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From Bertrand to Cournot via Kreps and Scheinkman:
a hazardous journey

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**From Bertrand to Cournot via Kreps and Scheinkman:
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Abstract

The minimal core of strategic decisions a firm has to make is three-fold: What to produce? At which scale? At what price? A full-fledged theory of oligopolistic competition should be able to embrace these three dimensions jointly. Starting from the Cournot-Bertrand dispute and the stream of research it gave birth to, this survey shows that we are far from having such a theory at our disposal today. Many papers cover two dimensions out of three and display insightful results but no paper satisfactorily addresses the complete picture. I discuss the limitations of the different approaches that have been undertaken. This discussion sets a clear agenda for further theoretical research on the oligopoly front.

Keywords: Bertrand, Cournot, Edgeworth, product differentiation, capacity constraints.

JEL classification: L13.

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1 Introduction

When presenting the foundations of oligopoly theory, standard textbooks usually contrast two modes of competition: quantity competition, initiated by Cournot (1838) and price competition, put forward by Bertrand (1883). Standard textbooks almost immediately insist that neither Cournot nor Bertrand should be viewed as literal descriptions but rather understood as proxies to the actual competition where firms choose both quantities *and* prices. Which of the two models best describes the nature of competition must be a concern because if... “we want to explain market behaviour, we should better have a good idea as to what is the appropriate model of competition.” (Belleflamme and Peitz, 2010, p.66). The answer depends thus on each industry’s peculiarities.

Starting from these premisses, it is natural to question the link that may exist between the two modes of competition. Kreps and Scheinkman (1983) consider a duopoly stage-game with homogeneous goods where firms commit to capacity levels in the first stage and compete in prices in the second stage. The unique subgame perfect equilibrium in their game replicates the Cournot outcomes. Hence, Kreps and Scheinkman (1983) offer a convenient shortcut to reconcile the Cournot and Bertrand approaches of oligopoly competition. More and more scholars hence view Cournot as a reduced-form of capacity-then-price competition models, *in general*. Nowadays, a standard line of reasoning invokes Kreps and Scheinkman (1983) to argue that when production capacities are difficult to adjust instantaneously, the Cournot model is appropriate while the analysis proposed by Bertrand should be retained in the alternative cases (Cabral (2000) is a good example in this respect [see p. 101-114]).

The first claim of the present paper is that relying on Kreps and Scheinkman (1983) to reconcile price and quantity competition *in general terms* is abusive, at least for two reasons. First, in a world with homogeneous goods, the Kreps and Scheinkman result is as famous as it is fragile: it is for instance not robust to alternative rationing rules (Davidson and Deneckere (1986)), it is not robust to the presence of asymmetric marginal costs (Deneckere and Kovenock (1992)), it is not entirely robust to the presence of more than two firms (De Francesco and Salvadori (2010)), it is not robust to sequentiality of quantity commitments quantity setting (Allen et al. (2000)). It is neither robust to a ban on consumers’ rationing. Second, the robustness of the Kreps and Scheinkman’s result to the presence of product differentiation is still an open question. In this paper we shall in particular highlight the fact that a detailed analysis of price competition in the presence of product

differentiation and capacity constraints remains to be done.

Historically, two ways out of the Bertrand paradox have been explored in the literature in order to restore those equilibrium markups characterizing Cournot outcomes. The first one, building on Edgeworth's intuition, highlights the role of having constant marginal costs in achieving Bertrand's outcomes. Kreps and Scheinkman (1983) and more generally the literature dealing with capacity constrained price competition belongs to this vein. The second one builds on Hotelling (1929)'s intuition and puts forward product differentiation as a means to relax price competition. Lots of papers have been written along each of these two directions taken separately but the contributions that dealt with the two dimensions simultaneously can (almost) be counted on the fingers of one hand. There is thus a huge hole in the oligopoly theory literature: almost no equilibrium characterization exists for the very large class of pricing games with product differentiation and capacity constraints (or more generally, locally decreasing returns to scale). The second claim of this paper is then that in order to reconcile Cournot and Bertrand's approaches to competition, we definitely have to fill this hole.

The main aim of this paper is therefore to provide a guided tour to the literature dealing with price competition, capacity constraints and product differentiation. Different routes have been taken, some partial results already exist but many misunderstandings exist as well. The most frequent confusion in this respect is the widespread belief according to which the presence of product differentiation allows to dispense with rationing issues because demands are well defined for all prices, and therefore that product differentiation per se restores the existence of pure strategy equilibria in capacity-constrained pricing games (see for instance Maggi (1996), p.242 , or Adler and Hanany (2009)).

The next section briefly recalls the main insights that must be retained from the original Bertrand and Cournot models. Section 3 summarizes the main contributions that can be derived separately from the analysis of Edgeworth (insisting on the role of consumers' rationing) and Hotelling. Section 4 summarizes the main results of the capacity constrained pricing games with product differentiation, according to whether consumers' rationing is allowed or not. Section 5 concludes.

2 From Cournot to Bertrand

Cournot builds a model where firms sell a homogeneous product and set quantities strategically. In equilibrium, firms retain market power and this is materialized by the fact that the market price exceeds firms' marginal cost. By contrast, Bertrand considers the case where two firms sell homogeneous products but set prices strategically. In equilibrium, firms sell at marginal cost, so that no market power is retained.

It is extremely important at this step to understand the differences between the Cournot and the Bertrand approaches. There is indeed much more than a simple change of strategic variable when going from Cournot to Bertrand. As nicely summarized by Daughety (1988), one actually goes from a one-stage game to a two-stage game. To be more precise, the analysis proposed by Cournot hangs on a Walrasian black-box mechanism that clears the market, given the quantities set by the firms. The underlying mechanism ensures that whatever the quantities dumped on the market, these quantities will be entirely sold at the highest possible price (determined as the market clearing price given quantities supplied and the demand function). Under Bertrand competition, firms set prices first, then consumers compare prices and decide where to buy. Consumers are thus active players (though not strategic players) under Bertrand whereas they are simply not present under Cournot: the market demand is needed only to turn quantities into a unique price with the help of the Walrasian auctioneer. Put differently, the market allocation process is fully explicit under Bertrand (consumers compare prices and then "go" to the shop to buy a product they indeed get) whereas no comparable process can be constructed under Cournot.

More precisely, it is essential for Cournot outcomes to obtain that consumers are not players at all. In order to allow for a meaningful comparison between price and quantity competition, Daughety (1988) considers two games with firms and consumers who make their decisions simultaneously. In both games, consumers play a mixed strategy to decide where to buy while firms set quantities in the first game, and set prices in the second. It is then shown that the two games, the quantity setting one and the price setting one, display identical equilibrium outcomes, which are neither the Cournot nor the Bertrand ones but the collusive ones!

Summing up, Bertrand and Cournot should actually not be viewed as two variations on a common game, differing by the strategic variable retained by the firms but rather as two games that differ by their timing and

numbers of players.¹ Because it allows for a much richer set of interactions among players, the approach proposed by Bertrand may look more attractive. However the mere idea that two firms is enough to restore competitive equilibrium outcomes very early called for critiques. The essence of the so-called Bertrand paradox is quite simple: consumers are too powerful in putting pressure on the firms' decisions. Scholars therefore explored various reasons why consumers would be less powerful and started to study the extent to which firms are likely to relax price competition through strategic commitments.

Modern industrial organization extensively studied stage-games where firms make strategic commitments at a pre-market competition stage, followed by the market competition stage where price and sales are realized. Two instruments in particular have been studied in the literature: product differentiation and capacity commitments. These are the most direct ways to weaken consumers' market power. In this last respect it is then amazing to see that, although being so strongly connected, these two lines of research have evolved almost entirely separately one from the other: capacity-price competition models are (almost) invariably developed in markets for homogeneous goods whereas product differentiation models are (almost) invariably developed under a strictly constant marginal cost assumption. This is damaging. First, from an applied economics perspective, it is desirable to enrich basic theoretical models with product differentiation and various forms of quantitative constraints since real world is indeed populated by oligopolies where firms sell differentiated goods produced under (locally) decreasing returns to scale. Second, from a theoretical point of view, we are indeed very far from a complete theory of oligopoly competition that would connect Cournot and Bertrand competition in more general contexts than those explored by Kreps and Scheinkman (1983).

3 Relaxing price competition

In this section, I start by briefly summarizing the essence of Edgeworth's critique to Bertrand's approach. I then expose the main features that distinguish the Bertrand-Edgeworth approach to capacity-constrained price com-

¹In this respect, recent models with mixed Cournot-Bertrand behaviour are embarrassing. Tremblay (2011) for instance considers a duopoly with one firm setting quantity and the other setting price. Relying on a Bowley (1924) type of product differentiation, it is of course possible to put a standard market demand in the payoffs function of one firm and the corresponding inversed one in the other's payoff function. However, it is really hard to figure out how the market allocation works in this setting.

petition from the Bertrand approach (where consumer rationing is forbidden). Finally, I summarize Hotelling’s contributions to the debate and stress the key limitation of the approach he initiated.

3.1 Capacity constraints and the Edgeworth’s critique

In the presence of decreasing returns to scale, the case for the Bertrand paradox becomes weaker. This has been shown very early by Edgeworth (1897- translated in 1925) but has been largely neglected in the ensuing literature until modern industrial organization addressed the issue along two directions. The first one exactly follows Edgeworth and explores price competition in models where firms face increasing marginal costs and may decide to ration consumers whenever it is profitable for them to do so. The extreme case is simple to figure out: if firms face rigid capacity constraints, they simply cannot produce beyond capacities and must therefore turn some consumers away if demand exceeds capacity. If it is possible to meet any level of demand, but at an increasing marginal cost, firms may nevertheless find it more profitable to ration consumers. This line of research is coined as *Bertrand-Edgeworth* competition and accommodates the presence of rigid capacity constraints (i.e., producing beyond capacities is not feasible) as well as weaker forms of quantitative constraints like increasing marginal costs. This first approach, formalized in modern terms in Levitan and Shubik (1972) has received considerable interest very early after Kreps and Scheinkman’s (1983) contribution (a key author in this respect being Dan Kovenock).

The second line of research, referred to as *Bertrand* competition assumes that firms *must* produce to satisfy demand, i.e. that *they are not allowed to ration*. For this approach to make sense, one needs that producing beyond capacities is indeed feasible, hence capacities *must* be “permeable”, or soft. This second approach is therefore most often studied in models where marginal cost is defined either as a smoothly increasing function or a step function with an upward (but finite) jump at the capacity level. This approach, initiated by Dastidar (1995) only gained in popularity after Maggi (1996) AER paper. Recent papers like Besanko et al (2010) contributed to finally establish this modelling strategy as a standard one.²

These two approaches are actually quite different, and, in my opinion, the differences are largely overlooked. In the next section I build an example

²Cabon-Dhersin and Drouhin (2014) is the most recent example that follows such a strategy.

that highlights these differences and explain why these two approaches are in no case equivalent, nor even substitute one for another.

3.2 Why Bertrand differs from Bertrand-Edgeworth: an example

Let us assume the following setup: two firms sell a homogeneous good in a market defined by the linear demand $D(p) = 1 - p$. We assume that firms share the market equally in case of a price tie. Marginal cost is constant and equal to 0. Neglecting any form of capacity constraints, the unique Nash equilibrium is defined by $(0, 0)$: firms end up with zero profits and we have the Bertrand paradox.

Suppose now that one of the two firms, say firm 1, faces an exogenous capacity constraint at level $k_1 = \frac{3}{4}$, i.e., installed capacity is large enough to satisfy firm 1's demand at the equilibrium candidate but too small to meet aggregate demand under marginal cost pricing.

- In a Bertrand-Edgeworth model, $(0, 0)$ is not a Nash equilibrium anymore, even though firm 1 is clearly not capacity-constrained at the equilibrium candidate. The marginal cost pricing equilibrium is destroyed because there exists a profitable deviation for the unconstrained firm. By raising her price, firm 2 increases demand addressed to firm 1; k_1 becomes binding, so that firm 1 rations consumers; as soon as some of these rationed consumers turn back to firm 2, this firm enjoys a strictly positive profit; hence the deviation is profitable. *Rationing generates spillovers that induce profitable upwards deviations.* It is enough to have only one capacity-constrained firm for this mechanism to be at work.
- By contrast, in a Bertrand model, the presence of this capacity constraint does not affect the original pure strategy equilibrium at all. *Under Bertrand competition, rationing is indeed forbidden.* It immediately follows that the kind of spillovers we have just mentioned above are totally absent here. The fact that firm 1 is capacity constrained does not affect at all firm 2's strategic incentives. The only channel through which the capacity level could affect equilibrium rests on the cost that firm 1 has to bear when forced to meet demand beyond capacity. Suppose that selling beyond capacity involves a marginal cost of $\alpha > 0$, then when undercutting p_2 , firm 1 will incur a loss on sales $D(p_2 - \epsilon) - k_1$ whenever $p_2 \leq \alpha$. In other words, if capacity constraints matter under Bertrand competition, it is only because they temper the

incentives faced by the constrained firm to undercut. If only one firm is capacity constrained, like in our example, the unconstrained firm still has the incentive to undercut any rival's price and we end up at marginal cost pricing in equilibrium.

This example illustrates the mechanisms at work in the two approaches. Under Bertrand-Edgeworth competition the capacity limitation introduces upwards profitable deviations whereas under Bertrand competition it partially reduces incentives to undercut. The unconstrained firm's strategic incentives are affected under Bertrand-Edgeworth whereas only the constrained firm's are under Bertrand. As reminded in the next subsection, product differentiation also weakens firms' incentives to undercut.

3.3 Product Differentiation

Hotelling's *Stability in competition* paper (1929) is rightly celebrated for being the first one to formalize a critique against Bertrand's paradox based on product differentiation. His Main Street metaphor puts forward a simple idea: the presence of product differentiation restores continuity in the demands addressed to each firm. Demand continuity is then shown to restore stability in competition and firms end up selling above marginal cost in equilibrium.

Hotelling's contribution is actually twofold. First, he puts forward product differentiation as a means to relax price competition and second, by exploring optimal location choices by the firms, he paves the way to strategic commitment games. It turns out that his minimal differentiation result is formally flawed, as shown by d'Aspremont et al. (1979). Nevertheless, Hotelling must be considered as the founding father of modern industrial organization.

The original contribution of Hotelling gave birth to a very large set of theoretical papers that contributed to improve our understanding of product differentiation and strategic competition along numerous dimensions. Minimal, Maximal, Intermediate differentiation principles have been established, on the line, the circle, the triangle, the tube, the square, cube and hypercube³... Nowadays, the quadratic transportation cost version of the Hotelling model with extreme locations is accepted as the canonical model for modelling oligopoly price competition model when positive equilibrium markups are necessary for the theoretical point to be made.⁴

³Popular rumor even reports of Phd students working Hotelling on a Klein bottle.

⁴See for instance the recent literature on two-sided markets (Armstrong (2006)).

Notice that other models of product differentiation have been developed in parallel to the address-model literature inspired by Hotelling. In particular, the so-called Bowley (1924) type model of differentiation has been used recurrently in the literature. These models are based on the preferences of a representative consumer. They are particularly well-suited to compare Bertrand and Cournot since they easily accommodate the inversion of demand functions. Unfortunately, they are ill-suited to study strategic commitments in product varieties. More recently, discrete choice models of product differentiation have also been extended to deal with multi-dimensional product differentiation and probabilistic choices (see Anderson et al (1992)). This approach is particularly well-suited for empirical work. These different approaches to price competition under product differentiation have in common that they are invariably built under a strictly constant marginal cost assumption.

There is a sense in which Hotelling (and his “address-models” followers) as well as other models of product differentiation work too well in relaxing price competition. To some extent, this approach almost totally obscured the other way out of the Bertrand paradox, put forward by Edgeworth: the presence of capacity constraints. Even though Edgeworth explicitly suggests that his intuition extends to the case of differentiated products, this remained largely unnoticed. No effort has been made to study price competition with differentiation under non constant returns to scale. *As a result, the whole literature of strategic product differentiation very much lacks robustness:* strictly speaking, all available characterizations of Nash equilibria are valid only under the assumption that marginal costs are constant.

Having summarized the main mechanisms at work when one considers the presence of capacity limitations or product differentiation separately we are now ready to explore the literature that mixes the two dimensions.

4 Capacity-constrained price competition and product differentiation. A (subjective) review of the recent advanced literature

Capacity-constrained price competition models that include product differentiation typically consider stage games where firms make strategic commitments in a first stage, before going to the market where prices are chosen (market competition stage). As already mentioned, capacity-constrained price competition may be modelled by relying on either Bertrand or Bertrand-Edgeworth competition. As exemplified in Section 3.2 these are two very

different modes of price competition. Moreover, in a set-up where firms choose capacity levels and/or product attributes before price competition takes places, we may end up with two qualitatively different classes of pricing subgames: those exhibiting product differentiation and those exhibiting homogeneous goods. These classes radically differ because the first ones belong to the class of games with continuous payoffs whereas the second ones display discontinuous payoffs.

The discussion of this section is therefore organized in two subsections. The first one covers Bertrand-Edgeworth competition and the second one Bertrand competition. In each subsection, I discuss first the properties of the different classes of price subgames according to whether product differentiation prevails or not and second, I discuss non-price commitments made prior to the market competition stage, i.e., at the first stage of the game.

4.1 Bertrand-Edgeworth competition

In his famous critique of Bertrand's paradox, Edgeworth puts forward the idea that prices are actually likely to cycle in the presence of capacity constrained firms and that firms will secure positive profits. Interestingly enough, he also explicitly mentions that the presence of imperfect substitutes, instead of homogeneous goods, does not change the analysis qualitatively, i.e., cycles are robust to the presence of product differentiation but their width is inversely related to the degree of product differentiation. To quote Edgeworth (p.121): *It will be readily understood that the extent of indeterminateness diminished with the diminution of the degree of correlation between the articles.* Summing up, the presence of quantitative constraints qualitatively affects price competition be it under product homogeneity or differentiation.

4.1.1 Market competition stage

• The homogeneous good case

Let us start with the case of **homogeneous products**. Modern industrial organization, armed with the tools of game theory, formalized Edgeworth intuition with the help of mixed strategy equilibria. A pioneering contribution on the **Bertrand-Edgeworth side** is Levitan and Shubik (1972) who opens the (now famous) way from Bertrand to Cournot as finalized by Kreps and Scheinkman (1983). This literature shows for the homogeneous good case that under Bertrand-Edgeworth competition, pure strategy equilibria most often fail to exist but mixed strategy ones do exist and can be

characterized in many cases. The properties of these equilibria have been widely studied as well as those of the associated payoffs. Very often, the equilibrium support of price is continuous, with possibly one atom at the upper bound. A key result regarding equilibrium payoffs is that at least one firm, typically the larger capacity firm, is held down to its minmax payoff (Deneckere and Kovenock (1996)).⁵

• **The case of product differentiation**

As far as product differentiation is concerned, the picture looks very different. **Within the context of Bertrand-Edgeworth competition**, very few positive results exist. Directly elaborating on Edgeworth's intuition, Benassy (1988) characterizes the domain of capacity-differentiation levels for which the standard Bertrand-Hotelling pure strategy equilibrium is preserved. The presence of differentiation is not sufficient to ensure per se the existence of a pure strategy equilibrium for all levels of capacities. Demand spillovers at work when rationing prevails are smoother in the presence of product differentiation but they are still present. They may break demand quasi-concavity and therefore that of firms' payoffs. The general intuition is thus quite simple: the smaller the capacity levels the larger the degree of differentiation it takes to preserve the existence of a pure strategy equilibrium. This result is rephrased in a different context by Canoy (1996). Of course, this result also implies that outside of this domain, the usual equilibrium does not exist.

The continuity of the payoff function also introduces another (more subtle) difficulty. Starting from the idea that in equilibrium firms should not quote a price such that they are strictly rationed, it is tempting to take it as given that in equilibrium demands addressed to the firms must equate their installed capacities and simply invert demand functions, replacing demand levels by installed capacity levels. The continuity of demand functions allows indeed for well-defined inverse demand functions. The argument is sound if one restricts attention to pure strategy equilibria and if such equilibria exist. But of course they tend not to exist. Several authors fell into this trap and characterize candidate equilibria that are not robust to unilateral deviations rather than true equilibria. Examples are to be found for instance in Encaoua et al. (1992), Bouet (2001), Lahmandi (2000).

While the non-existence issue has been widely studied, very few positive results exist. Since the presence of product differentiation restores conti-

⁵Seminal contributions are Davidson and Deneckere (1986), Osborne and Pitchik (1986), Deneckere and Kovenock (1991), Deneckere and Kovenock (1996), Allen et al. (2000).

nuity of the payoff functions, the existence of a mixed strategy equilibrium is not problematic at all. The difficulty lies in the characterization of this equilibrium, or actually, equilibria, since uniqueness is quite problematic as well. Very few characterizations are available in the literature, and most of them only cover asymmetric cases, i.e., cases where one firm only possibly faces a binding capacity within the relevant range of prices.⁶ In a trade context, Krishna (1989) offers an original characterization for a case where only one firm is capacity constrained, within a limited domain of capacity levels around the free trade equilibrium level. In equilibrium, the unconstrained firm mixes over two atoms while the constrained one plays a pure strategy. Boccard and Wauthy (2006) characterize firms' equilibrium payoffs and bounds the support of equilibrium prices in a Hotelling model for the full domain of capacity. Boccard and Wauthy (2011) extend this result to a setup where both firms are capacity constrained. They show that the equilibrium support of prices is finite. Boccard and Wauthy (2013) consider a game with asymmetric capacities and vertical differentiation. These three papers share two main features. First, equilibrium payoffs of the two firms tend to increase in a mixed strategy equilibrium as compared to the corresponding unconstrained (pure strategy) price equilibrium candidate. Second, none of these papers manages to provide an explicit characterization of equilibrium distributions. All in all, it is thus only fair to say that we are almost completely ignorant in this segment of the literature.⁷

4.1.2 Precommitments.

There are two obvious dimensions along which to endogenize the parameters that govern price competition at the market stage: the degree of product differentiation and the capacity levels. As already mentioned, the theoretical literature on product differentiation commitments is immense. By contrast, the literature on endogenous capacity choices is more limited.⁸ The joint

⁶Furth and Kovenock (1993) are an exception but they consider a sequential pricing game, instead of a simultaneous one. Cabral et al. (1998) reports other (unpublished) results for a simultaneous game but I did not manage to obtain a copy of their (though available upon request) manuscript.

⁷This is damaging not only for the analysis of capacity constrained pricing games of product differentiation. It turns out indeed that other classes of games display the same features, namely pricing games with uninformed buyers and pricing games with switching costs. In these models also, upwards deviations destroy the existence of pure strategy equilibria, many results have been established for the case of discontinuous games but their extension to the class of continuous ones remains to be done.

⁸Important references, in addition to those already quoted are Deneckere and Kovenock (1991), Osborne and Pitchik (1986), to name a few.

study of capacity-differentiated commitments is a fortiori even less explored. In view of the limitations of the Bertrand-Edgeworth literature pointed out in the preceeding sections, we already know that very few results are to be expected regarding optimal choices made prior to price competition.

The most obvious research question pertains to the generalization of Kreps and Scheinkman (1983) to markets with differentiated products. Put differently: when we consider a model of product differentiation where inverting demand is feasible, i.e., where the Cournot equilibrium with product differentiation can be computed, do we expect that Cournot outcomes obtain in a subgame perfect equilibrium of the stage game where firms commit to capacities before competing in prices? Based on the existing literature, two results are easy to establish. First, in the domain where capacity levels are arbitrarily small, firms actually set market clearing prices in the unique equilibrium and are locally induced to increase their capacities. Locally, firms tend to behave like Cournot players when capacity levels are quite small. Second, in the domain where capacities are “large”, i.e., sufficiently above realized demand at the unconstrained price equilibrium, firms are induced to reduce their capacities, at least down to the level where the pure strategy equilibrium is destroyed (a result that directly follows from Krishna (1989)). Hence, whether from below or from above the capacity range, firms’ incentives tend to push capacities at least at the frontier of the domain where pure strategy equilibria do not exist. Since no general characterization of mixed strategy equilibria exist, almost nothing more can be said at this step.

The full generalization of Kreps and Scheinkman being out of scope for the moment, let us consider less ambitious setups. It turns out that the model of quality differentiation popularized by Tirole (1988) as a mash-up of Mussa and Rosen (1978) and Gabszewicz and Thisse (1979) offers a convenient vehicle for this task. Indeed under a uniform distribution of consumers’ taste and a large degree of consumers’ heterogeneity, this model displays the linear version of Bertrand and Cournot duopolies for the whole domain of product differentiation. It therefore allows to study some relevant branches of the complete capacity-differentiation-price stage game. Bocard and Wauthy (2013) consider the case where only one firm is allowed to limit its production capacity and shows that under efficient rationing the unconstrained firm’s payoffs is held down to its minmax payoff whatever the degree of product differentiation.⁹ When product differentiation tends to zero, this payoff converges to the Kreps and Scheinkman payoffs of the rel-

⁹A result which is clearly reminiscent of Deneckere and Kovenock (1992) for the case of homogeneous goods.

evant subgames. Relying on the same model, Boccard and Wauthy (2013) also partially address the related question of the endogenous level of product differentiation under capacity limitations. Their results clearly suggests that in a model where both capacities and qualities are endogenous, we may expect that the homogeneous good Cournot equilibrium prevails as a subgame perfect equilibrium outcome. At this stage, however, this is nothing more than a conjecture.

4.2 Bertrand competition

4.2.1 Market competition stage

- **The Homogeneous good case**

The formal analysis of Bertrand competition models in the presence of capacity constraints is more recent and dates back to Dastidar (1995) who studies a Bertrand model with homogeneous goods where firms face strictly convex costs. Many variations on this model have been studied that preserve his key result: instead of having a unique zero profit pure strategy equilibrium, we end up with a continuum of symmetric pure strategy equilibria where firms set identical prices and enjoy positive profits. The intuition is straightforward: below some critical price level, undercutting the other's price is not profitable anymore because meeting demand levels beyond capacities is too costly. As a result, matching the other's price becomes a best reply. In a very recent paper, Cabon-Dhersin and Drouhin (2014) consider a stage-game where the capacity level is chosen in the first stage so as to ensure that the collusion is the unique payoff-dominant pure strategy Nash equilibrium.

- **The case of product differentiation**

Within the context of Bertrand competition, Maggi (1996) popularized the stepwise version of Bertrand competition in which total cost is piecewise linear with a kink at the level of the installed capacity. He relies on a Bowley type of product differentiation. Notice first that when reading Maggi's paper, it is not entirely clear from the start which branch of the literature the model belongs to. We are indeed told that *"It may be interesting to relate this result to the well-known fact that pure-strategy equilibria (in prices) often fail to exist in Bertrand competition games with homogeneous goods and rigid capacity constraints (e.g., Kreps and Scheinkman, 1983). The nonexistence problem is caused by two features of these models. One is product homogeneity, which implies a discontinuity in the residual demand*

function. The other is that the production cost is infinite for output in excess of capacity. This implies that consumers must be rationed for certain ranges of prices, and the demand “spillovers” from one firm to the other may imply profit functions that are not quasi-concave. In the model proposed here, neither of those features is present, since I assume differentiated goods and firms do not have to ration consumers (since the marginal cost of production is finite for any production level); hence, profit functions are continuous and quasi-concave, and a unique pure-strategy equilibrium exists.” (Maggi 1996, p. 242) The ambiguity comes from Maggi’s claim that in his model firms *do not have to ration consumers*. Actually, what is crucial for the model to go through is that *firms are not allowed to ration*, which is quite a different statement. In Maggi’s model indeed, firms would always find it optimal to ration if they were allowed to do so. Thus the reason we do not have the “non-existence problem” here is simply that we do not have Bertrand-Edgeworth competition. Product differentiation actually plays no role in solving the “non existence problem”. This is a consequence of the ban on rationing, as established previously in Dastidar (1995). Product differentiation is, however, essential in ensuring uniqueness of the pure strategy equilibrium. Indeed, as mentioned earlier in this paper, Bertrand competition (i.e. models where rationing is forbidden) generically yields multiple pure strategy equilibria in the case of homogeneous goods. Once properly understood, it is hardly surprising that a combination of product differentiation, “soft” capacity constraints and banning of rationing yields a unique pure strategy price equilibrium. The ban of rationing prevents demands spillovers (this ensures that upward deviations are not profitable) whereas product differentiation smoothes downward deviations along well-defined demands.

Moreover, the resulting equilibrium payoffs displays a strong Cournotian flavour. In these models, firms must produce to satisfy demand and thus must produce beyond capacities if required. However, “in all cases, k represents the maximum efficient scale” (Maggi (1996, p. 241)). In other words, firms are not enclined to produce beyond installed capacity but prefer to sell at the capacity clearing price, defined as the largest price such that demand equals capacity, given the other’s price. Obviously, it is sufficient that the penalty incurred when selling beyond capacity is large enough to mechanically enforce the Cournot outcomes. Besanko et al. (2010) builds on Maggi’s model to study the dynamics of capacity accumulation and withdrawal in industries. Within their dynamic setup, relying on Bertrand competition with product differentiation is instrumental in ensuring well-defined, and simple to compute, equilibrium payoffs in all possible price subgames. However,

the behaviour of their model at the no-differentiation limit is questionable since in this case equilibrium uniqueness is lost.

4.2.2 Precommitments.

In a **Bertrand competition** setup, i.e. a setup where rationing is forbidden, Boccard and Wauthy (2010) consider a stage game where capacities and quality levels are fully endogenized. This paper studies a three stage game where first, quality levels are chosen, then capacity levels, and finally price levels. The second and third stage subgames therefore cover the generalized capacity-price competition domain of Bertrand competition. They show that Cournot outcomes obtain in all subgames with non-degenerate product differentiation, a result which points in the same direction as Maggi (1996). However, they also show that in the subgame perfect equilibrium of the full game products must be homogeneous. At the no-differentiation limit, a continuum of equilibria exists, including the collusive one. When quality choice is then endogenized, this collusive, homogeneous good, outcomes turn out to be a subgame perfect equilibrium outcome. In other words, under Bertrand competition, the pseudo generalization¹⁰ of Kreps and Scheinkman to differentiated markets does not display Cournot outcomes.

4.3 Comparison

When products are homogeneous, equilibrium outcomes in pricing subgames differ from Bertrand-Edgeworth to Bertrand as follows: a mixed strategy equilibrium prevails in the first case while a continuum of pure strategy equilibria prevail in the second case.

Under product differentiation, the difference between Bertrand-Edgeworth and Bertrand is also striking: on the one hand we have possibly multiple mixed strategy equilibria that we cannot characterize for a large domain of parameters and on the other hand we have a unique pure strategy equilibrium. It seems that the presence of product differentiation acts like a dividing line in each sub-branch of the literature:

1. In the case of Bertrand-Edgeworth competition, the division is between what is known (properties of mixed strategy equilibria and the associated payoffs under product's homogeneity) and what remains to be

¹⁰We refer to pseudo generalization because rationing is forbidden; an assumption which, as we have shown, defines a game which is entirely different from the Kreps and Scheinkman one.

done (a thorough analysis of mixed strategy equilibria under product differentiation);

2. In the case of Bertrand competition, the division is between multiplicity (in the case of homogeneous products) and uniqueness (in the case of differentiated products) of pure strategy equilibria.

Moreover, the connections between the two treatments need to be clarified. The bottom line is therefore the treatment we allot to rationing. Ideally, one would like to let rationing result from an optimal decision of the firms, rather than from an ad hoc assumption. Obviously, fully addressing this question is out of reach today since we do not have a complete characterization of equilibrium prices and payoffs in most subgames with rationing, in particular close to the frontier between product differentiation and product homogeneity.

To the best of my knowledge, no effort has been made to endogenize whether to ration or not.¹¹ Referring to the approach proposed by Maggi where soft capacity constraints are modeled as stepwise marginal costs, we may contemplate to endogenize the height of the step. In Boccard and Wauthy (2010), the height of the step is assumed to be very large in order to ensure that the penalty associated to undercutting is prohibitively large. So that rationing consumers is optimal, if allowed. Clearly enough, if capacity constraints are understood as a disciplining device, one may indeed think that committing to the largest penalty in case of undercutting is a likely outcome. But again this is a conjecture. On the other hand it is easy to show that, given installed capacities, at least one firm will very often find it optimal to commit to ration consumers. For instance, this is obviously the case in the example I proposed in Section 3.2.

5 Conclusion

The minimal core strategic decisions to be made by firms as a threefold issue: What to produce (product differentiation)? At which scale (choice of capacities)? At which price (market competition)? Arguably, the first two issues are decided before the third one. Whether the first is decided before the second or not is debatable. What is clear however is that a really satisfactory theory of oligopolistic competition should be able to address these

¹¹Of course the Bertrand-Edgeworth literature explored the robustness of Kreps and Scheinkman's conclusions to the choice of rationing rule but the choice between Bertrand-Edgeworth and Bertrand is another issue.

three issues within a self-contained model. Our survey is quite disappointing in this respect: we do not have such a theory at our disposal. Many authors dealt with all possible combinations of two issues out of three ones. They obtain very useful results. However, very few papers tried to address the three dimensions in a unified setup and those who did ended up with rather limited results up to now.

It turns out that even the nature of the similarities and differences between price and quantity competition are not entirely understood. Introducing capacity constraints in price competition games offers a way to connect Cournot and Bertrand outcomes. I have argued however that lots remain to be done in order to better understand and generalize the insights of capacity-price competition. Bertrand competition forbids consumers' rationing. This proves useful in ensuring the existence of pure strategy equilibria. However, the behaviour of the model at the no differentiation limit is problematic, at least from a theoretical viewpoint. Moreover, it is not clear that firms would always find it optimal to commit not to turn consumers away. On the other hand, the behaviour of Bertrand-Edgeworth approach seems more promising at the frontier between differentiated and homogeneous goods. However, much too little is known regarding the characterization of mixed strategy equilibria in capacity-constrained game with product differentiation.

At this step, the research agenda is thus quite clear: it is urgent to devote more efforts to analyze in full depth the class of Bertrand-Edgeworth pricing games with product differentiation.

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