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Licensing to vertically related markets

Eva-Maria Scholz



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Center for Operations Research
and Econometrics

Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium

<http://www.uclouvain.be/core>

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Eva-Maria SCHOLZ ¹

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Abstract

We analyse the problem of a non-producing patentee who licenses an essential process innovation to a vertical Cournot oligopoly. The vertical oligopoly is composed of an upstream and a downstream sector which may differ in their efficiency or, in other words, in the benefit they derive from the innovation. In this framework we characterise the optimal licensing contract in terms of the licensing revenue maximising policy (fixed-fee or per-unit royalty) and sector (upstream and/or downstream sector). First, it is shown that under a fixed-fee contract licensing to the less efficient industry sector may be the patentee's licensing revenue maximising strategy. Here, licensing to a less efficient downstream market is all the time optimal in terms of consumer surplus and aggregate economic welfare. Conversely, licensing to a less or equally efficient upstream industry is potentially inefficient. Second, our findings reveal that the optimal licensing policy is sector dependent. A per-unit royalty contract may dominate a fixed-fee policy on the downstream market in terms of licensing revenues, while offering a per-unit royalty contract to the upstream industry is never optimal. As a third and final point we address the case of licensing to both industry sectors. Here we also identify conditions under which two-sector licensing of both sectors is less profitable than one-sector licensing of a single industry (and vice versa).

Keywords: licensing contracts, fixed-fee, royalties, vertical Cournot oligopoly.

JEL classification: D43, L13, O31, O34.

¹ Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium.
E-mail: eva-maria.scholz@uclouvain.be

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1 Introduction

There is by now a large theoretical literature which attests the influence of market structure on the design of licensing agreements. Based on the seminal work of Arrow (1962), this literature predominantly focuses on comparing fixed-fee and per-unit royalty licensing schemes in terms of the patentee's licensing revenues. Whereas the early literature concentrates on a setting in which a non-producing (outside) patentee licenses to a homogeneous good oligopoly under either Bertrand or Cournot competition (Kamien and Tauman (1986), Kamien et al. (1992)), more recent approaches also take into account various forms of product differentiation (Erkal (2005), Kabiraj and Lee (2011), Wang and Yang (1999), Wang (2002)), asymmetries (Beggs (1992), Gallini and Wright (1990)) or the fact that the patentee may himself compete in the product market (inside patentee) (Kamien and Tauman (2002), Wang (1998)). Those more recent approaches are to a large part motivated by an apparent contradiction between early licensing theory and empirical evidence (Bousquet et al. (1998), Macho-Stadler et al. (1996), Rostoker (1983-1984)). Whereas the latter documents a widespread use of per-unit royalty licensing contracts, the early licensing literature shows that a fixed-fee contract provides an outside patentee with higher licensing revenues than a per-unit royalty scheme.

Despite its vastness, the theoretical licensing literature suffers from an important shortcoming. It neglects the fact that technology licensing agreements are particularly important for knowledge-intensive industries (Anand and Khanna (2000), Arora and Gambardella (2010), Gambardella and McGahan (2010), Walter (2012)). Knowledge-intensive industries, however, equally distinguish themselves by their vertically separated market structure, which, with view on the preceding discussion, is more than likely to exert an important influence on the design of a licensing agreement. In particular, the scarcity of the theoretical literature on technology transfer or licensing to such vertically separated industries brings with it that little is known regarding the optimal transfer strategy or the licensing incentives of independent innovators in those market environments.³ To shed some light on this topic we study the optimal licensing strategy of a non-producing (outside) patentee who licenses a cost-reducing innovation to a vertical Cournot oligopoly. We show that it is indeed crucial to take a vertically separated industry structure into account, not only *per se*, but also in so far as it is important to distinguish which layer of the vertical structure is licensed.

To be more precise, the focus of this paper is on a framework in which a non-producing innovator licenses a cost-reducing innovation to a vertical Cournot oligopoly which is composed of an upstream and a downstream sector.⁴ Firms on each sector have potential access to the innovation and are either offered a per-unit royalty or a fixed-fee contract. A key point of

³The importance of independent innovators is documented in the literature. Cesaroni (2003) reports that independent technology suppliers cover almost 70 % of the total market for licensing in the Chemical industry. Similarly, most of the technology transfer in functional design inventions or design modules in the Semiconductor industry involves chipless firms (Linden and Somaya (2003)).

⁴For completeness, in this paper we study the licensing of essential innovations. The innovation being essential implies that any unlicensed firm realises non-positive market revenues. Focusing on essential innovations allows us to present our findings in a systematic way. However, our results are not exclusive to the licensing of essential innovations (see Section E in the appendix). A precise definition and further motivation are given in Section 2.2.

our analysis is the fact that we allow for cross-sector efficiency differences. Meaning, the cost-reduction embodied in the innovation may vary across sectors. Or in other words, the upstream and the downstream sector are allowed to differ in terms of their capacity to incorporate the innovation in their production processes. In this setting we explore the design of the optimal licensing contract. In particular, we aim at clarifying which policy (fixed-fee or per-unit royalty) and which sector (upstream and/or downstream sector) maximise the licensing income of the patentee and how this ranking is related to the pre-licensing degree of competition, the pre-licensing market revenues or the efficiency of the two concerned markets. It is shown that the optimal licensing contract may involve licensing to the less efficient industry sector, downstream per-unit royalty contracts as well as one-sector licensing to a single sector or two-sector licensing to both markets.

Before presenting the three main results of our paper in more detail, we introduce two examples which serve to illustrate our findings. Those examples are developed further in the remainder of this paper. In both examples, the technology corresponds to quality control solutions in the food industry. The upstream sector may for instance correspond to the agricultural industry, while the downstream sector refers to the market of final good producers, i.e., the food processing industry. The downstream firms source their inputs, e.g., dairy products, from the upstream industry and then processes them to produce the final product. In the first scenario we have in mind a technology that enables firms in either industry to assess the quality of the input good which is supplied from the upstream to the downstream sector. It is safe to assume that firms in both industries are potentially interested in obtaining access to such quality control technology. However, intuitively, a potential licensee's perceived benefit from such a technology may be decreasing in the number of firms in the other sector conducting quality controls (*subadditivity* of the innovation). The more upstream firms are controlling the quality of the input good, the lower the direct benefit of the technology for the downstream firms (and vice versa). In the second case, the final technology is not developed yet, however, its technological basis is patented. The final technologies allow firms to assess the quality of their sector-specific product (i.e., input good and final good). Thus, although the technological basis of the quality control solutions may be the same for both markets, adjustments of the final product are most likely necessary to account for the specific needs of each industry. The agricultural sector may predominantly be interested in tests to detect traces of hormones or antibiotics, while the food processing industry may primarily necessitate solutions to control the vitamin content or to detect allergens in the final product. Depending on a variety of factors related to financial and strategic aspects, the patentee's development or commercialisation strategy may then focus on both or on a single industry sector.

Our paper has three main results. First, our findings show that for a per-unit royalty policy, the more efficient industry provides the patentee with higher licensing revenues. Surprisingly, this result does not necessarily apply under a fixed-fee policy. Instead, for a fixed-fee policy, licensing to the less efficient industry may be the optimal strategy of a licensing income maximising patentee. This result applies regardless of whether the technology is transferred to a

single or to both industry sectors.⁵ Taken together, our findings suggest that in vertically related industries, a fixed-fee, and not a royalty rate, functions as an *insurance* or *risk-reducing* mechanism. Focusing innovation or technology transfer efforts, for example, on the downstream sector may be optimal in terms of licensing income, even though the upstream sector derives a higher benefit from the innovation. For the case of one-sector licensing to a single industry sector we further demonstrate that in order for it to be optimal to license to a less efficient upstream (downstream) industry, the upstream market has to be sufficiently concentrated (competitive) relative to the downstream market. Under two-sector licensing to both industry sectors, the conditions on the upstream market structure are reversed.

The results of our analysis crucially depend on a distinct reaction of both sectors to the transfer of the technology. In particular, licensing to the downstream industry entails a demand independent upward shift in the level of the intermediary input price. This upward shift, termed *raising rival's cost effect*, also plays an important role in Mukherjee (2002). The author studies the licensing strategy of a producing innovator within a downstream Cournot duopoly under a fixed-fee contract. It is shown that licensing in the downstream market is profitable provided it encourages entry in the previously monopolised upstream industry. The result is based on the presence of a demand independent upward shift in the input price that follows the technology transfer to the competitor. That is, by licensing the technology the patentee not only faces a more efficient industry rival but equally a higher intermediary input price. As a consequence, entry in the upstream market is necessary to render licensing within the downstream industry profitable. In our framework the upward shift leads to a different profitability of the the upstream and the downstream sector in terms of licensing revenues. Everything else being equal, the upward shift reduces downstream fixed-fee licensing income and by this indirectly confers an advantage to the upstream industry. Therefore, to render a one-sector technology transfer to a less efficient downstream market profitable, a sufficient degree of upstream competition is required (the drop in the input price that follows from an increased degree of upstream competition balances the upward shift in the input price that follows a downstream technology transfer).⁶

It is clear that the presence of private incentives for a technology transfer to a less efficient market does not necessarily imply its social desirability. To explore the welfare implications of the optimal licensing contract we contrast the patentee's fixed-fee licensing strategy with the optimal contract in terms of consumer surplus and aggregate economic welfare. It is shown that one-sector licensing to a less efficient downstream market is all the time optimal in terms of consumer surplus and economic welfare. Regarding the upstream industry a conflict between the private incentives of the patentee and those of the consumers or the aggregate economy may

⁵In the case of a two-sector technology transfer to both industry sectors, licensing revenues may be seen as a way to measure incentives to innovate. Thus, our findings show that it may be the less efficient market that provides the patentee with larger innovation incentives e.g. regarding the development of second generation test devices, technology upgrades or other subsequent innovation efforts.

⁶In our eyes, in this context, one-sector licensing is the more interesting case. Under one-sector licensing the patentee is able to avoid the upward shift in the intermediary input price by transferring the technology to the upstream market. It follows that by deriving conditions under which a downstream technology transfer is licensing revenue maximising, despite a lower degree of downstream efficiency, we identify scenarios in which the patentee not only licenses to the less efficient sector, but also accepts the demand independent upward shift (and still adopts a revenue maximising strategy).

arise.

Second, the optimal one-sector licensing policy (fixed-fee vs per-unit royalty) is sector specific. Concerning the upstream industry the results of the traditional licensing literature apply and a fixed-fee contract provides larger licensing revenues than a per-unit royalty contract (Kamien and Tauman (1986), Kamien et al. (1992)). In contrast, regarding the downstream industry offering a royalty based contract may be optimal. Our paper thus adds to the previously cited licensing literature by providing another rationale for the empirically observed popularity of per-unit royalty contracts.⁷ Also this result is based on the distinct reaction of both sectors to the technology transfer. To be more precise, under a per-unit royalty contract, the patentee is able to mitigate the importance of the demand independent upward shift in the input price that follows a downstream technology transfer. In fact, the upward shift is shown to be zero at the optimal royalty rate. Under two-sector licensing, a per-unit royalty policy is never employed.

As a third and final point we address the case of two-sector licensing. Apart from the results already discussed in the preceding paragraphs, we also identify conditions which render a one-sector technology transfer the patentee's optimal licensing strategy. In general, there are many reasons for why an innovator's development or commercialisation strategy may (initially) be concentrated on a specific industry. The former are related to strategic and financial arguments and are further discussed in Section 2. In this paper we primarily focus on the aspect that the benefit embodied in the innovation may be lower under two-sector licensing. As such, we show that the patentee optimally transfers the innovation to a single industry (as compared to both sectors), given the innovation is characterised by a sufficient degree of *subadditivity*.

This paper is organised as follows. Section 2 introduces the general modelling framework. Here Section 2.1 and Section 2.2 focus on the specifics of a fixed-fee and a per-unit royalty licensing game. In this context, we first derive the subgame perfect Nash equilibria of the respective licensing games. Next we compare the upstream and the downstream market in terms of licensing revenues for a given licensing policy. Section 2.3 derives the optimal sector specific licensing policies (fixed-fee vs per-unit royalty) for a given industry sector. Under the optimal sector specific licensing policies we then re-assess the ranking of the upstream and the downstream market in terms of licensing revenues and derive the optimal licensing contract. To illustrate the results of our analysis, Section 2.4 introduces three polar cases (equal efficiency, pre-licensing market size or market revenues). Section 2.5 studies welfare aspects of the licensing revenue maximising fixed-fee contract. As a final point, Section 3 analyses the case of two-sector licensing. Section 4 concludes.

2 The Model

We consider a vertical Cournot oligopoly with an upstream (m) and a downstream (n) sector. In the following the sector is denoted by s with $s \in \{m, n\}$. On the upstream sector M firms

⁷Here our paper extends the results of Chang et al. (2013) to a more general framework. In Chang et al. (2013) an outside patentee transfers a cost-reducing technology to a downstream Cournot duopoly, the upstream firm holds a monopoly position. The authors focus on the superiority of two different licensing policies (fixed-fee and per-unit royalty) in terms of the patentee's licensing revenues as well as on welfare considerations.

are active and produce a homogeneous product which serves as an input for the downstream industry on a one-to-one basis. Similarly, on the downstream market N firms produce a final homogeneous good. For each sector, marginal costs of production are given by $c_i^{s,\alpha}$ where $\alpha \in \{l, u\}$ denotes the licensing status of a firm. As such, $\alpha = l$ if firm i belongs to \mathcal{L}^s , the set of licensed firms of sector s . Next to their marginal cost of production, downstream firms also face the input price w . The input price is determined upstream and taken as given by the downstream players. Upstream quantities are denoted by x_i^α , q_i^α refers to the individual downstream quantities. Finally, the inverse demand for the final product is of a linear form and given by $p(Q) = a - Q$ with $Q = \sum_{i=1}^N q_i^\alpha$ aggregate industry output. The setting is formalised as a standard two stage game and solved by employing the solution concept of a subgame perfect Nash Equilibrium (SPNE).

Firms in each sector have potential access to a cost-reducing technology. It is assumed that the technology is offered by a non-producing (outside) patentee, which is not active in either sector of the vertical structure. The licensing contracts offered include either a per-unit royalty rate (r^s) or a fixed-fee (f^s). In a first step we analyse the case of one-sector licensing to either level of the vertical structure. Meaning, the patentee may transfer the innovation to either the upstream or the downstream industry, but not to both markets at the same time. We then extend our analysis to two-sector licensing.

In general, there are various reasons for why an innovator's development or commercialisation strategy may (initially) focus on one particular industry sector. Those reasons include strategic arguments (first-mover advantage, test of marketability) as well as financial arguments (lack of funds for the launch of large scale commercialisation/development campaigns, industry response is uncertain). For instance, regarding the licensing of general purpose technologies such as IT or advanced materials, notice that their general nature implies that those technologies may be relevant for a variety of different industries (Gambardella and McGahan (2010), Maine and Garnsey (2006)).⁸ However, at the same time, their general nature brings with it that customised R&D and/or complementary innovations are most likely necessary for each target market. Furthermore, the requirements of the target industry's regulatory environment need to be addressed. For the patentee this implies significant uncertainty as well as high and long term financial investments. As a consequence, the patentee may want or have to focus his development or commercialisation strategy on a specific market (at least in the short run). What is more, two-sector licensing to both markets may be less profitable compared to one-sector licensing due to the nature of the innovation. This is for instance the case when the benefit embodied in the innovation is lower under two-sector licensing (*subadditivity* of the innovation). In this context note that as the cost reduction enters the profit function of a licensee linearly, the innovation may either be seen as cost-reducing or demand enhancing. In the extreme case, it may be assumed that the innovation once employed by one sector yields no further direct benefits to the other sector. This is for instance the case when the innovation takes on the form of a technology which is used to treat the intermediary input. Assuming that the treat-

⁸According to Arora and Gambardella (2010) non-producing innovators such as technology based firms have a strong incentive to offer generic technology with potential application in a variety of different sectors.

ment may be done either upstream or downstream, two-sector licensing becomes redundant. Finally, the potential licensees themselves may request a one-sector technology transfer (Arora and Gambardella (2010)).

The process innovation lowers the unit cost of a licensed firm by some θ_s with $\theta_s > 0$. This implies that we allow for cross-sector efficiency differences. In other words, the upstream and the downstream sector may differ in terms of the benefit they derive from the innovation. It follows that the licensing status dependent marginal cost of production of an agent in sector s are specified as

$$c_i^{s,\alpha} = \begin{cases} c^s - \theta_s + r^s & i \in \mathcal{L}^s, \\ c^s & i \notin \mathcal{L}^s. \end{cases} \quad (1)$$

Intuitively, $r^s = 0$ in the case of a fixed-fee licensing policy. We further restrict $\theta_s \leq c^s$.

The licensing game is modelled as the following three stage game. On the first stage, the patentee announces the sector specific licensing contract, which, depending on the licensing policy chosen, specifies the royalty rate r^s or the fixed-fee f^s . Under one-sector licensing, licensing contracts are offered to a single industry sector, whereas under two-sector licensing the number of licensing contracts offered is strictly positive for both markets. Subsequently, on the second stage of the game, firms in the licensed industry independently and simultaneously decide about acceptance and refusal of the proposed contract. On the last stage of the game Cournot competition takes place. This is the two stage game described above. The licensing game is solved via backward induction.

In the remainder of this section we solve the Cournot game. With this benchmark model in place we then focus on licensing aspects. Here Section 2 studies one-sector licensing, Section 3 analyses a two-sector technology transfer.

On the downstream market, each firm chooses its individual level of production, q_i^α , in order to maximise its profit function $\pi_i^{n,\alpha}(q_i^\alpha, Q_{-i}) = (p(Q) - c_i^{n,\alpha} - w)q_i^\alpha$. Here $p(Q) = a - Q$ refers to the inverse demand function for the final product with $Q = \sum_{i=1}^N q_i^\alpha$, w denotes the input price.

The first order conditions of the downstream firms' optimisation problem then amount to

$$a - Q - q_i^\alpha - (c_i^{n,\alpha} + w) = 0. \quad (2)$$

Define $C^s = \sum_{i=1}^{|\mathcal{S}|} c_i^{s,\alpha}$ as the sum of the marginal costs of sector s . Then, summing (2) for the group of licensed and unlicensed downstream players yields the derived demand $w(Q)$ with

$$w(Q) = a - \frac{1}{N}((N+1)Q + C^n). \quad (3)$$

On the upstream market each firm maximises its profit function $\pi_i^{m,\alpha}(x_i^\alpha, X_{-i}) = (w(X) - c_i^{m,\alpha})x_i^\alpha$ with respect to x_i^α . It is assumed that firms face no capacity constraints so that $Q = X$.

The first order conditions of the upstream firms optimisation problem are

$$w(X) - \left(\frac{N+1}{N}\right)x_i^\alpha - c_i^{m,\alpha} = 0. \quad (4)$$

Summing (4) for the group of licensed and unlicensed upstream firms yields

$$w(X) = \frac{1}{M} \left(\left(\frac{N+1}{N}\right)X + C^m \right). \quad (5)$$

with $X = \frac{MN}{(M+1)(N+1)} \left(a - \frac{C^m}{M} - \frac{C^n}{N} \right)$.

Lemma 1 Define $C^s = \sum_{i=1}^{|S|} c_i^{s,\alpha}$ as the sum of the marginal costs of sector s . Then, $\forall s \in \{m, n\}$, $\frac{\partial X}{\partial C^s} < 0$ and $\frac{\partial p}{\partial C^s} > 0$. However, $\frac{\partial w}{\partial C^m} > 0$, while $\frac{\partial w}{\partial C^n} < 0$.⁹

From Lemma 1 it follows that a decrease in the sum of the marginal costs of the downstream industry not only results in a decrease of the final good price, but, at the same time, entails an increase in the intermediary input price. Put simply, downstream firms are hurt twice by the transfer of the innovation to their sector in terms of market revenues.¹⁰

On the downstream market the presence of such a *raising rival's cost effect* (RRC)¹¹ implies that, everything else being equal, market revenues of licensed and unlicensed firms are lower than in the absence of any such effect. This RRC is particularly detrimental to those firms which do not obtain a licensing contract. The latter not only face a segment of more efficient rivals (and consequently a lower p and a higher w), but they also lack any direct benefit stemming from the innovation in terms of reduced marginal costs of production.

2.1 Royalty Licensing Policy

Basic results of the licensing game

We first consider the case of a per-unit royalty contract. The corresponding licensing game is solved in section A of the appendix, its relevant characteristics are summarised in Lemma 2.

Lemma 2 For a non-drastic innovation, the pure strategy SPNE of the royalty licensing game has the following features:

- i. The innovation is licensed to $l^s = |S|$ players.
- ii. The royalty rate amounts to $r^s = \theta_s$.
- iii. The patentee's licensing revenues are $\pi^{P,r,s} = r^s X$.

⁹For a general demand function this is shown by Banerjee and Lin (2003).

¹⁰For a non-drastic innovation and a fixed-fee policy, the licensees net market revenues are lower than their market revenues in the pre-licensing equilibrium. For a general demand function this is for instance shown by Kamien et al. (1992).

¹¹We follow Banerjee and Lin (2003) and Mukherjee (2002) and term the upward shift in $w(Q)$ *raising rivals' cost effect*. Following the acceptance of a licensing contract by a single firm, the intermediary input price not only increases for this firm, but equally for this firm's rivals.

From Lemma 2 it is apparent that a revenue maximising patentee optimally licenses to the more efficient sector. There is consequently a clear ranking of the two industries with respect to licensing income. Note that this ranking is independent of the degree of competition or the pre-licensing profits on both markets. In other words, whether the patentee licenses to the pre-licensing more or less competitive or profitable industry has no implications in terms of the licensing revenues obtained.¹² It follows that the optimal licensing contract specifies a strictly larger royalty rate for the more efficient sector. Depending on the size of both industries, the number of actual licensees (number of firms in the more efficient industry) may be higher or lower than the number of hypothetical licensees (number of firms in the less efficient industry).

The intuition behind this result is simple. The patentee's licensing income amounts to the product of the royalty rate and the aggregate production of the licensed firms. The optimal royalty rate equals the cost reduction so that the costs associated with obtaining a licensing contract just offset the benefit embodied in the innovation. Every firm is therefore indifferent between accepting and rejecting the proposed licensing contract and continues producing as prior to the transfer of the technology. As we abstract from capacity constraints, the level of production is then the same on both markets and equal to the pre-licensing output. Therefore, the only source of differentiation between both sectors in terms of the patentee's licensing revenues is the sector specific royalty rate.

2.2 Fixed-Fee Licensing Policy

Basic results of the licensing game

Let us now consider a fixed-fee policy. First off, in this paper we focus on essential, but non-drastic process innovations. This implies that the innovation θ_s is sufficiently large such that every unlicensed firm realises zero market revenues. At the same time, the innovation is not too large such that in equilibrium strictly more than one firm obtains a licensing contract. Focusing on essential, but non-drastic process innovations allows us to present the results in a clean and systematic way. We want to stress that our findings are not exclusive to the case of an essential innovation (see also Section E in the appendix). Also, in our eyes this focus is not necessarily restrictive. For instance, in the context of the example of quality monitoring technologies, those controls are often a legal obligation. Thus, without access to such a technology firms are not able to operate in the market as their costs of guaranteeing a certain quality standard are too high.

The SPNE of the fixed-fee licensing game is obtained by solving the corresponding licensing game via backward induction (see Section B.1 in the appendix). Some of its relevant characteristics are given in Lemma 3.

Lemma 3 *For an essential, but non-drastic innovation, the pure strategy SPNE of the fixed-fee licensing game has the following features:*

¹²This result is due to the absence of capacity constraints. For $Q \neq X$ and $r^m = r^n$ licensing revenues are higher for the industry with the larger output (absent any other effects).

- i. The innovation is licensed to l^s players with l^s such that $\pi_i^{s,u}(l^s) = 0$.
- ii. The licensing fee amounts to $f^s(l^s) = \pi_i^{s,l}(l^s) - \pi_i^{s,u}(l^s - 1)$.
- iii. The licensing revenues of the patentee are $\pi^{P,f,s} = f^s(l^s)l^s$.

The optimal fixed-fee licensing policy

In the case of vertically related markets it is a priori unclear which sector provides the patentee with higher fixed-fee licensing revenues. The more efficient sector? The ex-ante more competitive industry? Or the industry with the ex-ante higher market revenues?¹³ In a standard licensing framework, Kamien and Tauman (1986) show that the patentee's fixed-fee licensing profits are increasing in the number of players on a market. This result continues to apply in the present scenario. However, it does not necessarily imply that it is the industry with the ex-ante higher degree of competition that yields higher licensing revenues. This is due to the fact that in vertically related markets a higher degree of competition in one sector has a positive spillover effect on the other sector (the two markets are connected via the input price). Also, in contrast to a per-unit royalty, the relationship between the relative efficiency of a sector and the ranking of the two markets in terms of licensing revenues is a priori undetermined for a fixed-fee policy. Clearly, the patentee's licensing income within a given sector is increasing in the sector's efficiency. However, as we will demonstrate below, under a fixed-fee policy the patentee does not necessarily license to the more efficient industry.

We begin this section by introducing the main result of our paper, namely that licensing to a less efficient industry sector may be the optimal strategy of an outside patentee in terms of licensing income. In this context, we first derive conditions on the relative efficiency parameters of both sectors such that licensing to a less efficient market is optimal. In a next step we then characterise those intervals by comparing the fixed-fee as well as the number of licensing contracts across sectors.

First, a technology transfer may be licensing revenue maximising even though the target industry is equipped with less developed capacities to utilise the technology in question (given the efficiency deficit is not too large). In order for it to be optimal to license to a less efficient downstream industry, this industry has to be less competitive than the upstream market prior to the transfer of the technology. In contrast, a less efficient, yet licensed upstream industry may either be more or less competitive in the pre-licensing equilibrium.

Proposition 4 *For a fixed-fee contract, the patentee may maximise licensing revenues by licensing to the less efficient sector. This result applies given the efficiency deficit is not too large. Further, for a less efficient upstream (downstream) market, the upstream market has to be sufficiently concentrated (competitive) relative to the downstream industry. Hence, for $M \leq M_1(N)$ and $\theta_n \in [\theta_m, \delta_{fe}^\pi \theta_m]$, $\pi^{P,f,m} \geq \pi^{P,f,n}$, whereas for $M > M_1(N)$ and $\theta_n \in [\delta_{fe}^\pi \theta_m, \theta_m]$, $\pi^{P,f,n} > \pi^{P,f,m}$.*

¹³In a vertical Cournot oligopoly those thresholds do not coincide. An equal degree of competition implies $M = N$, whereas equal pre-licensing market revenues necessitate $M = \sqrt{N(N+1)}$.

Proof From the outcome of the fixed-fee licensing game it directly follows that $\pi^{P,f,m} \geq \pi^{P,f,n}$ reduces to $\theta_n \leq \delta_{fee}^\pi \theta_m$ (see (11) and (16)). Consequently, $\pi^{P,f,m} \geq \pi^{P,f,n}$ and $\theta_m \leq \theta_n$ hold for any $\theta_n \in [\theta_m, \delta_{fee}^\pi \theta_m]$. Such an interval exists if and only if $\delta_{fee}^\pi \geq 1$. It can be shown that $\delta_{fee}^\pi \geq 1$ for any $M \leq M_1(N)$. Quantitative expressions and numerical examples of the relevant thresholds are provided in Section B.2 of the appendix. \square

Depending on the size of the upstream relative to the downstream market there exist thus two intervals for θ_n in which licensing to the less efficient industry provides the patentee with a higher licensing income. Notice, that the relevant threshold on the size of the upstream industry, $M_1(N)$, is strictly larger than N .

It is intuitive that in order for it to be optimal to license to a less efficient industry, this industry must dispose over other advantages which balance its efficiency deficit in terms of the patentee's licensing income. In the given context, a key factor is the different reaction of both sectors to the transfer of the technology. As such, licensing to the downstream industry entails a demand independent upward shift in the intermediary input price that confers an advantage to the upstream industry in terms of licensing revenues. To illustrate this result further, assume for a moment that both industry sectors are equivalent in terms of their pre-licensing market revenues ($M \leq M_1(N)$ holds) and their efficiency, i.e. $\theta_m = \theta_n$. Then, a transfer of the technology to the downstream industry entails an upward shift in the intermediary input price, whereas such an effect is absent for an upstream technology transfer. This upward shift lowers downstream market and licensing revenues and by this renders licensing to the upstream market relatively more attractive. Consequently, for equivalent pre-licensing market revenues and equivalent efficiency parameters across sectors, a revenue maximising patentee optimally transfer an innovation to the upstream industry. In fact, $\forall M < M_1(N)$ and $\theta_m = \theta_n$, $\pi^{P,f,m} > \pi^{P,f,n}$. Therefore, θ_m may drop below θ_n just to the point where $\pi^{P,f,m} = \pi^{P,f,n}$, i.e. to $\theta_n = \delta_{fee}^\pi \theta_m$.

Assume again that $\theta_m = \theta_n$ and $M \leq M_1(N)$. Then, as the pre-licensing number of upstream firms increases, both, the actual upstream licensing income as well as the potential downstream licensing income increase (the sectors are connected via the input price). It can be shown that the patentee's downstream licensing revenues increase at a faster rate in M than the upstream licensing revenues, i.e. $\frac{\partial \pi^{P,f,n}}{\partial M} \geq \frac{\partial \pi^{P,f,m}}{\partial M}$, for any relevant θ_n as specified in Proposition 4. Then, from $M_1(N)$ onward, licensing to the downstream industry becomes optimal. The reason is that for any such M the upstream market is sufficiently competitive (and the decrease in the input price due to the higher degree of upstream competition by this sufficiently large) as to counteract the upward shift in the input price induced by the downstream technology transfer.

In the remainder of this section we characterise the optimal licensing contracts by means of a cross-sector comparison. Meaning, we analyse how the actual licensing contract that is offered to the less efficient industry compares to the one that would be offered to the other, more efficient, market.

Proposition 5 *Assume that licensing to a less efficient upstream industry maximises licensing revenues. Then, the optimal licensing contract specifies ($l^m \geq l^n$, $f^m < f^n$). Meaning, the*

number of actual licensees in the less efficient industry is weakly larger than the hypothetical number of licensees that would result if the more efficient sector had been licensed. Similarly, the actual fixed-fee in the less efficient industry is strictly below the hypothetical fixed-fee for the more efficient market. Assume that licensing to a less efficient downstream industry maximises licensing revenues. Then, the optimal downstream contract sets $(l^m \geq l^n, f^m < f^n)$, $(l^m < l^n, f^m < f^n)$ or $(l^m < l^n, f^m \geq f^n)$.

Proof Before starting with the analysis, we need to derive some preliminary results. From Lemma 3 it follows that $l^m \geq l^n$ for $\theta_n \geq \delta_{fee}^l \theta_m$ and $f^m \geq f^n$ for $\theta_n \leq \delta_{fee}^f \theta_m$. Here, $\forall M, N$ $\delta_{fee}^l < 1$ and $\delta_{fee}^f < 1$. We can then rank the three thresholds $(\delta_{fee}^\pi, \delta_{fee}^f, \delta_{fee}^l)$ and show that $\delta_{fee}^\pi \geq \delta_{fee}^f \geq \delta_{fee}^l \forall M \leq M_2(N)$. Notice that $N < M_1(N) < M_2(N) \forall N$. Quantitative expressions of the relevant thresholds are provided in Section B.2 of the appendix.

Let us start with the upstream industry. $\pi^{P,f,m} \geq \pi^{P,f,n}$ requires $\theta_n \leq \delta^\pi \theta_m$. For there to exist an interval for θ_n in which $\theta_n \in [\theta_m, \delta^\pi \theta_m]$, $\delta^\pi \geq 1$ and therefore $M \leq M_1(N)$ has to hold. Due to the fact that $M_1(N) < M_2(N)$ it follows that $\delta^\pi \geq \delta^f \geq \delta^l$ for any $M \leq M_1(N)$. Taking into account that $\delta^f < 1 \forall M, N$, $l^m \geq l^n$ and $f^m < f^n$ follow from $\theta_n \geq \theta_m$.

For the downstream market, $\pi^{P,f,n} \geq \pi^{P,f,m}$ requires $\theta_n \geq \delta^\pi \theta_m$. For there to exist an interval for θ_n in which $\theta_n \in [\delta_{fee}^\pi \theta_m, \theta_m)$, $\delta_{fee}^\pi < 1$ and therefore $M > M_1(N)$ is required. First, assume $M \in (M_1(N), M_2(N)]$. Then, $\delta_{fee}^\pi \geq \delta_{fee}^f \geq \delta_{fee}^l$ and $\theta_n \geq \delta_{fee}^\pi \theta_m$ implies $l^m \geq l^n$ and $f^m < f^n$. Next, assume $M > M_2(N)$ so that $\delta_{fee}^l > \delta_{fee}^f > \delta_{fee}^\pi$. Then, either $\theta_n \in [\delta_{fee}^l \theta_m, \theta_m)$ and $l^m \geq l^n$ and $f^m < f^n$; or $\theta_n \in [\delta_{fee}^f \theta_m, \delta_{fee}^l \theta_m)$ and $l^m < l^n$ and $f^m < f^n$; or $\theta_n \in [\delta_{fee}^\pi \theta_m, \delta_{fee}^f \theta_m)$ and $l^m < l^n$ and $f^m \geq f^n$. \square

Proposition 5 illustrates that it is important to differentiate between upstream and downstream technology transfer. As such, we reach different conclusions with respect to the design of the optimal fixed-fee licensing contract, depending on whether the innovation is transferred to a less efficient upstream or downstream market. Whereas licensing to a less efficient upstream market implies a larger number of licensees and a smaller fixed-fee than if the innovation had been transferred to the more efficient downstream market, this issue is less clear cut for a less efficient, but licensed downstream industry. In the following we provide some intuition for our results.

As it has been argued previously, the upward shift in the intermediary input price that follows a downstream technology transfer is particularly detrimental to the group of unlicensed firms. Those firms not only face a lower final good price, but equally a higher input price, while at the same time lacking the direct benefit from the innovation in form of the cost reduction. As a result, the number of firms accepting the proposed licensing contract is lower on the downstream market as compared to the upstream industry and $l^m \geq l^n$.¹⁴ In the same vein, the presence of an upward shift in the input price implies a fiercer competition for licensing contracts in the downstream sector. As a corollary, the downstream fixed-fee strictly exceeds the upstream

¹⁴ As it is shown in the appendix, for an essential innovation l^s is such that $\pi_i^{s,u}(l^s) = 0$. Then $\pi_i^{m,u} \geq \pi_i^{n,u}$ and $\frac{\partial \pi_i^{s,u}}{\partial l^s} < 0$ imply $l^m \geq l^n$. Meaning a lower number of licensees is required downstream in order to ensure zero market revenues of any unlicensed firm when l^s players are licensed.

fixed-fee and $f^n > f^m$. Those results apply for a sufficiently small efficiency deficit, i.e. for $\theta_n \in [\theta_m, \delta_{fee}^\pi \theta_m]$ or $\theta_n \in [\delta_{fee}^\pi \theta_m, \theta_m]$.

In the following, let us assume that $\theta_n \in [\delta_{fee}^\pi \theta_m, \theta_m]$ and $M > M_1(N)$ such that the patentee licenses to a less efficient downstream industry. Then, as θ_n decreases (keeping θ_m fixed) the number of downstream licensees increases, whereas the associated fixed-fee decreases. Those dynamics can be rationalised by the observation that the market profit of a licensee ($\pi_i^{s,l}(l^s)$) as well as his outside option ($\pi_i^{s,u}(l^s - 1)$) decrease as θ_s falls. However, the market profit decreases at a faster rate such that a lower level of the sector specific efficiency parameter implies a smaller fixed-fee. All in all, the fixed-fee is less responsive to changes in θ_s than the number of licensed players. That is why the optimal licensing contract evolves from $(l^m \geq l^n, f^m < f^n)$ over $(l^m < l^n, f^m < f^n)$ to $(l^n < l^m, f^m \geq f^n)$ as θ_n decreases.

2.3 The Optimal Licensing Contract

So far we contrasted the upstream and the downstream sector in terms of their licensing contracts and the patentee's licensing income for a given licensing policy. In this section we instead focus on comparing a fixed-fee and a per-unit royalty contract for a specific industry.

It is a well-known, yet much debated fact that in a standard licensing game a fixed-fee contract dominates a per-unit royalty policy in terms of the patentee's licensing revenues (Kamien and Tauman (1986)). The result stands somewhat at odds to the empirical evidence, which supports a widespread use of royalty based schemes for the traditional technology transfer (Bousquet et al. (1998), Macho-Stadler et al. (1996), Rostoker (1983-1984)). This contradiction between theoretical prediction and empirical evidence strongly contributed towards a newly awakened interest in the study of licensing contracts. In the course of this development it has been shown that the superiority of a fixed-fee over a per-unit royalty in terms of licensing revenues depends crucially on a number of assumptions, among other, Cournot competition (Filippini (2005), Kamien and Tauman (1986)), homogeneous products (Erkal (2005), Wang (2002)), as well as the treatment of the number of licensees as a continuous variable (Sen (2005), Sen and Tauman (2012)).

In the following we show that even in a framework in which firms produce homogeneous products, compete *à la Cournot* and in which the number of licensees is treated as a continuous variable, a per-unit royalty contract may yield larger licensing revenues than a fixed-fee policy. In this context our analysis not only provides a reason for the empirically observed popularity of royalty licensing contracts, but also emphasises that it is crucial to distinguish between upstream and downstream technology transfer. Subsequently we take the analysis of the optimal licensing contract one step further and compare the patentee's licensing income across sectors while assuming that the optimal sector specific licensing policies are offered.

A simple comparison of the patentee's licensing revenues under a fixed-fee (Lemma 3) and a per-unit royalty contract (Lemma 2) for the upstream and the downstream industry shows that it is indeed crucial to distinguish between upstream and downstream technology transfer in terms of the optimal licensing policy.

Proposition 6 *For the upstream market a per-unit royalty contract is never optimal in terms of licensing revenues, i.e., $\pi^{P,f,m} > \pi^{P,r,m} \forall M, \forall N$. In contrast, for the downstream sector, a per-unit royalty contract may be optimal for sufficiently concentrated upstream and downstream markets, i.e., $\pi^{P,r,n} \geq \pi^{P,f,n} \forall M \leq M_3(N), \forall N < 2.078$ with $M_3(N) < M_1(N)$ (at $N = 2.078$ $\pi^{P,r,n} \geq \pi^{P,f,n}$ for $M = 1$ ¹⁵).*

What is the intuition behind this result? Opposed to a fixed-fee, a per-unit royalty is a variable part in the optimisation problem of a licensee. For any strictly positive royalty rate this leads to a relative downward distortion of the optimal production levels. In fact, at the licensing equilibrium the patentee equates the royalty rate to the sector specific cost reduction. As a result, firms are indifferent between accepting and rejecting the proposed licensing contract and continue producing as they did prior to the transfer of the technology. Clearly, under a fixed-fee policy this effect is absent. It follows that an outside patentee generally maximises licensing profits by offering a fixed-fee, instead of a per-unit royalty contract.

When licensing to the downstream industry a new effect is present. The technology transfer to the downstream market triggers a demand independent upward shift in the intermediary input price. This upward shift implies lower downstream market and licensing revenues than in its absence. Notice now that by offering a royalty licensing contract the patentee is able to mitigate the importance of this effect. At the optimum, the royalty rate is set equal to the sector specific cost reduction which in turn results in a zero upward shift.¹⁶ Thus, regarding the downstream industry, the positive effects of a royalty contract in terms of a zero upward shift in the intermediary input price may outweigh its negative consequences in form of a relative downward distortion of the optimal production levels. However, to do so, the upward shift needs to be sufficiently important.¹⁷

Nevertheless, for the upstream industry, our previously derived result still applies (qualitatively). As such, it can be shown that also under downstream per-unit royalty contracts, fixed-fee licensing to a less efficient upstream market may be licensing revenue maximising. In contrast, a technology transfer to a less efficient downstream sector is never optimal.

Proposition 7 *Assume that the patentee optimally offers a per-unit royalty contract to the downstream industry. Then, a technology transfer to a less efficient downstream sector is never optimal in terms of licensing income. In contrast, fixed-fee licensing to a less efficient upstream industry may maximise licensing revenues, i.e., $\pi^{P,f,m} \geq \pi^{P,r,n}$ if and only if $\theta_n \leq \delta_{royalty}^\pi \theta_m$ with $\delta_{royalty}^\pi = \frac{M+2}{M+1}$ and $\delta_{royalty}^\pi \in (1, \delta_{fee}^\pi)$.*

With the results of Proposition 4, 6 and 7 in mind we can now define the patentee's optimal licensing strategy.

¹⁵In Proposition 6, $M_3(N) = \frac{2+N^2-N+(N+1)\sqrt{N^2+4}}{2N^2}$.

¹⁶This can be directly seen from (3); $w(Q) = a - (\frac{N+1}{N})Q - c^n + \frac{l^n}{N}(\theta_n - r^n)$.

¹⁷Under Proposition 6, $\frac{l^n}{N}$ is bounded from below by 0.4812. At the same time the countervailing force in form of a competitive upstream sector is restricted. This can also be seen from the integer solutions to Proposition 6. The latter are $N = 1, M = 1, 2, 3$ and $N = 2, M = 1$. Chang et al. (2013) restrict their attention to $N = 2, M = 1$.

Proposition 8 *Depending on the market structure and the efficiency parameters of the upstream and the downstream market the patentee's optimal one-sector licensing strategy may feature downstream per-unit royalty contracts (provided the upstream and the downstream market are sufficiently concentrated) or fixed-fee licensing to either industry sector. For a fixed-fee policy, one may observe licensing to a less efficient upstream or downstream industry (for a respectively sufficiently concentrated or competitive upstream market). More precisely,*

- i. For $M \leq M_3(N)$, $N < 2.078$ and
 - $\theta_n \leq \delta_{royalty}^\pi \theta_m$: fixed-fee upstream.
 - $\theta_n > \delta_{royalty}^\pi \theta_m$: per-unit royalty downstream.
- ii. For $M \leq M_3(N)$, $N \geq 2.078$ or $M \in (M_3(N), M_1(N)]$, $N < / \geq 2.078$ and
 - $\theta_n \leq \delta_{fee}^\pi \theta_m$: fixed-fee upstream.
 - $\theta_n > \delta_{fee}^\pi \theta_m$: fixed-fee downstream.
- iii. For $M > M_1(N)$, $N < / \geq 2.078$ and
 - $\theta_n < \delta_{fee}^\pi \theta_m$: fixed-fee upstream.
 - $\theta_n \geq \delta_{fee}^\pi \theta_m$: fixed-fee downstream.

Figure 1 provides a rough overview of the different cases featured in Proposition 8. The asterisk indicates that the licensed industry may be the less efficient one.

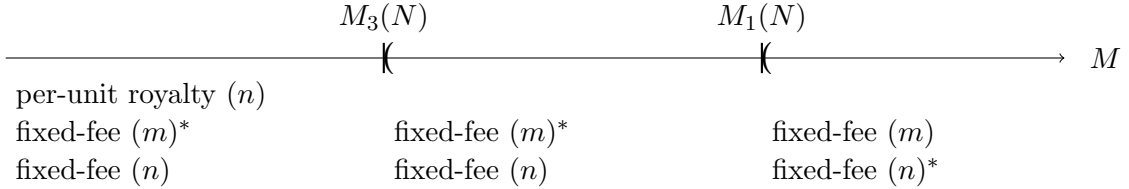


Figure 1: Proposition 8 - The optimal licensing contract

To illustrate Proposition 8, the next section studies three polar cases. As such we analyse the ranking of the different licensing policies (fixed-fee vs per-unit royalty) and industries (upstream vs downstream sector) in terms of the patentee's licensing income given an equal efficiency, an equal pre-licensing market size or equal pre-licensing market revenues across sectors.

2.4 Some Polar Cases

Equal efficiency

Take the case of an independent research laboratory which offers solutions for quality control in the food industry. In this example, the upstream sector may for instance correspond to the agricultural industry, while the downstream sector refers to the market of final good producers, i.e. the food processing industry. The downstream firms source their inputs, e.g. dairy products,

from the upstream industry and then processes them to produce the final product. Intuitively, both industry sectors may be interested in obtaining access to quality control solutions offered by the research institute and it is assumed that both markets are characterised by equivalent capacities to utilise such a technology. Here, the technological basis of the quality control solutions may be the same for both sectors, however, final adjustments of the product are most likely necessary in order to address the specific industry needs. The agricultural sector may predominantly be interested in tests to detect traces of hormones or antibiotics, while the food processing industry may primarily necessitate solutions to control the vitamin content or to detect allergens in the final product. Assuming that the only feasible option is a one-sector technology transfer, the research institute then faces the following questions. For which industry sector should the (final) technology be developed first? How should the optimal contract be structured? Meaning, is it a fixed-fee or a per-unit royalty policy which maximises licensing revenues? Figure 2 illustrates the following discussion.

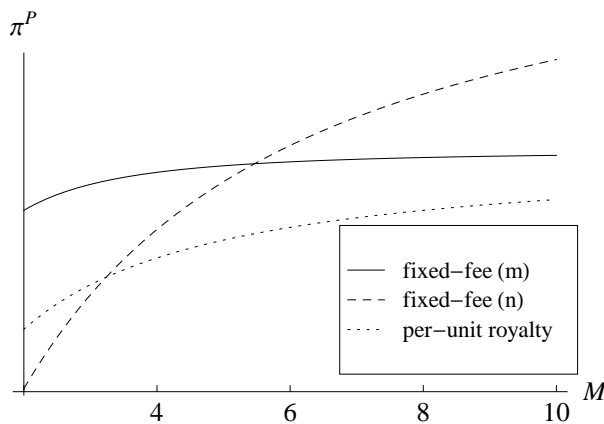


Figure 2: The optimal licensing contract (Graph for $A = 1, N = 1, \theta_m = \theta_n = \frac{2}{5}$)

First, for a per-unit royalty scheme the patentee is indifferent between licensing to the upstream and licensing to the downstream market. In our example the agricultural sector and the food processing industry are therefore equally likely to be offered the technology. Under a fixed-fee contract, the cross-sector comparison of licensing revenues is reduced to a comparison of market size, due to the assumption of equal efficiency parameters across sectors. For $M \leq M_1(N)$ the patentee prefers to license the upstream industry, while for $M > M_1(N)$ the downstream market yields higher licensing revenues. Due to the fact that $N < M_1(N)$, $M > M_1(N)$ implies that the less competitive sector provides the patentee with more pronounced incentives to innovate. In other words, whenever it is optimal to license to the downstream industry, this industry is less competitive than the upstream market prior to the transfer of the innovation. For $M \leq M_1(N)$ this issue is less clear cut. Either $M \leq N$ and a less competitive upstream sector yields a higher licensing income or $M \in (N, M_1(N)]$ and a more competitive upstream sector dominates in terms of licensing revenues. In our example this implies that the research institute should focus its strategy on the agricultural market provided this industry is sufficiently concentrated (notice that the agricultural market may either be more or less concentrated than the

food processing industry). The opposite result applies whenever the food processing industry is the relatively more concentrated market.

In terms of the optimal sector specific licensing policies, notice that, regardless of θ_s , per-unit royalty contracts may be optimal for the downstream industry, but never maximise upstream licensing revenues. Consequently, for sufficiently concentrated industries, per-unit royalty contracts may be observed in the food processing industry, whereas the agricultural market is all the time offered a fixed-fee contract. For an equal efficiency across sectors it then follows that for a downstream royalty contract and an upstream fixed-fee contract the research institute in our example focuses on providing quality control solutions for the agricultural market.

Equal pre-licensing industry size or equal pre-licensing market revenues

Before starting with the analysis note that by the nature of a vertical Cournot oligopoly $\pi_i^m > \pi_i^n$ for $M = N$. Meaning for an equal market size, the upstream pre-licensing market revenues exceed those on the downstream market. Hence, in order to ensure that the upstream industry is not given any advantage in terms of its pre-licensing profitability, this industry is required to be more competitive than the downstream market. More specifically, $M \geq \sqrt{N(N+1)}$ is required for $\pi_i^m \leq \pi_i^n$ to hold.

Assume that in the given framework the process innovation is employed to model an electronic B2B marketplace which connects suppliers and buyers in a given industry. In this context, the innovator may be seen as the manager responsible for the platform's launch or its further development and the associated pricing strategy. We focus on the latter, that is on the development of an existing trading platform via the introduction of value-added services or premium membership options.¹⁸ In this framework $|S|$ then refers to the number of firms that are active on the platform. It is assumed that prior to the introduction of premium features, both sides are comparable regarding their industry structure. Meaning, the demand and the supply side either feature an equal number of firms or are equivalent in terms of their pre-licensing market revenues. In the final stages of the marketplace's development the decision has to be taken whether the marketplace's premium packages should primarily be targeted at the supply or the demand side. Should the design of the marketplace's premium packages yield predominantly benefits to buyers by featuring demand side related management tools (automated procurement system, storage of procurement related information) or should it target the suppliers by incorporating supply side related management tools (demand forecasting, supply management tools)? Also what is the optimal pricing policy (membership vs transaction fee)?

Under a transaction fee, value-added services are offered to the market with the more developed capacities to use the latter. Although this result might appear straight-forward, it is

¹⁸In our paper we focus on the licensing of essential innovations. In the context of the example of an electronic B2B marketplace or procurement software this implies that the unlicensed firms may be seen as no longer important in terms of competition. For instance, according to the statement of the website operators, the majority of trade on an electronic marketplace offering premium membership options is realised with premium buyers or suppliers. Those premium features range from a preferred treatment regarding new buy or supply leads over the provision of a higher visibility on the website to the provision of identity verification services. Also the manual sourcing of c-items or other (indirect) input goods appears to us as a time-consuming and costly task which may only be possible for small firms.

by now known that it does not necessarily apply under a flat membership fee. Notice that for equivalent market size or revenues $M < M_1(N)$ is all the time satisfied. Coming back to our example, this brings with it that for structurally similar buyer and supplier industries a marketplace's value-added services may target the supply side even if the latter is relatively less efficient. This result is in line with the findings of our analysis of existing B2B marketplaces. Especially vertical B2B e-marketplaces appear to predominantly ask the supply side a monthly or annual membership fee, while the demand side may join the marketplace for free (similarly, premium membership options are almost exclusively offered to the supply side).¹⁹

Let us now focus on the platform provider's choice between transaction and flat membership fees. Applying the results of our previous analysis we find that the platform provider optimally asks a flat membership fee from the supply side, while regarding the demand side a transaction fee may present the optimal payment scheme. Nevertheless, unless there is a single buyer prior to the introduction of value-added services, such transaction based payment schemes never maximise platform revenues. This result confirms our observation that royalty contracts are rarely employed on B2B marketplaces (see also Stockdale and Standing (2002)). What is more, similar to the previous example it can be observed that whenever a transaction fee is optimally employed for the buyer segment, focusing one's strategy on this market side is never optimal.

2.5 Welfare Analysis

The previous sections showed that under a fixed-fee policy an outside patentee may maximise licensing revenues via a technology transfer to the less efficient industry sector. It is clear that the presence of private incentives for such a technology transfer does not necessarily imply its social desirability. To analyse this issue, we contrast the private incentives of the patentee, as reflected by the licensing contract chosen, with the optimal contract in terms of consumer and aggregate economic surplus. Quantitative expressions of the relevant economic quantities (consumer surplus, producer surplus, aggregate welfare) can be found in Section C of the appendix.

Proposition 9 *The consumer surplus under licensing to the upstream sector ($CS_{fee,m}$) exceeds the consumer surplus obtainable under licensing to the downstream sector ($CS_{fee,n}$), provided the upstream industry is sufficiently concentrated. That is, $CS_{fee,m} \geq CS_{fee,n} \forall M \leq N+1, \forall N$.*

The positive effects of the innovation for the consumers in terms of a lower final good price (or higher aggregate industry output) depend on the average efficiency of the licensed industry sector as measured by $\frac{l^s}{|S|}\theta_s$. When licensing to the downstream industry, the RRC partially offsets the benefits to the consumers and it follows that $\frac{l^m}{M}\theta_m \geq \frac{l^n}{N}\theta_n$ for any $M \leq N+1$.

Hence, for any $M \in (N+1, M_1(N)]$ an inefficiency may arise. Within this interval, licensing to a less or equally efficient upstream industry may be optimal in terms of licensing revenues, whereas consumer surplus is maximised for a technology transfer to the downstream industry. A similar result applies in terms of aggregate economic welfare.

¹⁹See e.g. www.paperindex.com or www.solarexchange.com.

Proposition 10 Define aggregate welfare under fixed-fee licensing to sector s ($W_{fee,s}$) as the sum of consumer surplus, producer surplus and licensing revenues. Then, licensing to a less efficient downstream market is all the time optimal in terms of aggregate welfare. In contrast, when licensing to a less efficient upstream market an inefficiency may arise. This inefficiency increases (decreases) as θ_m (θ_n) increases. More precisely,

- i. $\forall M > M_1(N), \forall N$ and $\theta_n \in [\delta_{fee}^\pi \theta_m, \theta_m)$ (licensing to a less efficient downstream market is optimal) $W_{fee,n} > W_{fee,m}$.
- ii. $\forall M \leq M_1(N), \forall N > 1.618$ and $\theta_n \in [\theta_m, \delta_{fee}^\pi \theta_m]$ (licensing to a less efficient upstream market is optimal) either of the following three subcases applies: I) $\forall M \leq N - \frac{1}{N}$, $W_{fee,m} > W_{fee,n}$. II) $\forall M \in (N - \frac{1}{N}, N + 1]$, $W_{fee,m} \geq / < W_{fee,n}$. Hereby, $W_{fee,m} > W_{fee,n}$ if $PS_{fee,m}^m \geq PS_{fee,n}^n$. III) $\forall M \in (N + 1, M_1(N))$, $W_{fee,n} > W_{fee,m}$ (at $M = M_1(N)$, $W_{fee,n} > W_{fee,m}$ for $\theta_n = \theta_m$).

Proposition 10 implies that also in terms of aggregate economic surplus it is crucial to make a distinction between upstream and downstream technology transfer. While a technology transfer to a less efficient downstream market is all the time optimal in terms of aggregate welfare, a similar result does not necessarily apply regarding a less efficient upstream industry. In particular, for $M \in (N + 1, M_1(N))$ and $\theta_n \in [\theta_m, \delta_{fee}^\pi \theta_m]$ the private incentives of the patentee collide with those of the aggregate economy. For any such M the patentee licenses to the less efficient upstream market (Proposition 4), whereas the economy as a whole is better off under a technology transfer to the more efficient downstream industry. The main driver of this result is the group of consumers. According to Proposition 9, consumer surplus is larger under licensing to the downstream market whenever $M > N + 1$.

With view on the definition of a welfare maximising innovation policy the results of this section suggest that the attention of public decision makers should be primarily focused on supporting the development of downstream technology. A technology transfer to less efficient downstream markets is all the time optimal in terms of consumer surplus and aggregate economic welfare. This conclusion is particularly important keeping in mind that an outside innovator is less likely to target those industries. As it was shown previously, a technology transfer to a less efficient downstream industry never maximises licensing revenues when this industry is structurally comparable to its upstream sector (e.g. for $M = N$ or $M = \sqrt{N(N + 1)}$).

3 Two-sector Licensing

Up to now we studied one-sector licensing. By this we implicitly imposed that transferring the innovation to both markets at the same time is either not a feasible or not a profitable licensing strategy (see Section 2 for a detailed discussion). What if we relax the feasibility assumption and let the patentee choose between transferring the innovation to a single or to both sectors of the vertical structure? In particular, under what conditions is a two-sector transfer to the upstream and the downstream market more profitable than a one-sector transfer

to either industry? Moreover, measuring incentives to innovate by licensing revenues, which industry sector provides the patentee with larger incentives to innovate, for instance, regarding the development of second generation test devices, technology upgrades or other subsequent innovation efforts?

In the context of one-sector licensing we showed that the upstream and the downstream market react differently to the transfer of a cost-reducing technology to their market. A such, downstream technology transfer features a demand independent upward shift in the intermediary input price, whereas such an effect is absent for upstream technology transfer. Everything else being equal, this renders the development of upstream technology more attractive. As a consequence, licensing to the upstream market may be the patentee's revenue maximising strategy, even when this market is characterised by a lower efficiency. In this context it was then discussed what conditions on the market structure of the two sectors balance the upward shift and render licensing to a less efficient downstream market profitable.

Under two-sector licensing the patentee clearly cannot avoid the upward shift in the input price. This confers an additional advantage to the upstream market in terms of licensing revenues. As it is argued in the previous paragraph, upstream technology transfer does not feature any adverse effects which reduce this sector's profitability from the perspective of the patentee. At the same time, the presence of an upward shift in the input price creates a spillover on the upstream market which increases upstream licensing revenues. Taken together this renders upstream technology transfer even more profitable than in the case of a one-sector licensing. Nevertheless, as in the case of one-sector licensing, we are able to identify conditions under which a less efficient downstream market provides the patentee with larger incentives to innovate.

3.1 Royalty Licensing Policy

In analogy to our analysis of the patentee's optimal one-sector technology transfer strategy, we begin the study of two-sector licensing with a per-unit royalty policy. In the following the subscript b refers to the quantities of the two-sector licensing game.

Lemma 11 *For a non-drastic innovation, the pure strategy SPNE of the two-sector royalty licensing game has the following features:*

- i. The innovation is licensed to $l_b^m = M$ and $l_b^n = N$ players.*
- ii. The royalty rates amount to $r_b^m = \theta_{m,b}$ and $r_b^n = \theta_{n,b}$.*
- iii. The patentee's aggregate licensing revenues are $\pi_b^{P,r} = (r_b^m + r_b^n)X$.*

From Lemma 11 it is apparent that as in the case of one-sector licensing, the more efficient sector provides the patentee with larger incentives to innovate. Under a per-unit royalty policy, we have therefore a clear ranking of the upstream and the downstream sector in terms of licensing revenues which does not depend on whether we are in the case of one-sector or two-sector technology transfer.

To clarify, $\theta_{s,b}$ does not need to coincide with its counterpart in the one-sector licensing game. In particular, as we will argue below, $\theta_{s,b}$ does not need to be constant, but may, for instance, be interpreted as a decreasing function of l_b^{-s} (*subadditivity* of the innovation). To illustrate this point, let us come back to the example of quality control technology for the food industry. To recall, we referred to the upstream sector as the agricultural industry and to the downstream sector as the food processing industry. Downstream firms source their input requirements, e.g. dairy products, from the upstream market. Here we alter the example slightly and assume that the non-producing research institute provides a quality control technology which enables upstream and downstream firms to test the quality of the input good. It is in our eyes intuitive that in this case a licensee's potential benefit from such a technology is decreasing in the number of firms on the other market conducting quality controls. For example, the more upstream firms are testing the quality of the input good, the lower the direct benefit the technology yields to a downstream firm.

By comparing $\pi^{P,r}$ and $\pi_b^{P,r}$ we can then derive conditions on $\theta_{s,b}$ under which the patentee prefers one-sector to two-sector licensing and vice versa. Note, however, that those conditions do not take into account the costs associated with the technology transfer. As we argue in Section 2, it may very well be the case that the patentee faces high transaction costs, which more than outweigh the additional benefit of licensing to both industries.

Proposition 12 *A necessary condition for one-sector licensing to yield higher licensing revenues than a two-sector transfer is $\theta_{s,b} < \theta_s$ (abstracting from other conditions). One possibility for the latter to hold is that the innovation is characterised by subadditivity. Meaning the perceived benefit from the innovation is decreasing in the number of licensees on the other sector. To be more precise, one-sector licensing is the patentee's revenue maximising strategy if and only if $\max\{\theta_m, \theta_n\} \geq \theta_{m,b} + \theta_{n,b}$. Conversely, for $\max\{\theta_m, \theta_n\} < \theta_{m,b} + \theta_{n,b}$ the patentee strictly prefers to offer the innovation to both industry sectors at the same time (if possible).*

The following example illustrates Proposition 12. Assume that firms of the agricultural industry have more developed capacities to incorporate the technology in their production processes ($\theta_m > \theta_n$ and $\theta_{m,b} > \theta_{n,b}$). Further, under a two-sector technology transfer, $\theta_{s,b} \leq \theta_s$. To make the example more concrete, let us take $\theta_m = 1$ and $\theta_n = \frac{1}{2}$. In the case of a one-sector technology transfer the patentee would consequently transfer the technology to the upstream market. Assume that under two-sector licensing $\theta_{n,b} = \frac{1}{4}$. Then, for any $\theta_{m,b} \leq \frac{3}{4}$ a one-sector transfer to the upstream market is more profitable than a simultaneous transfer to both sectors (and vice versa for $\theta_{m,b} \in (\frac{3}{4}, 1]$).

3.2 Fixed-Fee Licensing Policy

Basic results of the licensing game

As a final point we consider the two-sector fixed-fee licensing game. We derive its SPNE in section D.1 of the appendix, Lemma 13 summarises the result.

Lemma 13 *For an essential, but non-drastic innovation, the pure strategy SPNE of the two-sector fixed-fee licensing game has the following features:*

- i. *The innovation is licensed to l_b^m and l_b^n players with l_b^s such that $\pi_{i,b}^{s,u}(l_b^m, l_b^n) = 0$.*
- ii. *The licensing fee amounts to $f_b^s(l_b^m, l_b^n) = \pi_{i,b}^{s,l}(l_b^m, l_b^n) - \pi_{i,b}^{s,u}(l_b^s - 1, l_b^{-s})$.*
- iii. *The licensing revenues of the patentee are $\pi_b^{P,s} = f_b^m(l_b^m, l_b^n)l_b^m + f_b^n(l_b^m, l_b^n)l_b^n$.*

From Lemma 13 it is apparent that all relevant quantities of the licensing game not only depend on l^s but in addition on l^{-s} . More precisely, under two-sector licensing the number of licensees in a given industry sector not only exerts a negative within-sector externality, but also a positive cross-sector externality on the firms on the other, vertically related industry.²⁰ Meaning, the fixed-fee(s), the number of licensees and by this also the patentee's licensing income for a given industry sector increase in the number of licensees of the other industry sector.

Proposition 14 *The sector specific fixed-fee is decreasing in the number of licensees of a sector. Further, under two-sector licensing, the sector specific fixed-fee and the sector specific number of licensing contracts are increasing in the number of licensees of the other sector. Consequently, under two-sector licensing, $\frac{\partial f_b^s(l_b^m, l_b^n)}{\partial l_b^{-s}} > 0$, $\frac{\partial l_b^s(l_b^{-s})}{\partial l_b^{-s}} > 0$, while $\frac{\partial f_b^s(l_b^m, l_b^n)}{\partial l_b^s} < 0$ for $l_b^s, l_b^{-s} > 0 \forall s \in \{m, n\}$.*

This point deserves further attention. First, as the sector specific fixed-fee is decreasing in the number of licensees of a given sector, but increasing in the number of firms on the other market, determining the optimal licensing contract is not a trivial problem. Notice that increasing the licensing income for a given sector by increasing the number of licensees on the other market implies at the same time a lower fixed-fee for this other market (and vice versa). As the indirect cross-sector effect of l_b^{-s} is less strong than any direct within-sector effect, it is not straightforward that licensing to both markets is optimal. Second, it is easily seen that by the nature of a per-unit royalty policy, those spillovers are absent for the latter. It follows that the patentee has no incentive to offer a royalty licensing contract when both industries are licensed at the same time.

By comparing $\pi^{P,f,s}$ and $\pi_b^{P,f,s}$ we can again identify conditions on $\theta_{s,b}$ under which the patentee prefers one-sector to two-sector licensing and vice versa.

Proposition 15 *For one-sector licensing to be more profitable than a two-sector technology transfer, the innovation has to be characterised by a sufficient degree of subadditivity. As such, one-sector licensing to a single industry sector is the patentee's revenue maximising strategy given either*

- i. $\theta_m \geq (\frac{N+1}{N})(\theta_{m,b} + \lambda\theta_{n,b})$ (for $\pi^{P,f,m} \geq \pi^{P,f,n}$) or
- ii. $\theta_n \geq \frac{G}{M}(\theta_{n,b} + \frac{1}{\lambda}\theta_{m,b})$ (for $\pi^{P,f,n} > \pi^{P,f,m}$)

²⁰Similar dynamics, although in the context of two-sided markets are observed by Belleflamme and Toulemonde (2009) (negative intra-group and positive inter-group externalities).

with $\lambda = \frac{2H((M+1)N - \frac{1}{N})}{M(M+2)(N+1)^2}$ hold true (H is defined in Section B.2).

To illustrate Proposition 15, take the example of a vertical duopoly ($M = N = 2$). Assume that $\theta_m = 1$ and $\theta_n = \frac{11}{10}$ so that under one-sector licensing, licensing to the less efficient upstream market is optimal (Proposition 4). As in the previous example we impose a subadditivity of the innovation so that $\theta_{s,b} \leq \theta_s$. Let us assume that $\theta_{n,b} = \frac{11}{40}$ (perceived benefit of the innovation decreases by a factor of 4 under two-sector licensing). Then, for any $\theta_{m,b} \leq \frac{4}{10}$ (perceived benefit decreases by a factor of at least 2.5) one-sector licensing to the upstream market dominates two-sector licensing to the upstream and the downstream market (and vice versa for $\theta_{m,b} \in (\frac{4}{10}, 1]$).

The optimal fixed-fee licensing policy

The remainder of this section focuses on extending the previously derived results (one-sector licensing, Proposition 4) to the case of two-sector licensing. Assuming that it is optimal to license to both markets at the same time, we ask which industry provides the patentee with larger incentives to innovate and further how the latter is related to efficiency and market structure of the upstream and the downstream sector.

To begin with, it can be established that also under two-sector licensing the less efficient market may provide the patentee with larger incentives to innovate. Proposition 16 summarises those cases for the upstream and the downstream industry (see Section D.2 of the appendix for numerical illustrations).

Proposition 16 *For a fixed-fee contract, the less efficient sector may provide the patentee with larger incentives to innovate. This result applies given the efficiency deficit is not too large. Further, for a less efficient upstream (downstream) market, the upstream industry has to be sufficiently competitive (concentrated) relative to the downstream industry. Hence, for $M \geq N - \frac{1}{N}$ and $\theta_{n,b} \in [\theta_{m,b}, \delta_{fe,b}^{\pi} \theta_{m,b}]$, $\pi_b^{P,f,m} \geq \pi_b^{P,f,n}$ (and vice versa).²¹*

In terms of its qualitative results, Proposition 16 is equivalent to Proposition 4. That is to say, also under two-sector licensing the less efficient market may yield higher licensing revenues. In particular, notice that for $M = N$ or $M = \sqrt{N(N+1)}$ the implications of the model are equivalent regardless of whether the technology is licensed to a single or to both industry sectors. As such, for an equal pre-licensing industry size or market revenues a less efficient downstream market never provides higher incentives to innovate, whereas a less efficient upstream industry may do so.

Nevertheless, the general conditions for the result to hold differ depending on whether one or both sectors are licensed. While Proposition 4 places an upper bound on the size of the upstream market ($M \leq M_1(N)$), its pendant under two-sector licensing specifies a lower bound ($M \geq N - \frac{1}{N}$).

First, how can one rationalise the lower bound on M under two-sector licensing? In general, the patentee's fixed-fee licensing income for a given industry increases in market size. Without the presence of the RRC this would imply that the upstream market provides larger licensing

²¹Quantitative expressions of the relevant thresholds are given in section D.2 of the appendix.

revenues whenever $M \geq N$. Under two-sector licensing, however, the upward shift in the intermediary input price that follows downstream technology transfer cannot be avoided. As it was argued previously, this renders upstream technology transfer even more profitable than under one-sector licensing. As a corollary, the relevant threshold on M decreases proportionally to the RRC.²²

It remains to clarify why the conditions on the size of the upstream market differ depending on whether the technology is transferred to a single or to both industry sectors (upper vs lower bound on M)? Or, in other words, why does downstream technology transfer provide higher licensing revenues under two-sector licensing when the upstream market is sufficiently concentrated, while the opposite holds true under one-sector licensing. In general, a low degree of competition is disadvantageous in terms of licensing revenues. This follows directly from the fact that the patentee's licensing income for a given industry sector is increasing in market size. Hence, for an upstream industry that is sufficiently concentrated relative to the downstream market, the downstream market yields larger licensing revenues under two-sector licensing. Under one-sector licensing the patentee can avoid the demand independent upward shift in the intermediary input price by licensing to the upstream market. It follows that for the patentee to choose to license to the downstream market, a sufficiently large degree of upstream competition is necessary to balance the RRC in terms of licensing revenues.

4 Conclusion

The asymmetric market structure inherent to most knowledge-intensive industries exerts an important influence on the design of licensing agreements. To analyse this issue further we study the problem of an outside patentee who licenses an essential process innovation to a vertical Cournot oligopoly. The vertical oligopoly is composed of an upstream and a downstream sector, which may differ in terms of their capacity to utilise the innovation. It is assumed that the innovation may be transferred to either industry by means of a per-unit royalty or a fixed-fee policy. In this framework we explore the design of the optimal licensing contract in terms of the optimal licensing policy (per-unit royalty vs fixed-fee) and sector (upstream and/or downstream market).

First, under a per-unit royalty contract we derive a rather intuitive result. The more efficient industry yields strictly larger licensing revenues to a non-producing patentee. This result does not necessarily apply under a fixed-fee policy. Instead, transferring the innovation to the less efficient market may be the licensing income maximising strategy. Here, a technology transfer to a less efficient downstream market is all the time optimal in terms of consumers surplus and aggregate economic welfare. In contrast, licensing to a less or equally efficient upstream industry is potentially inefficient. Second, we show that the optimal licensing contract is sector specific under one-sector licensing to either industry sector. Regarding the upstream market the results of the traditional licensing literature apply and an essential innovation is optimally transferred via a fixed-fee as opposed to a per-unit royalty policy. In contrast, on the downstream market

²²From (3) it follows that the upward shift takes on the form of $\frac{l_n^n}{N}\theta_{n,b}$ ($w(Q) = a - (\frac{N+1}{N})Q - c^n + \frac{l_n^n}{N}\theta_n$).

an outside patentee may maximise licensing revenues via a per-unit royalty contract. As a third and final point we address the case of two-sector licensing. Here we show, among other things, that the patentee may prefer to transfer the technology to a single industry sector (as compared to both sectors) provided the innovation is characterised by a sufficient degree of *subadditivity*.

Our findings are not only relevant from a purely theoretical point of view. Instead, the results of this work may equally provide guidance in the context of managerial decision processes or innovation policy initiatives. Measuring the incentives to innovate of a third party by licensing revenues, we analyse whether it is an upstream or a downstream industry that provides a non-producing patentee such as an independent research laboratory or think tank with higher incentives to innovate. As a corollary, our work further clarifies whether it is the development of upstream or downstream technology that necessitates public sector support as well as how this question is related to market structure and efficiency of the industries concerned.

Throughout this paper we illustrate how our findings may support public innovation policy initiatives or strategic management decisions. Regarding the latter our proposed modelling framework not only has an interpretation concerning the classic technology transfer, but also concerning electronic B2B marketplaces or e-commerce software. The latter represent business sectors in which innovation not only requires a significant initial investment, but which are equally characterised by a high failure rate (Brunn et al. (2002), Ravichandran et al. (2007)). This makes the definition of a successful business strategy, particularly in the early stages, i.e., regarding the launch of an electronic marketplace or B2B software, crucial. In this context, our paper provides a first step to answering why different marketplaces may find it profitable to adopt different strategies. In particular, the results of this paper reveal whether a transaction or an access fee provides higher revenues for the platform provider and further on which side of the market (demand and/or supply side) such a platform should focus on in order to generate liquidity.

A Royalty Licensing Policy

Assume that there are $l^s \leq |S|$ licensees in sector s with $s \in \{m, n\}$. A licensee's payoff function is given by $\pi_i^{s,l}(c^s + r^s - \theta_s)$. It is easily seen that $\pi_i^{s,l}(c_i^{s,l}) \geq \pi_i^{s,u}(c_i^{s,u})$ for any $r^s \leq \theta_s$. This implies that every firm in the licensed sector will accept the proposed licensing contract for any $r^s \leq \theta_s$. Hence, $l^s = |S|$.

It follows that for a royalty licensing contract with $r^s \leq \theta_s$ the patentee's licensing income amounts to $\pi^{P,s} = r^s X(r^s)$. From the first order condition it is easily seen that $r^s \leq \theta_s$ implies that $\pi^{P,s}$ is strictly increasing in r^s for any non-drastic innovation. It is therefore optimal to set $r^s = \theta_s$.

Proof $\frac{\partial \pi^{P,s}}{\partial r^s} > 0$ if and only if $r^s < \frac{A+\theta_s}{2}$. Due to the fact that we impose $r^s \leq \theta_s$ it is sufficient to show that $\theta_s < \frac{A+\theta_s}{2}$. The latter condition reduces to $\theta_s < A$.

Taking into account that for an innovation to be non-drastic $\theta_m < A$ (licensing to the upstream sector) or $\theta_n < \frac{MA}{M+1+\frac{1}{N}} < A$ (licensing to the downstream sector) is required, $\frac{\partial \pi^{P,s}}{\partial r^s} > 0$ follows for any non-drastic innovation and $r^s \leq \theta_s$. \square

Note that in the SPNE with $r^s = \theta_s$, $l^s = |S|$ has to hold. Otherwise, by setting $r^s = \theta_s - \delta$ with δ arbitrarily close to, but strictly larger than zero, the patentee is able to ensure $l^s = |S|$ and as a consequence higher licensing revenues.

B Fixed-Fee Licensing Policy

B.1 Lemma 3

The following results are obtained for a linear demand schedule with the inverse demand function $p(Q) = a - \sum_{i=1}^N q_i^\alpha$. For notational convenience we define $A = a - c^m - c^n$.

Licensing to the upstream sector

Assume $l^m \leq M$ players are licensed on the upstream sector, while $l^n = 0$ on the downstream market. Then,

$$x_i^\alpha = \begin{cases} \frac{N(A+(M-l^m+1)\theta_m)}{(M+1)(N+1)} & i \in \mathcal{L}^m, \\ \frac{N(A-l^m\theta_m)}{(M+1)(N+1)} & i \notin \mathcal{L}^m \end{cases} \quad (6)$$

and $\pi_i^{m,\alpha} = (\frac{N+1}{N})(x_i^\alpha)^2$.

Firms accept any proposed licensing contract as long as the fixed-fee does not exceed their willingness to pay for the process innovation. Assuming that licensing contracts cannot be re-offered, the fixed-fee is then derived as

$$\begin{aligned} f^m(l^m) &= \pi_i^{m,l}(M, l^m) - \pi_i^{m,u}(M, l^m - 1) \\ &= \frac{MN(\theta_m)^2}{(M+1)^2(N+1)}(M + 2(\frac{A}{\theta_m} - l^m + 1)). \end{aligned} \quad (7)$$

By definition, an essential innovation is licensed to l^m players such that $\pi_i^{m,u}(l^m) \leq 0$. Given (6) this implies

$$l^m = \frac{A}{\theta_m}. \quad (8)$$

For the innovation to be essential and non-drastic $\theta_m \in [\frac{2A}{M+2}, A]$.²³

Given (8) it is easily established that

$$f^m(l^m) = \frac{MN(\theta_m)^2}{(M+1)^2(N+1)}(M+2). \quad (10)$$

²³To see this, note that in general, in order to determine the optimal (interior) l^m , the patentee solves

$$\max_{l^m} \pi^{P,f,m}(l^m) = f^m(l^m)l^m. \quad (9)$$

The interior equilibrium (l^{m*}), which is given by the solution to $l^{m*}f'(l^{m*}) + f^m(l^{m*}) = 0$, is characterised by $\pi_i^{m,l}(l^{m*}) > 0$ and $\pi_i^{m,u}(l^{m*}) > 0$. Hence, in the interior equilibrium, the industry is characterised by a mixed technology, meaning licensed and unlicensed firms obtain strictly positive profit. Next to the interior equilibrium there are two additional equilibria at the boundary. One at $l^m = M$ and one at $l^m = \frac{A}{\theta_m}$ (essential innovation). It follows that in order to be in the case of an essential, but non-drastic innovation θ_m has to be such that $\pi_i^{m,u}(l^{m*}(\theta_m)) \leq 0$ and $l^m(\theta_m) > 1$. In other words, $\theta_m \in [\frac{2A}{M+2}, A]$ is required.

Hence,

$$\pi^{P,m,f} = \frac{MNA\theta_m}{(M+1)^2(N+1)}(M+2). \quad (11)$$

Licensing to the downstream sector

Assume $l^n \leq N$ players are licensed on the downstream sector, while $l^m = 0$ on the upstream market. Then,

$$q_i^\alpha = \begin{cases} \frac{MA - Gl^n\theta_n + \theta_n(M+1)(N+1)}{(M+1)(N+1)} & i \in \mathcal{L}^m, \\ \frac{MA - Gl^n\theta_n}{(M+1)(N+1)} & i \notin \mathcal{L}^m \end{cases} \quad (12)$$

and $\pi_i^{n,\alpha} = (q_i^\alpha)^2$. Here, $G = M + 1 + \frac{1}{N}$.

Similar to the previous case, it can be established that

$$\begin{aligned} f^n(l^n) &= \pi_i^{n,l}(N, l^n) - \pi_i^{n,u}(N, l^n - 1) \\ &= \frac{2((M+1)N - \frac{1}{N})(\theta_n)^2}{(M+1)^2(N+1)^2} \left(\frac{MA}{\theta_n} - l^n G + H \right) \end{aligned} \quad (13)$$

with $H = M + 1 + \frac{1}{2}((M+1)N + \frac{1}{N})$.

An essential innovation is licensed to l^n players (with $l^n > 1$) such that $\pi_i^{n,u} \leq 0$. Given (12) this implies

$$l^n = \frac{MA}{G\theta_n}. \quad (14)$$

In analogy to the previous case it can be established that $\theta_n \in [\frac{2MA}{(N+2)(M+1) + \frac{1}{N}}, \frac{MA}{G})$ is required for the innovation to be essential, but non-drastic.

It is then readily established that

$$f^n(l^n) = \frac{2(\theta_n)^2}{(M+1)^2(N+1)^2} ((M+1)N - \frac{1}{N})H \quad (15)$$

and

$$\pi^{P,n,f} = \frac{2MA\theta_n}{(M+1)^2(N+1)^2} ((M+1)N - \frac{1}{N}) \frac{H}{G}. \quad (16)$$

B.2 Proposition 4/5

Thresholds

The thresholds given in Proposition 4 and Proposition 5 are

- $\delta_{fee}^\pi = \frac{N(N+1)(M+2)G}{2H((M+1)N - \frac{1}{N})}$,
- $\delta_{fee}^l = \frac{M}{G}$,
- $\delta_{fee}^f = \sqrt{\frac{MN(N+1)(M+2)}{2H((M+1)N - \frac{1}{N})}}$,

- $M_1(N) = \frac{2+N^3+N+(N+1)\sqrt{4+5N^2+2N^3+N^4}}{2N^2}$,
- $M_2(N)$ is the solution to $N(N+1)(M+2)G^2 \geq 2MH((M+1)N - \frac{1}{N})$ (note that the existence of a solution for $M_2(N)$ requires $M > M_1(N)$).

As defined previously, $G = M + 1 + \frac{1}{N}$ and $H = M + 1 + \frac{1}{2}((M+1)N + \frac{1}{N})$.

Simulations

Table 1 provides numerical values for the relevant thresholds for different sizes of the upstream and the downstream industry.

M, N	δ_{fee}^l	δ_{fee}^f	δ_{fee}^π	$M_2(N)$	$M_1(N)$
$M = N = 2$	0.571	0.836	1.222	9.817	4.306
$M = N = 3$	0.692	0.871	1.096	11.133	4.792
$M = N = 10$	0.901	0.954	1.009	24.311	11.210
$M = 2, N = 3$	0.600	0.850	1.204	11.133	4.792
$M = 2, N = 5$	0.625	0.875	1.224	14.645	6.442
$M = 2, N = 10$	0.645	0.903	1.264	24.311	11.210
$M = 3, N = 2$	0.667	0.853	1.091	9.817	4.306
$M = 5, N = 2$	0.769	0.863	0.969	9.817	4.306
$M = 10, N = 2$	0.870	0.868	0.865	9.817	4.306

Table 1: Numerical values for Proposition 4 and 5

C Welfare Analysis

Consumer surplus and producer surplus of firms in the unlicensed and the licensed industry sector under either fixed-fee licensing to the upstream market or the downstream market amount to:

- $CS_{fee,m} = \frac{1}{2}(\frac{NA}{N+1})^2$ or $CS_{fee,n} = \frac{1}{2}(\frac{MNA}{(M+1)N+1})^2$,
- $PS_{fee,m}^n = \frac{NA^2}{(N+1)^2}$ or $PS_{fee,n}^m = \frac{MN(N+1)A^2}{((M+1)N+1)^2}$,
- $PS_{fee,m}^m = \frac{NA\theta_m}{(M+1)^2(N+1)}$ or $PS_{fee,n}^n = \frac{MGA\theta_n}{(M+1)^2(N+1)^2}$.

D Two-sector Licensing

D.1 Proposition 13

Assume that $l_b^m \leq M$ and $l_b^n \leq N$ players are licensed on the upstream and on the downstream market. Denote by $x_{i,b}^\alpha$ and $q_{i,b}^\alpha$ their corresponding production levels. Then,

$$x_{i,b}^\alpha = x_i^\alpha + \frac{l_b^n \theta_{n,b}}{(M+1)(N+1)} \quad (17)$$

and

$$q_{i,b}^\alpha = q_i^\alpha + \frac{l_b^m \theta_{m,b}}{(M+1)(N+1)} \quad (18)$$

where x_i^α and q_i^α are given by (6) and (12).

As before

$$\pi_{i,b}^{m,\alpha} = \left(\frac{N+1}{N}\right)(x_{i,b}^\alpha)^2 \quad \text{and} \quad \pi_{i,b}^{n,\alpha} = (q_{i,b}^{\alpha,b})^2. \quad (19)$$

Firms accept any proposed licensing contract as long as the fixed-fee does not exceed their willingness to pay for the innovation. Assuming that licensing contracts cannot be re-offered, the fixed-fees are derived as

$$f_b^m(l_b^m, l_b^n) = \frac{MN(\theta_{m,b})^2}{(M+1)^2(N+1)} [M + 2\left(\frac{A + \frac{l_b^n}{N}\theta_{n,b}}{\theta_{m,b}} - l_b^m + 1\right)] \quad (20)$$

and

$$f_b^n(l_b^m, l_b^n) = \frac{2((M+1)N - \frac{1}{N})(\theta_{n,b})^2}{(M+1)^2(N+1)^2} \left(\frac{M(A + \frac{l_b^m}{M}\theta_{m,b})}{\theta_{n,b}} - l_b^n G + H\right). \quad (21)$$

By definition an essential innovation is licensed to l_b^s players such that $\pi_{i,b}^{s,u}(l_b^m, l_b^n) \leq 0$. This implies,

$$l_b^m(l_b^n) = \frac{A + \frac{l_b^n}{N}\theta_{n,b}}{\theta_{m,b}} \quad \text{and} \quad l_b^n(l_b^m) = \frac{M(A + \frac{l_b^m}{M}\theta_{m,b})}{\theta_{n,b}}. \quad (22)$$

Consequently, in the symmetric equilibrium

$$l_b^m = \frac{A}{\theta_{m,b}} \left(1 + \frac{1}{N}\right) \quad \text{and} \quad l_b^n = \frac{A}{\theta_{n,b}} \quad (23)$$

with $\theta_{m,b}$ such that $l_b^m(\theta_{m,b}) \in (1, \frac{1}{2}(M+2 + \frac{2}{MN}\frac{1}{\Theta})]$ and $\theta_{n,b}$ such that $l_b^n(\theta_{n,b}) \in (1, \frac{1}{2}((N+2)(M+1) + \frac{1}{N} + \Theta)]$ where $\Theta = \frac{\theta_{m,b}}{\theta_{n,b}} \left(\frac{N+1}{(M+1)N - \frac{1}{N}}\right)$.

It is then easily established that

$$\pi_b^{P,m,f} = \frac{MA\theta_{m,b}}{(M+1)^2} (M+2) \quad (24)$$

and

$$\pi_b^{P,n,f} = \frac{2A\theta_{n,b}}{(M+1)^2(N+1)^2} \left((M+1)N - \frac{1}{N}\right)H. \quad (25)$$

D.2 Proposition 16

Thresholds

The thresholds given in Proposition 16 are

- $\delta_{fee,b}^\pi = \frac{M(M+2)(N+1)^2}{2H((M+1)N - \frac{1}{N})}$,
- $\delta_{fee,b}^l = \frac{N}{N+1}$,
- $\delta_{fee,b}^f = \delta_{fee}^f$.

Simulations

Table 2 provides numerical values for the relevant thresholds for different sizes of the upstream and the downstream industry.

M, N	$\delta_{fee,b}^l$	$\delta_{fee,b}^f$	$\delta_{fee,b}^\pi$	$N - \frac{1}{N}$
$M = N = 2$	0.667	0.836	1.047	1.500
$M = N = 3$	0.750	0.871	1.012	2.667
$M = N = 10$	0.909	0.954	1.000	9.900
$M = 2, N = 3$	0.750	0.850	0.963	2.667
$M = 2, N = 5$	0.833	0.875	0.918	4.800
$M = 2, N = 10$	0.909	0.903	0.897	9.900
$M = 3, N = 2$	0.667	0.853	1.091	1.500
$M = 5, N = 2$	0.667	0.863	1.118	1.500
$M = 10, N = 2$	0.667	0.868	1.129	1.500

Table 2: Numerical values for Proposition 16

E Non-Essential Innovations

In the following we provide some evidence on the licensing of non-essential innovations.

We argued in Section B that next to $l^m = \frac{A}{\theta_m}$ (or $l^n = \frac{MA}{G\theta_n}$) the patentee's optimisation problem has two additional equilibria. The interior equilibrium at l^{s*} and another one at the boundary at $l^s = |S|$. For completeness, $l^{m*} = \frac{1}{4}(M + \frac{2A}{\theta_m} + 2)$ and $l^{*,n} = \frac{1}{2G}(\frac{MA}{\theta_n} + H)$. In the interior equilibrium the economy is one with a mixed technology in which licensed and unlicensed firms produce positive quantities. Which equilibrium applies depends on the size of the innovation. For sufficiently small innovations the entire industry is licensed, for intermediate values of the innovation a strict subset of firms is licensed. As a result, when licensing to the upstream market, the patentee obtains

$$\pi^{P,f,m} = \begin{cases} \frac{M^2 N (M + 2(\frac{A}{\theta_m} - M + 1)) \theta_m^2}{(M+1)^2 (N+1)} & \theta_m \leq \frac{2A}{3M-2}, \\ \frac{MN(M + \frac{2A}{\theta_m} + 2) \theta_m^2}{8(M+1)^2 (N+1)} & \theta_m \in (\frac{2A}{3M-2}, \frac{2A}{M+2}), \end{cases} \quad (26)$$

depending on the size of the innovation.

Similarly, when licensing to the downstream market

$$\pi^{P,f,n} = \begin{cases} \frac{2N((M+1)N - \frac{1}{N})(\frac{MA}{\theta_n} - (M+1)N - 1 + H) \theta_n^2}{(M+1)^2 (N+1)^2}, \\ \frac{((M+1)N - \frac{1}{N})(\frac{MA}{\theta} + H)^2 \theta_n^2}{2(M+1)^2 (N+1)^2 G} \end{cases} \quad (27)$$

for respectively $\theta_n \leq \frac{2MA}{(M+1)(3N-2)+4-\frac{1}{N}}$ and $\theta_n \in (\frac{2MA}{(M+1)(3N-2)+4-\frac{1}{N}}, \frac{2MA}{2H})$.

E.1 Licensing to the Less Efficient Market

Here we focus on upstream technology transfer. For simplicity we set $A = 1$.

Assume first that the innovation is such that the entire upstream industry is licensed ($l^m = M$). The following table summarises some scenarios in which licensing to a less efficient upstream market is optimal for different constellations of M and N .

M, N	$\theta_m \in$	$\theta_n \in$
3, 3	(0, 0.165]	$[\theta_m, 0.529 - 0.01\sqrt{2835 - 11016\theta_m + 5508\theta_m^2}]$
2, 3	0.066	0.066
	0.162	0.162
	(0.066, 0.142]	$[\theta_m, 0.429 - 0.014\sqrt{936 - 4032\theta_m}]$
	(0.142, 0.162)	$[\theta_m, 0.162]$

Table 3: Licensing to a less efficient upstream market, $l^m = M$

Assume now that the innovation is such that a strict subset of upstream firms is licensed ($l^m = l^{m*}$). The following table summarises some cases in which licensing to a less efficient upstream market is optimal for different constellations of M and N .

M, N	$\theta_m \in$	$\theta_n \in$
3, 3	(0.286, 0.295)	$[\theta_m, 0.295]$
2, 3	-	-
3, 2	(0.286, 0.308]	$(0.308, 0.008\sqrt{2592 + 12960\theta_m + 16200\theta_m^2} - 0.364]$
	(0.308, 0.330)	$[\theta_m, 0.008\sqrt{2592 + 12960\theta_m + 16200\theta_m^2} - 0.364]$
	0.330	$[0.330, 0.364]$
	(0.330, 0.364)	$[\theta_m, 0.364]$

Table 4: Licensing to a less efficient upstream market, $l^m = l^{m*}$

E.2 Fixed-Fee versus Per-Unit Royalty

Here we only provide evidence for the downstream industry. For the upstream industry a per-unit royalty scheme is never optimal. We further restrict our attention to the integer solutions.

Lemma 17 *Assume that the innovation is licensed to the entire downstream industry so that $l^n = N$. Then, the patentee optimally offers a per-unit royalty contract to the downstream industry whenever $N = 1$, $M \geq 1$ and $\theta_n \leq \frac{2MA}{(M+1)(3N-2)+4-\frac{1}{N}}$.*

Lemma 18 *Assume that the innovation is licensed to a subset of downstream firms so that $l^n = l^{*,n}$. Then, the patentee optimally offers a per-unit royalty contract to the downstream industry whenever $N = 2$, $M = 1$ and $\theta_n \in [\frac{A(484-16\sqrt{30})}{2023}, \frac{2MA}{2H})$.*

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