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**CORE**

**DISCUSSION PAPER**

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## Alliance Formation in a Vertically Differentiated Market

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### Abstract

This paper studies how the possibility for firms to sign collusive agreements (as for instance being part of alliances, cartels and mergers) may affect their quality and price choice in a market with vertically differentiated goods. For this purpose we model the firm decisions as a three-stage game in which, at the first stage, firms can form an alliance via a sequential game of coalition formation and, at the second and third stage, they decide simultaneously their product qualities and prices, respectively. In such a setting we study whether there exist circumstances under which either full or partial collusion can be sustained as a subgame perfect Nash equilibrium of the coalition formation game. Also, we analyse the effects of different coalition structures on equilibrium qualities, prices and profits accruing to firms. It is shown that only intermediate coalition structures arise at the equilibrium, with the bottom quality firm always included. Moreover, all equilibrium price and quality configurations always coincide with that observed in the duopoly case, with only two quality variants on sale.

**Keywords:** Vertically differentiated market, endogenous alliance formation, coalition structures, price collusion, grand coalition, coalition stability, sequential games of coalition formation.

**JEL Classification:** D42, D43, L1, L12, L13, L41.

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# 1 Introduction

This paper studies how the possibility for firms to sign collusive agreements (as for instance being part of alliances, cartels and mergers) may affect their quality and price choice in a market with vertically differentiated goods. It is worth mentioning that, although collusive agreements among firms are rarely persecuted by courts on quality grounds, the issue of goods quality frequently arises in anti-trust trials.<sup>1</sup> One case of quality restriction related to firms' collusive agreements is, for instance, reported by Yanich (2010, 2011, 2013) who analyses in detail the Shared Service Agreements (SSA) signed by some TV channels in US, as CBS, NBC, FOX, CW, with the aim to limit their duplicative broadcasting costs and coordinating their journalists' crew and editorial staff. Yanich (2011) reports how a major effect of these agreements is a drastic reduction of broadcast news varieties. The data gathered from a sample of 17 channels also shows that, in general, ownership concentration may significantly explain the average diversity of broadcast news, with consolidated stations producing less local content than independent channels.<sup>2</sup>

Somehow opposite results were obtained studying a sample of US radio stations. Berry and Waldfogel (2001) found evidence that the mergers followed to the 1996 Telecommunications Act drastically reduced the number of stations without, however, reducing their apparent product variety and, actually, slightly increasing it. Similar evidence is reported by Sweeting (2010) for the radio music industry and by George (2007) for the US newspapers.<sup>3</sup>

However, sparse evidence suggests that, in many cases, the entry of new competitors in the market pushes the incumbent to reduce his product variants. Johnson and Myatt (2003) detail a few cases in which the incumbent adjusts its product line in response to a new entrant, in particular withdrawing some of the products from the market ("pruning" its product lines).<sup>4</sup>

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<sup>1</sup>This is likely to depend on the difficulty for the court to ascertain whether quality re-shuffling ultimately depends on firms' collusion. See, for instance, McMillan (2015, pp.1921-1922) who analyses all recent US antitrust trials in which quality issues arise. As a matter of fact, US Antitrust Law prohibits "all unreasonable restraints on trade, regardless of whether they are based on price, quantity or quality".

<sup>2</sup>The channels participating to SSAs are often reported to simulcast identical content. Such content is, however, judged of higher quality by the public than before the agreements (using average advertising prices as quality indicators). See Yanich (2010).

<sup>3</sup>Using data on the assignment of reporters to topical areas at 706 newspapers in the US, George (2007) observes that differentiation increases with ownership concentration. Sweeting (2010) finds instead that those firms that buy competing stations tend to emphasize "service differentiation" among themselves as well as from the competing stations.

<sup>4</sup>For instance Timex recently removed a few of its lower-priced watches from the Indian wristwatch market and Mitsubishi phased-out the low-end versions of its Trium mobile phones. Many more cases of incumbent firms "pruning" their product lines are reported in Johnson and Myatt (2003).

A new entrant in the market can either lead to the incumbent's exit from lower markets or, alternatively, force him to introduce a low quality fighting brand,<sup>5</sup> which allows the firm to be competitive in lower markets, still preserving margins on the high quality good.<sup>6</sup>

In some other cases, merging firms have been observed to re-shape their products quality in more sophisticated ways. Giraud-Heraud *et al.* (2003) describes the case of European mineral water market consolidation process occurring in the 90s, and driving its main actors (Nestlé and Danone) to reshuffle their brands so as to form portfolios of directly substitutable products.

Whatever the strategies adopted by the firms, the formation of alliances between firms appears as strictly interlinked with the choice of their product quality. In order to study the nature of this link, in this paper we introduce a model of endogenous alliance formation in which firms can endogenously select the quality and price of their products. As in some of the cases mentioned above, in our model colluding firms may possess an incentive to reduce the variants on sale, to soften the existing price competition among firms in the alliance, as well as that with those competing outside. We also find that in some cases an alliance of firms playing against an independent firm can find profitable to adopt a *leapfrogging* strategy, so as to enhance the vertical differentiation within the alliance and in the market as a whole. In particular, using a three-firm model, we are able to characterize all equilibrium alliance structures arising in a vertically differentiated market. In particular, we show that when the process of alliance formation is sequential (as in a bargaining model *à la* Rubinstein, 1982), the temptation of every firm to free-ride and to remain independent can lead the firms to form in equilibrium only intermediate coalition structures, with only two qualities remaining on sale, whereas the whole industry alliance turns out to be unstable.

## 1.1 Related Literature

The relationship between *collusive agreements* and *vertical product differentiation* was formerly analysed by Hackner (1994). In his work, the key question is whether *price collusion* is more likely to arise when products are close substitutes or, rather, when they are highly differentiated. In a natural duopoly setting, he finds that monopoly pricing is easier to sustain in markets in which products are similar. Further, he proves that the incentive to deviate from a

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<sup>5</sup>For a duopoly with quantity competition Johnson and Myatt (2003) show that these two cases can be, in turn, due to the decreasing marginal revenues or nonmonotone marginal revenue of the firms.

<sup>6</sup>After Hewlett-Packard's entry into the market, IBM introduced its LaserJet IIP, a lower-quality substitute for IBM LaserPrinter. Similarly, after the entry of low-cost airlines British Airways initially concentrated its efforts on the high-segment of the market, deciding afterward to establish Go, its low-cost airline supposed to compete with Easyjet and Ryanair in the economy flight market.

collusive agreement is always stronger for the high-quality firm. The main reason is that when the quality gap between products is significant, the profit of the top-quality firm is already high under no collusion, so that its incentive to collude is weak. As the quality gap decreases and the noncooperative payoff become smaller for the high-quality firm, reaching a collusive agreement gets more and more attractive. Along the same research line, Ecchia and Lambertini (1997) study how the stability of price collusion in a duopoly setting is affected by the introduction of a minimum quality standard. They observe how the introduction of a welfare-maximizing minimum quality standard makes collusive agreements more difficult to sustain. This is because the existence of a standard decreases product differentiation by providing the bottom quality firm with a stronger incentive to break the agreement.<sup>7</sup>

There are two common traits in these works. First, (i) the degree of product differentiation does not change after a coalition has formed, since the collusive behavior is restricted to pricing. The former assumption is a natural entry point in the literature on cartel stability under product differentiation, as it enables to disentangle the effect of quality gap on the stability of a cartel. Further, conceiving collusion in terms of pricing is particularly reasonable in a short-run perspective. Still, it leaves unexplored a companion question, namely the effect of the cartel on product differentiation. This analysis is particularly pregnant in a long-run perspective since one cannot exclude that in a more extended time span a coalition (typically a cartel or a merger) entails structural changes, such as relocations of production facilities, or adjustment in the product range and quality.

Secondly, (ii) the market is populated by two firms so that it turns out to be fully monopolized by a *grand coalition* in the case of cooperation between firms.<sup>8</sup> While considering at the start a duopoly enables to detail the effects of a full cooperation, casual observations show that, there exist circumstances under which firms choose to form an *intermediate* alliance (i.e. one including a subset of firms in the market) rather than the *grand coalition*. While in an intermediate alliance, colluding firms compete against some rivals outside the coalition so that a noncooperative behavior is still preserved. Of course, *a priori* the effects of a partial alliance are not equivalent to those observed when all agents collude and mimic a monopolist.

To the best of our knowledge, the possibility that firms cooperate both along a price dimension and a quality dimension in a vertically differentiated market has been investigated only by Lambertini (2000). He studies how the cartel stability is related to R&D activity in a

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<sup>7</sup>In Hackner, the opposite holds since, due to the cost structure, in his model the asymmetry in profits gives an advantage to the high quality firm.

<sup>8</sup>The grand coalition is the one formed by *all* firms in the market.

duopoly with convex costs, and assumes that the collusive quality choice can occur either under price or quantity-setting behaviour.<sup>9</sup> The issue concerned with the alliance formation when more than two firms are active in a vertically differentiated market is however still unexplored, like so the impact of partial collusion on the market equilibrium. The introduction of an intermediate quality firm sheds light on some interesting features of the coalition formation process. As far as we know, the only model of vertical differentiation with three independent firms competing in quality and price is provided by Scarpa (1998).<sup>10</sup> Considering the role of a minimum quality standard, Scarpa (1998) stresses that the demand level of a firm in a vertically differentiated market depends on quality and price of adjacent firms in the product space. This property, reminiscent of a spatial competition approach, is rather interesting when considering the rationale adopted by the colluding firms to define the optimal range of variants. Indeed, since only adjacent variants compete against each other, under partial collusion defining the optimal set of products to market requires to put in balance *the cannibalization effect* that a variant produced by the coalition may exert *within* the coalition with the possibility that this variant *steals* consumers from the rival firm (henceforth *stealing effect*).<sup>11</sup>

## 1.2 Our Paper

In the present paper we remove both assumptions that collusion only develops along a price dimension and that the market is a duopoly. To this aim, we consider a vertically differentiated setting in which *three* firms produce different variants of the same product.

More specifically, we introduce a three-stage game where, at the first stage, every firm expresses its willingness to form an alliance or, alternatively, to play as singleton. An alliance can either contains all firms in the market (*grand coalition*) or only a subset of them (two firms colluding against the third one playing alone). As in Bloch (1995, 1996) and Ray and Vohra (1999) we assume that the coalition formation game is *sequential*. Differently from them, we assume that every firm proposes not only an alliance, but also a division of the coalition joint payoff.<sup>12</sup> Each recipient of the proposal can either accept or reject the offer and, in case of

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<sup>9</sup> A different strand of literature considers the possible impacts of R&D joint ventures on product market collusion. See on this, Martin (1995) and Lambertini *et al.* (2002).

<sup>10</sup> Pezzino (2010) analyses quantity competition among three firms in a vertically differentiated market.

<sup>11</sup> These effects resembles the so called *peer effect* and *pecking order effect*. The peer effect takes place when joining organization with high-quality agents increases the payoff of its members. This effects explains why outstanding researchers tend to join top research department. On the other hand, the pecking order effect takes place when the payoff an individual gets depends on his/her relative position in a ranking. Typically, people at the top in the pecking order have a greater chance to obtain further internal promotions.

<sup>12</sup> Bloch (1995, 1996) and Ray and Vohra (1999) assume a fixed allocation rule within each coalition. See

rejection, it becomes its turn to make a proposal. The game is assumed finite-horizon and every firm only possesses one turn of proposal.<sup>13</sup> Once a coalition structure has formed, at the second stage firms decide simultaneously the optimal quality of their products. When considering this issue, we take into account how the full or partial collusion among firms may affect their incentives to differentiate products in the market. Choosing the optimal quality after colluding, in turn, affects their incentives to collude. Finally, at the third stage, firms set simultaneously prices. When in an alliance, quality and price are set so as to maximize the joint profits of firms which belong to it. Notice also that, when colluding, firms can choose at the second stage (resp. third stage) to produce a quality so low (resp. to quote a price so high) that no consumer is willing to buy it. This is equivalent to stop producing the variant, thereby reducing the range of products sold at equilibrium.

We study whether there exist circumstances under which a partial collusion is preferred over the grand coalition (or noncooperation) and analyse the effects of such intermediate coalition structures arising at equilibrium in terms of quality, price and profits accruing to the firms.

We find that the incentive for firms to form a grand coalition is always dominated by that of colluding in intermediate coalition structures, which in our model emerge as subgame Nash equilibria of the sequential game of coalition formation. Furthermore, we prove that all equilibrium coalitions always contains the bottom quality firm which, in all cases, drops its low-quality variant from the market. In particular, whoever is the additional player included in coalition (either the intermediate or the top quality firm), equilibrium price and quality configurations always coincide with that observed in the case of a duopoly, with a high-quality firm competing against a low-quality rival, as in Motta (1993). At first sight, this result seems to be counterintuitive. A natural conjecture when considering that players producing different variants collude is that either the range of variants or the quality gap between variants in the market changes with the players involved in the alliance. We find on the contrary that only profits accruing to the single players change with the type of partial collusion, range of products, quality gap and price being unchanged. Indeed, the cannibalization effect and the stealing effect induce the coalition, whatever its members, to withdraw from the market the lowest quality variant between the set which can be produced *a priori*. Interestingly, depending on the intensity of these effects, in some circumstances this variant is withdrawn from the

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also Belleflamme (2000) for an extension of the model to the formation of asymmetric alliances, and also Bloch (2003, 2004), Marini (2009) and Currarini and Marini (2014) for extensive surveys on alliance formation models applied to oligopoly.

<sup>13</sup>Both Bloch's (1995, 1996) and Ray and Vohra's (1999) models are, instead, infinite-horizon.



market at the price stage, in some other circumstances at the quality stage. In addition, our results on the stability of intermediate coalition structures are fully in line with many theoretical and experimental studies on coalition formation in triads of heterogeneous individuals, i.e. possessing different skills or fighting ability (e.g. Caplow 1956, 1959, 1968, Vinacke and Arkoff 1957, Gamson 1961). As a matter of facts, a central conclusion of these studies is that “weakness is strength” (see, for instance, Mesterton-Gibbons *et al.* 2011, p.189), with this meaning that less-powered individuals have usually more chances to be part of a coalition. We obtain the same result with firms competing in a vertically differentiated market.<sup>14</sup>

The structure of the paper is as follows. Section 2 briefly introduces the paper setting. Section 3 describes in detail the various equilibrium market configurations, the *noncooperative* case,<sup>15</sup> the *fully collusive* case and all different cases of *partial collusion*. Section 4 characterizes all equilibria of the alliance formation game. Section 5 briefly concludes. Most of the proofs are gathered in the Appendix.

## 2 The Model

As mentioned in the introduction, firms are assumed to play a three-stage game: (i) an *alliance formation* (sub)game (stage 1) assumed sequential; (ii) a *market* (sub)game including a *quality stage* (stage 2) and a *price stage* (stage 3). The next section is devoted to introduce the alliance formation game.

### 2.1 The Alliance Formation Game

The game of alliance formation occurs at the first stage of the game, before firms choose qualities and prices. Following Bloch (1995, 1996) and Ray and Vohra (1999) we model the process of coalition formation as a *sequential* unanimity game in which, in a given order, firms propose to their rivals an alliance to which they belong. The firm with the lowest index among those receiving the proposal may, in turn, either accept or reject it. In case of acceptance, the turn passes to the subsequent firm in the proposed alliance and, if all firms accept, the alliance is irrevocably formed and its members decide cooperatively qualities and prices. If, alternatively, one of the firms rejects the offer, it becomes its turn to make a proposal and the game continues with the same logic until a given coalition structure is obtained. Differently

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<sup>14</sup>Note that in a repeated Cournot game with three heterogeneous firms, Garella and Richelle (1999) obtain that only one stable cartel exists, always containing the firm with the highest average cost.

<sup>15</sup>Part of this analysis is also contained in Scarpa (1998).

from Bloch (1996) and Ray and Vohra (1999) and following Selten (1981) and Chatterjee *et al.* (1993) we let the allocation rule be part of the bargaining process.<sup>16</sup> Specifically, when it is its turn to offer, a firm proposes both an *alliance* and a *division* of the alliance profit among its members. A second distinction of our game with respect to Bloch's (1996) and Ray and Vohra's (1999) is that in our case the alliance formation game is a one-shot game in which every player can make *at most* one proposal. This means that once one player has proposed an alliance and has been rejected, he can enter a given alliance only if it is proposed by someone else, remaining singleton otherwise. For this game, we look for all profiles of strategies which are subgame perfect Nash equilibria.

Formally, our *alliance formation game* is a triple  $G = (N, \{\Sigma_k, \Pi_k\}_{k \in N})$ , with player set  $N = \{1, 2, 3\}$ , strategy set  $\Sigma_k$  and payoff  $\Pi_k(\sigma) : \Sigma \rightarrow \mathcal{R}$ . For every firm (player)  $k \in N$ , a strategy  $\sigma_k \in \Sigma_k$  defines the *actions*  $a_k \in A_k$  available at any node (or information set  $I_k \in \mathcal{I}_k$ ) in which it is its turn to play. In our game, an *action* for a firm  $k \in N$  can either be an element of the set  $\{Yes, No\}$  coming in response to another firm's proposal  $p_j$  for  $j \neq k$  or, in turn, a proposal  $p_k = (S, \Pi)$  including an alliance  $S \subset N$  to which  $k$  belongs and a division  $\Pi \in \mathcal{R}^{|S|}$  of the alliance joint profit  $\Pi_S$ , such that  $\sum_{h \in S} \Pi_h = \Pi_S$ . Thus, for a firm a *strategy*  $\sigma_k \in \Sigma_k$  is a mapping from its information sets to the set of its feasible actions  $A_k$  available therein, namely,  $f(I_k) : \mathcal{I}_k \rightarrow A_k$ , where  $A_k \subset \left( (2^N \setminus \{\emptyset\}, \mathcal{R}^{|S|}) \cup \{Yes, No\} \right)$ , with the property that a proposal  $p_k \in (2^N \setminus \{\emptyset\}, \mathcal{R}^{|S|})$  can be made by a firm only if, at its turn of play, there are no other players' proposals on the floor and the firm itself has not already made a proposal. That is, for every firm  $k \in N$  the action available at every information set  $I_k^t$  is  $a_k(I_k^t) = p_k$  if both  $p_j(I_j^t) = \emptyset$  for  $j \neq k$  and  $p_k(\{I_k^\tau\}_{\tau < t}) = \emptyset$  for any previous information set, and  $a_k(I_k^t) \in \{Yes, No\}$  otherwise. Note that every *strategy profile*  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  of  $G$  induces an *outcome*  $O(\sigma) = (C(\sigma), \Pi(\sigma))$ , namely a coalition structure  $C \in \mathcal{C}$  and a profile of payoffs  $\Pi = (\Pi_1, \Pi_2, \Pi_3)$  assigned to the firms in  $C$ . The payoff of every firm  $\Pi_k(p(v)) \in \Pi$  is obtained by associating to each coalition structure  $C$  a price-quality equilibrium profile  $p(v)$  which will be described in Section 3. As last step, we need to define a *subgame perfect Nash equilibrium* (SPE) of the alliance formation game and, accordingly, a notion of *stable coalition structure*.

**Definition 1** *A subgame perfect Nash equilibrium (SPE) of the alliance formation game is a strategy profile  $\sigma$  such that, for every firm  $k \in N$ , for every proper subgame  $G' \subset G$ , and for every  $\sigma_k \in \Sigma_k$ ,  $\Pi_k(\sigma_k^*, \sigma_{-k}^*) \geq \Pi_k(\sigma_k, \sigma_{-k}^*)$ .*

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<sup>16</sup>The same assumption is also made in Moldovanu's (1992).three-player coalition formation game.

**Definition 2** A coalition structure  $C \in \mathcal{C}$  (a partition of the  $N$  firms) is stable if and only if it is sustained by a SPE  $\sigma^*$  of the alliance formation game, namely,  $C = C(\sigma^*)$ .

## 2.2 The Market

Let us assume an uncovered market initially populated by three firms,  $k = 1, 2, 3$  selling three vertically differentiated goods  $v_H, v_M, v_L$  with  $v_H > v_M > v_L$ .<sup>17</sup> Also,  $v_i \in [0, \bar{v}]$ , where  $\bar{v} \in \mathcal{R}_+$  is the highest quality level which is technologically feasible.<sup>18</sup> There exists a quality specific fixed cost, say  $C_i = \frac{1}{2}v_i^2$ . Consumers are indexed by  $\theta$  and uniformly distributed in the interval  $[0, 1]$  with density 1.<sup>19</sup> The parameter  $\theta$  captures consumers' willingness to pay (henceforth WTP) for quality: the higher  $\theta$ , the higher the corresponding WTP. Each consumer can either buy one variant or not buying at all. Formally, consumers' utility can be written as

$$U(\theta) = \begin{cases} \theta v_i - p_i & \text{if he/she buys variant } i \\ 0 & \text{if he/she refrains from buying.} \end{cases}$$

From the above formulation, the consumer indifferent between buying variant  $i$  and not buying is:

$$\theta_i = \frac{p_i}{v_i},$$

while the consumer indifferent between buying variant  $i$  and  $i + 1$  is:<sup>20</sup>

$$\theta_i = \frac{p_i - p_{i+1}}{v_i - v_{i+1}}.$$

Without loss of generality, we can assume that in the noncooperative setting, firm 1 is at the top, firm 2 at the intermediate level and firm 3 at the bottom of the quality ladder. Thus, firms 1, 2 and 3 produce variant  $v_H, v_M$  and  $v_L$ , respectively.

Of course, since qualities are endogenously defined it can happen that under coalition this apparently innocuous quality assignment no longer holds. For example, one can observe that the when firm 3 is involved in a collusive agreement, it finds it profitable to fix its quality in such a way that it leapfrogs rival 2 in terms of quality. In order to capture this possibility

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<sup>17</sup>Since the market is always *endogenously* uncovered in the case of full collusion, the assumption of uncovered market, that some of the consumers refrain from buying goods, appears in our model as the most natural one (cfr. Section 3.3).

<sup>18</sup>We share this assumption on the quality interval with Wauthy (1996).

<sup>19</sup>Considering an interval  $[0, m]$  simply leads to the addition of a parameter on which prices, quantities and quality levels depend linearly, with no substantial changes in the payoff structure (see for instance Scarpa 1998).

<sup>20</sup>We easily deduce the expression of the indifferent consumer from:  $U_L(\theta) = U_M(\theta)$  and  $U_M(\theta) = U_L(\theta)$

without weighting the notation down, we write the demand function for firms  $k = 1, 2, 3$ , when producing  $v_H$ ,  $v_M$ , and  $v_L$  respectively as:

$$\begin{aligned} D_H &= (1 - \theta_H) \\ D_M &= (\theta_H - \theta_M) \\ D_L &= (\theta_M - \theta_L). \end{aligned}$$

Then, the corresponding profit functions are:

$$\Pi_H = \left(1 - \frac{p_H - p_M}{v_H - v_M}\right) p_H - \frac{1}{2} v_H^2 \quad (1)$$

$$\Pi_M = \left(\frac{p_H - p_M}{v_H - v_M} - \frac{p_M - p_L}{v_M - v_L}\right) p_M - \frac{1}{2} v_M^2 \quad (2)$$

$$\Pi_L = \left(\frac{p_M - p_L}{v_M - v_L} - \frac{p_L}{v_L}\right) p_L - \frac{1}{2} v_L^2. \quad (3)$$

Note that we will add a subscript  $k$  to each generic function  $F$  and variable  $f$ , namely  $F_{k,i}$  and  $f_{k,i}$ , only when the above quality assignment does not hold. For example, if under a collusive agreement it is firm 3 to produce the intermediate quality variant (instead of firm 2 as in the noncooperative setting), we use notation  $D_{3,M}$  to denote the demand function for firm 3 when producing the intermediate quality  $v_{3,M}$  and  $\Pi_{3,M}$  for corresponding profits.

### 3 Equilibrium Analysis: Prices and Qualities

Since the whole game is solved backward, for the sake of simplicity we can start characterizing the two final stages of the game. In particular, we firstly present the case in which firms decide *noncooperatively* prices and qualities (*non cooperative equilibrium*); secondly, we turn to the case in which the grand coalition decide prices and qualities (*full collusion*); finally, we look at what happens when firms form intermediate coalitions (*partial collusion*). Since prices are more easily adjusted than qualities, it is reasonable to assume that firms define qualities at the second stage (quality choice stage) and fix prices at the third one (price stage).

The game is solved by backward induction. So, we consider first the price stage under the assumption that qualities have been fixed. Then, we move to the quality stage.

### 3.1 Noncooperative equilibrium

In this section, we briefly summarize price and quantity equilibrium obtained when the three firms compete in the market against each other, while referring the interesting reader to Scarpa (1998) for further details. We assume that at the first stage, no collusive agreement has been reached so that firms decide their quality and then their price in a fully noncooperative fashion.

#### 3.1.1 Price stage

At the price stage, given that costs are fixed, we can study the noncooperative price behaviour of the three firms by simply characterizing their revenue functions in the quality spectrum: (i) top quality  $H$ , (ii) intermediate quality  $M$  and (iii) bottom quality  $L$ . Let us assume here, without loss of generality, that firm 1 has chosen at the quality stage to produce the top quality, firm 2 the intermediate quality and firm 3 the bottom quality.

Thus differentiating (1), (2) and (3) w.r.t  $p_H$ ,  $p_M$ , and  $p_L$ , respectively, we can easily derive all firms' best-replies as:

$$p_H(p_M) = \frac{1}{2} (p_M + (v_H - v_M)), \quad (4)$$

$$p_M(p_H, p_L) = \frac{1}{2} \frac{(p_H(v_M - v_L) + p_L(v_H - v_M))}{v_H - v_L} \quad (5)$$

and

$$p_L(p_M) = \frac{1}{2} p_M \frac{v_L}{v_M}. \quad (6)$$

As stressed by Scarpa (1996), the best-reply function of a firm depends on the quality and price of the firm itself and of its neighboring rivals, while products that are farther away in the product space do not play any role. From the above, equilibrium prices  $p_i$  at the price stage are obtained as:

$$p_H^*(v_H, v_M, v_L) = \frac{1}{2} \frac{(v_H - v_M)(4v_H v_M - v_H v_L - 3v_L v_M)}{(4v_H v_M - v_H v_L - 2v_L v_M - v_M^2)} \quad (7)$$

$$p_M^*(v_H, v_M, v_L) = \frac{(v_H - v_M)(v_M - v_L)v_M}{(4v_H v_M - v_H v_L - 2v_L v_M - v_M^2)} \quad (8)$$

$$p_L^*(v_H, v_M, v_L) = \frac{1}{2} \frac{(v_H - v_M)(v_M - v_L)v_L}{(4v_H v_M - v_H v_L - 2v_L v_M - v_M^2)}, \quad (9)$$

with corresponding profits

$$\Pi_H(p^*(v_H, v_M, v_L)) = \frac{1}{4} \frac{(v_H - v_M)(v_H v_L - 4v_H v_M + 3v_L v_M)^2}{(v_M^2 + v_H v_L - 4v_H v_M + 2v_L v_M)^2} - \frac{1}{2} v_H^2 \quad (10)$$

$$\Pi_M(p^*(v_H, v_M, v_L)) = v_M^2 \frac{(v_H - v_M)(v_M - v_L)(v_H - v_L)}{(v_M^2 + v_H v_L - 4v_H v_M + 2v_L v_M)^2} - \frac{1}{2}v_M^2 \quad (11)$$

$$\Pi_L(p^*(v_H, v_M, v_L)) = \frac{1}{4} \frac{v_L (v_H - v_M)^2 (v_M - v_L) v_M}{(v_M^2 + v_H v_L - 4v_H v_M + 2v_L v_M)^2} - \frac{1}{2}v_L^2, \quad (12)$$

where  $p^* = (p_H^*, p_M^*, p_L^*)$  denote Nash equilibrium prices of firms obtained at the price stage (stage 3). Let us now consider the choice of quality levels by firms in the interval  $[0, \bar{v}]$ .

### 3.1.2 Quality stage

In order to characterize the Nash equilibrium quality choices occurring at the second stage, it suffices to maximize payoff function (10), (11) and (12) w.r.t. quality  $v_H, v_M, v_L$ , respectively, thereby getting:

$$v_H^* = 0.2526, \quad v_M^* = 0.0497, \quad v_L^* = 0.0095. \quad (13)$$

Moreover, the corresponding subgame perfect Nash equilibrium prices  $p^*(v^*)$  and profits  $\Pi_i(p^*(v^*))$ , for  $v^* = (v_H^*, v_M^*, v_L^*)$ , are immediately obtained as:

$$p_H^*(v^*) = 0.10601, \quad p_M^*(v^*) = 0.0091297, \quad p_L^*(v^*) = 0.00087255, \quad (14)$$

$$\Pi_H(p_H^*(v^*)) = 0.023489, \quad \Pi_M(p_H^*(v^*)) = 0.0012491, \quad \Pi_L(p_H^*(v^*)) = 0.000053956. \quad (15)$$

## 3.2 Collusion

By definition a collusive agreement can either involve the set of all firms, denoted  $N = \{1, 2, 3\}$  (*grand coalition*) or, alternatively, any other nonempty subset  $S \subset N$  of them, with  $S \in \mathcal{N}$ , where  $\mathcal{N} = 2^N \setminus \emptyset$  is the set of all nonempty coalitions of the  $N$  firms, in this case simply:

$$\mathcal{N} = (\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}).$$

Thus, while if the firms form the grand coalition they commit irrevocably to jointly set qualities and prices so as to maximize the sum of all firms' profits (full cooperation), in the second scenario (partial collusion), a smaller subset of firms jointly decide qualities and prices, again irrevocably, so as to maximize the sum of their own profits, while competing against a rival(s), if any. In general, we can describe any type of (full or partial) firm collusion or noncooperative behaviour by simply indicating the *coalition structure*  $C = (S_1, S_2, \dots, S_m)$  representing a

collection of firms in alliances having null intersection and summing up to  $N$ , with  $m \leq n$ . The set  $\mathcal{C}$  of all coalition structures  $C$  that can be formed by the three firms is, therefore, simply given by:

$$\mathcal{C} = ((\{1\}, \{2\}, \{3\}), (\{1, 2\}, \{3\}), (\{1\}, \{2, 3\}), (\{1, 3\}, \{2\}), (\{1, 2, 3\})).$$

The game is solved backward so that we first analyse the price and then the quality stage under the assumption that either the grand coalition or any other intermediate coalition structure have formed at the first stage. After the full characterization of market equilibrium in any of these cases, we wonder which type of collusion (if any) will prevail in equilibrium.

### 3.3 Full Collusion

Let us assume that, at the first stage, the firms have formed the grand coalition. In the following, we consider the price and then the quality decision.

#### 3.3.1 Price stage

When the grand coalition  $\{N\}$  forms, at the price stage each firm maximizes the sum of all firms' payoffs (1)-(3) for arbitrary levels of the quality chosen at the second stage. Without loss of generality, henceforth we can keep the quality assignment that at the quality stage firm 1 has chosen to produce the top quality, firm 2 the intermediate quality and firm 3 the bottom quality product, by symmetry with the assumption made in the noncooperative scenario. Thus, by the price maximization of the joint payoff of the grand coalition, the firm *fully-collusive optimal replies*  $p_L^c$ ,  $p_M^c$  and  $p_H^c$  are obtained as

$$p_H^c(p_M) = p_M + \frac{1}{2}(v_H - v_M), \quad (16)$$

$$p_M^c(p_H, p_L) = \frac{p_H(v_M - v_L) + p_L(v_H - v_M)}{v_H - v_L}, \quad (17)$$

and

$$p_L^c(p_M) = p_M \frac{v_L}{v_M}. \quad (18)$$

By solving the system (16)-(18), an fully collusive optimal prices profile  $p^{\{N\}}(v)$ , for  $v = (v_H, v_M, v_L)$ , is obtained as:

$$p_H^{\{N\}}(v_H) = \frac{1}{2}v_H, \quad p_M^{\{N\}}(v_M) = \frac{1}{2}v_H, \quad p_L^{\{N\}}(v_L) = \frac{1}{2}v_L. \quad (19)$$

Given the above prices, the market share of any firm at the price stage, turns out to be:

$$D_H(p^{\{N\}}(v)) = \frac{1}{2}, \quad D_M(p^{\{N\}}(v)) = 0, \quad D_L(p^{\{N\}}(v)) = 0. \quad (20)$$

It is immediate to see that, at the prices selected by the grand coalition, consumers are willing to buy only the top quality variant  $v_H$ , the demand for the intermediate and bottom variants being nil. Accordingly, the profit accruing to the grand coalition at the price stage are

$$\Pi^{\{N\}}(p^{\{N\}}(v)) = \frac{1}{4}v_H - \frac{1}{2}v_H^2.$$

### 3.3.2 Quality stage

In order to fully characterize the behaviour of the grand coalition, we can easily find its optimal quality, given by  $v_H^{\{N\}} = 0.25$ , so that profit obtains as:

$$\Pi^{\{N\}}(p^{\{N\}}(v^{\{N\}})) = 0.03125. \quad (21)$$

The logic underlying this finding has been well described by Mussa and Rosen (1978): “Serving customers who place smaller valuations on quality creates negative externalities for the monopolist that limit possibilities for capturing consumer surplus from those who do value quality highly. (p.306)”.<sup>21</sup> Rather interestingly, this finding does not depend on the initial assumption on the market coverage. Indeed, even if one would develop the above analysis under the alternative assumption that the market is covered, still at the price-quality equilibrium the grand coalition would offer only the top-quality, while serving half of the market.

Finally, it is worth remarking that, under a full collusive behaviour, the level of prices is, for all firms, always higher than under Nash equilibrium. This can be easily checked by the following simple reasoning: (i) Start with the Nash equilibrium price of firm 1 and let the remaining firms responding using their optimal collusive replies (16)-(17). (ii) Since comparing (4)-(5) with (16)-(17) it turns out that optimal cooperative replies are twice as sloped as the noncooperative best-replies and both upward sloping, as effect of (i) all  $N \setminus \{1\}$  firms will

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<sup>21</sup> Further discussion on this result are provided by Gabszewicz *et al.* (1982) and by Gabszewicz and Wauthy (2002) under the assumption of zero quality costs. Along the same research line, Acharia (1998) shows that when the cost for quality improvement is not too convex, a multiproduct monopolist offers only the top variant among the ones which *a priori* can be sold in the market. Indeed, if the costs are not so significant, offering the top variant only allows firms to escape from the cannibalization effect which would take place if the more than one variant would be sold at equilibrium. Finally, Lambertini (1997) analyses the Mussa-Rosen’s model with quality specific variable costs under the alternative assumption of full market coverage and partial market coverage.



increase their prices. (iii) Let now also firm 1 respond cooperatively using its cooperative optimal reply (18) and, as a result, it will increase its price. (iv) By continuing the adjustment process of all firms along their collusive optimal replies, since these are all contractions (due to inequality  $v_H > v_M > v_L$ ), a new price profile  $p^{\{N\}}$  will be reached with the property that  $p^{\{N\}} \gg p^*$ , where  $p^*$  is the corresponding profile of noncooperative Nash equilibrium prices.

### 3.4 Partial collusion

In this section we analyse all market configurations arising when partial collusion takes place among firms. We characterize three different market scenarios occurring, in turn, under the following coalition structures: (i)  $C_{1,23} = (\{1\}, \{2, 3\})$ , (ii)  $C_{13,2} = (\{1, 3\}, \{2\})$  and, finally (iii)  $C_{12,3} = (\{1, 2\}, \{3\})$ . Without loss of generality, we can start the analysis by assuming that, firms 1, 2 and 3 produce the high, intermediate and low quality variants, respectively. This assumption is in line with the quality assignment in the noncooperative setting. We then verify whether at the second stage of the game such an assignment remains optimal for firms in partial collusive agreements or if, conversely, a quality reversal can take place.

Before describing in detail the price and quality behaviour of firms under partial collusion, note that from (1)-(3) when either the bottom quality firm or the top quality firm collude in prices with their *direct* competitor, i.e. the intermediate quality firm, they just behave as in the fully collusive case, with optimal replies given by (16) and (18), respectively. On the other hand, when bottom and quality firms form a coalition, due to the structure of the vertical differentiation model, they set prices exactly as in the noncooperative case, with optimal replies given by (4) and (6). Thus, under partial collusion only the price behaviour of the firm producing the intermediate quality variant  $v_M$  (henceforth denoted *intermediate firm*) varies according on whether it is allied either with its left (lower quality) or with its right (higher quality) competitor. In particular, when the intermediate firm colludes with its left competitor, its first-order condition implies

$$\frac{\partial \Pi_M}{\partial p_M} + \frac{\partial \Pi_L}{\partial p_M} = \frac{2p_L - 2p_M}{v_M - v_L} + \frac{p_H - 2p_M}{v_H - v_M} = 0,$$

whereas, when it colludes with its right-competitor, it sets  $p_M$  such that

$$\frac{\partial \Pi_M}{\partial p_M} + \frac{\partial \Pi_H}{\partial p_M} = \frac{p_L - 2p_M}{v_M - v_L} + \frac{2p_H - 2p_M}{v_H - v_M} = 0.$$

As a result, the optimal reply of the *intermediate firm*,  $p_M^{lc}(p_L, p_H)$  in the left-partial (resp.

$p_M^{rc}(p_L, p_H)$  in the right-partial) collusion writes as

$$p_M^{lc}(p_L, p_H) = \frac{p_L(v_H - v_M) + \frac{1}{2}p_H(v_M - v_L)}{(v_H - v_L)} \quad (22)$$

$$(\text{resp. } p_M^{rc}(p_L, p_H) = \frac{\frac{1}{2}p_L(v_H - v_M) + p_H(v_M - v_L)}{(v_H - v_L)}). \quad (23)$$

### 3.4.1 Collusion between intermediate and bottom quality firms

We consider initially the scenario where at the first stage a collusive agreement has been reached by firm 2 and firm 3, with firm 1 playing as singleton against them. We assume, as a start, that variants  $v_M$  and  $v_L$  are produced by the colluding firms 2 and 3, respectively. Firm 1, outside the collusive agreement, produces the high quality variant  $v_H$ . This assumption is in line with the quality assignment holding in the noncooperative scenario. We can check later if this quality assignment is optimal at the equilibrium.

**Price stage** As coalition structure  $C_{1,23} = (\{1\}, \{2, 3\})$  forms, prices  $p_H$ ,  $p_M$  and  $p_L$  set at the last stage by firms 1, 2 and 3 are found from the maximization of the following objective functions

$$\begin{aligned} \Pi_{1,H} &= \left(1 - \frac{p_H - p_M}{v_H - v_M}\right) p_H \\ \Pi_{2,M} + \Pi_{3,L} &= \left(\frac{p_H - p_M}{v_H - v_M} - \frac{p_M - p_L}{v_M - v_L}\right) p_M + \left(\frac{p_M - p_L}{v_M - v_L} - \frac{p_L}{v_L}\right) p_L. \end{aligned}$$

Using (4), (18), and (22), the optimal replies are obtained, respectively, as

$$\begin{aligned} p_H^{pc}(p_M) &= \frac{1}{2}(p_M + (v_H - v_M)) \\ p_M^{pc}(p_H, p_H) &= \frac{p_L(v_H - v_M) + \frac{1}{2}p_H(v_M - v_L)}{(v_H - v_L)} \\ p_L^{pc}(p_M) &= \frac{v_L}{v_M} p_M. \end{aligned}$$

Therefore, the following equilibrium prices are set by firms:

$$\begin{aligned} p_H^{(\{1\},\{2,3\})}(v) &= \frac{2v_H(v_H - v_M)}{4v_H - v_M}, \\ p_M^{(\{1\},\{2,3\})}(v) &= \frac{v_M(v_H - v_M)}{4v_H - v_M}, \\ p_L^{(\{1\},\{2,3\})}(v) &= \frac{v_L(v_H - v_M)}{4v_H - v_M}, \end{aligned}$$

where  $v = (v_H, v_M, v_L)$ , with corresponding profits:

$$\begin{aligned} \Pi_H(p^{(\{1\},\{2,3\})}(v)) &= 4 \frac{v_H^2(v_H - v_M)}{(4v_H - v_M)^2} - \frac{1}{2}v_H^2, \\ \Pi_M(p^{(\{1\},\{2,3\})}(v)) &= \frac{v_H(v_H - v_M)v_M}{(4v_H - v_M)^2} - \frac{1}{2}v_M^2, \\ \Pi_L(p^{(\{1\},\{2,3\})}(v)) &= 0. \end{aligned}$$

Note that in this case the price of the low quality variant is set so high that no consumer is willing to buy this variant and, therefore,  $D_L^{\{2,3\}} = 0$ . Thus, firm 3 ceases to be active in the market: selling the bottom-quality variant would determine a cannibalization effect only within the coalition since the variant  $v_L$  would be in competition with the adjacent product  $v_M$ . Of course, it still plays a role in the coalition as the decision to stop producing benefits the coalition as a whole.<sup>22</sup>

**Quality stage** Then, moving to the quality stage and using the best reply functions, it is immediate to show that the top variant and the intermediate variant are respectively

$$v_H^{(\{1\},\{2,3\})} = 0.253\,31, \quad v_M^{(\{1\},\{2,3\})} = 0.04823\,8. \quad (24)$$

Given the above values, we can easily find the equilibrium prices as

$$p_H^{(\{1\},\{2,3\})} = 0.107\,66, \quad p_M^{(\{1\},\{2,3\})} = 0.010251, \quad (25)$$

with corresponding equilibrium profits:

$$\begin{aligned} \Pi_1^{(\{1\},\{2,3\})} &= \Pi_H^{(\{1\},\{2,3\})} = 0.024439, \\ \Pi_2^{(\{1\},\{2,3\})} + \Pi_3^{(\{1\},\{2,3\})} &= \Pi_M^{(\{1\},\{2,3\})} = 0.0015274. \end{aligned} \quad (26)$$

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<sup>22</sup>Its role will be clarified at the alliance formation stage.

We observe that at equilibrium, firm 1 produces the top quality while the coalition  $\{2, 3\}$  sells the intermediate quality and our assumption on the quality assignment is satisfied. Note also that the above findings coincide with those emerging in Motta (1993) where only two firms compete in a traditional duopoly setting. Indeed, coalition  $\{2, 3\}$  behaves like a multiproduct firm: since it withdraws from the market one of its variant, *it is as if* two single-product firms would be active in the market, each of them setting in a noncooperative way their quality and price. We resume these results in the next proposition.

**Proposition 1** *When firms 2 and 3 collude against firm 1 (playing as singleton), namely under coalition structure  $C_{1,23} = (\{1\}, \{2, 3\})$ , at the price stage colluding firms set a price so high for the low quality variant that no consumer is willing to buy it. Thus, at the two-stage partial collusive equilibrium only two variants are marketed and the equilibrium configuration in terms of quality and price coincides with that occurring in a traditional duopoly setting where only two firms compete.*

**Proof.** It directly follows by expressions (24) and (25) and by their comparison with results obtained, for instance, in Motta (1993). ■

Finally, it is worth remarking that this collusion benefits both the colluding firms and the rival 1 which plays as a singleton. Indeed, not only the lowest quality variant is dropped out from the market, but also the gap between the variants in the market is larger than the one emerging in the non cooperative setting with three independent firms: under partial collusion, the optimal quality of the intermediate variant is lower while the top quality is higher than the corresponding ones in the noncooperative setting. This relaxes price competition between firms thereby increasing the resulting profits.

### 3.4.2 Collusion between top and bottom quality firms

Let us move now to the case where the agreement at the first stage has been reached by firms 1 and 3, with firm 2 playing as singleton. Let us remind that, in line with the quality assignment made in the noncooperative scenario, we start assuming that at the second stage firms 1 and 3 have decided to produce variant  $v_H$  and  $v_L$ , respectively. Firm 2 sells the intermediate variant  $v_M$ . As usual, we will verify later whether this quality assignment holds at the SPNE.

**Price stage** To obtain the optimal prices set by the colluding firms 1 and 3 and by the firm 2 alone we need to take into account the fact that colluding firms 1 and 3 maximize the sum of their profits  $\Pi_H + \Pi_L$ , while 2 is only concerned with its own profit function  $\Pi_M$ . However,

since firm 1 and 3 are not close neighbours and are separated by firm 2, at the price stage their equilibrium prices coincides with those obtained in the noncooperative case:

$$\begin{aligned} p_H^{\{1,3\},\{2\}}(v_H, v_M, v_L) &= \frac{1}{2} \frac{(v_H v_L - 4v_H v_M + 3v_L v_M)(v_H - v_M)}{(v_H v_L - 4v_H v_M + 2v_L v_M + v_M^2)} \\ p_M^{\{1,3\},\{2\}}(v_H, v_M, v_L) &= \frac{(v_L - v_M)(v_H - v_M)v_M}{(v_H v_L - 4v_H v_M + 2v_L v_M + v_M^2)} \\ p_L^{\{1,3\},\{2\}}(v_H, v_M, v_L) &= \frac{1}{2} \frac{(v_L - v_M)(v_H - v_M)v_L}{(v_H v_L - 4v_H v_M + 2v_L v_M + v_M^2)} \end{aligned}$$

and profits are:

$$\begin{aligned} \Pi_H^{\{1,3\},\{2\}}(v_H, v_M, v_L) &= \frac{1}{4} \frac{(v_H - v_M)(v_H v_L - 4v_H v_M + 3v_L v_M)^2}{(v_H v_L - 4v_H v_M + 2v_L v_M + v_M^2)^2} - \frac{1}{2} v_H^2 \\ \Pi_M^{\{1,3\},\{2\}}(v_H, v_M, v_L) &= \frac{v_M^2 (v_H - v_L)(v_M - v_L)(v_H - v_M)}{(v_H v_L - 4v_H v_M + 2v_L v_M + v_M^2)^2} - \frac{1}{2} v_M^2 \\ \Pi_L^{\{1,3\},\{2\}}(v_H, v_M, v_L) &= \frac{1}{4} \frac{v_M (v_M - v_L)(v_M - v_H)^2 v_L}{(v_H v_L - 4v_H v_M + 2v_L v_M + v_M^2)^2} - \frac{1}{2} v_L^2. \end{aligned}$$

**Quality stage** We can now move to the quality stage. In order to identify the optimal qualities, notice that the revenue of coalition  $\{1, 3\}$  is monotonically decreasing in  $v_L$ , as

$$\frac{\partial (\Pi_H^{\{1,3\},\{2\}} + \Pi_L^{\{1,3\},\{2\}})}{\partial v_L} = \frac{1}{4} v_M^2 \frac{(v_H - v_M)^2 (v_M^2 + v_H v_L + 20v_H v_M - 22v_L v_M)}{(v_M^2 + v_H v_L - 4v_H v_M + 2v_L v_M)^3} < 0$$

Accordingly, at the quality stage the colluding firms 1 and 3 will find it profitable to set  $v_L = 0$ , whatever the quality chosen by the rival 2. The economic intuition underlying this finding is that the low quality variant and the intermediate variant are strategic complements. So, if the colluding firms would increase  $v_L$ , the independent firm producing  $v_M$  would increase the quality of its own variant, thereby making tighter the competition with the high quality producer.<sup>23</sup> Since the profit loss for firm  $L$  deriving from decreasing the low quality level is lower than the gain obtained by the high quality firm  $H$  when competition between  $v_M$  and  $v_H$  relaxes, then the colluding firms will optimally set  $v_L = 0$  and restrict their production choice to the high quality variant  $v_H$ .

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<sup>23</sup>See also Scarpa (1998), pg 669 for the same effect in a three-firm noncooperative setting.

As a result, from the first-order conditions obtained maximizing, in turn, the profit of coalition  $\{1, 3\}$  w.r.t to  $v_H$  and the profit of rival 2 w.r.t  $v_M$ , namely

$$\begin{aligned}\frac{\partial (\Pi_{1,H} + \Pi_{3,L})}{\partial v_H} &= \frac{(v_H v_M^3 - 64v_H^4 + 48v_H^3 v_M + 16v_H^3 - 12v_H^2 v_M^2 + 8v_H v_M^2 - 12v_H^2 v_M)}{(4v_H - v_M)^3} = 0 \\ \frac{\partial \Pi_{2,M}}{\partial v_M} &= \frac{(v_M^4 - 12v_H v_M^3 - 64v_H^3 v_M + 4v_H^3 + 48v_H^2 v_M^2 - 7v_H^2 v_M)}{(4v_H - v_M)^3} = 0\end{aligned}$$

given that we know that, at equilibrium  $v_L^{\{1,3\},\{2\}} = 0$ , we obtain the equilibrium optimal qualities set under coalition structure  $C_{13,2} = (\{1, 3\}, \{2\})$ :

$$v_H^{\{1,3\},\{2\}} = 0.25331, \quad v_M^{\{1,3\},\{2\}} = 0.048238. \quad (27)$$

It immediately follows that, equilibrium prices are:

$$p_H^{\{1,3\},\{2\}} = 0.10766, \quad p_M^{\{1,3\},\{2\}} = 0.010251, \quad (28)$$

with corresponding equilibrium profits:

$$\Pi_H^{\{1,3\},\{2\}} = 0.024439, \quad \Pi_L^{\{1,3\},\{2\}} = 0, \quad \Pi_M^{\{1,3\},\{2\}} = 0.0015274. \quad (29)$$

Again, the assumption that firms variants  $v_H$ ,  $v_M$  and  $v_L$  are associated to firms 1,2 and 3, respectively, is satisfied and the partial agreement between firm 1 and 3 can be fully characterized in the next proposition.

**Proposition 2** *When firms 1 and 3 collude against firm 2 that competes as singleton, namely under coalition structure  $C_{13,2} = (\{1, 3\}, \{2\})$ , at the quality stage the low quality variant is set equal to zero. Prices and qualities offered in equilibrium coincide with those observed under  $C_{23,1} = (\{2, 3\}, \{1\})$ .*

**Proof.** It directly follows by comparing expressions (24) and (25) with (27) and (28). ■

It is worth noting that from a market structure viewpoint, the formation of coalition structures  $C_{23,1}$  and  $C_{13,2}$  are equivalent, as both of them entail a duopoly structure with the same quality gap between variants. Still, the *rationale* underlying the equilibrium configuration in coalition  $C_{23,1}$  cannot be extended to  $C_{13,2}$ . In the former case, namely when the firms 2 and 3 compete against the top quality firm 1, the colluding firms decide at the price stage to fix a very high price for the bottom variant so that no consumer is willing to buy it, whatever

its quality. So, this finding would be observed even if firms would be prevented from defining endogenously the quality of their product. In this case, no ambiguous effect can be attributed to the low quality variant  $v_L$ . It is adjacent only to variant  $v_M$ . So, if it would be kept in the market, it would reap consumers to the other colluding player without playing any role in the competition against the top quality firm. Rather, in the latter scenario firms 1 and 3 decide to reduce the bottom quality to such an extent that the corresponding market share for this variant turns out to be nil. Of course, as this strategy is put in place at the quality stage, if qualities would be exogenously given, this finding would no longer be observed and all variants would be sold at equilibrium. When the coalition decides to withdraw variant  $v_L$  from the market, it takes into account two different effects. On one hand, since the low quality variant is adjacent to the intermediate variant, *ceteris paribus*, increasing its quality can enable the coalition to gain market share from the competitor 2 producing variant  $v_M$  and thus to benefit from higher profits for the bottom quality firm. On other hand, as these two variants  $v_M$  and  $v_L$  are strategic complements, the higher the quality of the bottom variant, the higher the optimal quality of the intermediate variant. This latter variant is in turn in direct competition with the top variant: since the lower the quality gap, the fiercer the price competition between players, the higher the intermediate quality, the lower, *ceteris paribus*, profits accruing to the top quality player. The loss for this player when the low quality is produced is higher than the gain obtained by the bottom producer so that the coalition stops producing the variant.

### 3.4.3 Collusion between top and intermediate quality firms

Finally, we characterize the equilibrium configuration when at the first stage of the game firm 1 and firm 2 decide to collude, with the bottom quality rival (here firm 3) playing as singleton. As usual, we assume that firms 1,2 and 3 produces variants  $v_H, v_M$  and  $v_L$ , respectively. Then, we check whether this assumption is satisfied at the quality stage. We consider first the price stage, qualities being taken at this stage as exogenous.

**Price stage** At the price stage, firms 1 and 2 maximize the sum of their own profits, namely  $\Pi_H + \Pi_M$ , whereas firm 3 is playing independently. Using (6), (16) and (23), the optimal replies under coalition structure  $C_{12,3} = (\{1, 2\}, \{3\})$  are obtained, respectively, as

$$\begin{aligned}
p_H^{pc}(p_M) &= p_M + \frac{1}{2}(v_H - v_M) \\
p_M^{pc}(p_L, p_H) &= \frac{\frac{1}{2}p_L(v_H - v_M) + p_H(v_M - v_L)}{(v_H - v_L)} \\
p_L^{pc}(p_M) &= \frac{1}{2} \frac{v_L}{v_M} p_M.
\end{aligned}$$

Thus, equilibrium prices are easily found as:

$$\begin{aligned}
p_H^{\{\{1,2\},\{3\}\}}(v_H, v_M, v_L) &= \frac{(4v_H v_M - v_H v_L - 3v_L v_M)}{2(4v_M - v_L)}, \\
p_M^{\{\{1,2\},\{3\}\}}(v_H, v_M, v_L) &= \frac{2v_M(v_M - v_L)}{4v_M - v_L}, \\
p_L^{\{\{1,2\},\{3\}\}}(v_H, v_M, v_L) &= \frac{v_L(v_M - v_L)}{4v_M - v_L}.
\end{aligned}$$

The corresponding firms' profits are:

$$\begin{aligned}
\Pi_H^{\{\{1,2\},\{3\}\}} &= \frac{1}{4} \frac{(4v_H v_M - v_H v_L - 3v_L v_M)}{(4v_M - v_L)} - \frac{1}{2} v_H^2, \\
\Pi_M^{\{\{1,2\},\{3\}\}} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_M^2, \\
\Pi_L^{\{\{1,2\},\{3\}\}} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_L^2.
\end{aligned}$$

**Quality stage** We saw before that at the price stage, when the coalition structure  $C_{12,3} = (\{1, 2\}, \{3\})$  formed, no variant is withdrawn from the market. Still, at the quality stage, it can be proved that a case of quality reversal occurs. This is done in the next proposition.

**Proposition 3** *In order to escape from the cannibalization effect taking place between adjacent variants, the colluding firms 1 and 2 enhance maximal differentiation between their products by setting the intermediate quality at the bottom of the quality ladder. The rival 3 leapfrogs firm 2, thereby producing a variant which lies now in the middle of the quality ladder.*

**Proof.** See the Appendix. ■

Notice that now, profit  $\Pi_{3,M}^{\{1,2\}}$  coincides with that obtained by firm 2 when producing variant  $v_M$  in coalition  $\{2, 3\}$ , namely  $\Pi_{2,M}^{\{2,3\}}$ . Thus, it follows that coalition  $\{1, 2\}$  produces top and bottom qualities against firm 3 producing instead the intermediate quality variant. So, due to



the leapfrogging by firm 3, the variants produced by the colluding firms 1 and 2 coincide now with those produced under the coalition structure  $C_{1,23} = (\{1\}, \{2, 3\})$  where firms 2 and 3 were the ones to collude. Moreover, in  $C_{12,3} = (\{1, 2\}, \{3\})$  the independent firm 3 produces now the variant that in the previous scenarios was sold by firm 2. In line with the analysis performed in the previous case, the optimal variants are immediately obtained here as:

$$v_{1,H}^{(\{1,2\},\{3\})} = 0.253, \quad v_{2,L}^{(\{1,2\},\{3\})} = 0 \quad v_{3,M}^{(\{1,2\},\{3\})} = 0.04824, \quad (30)$$

while the equilibrium profits write as:

$$\Pi_{1,H}^{(\{1,2\},\{3\})} + \Pi_{2,L}^{(\{1,2\},\{3\})} = 0.024439, \quad (31)$$

$$\Pi_{3,M}^{(\{1,2\},\{3\})} = 0.0015274.$$

Thus, one can state the following proposition.

**Proposition 4** *When firm 1 and firm 2 collude against firm 3 which colludes as singleton, namely under coalition structure  $C_{12,3} = (\{1, 2\}, \{3\})$ , at the quality stage firm 3 leapfrogs the adjacent rival 2 whose variant is no longer on sale in the market. The optimal qualities coincide with those occurring under the alternative coalition structures  $C_{13,2} = (\{1, 3\}, \{2\})$  and  $C_{1,23} = (\{1\}, \{2, 3\})$ .*

**Proof.** This follows directly by Proposition 3 and by comparing (27), (24) and (30). ■

For ease of exposition, we summarize in the following table the payoffs accruing to each firm or coalition in each feasible coalition structure.

$(\{1\}, \{2\}, \{3\})$	$\Pi_{1,H}^* = 0.023489$	$\Pi_{2,M}^* = 0.0012491$	$\Pi_{3,L}^* = 0.000053956$
$\{N\}$	$\Pi_{123,H}^{\{N\}} = 0.03125$		
$(\{1\}, \{2, 3\})$	$\Pi_{1,H}^{((1),(23))} = 0.024439$	$\Pi_{23,M}^{((1),(23))} = 0.0015274$	
$(\{1, 3\}, \{2\})$	$\Pi_{13,H}^{\{1,3\}\{2\}} = 0.024439$	$\Pi_{2,M}^{\{1,3\}\{2\}} = 0.0015274$	
$(\{1, 2\}, \{3\})$	$\Pi_{12,H}^{(\{1,2\},\{3\})} = 0.024439$	$\Pi_{3,M}^{(\{1,2\},\{3\})} = 0.0015274$	

Table 1 - Firm payoffs in every coalition structure.

It is worth remarking that the market structure (duopoly) arising under *partial collusion* does not vary with the coalition structure induced by the firms. Still, the profits accruing to firms depend on the coalitions to (against) which they belong (compete).

## 4 Equilibrium Analysis: Alliance Structures

### 4.1 The profitability and cooperative stability of the grand coalition

A first glance at the Table 1 shows that, in terms of firms' payoff, the grand coalition is by far the most profitable coalition structure in the vertical differentiated market. Before starting to analyse the stability of coalition structures in the sequential game, we may wonder whether the grand coalition is *stable* against coalitional deviations in a classical sense, namely whether there are feasible allocation of the monopoly profit that are in the *core* of the associated transferable utility game. In particular, we can easily associate to the vertical differentiated market a *partition function game*  $\rho = (N, v(S, C(S)))$ , where  $N$  is the set of firms and  $v(S, C(S)) \in \mathcal{R}$  is the worth associated to every coalition of firms  $S \subset N$  embedded in a given coalition structure  $C \in \mathcal{C}$  whose  $S$  is part. In our model, when an alliance  $S \subset N$  forms, its maximal payoff obtains when the remaining firms stick together in the complementary coalition  $\{N \setminus S\}$ .<sup>24</sup> Therefore, if the core of the partition function game  $\rho$  exists when every  $S \subset N$  is embedded in  $C = (S, N \setminus S)$ , it will *a fortiori* exist in any other coalition structure containing  $S$ . Let us state this result more formally.

**Definition 3** *The core of the partition function game  $(N, v(S, C))$  consists of all efficient profit allocations  $\Pi \in \mathcal{R}_+^{|N|}$  such that  $\sum_{k \in S} \Pi_k \geq v(S, C(S))$  for all  $S \subset N$  and for all  $C(S)$  in which  $S$  can be embedded.*

Thus, we can prove the following result.

**Proposition 5** *In the three-firm vertically differentiated market with endogenous qualities and prices, the core of the partition function game  $\rho = (N, v(S, C))$  is nonempty.*

**Proof.** See the Appendix. ■

The above result simply says that in a vertical differentiated market with three firms competing in prices, there would be room for cooperative agreements between firms. However, as we show in the next section, when the bargaining process is *sequential* and the firms possess

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<sup>24</sup>In a triopoly, the behaviour of firms outside a coalition  $S$  matters only when each individual firm  $k$  is competing with remaining firms in  $N \setminus \{k\}$ , that, in turn, can either play together, or stay as singletons. Moreover, from Section 3 we know that when two firms form a coalition they eliminate one of the variant either at the quality or at the price stage. Therefore, a firm playing as singleton prefers that its competitors merge rather than compete independently in the market: in game-theoretic terms there exist *positive coalition externalities* (see, for instance, Yi, 1997 and 2003).

a *finite* set of possibilities to propose coalitions and divisions of the joint profit to their rivals, the grand coalition can never be enforced in equilibrium. In particular, we show that only *intermediate coalition structures* can be enforced as subgame perfect equilibrium of the alliance formation game.

## 4.2 Stable Alliances Structures

In this section we characterize the equilibria of the sequential game of alliance formation. Since this game is sensitive to the identity of the initial player, we consider, in turn, the outcomes obtained when either firm 1, 2 and 3 starts the bargaining process. Let us first consider the case in which the firm producing the top-quality good (firm 1) is the initiator of the coalition formation game.

It can be proved the following:

**Proposition 6** *When firm 1 is the initiator of the sequential alliance formation game, the only stable coalition structure is  $C_{1,23} = (\{1\}, \{2, 3\})$ .*

**Proof.** See the Appendix. ■

In the next proposition, applying the same *rationale* as above, we can easily show that, when firm 2 is the initial player,  $C_{13,2} = (\{1, 3\}, \{2\})$  is the *only stable coalition structure*.

**Proposition 7** *When firm 2 is the initiator of the sequential alliance formation game, the only stable coalition structure is  $C_{13,2} = (\{1, 3\}, \{2\})$ .*

**Proof.** See the Appendix. ■

Notice that, in both above cases the initiator of the game is never part of an alliance at the equilibrium. Indeed, as shown in detail in the proofs of Proposition 6 and 7, the payoff for a firm to remain singleton (and rationally expecting that the other firms will prefer to collude) dominates that of being part of the grand coalition, since in this case the distribution of profits will be unfavourable for the initial proposer. The equilibrium profit accruing to either firm 1 or 2 when initiating the game and competing against an alliance is, therefore, larger than when they are part of the alliance itself. The optimal strategy is, therefore, to induce the remaining firms to collude.

A different result arises when firm 3 (the bottom quality one) begins the negotiation process. The reason is that, in this case, firm 3 cannot credibly commit to remain independent when

the remaining firms (1 and 2) prefer to play as singletons rather than forming an alliance (see Table 1). This is due to the fact that the alliance between firm 1 and 2 optimally leapfrogs the bottom quality firm, and ends up sharing the top quality firm duopoly payoff, which is lower than the sum of their profits under triopoly (cf. section 3.4.3). Under these circumstances, firm 3 will prefer to let firm 1 to play independently, and it will form an alliance with firm 2. This is shown in the next proposition.

**Proposition 8** *When firm 3 is the initiator of the sequential alliance formation game, the only stable coalition structure is  $C_{1,23} = (\{1\}, \{2, 3\})$ .*

**Proof.** See the Appendix. ■

It is worth noting that if the game initiator would be selected at random, the most likely outcome of the alliance formation game would certainly be that in which the coalition structure  $C_{1,23} = (\{1\}, \{2, 3\})$  forms, the other possible outcome implying the formation of  $C_{13,2} = (\{1, 3\}, \{2\})$ . Moreover, although at equilibrium the same coalition structure  $C_{1,23}$  forms both when either firm 1 or 3 starts the negotiation, there is a difference in term of rent extraction, in the two cases, for colluding firms 2 and 3: when firm 1 is the one starting the negotiation, firm 2 in alliance  $\{2, 3\}$  only receives its outside option  $\Pi_2 = \Pi_{2,M}^* = 0.0012491$ , whereas firm 3 is able to get a profit  $\Pi_3 = \Pi_{2M}^{(\{1\}, \{2,3\})} - \Pi_{2,M}^* = 0.0002783 > \Pi_{3,L}^*$ , exploiting its last-mover advantage in the sequential game. When, on the other hand, it is firm 3 to start the game, firm 2 in alliance  $\{2, 3\}$  receives  $\Pi_{2M}^{(\{1\}, \{2,3\})} - \Pi_{3,L}^* = 0.0014734 > \Pi_{2,M}^* = 0.0012491$ , while firm 3 only receives its noncooperative payoff  $\Pi_{3,L}^* = 0.000053956$ . In both cases, firm 1 receives its duopoly payoff  $\Pi_1 = 0.024439$ .

Surprisingly, in the alliance formation game, overall firm 2 enjoys a first-mover advantage, since when it starts the negotiation, it is able to enforce  $C_{13,2} = (\{1, 3\}, \{2\})$  and extract a profit  $\Pi_2 = \Pi_{2,M}^{(\{1,3\}, \{2\})} = 0.0015274$  higher than in all other cases. Moreover, this comes at expense of firm 1, which in coalition structure  $C_{13,2}$  only receives its noncooperative payoff  $\Pi_{1,H}^* = 0.023489$ .

Finally, it can be noticed that, since for any order of play our one-shot coalition formation game always sustains only one equilibrium alliance structure, the finite repeated version of game will generate similar outcomes. We condense these conclusions in the next corollary.

**Corollary 1** *If the alliance formation game is repeated for a finite number of periods, the coalition structures which are stable under the one-shot game would continue to be so, sustained by the same SPE strategy profile repeated at each period.*

Therefore, even in a repeated finite-horizon framework, our results on intermediate coalition structures would continue to hold.

## 5 Concluding Remarks

We have investigated the endogenous formation of alliances in a vertically differentiated market in which full or partial binding agreements among firms can be signed over prices and qualities of the products. We show that regardless of the high profitability of the full collusive agreement (i.e. the one signed by all firms in the market), such an arrangement can never be obtained in a (finite horizon) sequential negotiation process requiring the unanimity of firms. Conversely, we find that the sequential bargaining process enforces only partial collusive agreements, namely those involving subsets of firms. In particular, stable associations of firms always include the firm producing the bottom quality variant, which is, however, never sold by the coalition at equilibrium. Further, whatever the coalition structure arising at the equilibrium, the market moves from a triopoly to a duopoly with only two variants on sale. The rationale underlying this apparently surprising result can be found in the nature of competition among vertically differentiated firms. Indeed, under an intermediate coalition structure, the optimal set of products to market is defined by balancing *the cannibalization effect* within the coalition and the *stealing effect* occurring between the coalition and the firm outside. When the bottom quality is kept for sale in the market under a collusive agreement, the former effect always dominates the latter. As immediate consequence, this variant is withdrawn from the market and the equilibrium configuration coincides with that observed in the case of a duopoly in terms of price and quality gap between variants (Motta, 1993). In a complementary perspective, we can state that moving from a triopoly (observed in the noncooperative scenario) to a duopoly under partial collusion, firms can soften price competition in the market and magnify the quality differentiation between the variants kept on sale. Interestingly, this view is in line with the empirical findings of Sweeting (2010) in radio music industry. He finds that firms that buy competing stations tend to emphasize "service differentiation" among themselves and also from the competing stations. Partial collusion is thus a means to enhance *dynamic* competition *for* the market, while decreasing *static* competition in the market.

## 6

## Appendix: Omitted Proofs

### A.1 Proof of Proposition 3

**Proposition 3.** *In order to escape from the cannibalization effect taking place between adjacent variants, the colluding firms 1 and 2 enhance maximal differentiation between their products so that the intermediate quality is set at bottom of the quality ladder. The rival 3 leapfrogs firm 2, thereby producing a variant which lies now in the middle of the quality ladder.*

**Proof.** At the quality stage, firms' profits are:

$$\begin{aligned}\pi_H^{\{1,2\}} &= \frac{1}{4} \frac{(4v_H v_M - v_H v_L - 3v_L v_M)}{(4v_M - v_L)} - \frac{1}{2} v_H^2 \\ \pi_M^{\{1,2\}} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_M^2 \\ \pi_L^{\{1,2\}} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_L^2 \\ \pi_H^{\{1,2\}} + \pi_M^{\{1,2\}} &= \frac{v_H v_L^2 + 16v_H v_M^2 - 8v_L v_M^2 - v_L^2 v_M + 16v_L v_M^3 - 32v_M^4 - 2v_H^2 v_L^2 - 32v_H^2 v_M^2 - 2v_L^2 v_M^2 + 16v_H^2 v_L v_M - 8v_H v_L v_M}{4(v_L - 4v_M)^2}\end{aligned}$$

It is easy to see that, the coalition's profit  $\pi_H^{\{1,2\}} + \pi_M^{\{1,2\}}$  is monotonically decreasing in  $v_M$ , as

$$\frac{\partial \left( \frac{1}{4} \frac{(4v_H v_M - v_H v_L - 3v_L v_M)}{(4v_M - v_L)} + \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} (v_H^2 + v_M^2) \right)}{\partial v_M} - \frac{(4v_L^3 v_M + v_L^3 - 48v_L^2 v_M^2 + 20v_L^2 v_M + 192v_L v_M^3 - 256v_M^4)}{4(v_L - 4v_M)^3} < 0$$

So, the colluding firms find it profitable to set the quality of the intermediate variant at the minimum admissible value, say 0. By doing this, they choose to produce a variant which is at the bottom of the quality ladder. If the competitor 3 would keep his own variant at the same quality level, then he would obtain nil profits. Rather, choosing to produce an intermediate variant  $v_{3,M} > 0$  would yield positive equilibrium profits equal to

$$\pi_{3,M}^{\{1,2\}} = \frac{v_M^2 (v_H - v_L) (v_M - v_L) (v_H - v_M)}{(v_M^2 + v_H v_L - 4v_H v_M + 2v_L v_M)^2} > 0.$$

As this profit  $\pi_{3,M}^{\{1,2\}}$  is strictly positive for any  $v_H > v_M > v_L = 0$ , one can conclude that firm 3 finds it profitable to leapfrog the rival 2. ■

### A.2 Proof of Proposition 5

**Proposition 5.** *In the three-firm vertically differentiated market with endogenous qualities and prices, the core of the partition function game  $\rho = (N, v(S, C))$  is nonempty.*

**Proof.** Core allocations are individually-rational and group-rational profit division  $\Pi = (\Pi_1, \Pi_2, \Pi_3)$  of the efficient monopoly payoff  $v(N) = \Pi_{123,H}^{\{N\}} = 0.03125$ . Thus, the set of  $\Pi \in \text{Core}(\rho)$  must respect the following inequalities:

$$\begin{aligned}
\sum_{k=1}^3 \Pi_k &= v(N) = \Pi_{123,H}^{\{N\}} = 0.03125, \\
\Pi_1 + \Pi_2 &\geq v(\{1, 2\}, (\{1, 2\}, \{3\})) = \Pi_{12,H}^{\{1,2\},\{3\}} = 0.024439 \\
\Pi_1 + \Pi_3 &\geq v(\{1, 3\}, (\{1, 3\}, \{2\})) = \Pi_{13,H}^{\{1,3\},\{2\}} = 0.024439 \\
\Pi_2 + \Pi_3 &\geq v(\{2, 3\}, (\{1\}, \{2, 3\})) = \Pi_{23,M}^{((1),(23))} = 0.0015274 \\
\Pi_1 &\geq v(\{1\}, (\{1\}, \{2, 3\})) = \Pi_{1,H}^{((1),(23))} = 0.024439 \\
\Pi_2 &\geq v(\{2\}, (\{1, 3\}, \{2\})) = \Pi_{2,M}^{\{1,3\},\{2\}} = 0.0015274 \\
\Pi_3 &\geq v(\{3\}, (\{1, 2\}, \{3\})) = \Pi_{3,M}^{\{1,2\},\{3\}} = 0.0015274
\end{aligned}$$

which surely hold, since:

$$0.024439 + 0.0015274 + 0.0015274 = 0.027494 < 0.03125.$$

Note also that for every firm  $v(\{k\}, (\{k\}, \{N \setminus \{k\}\})) > v(\{k\}, (\{k\}, \{j\}, \{h\}))$  for any  $j, h \in N \setminus \{k\}$  and then the above inequality ensure also the individual stability when after a firm leaves the grand coalition, the coalition of remaining firms split-up in singletons. Thus all efficient payoff allocations  $\Pi = (\Pi_1, \Pi_2, \Pi_3)$  rewarding every firm *at least* its maximal deviating payoff and redistributing the remaining surplus  $Z$  among the three firms, namely

$$Z = \Pi_{123,H}^{\{N\}} - \Pi_{1,H}^{((1),(23))} - \Pi_{2,M}^{\{1,3\},\{2\}} - \Pi_{3,M}^{\{1,2\},\{3\}} = 0.0085006$$

belong to the core, which is, therefore, nonempty. ■

### A.3 Proof of Proposition 6

**Proposition 6.** *When firm 1 is the initiator of the sequential coalition formation game, the only stable coalition structure is  $C_{1,23} = (\{1\}, \{2, 3\})$ .*

**Proof.** The game can be solved backward. Firm's 1 available actions at the initial node (information set  $I_1^1 \in \mathcal{I}_1$ ) are the following (proposals):

$$A_1(I_1^1) = [(\{N\}, \Pi), (\{1, 2\}, \Pi), (\{1, 3\}, \Pi), (\{1\})].$$

Assume first that firm 1 proposes the grand coalition  $\{N\}$  associated to a given division of the efficient monopoly profit  $\Pi \in \Pi^{\{N\}}$  between the three firms. By the order of the game, firm 2 can either accept or reject. If it rejects the offer, it is its turn to make a proposal and can propose one of the following:

$$A_2(I_2^1) = [(\{N\}, \Pi), (\{1, 2\}, \Pi), (\{2, 3\}, \Pi), (\{2\})].$$

We know (by Table 2) that, for any associated payoff division, the coalition structure  $C_{12,3} = (\{1, 2\}, \{3\})$  is dominated by the choice of firm 1 and 2 to play as singletons, since

$$\Pi_{12,H}^{(\{1,2\},\{3\})} < \Pi_{1,H}^* + \Pi_{2,M}^*$$

and, therefore, when made, proposal  $p_2^{12} = (\{1, 2\}, \Pi)$  will always be rejected by firm 1. In this event, firm 1 has no more proposals to make. Thus, firm 3 can gain its highest payoff by proposing  $\{N\}$ , offering the noncooperative profits to 1 and 2 and get the difference  $\Pi_{123,H}^{\{N\}} - \Pi_{1,H}^* - \Pi_{2,M}^*$ , which is its most profitable outcome. To break the ties, we can initially assume that, when gaining equal payoffs the firms prefer to be in coalition rather than being as singletons (but the reasoning can be repeated when the case opposite holds). A similar outcome would be reached if, after a rejection, firm 2 proposes  $p_2^{23} = (\{2, 3\}, \Pi)$  or  $p_2^N = (\{N\}, \Pi)$ , which would be both refused by firm 3, willing to propose (as last proposer) the grand coalition, obtaining:  $\Pi_3 = \Pi_{123,H}^{\{N\}} - \Pi_{1,H}^* - \Pi_{2,M}^*$ . Analogously, if firm 2 accepts the grand coalition proposed by firm 1, it knows that, in its turn to play, firm 3 will always reject such proposal and propose, in turn, the grand coalition with a payoff allocation assigning to its rivals their Nash equilibrium payoffs. Thus, reasoning backward, firm 1 knows that, if it proposes the grand coalition, it would get at most its Nash equilibrium payoff. For this reason it can try to make alternative offers. Proposing  $p_1 = (\{1, 2\}, \Pi)$  is out of question, since player 2 would always reject it, and the game would return to the situation described above. Another chance for player 1 is to propose  $p_1^{13} = (\{1, 3\}, \Pi)$  that, in turn, firm 3 would reject with the aim to propose again  $\{1, 3\}, \Pi$ , offering to firm 1 its noncooperative outside option. Alternative proposals by firm 3 (after its rejection of  $\{1, 3\}$  proposed by firm 1) involving firm 2, as  $p_3^N = (\{N\}, \Pi)$  or  $p_3^{23} = (\{2, 3\}, \Pi)$  would be rejected by firm 2 to enforce, as last proposer, the grand coalition payoff. As a result, at the initial node the most profitable action for firm 1 is to propose  $p_1^1 = \{1\}$ , signalling its intention to play irrevocably as singleton. Doing this, it knows that when it is its turn to play, firm 2 can propose either  $p_2^{23} = (\{2, 3\}, \Pi)$  or  $p_2 = \{2\}$ . In the first case, firm 2 knows that firm 3 will prefer to reject its proposal in order to propose itself  $p_3^{23} = (\{2, 3\}, \Pi)$  offering



$\Pi_{2,M}^*$  to firm 2 and keeping the difference, since:  $\Pi_{23}^{(\{1\}\{2,3\})} - \Pi_{2,M}^* > \Pi_{3,L}^*$ . In the second case, namely if firm 2 proposes  $p_2^2 = \{2\}$ , a triopoly will form and firm 2 will receive  $\Pi_{2,M}^*$ . Since with equal payoffs firms prefer by assumption to be in coalition rather than being as singletons, in this subgame its choice will be  $p_2^{23} = \{\{2,3\}, \Pi\}$ . Therefore, the coalition structure  $C_{1,23} = (\{1\}, \{2,3\})$  is stable because is sustained by the following SNE strategy profile along the *equilibrium path*:<sup>25</sup>

$$\sigma^* = \left( \sigma_1^* = \{1\}, \sigma_2^* = \left( \{2,3\}, \Pi' \right), \sigma_3^* = \left( \text{No}, \{2,3\}, \Pi'' \right) \right),$$

where  $\Pi' = (\Pi'_2, \Pi'_3)$ , for  $\Pi'_2 = \Pi_{23}^{(\{1\}\{2,3\})} - \Pi_{3,L}^*$ ,  $\Pi'_3 = \Pi_{3,L}^*$ , and  $\Pi'' = (\Pi''_2, \Pi''_3)$ , for  $\Pi''_2 = \Pi_{2,M}^*$  and  $\Pi''_3 = \Pi_{23}^{(\{1\}\{2,3\})} - \Pi_{2,M}^*$ . When we assume, to break the ties, that with equal payoffs firms prefer to be singletons rather than being in coalition, the same coalition structure  $C_{1,23}$  can be enforced by a SPE of the coalition formation game with the difference that, along the equilibrium path,  $\Pi'_2 = \Pi_{23}^{(\{1\}\{2,3\})} - (\Pi_{3,L}^* + \epsilon)$ ,  $\Pi'_3 = \Pi_{3,L}^* + \epsilon$  and  $\Pi''_2 = \Pi_{2,M}^* + \epsilon$ ,  $\Pi''_3 = \Pi_{23}^{(\{1\}\{2,3\})} - (\Pi_{2,M}^* + \epsilon)$ , for  $\epsilon > 0$ . The same occurs in all other proposals with presence of coalitions. The reason is that to convince a firm to join a coalition it must receive something *more* (an  $\epsilon > 0$ ) than its noncooperative payoff. Therefore, the coalition structure  $C_{1,23}$  remains stable (namely sustained by a SPE strategy profile of the sequential coalition formation game) whatever the rule adopted to break ties. Finally, to see that  $C_{1,23}$  is the *only* stable coalition structure arising when firm 1 is the initiator of the game, note that any alternative strategy profile cannot be SPE just because firm 1 would always possess an incentive to profitably deviate proposing  $p_1 = \{1\}$  with the expectation to compete in a duopoly (namely under  $C_{1,23}$ ) with a payoff  $\Pi_{1,H}^{((1),(23))} = 0.024439$  rather than remaining with its triopoly profit  $\Pi_{1,H}^* = 0.023489$  (or in turn,  $\Pi_{1,H}^* + \epsilon$ ). ■

#### A.4 Proof of Proposition 7

**Proposition 7.** *When firm 2 is the initiator of the sequential coalition formation game, the only stable coalition structure is  $C_{13,2} = (\{1,3\}, \{2\})$ .*

**Proof.** As before the game can be solved backward. Firm 2 available actions at the initial node (information set  $I_2^1 \in \mathcal{I}_2$ ) are:

$$A_2(I_2^1) = [(\{N\}, \Pi), (\{1,2\}, \Pi), (\{2,3\}, \Pi), (\{2\})].$$

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<sup>25</sup>We have verbally described the *out of equilibrium path* actions which compose the SPE strategy profile  $\sigma^*$  and, therefore, for ease of simplicity, we do not repeat it here.

As before, if firm 2 proposes the grand coalition  $\{N\}$  associated to a given division of the monopoly profit  $\Pi \in \Pi^{\{N\}}$ , the next player, firm 1, rejects the offer to offer, in turn, one of the following:

$$A_1(I_1^1) = [(\{N\}, \Pi), (\{1, 2\}, \Pi), (\{1, 3\}, \Pi), (\{1\})].$$

Again, we know that coalition structure  $C_{12,3} = (\{1, 2\}, \{3\})$  is dominated by the choice of firm 1 and 2 to play as singletons and, therefore, proposal  $p_1^{12} = (\{1, 2\}, \Pi)$  will always be rejected by firm 2. If this occurs, firm 2 has no more proposals to make and, hence, firm 3 can propose  $\{N\}$ , obtaining  $\Pi_3 = \Pi_{123,H}^{\{N\}} - \Pi_{1,H}^* - \Pi_{2,M}^*$ , which is its most profitable outcome. Similar outcome would be reached if, after a rejection, firm 1 proposes, in turn,  $p_1^{13} = (\{1, 3\}, \Pi)$  or  $p_1^N = (\{N\}, \Pi)$ , which could be either refused or accepted by firm 2, but in any case the final payoff would be, for firm 1 and 2, their noncooperative outside options. Thus, reasoning backward, firm 2 knows that by proposing the grand coalition, it will receive at most its noncooperative payoff. Its alternative proposals are  $p_2^{12} = (\{1, 2\}, \Pi)$  which would always be rejected by firm 1, and the game would reach the same outcome described above, and  $p_2^{23} = (\{2, 3\}, \Pi)$  that, in turn, firm 3 would reject with the aim to propose itself  $p_3 = (\{2, 3\}, \Pi)$ , offering to firm 2 its noncooperative outside option. which is better than propose any other coalition containing firm 1, that would exploit its last mover advantage. Note that forming  $\{2, 3\}$  is, for firm 3, better than any other proposal involving firm 1, that could exploit in this case its last-mover advantage. Thus, at the initial node the most profitable action for firm 2 is to propose  $p_2^2 = \{2\}$ , knowing (by assumption) that firm 1 will prefer to be in coalition rather than being as singleton, proposing  $p_1^{13} = (\{1, 3\}, \Pi)$  rather than  $p_1^1 = \{1\}$ . Hence, the proposal  $p_1^{13}$  will be rejected by firm 3, which will propose, in turn,  $p_3^{13} = (\{1, 3\}, \Pi)$  offering  $\Pi_{1,H}^*$  to firm 1 and keeping the difference for it, since  $\Pi_{13}^{\{1,3\}\{2\}} - \Pi_{1,H}^* > \Pi_{3,L}^*$ . As a result, the coalition structure  $C_{13,2} = (\{1, 3\}, \{2\})$  is stable because is sustained by the following SNE strategy profile along the *equilibrium path*:<sup>26</sup>

$$\sigma^* = \left( \sigma_1^* = (\{1, 3\}, \Pi'), \sigma_2^* = \{2\}, \sigma_3^* = (\text{No}, \{1, 3\}, \Pi'') \right),$$

with  $\Pi' = (\Pi'_1, \Pi'_3)$ , where  $\Pi'_1 = \Pi_{13}^{\{1,3\}\{2\}} - \Pi_{3,L}^*$  and  $\Pi'_3 = \Pi_{3,L}^*$  and  $\Pi'' = (\Pi''_2, \Pi''_3)$ , where  $\Pi''_1 = \Pi_{1,H}^*$  and  $\Pi''_3 = \Pi_{13}^{\{1,3\}\{2\}} - \Pi_{1,H}^*$ . As in the proof of Proposition 6, when under equal payoffs, firms prefer to be singletons rather than being in coalition, coalition structure  $C_{13,2}$  can be enforced as a SPE of the coalition formation game for,  $\Pi'_1 = \Pi_{13}^{\{1,3\}\{2\}} - (\Pi_{3,L}^* + \epsilon)$

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<sup>26</sup> Again, for simplicity, we skip the description of all players' out of equilibrium actions.

and  $\Pi'_3 = \Pi_{3,L}^* + \epsilon$ ,  $\Pi''_1 = \Pi_{1,H}^* + \epsilon$  and  $\Pi''_3 = \Pi_{13}^{(\{1,3\},\{2\})} - (\Pi_{1,H}^* + \epsilon)$ , for  $\epsilon > 0$ , and, similarly for all other proposal involving coalitions outside the equilibrium path. Finally,  $C_{13,2}$  is the *only* stable coalition structure when firm 2 is the initiator just because in any alternative strategy profile firm 2 will always prefer to propose  $p_2^2 = \{2\}$  and compete in a duopoly with a payoff  $\Pi_{2,M}^{(\{1,3\},\{2\})} = 0.0015274$  rather than have its triopoly profit  $\Pi_{2,M}^* = 0.0012491$  (or in turn,  $\Pi_{2,H}^* + \epsilon$ ), which occur in all other subgames. ■

## A.5 Proof of Proposition 8

**Proposition 8.** *When firm 3 is the initiator of the sequential coalition formation game, the only stable coalition structure is  $C_{23,1} = (\{2, 3\}, \{1\})$ .*

**Proof.** Again in this proof we reason backward. Note that when firm 3 is the initiator of the game, the line of reasoning is slightly different than in the other two cases described in Proposition 6 and 7. Firm 3 available actions at the initial node (information set  $I_3^1 \in \mathcal{I}_3$ ) are:

$$A_3(I_3^1) = [(\{N\}, \Pi), (\{1, 3\}, \Pi), (\{2, 3\}, \Pi), (\{3\})].$$

To break ties assume initially that, with equal payoffs, firms prefer to be in coalition rather than being as singletons. We first note that if firm 3 proposes  $p_3^3 = \{3\}$ , the turn passes to player 1, who can either propose  $p_1^1 = \{1\}$ , in which case the game ends with  $C_{1,2,3} = (\{1\}, \{2\}, \{3\})$  or instead  $p_1^{12} = (\{1, 2\}, \Pi)$ , which again forces the game to end with  $C_{1,2,3} = (\{1\}, \{2\}, \{3\})$ , since  $C_{1,23} = (\{1\}, \{2, 3\})$  is dominated by  $C_{1,2,3}$  for both firm 1 and 2 (see Table 1). So differently from above, firm 3 is unable to enforce the formation of its complementary coalition  $N \setminus \{3\} = \{1, 2\}$  by signalling its willingness to play as singleton. Alternatively, when firm 3 proposes either  $p_3^N = (\{N\}, \Pi)$  or  $p_3^{13} = (\{1, 3\}, \Pi)$  it always induces the formation of coalition structure  $C_{1,23}$  with  $\Pi_1 = \Pi_{1H}^{(\{1\},\{2,3\})}$ ,  $\Pi_2 = \Pi_{2M}^{(\{1\},\{2,3\})} - \Pi_{3L}^*$  and  $\Pi_3 = \Pi_{3L}^*$ . The reason is that, by the order of the play, after both these proposals, the turn passes to firm 1 and the optimal strategy for firm 1 is to reject the proposal and announce  $p_1^1 = \{1\}$ , thus inducing proposal  $p_2^{23} = (\{2, 3\}, (\Pi))$  by firm 2 with  $\Pi_2 = \Pi_{2M}^{(\{1\},\{2,3\})} - \Pi_{3L}^*$  and  $\Pi_3 = \Pi_{3L}^*$ , that firm 3 will accept. Finally, if firm 3 proposes at the beginning of the game  $p_3^{23} = (\{2, 3\}, \Pi)$ , for any  $\Pi$  firm 2 would reject it to propose, in turn,  $p_2^{23} = (\{2, 3\}, \Pi)$ , again offering  $\Pi_3 = \Pi_{3L}^*$  to firm 3, which would be obliged to accept. Therefore, since by assumption with equal payoffs firms prefer to be in coalition, the game possesses as unique outcome the intermediate coalition structure  $C_{1,23} = (\{1\}, \{2, 3\})$ , which can be sustained as a SPE strategy profiles. Again, it can be easily seen that the game outcome does not change if, to break ties, we assume that

under equal payoffs firms prefer to play as singletons rather than in coalitions. ■

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