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The taxation of capital returns in overlapping generations models

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ABSTRACT

This paper shows that in the Diamond (1965) overlapping generations economy with production and capital savings, there is a period-by-period balanced fiscal policy supporting a steady state allocation that Pareto-improves upon the laissez-faire competitive equilibrium steady state (whether dynamically inefficient or efficient) without resorting to intergenerational transfers. The policy consists of taxing linearly (or subsidizing, in the dynamically efficient case) the returns to capital, while balancing the budget period by period through a lump-sum transfer (or tax, respectively) in second period. This intervention grants every generation the highest steady state utility attainable through markets (i.e. remunerating factors by their marginal productivities and without transfers) which under laissez-faire is not a competitive equilibrium outcome. A transition from the competitive equilibrium steady state to this other steady state is also Pareto-improving when the former is dynamically inefficient. The result disentangles from redistributive considerations the impact of the taxation of capital returns on steady state welfare, and thus provides a rationale for the taxation of capital returns that is based on efficiency considerations and not on redistributive goals.

1. Introduction

Whether the taxation of capital returns is a good or bad idea is a recurrent issue in the economics literature.[1] Arguments against and in favor are put forward on the grounds of, respectively, the inefficiencies that taxes may introduce in the allocation of resources (as in Chamley (1986), Judd (1987)),[2] and the fact that taxes on capital income can help undo inefficiencies due to the incompleteness of markets (e.g. oversaving as a self-insurance against uninsurable risks, as in Aiyagari (1995), Chamley (2001)). Actually, the conclusions depend on the framework in which the question is addressed, namely the neoclassical growth model or the overlapping generations model. In effect, in the absence of uncertainty, taxing capital returns distorts factor prices

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[1] See, for instance, Atkeson et al. (1999) and Conesa et al. (2007).

[2] As shown in Chamley (1986) and Judd (1987), there is no role for capital taxation in an infinitely-lived, neoclassical growth model, because the First Welfare Theorem holds in that setup, so that competitive equilibria are Pareto efficient. Specifically, in the neoclassical growth model, the resource constraint faced by the planner each period is (because of the constant returns to scale) the same as the market constraints faced by agents supplying capital and labor remunerated by their marginal productivities, so that competitive equilibrium allocations cannot be improved upon in a Pareto sense. This is not the case in the Diamond (1965) setup: there the marginal productivity of factors is remunerated each period to different generations (to young agents for their labor, and to old agents for their capital), as opposed to the same agent in the neoclassical growth model. As a consequence, competitive equilibria fail typically to be Pareto efficient in the absence of the adequate intergenerational transfers.

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in the neoclassical growth model, which creates inefficiencies – given the Pareto optimality of the laissez-faire competitive equilibrium allocations of this model.[3] In contrast, the breakdown of the first welfare theorem in the overlapping generations economy with production of Diamond (1965) prevents this argument to apply in that setup.

Diamond (1970) argued nevertheless that, in the overlapping generations setup, increasing the capital returns tax rate can (under some conditions) decrease the steady state utility of the representative agent, even if the taxes raised were given back as a lump-sum transfer to the same generation. Thus, a reduction or elimination of such tax would be Pareto-improving in overlapping generations economies too. More specifically, Diamond (1970) established this negative impact when agents can save only in capital,[4] and if the after-tax rate of return exceeds the growth factor of the population.[5] This result seems to extend to the overlapping generations setup the undesirability of any taxation of capital returns (although under the qualification above). Independently, Svensson (1986) provides, nonetheless, conditions for the introduction of a (n infinitesimal) capital income tax rate to be welfare improving instead, at a locally stable steady state when the taxes raised are transferred as a lump-sum to the young generation.[6]

As a matter of fact, both Diamond (1970) and Svensson (1986) provide partial results that relate to a more general question, which involves comparing three steady states: (i) the *competitive equilibrium steady state*, defined as usual, (ii) the *best market steady state* (the one maximizing the representative agent's utility among those remunerating factors by their marginal productivities and without transfers), and (iii) the *first-best steady state* (the one a planner unconstrained from remunerating factors by their marginal productivities would choose in order to maximize the representative agent utility). The general question is the following: does the laissez-faire competitive equilibrium steady state maximize the utility of the representative agent *among those remunerating factors by their marginal productivities and without transfers*? In other words, is the competitive equilibrium steady state the best market steady state? The answer to this question is negative.[7] Indeed, given that typically the laissez-faire competitive equilibrium steady state is distinct from the first-best steady state, all the agents could be better-off saving differently from their competitive equilibrium steady state levels. In effect, since the return to savings for any given generation is the marginal productivity of the aggregate capital next period (i.e. the aggregate savings today), all the members of a generation could, in principle, coordinate to manipulate the returns to their own savings in order to implement such improvement.[8] Of course under competitive conditions no agent has incentives to deviate from the competitive behavior and, as a consequence, this possibility of improvement is left unexploited at competitive equilibria. As a consequence, there is, typically, room for improving upon the competitive equilibrium steady state even without interfering with the working of markets or resorting to redistributing income, as the implementation of the first-best steady state typically requires.

Can some intervention decentralize as a competitive equilibrium the best market steady state? The answer to this second question is yes: when the laissez-faire competitive equilibrium steady state is dynamically inefficient,[9] (*i*) tax linearly each generation's capital returns at a rate that depends on the savings of the previous generation, and (*ii*) make a lump-sum transfer to the same generation equal to the result of applying to the previous generation returns the current generation's tax rate.[10] In doing so, no redistribution takes place at the steady state, and the government never incurs any deficit or surplus.[11] Does this contradict the result in Diamond (1970)? Was all this implicit in Svensson (1986)? The answer to both questions is negative. As a matter of fact, none of these two papers addresses the question above. Both Diamond (1970) and Svensson (1986) perform a local analysis (using the implicit function theorem) at the competitive equilibrium steady state to assess the welfare impact of changing slightly the tax rate on capital income. As a consequence, none of these papers characterizes the best market steady state (which requires the global analysis developed here instead), nor do they identify hence the exact policy allowing to implement it as a competitive equilibrium outcome. At any rate, the tax rate that implements the representative agent's utility maximizing market steady state is such that the after-tax return to savings *does not* exceed the rate of growth of the population, so that the result in Diamond (1970) does not apply.[12]

[3] There is still a role for capital taxation in the neoclassical growth model with infinitely-lived agents on the grounds of redistributive goals, as shown in Saez (2002).

[4] So that the intergenerational transfers required to attain the first-best steady state (the so-called golden rule) cannot take place.

[5] Although Diamond (1970) did not consider the symmetric case, if the returns to capital are instead linearly subsidized (and a lump-sum tax raised from the same generation), a decrease in the rate at which they are subsidized decreases the steady state utility of the representative agent if the after-subsidy rate of return *does not exceed* the growth factor of the population.

[6] The policies being different with respect to their lump-sum taxes or transfers, Svensson (1986) does not necessarily contradict Diamond (1970).

[7] Except, obviously, in the knife-edge case in which the laissez-faire competitive equilibrium steady state maximizes already the representative agent's utility among *all* feasible steady states (i.e. when it coincides with the first-best steady state).

[8] Note that, for an economy running from $-\infty$ to $+\infty$, if all generations behaved this way, then each generation's utility could be improved upon the laissez-faire competitive equilibrium steady state, even in the dynamically efficient case.

[9] That is to say, it over-accumulates capital compared to the first-best steady state.

[10] Contrarily, when the laissez-faire competitive equilibrium steady state is dynamically efficient (it under-accumulates capital with respect to the first-best) the returns to savings need rather to be subsidized and a second period lump-sum tax needs to be raised. The handing of the taxes raised to the same generation as a lump-sum makes clear that redistribution does not play any role here in the optimality of taxing capital income.

[11] Diamond (1970) considers a similar policy to assess the impact of taxes on capital returns in this setup. Nonetheless, in Diamond (1970) the lump-sum transfer matches the amount raised from that *same* generation, and is hence contingent to the decisions of the very agents that will receive it, which raises (and leaves unanswered) the question of why agents do not manipulate the policy. Here, on the contrary, the tax rates and transfers depend on past decisions, and therefore cannot be manipulated.

[12] In the dynamically efficient case, the subsidy rate that implements the best market steady state is such that the after-subsidy return to savings *does* exceed the rate of growth of the population, so that the extension of Diamond (1970) mentioned in footnote 5 would not apply either.

Why is this a relevant question? Firstly, since the proposed policy Pareto-improves upon the laissez-faire competitive equilibrium steady state of a model with a representative agent, the previous result provides a rationale for taxation that transcends redistributive goals. Thus the result contributes to the view that capital income taxation needs not be justified *only* on the grounds of redistributive arguments[13] or by the need to finance some public spending, but rather that it can be justified on the grounds of efficiency too.[14]

This efficiency role for capital taxation is a consequence of the assumption from Diamond (1970) according to which agents can only save in capital – more specifically, no money or asset bubble à la Tirole (1985), public debt as in Diamond (1965), or social security scheme allows for the intergenerational transfers needed to support the first-best steady state. In effect, while the first-best steady state can be implemented through intergenerational transfers, the result in this paper shows that this needs not be the only way of improving efficiency: even when no intergenerational transfer can be made, Pareto-improvements can be implemented through non-redistributive (at the steady state) taxes and transfers. Maintaining this assumption helps disentangling the impact on efficiency of taxes on capital returns from the redistributive considerations needed for the implementation of the first-best steady state. Thus, if redistribution is for any reason limited or difficult (and it certainly is, being as it is one of the contentious points at the heart of economics), there is still room for improvement in a market economy in which redistribution may not be a fully available tool of policy. The simple Diamond (1970) model considered here allows to make this point particularly transparently, but the point remains valid in a more general setup like the *perpetual youth* overlapping generations models à la Blanchard (1985).[15]

The rest of the paper proceeds as follows. Section 2 characterizes the competitive equilibria of the economy (mainly to fix notation) and shows that there exists a steady state that Pareto-improves upon the competitive equilibrium steady state, and a Pareto-improving transition to it (if dynamically inefficient). Section 3 shows that any competitive equilibrium steady state providing a smaller utility than the first-best steady state does not maximize the utility among market steady states either, i.e. among the steady states attainable through the market (Proposition 1). Section 4 characterizes the steady state that achieves this maximization (Proposition 2), and establishes that it is a laissez-faire competitive equilibrium outcome if, and only if, it coincides also with the first-best steady state. Actually, the three steady states above – namely the first-best steady state, the best market steady state, and the competitive equilibrium steady state – are either all identical or all distinct (Proposition 3). Section 5 characterizes the fiscal policy that decentralizes the best market steady state as a competitive equilibrium (Proposition 4). The policy requires either taxing or subsidizing the returns on savings depending on whether the best market steady state over-accumulates or under-accumulates capital with respect to the first-best (Proposition 5). A concluding section closes the paper.

2. Competitive equilibria

Consider the Diamond (1965) economy with a representative agent living for two periods, t and $t + 1$. When young at t, he supplies an amount of labor normalized to one for a wage w_t, which he splits between consumption c_0^t and savings s^t. He can lend s^t as capital for a return of r_{t+1}, and then consume $c_1^t = r_{t+1}s^t$ when old at $t + 1$.[16] Utility u from consumption is strictly increasing and concave, and future utility is discounted by a factor $\beta \in (0,1]$.[17] The agent's problem is then

$$\begin{aligned} &\max_{0\leqslant c_0^t, c_1^t, s^t} \; u(c_0^t) + \beta u(c_1^t) \\ &c_0^t + s^t \leqslant w_t \\ &c_1^t \leqslant r_{t+1}s^t \end{aligned} \tag{1}$$

given w_t and r_{t+1}. Under standard assumptions, the agent's optimal saving s^t is a function of w_t and r_{t+1} defined by the condition

$$\frac{1}{\beta}\frac{u'(w_t - s^t)}{u'(r_{t+1}s^t)} = r_{t+1} \tag{2}$$

equalizing the marginal rate of substitution of old-age to young-age consumption and the return to savings. The agent's optimal consumptions are then determined by s^t through his budget constraints.

[13] As, for instance, argued in Saez (2002) in an infinitely-lived agents setup.

[14] See also Erosa and Gervais (2002), where the need to tax capital returns follows from the fact that, in order to finance efficiently a flow of public expenditures, taxes should optimally be age-contingent. When this cannot be, capital taxes allows to mimic this age-dependence by having a different impact on the individual's ability to transfer wealth across periods along his life-cycle. In the two-period overlapping generations model considered here, there is no room for such an effect to take place, and still an active tax policy allows to improve upon the competitive equilibrium steady state.

[15] In Blanchard (1985) generations face an age-independent probability of survival into the next period, which on the one hand makes of their problem one over an infinite horizon, and on the other hand makes all past generations (in decreasing proportions) coexist at any point in time. In this case too, no competitive equilibrium implements the best steady state remunerating factors by their marginal productivities and without transfers, for the same reasons as here: competitive agents do not internalize the impact of their saving decisions on the marginal productivity of capital. A policy decentralizing the best market steady state would, however, be more demanding in this case, since it would require capital tax rates contingent to the agents' age (besides the period-by-period balancing lump-sum transfers or taxes).

[16] Without loss of generality, capital depreciates completely in one period.

[17] For notational convenience only. Separability and discounting are inessential in what follows.

Population grows by a factor $n > 0$ every period, and firms produce output from capital and labor through a constant returns to scale production function $F(K,L)$. Under perfect competition, the wage and the rental rate of capital are the marginal productivities of labor and capital respectively. Since at t the capital K_t available consists of the aggregate savings $n^{t-1}s^{t-1}$ at $t-1$, and aggregate labor L_t is n^t, then the wage and rental rate faced by generation t are

$$\begin{aligned} r_{t+1} &= F_K\left(\frac{s^t}{n},1\right) \\ w_t &= F_L\left(\frac{s^{t-1}}{n},1\right) \end{aligned} \tag{3}$$

Also, from the budget constraints, at any date t, of the n^{t-1} old agents and the n^t contemporaneous young agents, the feasibility of the allocation of resources follows (from the homogeneity of degree 1 of the production function).

The competitive equilibrium allocations are, therefore, characterized by the per capita savings dynamics

$$\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^{t-1}}{n},1\right)-s^t\right)}{u'\left(F_K\left(\frac{s^t}{n},1\right)s^t\right)} = F_K\left(\frac{s^t}{n},1\right) \tag{4}$$

In particular, a competitive equilibrium *steady state* of this overlapping generations economy is characterized by a constant sequence of per capita savings s^c satisfying the dynamics (4) above.[18]

Thus, if at some date t the available per capita savings made by generation $t-1$ is s^c,[19] then the only competitive equilibrium allocation the agents will be able to attain under laissez-faire is the allocation in which every generation $t' \geqslant t$ obtains a consumption profile $\left(c_0^c, c_1^c\right) = \left(F_L\left(\frac{s^c}{n},1\right)-s^c, F_K\left(\frac{s^c}{n},1\right)s^c\right)$. Nevertheless, this steady state is Pareto-dominated, if dynamically inefficient,[20] by the following one:

(1) generations $t' \leqslant t-1$ get the same consumption $\left(c_0^c, c_1^c\right)$ as before
(2) generation t obtains the consumption profile[21]

$$\begin{aligned} \tilde{c}_0 &= F_L\left(\frac{s^c}{n},1\right) - s^* \\ \tilde{c}_1 &= F_K\left(\frac{s^*}{n},1\right)s^* \end{aligned} \tag{5}$$

where s^* is the solution to

$$\begin{aligned} &\max_{0\leqslant c_0,c_1,s} u(c_0)+\beta u(c_1) \\ &c_0 + s = F_L\left(\frac{s}{n},1\right) \\ &c_1 = F_K\left(\frac{s}{n},1\right)s \end{aligned} \tag{6}$$

(3) and subsequent generations $t \geqslant t+1$ obtain

$$\begin{aligned} c_0^* &= F_L\left(\frac{s^*}{n},1\right) - s^* \\ c_1^* &= F_K\left(\frac{s^*}{n},1\right)s^* \end{aligned} \tag{7}$$

Note that in this new feasible allocation (i) the utility of generations $t' \leqslant t-1$ remains unchanged; (ii) the new allocation provides to all generations $t' \geqslant t+1$ the highest steady state utility attainable through the existing markets (hence not smaller than the competitive equilibrium steady state one); and (iii) the utility of generation t is higher than at the competitive equilibrium steady state, as long as $s^c > s^*$, which is the case when s^c is dynamically inefficient,[22] since

[18] The existence and uniqueness of a steady state s^c of (4) is not an issue, under mild conditions. For instance, for a Cobb-Douglas technology $F(K,L) = K^\alpha L^{1-\alpha}$ and a constant relative risk aversion utility $u(c) = \frac{c^{1-\rho}}{1-\rho}$ Eq. (8) becomes

$$\frac{1}{\beta}\left(\frac{\alpha n}{(1-\alpha)-nx}\right)^{\rho} = \frac{\alpha}{x}$$

(where $x = \left(\frac{s}{n}\right)^{1-\alpha}$) with the left-hand side increasing in $x > 0$ with a vertical asymptote at $\frac{1-\alpha}{n}$, and the right-hand side decreasing, so that a unique solution s^c exists in the interval $\left[0, n\left(\frac{1-\alpha}{n}\right)^{\frac{1}{1-\alpha}}\right]$

[19] Or, equivalently, generation $t-1$ is a first generation born old at date t endowed with s^c units of capital – and consumes hence $F_K\left(\frac{s^c}{n},1\right)s^c$.

[20] That is to say, if $s^c > s^g$, where s^g is the Golden Rule level of per capita savings that maximizes the net output per capita $F\left(\frac{s}{n},1\right) - s$ each period.

[21] Note that $\tilde{c}_0 > 0$ because of $s^c > s^g$, since this implies $s^c > s^*$, with s^* as defined in (6) (as established in the lemma in Appendix A) and hence that $\tilde{c}_0 > c_0^* \geqslant 0$.

[22] See Appendix A. Generation t gets a utility higher than all the other generations.

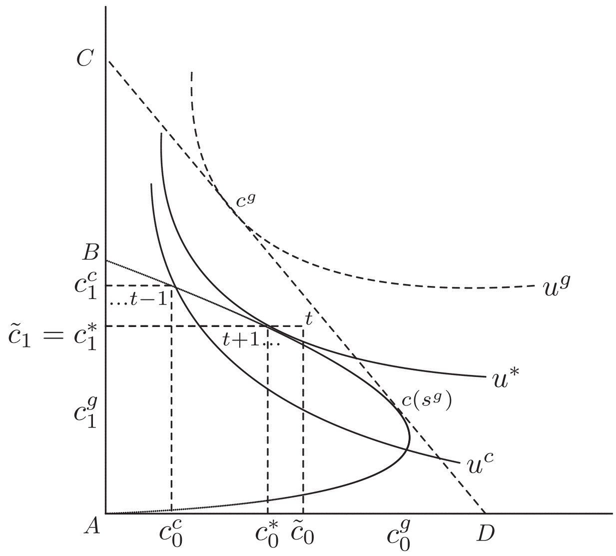

Fig. 1. First-best, constrained efficient, and competitive steady states.

$$\begin{aligned}
&u(c_0^c) + \beta u(c_1^c) \leqslant \\
&u(c_0^*) + \beta u(c_1^*) = u\left(F_L\left(\frac{s^*}{n}, 1\right)l - s^*\right) + \beta u\left(F_K\left(\frac{s^*}{n}, 1\right)s^*\right) \\
&\qquad\qquad\qquad < u\left(F_L\left(\frac{s^c}{n}, 1\right)l - s^*\right) + \beta u\left(F_K\left(\frac{s^*}{n}, 1\right)s^*\right) \\
&\qquad\qquad\qquad = u(\tilde{c}_0) + \beta u(\tilde{c}_1).
\end{aligned} \tag{8}$$

This is illustrated in Fig. 1, where the curve AB contains all possible profiles of consumption $(c_1(s), c_2(s)) = \left(F_L\left(\frac{s}{n}, 1\right) - s, F_K\left(\frac{s}{n}, 1\right)s\right)$ attainable through the markets at a steady state, for all possible initial levels of per capita savings s, and the line CD corresponding to the highest possible net output $D = F\left(\frac{s}{n}, 1\right) - s$ (attained at the first-best per capita level of savings s^g).

Before date t all generations get the competitive equilibrium steady state utility u^c from the profile of consumptions (c_0^c, c_1^c), and after t all generations get the best market steady state utility u^* from the profile of consumptions (c_0^*, c_1^*).[23] Generation t obtains an even higher utility from the consumptions $(\tilde{c}_0, \tilde{c}_1)$.[24]

Of course, the allocation above is not a competitive equilibrium allocation under laissez-faire, since the only competitive equilibrium per capita savings starting from the level s^c is precisely s^c, so that the economy will never attain this Pareto-improving allocation without public intervention. Can the economy attain it with some minimal public intervention (more specifically through the existing markets and without redistributing)? The answer to this question is yes, as established in the next sections.

3. Steady states

It is well known that the competitive equilibrium steady state may not maximize the utility of the representative agents among all feasible steady states, i.e. the steady state per capita savings s^c solution to (4) is typically distinct from the first-best per capita savings s^g following from

$$\begin{aligned}
&\max_{0 \leqslant c_1, c_2, s} u(c_1) + \beta u(c_2) \\
&c_1 + \frac{c_2}{n} + s = F\left(\frac{s}{n}, 1\right)
\end{aligned} \tag{9}$$

and characterized by

$$F_K\left(\frac{s^g}{n}, 1\right) = n \tag{10}$$

[23] The constraint of remunerating factors by their marginal productivities and making no transfers translates in Fig. 1 into having to stay on the AB curve.

[24] The first-best steady state utility u^g is only attainable through intergenerational transfers (unavailable in Diamond (1970)) that move the generations from $c(s^g)$ to c^g.

Whenever $s^c > s^g$ the laissez-faire competitive equilibrium steady state allocation over-accumulates capital with respect to the first-best steady state. On the contrary, if $s^c < s^g$ holds, markets inefficiently under-accumulate capital.[25]

Implementing the first-best steady state s^g requires allocating freely the output produced among the young and old irrespective of the productivity of the factors (labor and capital respectively) they provide. It is therefore not surprising that with so much power one could do better than the laissez-faire competitive outcome. But interestingly enough, one can do better even under the markets constraints. In effect, whenever the competitive equilibrium steady state is not the first-best steady state, then it is Pareto-dominated also among just those steady states that can be attained through the existing markets, i.e. those remunerating factors by their marginal productivities and allowing for no transfers. The reason why is that competitive agents fail to internalize the impact of their saving decisions on the return of their own savings, an effect that a planner can take into account even if constrained to remunerate factors by their marginal productivities and not to redistribute, i.e. even when facing the agents' budget constraints.

This is interesting because it indicates that, typically, there is room for improving upon the laissez-faire allocation even without interfering with the working of markets or without resorting to redistributing income across generations (as the implementation of the first-best steady state typically requires). This result is stated in the next proposition (proofs of all propositions are provided in Appendix A).

Proposition 1. *In a productive overlapping generations economy with only capital savings, the competitive equilibrium steady state is, whenever distinct from the first-best steady state, Pareto-dominated among the steady states attainable through the market.*

In other words, Proposition 1 establishes that, whenever the competitive equilibrium steady state is not the first-best steady state, there is other levels of savings, for all generations, that give every agent a higher utility than the competitive equilibrium steady state. Which is then the best steady state that is attainable through the existing markets for output, capital and labor? This question is addressed in the next section.

4. The best market steady state

If the laissez-faire competitive equilibrium steady state can be improved upon through the existing markets without redistributing income, which is the level of per capita savings that, if chosen by all agents, would maximize everyone's utility (without redistribution)? The next proposition characterizes this steady state.

Proposition 2. *In a productive overlapping generations economy with only capital savings, the best market steady state level of per capita savings s^* is characterized*[26] *by the condition*

$$\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^*}{n},1\right)-s^*\right)}{u'\left(F_K\left(\frac{s^*}{n},1\right)s^*\right)} = n\cdot\frac{F_K\left(\frac{s^*}{n},1\right)+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}} \tag{11}$$

In order to understand the meaning of condition (11), recall that the competitive equilibrium steady state could be Pareto-improved upon because in it the agents failed to take into account the impact that their own saving decisions had on their savings returns. In condition (11) this impact is accounted for by the derivative with respect to savings of the marginal productivity of capital, i.e. $F_{KK}\left(\frac{s^*}{n},1\right)$ in the right-hand side. As a matter of fact, should one drop these derivatives, condition (11) would revert to condition (4) at the steady state.

Note however that the best market steady state level of per capita savings s^* is not a laissez-faire competitive equilibrium outcome, unless it is actually optimal among *all* feasible steady states, i.e. unless it is the first-best steady state. In effect, the solution s^* to (11) does not satisfy the condition characterizing the competitive equilibrium steady state

$$\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s}{n},1\right)-s\right)}{u'\left(F_K\left(\frac{s}{n},1\right)s\right)} = F_K\left(\frac{s}{n},1\right) \tag{12}$$

unless

$$F_K\left(\frac{s^*}{n},1\right) = n\cdot\frac{F_K\left(\frac{s^*}{n},1\right)+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}} \tag{13}$$

that is to say, unless

$$F_K\left(\frac{s^*}{n},1\right) = n \tag{14}$$

[25] Only in the knife-edge case in which $s^g = s^c$ would the competitive equilibrium steady state coincide with the first-best steady state.

[26] If the utility function's boundary behavior guarantees interior solutions. Additive separability is inessential, and has been assumed for notational convenience only.

in other words, only if s^* satisfies the condition characterizing the first-best level of per capita savings s^g. But whenever the level of per capita savings s^* is distinct from the first-best level s^g, then both are distinct from the competitive equilibrium level s^c as well. Proposition 3 below states the precise way in which the three steady states relate to each other.

Proposition 3. *In a productive overlapping generations economy with only capital savings, either all per capita level of savings s^c, s^*, and s^g (at, respectively, the competitive equilibrium steady state, the best market steady state, and the first-best steady state) coincide, or they all are distinct.*

The previous proposition implies that, since typically the competitive equilibrium steady state is not the first-best one, it will not be the best allocation attainable through the markets either. But since the latter is not a competitive equilibrium outcome, reaching it requires some policy intervention. The question now is therefore which government intervention implements the best market steady state as a competitive equilibrium outcome. This question is addressed in the next section.

5. Decentralization of the best market steady state

Consider a government with the ability to tax (or subsidize) the agents capital income as well as to distribute a lump-sum transfer (or raise a lump-sum tax) when old. Specifically, letting $\tau_{t+1} > 0$ be one minus the tax rate at $t + 1$ (a subsidy if bigger than one), and T_{t+1} a lump-sum transfer (tax if negative), agent t's problem becomes

$$\begin{aligned} &\max_{0\leqslant c_0^t, c_1^t, s^t} u(c_0^t) + \beta u(c_1^t) \\ &c_0^t + s^t = w_t \\ &c_1^t = \tau_{t+1} r_{t+1} s^t + T_{t+1} \end{aligned} \tag{15}$$

and his optimal saving s^t is characterized by the condition

$$\frac{1}{\beta}\frac{u'(w_t - s^t)}{u'(\tau_{t+1} r_{t+1} s^t + T_{t+1})} = \tau_{t+1} r_{t+1} \tag{16}$$

for given w_t, r_{t+1}, τ_{t+1}, and T_{t+1}. The competitive equilibrium per capita savings dynamics, for given tax and transfer policy $\{\tau_t, T_t\}_t$ is then given by

$$\frac{1}{\beta}\frac{u'\left(F_L(\frac{s^{t-1}}{n}, 1) - s^t\right)}{u'(\tau_{t+1} F_K(\frac{s^t}{n}, 1) s^t + T_{t+1})} = \tau_{t+1} F_K\left(\frac{s^t}{n}, 1\right) \tag{17}$$

The next proposition characterizes the policy that implements as a competitive equilibrium the best market steady state.

Proposition 4. *In a productive overlapping generations economy with only capital savings, if savings returns are taxed at a rate $1 - \tau_{t+1}$ (a subsidy if $1 < \tau_{t+1}$) and a second period lump-sum transfer T_{t+1} (a lump-sum tax if negative) is introduced, determined as functions of the previous generation per capita savings s^{t-1} according to*

$$\begin{aligned} \tau_{t+1} &= \frac{n}{F_K(\frac{s^{t-1}}{n}, 1)} \cdot \frac{F_K\left(\frac{s^{t-1}}{n}, 1\right) + F_{KK}\left(\frac{s^{t-1}}{n}, 1\right)\frac{s^{t-1}}{n}}{n + F_{KK}(\frac{s^{t-1}}{n}, 1)\frac{s^{t-1}}{n}} \\ T_{t+1} &= (1 - \tau_{t+1}) F_K\left(\frac{s^{t-1}}{n}, 1\right) s^{t-1} \end{aligned} \tag{18}$$

if τ_{t+1} in (18) is positive, and $\tau_{t+1} = 0$ otherwise, then the competitive equilibrium steady state is the best market steady state. At such steady state the government keeps moreover a balanced budget every period.

To see why this policy supports the best market equilibrium, note that it makes depend the return to the agent's savings on past savings, internalizing thus at the steady state the impact that savings have on their own returns, which is at the origin of the typical inefficiency of the competitive equilibrium steady state. Specifically, it is the presence in the tax rate of the change of the marginal productivity of capital due to a change in savings, i.e. F_{KK}, that takes into account such effect. On the other hand, note that once the per capita savings reaches the first-best level s^g that equalizes the marginal productivity of capital to the growth factor of the population, $F_k = n$, (18) above prescribes $\tau = 1$ and $T = 0$, i.e. no tax or subsidy, as well as no lump-sum transfer or tax.

As for the practical implementation of such a policy, if the production function is a Cobb-Douglas function $F(K,L) = K^\alpha L^{1-\alpha}$ the tax/subsidy rate τ and lump-sum transfer/tax T depends on the population growth n, the share α of capital income in total income, and per capita savings s, all the three values directly observable. Note also that for the computation of both the rate and lump-sum values τ_{t+1} and T_{t+1} only information known at the time t they are chosen is used. Thus no generation can manipulate the policy, since it is determined by what the previous generation did.

If the economy is already at the competitive equilibrium steady state, a transition to the best market steady state is needed instead. Still there is a Pareto-improving policy implementing such a transition too, from any given $t+1$ onwards, to the best market steady state, namely

(1) announce at t that capital income will be taxed at date $t+1$ at a rate $1-\tau_{t+1}$ with

$$\tau_{t+1} = \frac{1}{F_K\left(\frac{s^*}{n},1\right)}\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^c}{n},1\right)-s^*\right)}{u'\left(F_K\left(\frac{s^*}{n},1\right)s^*\right)} \tag{19}$$

and that a lump-sum transfer T_{t+1} will be distributed to every old agent at $t+1$, with

$$T_{t+1} = (1-\tau_{t+1})F_K\left(\frac{s^*}{n},1\right)s^* \tag{20}$$

(2) tax capital income at every date $t' \geqslant t+2$ at a rate $1-\tau_{t'}$ with

$$\tau_{t'} = \frac{1}{F_K\left(\frac{s^*}{n},1\right)}\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^*}{n},1\right)-s^*\right)}{u'\left(F_K\left(\frac{s^*}{n},1\right)s^*\right)} \tag{21}$$

and make a lump-sum transfer $T_{t'}$ to every old agent at t', with

$$T_{t'} = (1-\tau_{t'})F_K\left(\frac{s^*}{n},1\right)s^*. \tag{22}$$

It is straightforward to check that this policy makes all generations $t' \geqslant t$ choose to save s^*. Moreover, the policy is balanced every period. Its implementation requires nonetheless to know the intertemporal rates of substitution for consumption at different consumption profiles, which is more demanding than the implementation of s^* (without transition) as a competitive equilibrium steady state in Proposition 4 above, since this only required to know the previous period per capita savings.[27]

Finally, to see that Diamond (1970) does not apply to the best market steady state found, note that at the best market steady state if $\tau^* < 1$, the net of tax returns rate $\tau^* F_K\left(\frac{s^*}{n},1\right)$ *does not* exceed the population growth rate n when $s^* > s^g$, since from

$$\tau^* = \frac{n}{F_K\left(\frac{s^*}{n},1\right)}\cdot\frac{F_K\left(\frac{s^*}{n},1\right)+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}} \tag{23}$$

it follows

$$\tau^* F_K\left(\frac{s^*}{n},1\right) = n\cdot\frac{F_K\left(\frac{s^*}{n},1\right)+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}} < n \tag{24}$$

because $F_K\left(\frac{s^*}{n},1\right) < n$ when $s^* > s^g$.[28] As a consequence, the result in Diamond (1970) does not apply to the tax (i.e. $\tau^* < 1$) that implements best market steady state.

6. Tax or subsidize savings?

Proposition 4 above establishes that the best market steady state can be decentralized by the appropriate tax or subsidy rate on savings returns coupled with some lump-sum transfer or tax. But when exactly is taxing or subsidizing savings required? The answer is provided by the next proposition establishing that, if the best market steady state over-accumulates capital with respect to the first-best steady state, then it can only be attained taxing savings returns and distributing second period lump-sum transfers. Conversely, if it under-accumulates capital with respect to the first-best steady state, it can only be attained subsidizing savings returns and raising a second period lump-sum tax.

Proposition 5. *In a productive overlapping generations economy with only capital savings, the best market steady state is decentralized as a competitive equilibrium taxing (resp. subsidizing) linearly capital income, along with a second period lump-sum transfer (resp. tax) if, and only if, it over-accumulates (resp. under-accumualtes) capital with respect to the first-best steady state.*

[27] Along with the elasticity α of output to capital and the growth factor of the population n.

[28] And $n + F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n} > 0$, as established in the proof of Proposition 2 in Appendix A (similarly, it can easily be established that if $\tau^* > 1$, the net of tax returns rate *does* exceed the population growth rate).

Should it seem counterintuitive that taxing in a distortionary way may improve upon the laissez-faire competitive steady state, note that the taxation of capital income aims at reducing the over-accumulation of capital (with respect to the unattainable first-best) from the laissez-faire competitive equilibrium level of per capita savings s^c to a smaller level s^*.[29] Reducing per capita savings below s^* is not efficient if factors are remunerated by their marginal productivities and no redistribution can take place. Similarly, subsidizing savings returns, but not up to the first-best level, allows to improve upon the laissez-faire in case it leads to excessive under-accumulation.

7. Concluding remarks

The main conclusion from this paper is that the taxation of capital returns can be justified by efficiency considerations alone. By disentangling from redistributive considerations the impact of the taxation of capital returns on steady state welfare, this paper establishes that in the absence of mechanisms allowing for the intergenerational transfers needed to attain the first-best steady state, a second-best can nonetheless be attained through capital income taxation and non-redistributive lump-sum transfers. The results thus provide a rationale for the taxation of capital returns that is based on welfare considerations independent of redistributive goals.

As for the generality of the results, it should be noted that the mechanism that drives results (i.e. that competitive agents fail to internalize the impact of their saving decisions in the aggregate on the return of their own savings) would still be there in more general overlapping generations models like, for instance, perpetual youth overlapping generations economies à la Blanchard (1985) with perfect annuities markets. Therefore, no competitive equilibrium will be able to implement the best steady state that still remunerates factors by their marginal productivities and makes no transfers. Nevertheless, designing a policy implementing this Pareto-improving steady state is a more delicate matter, since it would require age-contingent tax/subsidy rates (along with balancing lump-sum transfers or taxes). In effect, given that agents have a positive (albeit decreasing) probability of being alive at any future date, contemporaneous saving agents of all ages coexist at any point in time, and their savings must be taxed differently in order to replicate the choice of a constrained planner. The extension of the policy presented here to such a framework is left for future work.

A final remark on the assumption from Diamond (1970) about the absence of financial assets that would serve as alternative means of saving, or else of a social security implementing intergenerational transfers. Why is it interesting to know what is the best that can be done in the impossibility of intergenerational redistributions? Is it not clear that the first-best steady state can be attained by the adequate intergenerational transfers through monetary savings, public debt, or a social security? In principle, yes. But firstly, social security is far from being a universal institution, or from channeling all the agents' savings when it exists. And secondly, implementing the first-best steady state through a bubble asset like money or through rolled-over debt suffers from the indeterminacy of the agents' *choice* of their savings portfolio composition (not its level) between capital and a bubble asset or debt with identical returns. In effect, in the overlapping generations economy with production of Diamond (1965) the first-best steady state requires, as a competitive outcome, that the agents save both in capital *and* another intrinsically worthless asset – a bubble à la Tirole (1985) or public debt as in Diamond (1965) – to implement the necessary intergenerational transfers. In the absence of uncertainty the returns to capital and the financial asset have to be equal at equilibrium, so that the agents are indifferent about the composition of their savings portfolio. The latter is therefore not determined by their decisions, but by equilibrium conditions.[30] In other words, the implementation of the first-best steady state by means of, say, fiat money is only *implicitly* an equilibrium outcome, in the sense of being a collection of compatible optimal *decisions* from which it results, since the composition of the agents saving portfolio is decided, at first-best returns, by no one *in the model*.[31] Providing the missing element implicit in the decentralization of the first-best steady state as a monetary equilibrium is beyond the scope of this paper.

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[29] But it does not aim at its complete elimination, as the first-best would require, given the impossibility to implement the intergenerational transfers necessary to attain it.

[30] That the right amount of savings in terms of capital is the one that equalizes the marginal productivity of capital to the rate of growth of the population does not follow from the agent's (or firm's) decision problem. In the case there is uncertainty about the returns of the bubble asset, the agent's choice of the savings portfolio is well determined (see Dávila (2012)).

[31] Should this seem to be as innocuous an indeterminacy at equilibrium as that of the production plan of a firm with a constant returns to scale technology (which is assumed to result adjust to a demand that is well determined by prices), it is worth to notice that in this case in the money market, each period, *both* sides of the market are identical: the same representative agent facing the same indeterminacy. As a result, contrarily to what happens in the constant returns to scale case, there is no well-determined "other side of the market" able to anchor here the indeterminate side. Rather, in the savings portfolio problem the decisions of both sides of the money market are indeterminate each period at the first-best steady state equilibrium.

Appendix A

Proof of Proposition 1. The value $\phi(s)$ defined as

$$\phi(s) = u\left(F_L\left(\frac{s}{n},1\right) - s\right) + \beta u\left(F_K\left(\frac{s}{n},1\right)s\right) \tag{A1}$$

is the representative agent's utility at a steady state that remunerates factors by their marginal productivity, and with per capita savings s. The competitive equilibrium steady state utility is the value of $\phi(s)$ for $s = s^c$, the per capita savings solution to (4). For a constant returns to scale production function, ϕ is strictly decreasing (respectively increasing) at the competitive equilibrium steady state per capita savings s^c whenever it over-accumulates (resp. under-accumulates) capital with respect to the first-best steady state level s^g characterized by (10). In effect, note that

$$\phi'(s^c) = u'(c_0^c)\left[F_{LK}\left(\frac{s^c}{n},1\right)\frac{1}{n} - 1\right] + \beta u'(c_1^c)\cdot\left[F_K\left(\frac{s^c}{n},1\right) + F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}\right] \tag{A2}$$

where $c_0^c = F_L\left(\frac{s^c}{n},1\right) - s^c$ and $c_1^c = F_K\left(\frac{s^c}{n},1\right)s^c$ are the competitive equilibrium steady state profile of consumptions. But at the competitive equilibrium steady state it holds that

$$-u'(c_0^c) + \beta u'(c_1^c)F_K\left(\frac{s^c}{n},1\right) = 0 \tag{A3}$$

so that the derivative $\phi'(s^c)$ simplifies to

$$\phi'(s^c) = u'(c_0^c)F_{LK}\left(\frac{s^c}{n},1\right)\frac{1}{n} + \beta u'(c_1^c)F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n} \tag{A4}$$

Therefore, $\phi'(s^c) < (>) 0$ holds if, and only if,

$$\frac{1}{\beta}\frac{u'(c_0^c)}{u'(c_1^c)}\frac{1}{n} < (>) -\frac{F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}}{F_{LK}\left(\frac{s^c}{n},1\right)} \tag{A5}$$

or, equivalently – since the right-hand side is 1 because of the homogeneity of degree 1 of the neoclassical production function F, and the marginal rate of substitution $\frac{1}{\beta}\frac{u'(c_0^c)}{u'(c_1^c)}$ in the left-hand side is $F_K\left(\frac{s^c}{n},1\right)$ at the competitive steady state levels of consumption – if, and only if,

$$F_K\left(\frac{s^c}{n},1\right) < (>)\ n = F_K\left(\frac{s^g}{n},1\right). \tag{A6}$$

That is to say, $\phi'(s^c) < (>) 0$ holds if, and only if,

$$s^c > (<)\ s^g \tag{A7}$$

because of the decreasing marginal productivity of capital. □

Proof of Proposition 2. The steady state utility ϕ defined in (A1) is everywhere strictly concave because

$$\begin{aligned}\phi''(s) = {} & u''(c_0)\left[F_{LK}\left(\frac{s}{n},1\right)\frac{1}{n} - 1\right]^2 + u'(c_0)F_{LKK}\left(\frac{s}{n},1\right)\frac{1}{n^2} + \beta u''(c_1)\left[F_K\left(\frac{s}{n},1\right) + F_{KK}\left(\frac{s}{n},1\right)\frac{s}{n}\right]^2 \\ & + \beta u'(c_1)\left[\frac{2}{n}F_{KK}\left(\frac{s}{n},1\right) + F_{KKK}\left(\frac{s}{n},1\right)\frac{s}{n^2}\right] < 0\end{aligned} \tag{A8}$$

—where $c_0 = F_L\left(\frac{s}{n},1\right) - s$ and $c_1 = F_K\left(\frac{s}{n},1\right)s$ – given that all the terms are negative since

$$F_{LKK}(K,L) = (1-\alpha)\alpha(\alpha-1)K^{\alpha-2}L^{-\alpha} < 0 \tag{A9}$$

and

$$2F_{KK}(K,L) + F_{KKK}(K,L)K = \alpha^2(\alpha-1)K^{\alpha-2}L^{1-\alpha} < 0. \tag{A10}$$

Thus, the best market steady state level of capital s^* is characterized by the condition $\phi'(s^*) = 0$ or, equivalently, by (11) above, as long as $0 < s^* < F_L\left(\frac{s^*}{n},1\right)$, given that

$$n + F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n} = n + \alpha(\alpha-1)\left(\frac{s^*}{n}\right)^{\alpha-1} > 0 \tag{A11}$$

In effect, since s^* maximizes

$$\phi(s) = u\left(F_L\left(\frac{s}{n},1\right) - s\right) + \beta u\left(F_K\left(\frac{s}{n},1\right)s\right) \tag{A12}$$

and the derivative of the first term in the right-hand side in (A12) is null for $\frac{s}{n} = \left(\frac{\alpha(1-\alpha)}{n}\right)^{\frac{1}{1-\alpha}}$, i.e.

$$\frac{d}{ds}\left[F_L(\frac{s}{n},1) - s\right] = 0 \tag{A13}$$

for $\frac{s}{n} = \left(\frac{\alpha(1-\alpha)}{n}\right)^{\frac{1}{1-\alpha}}$ while the derivative of the second term is always positive, i.e.

$$\frac{d}{ds}\left[F_K\left(\frac{s}{n},1\right)s\right] = \alpha^2\left(\frac{s}{n}\right)^{\alpha-1} > 0 \tag{A14}$$

then $\phi'(s) > 0$ for $\frac{s}{n} = \left(\frac{\alpha(1-\alpha)}{n}\right)^{\frac{1}{1-\alpha}}$, so that, necessarily,

$$\frac{s^*}{n} > \left(\frac{\alpha(1-\alpha)}{n}\right)^{\frac{1}{1-\alpha}} \tag{A15}$$

as stated in (A11) above.

Finally, the steady state allocation giving a consumption $F_L(\frac{s^*}{n},1) - s^*$ when young and a consumption $F_K(\frac{s^*}{n},1)s^*$ when old is feasible since

$$F_L\left(\frac{s^*}{n},1\right) - s^* + \frac{1}{n}F_K\left(\frac{s^*}{n},1\right)s^* + s^* = F\left(\frac{s^*}{n},1\right) \quad \square \tag{A16}$$

Proof of Proposition 3. Assume $s^c = s^g$. Then

$$\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^g}{n},1\right) - s^g\right)}{u'\left(F_K\left(\frac{s^g}{n},1\right)s^g\right)} = F_K\left(\frac{s^g}{n},1\right) = n = n\cdot\frac{F_K\left(\frac{s^g}{n},1\right) + F_{KK}\left(\frac{s^g}{n},1\right)\frac{s^g}{n}}{n + F_{KK}\left(\frac{s^g}{n},1\right)\frac{s^g}{n}} \tag{A17}$$

so that $s^g = s^*$.

Assume $s^g = s^*$. Then

$$\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^*}{n},1\right) - s^*\right)}{u'\left(F_K\left(\frac{s^*}{n},1\right)s^*\right)} = n = F_K\left(\frac{s^g}{n},1\right) = F_K\left(\frac{s^*}{n},1\right) \tag{A18}$$

so that $s^* = s^c$.

Assume $s^* = s^c$. Then

$$F_K\left(\frac{s^c}{n},1\right) = \frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^c}{n},1\right) - s^c\right)}{u'\left(F_K\left(\frac{s^c}{n},1\right)s^c\right)} = n\cdot\frac{F_K\left(\frac{s^c}{n},1\right) + F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}}{n + F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}} \tag{A19}$$

from which

$$F_K\left(\frac{s^c}{n},1\right) = n \tag{A20}$$

so that $s^c = s^g$. $\square$

Proof of Proposition 4. If the tax rate and the lump-sum transfer are determined according to (18) above, then in the competitive equilibrium dynamics (17) the net of tax returns in the right-hand side becomes (whenever $\tau_{t+1} > 0$)[32]

$$\frac{F_K\left(\frac{s^t}{n},1\right)}{F_K\left(\frac{s^{t-1}}{n},1\right)}\cdot n\cdot\frac{F_K\left(\frac{s^{t-1}}{n},1\right) + F_{KK}\left(\frac{s^{t-1}}{n},1\right)\frac{s^{t-1}}{n}}{n + F_{KK}\left(\frac{s^{t-1}}{n},1\right)\frac{s^{t-1}}{n}} \tag{A21}$$

and the old-age consumption within the marginal utility in the denominator of the left-hand side is

[32] Note that the steady state net of tax saving returns $\tau^* F_K\left(\frac{s^*}{n},1\right)$ is positive,

$$\tau^* F_K\left(\frac{s^*}{n},1\right) = n\cdot\frac{F_K\left(\frac{s^*}{n},1\right) + F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n + F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}} = \frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^*}{n},1\right)l - s^*\right)}{u'\left(F_K\left(\frac{s^*}{n},1\right)s^*\right)} > 0$$

which guarantees that the agent's problem is well defined (in particular that the budget set is compact).

$$\tau_{t+1}F_K\left(\frac{s^t}{n},1\right)s^t+(1-\tau_{t+1})F_K\left(\frac{s^{t-1}}{n},1\right)s^{t-1} \tag{A22}$$

so that the steady state of the dynamics (17) is characterized precisely by the same condition (11) than the best steady state, i.e.

$$\frac{1}{\beta}\frac{u'\left(F_L\left(\frac{s^*}{n},1\right)l-s^*\right)}{u'\left(F_K\left(\frac{s^*}{n},1\right)s^*\right)}=n\cdot\frac{F_K\left(\frac{s^*}{n},1\right)+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}} \tag{A23}$$

From the definitions of the second period lump sum transfer T_{t+1} it follows trivially that, at the steady state, what the government withdraws from (respectively, injects to) each generation in a distortionary way is exactly offset by the resources it injects to (resp. withdraws from) that same generation in a non-distortionary way, so that the government's budget is balanced every period. □

Proof of Proposition 5. At the best market steady state level of per capita savings s^*, capital revenue is taxed (resp. subsidized) if the constant rate

$$\tau^*=\frac{n}{F_K\left(\frac{s^*}{n},1\right)}\cdot\frac{F_K\left(\frac{s^*}{n},1\right)+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}} \tag{A24}$$

is smaller (respectively, bigger) than 1, i.e. if, and only if,

$$n\cdot\frac{F_K\left(\frac{s^*}{n},1\right)+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}{n+F_{KK}\left(\frac{s^*}{n},1\right)\frac{s^*}{n}}<F_K\left(\frac{s^*}{n},1\right) \tag{A25}$$

which (since the denominator is positive, see Proposition 2) holds if, and only if,

$$F_K\left(\frac{s^*}{n},1\right)<n=F_K\left(\frac{s^g}{n},1\right) \tag{A26}$$

i.e. if, and only if

$$s^*>s^g \quad \square \tag{A27}$$

Lemma 1. *In the Diamond (1965) overlapping generations economy, it holds that*

$$s^c>s^*\iff s^c>s^g$$
$$s^c<s^*\iff s^c<s^g.$$

with s^c, s^*, and s^g being the per capita level of savings at, respectively, the competitive equilibrium steady state, the best market steady state, and the first-best steady state.

Proof 1. For $s^c > s^*$ if, and only if, $s^c > s^g$, note that a necessary and sufficient condition for $s^c > s^*$ is

$$1+F_K\left(\frac{s^c}{n},1\right)>n\cdot\frac{1+F_K\left(\frac{s^c}{n},1\right)+F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}}{n+F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}} \tag{A28}$$

—where the left-hand side is the slope of the representative agent indifference curve at the competitive equilibrium steady state profile of consumptions (c_0^c, c_1^c) (see Fig. 1), and the right-hand side is the slope at that same point of the curve $(c_0(s),c_1(s))$ of steady state consumption profiles attainable through the existing markets – and condition (A28) holds if, and only if

$$F_K\left(\frac{s^c}{n},1\right)<n=F_K\left(\frac{s^g}{n},1\right) \tag{A29}$$

i.e. if, and only if, $s^c > s^g$. This follows from the fact that $n+F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}>0$. In effect, should $n+F_{KK}\left(\frac{s^c}{n},1\right)\frac{s^c}{n}<0$ hold instead,[33] then condition (A28) would hold (so that $s^c > s^*$) if, and only if, $s^g > s^c$, which implies $s^g > s^c > s^*$. Nevertheless, the optimal policy to implement s^* (see Proposition 5) requires, in the case $s^g > s^*$, subsidizing the return to savings in order to *increase* (rather than reduce) savings from its laissez-faire competitive level s^c towards the first-best level s^g, i.e. $s^g > s^* > s^c$ instead of $s^g > s^c > s^*$. Finally, from $s^c = s^* \Leftrightarrow s^c = s^g$ in Proposition 3 and the previous result the rest of the statement follows. □

[33] In case of equality, the slope of $c(s)$ at s^c is vertical, so that it cannot be that $s^c > s^g$.

References

Aiyagari, S.R., 1995. Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting. Journal of Political Economy 103 (6), 1158–1175.

Atkeson, A., Chari, V.V., Kehoe, P.J., 1999. Taxing capital income: a bad idea. Federal Reserve Bank of Minneapolis Quarterly Review 23 (3), 3–17.

Blanchard, O.J., 1985. Debt, deficits and finite horizons. Journal of Political Economy 93, 223–224.

Chamley, G., 1986. Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica 54 (3), 607–622.

Chamley, C., 2001. Capital income taxation, wealth distribution and borrowing constraints. Journal of Public Economics 79 (1), 55–79.

Conesa, J.C., Kitao, S., Krueger, D., 2007. Taxing Capital? Not a Bad Idea After All!" NBER Working Paper 12880.

Dávila, J., 2012. Implementing the best steady state with savings in unbacked risky assets. Macroeconomic Dynamics. doi:10.1017/S1365100511000666.

Diamond, P.A., 1965. National debt in a neoclassical growth model. American Economic Review 55 (5), 1126–1150.

Diamond, P.A., 1970. Incidence of an interest income tax. Journal of Economic Theory 2 (3), 211–224.

Erosa, A., Gervais, M., 2002. Optimal taxation in life-cycle economies. Journal of Economic Theory 105, 338–369.

Judd, K.L., 1987. The welfare cost of factor taxation in a perfect-foresight model. Journal of Political Economy 95 (4), 675–709.

Saez, E., 2002. Optimal Progressive Capital Income Taxes in the Infinite Horizon Model. NBER Working Paper No. 9046.

Svensson, L.-G., 1986. Taxation of savings. Journal of Economics 46 (4), 421–426.

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