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Behavioral Welfare Economics and Redistribution†

By Marc Fleurbaey and Erik Schokkaert*

Behavioral economics has shaken the view that individuals have well-defined, consistent, and stable preferences. This raises a challenge for welfare economics, which takes as a key postulate that individual preferences should be respected. We argue, in agreement with Bernheim (2009) and Bernheim and Rangel (2009), that behavioral economics is compatible with consistency of partial preferences, and explore how the Bernheim-Rangel approach can be extended to deal with distributive issues. We revisit some key results of the theory in a framework with partial preferences, and show how one can derive partial orderings of individual and social situations. (*JEL* D03, D63, D71, H23)

One of the challenges for welfare economics is the formulation of adequate criteria to evaluate (re)distribution. Without such criteria, policy evaluation can only be based on the Pareto criterion. Pareto-improving policy measures are rare, however. Rejecting all other policies leads to a conservative defense of the status quo, while the Kaldor-Hicks criterion of potential Pareto improvements is lacking ethical content. Indeed, the existence of a "potential improvement" is not very relevant if the necessary compensations remain purely hypothetical. To go beyond these Pareto-type approaches, one needs a concept of interpersonally comparable well-being. Traditional welfare economics has struggled for a long time with the issue of interpersonal comparisons. Arrow's impossibility theorem has most often been interpreted as showing that the informational basis of ordinal preferences is insufficient to derive an ordering of social states. In the wake of Sen (1970), a large literature explored the consequences of going beyond such ordinal preference information and derived welfare criteria under different assumptions about interpersonal comparability and measurability of individual subjective welfare (for an overview of this so-called welfarist approach, see d'Aspremont and Gevers 2002).

The best way, or even the possibility, to measure subjective welfare in an interpersonally comparable way remains, however, a controversial question. Fortunately, recent developments in the theory of fair allocations have shown that the common interpretation of Arrow's theorem is wrong, and that an interpersonally comparable measure of subjective utility is not needed. According to these

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developments, fairness principles recommend to construct interpersonally comparable concepts of well-being that are actually based only on information about ordinal "noncomparable" individual preferences (for an overview, see Fleurbaey and Maniquet 2011). One attractive approach is based on the concept of equivalent income, which is firmly rooted in the tradition of money-metric utility (Samuelson 1974). This approach produces social criteria that respect individual preferences and are able to give some priority to the worse-off in the evaluation of public policies.

This so-called fairness approach offers a promising way out of Arrow's impossibility without necessitating the use of subjective utilities, but it does rest on the assumption that well-defined individual preferences exist. The findings of behavioral economics have cast doubt on this assumption. The existence of "behavioral anomalies" suggests that it is difficult to interpret individual choice behavior as the maximization of well-defined preferences. This has important implications for welfare economics. Some authors (Frey and Stutzer 2002; Kahneman, Wakker, and Sarin 1997; Kahneman and Sugden 2005; Kőszegi and Rabin 2008; Layard 2005) have advocated to focus on experience utility (and subjective happiness) rather than on decision utility. This would bring us back to the welfarist interpretation of Arrow's theorem. Other authors refuse to take this step and formulate preference- or choice-based welfare criteria (Bernheim and Rangel 2009; Bernheim 2009; Beshears et al. 2008; Choi et al. 2003; Dalton and Ghosal 2010; Rubinstein and Salant 2012; Salant and Rubinstein 2008). In most of these approaches, the proposed preference relations are imprecise or incomplete, and the question remains whether it is possible along this vein to go beyond the Pareto criterion.

In this paper, we bring together these two recent streams of literature. We examine if it is still possible to derive an interpersonally comparable concept of well-being and a tractable criterion for the evaluation of policies, when one works with an incomplete preference relation as defined, e.g., by Bernheim and Rangel (2009). We show that the answer to this question is positive. Using the incomplete individual preference relation proposed by Bernheim and Rangel (2009), we derive an incomplete ordering of personal situations in terms of well-being and we argue that this concept of well-being, which relies only on ordinal preferences, can be used for distributional judgments. Respect for individual preferences is the key value in our approach, and we explore how far one can go if one accepts this key value.

To set the scene, we summarize in Section II some relevant findings about the equivalent income in a setting with well-defined and complete preferences. We then briefly recall in Section III why behavioral welfare economics threatens approaches that involve standard individual preferences, including a social welfare approach that would invoke "authentic preferences" as the yardstick of well-being. Sections IV and V show how a theory of fair social choice, relative to interpersonal comparisons (Section IV) and social evaluation (Section V), can be developed for the case of incomplete preferences. Section VI concludes.

I. Interpersonal Comparisons and Social Evaluation with Well-Defined Preferences

We consider first the problem of evaluating social states in a setting where individuals have well-defined ideas about what a good life is. Relevant life dimensions (such as consumption, health status, job satisfaction, quality of interpersonal relations, etc.) are summarized in a vector $\mathbf{x} \in \mathcal{X}$, and we assume for simplicity that $\mathcal{X} = \mathbb{R}_+^\ell$. Each individual i has a (complete) preference ordering R_i over the vectors $\mathbf{x}_i$, which reflects her informed judgment about what makes a life good or bad: $\mathbf{x}_i \, R_i \, \mathbf{x}_i'$ if i weakly prefers the life described by $\mathbf{x}_i$ to the life described by $\mathbf{x}_i'$. Let $\mathbf{x}_i \, P_i \, \mathbf{x}_i'$ denote strict preference and $\mathbf{x}_i \, I_i \, \mathbf{x}_i'$ denote indifference. We assume that R_i is continuous and monotonic.[1] Let $\mathcal{D}$ be the set of preference orderings satisfying these assumptions.

We will proceed in two steps. First, we propose a method for interpersonal comparisons of individual well-being, relying only on ordinal preferences. Second, we consider a population with n individuals and we propose a method to derive social priorities, i.e., to rank allocations $(\mathbf{x}_1, \ldots, \mathbf{x}_n)$. Again, we will rely only on information about ordinal preferences. The two problems are obviously linked: the interpersonally comparable concept of well-being, derived in the first step, will play an essential role in the second step.

A. *Interpersonal Comparisons when Preferences Differ: Equivalent Income*

Let us first consider the issue of interpersonal comparisons, i.e., of ranking personal situations $(\mathbf{x}, R)$ in terms of well-being. The object to be constructed is a binary relation on such situations, that is denoted $\succcurlyeq$ (with asymmetric and symmetric parts $\succ$, $\sim$) and is required to be reflexive and transitive. To simplify the analysis, anonymity is assumed from the outset, so that the identity of individuals is not part of the description of situations $(\mathbf{x}, R)$. The statement $(\mathbf{x}, R) \succcurlyeq (\mathbf{x}', R')$ can be interpreted as stating that the well-being of an individual with preferences R in state $\mathbf{x}$ is at least as great as the well-being of an individual with preferences R' in state $\mathbf{x}'$.

Since we want to respect individual preferences, we impose the following preference principle as an essential requirement that interpersonal comparisons have to satisfy:

Preference principle: $(\mathbf{x}, R) \succeq (\mathbf{x}', R)$ if and only if $\mathbf{x} R \mathbf{x}'$.

This principle embodies the idea of individual sovereignty: if an individual prefers $\mathbf{x}$ to $\mathbf{x}'$, then the well-being ranking should reflect this personal preference. It also embodies the idea of respecting interpersonal comparisons across individuals sharing the same preferences. If two individuals have the same preferences R and agree that life situation $\mathbf{x}$ is better than life situation $\mathbf{x}'$, the well-being ranking should assign to the individual in $\mathbf{x}$ a larger well-being level than to the individual in $\mathbf{x}'$. Note that this preference principle is incompatible with ranking the individual life situations on the basis of subjective well-being (or happiness): it is very well possible that two individuals agree that $\mathbf{x}$ is better than $\mathbf{x}'$, and that at the same time the individual in situation $\mathbf{x}$ has a lower level of subjective happiness

[1] The monotonicity assumption makes the exposition easier but the main results of this section can also be derived without it—see, e.g., Fleurbaey, Schokkaert, and Decancq (2009). Vector inequalities will be denoted $\geq, >, \gg$.

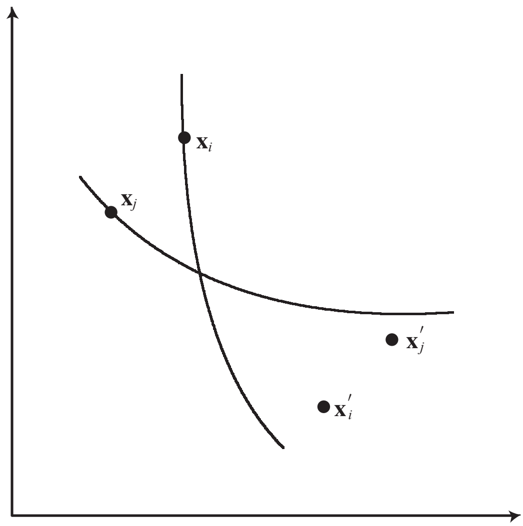

Figure 1. Incompatibility of the Preference and the Dominance Principles

than the individual in situation $\mathbf{x}'$, e.g., because she has more ambitious aspirations. Therefore, ranking individual situations on the basis of happiness does not respect individual preferences.[2]

The preference principle only bites if we have to compare the situations of two individuals with the same preferences. A more challenging problem of interpersonal comparisons arises when preferences differ. A natural starting point might seem to impose a dominance principle saying that, when a bundle $\mathbf{x}$ dominates another bundle $\mathbf{x}'$, the corresponding situation is preferable independently of the associated preferences:

Dominance principle: $(\mathbf{x}, R) \succcurlyeq (\mathbf{x}', R')$ if $\mathbf{x} \geq \mathbf{x}'$; $(\mathbf{x}, R) \succ (\mathbf{x}', R')$ if $\mathbf{x} \gg \mathbf{x}'$.

However, the preference principle and the dominance principle are incompatible. Indeed, the latter principle implies that $(\mathbf{x}, R)$ is as good as $(\mathbf{x}, R')$ for all $\mathbf{x}$ and all R, R', so that R plays no role in the evaluation of $(\mathbf{x}, R)$. Even the second part of the dominance principle is by itself incompatible with the preference principle. This is shown by the following example from Brun and Tungodden (2004). Assume $\mathcal{X} = \mathbb{R}^2_+$ and take $\mathbf{x}_i$, $\mathbf{x}_j$, $\mathbf{x}_i'$, $\mathbf{x}_j' \in \mathcal{X}$ and R_i R_j such that $\mathbf{x}_i \gg \mathbf{x}_j$, $\mathbf{x}_i' \ll \mathbf{x}_j'$, $\mathbf{x}_i' \, P_i \, \mathbf{x}_i$, and $\mathbf{x}_j \, P_j \, \mathbf{x}_j'$. Figure 1 illustrates this configuration. The preference principle implies that $(\mathbf{x}_i', R_i)$ is better than $(\mathbf{x}_i, R_i)$ and $(\mathbf{x}_j, R_j)$ is better than $(\mathbf{x}_j', R_j)$ while the dominance principle implies that $(\mathbf{x}_i, R_i)$ is better than $(\mathbf{x}_j, R_j)$ and $(\mathbf{x}_j', R_j)$ is better than $(\mathbf{x}_i', R_i)$. By transitivity, one obtains that $(\mathbf{x}_i, R_i)$ is better than $(\mathbf{x}_i, R_i)$, which is impossible.

[2] Fleurbaey, Schokkaert, and Decancq (2009) develop this basic insight and argue that happiness data can still be useful to recover information about individual ordinal preferences.

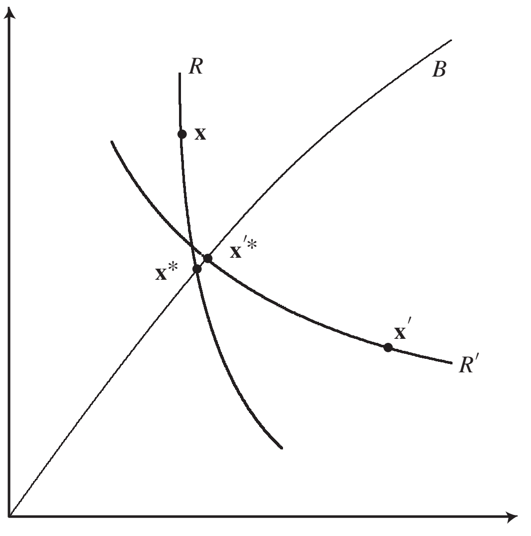

Figure 2. The Equivalence Approach

In order to cope with this incompatibility,[3] Fleurbaey, Schokkaert, and Decancq (2009) weaken the dominance requirement, accepting that it is enough if it is satisfied on a subset $\mathcal{B}$ of $\mathcal{X}$:

Restricted dominance principle: For all $\mathbf{x}, \mathbf{x}' \in \mathcal{B}$, $(\mathbf{x}, R) \succcurlyeq (\mathbf{x}', R')$ if $\mathbf{x} \geq \mathbf{x}'$; $(\mathbf{x}, R) \succ (\mathbf{x}', R')$ if $\mathbf{x} \gg \mathbf{x}'$.

As it turns out, requiring interpersonal comparisons to satisfy the preference principle in conjunction with this restricted dominance principle imposes a specific approach to interpersonal comparisons, namely, the *equivalence* approach. Let us explain this point. A set $\mathcal{B}$ is called a *monotone path* if $0 \in \mathcal{B}$, $\mathcal{B}$ is unbounded and connected, and for all $\mathbf{x}, \mathbf{x}' \in \mathcal{B}$, either $\mathbf{x} \geq \mathbf{x}'$ or $\mathbf{x} \leq \mathbf{x}'$. The equivalence approach consists in specifying a monotone path in $\mathcal{X}$, and comparing $(\mathbf{x}, R)$ and $(\mathbf{x}', R')$ by the relative positions of the vectors $\mathbf{x}^*$ and $\mathbf{x}'^*$ from the path such that $\mathbf{x} I \mathbf{x}^*$ and $\mathbf{x}' I' \mathbf{x}'^*$. This is illustrated in Figure 2, where $(\mathbf{x}, R)$ is declared inferior to $(\mathbf{x}', R')$ in this fashion, with the path given by the curve $\mathcal{B}$. The following proposition, proven by Fleurbaey, Schokkaert, and Decancq (2009), states formally that if one wants to respect the preference principle and the restricted dominance principle, one necessarily has to follow the equivalence approach to rank situations $(\mathbf{x}, R)$.

PROPOSITION 1: (Fleurbaey, Schokkaert, and Decancq 2009). *Let $\mathcal{B}$ be a subset of $\mathcal{X}$ such that for every $(\mathbf{x}, R)$ there is $\mathbf{x}^*$ in $\mathcal{B}$ such that $\mathbf{x} I \mathbf{x}^*$. The dominance principle restricted to $\mathcal{B}$, in conjunction with the preference principle, implies that $\succcurlyeq$ follows the equivalence approach with $\mathcal{B}$ as the monotone path.*

[3] An alternative approach would be to keep the dominance principle while relaxing (or giving up) the preference principle. This route is formally explored by Sprumont (2012).

As explained in Fleurbaey (2009), the equivalence approach is not new. It basically boils down to the idea of money-metric utility that was popular in the 1980s (Samuelson 1974; Deaton and Muellbauer 1980; Willig 1981; King 1983). The theory of fair allocations has given it an original axiomatic justification, however. Moreover, it is clear that Proposition 1 does allow for different ways of making interpersonal comparisons, since it does not fix the monotone path (just as money-metric utilities depend on the choice of reference prices). The recent literature has shown that there may be good ethical reasons to choose a specific monotone path (Fleurbaey and Maniquet 2011). Although some open questions remain, attractive solutions have been found for specific policy environments. One example (for health) will be described in the next subsection.

B. *Social Priorities*

We now move beyond interpersonal comparisons and consider the issue of evaluating social states. To obtain an intuitive identification of the monotone path, we work with a specific model in which the two relevant life dimensions are health and consumption (we follow Fleurbaey 2005). An individual situation is $\mathbf{x}_i = (c_i, h_i) \in \mathbb{R}_+ \times [0, 1]$. We keep the same assumptions about individual preferences R_i: they are assumed to be complete, monotonic, and transitive. The fixed population is $\mathcal{N} = \{1, \ldots, n\}$ and an allocation is denoted $\mathbf{x}_N = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$. The ranking of allocations from the point of view of social welfare will be denoted $\mathbf{R}$ (with asymmetric and symmetric parts $\mathbf{P}$ and $\mathbf{I}$), and will be assumed to be reflexive and transitive. Since we want this social ranking to depend on the profile of individual preferences $R_N = (R_1, \ldots, R_n)$, it is really a function $\mathbf{R}(R_N)$, but the argument will often be dropped to shorten notation.

It will turn out that one specific case of the equivalence approach will play an essential role in what follows. It is obtained by choosing the monotone path $\mathcal{B}$ as the set of all points in $\mathcal{X}$ with $h = 1$.[4] The *healthy-equivalent income* is then the quantity $E(\mathbf{x}_i, R_i)$, implicitly defined by the condition

$$\big(E(\mathbf{x}_i, R_i),\, 1\big)\, I_i(c_i, h_i), \tag{1}$$

i.e., it is the level of consumption that, combined with perfect health, would make the individual indifferent with his current situation (c_i, h_i). The concept is illustrated in Figure 3.

Fleurbaey (2005) then formulates three requirements that can be imposed on the social ranking $\mathbf{R}(R_N)$. Universal quantifiers are omitted whenever the meaning of the axiom is clear.

[4] This set does not include 0, and one can add the bundles such that $c = 0$ to have a full path. But the result recalled below only deals with bundles that are at least as good as $(0, 1)$ for every individual.

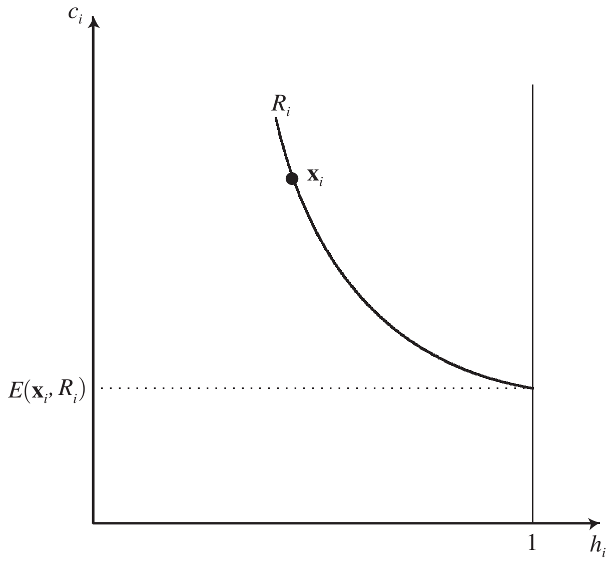

Figure 3. The Healthy-Equivalent Income

Weak Pareto: If for all i, $\mathbf{x}_i \, P_i \, \mathbf{x}_i'$, then $\mathbf{x}_N \, \mathbf{P}(R_N) \mathbf{x}_N'$.

Independence: If for all i, and all $\mathbf{q} \in \mathcal{X}$, $\mathbf{x}_i I_i \mathbf{q} \Leftrightarrow \mathbf{x}_i I_i' \mathbf{q}$ and $\mathbf{x}_i' I_i \mathbf{q} \Leftrightarrow \mathbf{x}_i' I_i' \mathbf{q}$, then $\mathbf{x}_N \mathbf{R}(R_N) \mathbf{x}_N'$ if and only if $\mathbf{x}_N \mathbf{R}(R_N') \mathbf{x}_N'$.

Pigou-Dalton: If there is i, j such that $h_i = h_j$ and $(c_i, h_i) = (c_i' - \delta, h_i') > (c_j' + \delta, h_j') = (c_j, h_j)$ for some $\delta > 0$ while $\mathbf{x}_k' = \mathbf{x}_k$ for all $k \neq i, j$, then $\mathbf{x}_N \mathbf{R}(R_N) \mathbf{x}_N'$ provided that either $R_i = R_j$ or $h_i = h_j = 1$.

The first axiom is standard. The second one defines the informational setting: it states that the social ranking of two allocations $\mathbf{x}$ and $\mathbf{x}'$ should be based only on information concerning the shape of the indifference curves through $\mathbf{x}_i$ and $\mathbf{x}_i'$ for all individuals i. This allows for richer information than Arrow's impossibility theorem, which would only consider individual pairwise preferences over $\mathbf{x}$ and $\mathbf{x}'$. However, it also implies—in line with the preference principle—that information on subjective welfare levels (i.e., the cardinalization of the utility function) is irrelevant.

The third axiom introduces some egalitarianism in the space of resources. At first sight one could think that it would make sense to impose the restriction that a transfer of consumption from the rich to the poor increases social welfare, under the condition that the rich and the poor are at the same health level. However, Fleurbaey and Trannoy (2003) have shown that this requirement is incompatible with the Pareto condition. The third condition above therefore restricts the application of resource transfers to individuals with the same preferences, or that have *perfect* health. This latter point is particularly important. The idea is that if two individuals are both perfectly healthy, then preferences "should not matter" in determining the desirability of an income transfer. With two individuals at the same mediocre health level, it may be legitimate for the richer to claim that he is in a worse situation when he cares more for his health. This reasoning is not at all convincing, however, if he is in perfect health.

Fleurbaey (2005) then derived the following result:

PROPOSITION 2: (Fleurbaey 2005) *If the social ordering* $\mathbf{R}(\cdot)$ *satisfies Weak Pareto*, *Independence*, *and Pigou-Dalton*, *then* $\mathbf{x}_N\,\mathbf{P}(R_N)\mathbf{x}'_N$ *whenever* $\min_i E(\mathbf{x}_i, R_i) > \min_i E(\mathbf{x}'_i, R_i)$.

There are two noteworthy features about Proposition 2. First, taking perfect health as the reference in the Pigou-Dalton condition is one example of how ethical considerations can supplement the finding of Proposition 1. Indeed, imposing this condition "fixes" the choice of the monotone path that we described in the previous subsection—as illustrated in Figure 3, the healthy-equivalent income is one specific case of the equivalence approach.

Observe that any alternative path would contain situations with less than perfect health, implying that, at such situations, the evaluation of well-being does not depend on preferences. This would be very questionable, because it is intuitive that when two individuals have the same bundle (c, h) with $h < 1$, the individual who cares more about health is worse off. The healthy-equivalent income is independent of health-consumption preferences only for healthy individuals, which seems much more acceptable.

Second, although we did not impose an extreme form of egalitarianism, the combination of the independence and Pigou-Dalton conditions imposes the maximin rule. This is reminiscent of a criticism that was raised against money-metric utilities by Blackorby and Donaldson (1988), namely, that in general they do not yield a quasi-concave social ordering over allocations. This problem disappears when the social ranking follows the maximin (or leximin) criterion. More on this can be found in Fleurbaey and Maniquet (2011).

II. Behavioral Economics: Shaking Preferences?

The equivalent income approach, as described in the previous section, crucially depends on preferences. More specifically, the previous results are based on the assumption that every individual has a well-defined complete preference ordering. While this has been the traditional assumption in welfare economics, it has been shaken by recent findings from behavioral economics. It is not our point here to give a complete overview of all behavioral anomalies that have been described in the literature, as there exist by now a lot of survey papers. Referring to just one of these that focuses on evidence from the field (DellaVigna 2009), one can distinguish *nonstandard preferences* (self-control problems in an intertemporal setting; the influence of default options and the endowment effect), *nonstandard beliefs* (economic agents overestimate their performance in tasks requiring ability, they expect small samples to exhibit large-sample statistical properties and they project their current preferences onto future periods) and *nonstandard decision making* (the neglect or overweighting of information because of limited attention; the use of suboptimal heuristics for choices out of menu sets; excess impact of others' beliefs; the possibly important role played by emotions such as mood and arousal). The findings from this literature suggest that preferences may not be well behaved—and that, even if standard preferences did exist, choice behavior cannot in any case be interpreted as the simple maximization of a fixed preference ordering. This raises difficult challenges for welfare economics.

One popular reaction in the behavioral literature has been to go back to experience utility (Frey and Stutzer 2002; Kahneman, Wakker, and Sarin 1997; Kahneman and Sugden 2005; Kőszegi and Rabin 2008; Layard 2005). The intuition behind this is that if people make mistakes, "decision utility" (the perceived utility on which decisions are based) and "experience utility" (the real after-decision utility) no longer coincide, and that in these circumstances it is better from the welfare point of view to focus on the "correct" outcomes. Yet, this move back to welfarism is a very controversial approach. In particular, subjective utility comparisons across individuals or even for a same individual at different dates are problematic, when the levels of utility to be compared involve different standards of evaluation. For instance, the subjective satisfaction of a given population may appear stable over time in spite of their judging that their situation has greatly improved, just because their standards of evaluation evolve with their situation, a phenomenon known as adaptation (or as the aspiration treadmill). Therefore ranking individual situations on the basis of happiness does not respect individual preferences (see also Bernheim 2009; Loewenstein and Ubel 2008). In particular, it may violate the preference principle introduced in the previous section.

The alternative approach is to keep preferences as the ultimate criterion for evaluating social states, but to take into account that the preference relation that can be derived from behavior is not standard if choices (or stated preferences)[5] are conflicting and context-dependent. An interesting way to model context-dependency has been proposed by Bernheim and Rangel (2009) and Salant and Rubinstein (2008).[6] They introduce the concept of a generalized choice situation $(\mathcal{A}, d)$, where $\mathcal{A}$ is the set of elements from which a choice has to be made and d is an "ancillary condition" (in the terminology of Bernheim and Rangel 2009) or a "frame" (in the terminology of Salant and Rubinstein 2008). A standard choice situation would be fully characterized by $\mathcal{A}$. Ancillary conditions (or frames) influence decisions but are (by definition) irrelevant for welfare. Examples of frames could be the specification of a default option or circumstances which lead to emotional arousal.[7] The set of all generalized choice situations of interest is given by $\mathbb{C}$. The choice-correspondence for individual i is then given by $C_i(\mathcal{A}, d) \subseteq A$ for all $(\mathcal{A}, d) \in \mathbb{C}$. Its interpretation is obvious: $\mathbf{x} \in C_i(\mathcal{A}, d)$ is an object that individual i may choose when facing

[5] Following the literature, we focus on choice in this part of the paper, but there is no reason to ignore other sources of data on preferences, such as stated preferences.

[6] Another interesting framework in terms of binary relations over swaps is proposed by Gustafsson (2011).

[7] Bernheim and Rangel (2009) and Salant and Rubinstein (2008) give many examples on how to cast the behavioral anomalies from the literature in the mold of generalized choice situations.

$(\mathcal{A}, d)$. The aim of this model is to make it possible that $C_i(\mathcal{A}, d) \neq C_i(\mathcal{A}, d')$. This is a key way in which behavioral "anomalies" can be integrated in this framework.[8]

While this is a convenient model, the literature disagrees about how to perform welfare analysis in this context. A first branch of the literature (Choi et al. 2003; Dalton and Ghosal 2010; Rubinstein and Salant 2012) considers that individuals do have "authentic" preferences that are well behaved, and that behavioral anomalies are just mistakes around this core preference ordering. If one has observations of individual behavior in different generalized choice situations, one can seek to derive information about authentic individual preferences over $\mathcal{A}$. The methodology is to apply a structural model of behavior to explain the observations $C_i(\mathcal{A}, d)$, i.e., to model how preferences together with frames determine choice. This structural model can then be used to derive a preference relation that is consistent with observed behavior conditional on the model used.

In principle, this first approach makes it possible to keep the concepts of welfare economics untouched. Once people's authentic preferences are estimated, one simply has to apply the standard criteria (e.g., cost-benefit analysis, or the criteria introduced in the previous section) to such preferences. Behavioral complications then interfere with the estimation of preferences and the prediction of the behavioral effects of policy interventions, but not with the application of welfare concepts. For instance Choi et al. (2003) and Carroll et al. (2009) consider a population afflicted with hyperbolic discounting in the choice of pension plans, and they eliminate present bias in the application of welfare criteria, while retaining hyperbolic discounting in the estimation of preferences and the analysis of behavior. This methodology is discussed in Beshears et al. (2008).

A difficulty with this approach is that the revealed preference relation is generally not identified precisely. In particular, it will depend on the specific behavioral model that is applied—and, very often it is difficult to identify the correct model from the observations, in the sense that the outcomes of two different behavioral models (with different underlying preference relations) are observationally equivalent (in terms of choices).[9]

As a matter of principle, one can even doubt that individuals "authentically" have a complete preference relation over all possible lives. Indeed, this would imply that they can order states with which they are not at all familiar. The psychological uncertainty about preferences may be expected to grow when one goes further away from the actual situation. To calculate, e.g., healthy-equivalent incomes, we need nonlocal information on the indifference curve. Is someone who has been chronically ill for a long time (or is handicapped since birth) able to evaluate trade-offs in a situation of (nearly) perfect health?[10]

It seems therefore unavoidable that welfare economics has to work with incomplete preference relations. As it turns out, this is also the key feature of the second

[8] Even without frame dependence, a choice mapping of the form $C(\mathcal{A})$ can accommodate menu dependence, as well as pairwise intransitive choice.

[9] See, e.g., Bernheim, Fradkin, and Popov (2011), for a comparison of four behavioral models of choices of pension plans.

[10] Note that we are referring here to ordinal preferences and not to the effect of adaptation leading to smaller changes in subjective satisfaction levels.

branch of the literature, to which we now turn. Bernheim and Rangel (2009) propose what they call a "libertarian" approach, because it only uses information about choices.[11] On this basis they define a series of incomplete welfare relations. The most attractive (and the one with which they themselves work extensively) is

$$\mathbf{x} P_i^* \mathbf{y} \quad \text{if and only if} \quad \text{for all } (\mathcal{A}, d) \in \mathbb{C} \text{ such that } \mathbf{x}, \mathbf{y} \in \mathcal{A}, \quad \text{one has } \mathbf{y} \notin C_i(\mathcal{A}, d).^{12}$$

Bernheim and Rangel (2009) have a counterexample showing that P_i^* is not necessarily transitive, but they show that P_i^* is acyclic. Imposing more structure on the space of alternatives may lead to P_i^* being transitive and, in fact, for almost all popular behavioral approaches, P_i^* is indeed transitive.

Bernheim and Rangel (2009) emphasize that their approach is only choice-based and does not assume the existence of an underlying preference relation. But their formalism is compatible with a variety of interpretations. As a matter of fact, they are well aware that, in a purely choice-based approach the relation P^* can be very coarse—and that in some generalized choice situations it is highly unlikely that a choice reveals something about welfare. The foreigner who is killed in a car accident in London, because he forgot to look right when crossing the road, does not reveal that he preferred being killed (unless there is other evidence in his life suggesting that he had suicidal tendencies). They therefore consider the possibility of refining P^* by using nonchoice information to discard information from some generalized choice situations as "suspect".[13] This move obviously goes some way in the direction of the first branch of the literature that seeks to elicit authentic preferences. For our purposes the common implication of the two approaches is the key insight: in most cases, the analyst (or the policymaker) will have to work with incomplete preferences.

For applied welfare analysis, Bernheim and Rangel (2009) introduce natural counterparts of the concepts of compensating and equivalent variation. We will focus on the former. Let us assume that the generalized choice situation can be written as $((\mathcal{A}(\boldsymbol{\alpha}, m), d)$, where $\boldsymbol{\alpha}$ is a vector of environmental parameters and m is a monetary transfer. Let us then consider a move from $((\mathcal{A}(\boldsymbol{\alpha}_0, 0), d_0)$ to $((\mathcal{A}(\boldsymbol{\alpha}_1, m), d_1)$. The compensating variation is the smallest value of m, such that for any $\mathbf{x} \in C(\mathcal{A}(\boldsymbol{\alpha}_0, 0), d_0)$ and $\mathbf{y} \in C(\mathcal{A}(\boldsymbol{\alpha}_1, m), d_1))$ the individual would be willing to choose $\mathbf{y}$ over $\mathbf{x}$. In a setting with incomplete preferences, the latter sentence is ambiguous, however. We can consider the compensation to be sufficient

[11] As already explained, one could also use additional information coming from stated preferences. The main point is to avoid making structural assumptions about how choice or preference behavior is determined.

[12] One can add the condition that there is at least one $(\mathcal{A}, d) \in \mathbb{C}$ such that $\mathbf{x}, \mathbf{y} \in \mathcal{A}$ for which $\mathbf{x} \in C_i(\mathcal{A}, d)$. This condition is always satisfied in Bernheim and Rangel (2009), because they assume that $(\{\mathbf{x}, \mathbf{y}\}, d) \in \mathbb{C}$ and the individual always selects some alternative in each generalized choice situation.

[13] Note that some ancillary conditions can be discarded even if they do not unambiguously involve biases, just because they are ethically unappealing. One example could be the importance of the reference situation—it is well known that people tend to focus on changes (gains and losses) rather than on the resulting final states and, moreover, that the feeling of loss looms larger than the feeling of gain. People will give larger subjective weight to avoiding the former than to experiencing the latter. One could argue that from the ethical point of view the status quo position should play a less prominent role, and definitely so if one is concerned about evaluating redistribution measures that are to the advantage of the poorest in society, i.e., where the rich lose and the poor gain. Of course, such an ethical argumentation may not necessarily be accepted by everyone.

when the new situation is unambiguously chosen over the old one, or when the old situation is not unambiguously chosen over the new one. This leads to two notions of compensating variation. The first, CV^{high}, is equal to

$$\inf\{m \,|\, \mathbf{y}\, P^{*} \mathbf{x} \text{ for all } m' \geqslant m, \mathbf{x} \in C(\mathcal{A}(\boldsymbol{\alpha}_0, 0), d_0) \text{ and } \mathbf{y} \in C(\mathcal{A}(\boldsymbol{\alpha}_1, m'), d_1)\}.$$

The second, CV^{low}, is equal to

$$\sup\{m \,|\, \mathbf{x}\, P^{*} \mathbf{y} \text{ for all } m' \leqslant m, \mathbf{x} \in C(\mathcal{A}(\boldsymbol{\alpha}_0, 0), d_0) \text{ and } \mathbf{y} \in C(\mathcal{A}(\boldsymbol{\alpha}_1, m'), d_1)\}.$$

It is easy to see that $CV^{high} \geq CV^{low}$.

In a setting with several individuals, a move from $\mathbf{x}$ to $\mathbf{y}$ is a Pareto improvement if $\mathbf{y}\, P_i^{*}\, \mathbf{x}$ for all i. If we do not define an interpersonally comparable concept of well-being, policy analysis remains restricted to looking for such Pareto improvements. These will be very rare indeed[14] and this usually motivates the use of the sum of compensating variations. While the compensating variation yields a specific measure of the welfare change for one individual, it is well known that simply adding compensating variations is not an acceptable welfare criterion if one cares about the distribution and if one wants to avoid cyclic decisions (Blackorby and Donaldson 1990). A setting with incomplete preferences does nothing to alleviate this criticism. This is why better measures of well-being, which allow for interpersonal comparisons, are needed, such as those studied in the previous section. As mentioned there, the equivalence approach is closely related to the concept of money-metric utility. In the next section we will show how the idea of upper and lower bounds just presented appears as the natural way to extend the equivalence approach to a setting with incomplete preferences.

The economic models that were introduced in the previous sections put more structure on the decision problem than the abstract and general approach of Bernheim and Rangel (2009). Let us therefore conclude this section by describing the form taken by preference relations in our approach.

We assume that individual preferences take the form of *partial* binary relations P^{*} defined on the set of relevant life dimensions $\mathcal{X}$, with $\mathcal{X} = \mathbb{R}_{+}^{\ell}$. Examples of life dimensions could be consumption, health status, job satisfaction, quality of interpersonal relations, etc. The expression $\mathbf{x} P^{*} \mathbf{y}$ means that $\mathbf{x}$ is strictly preferred to $\mathbf{y}$. We assume that P^{*} is transitive ($\mathbf{x} P^{*} \mathbf{y}$ and $\mathbf{y} P^{*} \mathbf{z}$ implies $\mathbf{x} P^{*} \mathbf{z}$) and irreflexive ($\mathbf{x} P^{*} \mathbf{x}$ for no $\mathbf{x} \in \mathcal{X}$). As noted before, transitivity is not a very strong requirement. In our setting it also makes sense—certainly as a first approach—to assume that preferences are monotonic ($\mathbf{x} > \mathbf{y}$ implies $\mathbf{x} P^{*} \mathbf{y}$) and continuous. We define continuity as meaning that the sets

$$UC(\mathbf{x}, P^{*}) = \{\mathbf{q} \in \mathcal{X} \,|\, \mathbf{q} P^{*} \mathbf{x}\}$$

[14] See Mandler (2012) for a thorough discussion of this problem. He shows that the introduction of a small amount of preference diversity across frames can in some examples cause every allocation to be Pareto optimal, and argues that this indecisiveness can only be resolved by introducing some notion of interpersonally comparable well-being.

and

$$LC(\mathbf{x},\, P^*) \;=\; \{\mathbf{q} \in \mathcal{X} | \mathbf{x} P^* \mathbf{q}\}$$

are open subsets of $\mathcal{X}$, and in addition $\mathbf{x} \in \partial UC(\mathbf{y},\, P^*)$ if and only if $\mathbf{y} \in \partial LC(\mathbf{x}, P^*)$, where $\partial UC(\cdot)$ denotes the lower boundary of $UC(\cdot)$ and $\partial LC(\cdot)$ the upper boundary of $LC(\cdot)$.[15] Let

$$NC(\mathbf{x},\, P^*) = \{\mathbf{q} \in \mathcal{X} \mid \text{neither } \mathbf{q} P^* \mathbf{x} \text{ nor } \mathbf{x} P^* \mathbf{q}\}$$

be the set of vectors which are not comparable to $\mathbf{x}$ by P^*. Continuity is important for our analysis in order to make sure that when $\mathbf{y}$ is in the interior of $NC(\mathbf{x},\, P^*)$, there is a refinement $\overline{P}^* \supsetneq P^*$ such that $\mathbf{y} \overline{P}^* \mathbf{x}$ and another $\overline{P}^{*\prime} \supsetneq P^*$ such that $\mathbf{x} \overline{P}^{*\prime} \mathbf{y}$.[16]

Under monotonicity and transitivity, $UC(\mathbf{x},\, P^*)$ is upper comprehensive, i.e., if $\mathbf{q} \in UC(\mathbf{x},\, P^*)$, and $\mathbf{q}' > \mathbf{q}$, then $\mathbf{q}' \in UC(\mathbf{x},\, P^*)$. Similarly, $LC(\mathbf{x},\, P^*)$ is lower comprehensive, i.e., if $\mathbf{q} \in LC(\mathbf{x},\, P^*)$ and $\mathbf{q}' < \mathbf{q}$, then $\mathbf{q}' \in LC(\mathbf{x},\, P^*)$.

Under these conditions, it is therefore enough to know $NC(\mathbf{x},\, P^*)$ in order to know $UC(\mathbf{x},\, P^*)$ and $LC(\mathbf{x},\, P^*)$. One has

$$UC(\mathbf{x},\, P^*) = \{\mathbf{q} \in \mathcal{X} \mid \mathbf{q} \notin NC(\mathbf{x},\, P^*) \text{ and } \exists \mathbf{q}' \in NC(\mathbf{x},\, P^*),\ \mathbf{q} > \mathbf{q}'\}$$

$$LC(\mathbf{x},\, P^*) = \{\mathbf{q} \in \mathcal{X} \mid \mathbf{q} \notin NC(\mathbf{x},\, P^*) \text{ and } \exists \mathbf{q}' \in NC(\mathbf{x},\, P^*),\ \mathbf{q} < \mathbf{q}'\}.$$

If $\mathbf{x} P^* \mathbf{y}$, then $LC(\mathbf{y},\, P^*) \subsetneq LC(\mathbf{x},\, P^*)$, as we now show. First, one cannot have $LC(\mathbf{x},\, P^*) = LC(\mathbf{y},\, P^*)$ because $\mathbf{y} \in LC(\mathbf{x},\, P^*)$ but $\mathbf{y} \notin LC(\mathbf{y},\, P^*)$. Second, suppose that $LC(\mathbf{y},\, P^*) \subseteq LC(\mathbf{x},\, P^*)$ does not hold. Let $\mathbf{z} \in LC(\mathbf{y},\, P^*) \backslash LC(\mathbf{x},\, P^*)$. One has $\mathbf{y} P^* \mathbf{z}$, which by transitivity implies $\mathbf{x} P^* \mathbf{z}$ and therefore $\mathbf{z} \in LC(\mathbf{x},\, P^*)$, a contradiction. Similarly, one shows that if $\mathbf{x} P^* \mathbf{y}$, then $UC(\mathbf{x},\, P^*) \subsetneq UC(\mathbf{y}, P^*)$.

In the next sections, it will be useful to address the following question. Consider two preferences $P^*, P^{*\prime}$ and two sets of points $\mathcal{Q}, \mathcal{Q}'$. Under what conditions does there exist a preference $P^{*\prime\prime}$, such that for all $\mathbf{q} \in \mathcal{Q}$, $NC(\mathbf{q},\, P^{*\prime\prime}) = NC(\mathbf{q},\, P^*)$ and for all $\mathbf{q}' \in \mathcal{Q}'$, $NC(\mathbf{q}',\, P^{*\prime\prime}) = NC(\mathbf{q}',\, P^{*\prime})$? A sufficient condition is that for all

[15] Under transitivity and monotonicity, assuming that $UC(\mathbf{x},\, P^*)$ and $LC(\mathbf{x},\, P^*)$ are open guarantees that the graph of P^* is open in $\mathcal{X}^2$ (see Bergstrom, Parks, and Rader 1976, theorem 1; Gerasimou forthcoming, theorem 5), which is the usual notion of continuity for incomplete preferences. But this does not prevent some forms of discontinuity, namely, the occurence of "poles" at some points: one could have a sequence $\mathbf{x}_n \to \mathbf{x}$ such that $y \in LC(\mathbf{x}_n,\, P^*)$ for all n but $\mathbf{y}$ belongs to the interior of $\mathcal{X} \backslash LC(\mathbf{x},\, P^*)$. Under the additional condition that $\mathbf{x} \in \partial UC(\mathbf{y},\, P^*)$ if and only if $\mathbf{y} \in \partial LC(\mathbf{x},\, P^*)$, such phenomenon cannot occur because $\mathbf{x}$ is in the interior of $NC(\mathbf{y},\, P^*)$ if and only if $\mathbf{y}$ is in the interior of $NC(\mathbf{x},\, P^*)$. Indeed, assume that $\mathbf{x}$ is in the interior of $NC(\mathbf{y},\, P^*)$. Then one cannot have $\mathbf{y} \in \partial NC(\mathbf{x}, P^*)$, which would require either $\mathbf{y} \in \partial UC(\mathbf{x},\, P^*)$ or $\mathbf{y} \in \partial LC(\mathbf{x},\, P^*)$. And one cannot have $\mathbf{y} \notin NC(\mathbf{x},\, P^*)$, which would require either $\mathbf{y} \in UC(\mathbf{x},\, P^*)$ or $\mathbf{y} \in LC(\mathbf{x},\, P^*)$.

[16] For instance, in $\mathbb{R}^2_+$, let $\mathbf{x} P^* \mathbf{y}$ if and only if either $x_1 + x_2 > \max\{y_1 + y_2, 1\}$, or $x_1 + x_2 \leq 1$, $y_1 + y_2 \leq 1$, $0.6x_1 + 0.4x_2 > 0.6y_1 + 0.4y_2$, and $0.4x_1 + 0.6x_2 > 0.4y_1 + 0.6y_2$. This P^* has an open graph but is not continuous in our sense. One has $(0.45, 0.45) \in NC((1,0), P^*)$—it is even in the interior—but it is impossible to find a refinement $\overline{P}^* \supsetneq P^*$ such that $(0.45, 0.45)\, \overline{P}^* (1,\, 0)$, because for any small positive epsilon, $(1, \varepsilon)\, P^* (0.45, 0.45)$ and therefore $(1, \varepsilon)\, \overline{P}^* (0.45, 0.45)$, which is incompatible with the continuity of $\overline{P}^*$ if $(0.45, 0.45)\, \overline{P}^* (1,\, 0)$.

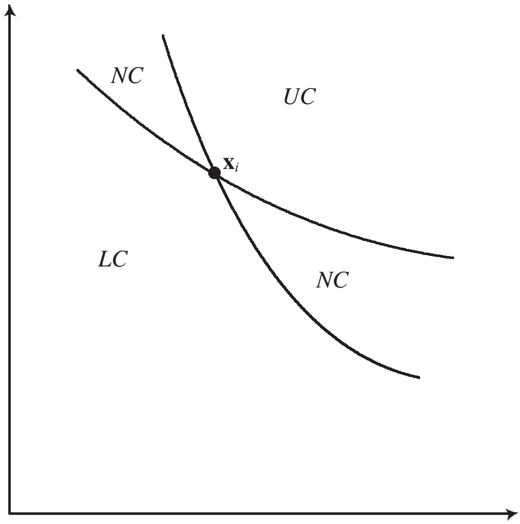

FIGURE 4. INDIFFERENCE CURVES FOR INCOMPLETE PREFERENCES

$\mathbf{q} \in \mathcal{Q}$, $\mathbf{q}' \in \mathcal{Q}'$, $NC(\mathbf{q}, P^*) \cap NC(\mathbf{q}', P^{*\prime}) = \varnothing$. This condition is an extension of the notion of noncrossing indifference curves.

The resulting indifference curves in the two-dimensional case are represented in Figure 4. For convenience, we have drawn them in a strictly convex way, but convexity is not necessary for our analysis. Note that our assumptions imply that individuals have finer preferences when comparing close alternatives: this seems a very natural assumption.

III. Interpersonal Comparisons with Incomplete Preferences

As in Section II, we take two steps in our extension of the equivalence approach to a setting with incomplete preferences. In this section, we consider the issue of interpersonal comparisons, i.e., of ranking personal situations $(\mathbf{x}, P^*)$ in terms of well-being. The object to be constructed is a binary relation on such situations, that is denoted $\succcurlyeq$ (with asymmetric and symmetric parts $\succ$, $\sim$) and is required to be reflexive and transitive (i.e., it is a pre-ordering), but not necessarily complete. In the next section, we will turn to the problem of evaluating social states.

The axioms and concepts from Section II can be adapted in a straightforward way:

Preference principle: $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^*)$ if $\mathbf{x} P^* \mathbf{x}'$.

Dominance principle: $(\mathbf{x}, P^*) \succcurlyeq (\mathbf{x}', P^{*\prime})$ if $\mathbf{x} \geq \mathbf{x}'$; $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ if $\mathbf{x} \gg \mathbf{x}'$.

Restricted dominance principle: For all $\mathbf{x}, \mathbf{x}' \in \mathcal{B}$, $(\mathbf{x}, P^*) \succcurlyeq (\mathbf{x}', P^{*\prime})$ if $\mathbf{x} \geq \mathbf{x}'$; $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ if $\mathbf{x} \gg \mathbf{x}'$.

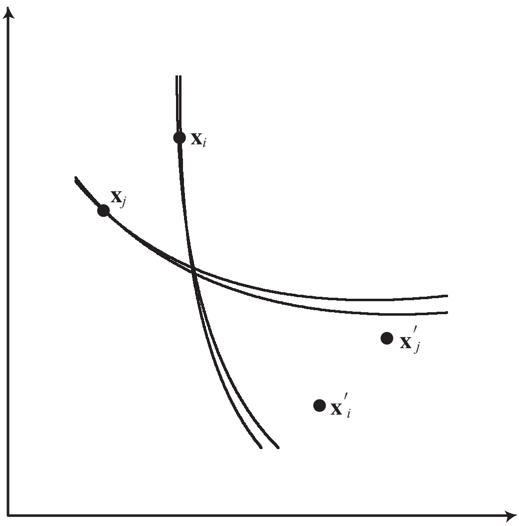

Figure 5. Incompatibility of the Preference and the Dominance Principles with Incomplete Preferences

It is easy to see that the incompatibility between the preference principle and the dominance principle extends to the case of incomplete preferences. The latter principle now implies that $(\mathbf{x}, P^*) \sim (\mathbf{x}, P^{*\prime})$ for all $\mathbf{x}$ and all $P^*, P^{*\prime}$, again making it impossible to take account of preferences. And the incompatibility between the preference principle and the second part of the dominance principle is illustrated by Figure 5, which is an obvious extension of the configuration of Figure 1.

However, as in Section II, there is no incompatibility between the restricted dominance principle and the preference principle. In fact, in the current framework, imposing both principles imply that the ordering $\succcurlyeq$ displays some important aspects of the equivalence approach.[17] To simplify the exposition, let $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*)$ denote the lowest element of $NC(\mathbf{x}, P^*) \cap \mathcal{B}$ and $E_{\mathcal{B}}^{\sup}(\mathbf{x}, P^*)$ the greatest element. These are well defined when $\mathcal{B}$ is a monotone path.

PROPOSITION 3: *Let $\mathcal{B}$ be a subset of $\mathcal{X}$ such that for all $(\mathbf{x}, P^*)$, $NC(\mathbf{x}, P^*) \cap \mathcal{B} \neq \varnothing$. If $\succcurlyeq$ satisfies the preference principle and the restricted dominance principle with respect to $\mathcal{B}$, then $\mathcal{B}$ is a monotone path and $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ whenever $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*) > E_{\mathcal{B}}^{\sup}(\mathbf{x}', P^{*\prime})$.*

It is useful to compare this result with the one that was given in Proposition 1 for the case of complete preferences. In the latter case, the relevant information was given by the intersection of B with the indifference sets. Proposition 3 extends this idea in a natural way: with incomplete preferences, the equivalence approach gathers the preorderings $\succcurlyeq$ such that how to rank $(\mathbf{x}, P^*)$ and $(\mathbf{x}', P^{*\prime})$ is fully determined by $NC(\mathbf{x}, P^*) \cap \mathcal{B}$ and $NC(\mathbf{x}', P^{*\prime}) \cap \mathcal{B}$. Proposition 3 remains conspicuously silent

[17] The proofs of all the following propositions are given in the Appendix.

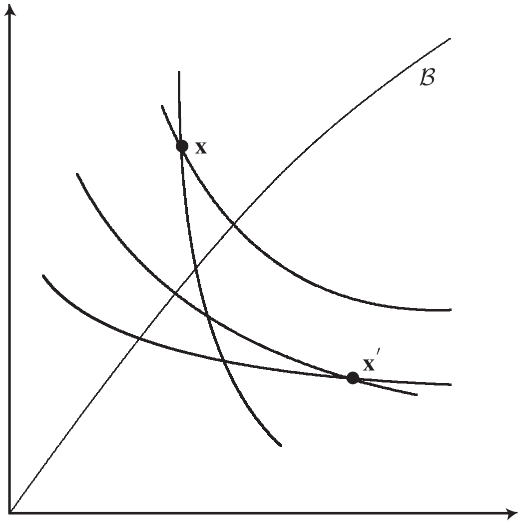

Figure 6. Case A

about how to rank situations in which $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*) \leq E_{\mathcal{B}}^{\sup}(\mathbf{x}', P^{*\prime})$.[18] As an example, compare the three cases in Figures 6–8. Proposition 3 enables us to say that $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ in case A—but it does not imply a similar conclusion in cases B and C.

The partial relation $\succ$ obtained in Proposition 3 can be made less incomplete when preferences P^* and $P^{*\prime}$ are refined. This is the route explored in the last section of Bernheim and Rangel's (2009) paper. Here we would like to propose an additional strategy, which makes it possible to refine the ordering $\succ$ without refining individual preferences. This strategy may be useful when refining individual preferences is not possible, or if after refinement efforts individual preferences still remain substantially incomplete.

Consider the idea that one should avoid ranking an individual as better off than another individual when the available information about his situation is compatible with his being unambiguously worse off. This intuition is captured by the following safety principle:

Safety principle: $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ if there exists $\overline{P^{*\prime}} \supseteq P^{*\prime}$ such that for all $\overline{P^*} \supseteq P^*$, $(\mathbf{x}, \overline{P^*}) \succ (\mathbf{x}', \overline{P^{*\prime}})$.

Specifically, the safety principle says that if a refinement of one individual's preferences may reveal him to be worse off than in the original situation, then we should

[18] More formally, assuming that $\mathcal{B}$ is a monotone path, every ranking such that: (i) $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ whenever $LC(\mathbf{x}, P^*) \cap UC(\mathbf{x}', P^{*\prime}) \cap \mathcal{B} \neq \varnothing$; (ii) $(\mathbf{x}\ P^*) \sim (\mathbf{x}, P^{*\prime})$ whenever $\mathbf{x} \in \mathcal{B}$; (iii) $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^*)$ whenever $\mathbf{x} P^* \mathbf{x}'$, satisfies the preference principle and the restricted dominance principle. This allows many different possible rankings when none of these three situations applies. In particular, such rankings may involve the equivalence approach w.r.t. other paths, or nonequivalence approaches.

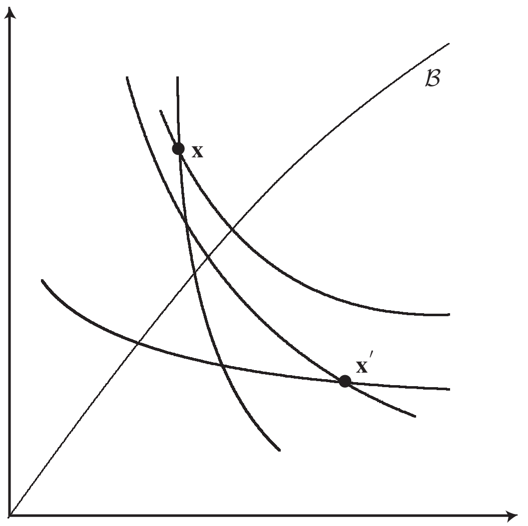

Figure 7. Case B

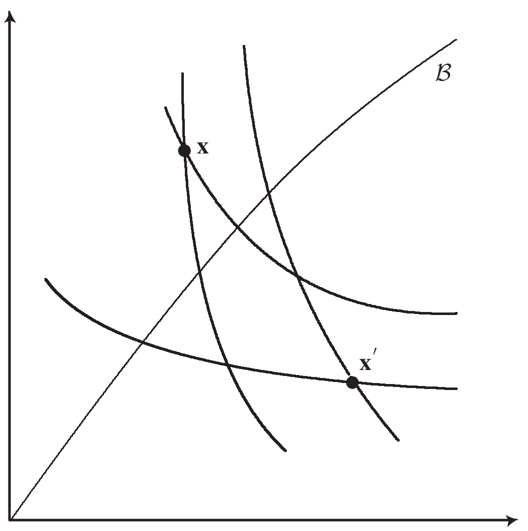

Figure 8. Case C

already consider him to be worse off in the latter. Of course, there are situations in which either individual can turn out to be worse off than the other when the information about both agents is refined. But the axiom deals with the case in which refining the information about one of them only may already determine that he is worse off. The main motivation for this axiom is that, even though it does not preclude mistakes in interpersonal comparisons, it prevents the evaluator from missing a situation in which the worse-off is really badly off. If the evaluator is wrong about the worse-off in a pairwise comparison, the true worse-off is not as badly off as he could be if the mistake was in the opposite direction. Imposing it leads to the following proposition.

PROPOSITION 4: *Let $\mathcal{B}$ be a subset of $\mathcal{X}$ such that for all $(\mathbf{x}, P^*)$, $NC(\mathbf{x}, P^*) \cap \mathcal{B} \neq \emptyset$. If $\succcurlyeq$ satisfies the preference principle and the restricted dominance principle with respect to $\mathcal{B}$, then $\mathcal{B}$ is a monotone path and, under the safety principle, $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ whenever $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*) > E_{\mathcal{B}}^{\inf}(\mathbf{x}', P^{*\prime})$.*

Returning to the examples in the figures, application of the safety principle now makes it possible to state that $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ also in cases B and C. However, case C illustrates a limitation of the safety principle. Indeed, refining $P^{*\prime}$ to $\overline{P^*}'$ could also lead to the opposite configuration—implying that $(\mathbf{x}, \overline{P^*}) \prec (\mathbf{x}', \overline{P^*}')$. This possibility is excluded by the following variant of the safety principle, which is perfectly symmetric with respect to the worse and the better situation:

Super safety principle: $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ if:

(i) there exists $\overline{P}^{*\prime} \supseteq P^{*\prime}$ such that for all $\overline{P}^* \supseteq P^*$, $(\mathbf{x}, \overline{P}^*) \succ (\mathbf{x}', \overline{P}^{*\prime})$ or there exists $\overline{P}^* \supseteq P^*$ such that for all $\overline{P}^{*\prime} \supseteq P^{*\prime}$, $(\mathbf{x}, \overline{P}^*) \succ (\mathbf{x}', \overline{P}^{*\prime})$;

(ii) there exists no $\overline{P}^{*\prime} \supseteq P^{*\prime}$ such that for all $\overline{P}^* \supseteq P^*$, $(\mathbf{x}, \overline{P}^*) \prec (\mathbf{x}', \overline{P}^{*\prime})$ and there exists no $\overline{P}^* \supseteq P^*$ such that for all $\overline{P}^{*\prime} \supseteq P^{*\prime}$, $(\mathbf{x}, \overline{P}^*) \prec (\mathbf{x}', \overline{P}^{*\prime})$.

Imposing super safety yields the following proposition.

PROPOSITION 5: *Let $\mathcal{B}$ be a subset of $\mathcal{X}$ such that for all $(\mathbf{x}, P^*)$, $NC(\mathbf{x}, P^*) \cap \mathcal{B} \neq \emptyset$. If $\succcurlyeq$ satisfies the preference principle and the restricted dominance principle with respect to $\mathcal{B}$, then $\mathcal{B}$ is a monotone path and, under the super safety principle, $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$ whenever $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*) \geq E_{\mathcal{B}}^{\inf}(\mathbf{x}', P^{*\prime})$ and $E_{\mathcal{B}}^{\sup}(\mathbf{x}, P^*) \geq E_{\mathcal{B}}^{\sup}(\mathbf{x}', P^{*\prime})$, with at least one strict inequality.*

Returning to the cases in Figures 6–8, we can now still draw conclusions in case B, but no longer in case C.

Let us further illustrate the interpretation of Propositions 3–5 for the specific case in which the two relevant life dimensions are health and consumption, i.e., an individual situation is $\mathbf{x}_i = (c_i, h_i) \in \mathbb{R}_+ \times [0, 1]$. We have argued before that in this case there are good reasons to choose a specific monotone path B, leading in the case of complete preferences to the concept of the healthy-equivalent income. When preferences are incomplete, upper and lower bounds extend this notion in a natural way. Defining

$$E^{\sup}(\mathbf{x}, P^*) = c^* \text{ such that } (c^*, 1) \in \partial UC(\mathbf{x}, P^*),$$

$$E^{\inf}(\mathbf{x}, P^*) = c^* \text{ such that } (c^*, 1) \in \partial LC(\mathbf{x}, P^*),$$

Propositions 3–5 provide the following simple operational criteria:

- (*preference principle*, *restricted dominance principle*) $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$, whenever $E^{\inf}(\mathbf{x}, P^*) > E^{\sup}(\mathbf{x}', P^{*\prime})$,
- (*preference principle*, *restricted dominance principle*, *safety principle*) $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$, whenever $E^{\inf}(\mathbf{x}, P^*) > E^{\inf}(\mathbf{x}', P^{*\prime})$,
- (*preference principle*, *restricted dominance principle*, *super safety principle*) $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$, whenever $E^{\inf}(\mathbf{x}, P^*) \geq E^{\inf}(\mathbf{x}', P^{*\prime})$ and $E^{\sup}(\mathbf{x}, P^*) \geq E^{\sup}(\mathbf{x}', P^{*\prime})$, with at least one strict inequality.

Note the close relationship with the concepts of compensating variation CV^{high} and CV^{low}, as proposed by Bernheim and Rangel (2009). However, healthy-equivalent incomes yield an interpersonally comparable measure of well-being, i.e., an evaluation of the individual's overall personal situation, and not only a monetary evaluation of a change in this personal situation. As shown in the following section, they can therefore be used for social evaluation in cases where the distribution matters.

IV. Social Evaluation with Incomplete Preferences

As in Section II, we work with a specific model in which the two relevant life dimensions are health and consumption. The fixed population is $\mathcal{N} = \{1, \ldots, n\}$ and an allocation is denoted $\mathbf{x}_N = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$. The incomplete individual preferences are denoted P_i^* and are assumed to be monotonic, transitive, irreflexive, and to satisfy the continuity property introduced at the end of Section III. The ranking of allocations from the point of view of social welfare will be denoted $\mathbf{R}$ (with asymmetric and symmetric parts $\mathbf{P}$ and $\mathbf{I}$), and will be assumed to be reflexive and transitive but not necessarily complete. Since we want this social ranking to depend on the profile of individual preferences $P_N^* = (P_1^*, \ldots, P_n^*)$, it is really a function $\mathbf{R}(P_N^*)$, but the argument will often be dropped to shorten notation.

We can easily adapt the axioms that were introduced in Section II. Universal quantifiers are omitted whenever the meaning of the axiom is clear.

Weak Pareto: If for all i, $\mathbf{x}_i P_i^* \mathbf{x}_i'$, then $\mathbf{x}_N \mathbf{P}(P_N^*) \mathbf{x}_N'$.

Independence: If for all i, $NC(\mathbf{x}_i, P_i^*) = NC(\mathbf{x}_i, P_i^{*\prime})$ and $NC(\mathbf{x}_i', P_i^*) = NC(\mathbf{x}_i', P_i^{*\prime})$, then $\mathbf{x}_N \mathbf{R}(P_N^*) \mathbf{x}_N'$ if and only if $\mathbf{x}_N \mathbf{R}(P_N^{*\prime}) \mathbf{x}_N'$.

Pigou-Dalton: If there is i,j such that $h_i = h_j$ and $(c_i, h_i) = (c_i' - \delta, h_i') > (c_j' + \delta, h_j') = (c_j, h_j)$ for some $\delta > 0$ while $\mathbf{x}_k' = \mathbf{x}_k$ for all $k \neq i,j$, then $\mathbf{x}_N \mathbf{R}(P_N^*) \mathbf{x}_N'$ provided that either $P_i^* = P_j^*$ or $h_i = h_j = 1$.

The first and third axioms are essentially the same as in subsection IIB. The second axiom now defines the informational setting in terms of the sets $NC(\mathbf{x}_i, P_i^*)$. This is the obvious extension of the corresponding axiom in the setting with complete preferences, stating that the social ranking should be based on information concerning the shape of the indifference curves through $\mathbf{x}_i$ and $\mathbf{x}_i'$ for all individuals.

We now have the following result.

PROPOSITION 6: *If the social ordering* $\mathbf{R}(\cdot)$ *satisfies Weak Pareto*, *Independence*, *and Pigou-Dalton*, *then* $\mathbf{x}_N \mathbf{P}(P_N^*)\mathbf{x}_N'$ *whenever* $\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i E^{\sup}(\mathbf{x}_i', P_i^*)$.

Proposition 6 extends the basic insights of Proposition 2 for the case of incomplete preferences. First, the combination of the axioms imposes to pick one specific choice of monotone path in the equivalence approach, i.e., to focus on the use of upper and lower bounds for the healthy-equivalent incomes. Second, it also imposes to give priority to the worst-off when ranking social states. The comparison of the worst-off across allocations appears compatible with Proposition 3, although the axiomatic route is different. Indeed, Proposition 6 can be read as saying that $\mathbf{x}_N P(P_N^*)\mathbf{x}_N'$ when for a pair i,j (which may be the same person), $E^{\inf}(\mathbf{x}_i, P_i^*) > E^{\sup}(\mathbf{x}_j', P_j^*)$, which is the sort of comparison made in Proposition 3. But, interestingly, the identification of the relevant worst-off person in the two allocations is different, as i has the lowest $E^{\inf}(\mathbf{x}_i, P_i^*)$ in $\mathbf{x}_N$ whereas j has the lowest $E^{\sup}(\mathbf{x}_j', P_j^*)$ in $\mathbf{x}_N'$.

The resulting ranking is of course very incomplete. It is possible to refine it by adding a safety axiom once again:

Super safety: If there is i and $P_i^{*\prime} \supseteq P_i^*$ such that $\mathbf{x}_N \mathbf{P}(P_i^{*\prime}, P_{N\setminus\{i\}}^{*\prime})\mathbf{x}_N'$ for all $P_{N\setminus\{i\}}^{*\prime} \supseteq P_{N\setminus\{i\}}^*$, and for no j and $P_j^{*\prime} \supseteq P_j^*$ one has $\mathbf{x}_N' \mathbf{P}(P_j^{*\prime}, P_{N\setminus\{j\}}^{*\prime})\mathbf{x}_N$ for all $P_{N\setminus\{j\}}^{*\prime} \supseteq P_{N\setminus\{j\}}^*$, then $\mathbf{x}_N \mathbf{P}(P_N^*)\mathbf{x}_N'$.

This axiom is similar to the super safety principle of the previous section. It makes it possible to refine the ordering but not in a very simple way, because the logic of refinement is quite different in the social evaluation context, as compared to interpersonal comparisons. In interpersonal comparisons, one can refine one agent's preferences without refining the other agent's preferences, therefore only one term of the comparison is altered. In the social context, refining one agent's preferences alters the evaluation of the two allocations to be compared. We can make two observations.

First, the (incomplete) relation defined by $\mathbf{x}_N \mathbf{P}(P_N^*)\mathbf{x}_N'$ if and only if $\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i E^{\inf}(\mathbf{x}_i', P_i^*)$ satisfies the four axioms. Second, one could have expected (on the basis of Proposition 5, applied to healthy-equivalent incomes) that if $\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i E^{\inf}(\mathbf{x}_i', P_i^*)$ and $\min_i E^{\sup}(\mathbf{x}_i, P_i^*) > \min_i E^{\sup}(\mathbf{x}_i', P_i^*)$, the super safety axiom, in conjunction with the other three, would imply that $\mathbf{x}_N \mathbf{P}(P_N^*)\mathbf{x}_N'$. This conjecture is wrong, however. To see this, consider a case in which there is one agent i who is far worse-off than the others, so that the evaluation depends only on his preferences. If $\mathbf{x}_i \in NC(\mathbf{x}_i', P_i^*)$, it may happen nonetheless that $E^{\inf}(\mathbf{x}_i, P_i^*) > E^{\inf}(\mathbf{x}_i', P_i^*)$ and $E^{\sup}(\mathbf{x}_i, P_i^*) > E^{\sup}(\mathbf{x}_i', P_i^*)$. This is compatible with finding $P_i^{*\prime} \supseteq P_i^*$ such that $\mathbf{x}_i P_i^{*\prime} \mathbf{x}_i'$ and $P_i^{*\prime\prime} \supseteq P_i^*$ such that $\mathbf{x}_i' P_i^{*\prime\prime} \mathbf{x}_i$. Therefore the super safety axiom has no bite in this case.[19]

[19] This example cannot occur if one assumes that every preference P^* is the intersection of a set $\mathcal{P}(P^*)$ of strict preference relations which are the asymmetric parts of monotonic, transitive, and complete relations, and that

We, however, obtain an interesting refinement, as follows.

PROPOSITION 7: *If the social ordering* $\mathbf{R}(\cdot)$ *satisfies Weak Pareto, Independence, Pigou-Dalton, and Super Safety, then* $\mathbf{x}_N \mathbf{P}(P_N^*) \mathbf{x}_N'$ *whenever* $\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i E^{\inf}(\mathbf{x}_i', P_i^*)$, *and for every j such that* $E^{\inf}(\mathbf{x}_j', P_j^*) < \min_i E^{\sup}(\mathbf{x}_i, P_i^*)$, $\mathbf{x}_j P_j^* \mathbf{x}_j'$.

The social ranking derived in this last proposition is still incomplete. Yet, the rankings obtained in this section are finer than the Pareto ranking proposed by Bernheim and Rangel (2009)—and they allow to introduce distributional considerations in welfare analysis, even if one only uses information about ordinal and noncomplete preferences. While it certainly would be worthwhile to explore further the potential contribution of imposing additional ethical requirements, the path that could be taken is clearly traced out.

V. Conclusion

We have argued in this paper that it is possible to define a concept of interpersonally comparable well-being that uses only information about ordinal preferences—even if these preferences are incomplete. Our paper therefore makes a contribution to two strands of the welfare economic literature. First, our introduction of incomplete preferences can be seen as an extension of the fair social choice approach. Second, we propose a method to define a normatively relevant concept of well-being as an extension of the Bernheim and Rangel (2009) approach to behavioral welfare economics. This makes it possible to go beyond Pareto efficiency and introduce distributional considerations into the welfare evaluation. Of course, for our approach to be meaningful it is necessary to assume that individuals do have preferences over different features of life. However, it is not necessary that these preferences are complete, nor that the analyst has perfect information about them.

The interpersonal comparisons and social rankings we derive are unavoidably incomplete. Yet, if one refines the individual preferences, one reaches the standard approach with equivalent incomes as a limiting case. Moreover, a more complete social ranking can also be obtained by imposing additional normative requirements. Further work should look for a definition of acceptable and feasible refinements—or for the development of better methods to measure preferences. As a matter of fact, any application of the approach described in this paper requires the estimation of equivalent incomes. Estimating equivalent incomes has shown to be feasible for the case of complete preferences. For that purpose one can use either happiness measures (Fleurbaey, Schokkaert, and Decancq 2009), stated preferences and

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every pair of preferences in $\mathcal{P}(P^*)$ satisfies the single-crossing property (i.e., the corresponding indifference curves cross at most once). But even under this domain restriction, the condition $\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i E^{\inf}(\mathbf{x}_i', P_i^*)$ and $\min_i E^{\sup}(\mathbf{x}_i, P_i^*) > \min_i E^{\sup}(\mathbf{x}_i', P_i^*)$ cannot be sufficient to ensure $\mathbf{x}_N \mathbf{P}(P_N^*) \mathbf{x}_N'$. For instance, one may have two agents i, j who are far worse off than the others, with $\mathbf{x}_i \in NC(\mathbf{x}_i', P_i^*)$ and

$$E^{\inf}(\mathbf{x}_i', P_i^*) < E^{\inf}(\mathbf{x}_i, P_i^*) < E^{\inf}(\mathbf{x}_j', P_j^*) < E^{\sup}(\mathbf{x}_j', P_j^*)$$
$$< E^{\sup}(\mathbf{x}_i, P_i^*) < E^{\sup}(\mathbf{x}_i', P_i^*) < E^{\inf}(\mathbf{x}_j, P_j^*) < E^{\sup}(\mathbf{x}_j, P_j^*).$$

One may in addition find $\overline{P}_i^* \supseteq P_i^*$, such that $E^{\sup}(\mathbf{x}_i, \overline{P}_i^*) < E^{\inf}(\mathbf{x}_i', \overline{P}_i^*) < E^{\inf}(\mathbf{x}_j', P_j^*)$, thereby forcing to prefer $\mathbf{x}_N'$ no matter how one refines $P_{N \setminus \{i\}}^*$.

contingent valuation studies (Fleurbaey et al. forthcoming) or revealed preferences (Bargain et al. forthcoming). Extending these empirical approaches to a setting with incomplete preferences is a natural next step.

The well-being concept we propose is very different from traditional "subjective utility" or "happiness." We do not aim at measuring "true" happiness, but at formulating a concept that is meaningful for policy evaluation. Both the choice of the monotone path used in the equivalence approach and the choice of axioms to be imposed in the social evaluation exercise are essentially normative. This is not a weakness, but rather an advantage of the approach. When one aims at policy evaluation, it is better to make the underlying value judgments as open as possible. Having an informed debate about such value judgments in a formal model has always been the main objective of social choice theory.

Appendix

PROOF OF PROPOSITION 3:

We first prove that $\mathcal{B}$ is a monotone path. As there is P^*, such that $NC(0, P^*) = \{0\}$, and as $\mathcal{B}$ is such that $NC(0, P^*) \cap \mathcal{B} \neq \varnothing$, necessarily $0 \in \mathcal{B}$.

Let $\mathbf{z}, \mathbf{z}' \in \mathcal{B}$ be such that neither $\mathbf{z} \geq \mathbf{z}'$ nor $\mathbf{z} \leq \mathbf{z}'$. There is P^* such that $\mathbf{z} P^* \mathbf{z}'$ and $P^{*\prime}$ such that $\mathbf{z}' P^{*\prime} \mathbf{z}$. By the preference principle, $(\mathbf{z}, P^*) \succ (\mathbf{z}', P^*)$ and $(\mathbf{z}', P^{*\prime}) \succ (\mathbf{z}, P^{*\prime})$. By the restricted dominance principle, $(\mathbf{z}, P^*) \sim (\mathbf{z}, P^{*\prime})$ and $(\mathbf{z}', P^*) \sim (\mathbf{z}', P^{*\prime})$. This violates transitivity.

The fact that for all $(\mathbf{x}, P^*)$, $NC(\mathbf{x}, P^*) \cap \mathcal{B} \neq \varnothing$, then directly implies that $\mathcal{B}$ is unbounded and connected.

Let $(\mathbf{x}, P^*)$, $(\mathbf{x}', P^{*\prime})$ be such that $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*) > E_{\mathcal{B}}^{\sup}(\mathbf{x}', P^{*\prime})$, which means that $LC(\mathbf{x}, P^*) \cap UC(\mathbf{x}', P^{*\prime}) \cap \mathcal{B} \neq \varnothing$. Let $\mathbf{z} \in LC(\mathbf{x}, P^*) \cap UC(\mathbf{x}', P^{*\prime}) \cap \mathcal{B}$. By the preference principle, $(\mathbf{x}, P^*) \succ (\mathbf{z}, P^*)$ and $(\mathbf{z}, P^{*\prime}) \succ (\mathbf{x}', P^{*\prime})$. By the restricted dominance principle, $(\mathbf{z}, P^*) \sim (\mathbf{z}, P^{*\prime})$. By transitivity, $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$.

PROOF OF PROPOSITION 4:

Let $(\mathbf{x}, P^*)$, $(\mathbf{x}', P^{*\prime})$ be such that $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*) > E_{\mathcal{B}}^{\inf}(\mathbf{x}', P^{*\prime})$, i.e.,

$$LC(\mathbf{x}, P^*) \cap \left(X \backslash LC(\mathbf{x}', P^{*\prime})\right) \cap \mathcal{B} \neq \varnothing.$$

As $LC(\mathbf{x}, P^*)$ is open (by continuity) and lower comprehensive, while $\mathcal{X} \backslash LC(\mathbf{x}', P^{*\prime})$ is closed and upper comprehensive, $LC(\mathbf{x}, P^*) \cap \left(\mathcal{X} \backslash LC(\mathbf{x}', P^{*\prime})\right) \cap \mathcal{B}$ is not a singleton and there exist $\mathbf{z} > \mathbf{z}'$ in $LC(\mathbf{x}, P^*) \cap \left(\mathcal{X} \backslash LC(\mathbf{x}', P^{*\prime})\right) \cap \mathcal{B}$. As $\mathbf{z}$ is not on the lower boundary of $\mathcal{X} \backslash LC(\mathbf{x}', P^{*\prime})$, i.e., the upper boundary of $LC(\mathbf{x}', P^{*\prime})$, by continuity $\mathbf{x}'$ is not on the lower boundary of $UC(\mathbf{z}, P^{*\prime})$, and therefore there is a refinement $\overline{P}^{*\prime} \supseteq P^{*\prime}$ such that $\mathbf{z} \in UC(\mathbf{x}', \overline{P}^{*\prime})$. For all refinements $\overline{P}^* \supseteq P^*$, $\mathbf{x} \overline{P}^* \mathbf{z}$ because $\mathbf{x} P^* \mathbf{z}$. By Proposition 3, $\mathbf{z} \in LC(\mathbf{x}, \overline{P}^*) \cap UC(\mathbf{x}', \overline{P}^{*\prime}) \cap \mathcal{B}$ implies that $(\mathbf{x}, \overline{P}^*) \succ (\mathbf{x}', \overline{P}^{*\prime})$. By the safety principle, $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$.

PROOF OF PROPOSITION 5:

Assume that $E_{\mathcal{B}}^{\inf}(\mathbf{x}, P^*) \geq E_{\mathcal{B}}^{\inf}(\mathbf{x}', P^{*\prime})$ and $E_{\mathcal{B}}^{\sup}(\mathbf{x}, P^*) \geq E_{\mathcal{B}}^{\sup}(\mathbf{x}', P^{*\prime})$, with at least one strict inequality. There are two possible cases, depending on which inequality is strict. First, let $(\mathbf{x}, P^*)$, $(\mathbf{x}', P^{*\prime})$ be such that $LC(\mathbf{x}', P^{*\prime}) \cap (\mathcal{X} \setminus LC(\mathbf{x}, P^*)) \cap \mathcal{B} = \varnothing$, $UC(\mathbf{x}, P^*) \cap (\mathcal{X} \setminus UC(\mathbf{x}', P^{*\prime})) \cap \mathcal{B} = \varnothing$, and $LC(\mathbf{x}, P^*) \cap (\mathcal{X} \setminus LC(\mathbf{x}', P^{*\prime})) \cap \mathcal{B} \neq \varnothing$ (i.e., the first inequality is strict).

Let $\mathbf{z} \in LC(\mathbf{x}, P^*) \cap (\mathcal{X} \setminus LC(\mathbf{x}', P^{*\prime})) \cap \mathcal{B}$ be such that $\mathbf{z}$ is not in the lower boundary of $\mathcal{X} \setminus LC(\mathbf{x}', P^{*\prime})$.

There is $\overline{P}^{*\prime} \supseteq P^{*\prime}$, such that for all $\overline{P}^* \supseteq P^*$, $\mathbf{z} \in LC(\mathbf{x}, \overline{P}^*) \cap UC(\mathbf{x}', \overline{P}^{*\prime}) \cap \mathcal{B}$ (implying $(\mathbf{x}, \overline{P}^*) \succ (\mathbf{x}', \overline{P}^{*\prime})$ by Proposition 3). This implies that there is no $\overline{P}^* \supseteq P^*$, such that for all $\overline{P}^{*\prime} \supseteq P^{*\prime}$, $(\mathbf{x}, \overline{P}^*) \prec (\mathbf{x}', \overline{P}^{*\prime})$.

It remains to check that there exists no $\overline{P}^{*\prime} \supseteq P^{*\prime}$, such that for all $\overline{P}^* \supseteq P^*$, $(\mathbf{x}, \overline{P}^*) \prec (\mathbf{x}', \overline{P}^{*\prime})$. This directly follows from $UC(\mathbf{x}, P^*) \cap (\mathcal{X} \setminus UC(\mathbf{x}', P^{*\prime})) \cap \mathcal{B} = \varnothing$.

By the super safety principle, $(\mathbf{x}, P^*) \succ (\mathbf{x}', P^{*\prime})$.

The case in which $LC(\mathbf{x}', P^{*\prime}) \cap (\mathcal{X} \setminus LC(\mathbf{x}, P^*)) \cap \mathcal{B} = \varnothing$, $UC(\mathbf{x}, P^*) \cap (\mathcal{X} \setminus UC(\mathbf{x}', P^{*\prime})) \cap \mathcal{B} = \varnothing$, and $UC(\mathbf{x}', P^{*\prime}) \cap (\mathcal{X} \setminus UC(\mathbf{x}, P^*)) \cap \mathcal{B} \neq \varnothing$ is dealt with similarly.

PROOF OF PROPOSITION 6:

Let $\mathbf{x}_N$, $\mathbf{x}'_N$ be such that $\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i E^{\sup}(\mathbf{x}'_i, P_i^*)$. Figure 9 illustrates the proof.

There exist $\hat{\mathbf{x}}_N$, $\hat{\mathbf{x}}'_N$, such that for all $i \in N$, $\hat{h}_i = \hat{h}'_i = 1$, $\mathbf{x}_i P_i^* \hat{\mathbf{x}}_i$, $\hat{\mathbf{x}}'_i P_i^* \mathbf{x}'_i$, and

$$\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i \hat{c}_i > \min_i \hat{c}'_i > \min_i E^{\sup}(\mathbf{x}'_i, P_i^*).$$

Moreover, one can construct $\hat{\mathbf{x}}_N$, $\hat{\mathbf{x}}'_N$ so that there is a unique i_0 such that $\hat{c}_{i_0} = \min_i \hat{c}_i$ and $\hat{c}'_{i_0} = \min_i \hat{c}'_i$, and so that $\hat{\mathbf{x}}'_i P_i^* \hat{\mathbf{x}}_i$ for all $i \neq i_0$.

There exist $\overline{\mathbf{x}}_{i_0}$, $\overline{\mathbf{x}}'_{i_0}$ such that $\overline{h}_{i_0} = \overline{h}'_{i_0} < 1$ and $\hat{\mathbf{x}}_{i_0} P_{i_0}^* \overline{\mathbf{x}}_{i_0} P_{i_0}^* \overline{\mathbf{x}}'_{i_0} P_{i_0}^* \hat{\mathbf{x}}'_{i_0}$. For each $i \neq i_0$, let $\overline{\mathbf{x}}_i$, $\overline{\mathbf{x}}'_i$ be such that $\overline{h}_i = \overline{h}'_i = \overline{h}_{i_0}$, $\overline{\mathbf{x}}'_i P_i^* \hat{\mathbf{x}}'_i$ and $\overline{c}'_i - \overline{c}_i = (\overline{c}_{i_0} - \overline{c}'_{i_0})/(n-1)$.

There exist $\overline{\mathbf{x}}'''_{i_0}$ such that $\overline{h}'''_{i_0} = 1$ and $\hat{\mathbf{x}}_{i_0} P_{i_0}^* \overline{\mathbf{x}}'''_{i_0} P_{i_0}^* \overline{\mathbf{x}}_{i_0}$. For each $i \neq i_0$, let $\overline{\mathbf{x}}''_i$, $\overline{\mathbf{x}}'''_i$ be such that $\overline{h}''_i = \overline{h}'''_i = 1$, $\hat{\mathbf{x}}_i > \overline{\mathbf{x}}'''_i > \overline{\mathbf{x}}''_i > \hat{\mathbf{x}}_{i_0}$ and $\overline{c}'''_i - \overline{c}''_i = (\hat{c}_{i_0} - \overline{c}'''_{i_0})/(2(n-1))$. Let $\overline{\mathbf{x}}''_{i_0} = (\hat{\mathbf{x}}_{i_0} + \overline{\mathbf{x}}'''_{i_0})/2$. One has $\overline{c}'''_i - \overline{c}''_i = (\overline{c}''_{i_0} - \overline{c}'''_{i_0})/(n-1)$.

Let $P_{i_0}^{*\prime} = P_{i_0}^*$ and for $i \neq i_0$, let $P_i^{*\prime}$ be such that $\overline{\mathbf{x}}'_i P_i^{*\prime} \hat{\mathbf{x}}'_i$, $\overline{\mathbf{x}}'''_i P_i^{*\prime}\, \overline{\mathbf{x}}_i$, $NC(\overline{\mathbf{x}}'_i, P_i^{*\prime}) \cap NC(\hat{\mathbf{x}}'_i, P_i^*) = \varnothing$, $NC(\hat{\mathbf{x}}_i, P_i^*) \cap NC(\overline{\mathbf{x}}''_i, P_i^{*\prime}) = \varnothing$, $NC(\overline{\mathbf{x}}_i, P_i^{*\prime}) \cap NC(\overline{\mathbf{x}}_{i_0}, P_{i_0}^{*\prime}) = \varnothing$.

For $i \neq i_0$, let $P_i^{*\prime\prime}$ be such that $NC(\overline{\mathbf{x}}'_i, P_i^{*\prime\prime}) = NC(\overline{\mathbf{x}}'_i, P_i^{*\prime})$, $NC(\overline{\mathbf{x}}_i, P_i^{*\prime\prime}) = NC(\overline{\mathbf{x}}_i, P_i^{*\prime})$, and for all $\mathbf{x}$ such that $\overline{\mathbf{x}}'_{i_0} \leq \mathbf{x} \leq \overline{\mathbf{x}}_{i_0}$, $NC(\mathbf{x}, P_i^{*\prime\prime}) = NC(\mathbf{x}, P_{i_0}^{*\prime})$.

Number the agents $i \neq i_0$ from 1 to $n-1$. By Pigou-Dalton,

$$(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}'_2, \ldots, \overline{\mathbf{x}}'_{n-1}, \overline{\mathbf{x}}'_{i_0} + \overline{\mathbf{x}}'_1 - \overline{\mathbf{x}}_1)\, \mathbf{R}(P_1^{*\prime\prime}, P_2^{*\prime}, \ldots, P_{n-1}^{*\prime}, P_1^{*\prime\prime}) \overline{\mathbf{x}}'_N,$$

and by independence,

$$(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}'_2, \ldots, \overline{\mathbf{x}}'_{n-1}, \overline{\mathbf{x}}'_{i_0} + \overline{\mathbf{x}}'_1 - \overline{\mathbf{x}}_1)\, \mathbf{R}(P_N^{*\prime}) \overline{\mathbf{x}}'_N.$$

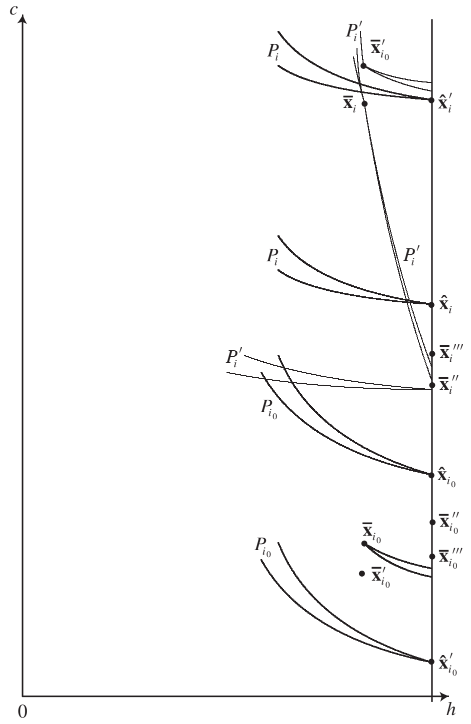

FIGURE 9. ILLUSTRATION OF THE PROOF OF PROPOSITION 6

Repeating this argument for agent 2, one obtains

$$\left(\overline{\mathbf{x}}_1,\ \overline{\mathbf{x}}_2,\ \overline{\mathbf{x}}_3', \ldots, \overline{\mathbf{x}}_{n-1}',\ \overline{\mathbf{x}}_{i_0}' + 2\left(\overline{\mathbf{x}}_1' - \overline{\mathbf{x}}_1\right)\right) \mathbf{R}(P_N^{*\prime})\overline{\mathbf{x}}_N'.$$

After applying this argument also to $i = 3, \ldots, n-1$, and noting that $(n-1) \times \left(\overline{\mathbf{x}}_1' - \overline{\mathbf{x}}_1\right) = \overline{\mathbf{x}}_{i_0} - \overline{\mathbf{x}}_{i_0}'$, one obtains $\overline{\mathbf{x}}_N \mathbf{R}(P_N^{*\prime})\overline{\mathbf{x}}_N'$.

By weak Pareto, $\overline{\mathbf{x}}_N''' \mathbf{P}(P_N^{*\prime})\overline{\mathbf{x}}_N$. By $n-1$ applications of Pigou-Dalton, $\overline{\mathbf{x}}_N'' \mathbf{R}(P_N^{*\prime})\overline{\mathbf{x}}_N'''$. By transitivity, $\overline{\mathbf{x}}_N'' \mathbf{P}(P_N^{*\prime})\overline{\mathbf{x}}_N'$.

Let $P_{i_0}^{*\prime\prime\prime} = P_{i_0}^*$ and for $i \neq i_0$, let $P_i^{*\prime\prime\prime}$ be such that $NC(\overline{\mathbf{x}}_i', P_i^{*\prime\prime\prime}) = NC(\overline{\mathbf{x}}_i', P_i^{*\prime})$, $NC(\overline{\mathbf{x}}_i'', P_i^{*\prime\prime\prime}) = NC(\overline{\mathbf{x}}_i'', P_i^{*\prime})$, and $NC(\hat{\mathbf{x}}_i, P_i^{*\prime\prime\prime}) = NC(\hat{\mathbf{x}}_i, P_i^*)$, $NC(\hat{\mathbf{x}}_i', P_i^{*\prime\prime\prime}) = NC(\hat{\mathbf{x}}_i', P_i^*)$.

By independence, $\overline{\mathbf{x}}_N'' \mathbf{P}(P_N^{*\prime\prime\prime})\overline{\mathbf{x}}_N'$.

By weak Pareto, $\hat{\mathbf{x}}_N \mathbf{P}(P_N^{*\prime\prime\prime})\overline{\mathbf{x}}_N''$ and $\overline{\mathbf{x}}_N' \mathbf{P}(P_N^{*\prime\prime\prime})\hat{\mathbf{x}}_N'$. By transitivity, $\hat{\mathbf{x}}_N \mathbf{P}(P_N^{*\prime\prime\prime})\hat{\mathbf{x}}_N'$.

By independence, $\hat{\mathbf{x}}_N \mathbf{P}(P_N^*)\hat{\mathbf{x}}_N'$. By weak Pareto, $\mathbf{x}_N \mathbf{P}(P_N^*)\hat{\mathbf{x}}_N$ and $\hat{\mathbf{x}}_N' \mathbf{P}(P_N^*)\mathbf{x}_N'$. By transitivity, $\mathbf{x}_N \mathbf{P}(P_N^*)\mathbf{x}_N'$.

PROOF OF PROPOSITION 7:

Let $\mathbf{x}_N$, $\mathbf{x}_N'$ be such that $\min_i E^{\inf}(\mathbf{x}_i, P_i^*) > \min_i E^{\inf}(\mathbf{x}_i', P_i^*)$, and for every j such that $E^{\inf}(\mathbf{x}_j, P_j^*) < \min_i E^{\sup}(\mathbf{x}_i', P_i^*)$, $\mathbf{x}_j P_j^* \mathbf{x}_j'$.

There is j_0 and a complete $\overline{P}_{j_0}^* \supseteq P_{j_0}^*$, such that $E(\mathbf{x}_{j_0}', \overline{P}_{j_0}^*) = \min_i E^{\inf}(\mathbf{x}_i', P_i^*)$. Take any $\overline{P}_i^* \supseteq P_i^*$ for all $i \neq j_0$. Necessarily, $\min_i E^{\sup}(\mathbf{x}_i', \overline{P}_i^*) = E(\mathbf{x}_{j_0}', \overline{P}_{j_0}^*) < \min_i E^{\inf}(\mathbf{x}_i, P_i^*) \leq \min_i E^{\inf}(\mathbf{x}_i, \overline{P}_i^*)$, implying that $\mathbf{x}_N \mathbf{P}(\overline{P}_N^*)\mathbf{x}_N'$ by Proposition 6.

Suppose there were k and $P_k^{*\prime} \supseteq P_k^*$, such that $\mathbf{x}_N' \mathbf{P}(P_k^{*\prime}, P_{N\setminus\{k\}}^{*'})\mathbf{x}_N$ for all $P_{N\setminus\{k\}}^{*'} \supseteq P_{N\setminus\{k\}}^*$. By the previous paragraph, it is impossible that $k \neq j_0$, because with $P_{j_0}^{*\prime} = \overline{P}_{j_0}^*$ one would then have $\mathbf{x}_N \mathbf{P}(P_k^{*\prime}, P_{N\setminus\{k\}}^{*'})\mathbf{x}_N'$. Therefore $k = j_0$. There is no loss in generality in assuming that all $P_i^{*\prime}$ $(i \in N)$ are complete[20] when one writes that $\mathbf{x}_N' \mathbf{P}(P_k^{*\prime}, P_{N\setminus\{k\}}^{*'})\mathbf{x}_N$ for all $P_{N\setminus\{k\}}^{*'} \supseteq P_{N\setminus\{k\}}^*$. Necessarily, $P_{j_0}^{*'} \neq \overline{P}_{j_0}^*$ and $E(\mathbf{x}_{j_0}', P_{j_0}^{*'}) \geq \min_i E^{\inf}(\mathbf{x}_i, P_i^*)$, otherwise $\mathbf{x}_N \mathbf{P}(P_k^{*\prime}, P_{N\setminus\{k\}}^{*'})\mathbf{x}_N'$ would be guaranteed.

For all j, such that $E^{\inf}(\mathbf{x}_j, P_j^*) < \min_i E^{\sup}(\mathbf{x}_i', P_i^*)$, let $P_j^{*\prime} \supseteq P_j^*$ be a (complete) ordering such that $E(\mathbf{x}_j, P_j^{*\prime}) = E^{\inf}(\mathbf{x}_j, P_j^*)$. This set of j may include j_0. Necessarily, $\min_i E^{\inf}(\mathbf{x}_i, P_i^{*\prime}) = \min_i E^{\sup}(\mathbf{x}_i, P_i^{*\prime})$, which can be denoted $E(\mathbf{x}_j, P_j^{*\prime})$, for one of these j.

Moreover, for all of them, $E(\mathbf{x}_j', P_j^{*\prime}) = E^{\inf}(\mathbf{x}_j', P_j^{*\prime}) = E^{\sup}(\mathbf{x}_j', P_j^{*\prime}) \geq \min_i E^{\sup}(\mathbf{x}_i', P_i^{*\prime})$.

Now, for all of them, $\mathbf{x}_j P_j^* \mathbf{x}_j'$, which implies $\mathbf{x}_j P_j^{*\prime} \mathbf{x}_j'$ and therefore $E(\mathbf{x}_j, P_j^{*\prime}) > E(\mathbf{x}_j', P_j^{*\prime})$, implying that $\min_i E^{\inf}(\mathbf{x}_i, P_i^{*\prime}) > \min_i E^{\sup}(\mathbf{x}_i', P_i^{*\prime})$, and therefore $\mathbf{x}_N \mathbf{P}(P_N^{*\prime})\mathbf{x}_N'$ by Proposition 6. One obtains a contradiction with the assumption that $\mathbf{x}_N' \mathbf{P}(P_N^{*\prime})\mathbf{x}_N$.

Therefore, super safety applies, and one concludes that $\mathbf{x}_N \mathbf{P}(P_N^*)\mathbf{x}_N'$.

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[20] Completeness of P^* means that there is a complete binary relation R^* such that $\mathbf{x}R^*\mathbf{y}$ if and only if not $(\mathbf{y}P^*\mathbf{x})$.

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