negotiation between player 1 and player 2 is large enough. Hence, player 3 may now have incentives to be linked to both players 1 and 2 although it is commonly known that player 2 is more patient than player 1.

We now provide sufficient conditions such that core–periphery networks are still pairwise stable when players have private information.

Proposition 5. *Suppose that $r_i=r_I>0$ for $i\in I=\{1,\dots,m\}$ and $r_j\in[\underline{r}_P,\overline{r}_P]$ for $j\in P=\{m+1,\dots,n\}$ $(0<\underline{r}_P\le\overline{r}_P)$ and that it is common knowledge that any player $i\in I$ is less patient than any player $j\in P$: $\underline{r}_P<\overline{r}_P<r_I$. Under the initiator procedure, if*

(i) $D(i,j)<\dfrac{-1}{\overline{r}_P}\log\left(\dfrac{\overline{r}_P}{r_I}\dfrac{r_I+\overline{r}_P}{\underline{r}_P+\overline{r}_P}\right)$ *and*

(ii) $D(j,i)<\dfrac{-1}{r_I}\log\left(\dfrac{2\overline{r}_P}{r_I+\overline{r}_P}\right)$,

then a network g such that $g^I\subseteq g$, $d_i(g)=1$ for all $i\in P$ and $\#C(g)=1$ is pairwise stable.

Condition (i) in Proposition 5 is a sufficient condition for player $j\in P$ for not adding a link to another player $k\in P$ in a core–periphery network g. It implies that if $j\in P$ and $k\in P$ are matched then buyer j prefers to negotiate with the player $i\in I$ is linked to rather than building the link jk and negotiating directly with k. For condition (i) to hold we need that $r_I-\overline{r}_P$ is large enough (for the right-hand side of the inequality being positive) and $\overline{r}_P-\underline{r}_P$ is not too large (for $D(i,j)$ being small enough). Condition (ii) in Proposition 5 is a sufficient condition for player $j\in P$ for not adding a link to another impatient player $l\in I$ $(l\neq i)$ in a core–periphery network g because if $i\in I$ is an intermediary (or the buyer) in a match where $j\in P$ is the seller then i prefers to negotiate with player $l\in I$ rather than negotiating directly with j. For condition (ii) to hold we need that $\overline{r}_P-\underline{r}_P$ is not too large (for $D(j,i)$ being small enough) and $r_I-\overline{r}_P$ is large enough. Similar conditions can be provided in case only impatient players have private information and it is commonly known that they are less patient than any patient player.

So, a core–periphery network is likely to be pairwise stable if all players do not have too much private information and impatient players are quite more impatient than patient players. Otherwise, players may prefer to add links for reducing the length of trading paths and so avoiding longer costly delays in reaching a global agreement.

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Appendix A. Bargaining with complete information

We consider the initiator procedure. Suppose that (s,b) is a pair randomly matched with s being the seller and b being the buyer and s and b are connected in the trading network g. The negotiation starts with the buyer b who first chooses one of her predecessors, say intermediary i_k, on a path from b to s to negotiate bilaterally a partial agreement. In the bilateral bargaining session (i_k,b), the negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model where players make alternate offers, with i_k making offers in even-numbered periods and b making offers in odd-numbered periods. The length of each period is Δ. The negotiation starts in period 0 and ends when one of the players accepts an offer and leads to a partial agreement. In case of perpetual disagreement in some bilateral session, all players get 0. Players have time preferences with constant discount factors, $\delta_i\in(0,1)$. A partial agreement specifies the share of the surplus, $0\le x_b\le 1$, for b to exit the game. Once b exits the game, i_k chooses one of her predecessors, say intermediary i_{k-1}, on a path from i_k to s such that b does not lie on the path. In the bilateral bargaining session (i_{k-1},i_k), the negotiation specifies the share of the surplus, $0\le x_{i_k}\le 1$, for i_k to exit the game. Once a partial agreement is reached between i_k and i_{k-1}, i_k exits the game. Then, i_{k-1} chooses one of her predecessors, say intermediary i_{k-2}, on a path from i_{k-1} to s such that i_k and b do not lie on the path. Once a partial agreement is reached between i_{k-1} and i_{k-2}, i_{k-1} exits the game; and so on until a partial agreement is reached between i_1 and s. An outcome consists of a sequence $(i_1,i_2,\dots,i_k)$ of intermediaries between s and b and $(k+1)$ partial agreements that specify player i's share of the surplus, $0\le x_i\le 1$, for $i\in\{s,i_1,i_2,\dots,i_k,b\}$, such that $x_s+x_{i_1}+\cdots x_{i_k}+x_b=1$. Each player only receives her share once all $(k+1)$ partial agreements have been reached. This multi-agent bilateral bargaining model is solvable by backward induction.

Let y_t be the surplus left to be shared among the remaining players after t bilateral bargaining sessions. So, y_0 is the initial surplus to be shared and is equal to 1; y_1 is the surplus left to be shared after buyer b has taken her share; y_2 is the surplus left to be shared after intermediary i_k and buyer b have taken their shares; y_3 is the surplus left to be shared after intermediary i_{k-1}, intermediary i_k and buyer b have taken their shares; y_k is the surplus left to be shared after intermediary i_2, intermediary $i_3,\dots,$ intermediary i_{k-1}, intermediary i_k and buyer b have taken their shares.

Consider first the pair (s, i_1). Intermediary i_1 chooses to negotiate with s only if there is no other player less patient than s linked to i_1 such that there is a path from this other player to s and players $\{i_2, i_3, \ldots, i_k, b\}$ do not lie on the path. The SPE partial agreement of the bilateral bargaining session (s, i_1) is $x_{i_1} = y_k\delta_{i_1}(1-\delta_s)/(1-\delta_s\delta_{i_1})$. Consider next the pair (i_1, i_2). Intermediary i_2 chooses to negotiate with i_1 only if there is no other player less patient than i_1 linked to i_2 such that there is a path from this other player to s and players $\{i_3, \ldots, i_k, b\}$ do not lie on the path. The SPE partial agreement of the bilateral bargaining session (i_1, i_2) is $x_{i_2} = y_{k-1}\delta_{i_2}(1-\delta_{i_1})/(1-\delta_{i_1}\delta_{i_2})$; and so forth. Consider finally the pair (i_k, b). Buyer b chooses to negotiate with i_k only if there is no other player less patient than i_k linked to b such that there is a path from this other player to s. The SPE partial agreement of the bilateral bargaining session (i_k, b) is $x_b = y_0\delta_b(1-\delta_{i_k})/(1-\delta_{i_k}\delta_b)$. Since $y_0 = 1$, buyer b will obtain at equilibrium

$$x_b^* = \frac{\delta_b(1-\delta_{i_k})}{1-\delta_{i_k}\delta_b}.$$

Since $y_1 = y_0 - x_b^*$, intermediate i_k will obtain at equilibrium

$$x_{i_k}^* = \frac{\delta_{i_k}(1-\delta_{i_{k-1}})}{1-\delta_{i_{k-1}}\delta_{i_k}}\frac{1-\delta_b}{1-\delta_{i_k}\delta_b}.$$

Since $y_2 = y_1 - x_{i_k}^*$, intermediate i_{k-1} will obtain at equilibrium

$$x_{i_{k-1}}^* = \frac{\delta_{i_{k-1}}(1-\delta_{i_{k-2}})}{1-\delta_{i_{k-2}}\delta_{i_{k-1}}}\frac{1-\delta_{i_k}}{1-\delta_{i_{k-1}}\delta_{i_k}}\frac{1-\delta_b}{1-\delta_{i_k}\delta_b};$$

and so on. Since $y_k = y_{k-1} - x_{i_2}^*$, intermediate i_1 will obtain at equilibrium

$$x_{i_1}^* = \frac{\delta_{i_1}(1-\delta_s)}{1-\delta_s\delta_{i_1}}\frac{1-\delta_{i_2}}{1-\delta_{i_1}\delta_{i_2}}\frac{1-\delta_{i_3}}{1-\delta_{i_2}\delta_{i_3}}\cdots\frac{1-\delta_{i_k}}{1-\delta_{i_{k-1}}\delta_{i_k}}\frac{1-\delta_b}{1-\delta_{i_k}\delta_b};$$

and seller s will obtain at equilibrium

$$x_s^* = \frac{1-\delta_{i_1}}{1-\delta_s\delta_{i_1}}\frac{1-\delta_{i_2}}{1-\delta_{i_1}\delta_{i_2}}\frac{1-\delta_{i_3}}{1-\delta_{i_2}\delta_{i_3}}\cdots\frac{1-\delta_{i_k}}{1-\delta_{i_{k-1}}\delta_{i_k}}\frac{1-\delta_b}{1-\delta_{i_k}\delta_b}.$$

It is customary to express the players' discount factors in terms of discount rates, $r_1 > 0, r_2 > 0, \ldots r_n > 0$, and the length of the bargaining period, Δ, according to the formula $\delta_i = \exp(-r_i\Delta)$. As Δ approaches zero, using l'Hopital's rule, the SPE outcomes $x_b^*, x_{i_k}^*, x_{i_{k-1}}^*, \ldots, x_{i_1}^*, x_s^*$ tend to the equilibrium outcomes given in Proposition 1.

Appendix B. Pairwise stable trading networks

Proof of Lemma 3. (i) First, we show that a patient player always wants to link to an impatient player if in the current network there are at least two patient players as intermediaries on the geodesic between the patient player and the impatient player. Remember that a geodesic between players i and j is a shortest path between these nodes; that is, a path with no more links than any other path between these nodes.

Suppose that $1 \in I$ and $\{2, 3, 4\} \subseteq P$. Take any network g_1 such that $\{12, 23, 34\} \subseteq g_1$, the path 1, 2, 3, 4 is a geodesic between 1 and 4, and the distance between player 4 and any other impatient player is greater than 3. We now show that 1 and 4 have incentives to add the link 14 to form $g_2 = g_1 + 14$.

Player 4 is winning when she is matched as a buyer to the impatient player 1 or to one of the patient players 2 and 3 or to any other player j such that in g_1 players 2 or 3 were on the equilibrium trading path between 4 and j. Notice that 4 is strictly winning because she will get a larger share of the surplus when bargaining with the impatient player 1 rather than bargaining with the patient player 3 or 2 depending to whom she is matched. If 4 is matched to a player j such that in g_1 players 2 or 3 are not on the equilibrium trading path between 4 and j, then 4 is indifferent between g_1 and g_2 because the link 14 will not be used. So, when 4 is the buyer she is never losing by adding the link 14 to g_1. When 4 is the seller, she is always better off when she is matched to a player j such that player 1 was lying on the trading path in g_1 since the trading path in g_2 will be shorter and 1 will end the sequence of bilateral bargaining sessions negotiating with 4. In addition, in g_2 player 4 is winning when she is matched as a seller to a player j such that player 1 was not lying on the trading path in g_1 and the length of the geodesic between 1 and j is shorter than the length of the geodesic between 4 and j. Otherwise, she is equal off. When 4 was an intermediary in g_1 for some match (i, j) then she is still an intermediary for the match (i, j) in g_2 and she is either better off or equal off depending if the equilibrium trading path is passing or not through the impatient player 1. Finally, it may happen that 4 is strictly winning in case she was not an intermediary in g_1 for some match (i, j) and now becomes in g_2 an intermediary for the match (i, j).

Player 1 is indifferent between g_1 and g_2 when he is the buyer because in both networks he has to negotiate with one patient player. When player 1 is the seller, he is either better off or equal off depending if the length of the equilibrium trading path becomes shorter or not in g_2. When 1 was an intermediary in g_1 for some match (i, j) then he is still an intermediary for the match (i, j) in g_2 and he is either better off or equal off depending if the length of the equilibrium path between him and the buyer becomes shorter or not. Finally, it may happen that 1 is strictly winning in case he was not an intermediary in g_1 for some match (i, j) and now becomes in g_2 an intermediary for the match (i, j). Thus, we conclude that both players 1 and 4 have incentives to add the link 14.

(ii) Second, we show that a patient player having links with at least two other patient players that are linked to the same impatient player has always incentives to link to the impatient player.

Suppose that $1 \in I$ and $\{2, 3, 4\} \subseteq P$. Take any network g_1 such that $\{12, 13, 24, 34\} \subseteq g_1$, the paths 1, 2, 4 and 1, 3, 4 are geodesics between 1 and 4, and the distance between player 4 and any other impatient player is greater or equal than 2. We now show that 1 and 4 have incentives to add the link 14 to form $g_2 = g_1 + 14$.

Player 4 is strictly winning when she is matched as a buyer to the impatient player 1 or to one of the patient players 2 and 3 or to any other player j such that there is a path between players 1 and j and player 4 does not lie on the path. The reason is the same as before. By adding the link 14 she will get a larger share of the surplus when bargaining with the impatient player 1 rather than bargaining with the patient player 3 or 2 depending to whom she is matched. Otherwise, if 4 is matched to a player j such that in g_1 players 2 or 3 are not on the equilibrium trading path between 4 and j, she is equal off because the link 14 will not be used. Player 4 is winning when she is matched as a seller to a player j such that player 1 is on the equilibrium trading path in g_1 since the trading path in g_2 will be shorter and 1 will end the sequence of bilateral bargaining sessions negotiating with 4. Otherwise, she is equal off. Player 4 is winning when she is an intermediary on trades whose equilibrium trading paths are passing through the impatient player 1 in g_1. Otherwise, player 4 is equal off.

Player 1 is equal off when he is the buyer because in both networks he has to negotiate with one patient player. However, player 1 is better off or equal off when he is the seller or an intermediary depending if the length of the equilibrium path between him and the buyer becomes shorter or not. Thus, we have that both players 1 and 4 have incentives to add the link 14.

(iii) Third, suppose that $1 \in I$ and $\{2, 3\} \subseteq P$. In any network g_1 such that $\{12, 23\} \subseteq g_1$, the path 1, 2, 3 is a geodesic between 1 and 3 in g_1, $d_2(g_1) = 2$, and the distance between the patient player 3 and any other impatient player is greater or equal than 2, players 1 and 3 have incentives to add the link 13 to form $g_2 = g_1 + 13$. For player 1, the reason is the same as in (i) or (ii). For player 3, simple computations allow us to show that the sum of the equilibrium payoffs when she is the buyer and the seller in the matches with players 1 and 2, are bigger in the network g_2 than in the network g_1. When matched with any other player $j \neq 1, 2$ such that there is a path between players 1 and j, player 3 as a buyer is better off because by adding the link 13 she will get a larger share of the surplus when bargaining with the impatient player 1 rather than bargaining with the patient player 2. Otherwise, she will be indifferent as a buyer. Player 3 is winning when she is matched as a seller to a player $j \neq 1, 2$ such that player 1 is on the equilibrium trading path in g_1 since the trading path in g_2 will be shorter and 1 will end the sequence of bilateral bargaining sessions negotiating with 3. Otherwise, she is equal off. Finally, notice that player 3 is never an intermediary.

(iv) Fourth, suppose that $1 \in I$ and $\{2, 3, 4\} \subseteq P$. In any network g_1 such that $\{12, 23, 24\} \subseteq g_1$, the path 1, 2, 3 is a geodesic between 1 and 3, the path 1, 2, 4 is a geodesic between 1 and 4, and the distance between the patient player 3 (4) and any other impatient player is greater or equal than 2, the patient players 3 and 4 have first incentives to add the link 34 to form $g_2 = g_1 + 34$ because players 3 and 4 will win as sellers when matched with 4 and 3 respectively. Adding the link 34 does not affect players 3 and 4 in the other matches. Once the link 34 is formed, the patient player 4 has now incentives to link to the impatient player 1 to form the network g_2+14. The reason is that now, at g_2, player 4 as a seller can never be worse off, even when she is matched with player 3 because player 3 will bargain directly with her instead with player 2. Notice that without adding first the link 34 to g_1, player 4 in the match with player 3 would have been worse off as a seller if the link 14 is added to g_1, because the equilibrium path would have been (3, 2, 1, 4) (with the link 14) instead of (3, 2, 4) (without the link 14). Notice that player 1 is equal off when he is the buyer, but he is better off or equal off when he is the seller or an intermediary (the reason is the same as in (i) or (ii)). Thus, player 1 agrees to add the link 14 to the network g_2.

From (i)–(iv) we conclude that a network g cannot be pairwise stable if there is some patient player that is not linked to at least one impatient player. □

Proof of Lemma 4. Consider any network g such that $\#C(g) = 1$, $N(g) = N$ and each patient player is linked to at least one impatient player.

(i) First, we will show that two impatient players $i, j \in I$ having a common impatient player $l \in I$ as neighbor (i.e. $il, jl \in g$ but $ij \notin g$) have incentives to link to each other in g to form $g+ij$. When i is the buyer, her payoff does not change by adding the link ij since she is already linked to another impatient player l (that is linked to j) with whom she can negotiate first. When i is the seller, her payoff does not change by adding the link ij for all trades such that player j is not the buyer nor an intermediary in g since the equilibrium trading path in $g + ij$ will be the same as the one in g. When i is the seller, she is winning by adding the link ij for all trades such that player j is either the buyer or an intermediary in g since the new equilibrium trading path in $g + ij$ will be shorter than the one in g avoiding one intermediary, namely player l. When i is an intermediary, she is winning by adding the link ij for all trades such that player j is either the buyer or a preceding intermediary in g since the new equilibrium trading path in $g + ij$ will be shorter than the one in g avoiding one intermediary, namely player l. Finally, when i is an intermediary, her payoff does not change by adding the link ij for all trades such that player j is not on the equilibrium trading path in g or is not a preceding intermediary in g. Similarly for player j. Hence, players i and j have incentives to add the link ij.

(ii) Next, we proceed from g by adding a link between any two impatient players having a common impatient player as neighbor until we cannot add such links and we end up with the new network $g' = g_I \cup g_P$ where

$$g_I = \left\{ij \in g^N \mid \text{ there is a path between } i \text{ and } j \text{ in } g \setminus g_P\right\}$$

and $g_P = \{ij \in g \mid i \in P \text{ or } j \in P\}$. Let $\Pi(g_I)$ be the partition of I induced by g_I. That is, $\pi \in \Pi(g_I)$ if and only if either there exists $h \in C(g_I)$ such that $\pi = N(h)$ or there exists $i \notin N(g_I)$ such that $\pi = \{i\}$. The set of impatient players is partitioned into coalitions such that all impatient players within each coalition are linked to each other and no impatient player from a given coalition is linked to an impatient player from another coalition.

We want now to prove that, in g', two impatient players i and j of different coalitions π_i and π_j in $\Pi(g_I)$ ($i \in \pi_i$ and $j \in \pi_j$) of fully connected players that are not linked to any patient player on the path between these two coalitions π_i and π_j have incentives to add the link ij to form the network $g' + ij$. When i is the seller she is winning for all trades where j or one of his coalition partner in π_j is the buyer or an intermediary since the new equilibrium trading path in $g' + ij$ will be shorter than the one in g' avoiding at least one patient intermediary; otherwise she is indifferent. When i is the buyer she is indifferent between $g' + ij$ and g'. When i is an intermediary in g' she is also an intermediary in $g' + ij$ and she is either equal off or better off (when the new equilibrium trading path in $g' + ij$ is shorter than the one in g' and avoids one patient preceding intermediary). Similarly for player j.

In addition, in g', two impatient players i and j of different coalitions π_i and π_j in $\Pi(g_I)$ ($i \in \pi_i$ and $j \in \pi_j$) of fully connected players that are linked to a patient player on the path between these two coalitions π_i and π_j have also incentives to add the link ij to form the network $g' + ij$. When i is the buyer or the seller she is either better off or equal off between $g' + ij$ and g' depending to whom she is matched. For instance, as a seller, she is better off when matched to someone such that the new equilibrium trading path in $g' + ij$ will be shorter than the one in g' avoiding at least one patient intermediary; otherwise she is indifferent. As a buyer, she is better off when matched to a player such that j was in the equilibrium trading path in g' and i was forced to bargain first with some patient player in g' while, now, in $g' + ij$ she can bargain directly with j avoiding the patient player; otherwise she is indifferent. When i is an intermediary in g' she is also an intermediary in $g' + ij$ and she is either equal off or better off or worse off. However, the losses she makes as an intermediary in some matches are easily compensated by the gains she makes as an intermediary in other matches. Precisely, player i can only make losses when she is an intermediary in matches between two patient players who are linked to impatient players both in π_i and in π_j, and these losses are compensated by the gains she makes when she is an intermediary in matches between those two patient players (as sellers) and impatient players (as buyers) from π_j.

(iii) Next, we repeat the process of step (ii) until we end up with network $g^I \cup g_P$ where all impatient players are linked to each other and all patient players have exactly the same links as in g. □

Proof of Lemma 5. From Lemmas 2–4 we know that the candidates for being pairwise stable are networks g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^I \subseteq g$, (iii) for each $i \in P$ there is $j \in I$ such that $ij \in g$. We now show that g cannot be pairwise stable if there is some link between two patient players that are linked to the same impatient player. Five cases have to be considered.

(a) In g the patient player i is only linked to one impatient player j. Suppose that we add the link ik to g to form $g + ik$ where $i, k \in P$. (a.1) If k is only linked to i and j then the link ik will never be used. Remember that the definition of pairwise stability incorporates the idea of infinitesimally small linking costs and implies that links never used are deleted. (a.2) If k is only linked to i and j and to another impatient player l then player i has incentives to delete the link ik because when the match is (i, j) player j will choose to negotiate first with the other impatient player l instead of negotiating directly with i. (a.3) If k is only linked to i and j and to another patient player that is only linked to j then the link ik will never be used. (a.4) If k is only linked to i and j and to another patient player that is linked to another impatient player ($\neq j$) then player i has incentives to delete the link ik for the same reason as in (a.2).

(b) In g the patient player i is only linked to impatient players $j, k \in I$ (at least two). Suppose that we add the link il to g to form $g + il$ where $i, l \in P$. (b.1) If l is only linked to i and j then player l has incentives to delete the link il because when the match is (l, j) player j will choose to negotiate first with the other impatient player k instead of negotiating directly with l. (b.2) If l is only linked to i and j and to another impatient player then the link il will never be used. (b.3) If l is only linked to i and j and to another patient player $m \in S$ that is only linked to j, then this patient player m has incentives to delete the link lm because when the match is (m, j) player j will choose to negotiate first with the other impatient player k instead of negotiating directly with m. (b.4) If l is only linked to i and j and to another patient player $m \in S$ that is linked to another impatient player n ($\neq j$), then the link lm is never used if $n \neq k$ and player m has incentives to delete the link lm if $n = k$ because when the match is (m, k) player k will choose to negotiate first with the other impatient player j instead of negotiating directly with m.

(c) In g the patient player i is only linked to one impatient player $j \in I$ and to a patient player $k \in P$ that is only linked to i and j. Suppose that we add the link il to g to form $g + il$ where $i, l \in P$. (c.1) In $g+il$ player k has incentives to delete the link ik since player k is in the position of player i in case (a.4) if the patient player l is linked to an impatient player $m \neq j$. (c.2) Otherwise, if the patient player l is linked to the impatient player j, the link ik in $g + il$ will never be used (like in case (a.3)).

(d) In g the patient player i is only linked to one impatient player $j \in I$ and to a patient player $k \in P$ that is only linked to i and to another impatient player $m \neq j$ ($kj \notin g$). Suppose that we add the link il to g to form $g + il$ where $i, l \in P$. (d.1) In $g + il$ player l has incentives to delete the link il if he is only linked to j because when the match is (l, j) player j will choose to negotiate first with the other impatient player m instead of negotiating directly with l. (d.2) In $g+il$ player l has also incentives to delete the link to another impatient player n if this link exists in $g+il$ with $lj \in g+il$ because when l is the seller there will always be an additional impatient intermediary on the trading path.

(e) In g the patient player i is only linked to one impatient player $j \in I$ and to a patient player $k \in P$ that is linked to i and j and to another impatient player $m \neq j$ ($kj \in g$). Suppose that we add the link il to g to form $g + il$ where $i, l \in P$. (e.1) If l is linked only to j and i then l has incentives to delete the link il to avoid this link being used when l is the seller (notice that l is never intermediary in $g + il$). (e.2) If l is linked only to j and i and to another impatient player n, then i has incentives to delete the link il either to avoid this link being used when i is the seller in case $n = m$, or because this link is not used in case $n \neq m, j$. □

Proof of Lemma 6. We now show that g cannot be pairwise stable if there is some link between two patient players that are not linked to the same impatient player. From Lemmas 2–5 we know which networks are the candidates for being pairwise stable networks. Hence, take any network g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^I \subseteq g$, (iii) for each $i \in P$ there is $j \in I$ such that $ij \in g$, (iv) $ij \notin g$ if $i, j \in P$ and there is some $k \in I$ such that $ik \in g$ and $jk \in g$.

(a) Suppose that $ik \in g$, $jl \in g$ and $ij \notin g$ where $i, j \in P$ and $k, l \in I$. Suppose that we add the link ij to g to form $g + ij$ where $i, j \in P$. (a.1) If i and j do not have other links then i has incentives to delete the link ij. By deleting the link ij she is only losing the payoff she obtains as an intermediary for the match (j, l) in $g + ij$. This loss is compensated by the gains she makes by shortening the trading path for the match (i, k) in g. (a.2) If j is linked to another impatient player (say $m \in I$) then i would have more incentives than in (a.1) to delete ij since she would earn less from the match (i, k) in $g + ij$ and she would get nothing from the matches (j, l) and (j, m). (a.3) If j is linked to another patient player ($\neq i$) then i has more incentives than in (a.1) to delete ij. (a.4) If i is linked to at least two impatient players then (i) if j is also linked to at least two impatient players then ij is not used, (ii) if j is linked to one impatient player then j has incentives to delete ij since j is in the position of i in case (a.2).

(b) The last case to be considered is when in g patient players are linked to all of them (that is, $g^P \subseteq g$) but each patient player is linked to a different impatient player. Suppose that $il \in g$, $jm \in g$ and $kn \in g$ where $i, j, k \in P$ and $l, m, n \in I$. Suppose that we add the link im to g to form $g + im$. For player i the link im only modifies her payoff from the match (i, l). With the link im the trading path is shorter and so, player i has incentives to add the link im. By adding the link im, player m makes additional gains from the matches (m, i) (i will bargain directly with him) and (k, i) for $k \neq j, k \in P$, (because, without the link im, player m will always be the second intermediary in the trading path while, with the link im, player m could be the first intermediary in the trading path) but he makes losses from the matches (i, k) for $k \neq j, k \in P$ (because, without the link im, player m will always be the second intermediary in the trading path while, with the link im, player m could be the third intermediary in the trading path). However, the losses are much smaller than the gains. In all other matches nothing changes for player m. Hence, player m has also incentives to add the link im to g, and so we have that g is not pairwise stable. Once we have added the link im to g, we have obtained a network $g + im$ where two strong players i and j are linked to the same weak player m and we know from Lemma 5 that such network cannot be pairwise stable. □

Proof of Lemma 7. From Lemmas 2–6 we know that the candidates for being pairwise stable are networks g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^I \subseteq g$, (iii) $ij \notin g$ if $i \in P$ and $j \in P$. We now show that g cannot be pairwise stable if some patient player $i \in P$ is linked to more than one impatient player. Suppose that in g player $i \in P$ is linked to two impatient players $k, l \in I$. When i is the buyer she is indifferent between g and $g - ik$. Notice that player i is never an intermediary in g nor in $g - ik$. Suppose now that i is the seller. When she is matched to an impatient player $m \neq k$ she is better off by deleting the link ik since the equilibrium trading path is shortened of one link, and when she is matched to the impatient player k she is indifferent between g and $g - ik$. When player i (as a seller) is matched to a patient player that is not linked to player k she is better off by deleting the link ik since the equilibrium trading path is shortened of one link, and when she is matched to a patient player that is linked to the impatient player k (and not to player l) she is indifferent between g and $g - ik$. Finally, when player i (as a seller) is matched to a patient player that is linked to player l she is better off by deleting the link ik since the equilibrium trading path is shortened of one link between two impatient players. □

Appendix C. Private information and maximum delay

Consider again the path $(i_0, i_1, i_2, \ldots, i_k, i_{k+1})$ that connects seller s (player i_0) to buyer b (player i_{k+1}). Players negotiate how to split the surplus via successive bilateral bargaining sessions in the following order: (i_k, i_{k+1}), (i_{k-1}, i_k), (i_{k-2}, i_{k-1}), …, (i_1, i_2), (i_0, i_1). Suppose now that the players have private information. They are uncertain about each others' discount factors. Player i's discount factor lies in the range $[\underline{\delta}_i, \overline{\delta}_i]$, where $0 < \underline{\delta}_i \le \overline{\delta}_i < 1$. The types are independently drawn from the interval $[\underline{\delta}_i, \overline{\delta}_i]$ according to the probability distribution p_i, $i \in N$.

Lemma 8. *Consider the sequence (i_k, i_{k+1}), (i_{k-1}, i_k), (i_{k-2}, i_{k-1}), …, (i_1, i_2), (i_0, i_1) of $k+1$ bilateral bargaining sessions with private information in which the probability distributions are common knowledge and in which the period length shrinks to zero. Under the initiator procedure, for any perfect Bayesian equilibria, the payoff of player i_{k+1-l} in each bilateral bargaining session (i_{k-l}, i_{k+1-l}) belongs to*

$$\left[\frac{\underline{\delta}_{i_{k+1-l}}\left(1-\overline{\delta}_{i_{k-l}}\right)}{1-\overline{\delta}_{i_{k-l}}\underline{\delta}_{i_{k+1-l}}}y_l,\ \frac{\overline{\delta}_{i_{k+1-l}}\left(1-\underline{\delta}_{i_{k-l}}\right)}{1-\underline{\delta}_{i_{k-l}}\overline{\delta}_{i_{k+1-l}}}y_l\right],$$

for $l = 0, \dots, k$, where y_l is the surplus left to be shared after players i_j $(j > k+1-l)$ have taken their shares.

This lemma follows from Watson (1998, Theorem 1).[19] Since we allow for general probability distributions over discount factors, multiplicity of perfect Bayesian equilibria (PBE) is not an exception (even when the game is almost with complete information).

In each bilateral bargaining session (i, j), the maximum real time player j would spend bargaining is the time $D(i, j)$ such that player j is indifferent between getting her lower bound PBE payoff at time 0 and getting her upper bound PBE payoff at time $D(i, j)$. Hence, the maximum number of bargaining periods player i_{k+1-l} would spend negotiating in the bilateral bargaining session (i_{k-l}, i_{k+1-l}), $I\left(m(i_{k-l}, i_{k+1-l})\right)$, is given by

$$\frac{\underline{\delta}_{i_{k+1-l}}\left(1-\overline{\delta}_{i_{k-l}}\right)}{1-\overline{\delta}_{i_{k-l}}\underline{\delta}_{i_{k+1-l}}}y_l = \left(\overline{\delta}_{i_{k+1-l}}\right)^{m(i_{k-l},i_{k+1-l})}\frac{\overline{\delta}_{i_{k+1-l}}\left(1-\underline{\delta}_{i_{k-l}}\right)}{1-\underline{\delta}_{i_{k-l}}\overline{\delta}_{i_{k+1-l}}}y_l,$$

from which we obtain

$$m(i_{k-l}, i_{k+1-l}) = \frac{1}{\log(\overline{\delta}_{i_{k+1-l}})}\log\left[\frac{\underline{\delta}_{i_{k+1-l}}}{\overline{\delta}_{i_{k+1-l}}}\frac{1-\overline{\delta}_{i_{k-l}}}{1-\underline{\delta}_{i_{k-l}}}\frac{1-\underline{\delta}_{i_{k-l}}\overline{\delta}_{i_{k+1-l}}}{1-\overline{\delta}_{i_{k-l}}\underline{\delta}_{i_{k+1-l}}}\right].$$

Notice that $I\left(m(i_{k-l}, i_{k+1-l})\right)$ is simply the integer part of $m(i_{k-l}, i_{k+1-l})$. It is customary to express the players' discount factors in terms of discount rates, $r_i > 0$, and the length of the bargaining period, Δ, according to the formula $\delta_i = \exp(-r_i\Delta)$. With this interpretation, player i's type is identified with the discount rate r_i, where $r_i \in [\underline{r}_i, \overline{r}_i]$. We thus have that $\underline{\delta}_i = \exp(-\overline{r}_i\Delta)$ and $\overline{\delta}_i = \exp(-\underline{r}_i\Delta)$. Note that $\overline{r}_i \ge \underline{r}_i$ since greater patience implies a lower discount rate. As Δ approaches zero, using l'Hopital's rule we obtain that

$$\begin{aligned} D(i_{k-l}, i_{k+1-l}) &= \lim_{\Delta\to 0}\left(m(i_{k-l}, i_{k+1-l})\cdot\Delta\right) \\ &= -\frac{1}{\underline{r}_{k+1-l}}\cdot\log\left[\frac{\underline{r}_{k-l}}{\overline{r}_{k+1-l}}\cdot\frac{\overline{r}_{k-l}+\underline{r}_{k+1-l}}{\underline{r}_{k-l}+\overline{r}_{k+1-l}}\right], \end{aligned}$$

which is a positive, finite number. Notice that $D(i_{k-l}, i_{k+1-l})$ converges to zero as $\underline{r}_i$ and $\overline{r}_i$ become close. We have $\partial D(i_{k-l}, i_{k+1-l})/\partial \underline{r}_{i_{k+1-l}} < 0$, $\partial D(i_{k-l}, i_{k+1-l})/\partial \overline{r}_{i_{k+1-l}} > 0$, $\partial D(i_{k-l}, i_{k+1-l})/\partial \underline{r}_{i_{k-l}} < 0$ and $\partial D(i_{k-l}, i_{k+1-l})/\partial \overline{r}_{i_{k-l}} > 0$. Given the equilibrium trading path $(s, i_1, i_2, \dots, i_k, b)$, the maximum real delay time in reaching a global agreement is $D(s, i_1, i_2, \dots, i_k, b) = D(s, i_1) + D(i_1, i_2) + \dots + D(i_k, b)$.

[19] Watson (1998) has characterized the set of perfect Bayesian equilibrium (PBE) payoffs which may arise in Rubinstein's alternating-offer bargaining game and constructed bounds (which are met) on the agreements that may be made. The bounds and the PBE payoffs set are determined by the range of incomplete information and are easy to compute because they correspond to the SPE payoffs of two bargaining games with complete information. These two games are defined by matching one player's most impatient type with the opponent's most patient type.

References

Abreu, D., Manea, M., 2012. Bargaining and efficiency in networks. J. Econom. Theory 147, 43–70.
Babus, A., Hu, T.W., 2015. Endogenous Intermediation in Over-the-Counter Markets. Mimeo, Northwestern University, USA.
Bala, V., Goyal, S., 2000. A strategic analysis of network reliability. Rev. Econ. Des. 5, 205–228.
Blume, L.E., Easley, D., Kleinberg, J., Tardos, E., 2009. Trading networks with price-setting agents. Games Econom. Behav. 67, 36–50.
Cai, H., 2003. Inefficient Markov perfect equilibria in multilateral bargaining. Econom. Theory 22, 583–606.
Calvo-Armengol, A., 2003. A decentralized market with trading links. Math. Social Sci. 45, 83–103.
Chae, S., Yang, J.-A., 1994. An N-person pure bargaining game. J. Econom. Theory 62, 86–102.
Condorelli, D., Galeotti, A., 2012. Endogenous Trading Networks. Mimeo, University of Essex, UK.
Condorelli, D., Galeotti, A., Renou, L., 2015. Bilateral Trading in Networks. Mimeo, University of Essex, UK.
Corominas-Bosch, M., 2004. Bargaining in a network of buyers and sellers. J. Econom. Theory 115, 35–77.
Craig, B., von Peter, G., 2014. Interbank tiering and money center banks. J. Financ. Intermed. 23, 322–347.
Cramton, P.C., 1992. Strategic delay in bargaining with two-sided uncertainty. Rev. Econom. Stud. 59, 205–225.
Dutta, B., Ghosal, S., Ray, D., 2005. Farsighted network formation. J. Econom. Theory 122, 143–164.
Easley, D., Kleinberg, J., 2010. Networks, Crowds and Markets: Reasoning About a Highly Connected World. Cambridge University Press, New York, NY, USA.
Elliott, M., 2015. Inefficiencies in networked markets. Amer. Econ. J.: Microecon. 7, 43–82.
Gale, D.M., Kariv, S., 2009. Trading in networks: a normal form game experiment. Amer. Econ. J.: Microecon. 1, 114–132.
Gofman, M., 2014. A Network-based Analysis of Over-the-Counter Markets. Mimeo, University of Wisconsin, USA.
Goyal, S., 2007. Connections: An Introduction to the Economics of Networks. Princeton University Press, Princeton, NJ, USA.
Goyal, S., Joshi, S., 2003. Networks of collaboration in oligopoly. Games Econom. Behav. 43, 57–85.
Goyal, S., Vega-Redondo, F., 2007. Structural holes in social networks. J. Econom. Theory 137, 460–492.
Herings, P.J.-J., Mauleon, A., Vannetelbosch, V., 2009. Farsightedly stable networks. Games Econom. Behav. 67, 526–541.
Hojman, D.A., Szeidl, A., 2008. Core and periphery in networks. J. Econom. Theory 139, 295–309.
Huang, C.-Y., 2002. Multilateral bargaining: conditional and unconditional offers. Econom. Theory 20, 401–412.
in't Veld, D., van der Leij, M., Hommes, C., 2014. The formation of a core periphery structure in heterogeneous financial networks. Tinbergen Institute Discussion Paper 2014-098, The Netherlands.
Jackson, M.O., 2008. Social and Economic Networks. Princeton University Press, Princeton, NJ, USA.
Jackson, M.O., Wolinsky, A., 1996. A strategic model of social and economic networks. J. Econom. Theory 71, 44–74.
Kranton, R.E., Minehart, D.F., 2001. A theory of buyer–seller networks. Amer. Econ. Rev. 91, 485–508.
Krishna, V., Serrano, R., 1996. Multilateral bargaining. Rev. Econom. Stud. 63, 61–80.
Li, D., 2010. A multilateral telephone bargaining game. Econom. Lett. 108, 43–45.
Manea, M., 2011. Bargaining in stationary networks. Amer. Econ. Rev. 101, 2042–2080.
Manea, M., 2015. Intermediation and Resale in Networks. Mimeo, MIT, USA.
Mauleon, A., Sempere-Monerris, J.J., Vannetelbosch, V., 2011. Networks of manufacturers and retailers. J. Econ. Behav. Organ. 77, 351–367.
Page Jr., F.H., Wooders, M., 2009. Strategic basins of attraction, the path dominance core, and network formation games. Games Econom. Behav. 66, 462–487.
Polanski, A., 2007. Bilateral bargaining in networks. J. Econom. Theory 134, 557–565.
Polanski, A., Vega-Redondo, F., 2014. Bargaining and Arbitrage in Endogenous Trading Networks. Mimeo, Bocconi University, Italy.
Rubinstein, A., 1982. Perfect equilibrium in a bargaining model. Econometrica 50, 97–109.
Siedlarek, J.P., 2015. Intermediation in Networks. Mimeo, University of Manheim, Germany.

Suh, S.-C., Wen, Q., 2006. Multi-agent bilateral bargaining and the Nash bargaining solution. J. Math. Econom. 42, 61–73.
Suh, S.-C., Wen, Q., 2009. A multi-agent bilateral bargaining model with endogenous protocol. Econom. Theory 40, 203–226.
Vannetelbosch, V., 1999. Rationalizability and equilibrium in N-person sequential bargaining. Econom. Theory 14, 353–371.
Wang, P., Watts, A., 2006. Formation of buyer–seller trade networks in a quality-differentiated product market. Canad. J. Econom. 39, 971–1004.
Watson, J., 1998. Alternating-offer bargaining with two-sided incomplete information. Rev. Econom. Stud. 65, 573–594.
Xiao, J., 2015. Bargaining Order in a Multi-person Bargaining Game. Mimeo, Department of Economics, University of Melbourne, Australia.