2013/47

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CORE

DISCUSSION PAPER

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and Econometrics

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Uncertain altruism and the provision of long term care

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August 2013

Abstract

This paper studies the role of private and public long term care (LTC) insurance programs in a world in which family assistance is uncertain. Benefits are paid in case of disability but cannot be conditioned (directly), due to moral hazard problems, on family aid. Under a topping up scheme, when the probability of altruism is high, there is no need for insurance. At lower probabilities, insurance is required, thought not full insurance. This can be provided either privately or publicly if insurance premiums are fair, and publicly otherwise. Moreover, the amount of LTC insurance varies negatively with the probability of altruism. With an opting out scheme, there will be three possible equilibria depending on the children's degree of altruism being "low," "moderate," or "very high". These imply: full LTC insurance with no aid from children, less than full insurance just enough to induce aid, and full insurance with aid. Fair private insurance markets can support the first equilibrium, but not the other two equilibria. Only a public opting-out scheme can attain them by creating incentives for self-targeting and ensuring that only dependent parents who are not helped by their children seek help from the government.

Keywords: long term care, uncertain altruism, private insurance, public insurance, topping up, opting out.

JEL Classification: H2, H5

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Helmuth Cremer and Pierre Pestieau gratefully acknowledge financial support from the Chaire "Marché des risques et creation de valeur" of the FdR/SCOR. We thank Craig Brett, Chiara Orsini, Grégory Ponthière and Erik Schokkaert for their comments.

1 Introduction

Long-term care (LTC) is the provision of assistance and services to people who, because of disabling illnesses or conditions, have limited ability to perform basic daily activities such as bathing and preparing meals. It is a problem mainly, though not exclusively, for the elderly. As people have come to live longer, the demand for LTC services by the elderly population has grown substantially in recent years—a trend likely to further increase and intensify in future. There are two related reasons for this. First, LTC needs start to rise exponentially from around the age of 80 years old; second, the number of persons aged 80 years and above are growing faster than any other segment of the population. As a consequence, the number of dependent elderly is expected to more than double by 2050 in most countries. This will exacerbate the current pressures on the demand for LTC services and lead to new challenges for these countries and their governments.[1]

There are, currently, three institutions that finance and provide LTC services: the family, the market, and the state. The majority of the dependent population receiving long-term care at home rely exclusively on assistance from family members, mainly women; this is often referred to as "informal care". This avenue for LTC provision is, unfortunately, facing a number of formidable challenges: drastic changes in family values, increasing number of childless households, mobility of children, and growing rate of market activity on the part of women (particularly those aged 50–65). As a consequence, the number of dependent elderly who will not be able to count on the assistance of family members is likely to increase. This creates a pressing demand on the other two institutions, the market and the state, to offer either a substitute or a complement to what the family has thus far been providing by way of long term care.

The aim of this paper is to highlight and study the challenge posed by the idea that

[1] For surveys on LTC and for more details on these estimates, see Cremer, Pestieau and Ponthière (2012) and Grabowski et al. (2012).

family solidarity is uncertain. There are multifold reasons for this. First, there are pure demographic factors such as widowhood, the absence or the loss of children. Divorce and migration can also be put in that category. Other causes are conflicts within the family or financial problems incurred by children, which prevent them from helping their parents. Whatever the reason, the possibility of solidarity default requires people to take appropriate steps such as purchasing private insurance, self-insuring or relying on some public insurance or assistance scheme. What makes the problem particularly daunting and ripe for government intervention is the fact that there exists no good insurance mechanism to protect individuals against all sorts of default of family altruism.[2]

We study the role of private and public insurance programs in a world in which family assistance is uncertain. We do this by modeling the behavior and welfare of one single generation of "parents" over their life cycle. When they are young, they work, consume, and save for their retirement. In retirement, they face a probability of becoming dependent. This probability is exogenously determined and parents cannot affect it through their behavior (when young or when old). If they become dependent, parents face yet another uncertainty. They may or may not receive assistance from their children. Many factors affect the children's behavior. Some causes of altruism default are purely exogenous, but others can be influenced by the parents. Investment in the children's education and inculcating values in them through one's own behavior are such mechanisms.[3] In this paper, we study the problem assuming an exogenously given probability for the default of altruism on the part of the children. We shall leave the setting where parents can affect this probability to another study.

An important feature of our study is that we do not rule out private insurance markets by fiat. Indeed, we allow for the possibility of parents insuring themselves against becoming dependent. Plainly, however, moral hazard problems preclude the

[2]The consequences of uncertain altruism for old age retirement have been studied by Chabbakatri *et al.* (1993) and Leroux and Pestieau (2013).

[3]On this, see on this Kotlikoff and Spivak (1981) or Cox and Stark (2005).

development of insurance markets against the default of altruism per se as opposed to dependency. For the same reasons, the government cannot condition its assistance to the old on the default of altruism; only on age-old dependency.[4] Within this framework, we provide answers to two broadly defined questions.

One question is the general need for insurance and how it should be provided: privately or publicly. We study the conditions under which private savings will or will not be enough for the three states of the world parents face in retirement (autonomy and dependency with or without assistance from children). In the latter case, we examine when parents can rely on private insurance markets to secure the extra resources they want in case of dependency (because of the possibility of altruism default on the part of the children). We also discuss the circumstances that call for the government to step in and provide the needed assistance.

The second broad question we address concerns the nature of public assistance. One possibility is for the government to provide all dependent parents with monetary help while allowing them to top this up as they see fit. Another possibility is for the government to provide every dependent parent who asks for it a "minimal" care facility. If this is deemed insufficient the parent will have to opt out and use his own resources, and their children's, to purchase whatever home care services he needs (without any help from the government). The dependent parent consumes either one or the other. We examine and compare the properties of these two schemes.

In searching a role for the government, we confine ourselves to scenarios wherein the cost of financing the LTC program is borne by the potential beneficiaries themselves (and not by their children or future generations). In this way, one can zero-in on the insurance reasons for such programs rather than compounding this with issues that arise from

[4] However, the government has a basic advantage over the private sector in that it can induce self-targeting. That is, it can adopt an opting out policy that ensure only the old who suffer from the default of altruism opt for this insurance. Exploiting this possibility, and investigating its properties, is an important feature of our paper.

wealth transfers across generations. Consequently, we model the behavior and welfare of one single generation of parents over their life cycle. Our interest in their children is limited only to their role in providing assistance to their parents. As a consequence, the welfare of the grown-up children does not figure in government's objective function; only the expected utility of parents. Nor do the children pay any taxes to finance the LTC program (otherwise, they become a "costless" source of taxation to provide benefits for their parents).

In answering the first set of questions we have asked, we find that the scheme the government adopts, "topping up" or "opting out", has an important bearing on the question of who should provide LTC; the market or the state. Specifically, under the topping up scheme, if the probability of altruism is high there is no need for insurance regardless of who provides it. All assistance is provided through one's children and private savings. At lower probabilities, LTC insurance is called for; albeit one that is less than full. Moreover, the amount of insurance varies negatively with the probability of altruism. If private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). At higher than fair insurance premiums, on the other hand, public assistance dominates private insurance.

With an opting out scheme, different types of equilibria emerge depending on the children's degree of altruism towards their parents. If the degree of altruism is "small" or "very large", the optimal solution is for the government to provide full LTC insurance for everyone. With a small degree of altruism, all children opt for the government plan providing no assistance of their own to their parents. With the very large degree of altruism, altruistic children do not consider the government's full insurance plan good enough and opt out of it. They provide their own assistance instead of what the government offers. The only option open to the parents of non-altruistic children is of course the government assistance. Interestingly, when the children's degree of altruism is "moderate," lying somewhere between small and very large, the best strategy for the government is to provide less that full LTC insurance. This will be just small enough to entice the altruistic children to substitute their own assistance for the government's.

As to the question of who should provide the insurance, we show that the two equilibria which entail assistance from altruistic children—arising when the children's degree of altruism is moderate and very large—can be supported only through the public opting out scheme. Private insurance markets cannot do the job even if they are fair. On the other hand, when children have a low degree of altruism so that the equilibrium is one without assistance, fair private insurance markets are just as good as public insurance.

Finally, comparing topping up and opting out, we show that opting out always dominates when children are sufficiently altruistic. This is because under opting out public LTC can be targeted to the parents whose children's turn out not to be altruistic. However, for lower levels of altruism, this more precise targeting comes at a price, namely that public LTC is distorted downward (to ensure continued aid from altruistic children). This makes the comparison ambiguous.

2 The model with topping up

Consider a setup wherein (i) in period 0, the government formulates and announces its tax/transfer policy; (ii) in period 1, a young working parent and a child appear on the scene and the parent decides on his present and future consumption levels; in period 2, the parent has grown old and is retired, and the child who has turned into a working adult decides if he wants to help his parents. We will not be concerned with what will happen to the grown-up child when he turns old, nor with any future generations. Two uncertain events give rise to the problem that we study. One concerns the health of the parent in old age. He may be either "dependent" or "independent". Denote the probability of dependency by π; naturally, the parent will be independent with probability $1-\pi$. We assume that π is exogenously given. The second source of uncertainty concerns the help that the parent might get from his children *if* dependent. Denote the probability that a child is altruistic towards his parents by p and the probability that he is not by $1-p$. We shall assume that p is also exogenously fixed.[5]

Parents have preferences over consumption when young, c, and consumption when old, d; there is no disutility associated with working. We assume, for simplicity, that the parents' preferences are quasilinear in consumption when young. With p fixed, labor supply is also predetermined as there is no other usage for the parents' time.

Government's policy consists of levying a tax at rate of τ on the parents' wage, w, to finance public provision of dependency assistance, g. We shall refer to g as the LTC insurance. Labor supply is fixed and equal to $\overline{T}$. As far as private insurance is concerned, it is easier to rule it out initially for modeling purposes. Having established the need for insurance, we examine if it can be decentralized through private insurance markets.

Denote the level of assistance an altruistic child would give his parents by a, savings by s, and set the rate of interest on savings at zero. At this point (up to Section 3) we assume that g is non exclusive in the sense that it can be topped up by a and s. One can then represent the parent's preferences by means of the expected utility,

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi\left[pH(s+g+a) + (1-p)H(s+g)\right]. \quad (1)$$

We shall assume that the grown-up children of dependent parents too have quasilinear preferences represented by

$$u = y - a + \beta H(s+g+a), \quad (2)$$

where y denotes their income with β being the degree of their altruism towards their parents.[6] Those who are not altruistic towards their parents have a $\beta = 0$.

[5] In a companion paper, Cremer, Gahvari and Pestieau (2012), we drop this assumption and allow parents to affect this probability by spending time with their children when they are growing up. There we also consider the case where parents are heterogeneous in their earning abilities.

[6] Throughout the paper we assume that β, and more generally individual preferences are common knowledge.

To determine the government's policy we start by studying the last stage of our decision making process. This is when the grown-up children decide on the extent of their help to their parents, if any.

2.1 Stage 3: The child's choice

The altruistic child allocates an amount a of his income y to assist his dependent parent (given the parent's savings s and the government's provision of g). Its optimal level, a^*, is found through the maximization of equation (2). The first-order condition with respect to a is, assuming an interior solution,[7]

$$-1 + \beta H'(s + g + a) = 0.$$

It follows from this condition that a^* satisfies

$$s + g + a^* = \left(H'\right)^{-1}\left(\frac{1}{\beta}\right) \equiv m(\beta). \tag{3}$$

Finally, differentiating (3) yields

$$\frac{\partial m}{\partial \beta} = \frac{-1}{\beta^2 H''(s + g + a)} > 0,$$

where the sign follows from concavity of $H(\cdot)$.[8]

[7] A sufficient condition for an interior solution is to have

$$\frac{du}{da}|_{a=0} = -1 + \beta H'(s + g) > 0,$$

or

$$H'(s + g) > \frac{1}{\beta}.$$

[8] Differentiating (3) results in

$$\frac{\partial m(\beta)}{\partial \beta} = \frac{\partial (s + g + a^*)}{\partial \beta}.$$

But we also have, from differentiating $H'(s + g + a^*) = 1/\beta$ with respect to β, that

$$H''(s + g + a^*)\frac{\partial (s + g + a^*)}{\partial \beta} = \frac{-1}{\beta^2}.$$

2.2 Stage 2: The parent's choice

Recall that the parent may experience two states of nature when retired: dependency with probability π and autonomy with probability $(1-\pi)$. Substituting for a^* from (3) in the parent's expected utility function (1), we have

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi\left[pH(m(\beta)) + (1-p)H(s+g)\right]. \qquad (4)$$

Maximizing EU with respect to s, and assuming an interior solution, the optimal value of s satisfies

$$(1-\pi)U'(s) + \pi(1-p)H'(s+g) = 1. \qquad (5)$$

Denote the solution to equation (5) by $s(p;g)$. Substituting $s(p;g)$ for s in (5), the resulting relationship holds for all values of p and g. Differentiating this relationship partially with respect to p and g yields,

$$\frac{\partial s}{\partial p} = \frac{\pi H'(s+g)}{(1-\pi)U''(s) + \pi(1-p)H''(s+g)} < 0, \qquad (6)$$

$$\frac{\partial s}{\partial g} = \frac{-\pi(1-p)H''(s+g)}{(1-\pi)U''(s) + \pi(1-p)H''(s+g)} < 0. \qquad (7)$$

Thus a parent's savings move negatively with the probability of altruism and with the government's level of assistance.

2.3 Stage 1: The optimal policy

The government chooses τ and g to maximize EU, as optimized by the parents in stage 2, subject to its budget constraint

$$\tau w\overline{T} = \pi g. \qquad (8)$$

Substituting for τ from (8) into the parents' optimized value of EU, the government chooses g to maximize

$$\begin{aligned}\pounds \equiv\ & w\overline{T} - \pi g - s(p;g) + (1-\pi)U(s(p;g)) + \\ & \pi\left[pH(m(\beta)) + (1-p)H(s(p;g)+g)\right]. \end{aligned} \qquad (9)$$

Differentiating $\pounds$ with respect to g yields, using the envelope theorem,

$$\frac{d\pounds}{dg} = \pi\left[(1-p)\,H'\left(s\left(p;g\right)+g\right)-1\right]. \tag{10}$$

There are two possible outcomes depending on the sign of $d\pounds/dg$ at $g=0$.

One possible outcome is that

$$(1-p)\,H'\left(s\left(p\right)\right)-1 \leq 0, \tag{11}$$

where $s\left(p\right) \equiv s\left(p;0\right)$. This condition holds in the neighborhood of $p=1$.[9] Under this condition, $g=\tau=0$ and no LTC insurance is called for. The consumption levels provided through one's own savings, and help from altruistic children, are sufficient (albeit, if $p<1$, ex post there will be some people with "insufficient" means).[10]

The second outcome occurs if

$$(1-p)\,H'\left(s\left(p\right)\right)-1 > 0. \tag{12}$$

This condition is necessarily satisfied for some p as long as $H'\left(s\left(0\right)\right) > U'\left(s\left(0\right)\right)$.[11] In this case, there will be an interior solution for g, and τ, characterized by

$$H'\left(s\left(p;g\right)+g\right) = \frac{1}{1-p} > 1. \tag{13}$$

[9] The condition is satisfied at $p=1$ and by continuity in the neighborhood of $p=1$.

[10] One explanatory reason mentioned for the LTC insurance puzzle is family solidarity (as well as the more well-known annuity puzzle); see Pestieau and Ponthière (2011). Our result suggests that even uncertain family solidarity may explain the puzzle.

[11] Observe that

$$\frac{d}{dp}\left[(1-p)\,H'\left(s\left(p\right)\right)-1\right] = -H'\left(s\left(p\right)\right)+(1-p)\,H''\left(s\left(p\right)\right)s'\left(p\right),$$

which is negative at $p=1$. Consequently, as p becomes smaller than one, $(1-p)\,H'\left(s\left(p\right)\right)-1$ initially increases so that at some point *might* change its sign. Condition $H'\left(s\left(0\right)\right)>1$ ensures that this will happen. In turn, from (5), this condition is equivalent to assuming that $H'\left(s\left(0\right)\right) > U'\left(s\left(0\right)\right)$. The need for this assumption arises because with $H''\left(s\left(p\right)\right)<0$ and $s'\left(p\right)<0$, the sign of the above derivative while initially negative is in general indeterminate so that it may not always remain negative. Even if it does, there is no guarantee that $(1-p)\,H'\left(s\left(p\right)\right)-1$ necessarily changes sign.

Consequently, there is less than full insurance.[12] Substituting from (13) into (5), it is also the case that

$$U'(s(p;g)) = 1. \tag{14}$$

Interestingly too, it follows from equation (13) and (14) that the publicly-provided LTC insurance g, and the tax rate τ, decrease with p. Specifically, differentiating (14) with respect to p yields[13]

$$\frac{dg}{dp} = \frac{H'(s+g)}{(1-p)H''(s+g)} < 0. \tag{15}$$

The intuition behind this result is easily understood. Providing more g for dependent parents who do not get help from their children is also accompanied by providing more g for dependent parents who do receive aid from their children. Now while parents bear all the cost of providing more g in terms of a loss in their present day consumption, the beneficiary of more g also includes the altruistic children who would have otherwise provided the extra g to their parents. Put differently, g will only provide extra benefits to parents when children are not altruistic which occurs with probability p. Consequently a higher p will result in a lower level of g.

Finally, with our findings that $dg/dp < 0$ and $g = 0$ at high values of p, there must exist a $p = \widehat{p}$ at which the switch from one outcome to the other occurs. This is when condition (11) holds as an equality:

$$H'(s(\widehat{p})) = \frac{1}{1-\widehat{p}}. \tag{16}$$

[12] Full insurance requires the equality of $H'(\cdot)$ with the marginal utility of current consumption which, with quasilinear preferences, is equal to one.

[13] Upon differentiating (14) one gets

$$U''(s(p;g))\left[\frac{\partial s}{\partial p} + \frac{\partial s}{\partial g}\frac{dg}{dp}\right] = 0.$$

It follows from this equation that

$$\frac{dg}{dp} = \frac{-\frac{\partial s}{\partial p}}{\frac{\partial s}{\partial g}} = -\frac{\frac{\pi H'(s+g)}{(1-\pi)U''(s)+\pi(1-p)H''(s+g)}}{\frac{-\pi(1-p)H''(s+g)}{(1-\pi)U''(s)+\pi(1-p)H''(s+g)}}.$$

Simplification results in (15).

Observe also that at this price, we have from equation (14) that

$$U'(s(\widehat{p})) = 1. \tag{17}$$

2.4 The question of private insurance

Assume $0 \leq p < \widehat{p}$ so that dependent parents need additional assistance beyond their own savings when assistance is not forthcoming from their children. The question is if they can secure this through private insurance markets. To answer this question, denote the amount of insurance the parent may buy against old-age dependency, if it is optimal to have private insurance markets, by θ and its unit price by q. First, observe that allowing private insurance markets does not change the consumption level of a parent receiving assistance from his children; it remains at $m(\beta)$.[14] Second, let e denote the government's assistance *above* the privately-purchased insurance, θ, so that $g = \theta + e$. Reconsidering the parents' optimization problem, one can show that savings depends on $\theta + e$ and not its division between public and private insurance.[15]

Now consider the expected utility of the parent, if private insurance markets are operative. Fix the total amount of LTC insurance provided both privately and publicly, for the given p, at its optimal level derived for when all insurance is provided by the government. That is set, $\theta + e = g^*$. Under this circumstance, parents' savings would

[14] Let e denote the government's assistance *above* the privately-purchased insurance, θ. The children's optimization level results in the first-order condition $-1 + \beta H'(s + \theta + e + a) = 0$, so that the total consumption of the parent, $s + \theta + e + a$, does not change.

[15] Expected utility of the parent is now written as

$$\begin{aligned} EU &= w(1-\tau)\overline{T} - q\theta - s + (1-\pi)U(s) + \\ &\quad \pi\left[pH(m(\beta)) + (1-p)H(s+\theta+e)\right]. \end{aligned}$$

This leads to the following first-order condition with respect s,

$$(1-\pi)U'(s) + \pi(1-p)H'(s+\theta+e) = 1.$$

Comparing this equation with (5) indicates this point.

continue to be what it was when all LTC insurance is provided publicly. We thus have

$$EU = w(1-\tau)\overline{T} - q\theta - s^* + (1-\pi)U(s^*) + \pi\left[pH(m(\beta)) + (1-p)H(s^*+g^*)\right], \tag{18}$$

where a "star" on top of a variable indicates its fixed optimal value. With the parents' purchasing θ, the government must finance only $e = g^* - \theta$ in assistance to dependent parents. This changes the government's budget constraint to

$$\tau w\overline{T} = \pi(g^* - \theta). \tag{19}$$

Substitute for τ from (19) into (18). Upon simplification, we have

$$\begin{aligned} EU &= w\overline{T} - \pi g^* + (\pi - q)\theta - s^* + (1-\pi)U(s^*) + \\ &\quad \pi\left[pH(m(\beta)) + (1-p)H(s^*+g^*)\right]. \end{aligned} \tag{20}$$

At $\theta = 0$, we have the optimal solution with insurance being provided solely by the government. The question is if having $\theta > 0$ benefits or harms the parents. To answer this question, differentiate (20) with respect to θ. This yields

$$\frac{\partial EU}{\partial \theta} = \pi - q. \tag{21}$$

The result in (21) makes perfect sense. Both the private sector and the government provide insurance against dependency and not lack of altruism. The per unit cost through the private sector is q and through the government, implicitly, π. If the private insurance price is "fair" then $q = \pi$ and one is just as good as another. Under this circumstance, private markets suffice and there is no additional need for government intervention. On the other hand, if $q > \pi$, providing assistance through private markets can only hurt the parents. Insurance should be provided only publicly.

The main results of this section are summarized in the following proposition.

Proposition 1 *Consider a topping up scheme.*

Let $s(p)$ solve

$$(1-\pi)\,U'(s(p))+\pi(1-p)\,H'(s(p))=1.$$

Define $\widehat{p}$ as the solution to

$$H'(s(p))=\frac{1}{1-p}.$$

(i) If

$$\widehat{p}\leq p\leq 1,$$

there is no need for insurance regardless of who provides it. All assistance is provided through one's children and private savings. In other words, even uncertain family solidarity may explain the LTC insurance puzzle.

(ii) If

$$0\leq p<\widehat{p},$$

there will be need for LTC insurance.

(a) If $q=\pi$, i.e., if private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). On the other hand, if $q>\pi$, public assistance dominates private insurance.

(b) The optimal amount of LTC insurance g is characterized by

$$H'(s(p;g)+g)=\frac{1}{1-p}>1,$$

and there is less than full insurance.

(c) The amount of LTC insurance g varies negatively with the probability of receiving aid from one's children, p.

3 Opting out

We now assume that, if made dependent, parents will need a particular good/service, x. Specifically, one can think of purchasing home care services using one's own, and one's children's, resources versus government provision of a minimum facility. The crucial point is that one consumes either one or the other. One cannot top up what the government provides.

Denote government's provision by z. Clearly, $z \geq s$; otherwise z will be of no use to the parents. When providing such a facility, the government taxes away the recipient's resources (savings and any private insurance that he may have purchased). Consequently, $x = s+a$ if LTC insurance is provided by the altruistic children and $x = z$ if it is provided by the government. Clearly, the dependent parents with non-altruistic children end up with a consumption level $x = z$. What the consumption level of parents with altruistic children will be is not clear; however. That depends on the behavior of these children. Whereas under a topping up scheme an altruistic child decides on how much to *supplement* the government's provision; under an opting out scheme, he would have to decide *between* his own assistance versus that of the government's. Thus the parents end up either with some assistance from their altruistic children or no assistance at all. As in the previous section, we start with stage 3 and the children's decision.

3.1 Stage 3: The child's choice

The optimal level of family assistance a^* is found from the maximization of

$$u = y - a + \beta H(s + a).$$

To have an interior solution for a, it must be the case that $(\partial u/\partial a)|_{a=0} = -1+\beta H'(s) > 0$; or $\beta > 1/H'(s)$. We assume this is the case. The value of a^* is then given by

$$s + a^* = (H')^{-1}\left(\frac{1}{\beta}\right) \equiv m(\beta), \tag{22}$$

where, as with the topping-up case, $m'(\beta) > 0$. Observe that while a^* is now given by $m(\beta) - s$ as opposed to $m(\beta) - (s + g)$ in the previous section, the actual consumption level of the dependent parent when he is being assisted by his children remains the same. Put differently, the child fully makes up for what the government does not provide.

Clearly, the altruistic child chooses a^* if and only if it gives him more utility than the option of no assistance. However, whereas in the topping up case $a^* = m(\beta) - (s+g)$ moved continuously with g; here a^* is independent of z and the child faces a discontinuous choice. This is settled by comparing his utility if he chooses his own assistance,

$$u = y - a^* + \beta H(s + a^*), \tag{23}$$

versus his utility if he goes with government's assistance for his parents,

$$u = y + \beta H(z). \tag{24}$$

The two yield the same utility level if

$$H(z) = H(s + a^*) - \frac{a^*}{\beta}, \tag{25}$$

where $a^* = m(\beta) - s$. Denote the solution to equation (25) by $\widehat{z}(\beta)$. If the z offered by the government falls short of $\widehat{z}(\beta)$, the altruistic child provides his parents with a^* in aid; if z exceeds $\widehat{z}(\beta)$, the child opts for no assistance. The crucial difference with the topping up case is that there the possibility of altruistic children not providing help can be ruled out by assuming that $\beta > 1/H'(s+g)$. Here neither $\beta > 1/H'(s)$ nor $\beta > 1/H'(z)$ is sufficient for this purpose. No matter how large β is if z exceeds $\widehat{z}(\beta)$, altruistic children will not provide any assistance to their parents.

Observe that $\widehat{z}(\beta)$ is an increasing function of β. To see this, substitute $\widehat{z}(\beta)$ for z in (25) to get

$$-a^* + \beta H(s + a^*) - \beta H(\widehat{z}) = 0.$$

Differentiating, using the envelope theorem, yields

$$\frac{d\widehat{z}(\beta)}{d\beta} = \frac{H(s + a^*) - H(\widehat{z})}{\beta H'(\widehat{z})} > 0.$$

Observe also that if an altruistic child opts to help his parents, he must be providing them with a higher consumption level than what they can get from the government.

That is, it must be the case that $s + a^* > z$.[16] An implication of this is that parents who receive aid from their children will never ask the government for help; the program is self-targeted.

3.2 Stage 2: The parent's choice

Assume first that altruistic children come to the help of their parents. With $s + a^* = m(\beta)$, the expected utility of the parent is

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi\left[pH(m(\beta)) + (1-p)H(z)\right]. \qquad (26)$$

The parent's first-order condition with respect to s then results in,[17]

$$U'(s) = \frac{1}{1-\pi}. \qquad (27)$$

Note that, unlike the topping up case, the solution for s is independent of the probability of altruism, p, and government's level of assistance, z. We denote the solution to equation (27) by s^*.

Next consider the case where altruistic children do not help their parents. The parent's expected utility is now given by

$$EU = w(1-\tau)\overline{T} - s + (1-\pi)U(s) + \pi H(z). \qquad (28)$$

This leads to the first-order condition,

$$U'(s) = \frac{1}{1-\pi}.$$

This is identical to the optimal condition for s when altruistic children help their parents, equation (27). We thus have the same solution, s^*.

[16] This follows because when the altruistic child decides to help his parent, it follows from (23)–(24) that

$$H(s+a) - H(z) > a/\beta > 0.$$

[17] An interior solution is guaranteed by assuming that $U'(0) > 1/(1-\pi)$.

3.3 Stage 1: The optimal policy

The determination of optimal policy is a more complicated undertaking under the opting out scheme as compared to the topping up scheme. The government chooses its policy anticipating what the children and parents do. The complicating factor here is the discontinuous manner with which the government can, through its policy choice, induce altruistic children to move from providing a given level of assistance to none, and vice versa. A determining factor in this is the value of $\widehat{z}$ which depends on β. Another factor is how z affects the parents' expected utility when they receive assistance from their children as compared to when they do not. We study the relevance and the implications of these factors in the next section.

4 Optimal policy under the opting out scheme

Two observations help in figuring out if the government may want to opt for an equilibrium with or an equilibrium without assistance from children (as well as the corresponding levels of the assistance). The first observation is that, for a given level of z, parents enjoy a higher expected utility with children's assistance than without.[18] This follows because help from children is "free" but help from the government comes at the cost of taxing the parents. The second observation is that the value of z that maximizes the expected utility of the parent conditional on the parent's receiving aid from his children is the same as the value of z that maximizes the expected utility of the parent conditional on the parent's not receiving aid from his children.

[18] For the same z, the expected utility of parents will be higher by

$$\pi p \left\{ \left[H\left(m\left(\beta\right)\right) - H\left(z\right) \right] + \left(z - s^{*}\right) \right\} > 0,$$

under an equilibrium with assistance from altruistic children as compared to no assistance.

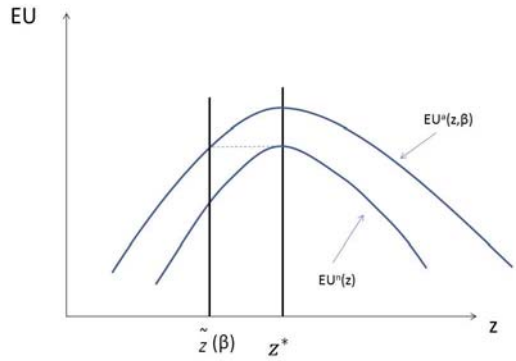

Figure 1: Parents' expected utility with and without probabilistic aid from children.

4.1 Altruistic children help

With targeted assistance, and help from altruistic children, the government's budget constraint changes to

$$\tau w\overline{T} + \pi (1-p) s = \pi (1-p) z, \tag{29}$$

as only parents who are not helped by their children ask for assistance (and pay the additional taxes).[19] Substitute for $\tau w\overline{T}$ from (29) into (26) to get

$$\begin{aligned} EU^a (z,\beta) &\equiv w (1-\tau) \overline{T} - s^* + (1-\pi) U (s^*) + \pi [pH (m (\beta)) + (1-p) H (z)] \\ &= w\overline{T} - \pi (1-p) z + \pi (1-p) s^* - s^* + \\ &\quad (1-\pi) U (s^*) + \pi [pH (m (\beta)) + (1-p) H (z)], \end{aligned} \tag{30}$$

where s^* is the solution to (27). Thus $EU^a (z,\beta)$ denotes the expected utility of a parent conditional on receiving probabilistic aid from his children when the government provision is z and the children's degree of altruism is β. Differentiating $EU^a (z,\beta)$

[19] Recall that when providing public care, the government taxes away the recipient's resources (savings and any private insurance that he may have purchased).

partially with respect to z yields

$$\frac{\partial EU^a(z,\beta)}{\partial z} = \pi(1-p)\left[H'(z)-1\right]. \tag{31}$$

Assuming $\partial EU^a(z,\beta)/\partial z|_{\tau=0} > 0$, or equivalently $H'(s^*) > 1$,[20] the value of z that maximizes $EU^a(z,\beta)$ is characterized by[21]

$$H'(z^*) = 1. \tag{32}$$

4.2 No help from altruistic children

With targeted assistance but no help from altruistic children, the government's budget constraint is

$$\tau w\overline{T} = \pi(z-s), \tag{33}$$

as it has to provide z for everyone who is dependent. Substituting for $\tau w\overline{T}$ from (33) into (28) yields

$$\begin{aligned} EU^n(z) &\equiv w(1-\tau)\overline{T} - s^* + (1-\pi)U(s^*) + \pi H(z) \\ &= w\overline{T} - \pi z + \pi s^* - s^* + (1-\pi)U(s^*) + \pi H(z), \end{aligned} \tag{34}$$

where s^* is, as previously, the solution to (27). Thus $EU^n(z)$ denotes the expected utility of a parent who receives no aid from his children when the government provision is z. Differentiating $EU^n(z)$ with respect to z yields

$$\frac{dEU^n(z)}{dz} = \pi\left[H'(z)-1\right].$$

Consequently, $z = z^*$ also maximizes $EU^n(z)$.

[20] Given that $U'(s^*) = 1/(1-\pi) > 1$, this condition is necessarily satisfied as long as $H'(s^*) > U'(s^*)$.

[21] Except for $p = 1$ at which $\tau = z = 0$ and the government does not have an optimization problem.

4.3 Three possible equilibria

Figure 1 shows the graphs for $EU^a(z, \beta)$ and $EU^n(z)$ in light of the above observations (for a given value of β). The figure also shows the value of $z < z^*$ at which the parents' expected utility when receiving probabilistic assistance from their children, $EU^a(z, \beta)$, is equal to $EU^n(z^*)$, the parents' expected utility at $z = z^*$ when they do not receive assistance. Denote this value by $\widetilde{z}(\beta)$; it is defined by $EU^a(\widetilde{z}(\beta), \beta) = EU^n(z^*)$ or

$$(1-p)\left[H(\widetilde{z}(\beta)) - \widetilde{z}(\beta)\right] = p\left[s^* - H(m(\beta))\right] + \left[H(z^*) - z^*\right].$$

The optimal value of z depends on the location of $\widehat{z}(\beta)$ in relation to $\widetilde{z}(\beta)$ and z^*. If $\widehat{z}(\beta)$ is to the left of $\widetilde{z}(\beta)$, the optimal solution for z is z^* with the solution entailing no aid from children; see Figure 2. If $\widehat{z}(\beta)$ is between $\widetilde{z}(\beta)$ and z^*, the optimal solution for z is $\widehat{z}(\beta)$; see Figure 3. By restricting z to $\widehat{z}(\beta)$, which is just low enough to entice the children to help, the government increases the parents' expected utility as compared to providing z^*.[22] Finally, if $\widehat{z}(\beta)$ is to the right of z^*, the government provides z^*; but in this case children also provide assistance (probabilistically); see Figure 4.

4.4 The degree of altruism and the equilibrium type

We have already seen that there are three possible equilibria: (a) z^* with no aid from children, (b) $\widehat{z}(\beta)$, and (c) z^* with probabilistic aid from children. We now prove that the type of equilibrium that emerges depends on the value of β, the children's degree of altruism towards their parents. Specifically, the equilibrium is of type (a) for "low" values of β, of type (b) for "moderate" values of β, and of type (c) for "very large" values of β.[23]

[22] We assume that the tie-breaker is to provide assistance whenever altruistic children are indifferent between providing and not providing assistance.

[23] It becomes clear below why we use the "very large" terminology.

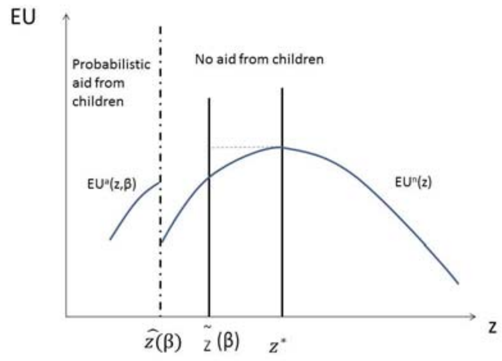

Figure 2: Equilibrium with no aid from children.

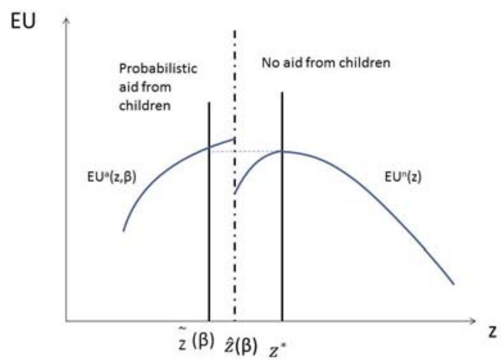

Figure 3: Constrained equilibrium with probabilistic aid from children.

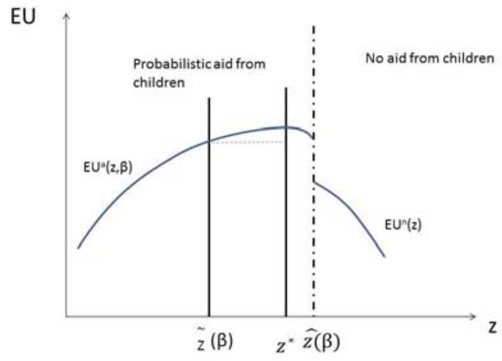

Figure 4: Unconstrained equilibrium with probabilistic aid from children.

Substitute $\widehat{z}(\beta)$ for z in (30), the expression for $EU^a(z,\beta)$, to get

$$\begin{aligned} EU^a(\widehat{z}(\beta),\beta) &= w\overline{T}+\pi(1-p)s^*-s^*+(1-\pi)U(s^*)- \\ &\quad \pi(1-p)\widehat{z}(\beta)+\pi[pH(m(\beta))+(1-p)H(\widehat{z}(\beta))]. \qquad (35) \end{aligned}$$

This shows the expected utility of parents at the maximum value of z beyond which their children, with a degree of altruism equal to β, will not provide them any assistance. For a given value of β, $EU^a(\widehat{z}(\beta),\beta)$ is represented in the figures by the point of intersection between the graphs of $EU^a(z,\beta)$ and $\widehat{z}(\beta)$. Clearly, by setting $z=\widehat{z}(\beta)$, the government can ensure the parents an expected utility equal to $EU^a(\widehat{z}(\beta),\beta)$. The other alternative for the government is to set $z=z^*$. If $\widehat{z}(\beta)<z^*$ there will be no aid forthcoming from the children so that the comparison would be between $EU^a(\widehat{z}(\beta),\beta)$ and $EU^n(z^*)$ (which is independent of β). On the other hand, if $\widehat{z}(\beta)>z^*$, there will be probabilistic aid from the children and the comparison would be between $EU^a(\widehat{z}(\beta),\beta)$ and $EU^a(z^*,\beta)$ (which also depends on β).

Recall from the discussion of children's choice in subsection 3.1 that $\beta>1/H'(s^*)$

to ensure the possibility of assistance from children. Let

$$\underline{\beta} \equiv \frac{1}{H'(s^*)}. \tag{36}$$

Starting at $\beta = \underline{\beta}$, we have $\widehat{z}\left(\underline{\beta}\right) = s^* < z^*$ and $EU^a\left(\widehat{z}\left(\underline{\beta}\right), \underline{\beta}\right) = EU^n\left(\widehat{z}\left(\underline{\beta}\right)\right) < EU^n(z^*)$. The solution is thus z^* with no assistance from children. Continuity implies that the solution remains the same in the neighborhood of $\beta = \underline{\beta}$.

Next differentiate $EU^a(\widehat{z}(\beta), \beta)$ with respect to β:

$$\frac{d}{d\beta} EU^a(\widehat{z}(\beta), \beta) = \pi \left\{ (1-p)\left[H'(\widehat{z}(\beta)) - 1\right] \frac{d\widehat{z}(\beta)}{d\beta} + pH'(m(\beta)) \frac{dm(\beta)}{d\beta} \right\}.$$

As long as $\widehat{z}(\beta) < z^*$, $H'(\widehat{z}(\beta)) > H'(z^*) = 1$ and the above expression is positive so that $EU^a(\widehat{z}(\beta), \beta)$ is increasing in β. The solution will thus change only when $EU^a(\widehat{z}(\beta), \beta)$ equals $EU^n(z^*)$ with $\widehat{z}(\beta) < z^*$. This is when $\widehat{z}(\beta) = \widetilde{z}(\beta)$. Denote the solution to this equation by $\beta = \widetilde{\beta}$; it is given by

$$(1-p)\left[H\left(\widehat{z}\left(\widetilde{\beta}\right)\right) - \widehat{z}\left(\widetilde{\beta}\right)\right] + pH\left(m\left(\widetilde{\beta}\right)\right) = H(z^*) - z^* + ps^*. \tag{37}$$

Hence z^* with no assistance from children is the optimal solution for all $\underline{\beta} < \beta < \widetilde{\beta}$.

Starting at $\beta = \widetilde{\beta}$, the solution changes to $\widehat{z}(\beta)$. It will remain so in the neighborhood of $\beta > \widetilde{\beta}$ because of continuity and the fact that $EU^a(\widehat{z}(\beta), \beta)$ is increasing in β at $\beta = \widetilde{\beta}$. As long as $\widehat{z}(\beta) < z^*$, $EU^a(z^*, \beta)$ is unattainable and the solution remains $EU^a(\widehat{z}(\beta), \beta)$. Now as β increases, both $EU^a(\widehat{z}(\beta), \beta)$ and $EU^a(z^*, \beta)$ increase. Nevertheless since $\widehat{z}(\beta)$ is increasing in β, at some point $\widehat{z}(\beta) = z^*$ so that $EU^a(\widehat{z}(\beta), \beta) = EU^a(z^*, \beta)$. Let $\widehat{\beta}$ denote the solution to $\widehat{z}(\beta) = z^*$. The question is what happens if $\beta > \widehat{\beta}$.

We now prove that $EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta) > 0$ whenever $\beta > \widehat{\beta}$ so that for these values of β we will have the z^* solution which entails probabilistic assistance from children. We have

$$EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta) = \pi(1-p)\left\{\widehat{z}(\beta) - H(\widehat{z}(\beta)) - [z^* - H(z^*)]\right\}.$$

Thus β affects $EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)$ only through z and not directly. But this function has a minimum value of zero at $\beta = \widehat{\beta}$ and will be positive otherwise.[24]

Finally, it is interesting to note that the z^* solution with probabilistic assistance from children occurs only if children exhibit an extremely high degree of altruism towards their parents. Specifically, if altruistic children consider government provision insufficient and substitute their own help, it must be the case that $a^* + s^* > z^*$. The concavity of $H(\cdot)$ then implies $H'(a^* + s^*) < H'(z^*)$. Now we have, from the first-order condition (22) for altruistic children providing help, $H'(a^* + s^*) = 1/\beta$. Additionally, we know that $H'(z^*) = 1$. Consequently, $1/\beta < 1$ so that $\beta > 1$. This means that children care about their dependent parents more than the parents care about themselves. One may refer to such children as "super altruistic".[25]

4.5 Full and less-than-full LTC insurance

Introduce g to denote the *net* contribution of the government to the dependent parent who seeks help. This is what the government provides net of the parent's savings, s; it is the LTC insurance. Hence[26]

$$g \equiv z - s. \tag{38}$$

Substituting from (38) into (32), while setting $s = s^*$, the optimal g is characterized by

$$H'(s^* + g^*) = 1,$$

[24] First- and second-order derivatives of $EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)$ with respect to β are

$$\begin{aligned}
\frac{\partial}{\partial\beta}\left[EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)\right] &= \pi(1-p)\left[1 - H'(\widehat{z}(\beta))\right]\frac{\partial\widehat{z}(\beta)}{\partial\beta}, \\
\frac{\partial^2}{\partial\beta^2}\left[EU^a(z^*, \beta) - EU^a(\widehat{z}(\beta), \beta)\right] &= \pi(1-p)\left\{-H''(\widehat{z}(\beta))\left[\frac{\partial\widehat{z}(\beta)}{\partial\beta}\right]^2 + \frac{\partial^2\widehat{z}(\beta)}{\partial\beta^2}\left[1 - H'(\widehat{z}(\beta))\right]\right\}.
\end{aligned}$$

At $\widehat{z}(\beta) = z^*$, the first-order derivative is equal to zero and the second-order derivative is positive.

[25] On the other hand, one could also argue that Alzheimer patients do not care much about what happens to them but their children want them to be cared for in a dignified and decent manner. We remain agnostic about this possibility and do not a priori rule out $\beta > 1$.

[26] If there is private insurance of the amount θ, the dependent parent will have $s + \theta + a$ if receiving assistance from his children, and z with the default of altruism. However, in this case θ is also taxed away so that $g = z - s - \theta$.

and we have full LTC insurance whenever the equilibrium is z^*, with or without aid from children.[27]

On the other hand, when equilibrium is the constrained $\widehat{z}(\beta)$, the solution entails less than full insurance. To see this, note that $\widehat{z}(\beta) > s^*$ so that there is positive LTC insurance. However, given that $\widehat{z}(\beta) < z^*$, we have

$$H'(\widehat{z}(\beta)) > H'(z^*) = 1.$$

Finally observe that, in contrast to the topping up scheme, the optimality of a positive LTC insurance no longer rests on the probability of altruism default being high (small p). With z^*, $\widehat{z}(\beta)$, and s^* not varying with p, the optimal level of LTC insurance g (whether $z^* - s^*$ or $\widehat{z}(\beta) - s^*$), is independent of p. The difference arises because under opting out, the government does not have to worry that the insurance intended for altruism default is automatically provided to all dependent parents. With no leakage of benefits to the parents who are helped by their altruistic children, and thus indirectly the altruistic children themselves, the probability of altruism does not matter.[28]The main results of this section are summarized in the following proposition.

Proposition 2 *Consider an opting-out LTC scheme. Altruistic children have a degree of altruism towards their parents equal to β. Let $\underline{\beta}$, defined by equation (36), denote the minimum value of β below which no children helps his parents. Let $\widehat{z}(\beta)$, defined by equation (25), denote the maximum value of public assistance z consistent with children, whose degree of altruism towards their parents is β, providing assistance to their parents.*

[27] The z^* solution with aid from children implies that children do not consider full insurance as "sufficient enough" and go on to substitute their own higher level of help for it.

[28] Except, of course, if $p = 1$ and no public LTC insurance is required. At $p < 1$, the calculus of LTC provision is as follows. Under topping up, providing one more dollar worth of g entails a marginal benefit of $\pi(1-p)H'(\cdot)$ and a marginal cost of π for a net benefit of $\pi[(1-p)H'(\cdot) - 1]$; see equations (9) and (10). Under opting out, there are two possibilities. With probabilistic aid from children, one extra dollar expenditure on z has a marginal benefit of $\pi(1-p)H'(\cdot)$ and a marginal cost of $\pi(1-p)$ for a net benefit of $\pi(1-p)[H'(\cdot) - 1]$; see equations (30) and (31) Without aid from altruistic children, the marginal benefit is $\pi H'(\cdot)$ and the marginal cost π resulting in a net benefit of $\pi[H'(\cdot) - 1]$; see equation (34). It is only in the topping up case that the sign of the net marginal benefit depends on p.

Let z^* denote the amount of z that entail full LTC insurance defined by $H'(z^*) = 1$. The expected utility of the parents who receive probabilistic aid from children, $EU^a(\widehat{z}(\beta), \beta)$, has the following properties:

(i) It is increasing at $\beta = \underline{\beta}$; it assumes a value equal to the expected utility of the parents who do not receive assistance from their children and are given z^* in public assistance at $\beta = \widetilde{\beta} > \underline{\beta}$, it attains its maximal value at $\widehat{\beta}$.

(ii) An opting out LTC scheme has three types of equilibria depending on the children's degree of altruism towards their parents: (a) For all $\underline{\beta} < \beta < \widetilde{\beta}$, the equilibrium is z^* with no assistance from children; (b) for all $\widetilde{\beta} < \beta < \widehat{\beta}$, the equilibrium is $\widehat{z}(\beta)$ so that probabilistic aid from children is forthcoming; and (c) for all $\beta > \widehat{\beta}$, the equilibrium is z^* with probabilistic assistance from children. This last equilibrium occurs only if $\beta > \widehat{\beta} > 1$ and children are "super altruistic."

(iii) The optimal amount of LTC insurance, $g^* = z - s^*$, is always positive and independent of the probability of altruism (as long as the probability of altruism is less than one). There is full insurance with equilibria z^* with and without probabilistic assistance from children; there is less than full insurance with the $\widehat{z}(\beta)$ equilibrium.

5 Opting out and private insurance

5.1 Solutions with no aid from children: $\underline{\beta} < \beta < \widetilde{\beta}$

Consider again the expected utility of the parent, if private insurance markets are operative, and fix the level of z at its optimal level, z^*. We have

$$EU = w(1-\tau)\overline{T} - s^* - q\theta + (1-\pi)U(s^*) + \pi H(z^*).$$

However, if parents buy θ in private insurance, given that the government will tax these resources away as well when providing z, we have

$$\tau w\overline{T} = \pi(z^* - s^* - \theta). \tag{39}$$

Substitute for τ from (39) into the expression for EU and simplify to get

$$EU = w\overline{T} - \pi\left(z^{*} - s^{*}\right) + \left(\pi - q\right)\theta - s^{*} + \left(1 - \pi\right)U\left(s^{*}\right) + \pi H\left(z^{*}\right). \tag{40}$$

At $\theta = 0$, we have the optimal solution when insurance is being provided solely by the government. To see if having $\theta > 0$ benefits or harms the parents, differentiate (40) with respect to θ. This results in

$$\frac{\partial EU}{\partial \theta} = \pi - q.$$

If the private insurance price is fair, we will have $q = \pi$. The private insurance will then be as good as public insurance. Otherwise, if $q > \pi$, public insurance will dominate. Intuitively, when no child helps his parents, public assistance is provided to all dependent parents. This makes its implicit price, when provided publicly, to be π again.

5.2 Solutions with probabilistic aid from children: $\beta > \widetilde{\beta}$

Consider again the expected utility of the parent, if private insurance markets are operative, and fix the level of z at its optimal level. Denote this by Z where Z stands for $\widehat{z}$ or z^{*} depending on whether we have an equilibrium with less than or full insurance. As previously, the consumption level of parents receiving assistance from their children remains the same at $m\left(\beta\right)$. Parents' savings will also remain at s^{*}. We have

$$EU = w\left(1-\tau\right)\overline{T} - q\theta - s^{*} + \left(1-\pi\right)U\left(s^{*}\right) + \pi\left[pH\left(m\left(\beta\right)\right) + \left(1-p\right)H\left(Z\right)\right],$$

where

$$\tau w\overline{T} = \pi\left(1-p\right)\left(Z - s^{*} - \theta\right). \tag{41}$$

Substitute for τ from (41) into the expression for EU. We have, upon simplification,

$$\begin{aligned} EU \;\; = \;\; & w\overline{T} - \pi\left(1-p\right)\left(Z - s^{*}\right) + \left[\pi\left(1-p\right) - q\right]\theta - s^{*} \\ & + \left(1-\pi\right)U\left(s^{*}\right) + \pi\left[pH\left(m\left(\beta\right)\right) + \left(1-p\right)H\left(Z\right)\right]. \end{aligned} \tag{42}$$

Again, at $\theta = 0$, we have the optimal solution when insurance is being provided solely by the government. Differentiating (42) with respect to θ yields

$$\frac{\partial EU}{\partial \theta} = \pi (1 - p) - q. \tag{43}$$

If the private insurance price is fair, $q = \pi$ so that $\partial EU/\partial \theta = -\pi p < 0$. Interestingly, then, even fair private insurance markets now cannot do as good a job as public insurance. Intuitively, to the extent that private insurance markets may exist, they do for the possibility of dependency and not the default of altruism. Thus the lowest rate they can offer is $q = \pi$. The opting out scheme, on the other hand, allows the government to effectively provide assistance against the default of altruism and that at an implicit price of $\pi (1 - p)$.[29] This is because only the parents who do not receive aid from their children request government's assistance even though all dependent parents can do so. Clearly, if $q > \pi$, providing assistance through private markets will be even worse.

The main results of this section are summarized in the following proposition.

Proposition 3 *Consider the opting-out LTC scheme described in Proposition 2:*

(i) If $\underline{\beta} < \beta < \widetilde{\beta}$ so that the equilibrium is z^* with no aid from children, the LTC insurance can be provided privately or publicly if private insurance is fair and publicly otherwise.

(ii) If $\beta > \widetilde{\beta}$ so that the equilibrium entails probabilistic aid from children, no private insurance is used even if insurance markets are fair. All LTC insurance, whether less-than-full or full, should be provided publicly.

6 Opting out versus topping up

So far we have assumed from the outset that the LTC policy was either of the topping up or the opting out type. To complete the picture we shall now examine the choice

[29] Fair insurance markets can duplicate this rate only if they can sell insurance against the default of altruism which is not possible.

between the two types of policies. Can we say that one approach always dominates the other or does the comparison depend on the parameters of the model?

It turns out that opting out always dominates when children are sufficiently altruistic and specifically when $\beta > \widehat{\beta}$ so that it involves an unconstrained solution with probabilistic aid.

To see this denote the optimal solutions under topping up and opting out schemes (with assistance from altruistic children), by superscripts OO and TU. From equations (13)–(14), the optimal level of government provision is characterized by

$$\begin{aligned} H'\left(s\left(p;g^{TU}\right)+g^{TU}\right) &= \frac{1}{1-p}>1, \\ U'\left(s\left(p;g^{TU}\right)\right) &= 1. \end{aligned}$$

Similarly, from equations (27), (32), and (38),

$$\begin{aligned} H'\left(s^{OO}+g^{OO}\right) &= 1, \\ U'\left(s^{OO}\right) &= \frac{1}{1-\pi}>1, \end{aligned}$$

where $s^{OO}=s^{*}$. Comparing the equations for $H'(\cdot)$ under the two schemes, and the equations for $U'(\cdot)$, it follows from the concavity of $H(\cdot)$ and $U(\cdot)$ that

$$\begin{aligned} s\left(p;g^{TU}\right)+g^{TU} &< s^{OO}+g^{OO}, \\ s\left(p;g^{TU}\right) &> s^{OO}. \end{aligned}$$

These results are interesting. They tell us that, assuming optimal government's policy, parents who are not helped by their children have a smaller consumption level if dependent and a higher consumption level if independent, under the topping up scheme as compared to the opting out scheme.

In turn, these inequalities imply

$$g^{TU}<g^{OO}.$$

Net assistance to dependent parents who receive no help from their children is always smaller under the topping up scheme as compared to opting out scheme. Finally, observe that one can always implement the optimal topping up policy under opting out with some resources left over (as the children will make up for the g^{TU} that will not be provided under an opting out scheme to the parents who are being assisted by their children). Hence

$$EU^{TU} < EU^{OO}.$$

Proposition 4 summarizes the results of this section.

Proposition 4 *Consider an opting out LTC wherein $\beta > \widehat{\beta}$ so that the equilibrium is z^* with probabilistic aid from children. Comparing this opting-out solution (OO) with the solution under topping up (TU), we have*

$$\begin{aligned} g^{OO} &> g^{TU}, \\ s^{OO} &< s\left(p; g^{TU}\right), \\ s^{OO} + g^{OO} &> s\left(p; g^{TU}\right) + g^{TU} \\ EU^{OO} &> EU^{TU}. \end{aligned}$$

Intuitively the dominance of OO does not come as a surprise. As shown in the previous section, under opting out public LTC can be targeted to the parents whose children's turn out not to be altruistic. In other words, under OO the public system can effectively provide insurance against the failure of altruism, while this is not possible with a *TU.*

However, this argument only goes through as long as the altruism is sufficiently high to ensure an unrestricted solution. For an intermediate degree of altruism $\widetilde{\beta} < \beta < \widehat{\beta}$ the more precise targeting comes at a price, namely that public LTC is distorted downward (to ensure continued aid from altruistic children). This makes the comparison ambiguous; the dominance result will by continuity go through for $\beta < \widehat{\beta}$ but sufficiently close. However, as β decreases further, one cannot rule out the possibility that the result is reversed.[30]

7 Concluding remarks

The idea of altruism default is not new. Yet there does not exist much work on it in economics and particularly in the area of long-term care where one expects it to have multiple implications. Even though the family continues to remain the prime source of LTC provision in the US and elsewhere, its role has been on the decline. Among the contributing factors in this trend are mobility of the children, increasing frequency of childless families, fading family norms, and higher labor force participation of middle-aged women. This phenomenon in conjunction with rising levels of life expectancy and an aging population pose challenging questions for policy makers concerning LTC provision. The current paper has studied the role of LTC insurance programs in a world in which family assistance is uncertain. And the main lesson that has emerged from it is that the type of insurance program the government may offer, "topping up" or "opting out," affects not only the desirability of private versus public insurance but also the desirability of insurance in general and its optimal size (whether private or public).

We have shown that under a topping up scheme, if the probability of altruism is high there is no need for insurance; public or private. At lower probabilities, LTC insurance is called for, albeit one that is less than full. The amount of insurance varies negatively with the probability of altruism. If private insurance markets for dependency are fair, private insurance will suffice (although public assistance is just as good). At higher than fair insurance premiums, public assistance dominates private insurance.

Studying an opting out scheme, the paper has shown the possibility of three equilibria depending on the children's degree of altruism towards their parents. With a "small"

[30] The comparison will now also depend on the level of p. Specifically, we know that when p is large enough the solution under TU involves no aid so that the solution under OO cannot be worse (no aid is also an option there).

degree of altruism, the optimal solution calls for full LTC insurance for everyone. Under this circumstance, even altruistic children provide no assistance to their parents. The required insurance may be provided privately if private insurance markets are fair.

Other types of equilibria emerge under "moderate" and "very large" degree of altruism. These equilibria can be supported only through the public opting out scheme. Private insurance markets cannot do the job even if they are fair. With the very large degree of altruism, the optimal solution is again one of full LTC insurance. Nevertheless altruistic children do not consider the public full insurance plan good enough and opt out of it; they provide their own assistance instead. Of course, the parents of non-altruistic children have no other option but public assistance. With the moderate degree of altruism, the optimal solution is one of less than full LTC insurance. This will be just small enough to entice the altruistic children to substitute their own assistance for the government's.

The most interesting feature of the opting out scheme is that it provides, indirectly, insurance against not only dependency but also the default of altruism. It does so by creating incentives for self-targeting; ensuring that only dependent parents who are not helped by their children seek help from the government. This feature cannot be replicated by private insurance markets. A word of caution is called for; however. The system works in part because the government has the ability to tax away the dependent elderly's resources (savings and any private insurance that they may have purchased). This a strong assumption. It is not at all clear that one's private assets are publicly observable to be taxed away at no cost.

In thinking about fruitful extensions of this work, a number of avenues come to mind. One is to allow for altruism default to be endogenously determined. Parents may be able to influence the likelihood of its occurrence by spending more time with their children instilling family values in them and fostering family solidarity. Another is to introduce heterogeneity in the model. Heterogeneity can take different forms: in parents' earnings, in the degree of altruism on the part of the children, and in dependency probability. All are realistic and worth studying. One other direction is to probe into the financing the public LTC schemes using a richer tool-box including non-linear income taxes. This is particularly needed in the presence of heterogeneity in earnings. Cremer *et al.* (2013) study a model wherein altruism default is endogenously determined while simultaneously introducing earnings heterogeneity. However, as in this paper, they assume that the financing comes from a proportional income tax levied on the parents.

Other research avenues include bringing the welfare of children explicitly into the analysis. In this paper, we have proceeded as if children live in a different community and that we are not concerned about their welfare. An assumption that has made us rule out their taxation as well. If they are in the story, one has to also find a satisfactory way for treating the altruistic component of their utility in the overall social welfare. Simple inclusion gives rise to the possibility of double counting.

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