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Vertical product differentiation and two-sided markets

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Vertical product differentiation and two-sided markets

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HIGHLIGHTS

- Using the vertical differentiation model to represent competition in two-sided markets.
- Show the existence of an interior solution with two platforms.
- Ranking of competing platforms by their "quality" (the size of the networks).

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ABSTRACT

We model platform competition in a market where products are characterized by cross network externalities. Consumers differ in their valuation of these externalities. Although the exogenous set-up is entirely symmetric, we show that platform competition induces a vertical differentiation structure that allows for the co-existence of asymmetric platforms in equilibrium. We establish this result in two set-ups: in the first one platforms commit to prices, in the second one they commit to network sizes.

1. Introduction

The recent literature on two-sided markets (see Rochet and Tirole (2006)) builds on network externalities. In markets where products are subject to network externalities, the number of product's users determines, at least partially, the perceived quality of the product. In two sided-markets, a product is best viewed as a platform on which different groups of users meet or trade. The externalities that benefit to one group typically originate in the number of participants from the other group: network externalities cross from one side to the other.

Relying on the product differentiation literature (Gabszewicz and Thisse, 1979), two products subject to network externalities could then be considered as vertically differentiated products whenever their number of users differs. In this respect, vertical differentiation seems endemic to the presence of consumption network externalities, which in turn suggests that models of vertical differentiation, as originally developed in Gabszewicz and Thisse (1979), could prove useful in modelling price competition in markets with network externalities.

The present note shows that heterogeneity among consumers can be naturally introduced by assuming that their preferences w.r.t. the size of the networks vary across the population. When the population of agents on both sides of the market is heterogeneous in its willingness to pay for network sizes, we show that asymmetric equilibria naturally emerge. In these equilibria, the two platforms are clearly ranked by size but nevertheless enjoy positive market shares and profits. On each side of the market, equilibrium outcomes resemble those obtained in standard models of vertical differentiation: one firm is perceived by all agents as better than the other but not all agents register to that firm because of the price differential. A dominated platform can survive by charging lower prices, without inducing the dominant platform to price aggressively and preempt the market. A key difference with standard models of vertical differentiation is that realized qualities are endogenous to the price decisions rather than exogenous.

2. The model

The specification of preferences we retain here are those of Mussa and Rosen (1978). There are three types of agents:

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- Platforms: they are denoted by i and sell product $i = 1, 2$. Product i is best viewed as a device that allows information exchange between agents. For the sake of illustration, we shall refer here to the credit cards' metaphor. Card issuers sell their product in two markets: the buyers' market and the merchants' market. The subscription fee paid by the buyers, as well as the fee paid to the platforms by the merchants, allow buyers and merchants to use the card as a means of payment.
- Buyers: they are denoted by their type θ. Types are uniformly distributed in the [0, 1] interval. The total number of buyers is normalized to 1. They possibly buy a card $i = 1, 2$ according to a utility function $U_i = \theta x_i - p_i$, with x_i denoting the number of merchants at platform i. Holding no credit card yields a utility level normalized to 0.[1]
- Merchants: they are denoted by their type γ. Types are uniformly distributed in the [0, 1] interval. Their total number is normalized to 1. When they accept card $i, i = 1, 2$, their utility is measured by $U_i^{'} = \gamma v_i - \pi_i$, with v_i denoting the number of cardholders holding card i. Refraining from accepting any card yields a utility level normalized to 0.

The present set-up is best viewed as a model where two vertically differentiated markets operate in parallel with the key feature that quality in one of the two markets is determined by outcomes in the other market: agents' participation on each side determines the perceived quality for the other side.

We consider two different games. In the first one, platforms commit to uniform unit prices, as a function of expectations about participation on the two sides of the market and we require that expectations are fulfilled in equilibrium. In the second one, we assume that prices are set after platforms have directly committed to network sizes and we require that those prices are set to implement committed sizes through optimal participation decisions by the two sides of the market.

3. Equilibrium analysis under price competition

In this section, we assume that platforms choose price, taking expectations as given. We may thus start by identifying the expression of demand for participation from both sides, defined as a function of the expected participation on the other side. Consider demands addressed to these two platforms by the merchants, with v_i^e denoting the expectation merchants have about the number of buyers at platform i, and π_i the price paid by the merchants to register to platform i. Assuming $v_2^e > v_1^e > 0$, we get:

$$D_1^x(\pi_1, \pi_2) = \frac{\pi_2 v_1^e - \pi_1 v_2^e}{v_1^e(v_2^e - v_1^e)}$$

$$D_2^x(\pi_1, \pi_2) = 1 - \frac{\pi_2 - \pi_1}{v_2^e - v_1^e}.$$

These are the demand functions in a vertical differentiation model with quality products defined exogenously by $v_2^e > v_1^e$.[2] A similar demand specification $D_i^v(p_1, p_2)$ can be defined for the buyers' market, given expectations $x_2^e > x_1^e$. Conditional on expectations $v_2^e > v_1^e > 0$, and $x_2^e > x_1^e > 0$, the payoff function of platform i is then derived as

$$p_i D_i^v(p_1, p_2) + \pi_i D_i^x(\pi_1, \pi_2), \quad i = 1, 2.$$

Formally, we define a Nash equilibrium in the two-sided market duopoly as follows:[3] A Nash Equilibrium is defined by two quadruples (p_i^*, π_i^*) and (v_i^*, x_i^*) with $i = 1, 2$, such that (i) given expectations $(v_1^*, v_2^*, x_1^*, x_2^*)$, $\left(p_{i,}^* \pi_i^*\right)$ is a best reply against $(p_{j,}^* \pi_j^*)$, $i \neq j$, and vice-versa; (ii) $D_i^v(p_1^*, p_2^*) = x_i^*$; $D_i^x(\pi_1^*, \pi_2^*) = v_i^*$, $i = 1, 2$.

We now derive the price equilibrium on the merchants' market, conditional on expectations $v_1^e < v_2^e$:

$$\pi_2(v_1^e, v_2^e) = \frac{2v_2^e(v_2^e - v_1^e)}{4v_2^e - v_1^e}$$

$$\pi_1(v_1^e, v_2^e) = \frac{v_1^e(v_2^e - v_1^e)}{4v_2^e - v_1^e},$$

with corresponding demands:

$$D_2^x(v_1^e, v_2^e) = \frac{2v_2^e}{4v_2^e - v_1^e}$$

$$D_1^x(v_1^e, v_2^e) = \frac{v_2^e}{4v_2^e - v_1^e}.$$

Obviously, the symmetry of our model allows us to directly infer the price equilibrium, conditional on expectations, $x_2^e > x_1^e$, on the merchants' market. We obtain

$$D_2^v(x_1^e, x_2^e) = \frac{2x_2^e}{4x_2^e - x_1^e}$$

$$D_1^v(x_1^e, x_2^e) = \frac{x_2^e}{4x_2^e - x_1^e}.$$

Then it remains to solve the model for fulfilled expectations, i.e. condition (ii) in the above definition of a Nash equilibrium. This is done by solving the system

$$x_2 = \frac{2D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)}$$

$$x_1 = \frac{D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)}.$$

Straightforward computations yield $x_1^* = v_1^* = \frac{2}{7}$ and $x_2^* = v_2^* = \frac{4}{7}$, and corresponding prices $\pi_1^* = p_1^* = \frac{2}{49}$, $\pi_2^* = p_2^* = \frac{8}{49}$.

Proposition 1. *The presence of heterogeneity on both markets allows for an interior equilibrium where both platforms enjoy strictly positive networks and profits. The quadruples* $(x_1^* = v_1^* = \frac{2}{7}, x_2^* = v_2^* = \frac{4}{7})$ *and* $(\pi_1^* = p_1^* = \frac{2}{49}, \pi_2^* = p_2^* = \frac{8}{49})$ *define the unique (up to permutation) interior equilibrium.*

This proposition clearly illustrates the links that relate markets with cross network externalities and vertically differentiated industries. When setting different prices, platforms attract different types of agents on both sides of the market and thereby fix the size of the networks. In equilibrium, the size of the network endogenously determines the willingness of the consumers to participate in one of the two platforms. Heterogeneity on both sides allows for the co-existence of two asymmetric platforms.

4. Equilibrium analysis under network commitment

An alternative route to solve the duopoly platform problem consists in formalizing it as a Cournot game, i.e. a game where firms commit to quantities rather than prices. In the present context, platforms commit to network sizes before participants make their decisions. Interestingly enough, this avenue has been entirely neglected by the recent literature on two sided-markets.

1 Multi-homing behaviour is ruled out in the present model. See Gabszewicz and Wauthy (2004) for a comparable model with multi-homing.

2 We do not consider explicitly the case where $v_1^e = v_2^e$ since under such expectations the equilibrium candidate displays zero profit. Notice also that we restrict attention to configurations of prices where the two firms enjoy a positive demand. Indeed, since $v_1^e > 0$, it cannot be the case that firm e is excluded from the market in equilibrium.

3 This definition essentially extends the definition of Katz and Shapiro (1985) to a context of multi-sided market.

Recall that under Cournot competition, it is assumed that there exists a coordination mechanism among consumers that ensures that the quantities sold are exactly those that firms committed to. Under standard Cournot competition, the mechanism is summarized by the Walrasian auctioneer, i.e. coordination is achieved by selecting the levels of prices such that consumers' demands *exactly* match firms' outputs. In a market with network externalities, the corresponding mechanism captures the extent to which firms can influence participants' expectations about their size (as argued in the appendix of Katz and Shapiro (1985)). Market clearing prices are then defined as the highest price levels that are jointly compatible with the committed sizes.[4] Obviously, the presence of this coordination mechanism will reinforce network externalities. As compared with the previous model, we may expect that a given set of prices will command a larger participation.

In our present set-up, we formalize this idea by considering a set of conditions that links a quadruple of network sizes to a corresponding quadruple of prices. Using the specification of the consumers' preferences, we may associate to any network size configuration $x_2 > x_1$ and $v_2 > v_1$ a corresponding set of equations involving the price levels that should be simultaneously satisfied to implement these sizes in equilibrium. Formally, we have:

$$v_2 = \frac{x_2 - x_1 - p_2 + p_1}{x_2 - x_1} \tag{1}$$

$$v_1 = \frac{x_2 p_1 - x_1 p_2}{x_1(x_2 - x_1)} \tag{2}$$

$$x_2 = \frac{v_2 - v_1 - \pi_2 + \pi_1}{v_2 - v_1} \tag{3}$$

$$x_1 = \frac{v_2 \pi_1 - v_1 \pi_2}{v_1(v_2 - v_1)}. \tag{4}$$

Taken side by side, these expressions can be inverted to express the corresponding prices as a function of the participation level on both sides. On the buyers' side for instance, one immediately gets:

$$p_2 = x_2(1 - v_2) - x_1 v_1 \tag{5}$$

$$p_1 = x_1(1 - v_2 - v_1). \tag{6}$$

Unsurprisingly, these expressions replicate the Cournotian system of inverse demands that would prevail in a vertically differentiated market where firms 1 and 2 would sell products of quality $x_1 < x_2$.

Comparable expressions obtain for the merchants' side:

$$\pi_2 = v_2(1 - x_2) - x_1 v_1 \tag{7}$$

$$\pi_1 = v_1(1 - x_2 - x_1). \tag{8}$$

Assuming that platforms commit to network sizes, the above expression can be understood as the highest prices quadruple at which the committed sizes would realize. We may then characterize optimal network sizes by solving the following Cournot game between platforms. We have

$$\Pi_1 = v_1 x_1(2 - v_1 - v_2 - x_1 - x_2) \tag{9}$$

$$\Pi_2 = x_2 v_2(2 - v_2 - x_2) - x_1 v_1(v_2 + x_2). \tag{10}$$

Maximizing over v_i, x_i, first order conditions are:

$$v_1 = \frac{2 - x_1 - x_2 - v_2}{2} \tag{11}$$

$$x_1 = \frac{2 - v_1 - v_2 - x_2}{2} \tag{12}$$

$$v_2 = \frac{2 - x_2}{2} - \frac{v_1 x_1}{2x_2} \tag{13}$$

$$x_2 = \frac{2 - v_2}{2} - \frac{v_1 x_1}{2v_2}. \tag{14}$$

Solving this system of equations, we obtain two quadruples of interior solutions, but only one of them satisfies the required hierarchy $x_2 > x_1$ and $v_2 > v_1$, namely:

$$x_1^* = v_1^* = \frac{2}{31}(6 - \sqrt{5}) \cong .242,$$

$$x_2^* = v_2^* = \frac{1}{31}(13 + 3\sqrt{5}) \cong .636.$$

Notice however that when platforms commit to network sizes, there is no a priori reason to rule out configurations where they announce identical sizes. The cases where $k_1 = k_2$, with $k = x, v$ can be solved easily. When committed sizes are identical on one side, this imposes the restriction that prices on the other side should be equal. Otherwise, all agents on that size would turn to the low price platform, which is not compatible with the implementation of committed sizes. We may then define a system of inverse demand for the case where platforms announce symmetric network sizes on the two sides of the market. Assuming $x_1 = x_2 = x$ we get:

$$p_1 = p_2 = x(1 - v_1 - v_2)$$

and assuming $v_1 = v_2 = v$, we get:

$$\pi_1 = \pi_2 = v(1 - x_1 - x_2).$$

Plugging these expressions into the payoff functions and maximizing for each platform over (v_i, x_i), we obtain a symmetric solution where $x_1^{**} = x_2^{**} = v_1^{**} = v_2^{**} = \frac{2}{5}$. In order to check whether this configuration can be sustained as a Nash equilibrium, we may consider either downward or upward deviations. Upward deviations lead us to the previous configuration, taking the point of view of the small platform. It is then immediate to see that such a deviation is not profitable since the relevant payoff remains unchanged for the small platform (defined by (8) and (6)). Considering upward deviations, it is also immediate to check with the system of best reply functions of platform 2 here above that a joint upward deviation on both side is always profitable. As a result we have to rule out our symmetric equilibrium candidate.

Finally we should check that the small platform in our asymmetric equilibrium candidate would not gain by matching the size of the large platform, i.e. restore a symmetric outcome. It is again easy to see that such a deviation cannot be profitable since this would imply that $k_1 + k_2 > 1$, for $k = v, x$, a total size that cannot be achieved at positive prices.

All in all we have thus proved the following proposition:

Proposition 2. *Let us assume that* $v_2 \geq v_1$ *and* $x_2 \geq x_1$*. Then, there exists a duopoly equilibrium in network size strategies given by* $x_1^* = v_1^* = \frac{2}{31}(6 - \sqrt{5})$ *and* $x_2^* = v_2^* = \frac{1}{31}(13 + 3\sqrt{5})$.

5. Final remarks

In a market with cross network externalities, it is often the case that participants within each group differ in their valuation of the externality. We have shown that the vertical differentiation model offers a natural vehicle to model platform competition in two-sided markets. In a market with membership externalities, prices set by the firms elicit participation on either side and thereby simultaneously determine platform quality. Because participants are

[4] Notice that it is possible to formalise a mechanism that would sustain this formation of prices as follows: Suppose that platforms are allowed to offer contracts by which they link prices to participation levels, i.e. the price participants end up paying is a particular selection of a menu of prices where the selection is based on realised participation. This amounts to insure participants on each side against default of coordination on the other side. Eqs. (5)–(8) here below are, by construction, examples of such contracts. They can be viewed as an application of the White and Weyl (2012) concept of insulated equilibrium.

heterogeneous, asymmetric platforms, i.e. platforms with different sizes, co-exist in the market. In this paper we have considered two different ways to capture the interaction between platforms strategies and participation decisions. When firms can commit to network sizes total participation is larger and the size of the large platform is also larger than in the model where firms have to set prices given expectations. Notice then that in the present set-up (where costs are zero) the extent and the composition of participation are direct proxies for total welfare. Total participation is larger in the second model, but, in addition, the distribution of participation is also better from a welfare point of view: since larger types (who generate larger total surplus) register to the large platform, which itself is larger in the second model than in the first one, it is immediate to see that total welfare is larger in the second model.

It would be interesting to explore the applicability of this set-up to more general types of externalities and more general tariffs.

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