Estimating and Bootstrapping Malmquist Indices*

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ABSTRACT

This paper develops a consistent bootstrap estimation procedure for obtaining confidence intervals for Malmquist indices of productivity and their decompositions. Although the exposition is in terms of input-oriented indices, the techniques can be trivially extended to the output orientation. The bootstrap methodology is an extension of earlier work described in Simar and Wilson (1996). Some empirical examples are also given, using data on Swedish pharmacies.

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1. INTRODUCTION

Färe *et al.* (1992) merge ideas on measurement of efficiency from Farrell (1957) and on measurement of productivity from Caves *et al.* (1982) to develop a Malmquist index of productivity change. Caves *et al.* define their input-based Malmquist productivity index as the ratio of two input distance functions, while assuming no technical inefficiency in the sense of Farrell. Färe *et al.* extend the Caves *et al.* approach by dropping the assumption of no technical inefficiency and devoloping a Malmquist index of productivity that can be decomposed into indices describing changes in technology and efficiency. We extend the Färe *et al.* approach by giving a statistical interpretation to their Malmquist productivity index and its components, and by presenting a bootstrap algorithm which may be used to estimate confidence intervals for the indices. This work will allow researchers to speak in terms of whether changes in productivity, efficiency, or technology are significant in a statistical sense.

The input-based Malmquist index of productivity developed by Färe *et al.* measures productivity change between time t_1 and t_2. This index, as well as its component indices describing changes in technology and efficiency, consists of ratios of input distance functions (a more rigorous description appears in the next section). However, Färe *et al.* do not distinguish between the underlying *true* distance functions and their *estimates.* For example, as a prelude to their equation (4) (page 88), they state that "the value of the distance function ... is obtained as the solution to the linear programming problem ...." Färe *et al.* are not alone in this regard; indeed, the literature on nonparametric efficiency measurement is filled with such statements. Lovell (1993) and others have labelled nonparametric, linear-programming based approaches to efficiency measurement as *deterministic,* which seems to suggest that these approaches have no statistical underpinnings. Yet, if one views production data as having been generated from a distribution with bounded support over the true production set, then efficiency, and changes in productivity, technology, and efficiency, are always measured relative to *estimates* of underlying, *true* frontiers, conditional on observed data resulting from the underlying (and unobserved) data-generating process. Consequently, the estimates researchers are interested in involve uncertainty due to sampling variation.

Simar and Wilson (1996) develop a bootstrap procedure which may be used to estimate confidence intervals for distance functions used to measure technical efficiency, and demonstrate that the key to statistically consistent estimation of these confidence intervals lies in the replication of the unobserved data-generating process. This paper extends those ideas to the case of Malmquist indices constructed from nonparametric distance function estimates using data from different time periods.

In the next section, we define the input-based Malmquist productivity index and the distance functions from which it is constructed, and describe how the productivity index can be decomposed into indices of efficiency change and technical shift. While we focus on input-based indices, one may trivially extend our results to output-based measures by merely modifying our notation. In section three, we examine how the Malmquist index and its component indices may be estimated nonparametrically using linear programming techniques. The bootstrap procedure is presented in section four. In section five, we illustrate the bootstrap estimation using a panel of data on Swedish pharmacies previously examined by Färe *et al.* Conclusions are given in the final section.

2. THE MALMQUIST INDEX AND ITS DECOMPOSITION

To begin, consider firms which produce m outputs from n inputs. Let $\boldsymbol{x} \in \mathbb{R}_+^n$ and $\boldsymbol{y} \in \mathbb{R}_+^m$ denote input and output vectors, respectively. The production possibilities set at time t is given by the closed set

$$\mathcal{P}^t = \{(\boldsymbol{x}, \boldsymbol{y}) \mid \boldsymbol{x} \text{ can produce } \boldsymbol{y} \text{ at time } t\}. \tag{2.1}$$

The boundary of this set is given by the intersection of $\mathcal{P}^t$ and the closure of its compliment, and may be represented as

$$\mathcal{B}^t = \left\{(\boldsymbol{x}, \boldsymbol{y}) \mid (\boldsymbol{x}, \boldsymbol{y}) \in \mathcal{P}^t,\ (\lambda\boldsymbol{x}, \boldsymbol{y}) \notin \mathcal{P}^t\ \forall\, 0 < \lambda < 1,\ (\boldsymbol{x}, \tau\boldsymbol{y}) \notin \mathcal{P}^t\ \forall\, \tau > 1\right\}. \tag{2.2}$$

The set $\mathcal{B}^t$ represents the technology faced by firms at time t, and is sometimes called the production frontier.

The set $\mathcal{P}^t$ may be described in terms of its section

$$\mathcal{X}^t(\boldsymbol{y}) = \left\{\boldsymbol{x} \in \mathbb{R}^n_+ \mid (\boldsymbol{x}, \boldsymbol{y}) \in \mathcal{P}^t\right\}, \tag{2.3}$$

which is the input requirement set. Shephard (1970) discusses assumptions one may reasonably make regarding $\mathcal{X}^t(\boldsymbol{x})$ (and hence $\mathcal{P}^t$ and $\mathcal{B}^t$); typical assumptions, which we adopt, are (i) $\mathcal{X}^t(\boldsymbol{y})$ is convex for all $\boldsymbol{y},\ t$; (ii) all production requires use of some inputs, *i.e.*, $0 \notin \mathcal{X}^t(\boldsymbol{y})$ if $\boldsymbol{y} \geq 0, \boldsymbol{y} \neq 0$; and (iii) both inputs and outputs are strongly disposable, *i.e.*, $\tilde{\boldsymbol{x}} \geq \boldsymbol{x} \in \mathcal{X}^t(\boldsymbol{y}) \Rightarrow \tilde{\boldsymbol{x}} \in \mathcal{X}^t(\boldsymbol{y})$ and $\tilde{\boldsymbol{y}} \geq \boldsymbol{y} \Rightarrow \mathcal{X}^t(\tilde{\boldsymbol{y}}) \subseteq \mathcal{X}^t(\boldsymbol{y})$.

Let subscripts $i, t,\ i = 1, \ldots, N,\ t = 1, 2$ denote observations on a particular firm i at time t. The Shephard (1970) input distance function for firm i at time t, relative to the technology existing at time t, is defined as

$$D_i^{t|t} \equiv \sup\left\{\theta > 0 \mid \boldsymbol{x}_{it}/\theta \in \mathcal{X}^t(\boldsymbol{y}_{it})\right\}. \tag{2.4}$$

Clearly, $D_i^{t|t} \geq 1$ iff $\boldsymbol{x}_{it} \in \mathcal{X}^t(\boldsymbol{y}_{it})$; therefore, $D_i^{t|t} \geq 1$. The distance function $D_i^{t|t}$ gives a normalized measure of distance from the ith firm's position in the input/output space at time t to the boundary of the production set at time t in the hyperplane where outputs remain constant. For distance measured relative to the contemporaneous technology as in (2.4), the distance function provides a measure of input technical efficiency at time t. The distance function $D_i^{t|t}$ measures the maximum feasible proportionate reduction at time t of the ith firm's inputs, holding output constant, subject to the technology at time t defined by (2.2). Firms where $D_i^{t|t} > 1$ are regarded as inefficient in a technical sense, while those where $D_i^{t|t} = 1$ are regarded as technically efficient.

Input distance functions may also be defined to measure normalized distance between a firm's position in the input/output space at time t_1 and some future technology at time t_2 by writing

$$D_i^{t_1|t_2} \equiv \sup\left\{\theta > 0 \mid \boldsymbol{x}_{it_1}/\theta \in \mathcal{X}^{t_2}(\boldsymbol{y}_{it_1})\right\}. \tag{2.5}$$

Alternatively, we may define $D_i^{t_2|t_1}$ by reversing the t_1, t_2 superscripts in (2.5); $D_i^{t_2|t_1}$ would measure normalized distance between a firm's position in the input/output space at time t_2 and a past technology at time t_1.

Following Caves *et al.* (1982), input-based Malmquist indexes measuring productivity change from time t_1 to time t_2 (relative to the technology at time t_1) may be defined as

$$\mathcal{M}_i(t_1, t_2|t_1) \equiv \frac{D_i^{t_2|t_1}}{D_i^{t_1|t_1}} \tag{2.6}$$

and (relative to the technology at time t_2) as

$$\mathcal{M}_i(t_1, t_2|t_2) \equiv \frac{D_i^{t_2|t_2}}{D_i^{t_1|t_2}}. \tag{2.7}$$

The Malmquist indices of Caves *et al.* (1982) can be defined so that one may compare two firms at a point in time or one firm over two periods; here, we compare one firm over two periods. In addition, Caves *et al.* assume $D_i^{t_1|t_1} = D_i^{t_2|t_2} = 1$; *i.e.,* they assume no technical inefficiency, which we allow.

The productivity index proposed by Färe *et al.* (1992) can be obtained by combining (2.6)–(2.7) by taking the geometric mean, yielding

$$\mathcal{M}_i(t_1, t_2) = \left(\frac{D_i^{t_2|t_1}}{D_i^{t_1|t_1}} \times \frac{D_i^{t_2|t_2}}{D_i^{t_1|t_2}} \right)^{(1/2)}. \tag{2.8}$$

Values $\mathcal{M}_i(t_1, t_2) < 1$ indicate improvements in productivity between t_1 and t_2, while values $\mathcal{M}_i(t_1, t_2) > 1$ indicate decreases in productivity from time t_1 to t_2 ($\mathcal{M}_i(t_1, t_2) = 1$ would indicate no change in productivity). Färe *et al.* then decompose this index into indices of efficiency and technology change by rewriting (2.8) as

$$\mathcal{M}_i(t_1, t_2) = \frac{D_i^{t_2|t_2}}{D_i^{t_1|t_1}} \times \left(\frac{D_i^{t_2|t_1}}{D_i^{t_2|t_2}} \times \frac{D_i^{t_1|t_1}}{D_i^{t_1|t_2}} \right)^{(1/2)}. \tag{2.9}$$

The ratio $D_i^{t_2|t_2}/D_i^{t_1|t_1}$ in (2.9) measures the change in input technical efficiency between periods t_1 and t_2; hence we can define an input-based index of efficiency change as

$$\mathcal{E}_i(t_1, t_2) \equiv \frac{D_i^{t_2|t_2}}{D_i^{t_1|t_1}}. \tag{2.10}$$

Values of $\mathcal{E}_i(t_1, t_2)$ less than (greater than) unity indicate improvements (decreases) in efficiency between t_1 and t_2.

The first ratio inside the parentheses in (2.9),

$$\mathcal{T}_i(t_1, t_2|t_2) \equiv \frac{D_i^{t_2|t_1}}{D_i^{t_2|t_2}}, \tag{2.11}$$

measures the position of the ith firm in input-output space at time t_2 relative to technologies at times t_1 and t_2. Thus, this ratio gives a measure of the shift in technology relative to the position of the ith firm at time t_2; Similarly, the second ratio inside the parentheses in (2.9),

$$\mathcal{T}_i(t_1, t_2|t_1) \equiv \frac{D_i^{t_1|t_1}}{D_i^{t_1|t_2}}, \tag{2.12}$$

measures the position of the ith firm in input-output space at time t_1 relative to technologies at times t_1 and t_2, and thus gives a measure of the shift in technology relative to the position of the ith firm at time t_1.[1] Hence, we can define an input-based measure of technical change as

$$\mathcal{T}_i(t_1, t_2) = \left(\frac{D_i^{t_2|t_1}}{D_i^{t_2|t_2}} \times \frac{D_i^{t_1|t_1}}{D_i^{t_1|t_2}} \right)^{(1/2)}, \tag{2.13}$$

which gives the geometric mean of two measures of the shift in technology from t_1 to t_2 in (2.11)–(2.12). As with $\mathcal{M}_i(t_1, t_2)$ and $\mathcal{E}_i(t_1, t_2)$, values of $\mathcal{T}_i(t_1, t_2)$ less than (greater than) unity indicate technical progress (regress) between times t_1 and t_2.[2]

3. ESTIMATING MALMQUIST INDICES

Unfortunately, the production set $\mathcal{P}^t$ is typically unobserved; similarly, $\mathcal{B}^t$ and $\mathcal{X}^t(\boldsymbol{x})$ are also unobserved, as are the values of the distance functions which appear in the Malmquist index in (2.9) and its components in (2.10) and (2.13). Similarly, the Malmquist index

[1] The measure $\mathcal{T}_i(t_1, t_2|t_1)$ is analgous to the measure of technical change used by Elyasiani and Mehdian (1990).

[2] Note that output distance functions can be defined similar to the input distance functions in (2.4)–(2.5). Hence output-based measures of productivity, efficiency, and technical change can be constructed by merely replacing the input distance functions in (2.9)–(2.13) with the corresponding output distance functions.

given in (2.9) and its components in (2.10) and (2.13) represent true values which must be estimated. Substituting estimators for the correpsonding true distance function values in (2.9)–(2.10) and (2.13) yields estimators $\widehat{\mathcal{M}}_i(t_1, t_2)$, $\widehat{\mathcal{E}}_i(t_1, t_2)$, and $\widehat{\mathcal{T}}_i(t_1, t_2)$ of the productivity, efficiency, and technology change indices, respectively.

Estimation of the input distance functions in (2.4)–(2.5) requires estimation of $\mathcal{P}^t$ and $\mathcal{X}^t(\boldsymbol{y})$. Given a sample $\mathcal{S} = \{(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}) \mid i = 1, \dots, N;\ t = 1, 2\}$ of observations on N firms in 2 periods, there are several ways in which $\mathcal{P}^t$ may be estimated. A common approach is to estimate $\mathcal{P}^t$ by the convex hull of the sample observations, which allows for the possibility of variable returns to scale:

$$\widehat{\mathcal{P}}^t = \left\{ (\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{m+n} \mid \boldsymbol{y} \leq \boldsymbol{Y}^t\boldsymbol{q},\ \boldsymbol{x} \geq \boldsymbol{X}^t\boldsymbol{q},\ \overrightarrow{\mathbf{1}}\boldsymbol{q}_i\ = 1,\ \boldsymbol{q} \in \mathbb{R}_+^N \right\}, \tag{3.1}$$

where $\boldsymbol{Y}^t = [\boldsymbol{y}_{1t} \dots \boldsymbol{y}_{Nt}]$, $\boldsymbol{X}^t = [\boldsymbol{x}_{1t} \dots \boldsymbol{x}_{Nt}]$, with $\boldsymbol{x}_{it}$ and $\boldsymbol{y}_{it}$ denoting $(n \times 1)$ and $(m \times 1)$ vectors of observed inputs and outputs, respectively, and where $\overrightarrow{\mathbf{1}}$ is a $(1 \times N)$ vector of ones and $\boldsymbol{q}$ is a $(N \times 1)$ vector of intensity variables.

As with $\mathcal{P}^t$, the set $\widehat{\mathcal{P}}^t$ may be described in terms of its section, namely

$$\widehat{\mathcal{X}}^t(\boldsymbol{y}) = \left\{ \boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{y} \leq \boldsymbol{Y}^t\boldsymbol{q},\ \boldsymbol{x} \geq \boldsymbol{X}^t\boldsymbol{q},\ \overrightarrow{\mathbf{1}}\boldsymbol{q}_i\ = 1,\ \boldsymbol{q} \in \mathbb{R}_+^N \right\} \tag{3.2}$$

which provides an estimator of $\mathcal{X}^t(\boldsymbol{y})$ in (2.3). Using $\widehat{\mathcal{X}}^t(\boldsymbol{y})$, estimators of $D_i^{t|t}$ and $D_i^{t_1|t_2}$ may be constructed by defining (respectively)

$$\widehat{D}_i^{t|t} \equiv \sup \left\{ \lambda > 0 \mid \boldsymbol{x}_{it}/\lambda \in \widehat{\mathcal{X}}^t(\boldsymbol{y}_{it}) \right\} \tag{3.3}$$

and

$$\widehat{D}_i^{t_1|t_2} \equiv \sup \left\{ \lambda > 0 \mid \boldsymbol{x}_{it_1}/\lambda \in \widehat{\mathcal{X}}^{t_2}(\boldsymbol{y}_{it_1}) \right\}. \tag{3.4}$$

These may be computed by solving (respectively) the linear programs

$$\left(\widehat{D}_i^{t|t}\right)^{-1} = \max \left\{ \lambda \mid \boldsymbol{y}_{it} \leq \boldsymbol{Y}^t\boldsymbol{q}_i,\ \lambda\boldsymbol{x}_{it} \geq \boldsymbol{X}^t\boldsymbol{q}_i,\ \overrightarrow{\mathbf{1}}\boldsymbol{q}_i\ = 1,\ \boldsymbol{q}_i \in \mathbb{R}_+^N \right\} \tag{3.5}$$

and

$$\left(\widehat{D}_i^{t_1|t_2}\right)^{-1} = \max\left\{\lambda \mid \boldsymbol{y}_{it_1} \leq \boldsymbol{Y}^{t_2}\boldsymbol{q}_i,\ \lambda\boldsymbol{x}_{it_1} \geq \boldsymbol{X}^{t_2}\boldsymbol{q}_i,\ \overrightarrow{\boldsymbol{1}}\boldsymbol{q}_i\ = 1,\ \boldsymbol{q}_i \in \mathbb{R}_+^N\right\}. \tag{3.6}$$

Estimates $\widehat{D}_i^{t_2|t_1}$ can be computed by solving the linear program obtained by reversing the t_1 and t_2 super- and subscripts in (3.6).

Alternatively, the production set $\mathcal{P}^t$ can be estimated by the conical hull of the sample observations:

$$\widetilde{\mathcal{P}}^t = \left\{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{m+n} \mid \boldsymbol{y} \leq \boldsymbol{Y}^t\boldsymbol{q},\ \boldsymbol{x} \geq \boldsymbol{X}^t\boldsymbol{q},\ \boldsymbol{q} \in \mathbb{R}_+^N\right\}. \tag{3.7}$$

As before, the set $\widetilde{\mathcal{P}}^t$ can be described in terms of its section

$$\widetilde{\mathcal{X}}^t(\boldsymbol{y}) = \left\{\boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{y} \leq \boldsymbol{Y}^t\boldsymbol{q},\ \boldsymbol{x} \geq \boldsymbol{X}^t\boldsymbol{q},\ \boldsymbol{q} \in \mathbb{R}_+^N\right\}, \tag{3.8}$$

which provides another estimator of $\mathcal{X}^t(\boldsymbol{y})$ in (2.3). Replacing $\mathcal{X}^t(\boldsymbol{y})$ in (2.4)–(2.5) with $\widetilde{\mathcal{X}}^t(\boldsymbol{y})$ leads to the linear programs

$$\left(\widetilde{D}_i^{t|t}\right)^{-1} = \max\left\{\lambda \mid \boldsymbol{y}_{it} \leq \boldsymbol{Y}^t\boldsymbol{q}_i,\ \lambda\boldsymbol{x}_{it} \geq \boldsymbol{X}^t\boldsymbol{q}_i,\ \boldsymbol{q}_i \in \mathbb{R}_+^N\right\} \tag{3.9}$$

and

$$\left(\widetilde{D}_i^{t_1|t_2}\right)^{-1} = \max\left\{\lambda \mid \boldsymbol{y}_{it_1} \leq \boldsymbol{Y}^{t_2}\boldsymbol{q}_i,\ \lambda\boldsymbol{x}_{it_1} \geq \boldsymbol{X}^{t_2}\boldsymbol{q}_i,\ \boldsymbol{q}_i \in \mathbb{R}_+^N\right\}. \tag{3.10}$$

The distance function estimators in (3.9)–(3.10) based on the conical hull can be substituted into (2.9)–(2.10) and (2.13) to yeild estimators $\widetilde{\mathcal{M}}_i(t_1, t_2)$, $\widetilde{\mathcal{E}}_i(t_1, t_2)$, and $\widetilde{\mathcal{T}}_i(t_1, t_2)$ of the productivity, efficiency, and technology change indices, respectively.

Other estimators are also possible; for instance, one could use the free disposal hull of the sample observations suggested by Deprins *et al.* (1984). Use of the conical hull in (3.7) implicitly assumes that the technology $\mathcal{B}^t$ exhibits constant returns to scale. Färe *et al.* (1992) as well as most others who have used Malmquist indices have assumed constant returns to scale, perhaps because allowing for variable returns by using the convex hull estimator does not guarantee solutions to (3.6) for all observations.

If the underlying true technology has constant returns to scale, then both $\widehat{\mathcal{P}}^t$ and $\widetilde{\mathcal{P}}^t$ will converge to $\mathcal{P}^t$ as $N \to \infty$. However, if the true technology has variable returns to scale, then $\widehat{\mathcal{P}}^t$ will converge to $\mathcal{P}^t$ as $N \to \infty$, but $\widetilde{\mathcal{P}}^t$ will not. Hence $\widetilde{\mathcal{P}}^t$ and the distance function estimators in (3.9)–(3.10) are inconsistent in a statistical sense when the true technology has variable returns to scale. Therefore, using distance function estimators based on the convex hull of the data may be a safer approach. In the empirical examples which appear below, we present results using both estimators.[3]

4. BOOTSTRAPPING THE MALMQUIST INDICES

The methodology for bootstrapping distance function estimators such as (3.3) presented in Simar and Wilson (1996) are easily adapted to the present case, except here the possible time-dependence structure of the data must be taken into account. As in our earlier work, we assume a data-generating process where firms randomly deviate from the underlying true frontier in a radial input direction. These random deviations are further assumed to result from inefficiency, and have density f. Bootstrapping involves replicating this data-generating process, generating an appropriately large number B of pseudo samples $\mathcal{S}^* = \{(\boldsymbol{x}_{it}^*, \boldsymbol{y}_{it}^*) \mid i = 1, \ldots, N;\ t = 1, 2\}$, and applying the original estimators to these pseudo samples. For example, if the distance function estimators based on the convex hull are used, this last step will yield bootstrap estimates $\left\{\widehat{D}_i^{t_1|t_1*}(b),\ \widehat{D}_i^{t_2|t_2*}(b),\ \widehat{D}_i^{t_1|t_2*}(b),\ \widehat{D}_i^{t_2|t_1*}(b)\right\}_{b=1}^{B}$ for each $i = 1, \ldots, N$. These estimates can then be used to construct bootstrap estimates $\widehat{\mathcal{M}}_i^*(t_1, t_2)(b)$, $\widehat{\mathcal{E}}_i^*(t_1, t_2)(b)$, and $\widehat{\mathcal{T}}_i^*(t_1, t_2)(b)$ (where $i = 1, \ldots, N$ and $b = 1, \ldots, B$) corresponding to (2.9)–(2.10) and (2.13), respectively, by replacing the true distance function values in (2.9)–(2.10) and (2.13) with their corresponding bootstrap estimates.

Once these bootstrap values have been computed, bias-corrected confidence intervals at

[3] In effect, each of the distance function estimators is computed by measuring the normalized radial distance from an observed point in the input/output space to the boundary of an estimate the production set. Korostelev *et al.* (1995) give the rate of convergence of $\widehat{\mathcal{P}}^t$ to $\mathcal{P}^t$ for some special cases.

the desired level of significance can be constructed using the methods described in Simar and Wilson (1996). To illustrate this methodology, consider the set of bootstrap estimates for the malmquist index for firm i: $\left\{\widehat{\mathcal{M}}^*_i(t_1,t_2)(b)\right\}_{b=1}^{B}$. The bootstrap bias estimate for the original estimator $\widehat{\mathcal{M}}_i(t_1,t_2)$ is

$$\text{Est. Bias}[\widehat{\mathcal{M}}_i(t_1,t_2)] = B^{-1}\sum_{b=1}^{b}\widehat{\mathcal{M}}^*_i(t_1,t_2)(b) - \widehat{\mathcal{M}}_i(t_1,t_2), \tag{4.1}$$

which is the empirical bootstrap analog of $E[\widehat{\mathcal{M}}_i(t_1,t_2)] - \mathcal{M}_i(t_1,t_2)$. Therefore, a bias-corrected estimate of $\mathcal{M}_i(t_1,t_2)$ may be computed as

$$\widehat{\widehat{\mathcal{M}}}_i(t_1,t_2) = \widehat{\mathcal{M}}_i(t_1,t_2) - \text{Est. Bias}[\widehat{\mathcal{M}}_i(t_1,t_2)]. \tag{4.2}$$

Of course, the bias-corrected estimate $\widehat{\widehat{\mathcal{M}}}_i(t_1,t_2)$ might have larger mean square error than the uncorrected estimate $\widehat{\mathcal{M}}_i(t_1,t_2)$, and so the correction in (4.2) should be used with caution. However, if the estimated variance of $\widehat{\mathcal{M}}_i(t_1,t_2)$ is much less than the square of the bias estimated from (4.1), then the bias correction in (4.1) is unlikely to increase mean square error.[4]

To obtain the bias-corrected confidence intervals, note that the bootstrapped values $\left\{\widehat{\mathcal{M}}^*_i(t_1,t_2)(b)\right\}_{b=1}^{B}$ give a biased approximation to the sampling distribution of $\widehat{\mathcal{M}}_i(t_1,t_2)$. To remove the bias, we compute

$$\widehat{\widehat{\mathcal{M}}}^*_i(t_1,t_2)(b) = \widehat{\mathcal{M}}^*_i(t_1,t_2)(b) - \Big(2\times \text{Est. Bias}[\widehat{\mathcal{M}}_i(t_1,t_2)]\Big) \quad \forall\, b = 1,\ldots,B. \tag{4.3}$$

Note that merely subtracting Est. Bias$[\widehat{\mathcal{M}}_i(t_1,t_2)]$ in (4.3) would center the distribution of the $\widehat{\mathcal{M}}^*_i(t_1,t_2)(b)$ on $\widehat{\mathcal{M}}_i(t_1,t_2)$, but $\widehat{\mathcal{M}}_i(t_1,t_2)$ must also be adjusted to remove bias; hence two times the estimated bias is the correct adjustment factor.[5] The values

[4] The variance of $\widehat{\mathcal{M}}_i(t_1,t_2)$ may be estimated by the sample variance of the bootstrap estimates $\left\{\widehat{\mathcal{M}}^*_i(t_1,t_2)(b)\right\}_{b=1}^{B}$. Note that we refer to $\widehat{\widehat{\mathcal{M}}}_i(t_1,t_2)$ as a bias-corrected, rather than an unbiased, estimator, since (4.2) involves only a first-order correction of the bias in $\widehat{\mathcal{M}}_i(t_1,t_2)$.

[5] The reader can indeed verify that $B^{-1}\sum_{b=1}^{b}\widehat{\mathcal{M}}^*_i(t_1,t_2)(b) = \widehat{\widehat{\mathcal{M}}}_i(t_1,t_2)$ to demonstrate that the correct adjustment factor is used in (4.3).

$\left\{\widehat{\widehat{\mathcal{M}}}_i^*(t_1,t_2)(b)\right\}_{b=1}^{B}$ provide a bias-corrected approximation of the sampling distribution of $\widehat{\widehat{\mathcal{M}}}_i(t_1,t_2)$.

Efron's (1982) percentile method involves sorting the values $\left\{\widehat{\widehat{\mathcal{M}}}_i^*(t_1,t_2)(b)\right\}_{b=1}^{B}$ by algebraic value and deleting $(1-\alpha/100)/2\times B$ elements from either end of the sorted array to obtain two-sided α-percent confidence intervals for the original estimator $\widehat{\mathcal{M}}_i(t_1,t_2)$.[6] The bootstrap methodology outlined by Simar and Wilson (1996) can also be used to construct confidence intervals for the distance function estimators used in estimating the Malmquist index. However, when bias-corrected bootstrap distance function estimates are computed analagously to (4.3), some of the bias-corrected bootstrap distance function values may be less than the corresponding original distance function estimate. Since all estimation is conditional on the sample data, the true (input) distance function values comprising the true Malmquist index in (2.9) cannot be less than the initial estimate obtained from (3.3) or (3.4). Thus, we condition on the observed data by omitting elements from the set $\left\{\widehat{\widehat{\mathcal{M}}}_i^*(t_1,t_2)(b)\right\}_{b=1}^{B}$ if the element is comprised of one or more bias-corrected bootstrap distance function estimates which are less than the corresponding original estimate.[7] Efron's percentile method is then applied using the remaining values of $\widehat{\widehat{\mathcal{M}}}_i^*(t_1,t_2)(b)$.[8]

The key to obtaining consistent bootstrap estimates of the confidence intervals lies in replicating the data-generating process. As discussed in Simar and Wilson (1996), resampling from the empirical distribution of the data to construct the pseudo samples will lead to inconsistent bootstrap estimation of the confidence intervals. However, using a

[6] Alternatively, one could employ the bias-corrected and accelerated (BC_a) described by Efron and Tibshirani (1993). However, since additional jackknife estimates are required to estimate the acceleration parameter in this method, the computational burden is likely to be too great for datasets with more than 100–200 observations.

[7] If $\widehat{D}_i > 1$ is the original distance function estimate and $\widehat{\widehat{D}}_i^*$ is the corresponding bias-corrected bootstrap estimate obtained by making a correction analogously to (4.3), then $\widehat{\widehat{D}}_i^* < \widehat{D}_i$ would suggest that the true frontier lies interior to the original estimate of the production frontier. The data, however, refute this possibility.

[8] We have illustrated the bootstrap procedure in terms of estimates of the productivity index; trivially changing the notation in (4.1)–(4.3) leads to bootstrap algorithms for the efficiency and technology change indices.

smooth bootstrap procedure as in Simar and Wilson (1996) will yield consistent estimates. When bootstrapping distance function estimates from a single cross-section of data, this may be accomplished by using a univariate kernel estimator of the density of the original distance function estimates, and then drawing from this estimated density to construct the pseudo samples $\mathcal{S}^*$ as in and Simar and Wilson (1996). In the present case, however, we have panel data, with the possibility of temporal correlation.[9] To preserve the temporal correlation, we use kernel methods to estimate the joint density of $\left\{(\widehat{D}_i^{t_1,t_1}, \widehat{D}_i^{t_2,t_2})\right\}_{i=1}^N$.

The bivariate kernel density estimator with bivariate kernel function $K(\cdot)$ and bandwidth h is given by

$$\widehat{f}(\boldsymbol{z}) = N^{-1}h^{-2}\sum_{i=1}^{N} K\left(\frac{(\boldsymbol{z} - \boldsymbol{Z}_i)}{h}\right), \tag{4.4}$$

where $\boldsymbol{z}$ has dimension (1×2) and $\boldsymbol{Z}_i$ is the ith row of the $(N \times 2)$ matrix containing the original data.[10] Note, however, that both $\widehat{D}_i^{t_1,t_1}$ and $\widehat{D}_i^{t_2,t_2}$ are bounded from below by unity. The density estimated from (4.4) can be shown to be inconsistent and asymptotically biased when the support of f is bounded.

To overcome this problem, we adapt the univariate reflection method described by Silverman (1986) to our bivariate case. For the case of univariate data $\{z_i\}_{i=1}^N$, bounded from below at unity, the reflection method involves using the univariate kernel density estimator to estimate the density of the original observations and their reflections $\{z_i'\}_{i=1}^N$ about unity, where $z_i' = 2 - z_i \ \forall \ i = 1, \ldots, N$. Truncating the resulting density estimate on the left at unity yields the desired density estimate for the univariate case (see Simar and Wilson, 1996, for an illustration). In the bivariate case, we proceed similarly, except that there are now two boundaries in two-dimensional space.

First, we form vectors

$$\boldsymbol{A} = \left[\, \widehat{D}_1^{t_1,t_1} \quad \ldots \quad \widehat{D}_N^{t_1,t_1} \,\right] \tag{4.5}$$

[9] For example, an inefficient firm in period one may be more likely to be inefficient in period two than a firm that is relatively more efficient in period one.

[10] ONe might prefer to use different bandwidths in each direction; however, this is not necessary if the data are rescaled by the decomposed sample covariance matrix as discussed below.

and

$$\boldsymbol{B} = \left[\, \widehat{D}_1^{t_2,t_2} \quad \ldots \quad \widehat{D}_N^{t_2,t_2} \,\right]. \tag{4.6}$$

To reflect the distance function values about the boundaries in two-dimensional space, we form the $(4N \times 2)$ matrix

$$\boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{A} & 2-\boldsymbol{A} & \boldsymbol{A} & 2-\boldsymbol{A} \\ \boldsymbol{B} & \boldsymbol{B} & 2-\boldsymbol{B} & 2-\boldsymbol{B} \end{bmatrix}'. \tag{4.7}$$

The matrix $\boldsymbol{\Delta}$ contains $4N$ pairs of values corresponding to the two time periods; the temporal correlation of this cloud of points is measured by the sample covariance matrix

$$\widehat{\boldsymbol{\Sigma}} = \begin{bmatrix} \hat{\sigma}_1^2 & \hat{\sigma}_{12} \\ \hat{\sigma}_{12} & \hat{\sigma}_2^2 \end{bmatrix} \tag{4.8}$$

of the columns of $\boldsymbol{\Delta}$.[11]

To generate the random deviates needed for the bootstrap, we do not have to actually estimate the density of the observations in $\boldsymbol{\Delta}$; rather, we use the method suggested by Silverman (1986) and analogous to that used for the univariate case in Simar and Wilson (1996). The Cholesky decomposition of $\widehat{\boldsymbol{\Sigma}}$ yields the lower triangular matrix $\boldsymbol{L} = \begin{bmatrix} \ell_1 & 0 \\ \ell_2 & \ell_3 \end{bmatrix}$, where $\boldsymbol{L}\boldsymbol{L}' = \widehat{\boldsymbol{\Sigma}}$, $\ell_1 = \hat{\sigma}_1$, $\ell_2 = \hat{\sigma}_{12}/\hat{\sigma}_1$, and $\ell_3 = \left(\hat{\sigma}_2^2 - \frac{\hat{\sigma}_{12}^2}{\hat{\sigma}_1^2}\right)^{1/2}$. Specifying the kernel function $K(\cdot)$ as bivariate normal with covariance matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, we draw $\boldsymbol{\epsilon}$, an $(N \times 2)$ matrix of independent, identically distributed standard normal deviates. Then compute

$$\boldsymbol{\epsilon}^* = \boldsymbol{\epsilon}\boldsymbol{L}', \tag{4.9}$$

which gives an $(N \times 2)$ matrix of normal deviates with the same correlation structure as the data in $\boldsymbol{\Delta}$.

[11] To visualize the reflection, let a_i, b_i represent the ith elements of $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively ($i = 1, \ldots, N$). Then for N firms at times t_1 and t_2, we have N points (a_i, b_i) lying northeast of the point $(1,1)$ in two-dimensional Euclidean space. These points may be reflected by taking the "mirror images" about vertical and horizontal lines passing through $(1,1)$. This leaves a boundary along the horizontal line passing through $(1,1)$ to the left of this point, and along the vertical line passing through $(1,1)$ below this point. Hence, an additional reflection is required, obtained by taking the mirror image of the points to the southeast of $(1,1)$ about the vertical line passing through $(1,1)$, or equivalently, by taking the mirror image of the points to the northwest of $(1,1)$ about the horizontal line passing through $(1,1)$. Hence $\boldsymbol{\Delta}$ in (4.7) has $4N$ rows.

Next, randomly draw with replacement N rows from $\boldsymbol{\Delta}$ to form the $(N \times 2)$ matrix $\boldsymbol{\Delta}^* = [\delta_{ij}]$, $i = 1, \ldots, N$, $j = 1, 2$ such that each row of $\boldsymbol{\Delta}$ has equal probability of selection. Let $\bar{\delta}_{\cdot j} = N^{-1} \sum_{i=1}^{N} \delta_{ij}$ for $j = 1, 2$. Then compute the $(N \times 2)$ matrix

$$\boldsymbol{\Gamma} = (1 + h^2)^{-1/2} \left(\boldsymbol{\Delta}^* + h\boldsymbol{\epsilon}^* - \boldsymbol{C} \begin{bmatrix} \bar{\delta}_{\cdot 1} & 0 \\ 0 & \bar{\delta}_{\cdot 2} \end{bmatrix} \right) + \boldsymbol{C} \begin{bmatrix} \bar{\delta}_{\cdot 1} & 0 \\ 0 & \bar{\delta}_{\cdot 2} \end{bmatrix}, \tag{4.10}$$

where $\boldsymbol{C}$ is an $(N \times 2)$ matrix of ones, which gives an $(N \times 2)$ matrix of bivariate deviates from the estimated density of $\boldsymbol{\Delta}$, scaled to have the first and second moment properties observed in the original sample represented by $\boldsymbol{\Delta}$. Finally, for each element γ_{ij} of $\boldsymbol{\Gamma}$, set

$$\gamma_{ij}^* = \begin{cases} \gamma_{ij}, & \text{if } \gamma_{ij} \geq 1 \\ 2 - \gamma_{ij}, & \text{otherwise.} \end{cases} \tag{4.11}$$

The resulting $(N \times 2)$ matrix $\boldsymbol{\Gamma}^* = [\gamma_{ij}^*]$ consists of two column-vectors of simulated distance function values. Pseudo samples $\mathcal{S}^*$ are then constructed by setting $\boldsymbol{x}_{it_j}^* = \gamma_{ij}^* \boldsymbol{x}_{it_j} / \widehat{D}_i^{t_j|t_j}$ and $\boldsymbol{y}_{it_j}^* = \boldsymbol{y}_{it_j}$ for $i = 1, \ldots, N$, $j = 1, 2$.[12]

The only remaining issue is the choice of the bandwidth, h. Tapia and Thompson (1978), Silverman (1978, 1986), and Härdle (1990) discuss considerations relevant to the choice of h; in general, for a given sample size, larger values of h produce more diffuse (*i.e.,* less efficient) estimates of the density, while very small values produce estimated densities with multiple modes. In the empirical examples which follow, we use Silverman's (1986) suggestion for bivariate data by setting

$$h = 0.96\hat{\sigma}N^{-1/6}, \tag{4.12}$$

where $\hat{\sigma}$ is average of the sample standard deviations of the elements in the columns of $\boldsymbol{\Delta}$.[13]

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[12] Computing $\boldsymbol{x}_{it_j} / \widehat{D}_i^{t_j|t_j}$ scales the input vector back to the ostensibly efficient level indicated by the estimated frontier; multiplying by γ_{ij}^* simulates a random deviation away from this frontier. If we were using output distance functions and the output-based Malmquist index, we would retain the original input vector in the pseudo sample and generate a new output vector.

[13] Alternatively, one might use the likelihood cross-validation procedure discussed by Silverman (1986) to choose h. The results in Simar and Wilson (1996) suggest, however, that the estimated confidence intervals are not very sensitive with respect to the choice of bandwidth.

5. EMPIRICAL EXAMPLES

Färe *et al.* (1992) describe annual data on 42 Swedish pharmacies from 1980 to 1989, which produce four outputs from four inputs. Färe *et al.* assume constant returns to scale and estimate distance functions using (3.9)–(3.10) to construct estimates $\widetilde{\mathcal{E}}_i(t_1, t_2)$, $\widetilde{\mathcal{T}}_i(t_1, t_2)$, and $\widetilde{\mathcal{M}}_i(t_1, t_2)$, which they report in Tables 1–3 of their article. Maintaining the assumption of constant returns to scale, we applied the bootstrap methods outlined in section four to obtain bias corrected estimates $\widetilde{\widetilde{\mathcal{E}}}_i(t_1, t_2)$, $\widetilde{\widetilde{\mathcal{T}}}_i(t_1, t_2)$, and $\widetilde{\widetilde{\mathcal{M}}}_i(t_1, t_2)$. Consistent with Färe *et al.,* we report the reciprocals of our estimates in Tables 1–3 (respectively) so that numbers greater than unity denote progress while numbers less than unity denote regress. Single asterisks (*) indicate cases where the indices are significantly different from unity at the .10 level; double asterisks (**) indicate cases where the indices are significantly different from unity at the .05 level.

While examining changes in efficiency, Färe *et al.* find (page 96) "five pharmacies (nos. 15, 32, 33, 35, and 39) to be efficient in all time periods," reflecting their failure to distinguish between true distance function values and their corresponding estimates. Consequently they report values of unity for efficiency change between all successive pairs of years in Table 1 of their article for these five pharmacies. By contrast, our bootstrap results in Table 1 below indicate that only three pharmacies (nos. 2, 14, and 33) had no significant changes in efficiency across the study period.

Turning to our results for the technical change index in Table 2, our bootstrap results generally support the statements made by Färe *et al.* with respect to technical change. Färe *et al.* state that "between 1981 and 1982 almost all pharmacies showed technical progress," and our results show that these changes are significant at the .05 level in all but two instances. They find only one pharmacy (no. 33) showing technical progress in all periods, but our results indicate that the changes are not significant in four periods.

Similarly, our results for the index of productivity change in Table 3 generally support statements made by Färe *et al.* Where Färe *et al.* find productivity gains in 259 cases and productivity losses in 119 cases, we find significant (at .10) gains in 226 cases, and significant (at .10) losses in 95 cases. Because of the bias correction afforded by our bootstrap methods, the magnitudes of the changes we report are different from those reported by Färe *et al.*

As noted in section three, assuming constant returns to scale and using the conical hull of the observed data to estimate the production set will yield statistically inconsistent distance function estimates when the true technology has nonconstant returns to scale. The convex hull estimator of the production set, however, converges to the true production set regardless of whether returns to scale are constant or otherwise. Therefore, lacking a formal test of returns to scale, using distance function estimators based on the convex hull as in (3.3)–(3.4) to construct the Malmquist index may be a safer approach.

Using the distance function estimators in (3.3)–(3.4) and the bootstrap methodology outlined in section four, we computed biased corrected estimates $\widehat{\widehat{\mathcal{E}}}_i(t_1, t_2)$, $\widehat{\widehat{\mathcal{T}}}_i(t_1, t_2)$, and $\widehat{\widehat{\mathcal{M}}}_i(t_1, t_2)$ of the efficiency, technology, and productivity change indices, respectively. Results are reported in Tables 4–6, where again single asterisks indicate cases where the indices are significantly different from unity at the .10 level, and double asterisks indicate cases where the indices are significantly different from unity at the .05 level.

Using the convex hull-based estimators indicates 195 significant changes in efficiency in Table 4, 19 more than were found when assuming constant returns to scale. The efficiency changes in Table 4 are generally smaller (*i.e.,* closer to unity) than those shown in Table 1. In addition, we find nine instances where the estimated efficiency change is significant in both Tables 1 and 4, but in opposite directions.

Using the convex hull-based estimators to estimate $\mathcal{T}_i(t_1, t_2)$ in Table 5 yields a few cases where the estimator is undefined. For these cases, the constraints of the linear program in (3.5) are infeasible; in such cases, no contraction or expansion of the input vector for a firm at time t_1 (t_2) can reach the production frontier estimated at time t_2 (t_1). Clearly, technical change is occurring in these cases, but the convex hull-based estimator of

$\mathcal{T}_i(t_1, t_2)$ is undefined. Similarly, since $\mathcal{M}_i(t_1, t_2) = \mathcal{E}_i(t_1, t_2) \times \mathcal{T}_i(t_1, t_2)$, the convex hull-based estimator of $\mathcal{M}_i(t_1, t_2)$ is undefined in the same cases. Fortunately, this problem arises in a relatively small number of cases.

The results for the convex hull-based estimators of $\mathcal{T}_i(t_1, t_2)$ and $\mathcal{M}_i(t_1, t_2)$ shown in Tables 5–6 are somewhat different than the corresponding estimates in Tables 2–3 based on the conical hull. In some cases, using the convex hull-based estimators yields larger estimated changes (*i.e.,* estimates farther from unity) than the conical hull-based estimators, while in other cases the reverse is true. There seems to be no clear pattern. To reiterate, the convex hull-based estimators are statistically consistent regardless of the shape of the true technology (provided it is convex), while the conical hull-based estimators are not consistent if returns to scale are nonconstant.

As noted in section four, use of the bias correction in (4.2) may increase mean square error. To check this possibility, we compared the bootstrap estimates of bias with the standard deviation of the bootstrap estimates; in almost all cases, the estimated bias exceeded the corresponding standard deviation measure by an order of magnitude or more. Hence, the bias corrections we employed are unlikely to increase mean square error.[14]

6. CONCLUSIONS

Malmquist indices have been widely used in recent years to examine changes in productivity, efficiency, and technology not only within a variety of industries, but across countries as well. In each case, researchers have provided point estimates, although clearly there must be uncertainty surrounding these estimates due to sampling variation. Our methodology outlined in the preceeding sections provides a tractible approach for consistently estimating confidence intervals. In addition, as illustrated in our empirical examples, our bootstrap methodology provides a correction for the inherent bias in nonparametric distance function estimates (and hence in estimates of Malmquist indices).

[14] We have omitted tables showing bias estimates, standard errors, *etc.* for individual firms in each year in the interests of parsimony. These results can be supplied on request.

Confidence intervals such as those estimated in our empirical examples are useful in interpreting estimates of Malmquist indices. As with any estimator, it is not enough to know whether the Malmquist index estimator indicates increases or decreases in productivity, but whether the indicated changes are significant in a statistical sense; *i.e.,* whether the result indicates a real change in productivity, or is an artifact of sampling noise. Our bootstrap procedure allows the researcher to make these distinctions.

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TABLE 1

Changes in Efficiency, 42 Swedish Pharmacies
(Constant returns to scale)

No.	1980/ 1981	1981/ 1982	1982/ 1983	1983/ 1984	1984/ 1985	1985/ 1986	1986/ 1987	1987/ 1988	1988/ 1989
1	1.0097**	0.8628**	1.1736**	1.0842**	0.9590**	0.9948	1.0173	1.0218	0.9960
2	1.0082	0.9507	1.0576	0.9937	1.0120	0.9975	0.9876	1.0100	1.0010
3	1.0242	0.9431	1.0752**	0.9912	0.9854	0.9932	1.0260	1.0083	0.9042**
4	1.0137	0.9086**	1.0332**	0.9921	0.9784*	0.9474**	0.9965	1.0638**	1.0229**
5	1.0796**	0.6975**	1.2137**	1.1802**	0.9624**	0.9264**	0.9983	1.0536**	0.9729**
6	1.0517**	0.8773**	1.1956**	0.9260**	1.0072	0.9970	0.9524**	1.0182	0.9679**
7	1.1741**	0.9919	0.9465**	1.0801**	0.9936	0.9682**	0.9900	1.1071**	0.9935
8	1.5896**	0.7400**	1.3454**	0.9604	1.0027	0.9878	1.0108	1.0139	1.0134
9	0.9794**	0.9463**	1.0257	1.2080**	0.9860	0.9103**	0.9681*	1.0093	0.9958
10	1.0210**	0.7991**	1.2473**	0.8821**	1.0999**	0.9630**	1.0305**	1.0233	1.0363**
11	0.9683**	0.9154**	1.2117**	0.9267**	1.0299**	1.0185	0.9986	1.0230	1.0266
12	0.9995	0.9498	1.1073	0.9752*	0.9652**	0.9584**	1.0449**	0.9057**	1.0344**
13	1.0282	0.9530	1.0504	0.9841	0.9414**	1.0422*	0.9721	1.0618**	1.0050
14	1.0091	0.9520	1.0409	0.9874	1.0051	0.9877	1.0041	1.0132	1.0113
15	1.0287	0.9436*	1.0599	0.9882	1.0053	0.9889	1.0058	1.0339	0.9924
16	1.0414**	0.9213**	1.1439**	0.9929	0.9402**	0.9968	0.9603**	1.0476**	1.0276**
17	1.0068	0.9281**	1.0677*	0.9860	1.0051	1.0005	0.9943	1.0559	1.0006
18	1.0170	0.9522	1.0364	0.9069**	1.0527**	1.0447**	0.9794*	1.0419**	0.9198**
19	1.1460**	0.9439	1.0820	0.9790	1.0057	0.9883	1.0041	1.0190	1.0066
20	0.9853	0.9768	1.0434**	0.9935	0.9752	0.9940	1.0006	1.0428*	1.0015
21	1.0085	0.8952**	1.0964**	0.9851*	0.9051**	0.9026**	1.0093	1.1893**	0.9993
22	0.9452**	0.8255**	1.2024**	0.9228**	0.9605**	0.9699*	1.0456**	1.0344*	1.0460**
23	1.0468**	0.8732**	1.1212**	0.9894	0.9963	1.0168	0.9884	1.0147	0.9986
24	1.0933**	0.9177**	1.0326*	0.9383**	1.0151*	1.0763**	0.9305**	1.0952**	0.9250**
25	1.0116	0.9102**	1.0318	1.0644**	0.9273**	1.0684**	0.9537	1.0141	1.0079
26	1.0004	0.9444*	1.0583	0.9909	1.0053	0.9739	1.0279	1.0118	1.0045
27	1.0924**	0.9066**	1.0260	0.9010**	1.0167*	0.9336**	1.0492**	0.9898	1.0718**
28	0.9999	0.8004**	1.1577**	1.0016	0.9595**	0.9871	1.0045	1.0714**	1.0416**
29	0.9405**	0.7841**	1.2133**	1.0473**	0.9608**	0.9874	0.9926	0.9820	1.1650**
30	1.0358**	0.9565	1.0147	0.8850**	1.0545**	0.9914	1.0503**	1.0403	1.0155
31	1.0182	0.9451*	1.0539	0.8825**	0.9805*	1.0219*	1.0537**	0.9771**	1.0136**
32	1.0128	0.9667	1.0297	0.9987	0.9940	1.0117	0.9819	1.0150	1.0262*
33	1.0087	0.9750	1.0242	0.9785	1.0217	0.9922	0.9851	1.0128	0.9999
34	1.0699**	0.9554	1.0337	0.9896	1.0081	0.9852	0.9024**	1.0984**	1.0524**
35	1.0213	0.9414	1.0636	0.9885	1.0089	0.9883	1.0040	1.0141	1.0082
36	1.0126	0.9589	1.0283	0.9273**	1.0510**	1.0290	1.0102	1.0102	1.0057
37	1.1263**	0.9004**	1.0064	1.0106	1.0254**	0.7784**	1.3017**	1.0027	1.0851**
38	1.0270	0.9790	1.0168	0.9919	0.9931	1.0062	1.0082	1.0060	0.9302**
39	1.0271	0.9422*	1.0613	0.9897	1.0056	0.9871	1.0048	1.0138	1.0072
40	0.8475**	0.8282**	1.1285**	0.9690**	1.3125**	0.9897	0.9960	1.0362*	1.0085
41	1.0195	0.9287*	1.0212	0.9582**	1.0101	1.0953**	0.8368**	0.9074**	1.0635**
42	1.0093	0.8983**	1.0927**	0.9927	1.0085	0.9783	0.9946	1.0703**	0.9547**

TABLE 2

Changes in Technology, 42 Swedish Pharmacies
(Constant returns to scale)

No.	1980/ 1981	1981/ 1982	1982/ 1983	1983/ 1984	1984/ 1985	1985/ 1986	1986/ 1987	1987/ 1988	1988/ 1989
1	0.8941**	1.1228**	0.9708**	0.9337**	1.0491**	0.9726	1.0270**	1.0193	1.0546**
2	0.9991	1.2207**	0.9266**	0.9723	1.0263	1.0207	1.2003**	1.1110**	1.0585*
3	0.9728	1.1489**	0.9438**	0.9467**	1.0546**	1.1112**	1.0057	1.0621**	1.0951**
4	0.9480**	1.1173**	0.9772**	0.9486**	1.0438**	1.0471**	1.1022**	1.0347**	1.0260**
5	0.9365**	1.4611**	0.8638**	0.7994**	1.0638**	1.1522**	1.0560**	1.0609**	1.1357**
6	0.9056**	1.1847**	0.9306**	0.8220**	1.0553**	1.0425**	1.1049**	1.0509**	1.0212**
7	0.8453**	1.0699**	1.0177	0.9017**	1.0437**	1.0506**	1.1201**	1.0235**	1.0062
8	0.9137**	1.4596**	0.7974**	1.1559**	1.0493*	1.1371**	0.9761	1.0385**	1.0362
9	0.9568**	1.1772**	0.9730	0.8331**	1.0298	1.0925**	1.0848**	1.0971**	1.0881**
10	0.9248**	1.3324**	0.8598**	1.0707**	1.0617**	1.1034**	1.0632**	1.0213*	1.0292**
11	0.9010**	1.1663**	0.9620	0.7607**	1.0480**	1.0697**	1.0979**	1.0414**	1.1361**
12	1.0405**	1.8305**	0.6660**	1.0051	1.0954**	1.0770**	1.1018**	1.0172	1.0107
13	1.0011	1.2127**	0.9171**	1.0007	0.9807	1.0465**	1.1489**	0.9802	1.0532**
14	1.0679**	1.2598**	1.0079	1.0354	0.9424**	1.0761**	1.0515*	1.1183**	0.9626
15	0.9901	1.1473**	0.8792**	1.0539	1.0262	0.9962	0.9904	0.8741**	1.0805**
16	0.9779**	1.1670**	0.9380**	0.9398**	1.0526**	1.0049	1.0531**	1.0431**	1.0316**
17	1.0516**	1.0952**	1.0031	0.9560	1.0282	0.9822	1.0626**	0.8868**	1.0648**
18	0.9579**	1.1932**	0.8990**	0.9658**	1.0531**	0.9545**	1.0222*	0.9483**	1.0385**
19	0.8141**	1.2018**	0.9643	1.1408**	1.0851**	1.0890**	0.9863	0.9352**	0.9869
20	0.9649**	1.1605**	1.0239	1.0620**	1.1187**	1.0280	1.0130	0.9543**	1.0532**
21	0.9597	1.1790**	0.9200**	1.0853**	1.1211**	1.0768**	1.0715**	1.0020	1.0358**
22	1.0186	1.1804**	0.8525**	1.0031	1.0040	0.9748*	0.9919	0.9355**	1.0942**
23	0.6550**	1.1221**	0.9175**	0.9886	1.0922**	0.9340**	1.0677**	0.9919	1.0352
24	0.9107**	1.1019**	0.9653**	0.9439**	1.0576**	1.0255*	1.1878**	1.0664**	1.1442**
25	0.8717**	1.2927**	0.9579**	0.9081**	1.0767**	0.9301**	1.1375**	1.0128	1.0643
26	1.0294*	1.0972**	0.9682	0.9812	1.1489**	0.8530**	1.2372**	1.0779**	1.0941**
27	0.9117**	1.1146**	1.0174	0.9384**	1.0567**	1.0353**	1.0203	1.0425**	1.0314**
28	0.9086**	1.3033**	0.8771**	0.9131**	1.0880**	0.9653**	1.0334**	1.0125	1.0696**
29	0.8835**	1.2877**	0.8520**	0.9232**	1.0466**	1.0112	1.0496**	1.0274**	1.0369**
30	0.9090**	1.1541**	1.1084**	0.9133**	1.0810**	1.0496**	1.0142	1.0541**	1.0058
31	0.9553	1.1656**	1.0127	0.8278**	1.0942**	1.0455**	1.1143**	1.0393**	1.0436**
32	0.9737	1.0995**	1.0392**	0.9879	1.0880**	1.0062	1.1270**	0.9997	0.9639**
33	1.0338**	1.0670	1.0359	1.0767**	0.9893	1.1238**	1.1901**	1.0082	1.1178**
34	0.8978**	1.1647**	1.0707**	0.9434	0.9259**	1.0331	0.8894**	1.0441**	1.0290**
35	1.3264**	0.8306**	0.9769	1.0993**	0.9565*	0.9756	0.9672	0.9949	0.9160**
36	0.9682	1.0854**	1.0141	0.9152**	1.0252	1.0028	1.0519*	1.0266*	0.9707*
37	0.9464**	1.2658**	0.9863	0.9616**	1.0732**	1.0351**	1.0456**	0.9817	1.0505**
38	1.0063	1.2297**	1.0101	0.9361**	1.0613**	1.1261**	1.0414	1.0618**	1.1110**
39	1.0047	1.1695**	1.0804**	0.8600**	0.9827	1.0392	1.1632**	1.0160	1.0089
40	1.1326**	1.2726**	0.9067**	1.0110	0.9784*	1.1064**	1.0947**	1.0908**	1.0760**
41	1.0782**	0.9925	0.9426**	1.0457**	0.9644**	1.0911**	1.1274**	0.9916	1.0553**
42	1.0673**	1.4503**	0.9225**	0.9974	1.0387	1.0361	1.0150	1.0564**	1.0442**

TABLE 3

Changes in Productivity, 42 Swedish Pharmacies
(Constant returns to scale)

No.	1980/ 1981	1981/ 1982	1982/ 1983	1983/ 1984	1984/ 1985	1985/ 1986	1986/ 1987	1987/ 1988	1988/ 1989
1	0.9025**	0.9678**	1.1380**	1.0121	1.0058	0.9671**	1.0441**	1.0407**	1.0495**
2	1.0049	1.1518**	0.9716*	0.9642**	1.0375**	1.0173	1.1816**	1.1208**	1.0583**
3	0.9912**	1.0804**	1.0127**	0.9377**	1.0386**	1.1028**	1.0286**	1.0702**	0.9894
4	0.9606**	1.0149**	1.0095**	0.9409**	1.0210**	0.9918	1.0982**	1.1004**	1.0493**
5	1.0097	1.0141**	1.0444**	0.9424**	1.0234**	1.0667**	1.0526**	1.1167**	1.1046**
6	0.9523**	1.0389**	1.1122**	0.7609**	1.0627**	1.0393**	1.0517**	1.0695**	0.9882
7	0.9915	1.0604**	0.9627**	0.9738**	1.0369**	1.0171	1.1083**	1.1324**	0.9995
8	1.4518**	1.0763**	1.0699**	1.1045**	1.0508**	1.1212**	0.9842	1.0514**	1.0484**
9	0.9366**	1.1123**	0.9968	1.0060	1.0150	0.9942	1.0498**	1.1067**	1.0834**
10	0.9441**	1.0615**	1.0696**	0.9437**	1.1673**	1.0620**	1.0951**	1.0446**	1.0660**
11	0.8724**	1.0669**	1.1641**	0.7027**	1.0791**	1.0891**	1.0960**	1.0650**	1.1646**
12	1.0396**	1.7091**	0.7257**	0.9796**	1.0571**	1.0317**	1.1506**	0.9206**	1.0452**
13	1.0271	1.1467**	0.9567**	0.9834**	0.9228**	1.0899**	1.1158**	1.0401**	1.0576**
14	1.0765**	1.1885**	1.0420**	1.0173**	0.9450**	1.0602**	1.0524**	1.1306**	0.9714**
15	1.0113**	1.0677**	0.9217**	1.0362**	1.0290**	0.9827**	0.9931	0.9007**	1.0710**
16	1.0178**	1.0727**	1.0714**	0.9329**	0.9892	1.0013	1.0108	1.0922**	1.0599**
17	1.0572**	1.0143	1.0693**	0.9391**	1.0317**	0.9814**	1.0549**	0.9323**	1.0654**
18	0.9727**	1.1259**	0.9279**	0.8749**	1.1079**	0.9969	1.0010	0.9876**	0.9547**
19	0.9282**	1.1184**	1.0293	1.1052**	1.0884**	1.0731**	0.9872**	0.9508**	0.9920
20	0.9493**	1.1310**	1.0667**	1.0535**	1.0875**	1.0207**	1.0119	0.9918	1.0542**
21	0.9669**	1.0544**	1.0066	1.0683**	1.0143	0.9715**	1.0808**	1.1909**	1.0351**
22	0.9617**	0.9697**	1.0209**	0.9250**	0.9641**	0.9450**	1.0367**	0.9669**	1.1439**
23	0.6808**	0.9787	1.0271	0.9760	1.0866**	0.9480**	1.0537**	1.0051	1.0323**
24	0.9950	1.0101*	0.9953	0.8854**	1.0735**	1.1035**	1.1040**	1.1670**	1.0576**
25	0.8811**	1.1736**	0.9875	0.9663**	0.9976	0.9930	1.0787**	1.0246**	1.0704**
26	1.0267**	1.0216**	1.0141**	0.9678**	1.1519**	0.8293**	1.2655**	1.0887**	1.0975**
27	0.9954	1.0087	1.0419**	0.8452**	1.0742**	0.9661**	1.0700**	1.0317**	1.1052**
28	0.9083**	1.0418**	1.0141**	0.9142**	1.0434**	0.9524**	1.0376**	1.0842**	1.1131**
29	0.8307**	1.0080**	1.0323**	0.9667**	1.0054	0.9981	1.0414**	1.0085	1.2079**
30	0.9414**	1.0993**	1.1225**	0.8061**	1.1395**	1.0400**	1.0646**	1.0949**	1.0203*
31	0.9701**	1.0880**	1.0574**	0.7280**	1.0724**	1.0679**	1.1738**	1.0152	1.0576**
32	0.9845**	1.0579**	1.0664**	0.9851	1.0807**	1.0167*	1.1044**	1.0137*	0.9872
33	1.0412**	1.0372**	1.0582**	1.0508*	1.0084	1.1137**	1.1699**	1.0195**	1.1157**
34	0.9587**	1.1051**	1.1004**	0.9289**	0.9310**	1.0142**	0.7987**	1.1463**	1.0825**
35	1.3497**	0.7747**	1.0269**	1.0827**	0.9621**	0.9611**	0.9677**	1.0068*	0.9220**
36	0.9786**	1.0336**	1.0376**	0.8457**	1.0769**	1.0301	1.0613**	1.0362**	0.9754**
37	1.0654**	1.1375**	0.9916	0.9714**	1.1000**	0.8055**	1.3605**	0.9838**	1.1399**
38	1.0312	1.2005**	1.0253**	0.9272**	1.0511**	1.1320**	1.0490**	1.0654**	1.0333**
39	1.0241**	1.0862**	1.1346**	0.8475**	0.9854**	1.0222**	1.1644**	1.0275**	1.0143**
40	0.9594**	1.0501**	1.0199*	0.9779**	1.2830**	1.0945**	1.0896**	1.1294**	1.0846**
41	1.0947**	0.9180**	0.9599**	1.0010	0.9738**	1.1936**	0.9420**	0.8992**	1.1216**
42	1.0757**	1.2994**	1.0041	0.9868	1.0460**	1.0112	1.0090	1.1301**	0.9966

TABLE 4

Changes in Efficiency, 42 Swedish Pharmacies
(Variable returns to scale)

No.	1980/ 1981	1981/ 1982	1982/ 1983	1983/ 1984	1984/ 1985	1985/ 1986	1986/ 1987	1987/ 1988	1988/ 1989
1	0.9970	0.9013**	1.1193**	1.0868**	0.9731**	1.0052	1.0254**	0.9906*	0.9998
2	1.0018	0.9597*	1.0376*	0.9992	1.0099	1.0140	0.9838**	1.0046	0.9999
3	1.0118	0.9784	1.0271*	0.9879	0.9993	0.9912	1.0212	1.0002	0.9801
4	1.0154**	0.9081**	1.0653**	0.9755**	0.9607**	0.9699**	1.0348**	1.0833**	0.9553**
5	1.0012	0.6862**	1.2970**	1.1340**	1.0060	0.9182**	0.9582**	1.0171**	0.9806**
6	1.0255**	0.8658**	1.1738**	0.9609*	1.0299**	0.9733**	0.9713**	1.0124*	0.9495**
7	1.1447**	1.0083	0.9760**	1.0389**	0.9932	0.9784**	1.0031	1.0696**	1.0005
8	1.5803**	0.7715**	1.2839**	0.9790	1.0031	0.9865	1.0173	1.0076	1.0057*
9	0.9402**	0.9311**	1.0009	1.1930**	1.0230**	0.8961**	0.9656**	1.0033	0.9913**
10	0.9996	0.8195**	1.2107**	0.9019**	1.0735**	0.9659**	1.0731**	0.9958	1.0130**
11	1.0169*	0.9621*	1.0287*	0.9988	1.0023	0.9900	1.0154	1.0053	0.9969
12	1.0015	0.9684	1.0641**	1.0021	0.9775*	0.9511**	1.0662**	0.8735**	1.0352**
13	1.0196*	0.9538	1.0465**	0.9978	1.0215	0.9798	1.0039	1.0092	1.0198*
14	0.9969	0.9545	1.0484**	0.9960	1.0025	0.9883	1.0158	1.0072	0.9995
15	1.0167*	0.9549*	1.0480**	0.9962	1.0032	0.9873	1.0149	1.0133	0.9936
16	0.9940	1.0112	1.0812**	1.0271**	0.9961	0.9790**	1.0438**	0.9974	0.9491**
17	1.0175*	0.9634	1.0291	1.0008	1.0031	0.9874	1.0146	1.0074	1.0031*
18	1.0130*	0.9411*	1.0547**	0.9141**	1.0581**	1.0511**	1.0002	0.9876	0.9991
19	1.0015	0.9554	1.0602**	0.9857	1.0027	0.9868	1.0154	1.0076	1.0002
20	0.9878	0.9811*	1.0309**	0.9914	0.9934	0.9883	1.0121	1.0156	1.0091**
21	0.9997	0.9001**	1.0953**	0.9815**	0.9244**	0.9025**	1.0131	1.1677**	0.9906**
22	0.9402**	0.8626**	1.1488**	0.9362**	0.9525**	0.9683**	1.0536**	1.0251**	1.1035**
23	1.0122	0.9538	1.0519**	0.9962	1.0017	0.9872	1.0142	1.0088	0.9995
24	1.0457**	0.9270**	1.0306**	0.9545**	1.0031	1.0905**	0.9785**	1.0136	1.0109**
25	1.0152*	0.9268**	1.0292**	1.0365**	0.9466**	1.0593**	0.9757*	1.0105	0.9999
26	1.0012	0.9555*	1.0459**	1.0000	1.0018	0.9762	1.0287	1.0066	0.9995
27	1.0270**	0.9167**	1.0446**	0.9446**	1.0277**	0.9614**	1.0884**	0.9932	0.9655**
28	1.0029	0.8576**	1.0796**	1.0176**	0.9715**	1.0029	1.0176**	1.0768**	0.9784**
29	0.9532**	0.7678**	1.2568**	0.9999	0.9818**	0.9874**	1.0022	0.9725**	1.1404**
30	1.0144*	0.9552	1.0468**	0.9247**	1.0872**	0.9590**	1.0204**	1.0251	0.9986
31	1.0065	0.9544	1.0433**	0.8716**	0.9960	1.0154*	1.0633**	1.0342**	0.9441**
32	1.0081	0.9748	1.0183	1.0008	1.0021	1.0002	0.9998	1.0073	1.0198**
33	1.0053	0.9800	1.0181	0.9904	1.0164	0.9868	1.0018	1.0070	0.9983
34	1.0195*	0.9626*	1.0316*	0.9970	1.0036	0.9860	1.0212	1.0304**	0.9980
35	1.0163*	0.9540	1.0478**	0.9992	1.0035	0.9876	1.0146	1.0091	0.9985
36	1.0075	0.9600*	1.0346*	0.9620*	1.0480**	0.9915	1.0047	1.0098	1.0019
37	1.1010**	0.9327**	0.9877*	0.9975	1.0412**	0.8148**	1.2530**	1.0196**	1.0347**
38	1.0162*	0.9780	1.0149	1.0125	0.9897	0.9933	1.0213	0.9975	0.9446**
39	1.0145*	0.9553	1.0460**	0.9981	1.0028	0.9866	1.0156	1.0079	0.9986
40	0.8977**	0.8567**	1.1206**	1.1387**	1.0447**	1.0005	1.0061	1.0013	0.9990
41	1.0148*	0.9729	1.0254	0.9961	1.0025	0.9875	1.0158	1.0064	0.9983
42	1.0109	0.9504*	1.0466**	0.9985	1.0031	0.9956	1.0149	1.0081	1.0006*

TABLE 5

Changes in Technology, 42 Swedish Pharmacies
(Variable returns to scale)

No.	1980/ 1981	1981/ 1982	1982/ 1983	1983/ 1984	1984/ 1985	1985/ 1986	1986/ 1987	1987/ 1988	1988/ 1989
1	0.8916**	1.0745**	0.9882	0.9351**	1.0491**	0.9852	0.9710**	1.0470**	1.0493**
2	1.0132	1.2274**	0.9231**	0.9968	1.0116	1.0001	1.1447**	1.0997**	1.0487**
3	0.9872	1.1375**	0.9883	0.9698*	1.0678**	1.0801**	1.0016	1.0890**	1.0580**
4	0.9510**	1.1173**	0.9498**	0.9697**	1.0607**	1.0017	1.0395**	0.9943	1.0787**
5	1.0334**	1.5258**	0.8306**	0.8096**	1.0137	1.1621**	1.1384**	1.0681**	1.1208**
6	0.9211**	1.2057**	0.9407**	0.7735**	1.0383**	1.0711**	1.0798**	1.0170**	1.0287**
7	0.8582**	1.0562**	0.9844**	0.9311**	1.0223**	1.0312**	1.0752**	1.0356**	1.0040
8	0.9214**	1.4006**	0.8046**	1.1409**	1.0424*	1.1993**	0.9672	1.0661**	1.0429**
9	0.9821**	1.2370**	0.9865	0.8430**	0.9904	1.1073**	1.0799**	1.0790**	1.0834**
10	1.1575**	1.5406**	0.9225**	1.0350**	1.0564**	1.0831**	1.0025	1.0195**	1.0381**
11	0.9720**	1.0847**	1.2123**	0.7523**	1.0017	1.0446*	0.9786	1.0390**	1.1282**
12	—	1.9463**	0.7065**	0.9448**	1.1747**	1.1157**	1.1943**	—	1.0093
13	1.0946**	1.3249**	0.9020**	0.9821	0.8926**	1.1273**	1.2330**	0.9873	0.9695**
14	—	—	—	—	—	—	—	—	—
15	1.0014	1.1779**	0.8879**	1.0142	1.0750**	1.0312	1.0191	0.8017**	1.0710**
16	1.0380**	1.0509**	0.9385**	0.9784**	0.9688**	0.9976	0.9348**	1.0155**	1.0969**
17	—	1.0742**	—	—	—	—	—	—	—
18	0.9612**	—	0.7569**	0.9842	1.0342**	0.9994	1.0745**	0.7940**	—
19	0.7637**	1.3461**	0.9711**	1.1183**	—	—	—	—	0.9692**
20	0.9725**	1.1808**	1.0351**	1.0913**	1.0858**	1.0514	1.0099	0.9249**	1.0519**
21	0.9628**	1.1818**	0.9179**	1.0847**	1.0910**	1.0681**	1.0521**	0.9943	1.0525**
22	1.0183**	1.1154**	0.8623**	1.0083	1.0202**	0.9691**	0.9882	0.9640**	1.0127**
23	0.8514**	1.1388**	0.6309**	0.9758*	—	—	—	—	—
24	0.9342**	1.1029**	0.9558**	0.9749**	1.0131	0.9883	1.1013**	1.0884**	1.0430**
25	0.8554**	1.2781**	0.9587**	0.9206**	1.0537**	0.9422**	1.1302**	1.0881**	1.0891**
26	1.0284*	1.0638*	0.9765	0.9815	1.1990**	0.8469**	1.2385**	1.1940**	1.1413**
27	0.9896	1.1020**	0.9835*	0.9209**	1.0164**	0.9971	0.9609**	0.9968	1.1184**
28	0.9167**	1.2145**	0.9338**	0.9053**	1.0586**	0.9603**	1.0279**	1.0272**	1.1538**
29	0.8949**	1.3261**	0.8272**	0.9071**	1.0269**	1.0035	1.0281**	1.0448**	1.0668**
30	—	—	—	—	—	—	1.0566**	—	—
31	0.9604**	1.1745**	1.0265	0.8241**	1.0668**	1.0406**	1.0758**	0.9943	1.0875**
32	1.1933**	1.1855**	1.0563**	0.9202**	1.1601**	1.0771**	1.2018**	1.0794**	0.9533**
33	1.1173**	1.1074**	1.0764**	1.0711**	1.0099	1.1426**	1.2681**	1.0454**	—
34	0.8694**	1.2972**	1.0977**	0.8174**	0.8835**	1.0096	0.8044**	1.0179**	1.0660**
35	—	—	—	—	—	—	—	—	—
36	0.9706**	1.0894**	1.0116	0.8962**	0.9685**	1.0726**	1.1153**	1.0543**	1.0096
37	0.9696**	1.2421**	0.9940	0.9694**	1.0351**	1.0169	1.0086	0.9667**	1.0708**
38	—	—	1.0147	0.9318**	1.0422**	1.1198**	1.0305	1.0563**	1.0903**
39	1.0366**	1.1448**	1.0700**	0.8757**	0.9800	1.0448*	1.1910**	1.0141	1.0059
40	1.1046**	1.2252**	0.8818**	1.0283**	1.1074**	1.0271	1.0530**	1.0699**	1.1014**
41	0.9711**	0.9661	0.9228**	1.1802**	0.9287**	1.1245**	0.9332**	0.9764	1.1055**
42	—	—	1.1486**	0.9195**	—	0.8984**	1.0948**	1.1324**	1.0609**

TABLE 6

Changes in Productivity, 42 Swedish Pharmacies
(Variable returns to scale)

No.	1980/ 1981	1981/ 1982	1982/ 1983	1983/ 1984	1984/ 1985	1985/ 1986	1986/ 1987	1987/ 1988	1988/ 1989
1	0.8889**	0.9681**	1.1058**	1.0160*	1.0205*	0.9901	0.9953	1.0361**	1.0488**
2	1.0138**	1.1696**	0.9525**	0.9933	1.0206**	1.0135	1.1249**	1.1040**	1.0480**
3	0.9955	1.1094**	1.0126**	0.9568**	1.0662**	1.0699**	1.0206**	1.0887**	1.0355**
4	0.9656**	1.0144*	1.0116**	0.9457**	1.0186**	0.9714**	1.0754**	1.0770**	1.0305**
5	1.0334**	1.0441**	1.0747**	0.9174**	1.0196**	1.0663**	1.0898**	1.0861**	1.0988**
6	0.9445**	1.0434**	1.1025**	0.7413**	1.0693**	1.0422**	1.0485**	1.0293**	0.9767**
7	0.9820	1.0644**	0.9604**	0.9670**	1.0152**	1.0088	1.0783**	1.1075**	1.0045
8	1.4556**	1.0778**	1.0316**	1.1140**	1.0448**	1.1808**	0.9815**	1.0731**	1.0482**
9	0.9234**	1.1510**	0.9869**	1.0053	1.0131	0.9920	1.0423**	1.0824**	1.0739**
10	1.1569**	1.2601**	1.1147**	0.9330**	1.1339**	1.0456**	1.0753**	1.0150**	1.0514**
11	0.9854	1.0369**	1.2435**	0.7495**	1.0032	1.0325**	0.9925	1.0436**	1.1238**
12	—	1.8632**	0.7454**	0.9458**	1.1474**	1.0609**	1.2723**	—	1.0446**
13	1.1089**	1.2505**	0.9366**	0.9764**	0.9098**	1.1020**	1.2370**	0.9948	0.9875
14	—	—	—	—	—	—	—	—	—
15	1.0125**	1.1120**	0.9231**	1.0069	1.0776**	1.0158**	1.0322**	0.8111**	1.0634**
16	1.0316**	1.0617**	1.0140	1.0047	0.9648**	0.9764**	0.9752**	1.0128**	1.0410**
17	—	1.0319*	—	—	—	—	—	—	—
18	0.9726**	—	0.7915**	0.8987**	1.0941**	1.0502**	1.0743**	0.7829**	—
19	0.7630**	1.2711**	1.0227**	1.0990**	—	—	—	—	0.9686**
20	0.9596**	1.1574**	1.0661**	1.0798**	1.0772**	1.0376**	1.0209**	0.9383**	1.0608**
21	0.9622**	1.0629**	1.0039	1.0641**	1.0083	0.9638**	1.0656**	1.1606**	1.0426**
22	0.9570**	0.9598**	0.9888	0.9432**	0.9714**	0.9382**	1.0409**	0.9880	1.1173**
23	0.8593**	1.0748**	0.6598**	0.9704**	—	—	—	—	—
24	0.9768**	1.0216**	0.9842	0.9303**	1.0161	1.0776**	1.0774**	1.1025**	1.0537**
25	0.8679**	1.1834**	0.9862	0.9534**	0.9971	0.9974	1.1005**	1.0979**	1.0878**
26	1.0280**	1.0052	1.0136**	0.9782**	1.2003**	0.8258**	1.2723**	1.2003**	1.1397**
27	1.0162	1.0089	1.0261**	0.8698**	1.0445**	0.9584**	1.0455**	0.9900**	1.0798**
28	0.9192**	1.0412**	1.0077**	0.9208**	1.0283**	0.9628**	1.0457**	1.1061**	1.1271**
29	0.8529**	1.0165**	1.0380**	0.9068**	1.0081*	0.9905	1.0300**	1.0160**	1.2164**
30	—	—	—	—	—	—	1.0777**	—	—
31	0.9653**	1.1094**	1.0638**	0.7169**	1.0622**	1.0564**	1.1438**	1.0283**	1.0266**
32	1.2005**	1.1511**	1.0733**	0.9191**	1.1618**	1.0763**	1.2003**	1.0864**	0.9709**
33	1.1221**	1.0828**	1.0945**	1.0591**	1.0250**	1.1260**	1.2694**	1.0514**	—
34	0.8822**	1.2408**	1.1280**	0.8123**	0.8861**	0.9931	0.8173**	1.0487**	1.0637**
35	—	—	—	—	—	—	—	—	—
36	0.9761**	1.0377**	1.0418**	0.8599**	1.0148	1.0604**	1.1197**	1.0635**	1.0109**
37	1.0673**	1.1581**	0.9813	0.9667**	1.0776**	0.8285**	1.2634**	0.9853**	1.1078**
38	—	—	1.0271	0.9421**	1.0307**	1.1116**	1.0506**	1.0520**	1.0299**
39	1.0460**	1.0810**	1.1116**	0.8715**	0.9817**	1.0281**	1.2074**	1.0203**	1.0035
40	0.9914	1.0479**	0.9868	1.1701**	1.1549**	1.0270**	1.0589**	1.0704**	1.0997**
41	0.9813	0.9351**	0.9433**	1.1729**	0.9302**	1.1078**	0.9460**	0.9818	1.1027**
42	—	—	1.1950**	0.9152**	—	0.8936**	1.1095**	1.1406**	1.0602**