Connections Among Farsighted Agents

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Abstract

We study the stability of social and economic networks when players are farsighted. In particular, we examine whether the networks formed by farsighted players are different from those formed by myopic players. We adopt the notion of pairwise farsightedly stable sets (Herings, Mauleon, and Vannetelbosch 2009). We first show that under the componentwise egalitarian allocation rule, the set of strongly efficient networks and the set of pairwise (myopically) stable networks that are immune to coalitional deviations are the unique pairwise farsightedly stable set if and only if the value function is top convex. We then investigate in some classical models of social and economic networks whether the pairwise farsightedly stable sets of networks

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coincide or not with the set of pairwise (myopically) stable networks and the set of strongly efficient networks.

1. Introduction

The network structure of social interactions influences a variety of behaviors and economic outcomes, including the formation of opinions, decisions on which products to buy, investment in education, access to jobs, social mobility, informal borrowing and lending, favor exchange, and participation in loan programs.[1] A simple way to analyze the networks that one might expect to emerge in the long run is to examine the requirement that individuals do not benefit from altering the structure of the network. Jackson and Wolinsky (1996) have proposed the notion of pairwise stability. A network is pairwise stable if no individual benefits from severing one of her links and no two individuals benefit from adding a link between them, with one benefiting strictly and the other at least weakly. Pairwise stability is a myopic definition. Individuals are not farsighted in the sense that they do not forecast how others might react to their actions. For instance, the adding or severing of one link might lead to subsequent addition or severing of another link. If individuals have very good information about how others might react to changes in the network, then these are things one wants to allow for in the definition of the stability concept. For instance, a network could be stable because individuals might not add a link that appears valuable to them given the current network, as that might in turn lead to the formation of other links and ultimately lower the payoffs of the original individuals.

Herings *et al.* (2009) have proposed the notion of pairwise farsightedly stable sets of networks that predict which networks one might expect to emerge in the long run when individuals are farsighted.[2] A set of networks G is pairwise farsightedly stable (i) if all possible pairwise deviations from any network $g \in G$ to a network outside G are deterred by the threat of ending worse off or equally well off, (ii) if there exists a farsighted improving path from any network outside the set leading to some network in the set,[3] and

[1] See Jackson (2008) or Goyal (2007) for a comprehensive introduction to the theory of social and economic networks.

[2] Other approaches to farsightedness in network formation are suggested by the work of Chwe (1994), Xue (1998), Herings, Mauleon, and Vannetelbosch (2004), Mauleon and Vannetelbosch (2004), Page, Wooders, and Kamat (2005), Dutta, Ghosal, and Ray (2005), and Page and Wooders (2009).

[3] A farsighted improving path is a sequence of networks that can emerge when players form or sever links based on the improvement the end network offers relative to the current network. Each network in the sequence differs by one link from the previous one. If a link is added, then the two players involved must both prefer the end network to the current network, with at least one of the two strictly preferring the end network. If a link is deleted, then it must be that at least one of the two players involved in the link strictly prefers the end network.

(iii) if there is no proper subset of G satisfying Conditions (i) and (ii). A nonempty pairwise farsightedly stable set always exists. Herings *et al.* (2009) have provided a full characterization of unique pairwise farsightedly stable sets of networks. Contrary to other pairwise concepts, pairwise farsighted stability yields a Pareto dominant network, if it exists, as the unique outcome.[4]

The objective of this paper is twofold. First, we provide some primitive conditions on value functions and allocation rules so that the set of strongly efficient networks is the unique pairwise farsightedly stable set. We find that, under the componentwise egalitarian allocation rule, the set of strongly efficient networks and the set of pairwise myopically stable networks that are immune to coalitional deviations are the unique pairwise farsightedly stable set if and only if the value function is top convex. A value function is top convex if some strongly efficient network also maximizes the per-capita value among individuals.

Second, we investigate in some classical models of social and economic networks whether the pairwise farsightedly stable sets of networks coincide with the set of pairwise myopically stable networks and the set of strongly efficient networks. We reconsider three classical models of network formation in which the aforementioned primitive conditions on value functions and/or allocation rules break down: the Jackson and Wolinsky (1996) symmetric connections model, the Corominas-Bosch (2004) model of trading networks with bilateral bargaining, and the Kranton and Minehart (2001) model of buyer-seller networks. We have chosen to analyze those models because they have different features. The symmetric connections model describes a situation where *homogeneous* individuals obtain payoffs not only from direct but also from *indirect* connections (where links represent social relationships between individuals such as friendships), while the models of buyer-seller networks formalize situations where *heterogeneous* individuals (sellers and buyers) bargain over prices for trade (where *direct* links are necessary for a transaction to occur). We find that, in the symmetric connections model, farsightedness does not eliminate the conflict between stability and strong efficiency that may occur when costs are intermediate. However, farsightedness helps to reduce the conflict when costs are large enough. In the bargaining model of Corominas-Bosch (2004), myopic or farsighted notions of stability sustain the set of strongly efficient networks when the costs of maintaining links are not too large. In the Kranton and Minehart (2001) model, pairwise farsighted stability may sustain the strongly efficient network while pairwise myopic stability only sustains networks that are strongly inefficient or even Pareto dominated.

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[4] Herings *et al.* (2009) have also studied the relationship between pairwise farsighted stability and other concepts. Any von Neumann-Morgenstern pairwise farsightedly stable set is a pairwise farsightedly stable set. But, a von Neumann-Morgenstern pairwise farsightedly stable set may fail to exist. Pairwise farsightedly stable sets have no relationship to either largest pairwise consistent sets or sets of pairwise stable networks.

The paper is organized as follows. In Section 2, we introduce some notations, basic properties and definitions for networks. In Section 3, we define the notions of pairwise myopically stable sets and of pairwise farsightedly stable sets. In Section 4, we look at the relationship between farsighted stability and efficiency of networks. In Section 5, we reconsider the Jackson and Wolinsky (1996) symmetric connections model, the Corominas-Bosch (2004) model of trading networks, and the Kranton and Minehart (2001) model of buyer-seller networks. In Section 6, we conclude.

2. Networks

Let $N = \{1, \dots, n\}$ be the finite set of players who are connected in some network relationship. The network relationships are reciprocal and the network is thus modeled as a nondirected graph. Individuals are the nodes in the graph and links indicate bilateral relationships between individuals. Thus, a network g is simply a list of which pairs of individuals are linked to each other. We write $ij \in g$ to indicate that i and j are linked under the network g. Let g^S be the set of all subsets of $S \subseteq N$ of size 2.[5] So, g^N is the complete network. The set of all possible networks or graphs on N is denoted by $\mathbb{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $g|_S = \{ij | ij \in g$ and $i \in S, j \in S\}$. Thus, $g|_S$ is the network found deleting all links except those that are between players in S. For any network g, let $N(g) = \{i | \exists j$ such that $ij \in g\}$ be the set of players who have at least one link in the network g. A path in a network $g \in \mathbb{G}$ between i and j is a sequence of players $i_1, \dots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \dots, K-1\}$ with $i_1 = i$ and $i_K = j$. A nonempty network $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. The set of components of g is denoted by $C(g)$.

A value function is a function $v : \mathbb{G} \to \mathbb{R}$ that keeps track of how the total societal value varies across different networks. The set of all possible value functions is denoted by $\mathcal{V}$. An allocation rule is a function $Y : \mathbb{G} \times \mathcal{V} \to \mathbb{R}^N$ that keeps track of how the value is allocated among the players forming a network. It satisfies $\sum_{i \in N} Y_i(g, v) = v(g)$ for all v and g.

Jackson and Wolinsky (1996) have proposed a number of basic properties of value functions and allocation rules. A value function is *component additive* if $v(g) = \sum_{h \in C(g)} v(h)$ for all $g \in \mathbb{G}$. Component additive value functions are the ones for which the value of a network is the sum of the value of its components. Given a permutation of players π and any $g \in \mathbb{G}$,

[5] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, # will refer to the notion of cardinality.

let $g^{\pi} = \{\pi(i)\pi(j)|ij \in g\}$. Thus, g^{π} is a network that is identical to g up to a permutation of the players. A value function is *anonymous* if for any permutation π and any $g \in \mathbb{G}$, $v(g^{\pi}) = v(g)$.

For a component additive v and network g, the *componentwise egalitarian allocation rule* Y^{ce} is such that for any $h \in C(g)$ and each $i \in N(h)$, $Y_i^{ce}(g, v) = v(h)/\#N(h)$. For a v that is not component additive, $Y^{ce}(g, v) = v(g)/n$ for all g; thus, Y^{ce} splits the value $v(g)$ equally among all players if v is not component additive.

In evaluating societal welfare, we may take various perspectives. A network g is *Pareto efficient* relative to v and Y if there does not exist any $g' \in \mathbb{G}$ such that $Y_i(g', v) \geq Y_i(g, v)$ for all i with at least one strict inequality. A network $g \in \mathbb{G}$ is *strongly efficient* relative to v if $v(g) \geq v(g')$ for all $g' \in \mathbb{G}$. This is a strong notion of efficiency as it takes the perspective that the value is fully transferable.

A simple way to analyze the networks that one might expect to emerge in the long run is to examine the requirement that agents do not benefit from altering the structure of the network. A weak version of such a condition is the pairwise stability notion defined by Jackson and Wolinsky (1996). A network is pairwise stable if no player benefits from severing one of her links and no two players benefit from adding a link between them, with one benefiting strictly and the other at least weakly. Formally, a network g is pairwise stable with respect to value function v and allocation rule Y if (i) for all $ij \in g$, $Y_i(g, v) \geq Y_i(g - ij, v)$ and $Y_j(g, v) \geq Y_j(g - ij, v)$, and (ii) for all $ij \notin g$, if $Y_i(g, v) < Y_i(g + ij, v)$ then $Y_j(g, v) > Y_j(g + ij, v)$.

3. Pairwise Farsightedly Stable Sets of Networks

Herings *et al.* (2009) have proposed the notion of pairwise myopically stable sets of networks which is a generalization of Jackson and Wolinsky (1996) pairwise stability notion. Pairwise stable networks do not always exist. A pairwise myopically stable set of networks is a set such that (i) from any network outside this set, there is a myopic improving path leading to some network in the set, (ii) each deviation outside the set is deterred because the deviating players do not prefer the resulting network, and (iii) there is no proper subset satisfying (i) and (ii). The notion of a myopic improving path was first introduced in Jackson and Watts (2001). A myopic improving path is a sequence of networks that can emerge when players form or sever links based on the improvement the resulting network offers relative to the current network. Each network in the sequence differs by one link from the previous one. If a link is added, then the two players involved must both prefer the resulting network to the current network. If a link is deleted, then it must be that at least one of the two players involved in the link prefers the resulting network.

Jackson and Watts (2001) have defined the notion of a closed cycle. A set of networks C is a *cycle* if for any $g \in C$ and $g' \in C \backslash \{g\}$, there exists a myopic improving path connecting g to g'. A cycle C is a maximal cycle if it is not a proper subset of a cycle. A cycle C is a *closed cycle* if no network in C lies on a myopic improving path leading to a network that is not in C. A closed cycle is necessarily a maximal cycle. Herings *et al.* (2009) have shown that the set of networks consisting of all networks that belong to a closed cycle is the unique pairwise myopically stable set.

A *farsighted improving path* is a sequence of networks that can emerge when players form or sever links based on the improvement the end network offers relative to the current network. Each network in the sequence differs by one link from the previous one. If a link is added, then the two players involved must both prefer the end network to the current network, with at least one of the two strictly preferring the end network. If a link is deleted, then it must be that at least one of the two players involved in the link strictly prefers the end network. Formally, a farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of graphs $g_1, \ldots,$ g_K with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either: (i) $g_{k+1} = g_k - ij$ for some ij such that $Y_i(g_K, v) > Y_i(g_k, v)$ or $Y_j(g_K, v) > Y_j(g_k, v)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $Y_i(g_K, v) > Y_i(g_k, v)$ and $Y_j(g_K, v) \geq Y_j(g_k, v)$. For a given network g, let $F(g)$ be the set of networks that can be reached by a farsighted improving path from g. Notice that $F(g)$ may contain many networks and that a network $g' \in F(g)$ might be the endpoint of several farsighted improving paths starting in g.

We now introduce a solution concept due to Herings *et al.* (2009), the pairwise farsightedly stable set.

Definition 1: *A set of networks $G \subseteq \mathbb{G}$ is pairwise farsightedly stable with respect v and Y if*

(i) $\forall g \in G$,
(ia) $\forall ij \notin g$ *such that* $g + ij \notin G$, $\exists g' \in F(g + ij) \cap G$ *such that* $(Y_i(g', v), Y_j(g', v)) = (Y_i(g, v), Y_j(g, v))$ *or* $Y_i(g', v) < Y_i(g, v)$ or $Y_j(g', v) < Y_j(g, v)$,
(ib) $\forall ij \in g$ *such that* $g - ij \notin G$, $\exists g', g'' \in F(g - ij) \cap G$ *such that* $Y_i(g', v) \leq Y_i(g, v)$ *and* $Y_j(g'', v) \leq Y_j(g, v)$,
(ii) $\forall g' \in \mathbb{G} \setminus G$, $F(g') \cap G \neq \varnothing$,
(iii) $\nexists G' \subsetneq G$ *such that G' satisfies Conditions (ia), (ib), and (ii).*

Condition (i) in Definition 1 requires the deterrence of external deviations. Condition (ia) captures that adding a link ij to a network $g \in G$ that leads to a network outside G, is deterred by the threat of ending in g'. Here g' is such that there is a farsighted improving path from $g + ij$ to g'. Moreover, g' belongs to G, which makes g' a credible threat. Condition (ib) is a similar requirement, but then for the case where a link is severed. Condition (ii) in Definition 1 requires external stability and implies that the networks within the set are robust to perturbations. From any network outside of G there is a farsighted improving path leading to some network in G. Condition (ii) implies that if a set of networks is pairwise farsightedly stable, it is nonempty. Notice that the set $\mathbb{G}$ (trivially) satisfies Conditions (ia), (ib), and (ii) in Definition 1. This motivates the requirement of a minimality condition, namely, Condition (iii). Herings *et al.* (2009) have shown that a pairwise farsightedly stable set of networks always exists.

A network g strictly Pareto dominates all other networks if g is such that for all $g' \in \mathbb{G} \setminus \{g\}$ it holds that, for all i, $Y_i(g, v) > Y_i(g', v)$. Although the network that strictly Pareto dominates all others is pairwise stable, there might be many more pairwise stable networks. Herings *et al.* (2009) have shown that, if there is a network g that strictly Pareto dominates all other networks, then $\{g\}$ is the unique pairwise farsightedly stable set.

4. Farsighted Stability and Efficiency

We now provide some alternative primitive conditions on value functions and allocation rules so that the set of strongly efficient networks is the unique pairwise farsightedly stable set. It will turn out that under the conditions we will impose the notion of pairwise farsighted stability refines the notion of pairwise stability by eliminating the inefficient pairwise stable networks.

A value function v is *top convex* if some strongly efficient network also maximizes the per-capita value among players. Let $\rho(v, S) = \max_{g \subseteq g^S} v(g)/\#S$. The value function v is *top convex* if $\rho(v, N) \geq \rho(v, S)$ for all $S \subseteq N$.

PROPOSITION 1: *Consider any anonymous and component additive value function v. The set of strongly efficient networks $E(v)$ is the unique pairwise farsightedly stable set under the componentwise egalitarian allocation rule Y^{ce} if and only if v is top convex.*

The proof of all propositions can be found in the Appendix. The intuition behind the proof of this proposition is as follows. Take any anonymous and component additive value function v. First, if the value function v is top convex then all components of a strongly efficient network must lead to the same per-capita value. In addition, under the componentwise egalitarian allocation rule Y^{ce}, any strongly efficient network Pareto dominates all inefficient networks. Then, it is immediate that there are no farsighted improving paths emanating from a strongly efficient network, and that from any inefficient network there is a farsighted improving path to some strongly efficient network. Using Theorem 5 in Herings *et al.* (2009), which says that G is the unique pairwise farsightedly stable set if and only if $G = \{g \in \mathbb{G} \mid F(g) = \varnothing\}$ and for every $g' \in \mathbb{G} \setminus G$, $F(g') \cap G \neq \varnothing$, we have that the set of strongly efficient networks, $E(v)$, is the unique pairwise farsightedly stable set.

Second, if $E(v)$ is the unique pairwise farsightedly stable set, then we know from Theorem 5 in Herings *et al.* (2009) that there are no improving paths emanating from any strongly efficient network. It follows that all players receive the same allocation in any strongly efficient network under the componentwise egalitarian allocation rule and that all players receive more in any strongly efficient network than in any inefficient network. Otherwise, there would exist a farsighted improving path from the strongly efficient network $g \in E(v)$. Thus, we have that v is top convex.

Jackson and van den Nouweland (2005) have shown that the set of strongly efficient networks coincides with the set of strongly stable networks under the componentwise egalitarian allocation rule if and only if v is top convex.[6] Hence, the set of strongly stable networks is the unique pairwise farsightedly stable set under the componentwise egalitarian allocation rule if and only if the value function is top convex. So, pairwise farsighted stability selects under Y^{ce} the pairwise stable networks that are immune to coalitional deviations if and only if v is top convex.

Notice that top convexity is a condition that is satisfied in some natural situations. For instance, the value function of Jackson and Wolinsky (1996) symmetric connections model is top convex for all values of $\delta \in [0, 1)$ and $c \geq 0$, so that all strongly efficient networks with respect to v form the unique pairwise farsightedly stable set with respect to Y^{ce} and v.[7]

5. Two Models of Social and Economic Networks

We now investigate in some classical models of social and economic networks whether the pairwise farsightedly stable sets of networks coincide with the set of pairwise (myopically) stable networks and the set of strongly efficient networks. We reconsider three classical models of network formation when the aforementioned primitive conditions on value functions and/or allocation rules break down. In the Jackson and Wolinsky (1996) symmetric connections model and in the Corominas-Bosch (2004) model of trading networks with bilateral bargaining, the value function is top convex but the allocation rule is not componentwise egalitarian. In the Kranton and Minehart (2001)

[6] Jackson and van den Nouweland (2005) have proposed a refinement of pairwise stability where coalitionwise deviations are allowed: the strongly stable networks. A strongly stable network is a network which is stable against changes in links by any coalition of individuals. Strongly stable networks are Pareto efficient and maximize the overall value of the network if the value of each component of a network is allocated equally among the members of that component.

[7] Provided that n is even, the value function of the Jackson and Wolinsky (1996) coauthor model is top convex as the strongly efficient network always involves pairs of players who are linked to each other. The value function of the Herings *et al.* (2009) criminal networks model is top convex too. Finally, the value function of Bramoullé and the Kranton (2007) risk sharing networks model is top convex when the utility function is quadratic.

model of buyer-seller networks, the value function violates top convexity and the allocation rule is not componentwise egalitarian.

5.1. The Symmetric Connections Model

In the Jackson and Wolinsky (1996) symmetric connections model, players form links with each other in order to exchange information. If player i is connected to player j by a path of t links, then player i receives a payoff of δ^t from her indirect connection with player j. It is assumed that $0 < \delta < 1$, and so the payoff δ^t decreases as the path connecting players i and j increases; thus information that travels a long distance becomes diluted and is less valuable than information obtained from a closer neighbor. Each direct link ij results in a cost c to both i and j. This cost can be interpreted as the time a player must spend with another player in order to maintain a direct link. Player i's payoff from a network g is given by

$$Y_i(g) = \sum_{j \neq i} \delta^{t(ij)} - \sum_{j:ij \in g} c,$$

where $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j). Here the value of network g equals $v(g) = \sum_{i \in N} Y_i(g)$ and is top convex. Let g^* denote a star network encompassing everyone and $g^{\varnothing}$ be the empty network (no links).

Jackson and Wolinsky (1996) have shown that the unique strongly efficient network is (i) the complete network g^N if $c < \delta(1-\delta)$, (ii) a star encompassing everyone if $\delta(1-\delta) < c < \delta + ((n-2)/2)\delta^2$, and (iii) the empty network if $\delta + ((n-2)/2)\delta^2 < c$. For $c < \delta(1-\delta)$, the unique pairwise stable network is the complete network g^N. For $\delta(1-\delta) < c < \delta$, a star encompassing all players is pairwise stable, but not necessarily the unique pairwise stable network. For $\delta < c$, any pairwise stable network which is nonempty is such that each player has at least two links and thus is inefficient. Thus, there is a conflict between efficiency and pairwise stability for a large range of the parameters. Indeed, only for $c < \delta(1-\delta)$, there is no conflict between the efficient and the pairwise stable networks. When $\delta(1-\delta) < c < \delta$, the efficient network is pairwise stable, but there are other pairwise stable networks that are not efficient. For $\delta < c < \delta + ((n-2)/2)\delta^2$, the efficient network is never pairwise stable. And, finally, for $\delta + ((n-2)/2)\delta^2 < c$, the efficient network is pairwise stable, but there could be other pairwise stable networks that are not efficient.[8] Let $\overline{c}(n)$ be the highest cost of link formation such that the payoff of all players is nonnegative in at least one network other than the empty network. Formally, $\overline{c}(n) = \max\{c \in \mathbb{R} \mid \exists\, g \in \mathbb{G}$ such that $g \neq g^{\varnothing}$ and

[8] If $\delta > c > n(\delta - \delta^{n-1})$ then the myopically stable set consists only of pairwise stable networks. Similarly, if c is very small or very large then the symmetric connections model has no cycles. See Jackson and Watts (2001).

$Y_i(g) \geq 0 \forall i \in N\}$. In the Appendix we show that $\delta < \overline{c}(n) < \delta + ((n-2)/2)\delta^2$ if $n \geq 4$.

PROPOSITION 2: *Take the symmetric connections model.*

- *For $c < \delta(1-\delta)$, a set consisting of the complete network, $\{g^N\}$, is the unique pairwise farsightedly stable set.*
- *For $\delta(1-\delta) < c < \delta$, every set consisting of a star network encompassing all players, $\{g^*\}$, is a pairwise farsightedly stable set of networks, but they are not necessarily the unique pairwise farsightedly stable sets.*
- *For $c > \delta$, a set consisting of the empty network, $\{g^\varnothing\}$, is the unique pairwise farsightedly stable set if $c > \overline{c}(n)$, while $\{g^\varnothing\}$ is not a pairwise farsightedly stable set if $c \leq \overline{c}(n)$.*

Proposition 2 shows that replacing myopic by farsighted players in the symmetric connections model does not eliminate the conflict between strong efficiency and stability but, sometimes it may help to reduce it. For instance, when $\delta + ((n-2)/2)\delta^2 < c$, a set consisting of the unique strongly efficient network is the unique pairwise farsightedly stable set while other networks may be pairwise stable.[9] Thus, pairwise farsighted stability may single out the strongly efficient network even though the allocation rule is not componentwise egalitarian. In fact, each player receives the same payoff in the empty network $g^\varnothing$ (so no player would like to take the role of another player in $g^\varnothing$) as if the componentwise egalitarian allocation rule was used. It follows that in any network g other than the empty network, there is some player with a negative payoff who prefers $g^\varnothing$ and hence there is no farsighted improving path from $g^\varnothing$ to g but there is a farsighted improving path from g to $g^\varnothing$ (players with negative payoffs in g have incentives to delete their links looking forward to $g^\varnothing$). That is, $F(g^\varnothing) = \varnothing$ and $g^\varnothing \in F(g)$for all $g \neq g^\varnothing$. Regarding the relationship between pairwise stability and pairwise farsighted stability, we observe that the concept of pairwise stability is quite robust to the introduction of farsighted players because, for a large range of parameters, we have that pairwise stable networks belong to pairwise farsightedly stable sets.

Watts (2001) has analyzed the process of network formation in a dynamic framework where pairs of myopic players meet and decide whether or not to form or sever links with each other based on the improvement the resulting network offers relative to the current network. If the benefit from maintaining an indirect link is greater than the net benefit from maintaining a direct link (case (ii) of Proposition 2), then it is difficult for the strongly efficient network (which is the star network) to form. In fact, starting at the empty network, the strongly efficient network only forms if the order in which the

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[9] For instance, Jackson and Wolinsky (1996) have shown that a "tetrahedron" involving 16 players is pairwise stable but is not strongly efficient.

players meet takes a particular pattern. Moreover, as the number of players increases it becomes less likely that the strongly efficient network forms. These results contrast with ours, for such parameter values, since every set consisting of a star network is a pairwise farsightedly stable set whatever the number of farsighted players. Thus, it is not unlikely that forward-looking players will increase the chances of the star forming.

5.2. Buyer-Seller Networks

5.2.1. A Model of Trading Networks with Bilateral Bargaining

Corominas-Bosch (2004) has developed a simple model of trading networks with bilateral bargaining. The market consists of m sellers $1, 2, \ldots, m$ and m buyers $m+1, m+2, \ldots, 2m$. We denote the set of buyers as B and the set of sellers as S. Each seller owns a single object to sell that has no value to the seller. Buyers have a valuation of 1 for an object and do not care from whom they purchase the good. If a seller and a buyer trade at price p , the seller receives a payoff of p and the buyer a payoff of $1-p$. Agents are embedded in a network that links sellers and buyers, and trade is only possible among linked agents. That is, a link in the network represents the opportunity for a buyer and a seller to bargain and potentially exchange an object.[10] Let $\mathbb{G}(S, B) = \{g \in \mathbb{G} \mid ij \in g \Leftrightarrow i \in S \text{ and } j \in B\}$ be the set of feasible buyer-seller networks. Agents incur a cost of maintaining each link equal to c_s for sellers and to c_b for buyers. So the payoff to an agent is her payoff from any trade on the network, less the cost of maintaining any links that she is involved with.

In the first period, sellers simultaneously call out prices. A buyer can only select from the prices that she has heard called out by the sellers to whom she is linked. Buyers simultaneously respond by either choosing to accept a single price offer received or rejecting all price offers received.[11] At the end of the period, trades are made and buyers and sellers who have traded are cleared from the market. In the next period, the situation reverses and buyers call out prices. These are then either accepted or rejected by the sellers connected to them. Each period the role of proposer and responder alternates and this process repeats itself until all remaining buyers and sellers are not linked to each other. Buyers and sellers are impatient so that a transaction at price p in period t is worth $\delta^t p$ to a seller and $\delta^t(1-p)$ to a buyer with $0 < \delta < 1$ being the common discount factor. In a subgame perfect

[10] A link is necessary between a buyer and a seller for a transaction to occur, but if an agent has several links, then there are several possible trading patterns. The network structure essentially determines the bargaining power of buyers and sellers.

[11] If there are several sellers who have called out the same price and/or several buyers who have accepted the same price, and there is any discretion under the given network connections as to which trades should occur, then there is a careful protocol for determining which trades occur. The protocol is essentially designed to maximize the number of transactions.

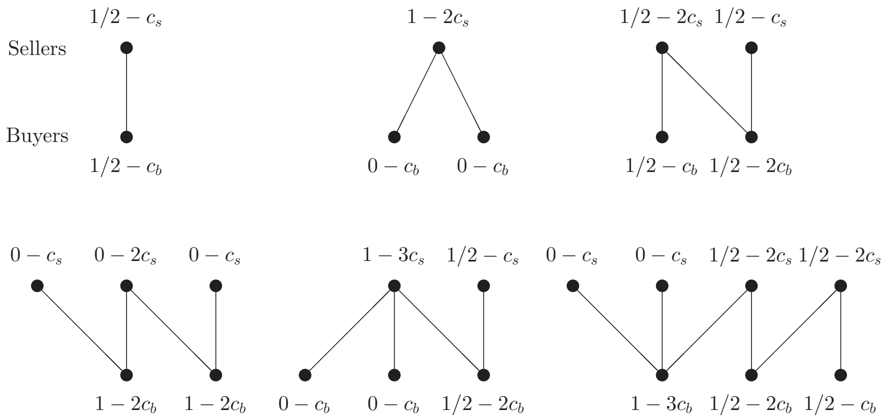

Figure 1: Limit payoffs in the Corominas-Bosch (2004) model for some networks.

equilibrium with very patient agents (δ close to 1), there are effectively three possible outcomes for any given agent (ignoring the costs of maintaining links): either she gets all the available gains from trade (1), or half of the gains from trade (1/2), or none of the available gains from trade (0). Corominas-Bosch (2004) has provided an algorithm that subdivides any network into three types of subnetworks: those in which a set of sellers are collectively linked to a larger set of buyers (sellers obtain 1 as payoffs, and buyers receive 0); those in which the collective set of sellers is linked to the same-sized collective set of buyers (each receives 1/2); and those in which sellers outnumber buyers (sellers receive 0, and buyers get 1).[12] In Figure 1 we give the limit payoffs of the Corominas-Bosch model for some networks. The value function is simply the sum of the payoffs (including the cost of maintaining links) and is top convex.

Let G_2 be the set of all buyer-seller networks consisting of pairs and so that the maximum number of potential pairs must form. That is, $G_2 = \{g \in \mathbb{G}(S, B) \mid l_i(g) = 1 \forall i \in S \cup B\}$, where $l_i(g)$ is the number of links player i has in g. Jackson (2003) has shown that, in the Corominas-Bosch model with

[12] The algorithm works as follows. Step 1a: Identify groups of two or more sellers who are all linked only to the same buyer. Regardless of that buyer's other connections, eliminate that set of sellers and buyer (with the buyer obtaining 1 and the sellers receiving 0). Step 1b: On the remaining network, repeat step 1a but with the role of buyers and sellers reversed. Step k: Proceed inductively in k, each time identifying subsets of at least k sellers who are collectively linked to some set of fewer-than-k buyers, or some collection of at least k buyers who are collectively linked to some set of fewer-than-k sellers. End: When all such subgraphs are removed, the buyers and sellers in the remaining network are such that every subset of sellers is linked to at least as many buyers and vice versa, and the buyers and sellers in that subnetwork get 1/2.

$1/2 > c_s > 0$ and $1/2 > c_b > 0$, the set of pairwise stable networks is G_2 which is exactly the set of strongly efficient networks.[13] The intuition for this result is straightforward. An agent having a payoff of 0 cannot have any links since by deleting a link she could save the link cost and not lose any benefit. So, all agents who have links must obtain payoffs of 1/2 (ignoring the costs of maintaining links). Then, we can show that if there are extra links in such a network relative to the strongly efficient network which consists of a maximal number of disjoint linked pairs, some links could be deleted without changing the payoffs from trade but saving link costs. Thus, a pairwise stable network must consist of linked pairs, and the maximum number of potential pairs must form. Notice that if $1/2 < c_s$ and/or $1/2 < c_b$ then the empty network is the unique pairwise stable network. The empty network is strongly efficient only if $c_s + c_b \geq 1$.

PROPOSITION 3: *In the Corominas-Bosch model with* $1/2 > c_s > 0$ *and* $1/2 > c_b > 0$*, the set* G_2 *is the unique pairwise farsightedly stable set of networks.*

In the Corominas-Bosch (2004) model of trading networks with bilateral bargaining, the value function is top convex and pairwise farsighted stability singles out the strongly efficient network even though the allocation rule is again not componentwise egalitarian. Myopic or farsighted notions of stability sustain the set of strongly efficient networks G_2 when the costs of maintaining links are not too large.[14] The intuition is that each buyer obtains the same payoff $(1/2 - c_b)$ in any efficient network, each seller obtains the same payoff $(1/2 - c_s)$ in any efficient network, and in any other network g there is at least one linked seller or buyer who obtains a payoff strictly less than $1/2 - c_s$ or $1/2 - c_b$. Hence, there is no farsighted improving from any efficient network to such network g but there is a farsighted improving path from g to some efficient network. That is, $F(g') = \emptyset$ for all $g' \in G_2$, and for each $g \notin G_2$ there is some $g' \in G_2$ such that $g' \in F(g)$.

Notice that if $1/2 < c_s$ and/or $1/2 < c_b$ then a set consisting of the empty network is obviously the unique pairwise farsightedly stable set. In that case, on at least one side of the market (buyers or sellers) agents who have some link in any network receive a payoff strictly less than 0 and thus are willing to delete their links looking forward to the empty network. It also implies that there are no farsighted improving paths emanating from the empty network.

[13] G_2 is also the myopically stable set.

[14] Myopic and farsighted notions of stability still sustain the set of strongly efficient networks when $\#S \neq \#B$. But then, each set of all networks consisting of the maximum number of potential pairs among $\min\{\#S, \#B\}$ sellers and $\min\{\#S, \#B\}$ buyers is a pairwise farsightedly stable set.

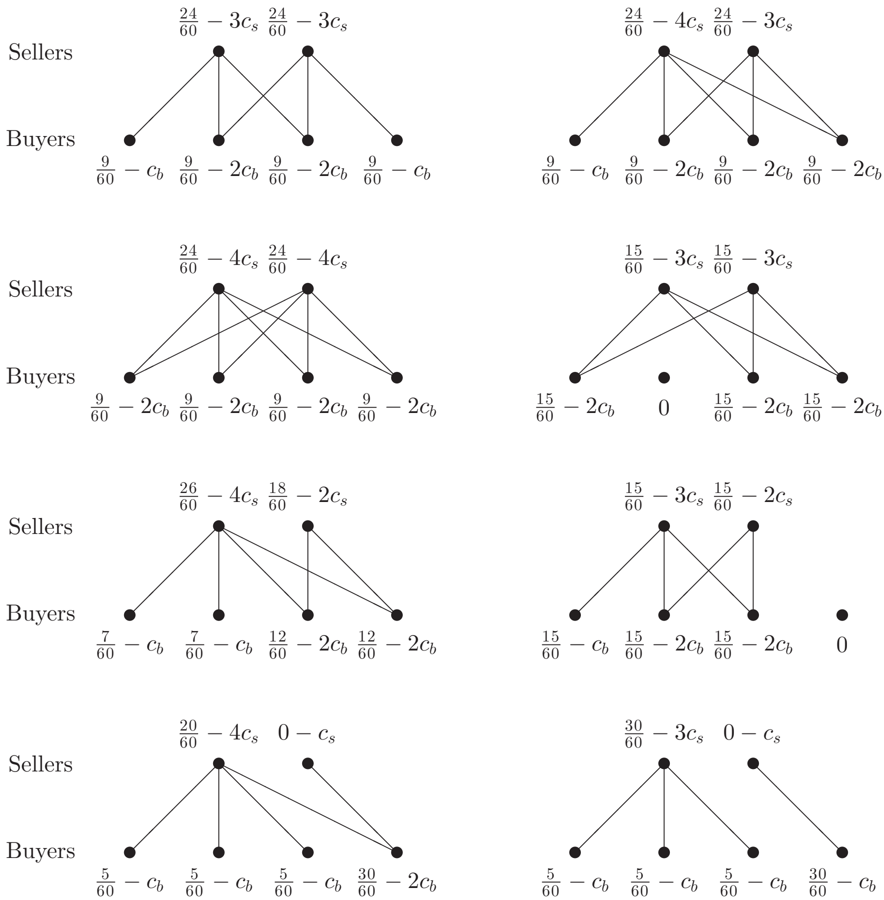

Figure 2: Payoffs in the Kranton and Minehart (2001) model for selected networks.

5.2.2. A Model of Buyer-Seller Networks

The Kranton and Minehart (2001) model of buyer-seller networks is similar to the Corominas-Bosch model except that the valuations of the buyers for an object are random and the determination of prices is made through an auction rather than alternating-offer bargaining. Consider a version of the model with one seller ($\#S = 1$) and some potential buyers ($\#B \geq 1$). So, there is one seller who has an indivisible object for sale and b potential buyers who have utilities for the object, denoted u_i, which are uniformly and independently distributed on $[0, 1]$. The object to sell has no value to the seller. Each buyer knows her own valuation, but only the distribution over the buyers' valuations. The seller also knows only the distribution of buyers' valuations. The object is sold by means of a standard second-price auction.

Only the buyers who are linked to the seller participate to the auction. The number of buyers linked to the seller is given by $l(g)$. For a cost per link of c_s to the seller and c_b to the buyer, the allocation rule for any network g with $l(g) \geq 1$ links between the buyers and the seller is

$$Y_i(g) = \begin{cases} 1/\,[l(g)\,(l(g)+1)] - c_b & \text{if } i \text{ is a linked buyer,} \\ (l(g)-1)\,/\,(l(g)+1) - l(g)\,c_s & \text{if } i \text{ is the seller,} \\ 0 & \text{if } i \text{ is a buyer without any links,} \end{cases},$$

and is not componentwise egalitarian. The value function is $v(g) = l(g)/(l(g) + 1) - l(g)(c_s + c_b)$, which is simply the expected value of the object to the highest valued buyer less the cost of links, and violates top convexity. Let l_s^* be the number of links l such that $2/[l(l + 1)] \geq c_s$ and $2/[(l+1)(l+2)] < c_s$, which is the number of links that maximizes the seller's payoff. Let l_b^* be the number of links l such that $1/[l(l + 1)] \geq c_b$ and $1/[(l + 1)(l + 2)] < c_b$, which is the maximal number of links up to which buyers make positive payoffs. A network g such that $l(g) = \min\{l_s^*, l_b^*\}$ is pairwise stable. Notice that if $2/[l_s^*(l_s^* + 1)] = c_s$, $1/[l_b^*(l_b^* + 1)] = c_b$, and $l_s^* = l_b^*$ then $g - ij$ such that $l(g) = \min\{l_s^*, l_b^*\}$ is pairwise stable too. Strongly efficient networks are not necessarily pairwise stable.[15] If $c_s = 0$ then the pairwise stable networks are exactly the efficient ones.[16]

PROPOSITION 4: *In the Kranton and Minehart model with one seller,*

- *If* $1/[l_b^*(l_b^* + 1)] > c_b$ *or* $1/[l_b^*(l_b^* + 1)] = c_b$ *and* $l_b^* > l_s^*$, *then* $\{g\}$ *with* $g \in G_1 = \{g \in \mathbb{G}(\{s\}, B) \mid l(g) = \min\{l_s^*, l_b^*\}\}$ *are the unique pairwise farsightedly stable sets.*
- *If* $1/[l_b^*(l_b^* + 1)] = c_b$ *and* $2/[l_s^*(l_s^* + 1)] > c_s$ *and* $l_b^* \leq l_s^*$, *then* $G_1 = \{g \in \mathbb{G}(\{s\}, B) \mid l(g) = \min\{l_s^*, l_b^*\}\}$ *is the unique pairwise farsightedly stable set.*
- *If* $1/[l_b^*(l_b^* + 1)] = c_b$ *and* $2/[l_s^*(l_s^* + 1)] = c_s$ *and* $l_b^* < l_s^*$, *then* $G_1 = \{g \in \mathbb{G}(\{s\}, B) \mid l(g) = \min\{l_s^*, l_b^*\}\}$ *is the unique pairwise farsightedly stable set.*
- *If* $1/[l_b^*(l_b^* + 1)] = c_b$, $2/[l_s^*(l_s^* + 1)] = c_s$ *and* $l_s^* = l_b^*$, *then* $G_1 \cup G_{-1}$ *with* $G_{-1} = \{g \in \mathbb{G}(\{s\}, B) \mid l(g) = l_s^* - 1\}$ *is the unique pairwise farsightedly stable set.*

Pairwise myopically or farsightedly stable networks may not be strongly efficient; however, they are Pareto efficient. When there are more sellers it is possible for nontrivial pairwise myopically stable networks to be Pareto inefficient. Consider a population with two sellers and four buyers. Let agents

[15] For instance, if $c_s = c_b = 1/100$ then the pairwise stable networks have 10 links, while networks with only 6 links are the strongly efficient ones.
[16] There are no cycles. The myopically stable set consists only of pairwise stable networks.

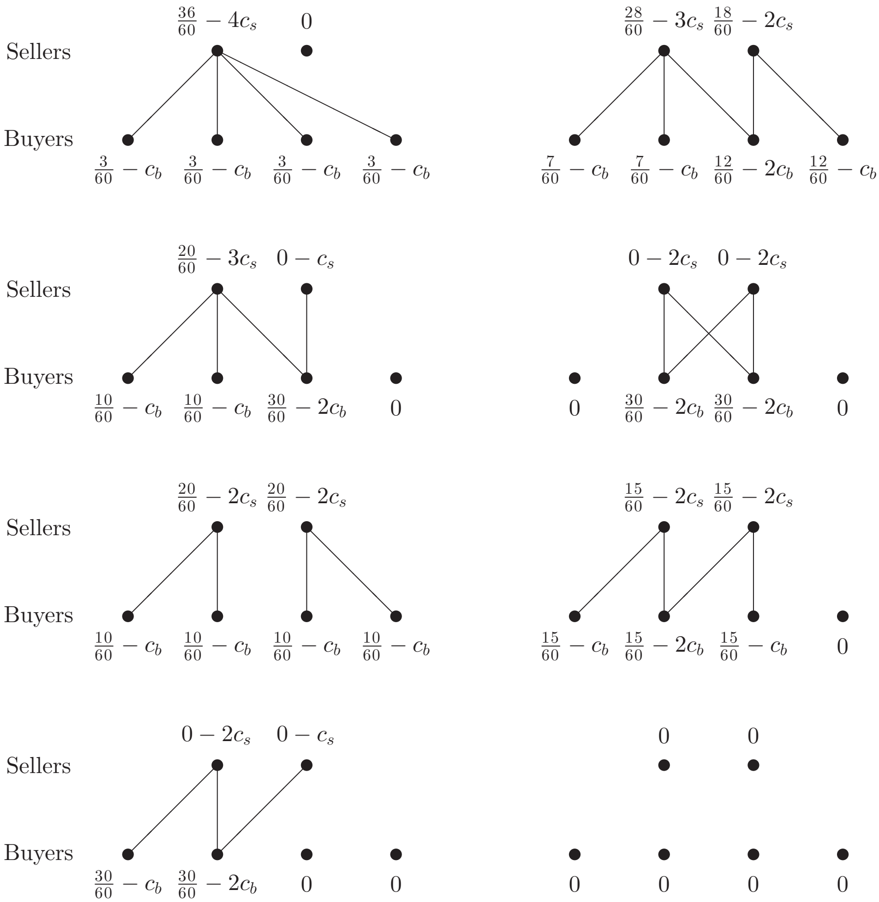

Figure 3: Payoffs in the Kranton and Minehart (2001) model for selected networks.

1 and 2 be the sellers and 3, 4, 5, and 6 be the buyers. Some straightforward but tedious calculations lead to the payoffs which are given in Figures 2 and 3 for selected networks.

For instance, when $c_s = 5/60$ and $c_b = 1/60$, there are three types of pairwise stable networks: the empty network, networks that look like {13, 14, 15, 16}, and networks that look like {13, 14, 15, 24, 25, 26}. Both the empty network and {13, 14, 15, 24, 25, 26} are not Pareto efficient, while {13, 14, 15, 16} is. The empty network and the network {13, 14, 15, 24, 25, 26} are Pareto dominated by the network {13, 14, 25, 26}. In addition, the network {13, 14, 15, 16} is not strongly efficient. The network {13, 14, 25, 26} is strongly efficient but is not pairwise stable since agents 1 and 5 have incentives to add a link. However, the network {13, 14, 25, 26} is pairwise farsightedly stable. Indeed, we have that $G' = \{g | d_1(g) = d_2(g) = 2$, and

$d_3(g) = d_4(g) = d_5(g) = d_6(g) = 1\}$ is a pairwise farsightedly stable set since for every $g' \notin G'$ we have $F(g') \cap G' \neq \varnothing$ and for every $g \in G'$, $F(g) \cap G' = \varnothing$. Thus, contrary to pairwise stability, pairwise farsighted stability may sustain strongly efficient networks when there are more than one seller. So, in the Kranton and Minehart (2001) buyer model of buyer-seller networks, the allocation rule is not componentwise egalitarian and the value function violates top convexity. Then, no general conclusions hold, and pairwise myopically or farsightedly stable networks may not be strongly efficient depending on the number of sellers and buyers. One open question is whether Pareto inefficient networks could belong to some pairwise farsightedly stable set with many sellers and buyers.

6. Conclusion

We have studied the stability of social and economic networks when players are farsighted. In particular, we have first shown that under the componentwise egalitarian allocation rule, the set of strongly efficient networks and the set of pairwise (myopically) stable networks that are immune to coalitional deviations are the unique pairwise farsightedly stable set if and only if the value function is top convex. We have then examined whether the networks formed by farsighted players are different from those formed by myopic players in the Jackson and Wolinsky (1996) symmetric connections model, in the Corominas-Bosch (2004) model of trading networks with bilateral bargaining, and in the Kranton and Minehart (2001) model of buyer–seller networks.

Appendix

Proof of Proposition 1: Take any anonymous and component additive value function v.

($\Leftarrow$)

Top convexity implies that all components of a strongly efficient network must lead to the same per-capita value (if some component led to a lower per-capita value than the average, then another component would have to lead to a higher per-capita value than the average which would contradict top convexity). Top convexity also implies that under the componentwise egalitarian allocation rule any $g \in E(v)$ Pareto dominates all $g' \notin E(v)$. Then, it is immediate that $F(g) = \varnothing$ and that $F(g') \cap E(v) \neq \varnothing$ for all $g' \in \mathbb{G} \setminus E(v)$. Using Theorem 5 in Herings *et al.* (2009), which says that G is the unique pairwise farsightedly stable set if and only if $G = \{g \in \mathbb{G} \mid F(g) = \varnothing\}$ and for every $g' \in \mathbb{G} \setminus G$, $F(g') \cap G \neq \varnothing$, we have that $E(v)$ is the unique pairwise farsightedly stable set.

($\Rightarrow$)

Since $E(v)$ is the unique pairwise farsightedly stable set, we have $F(g) = \varnothing$ for all $g \in E(v)$. We will show that this implies that under the componentwise egalitarian allocation rule (i) $Y_i^{ce}(g, v) = Y_j^{ce}(g, v) = Y_i^{ce}(g', v) = Y_j^{ce}(g', v)$ for all $i, j \in N$ and for all $g, g' \in E(v)$; (ii) $Y_i^{ce}(g, v) \geq Y_i^{ce}(g', v)$ for all $i \in N$, for all $g \in E(v)$, for all $g' \notin E(v)$. Thus, v is top convex.

(i.a) Suppose first that $Y_i^{ce}(g, v) \neq Y_j^{ce}(g, v)$ for some $g \in E(v)$ and some pair of players i, j. Without loss of generality, assume that $Y_i^{ce}(g, v) > Y_j^{ce}(g, v)$. Notice that $i \in N(g)$, since if it was not the case, player j could delete successively links from the network g to reach a network g' satisfying $Y_j^{ce}(g', v) \geq Y_i^{ce}(g, v) > Y_j^{ce}(g, v)$. This would then contradict the fact that $F(g) = \varnothing$. Let $h \subseteq g$ be such that $i \in N(h)$. Let $N_i(g) = \{l \in N | il \in g\}$. To see that our assumption leads to a contradiction, let us construct a path from the network g to the network $\tilde{g} = g - \{jl \mid jl \in g\} + \{jl \mid l \in N_i(g)\} - \{il \mid l \in N_i(g)\}$ as follows. From the network g, player j successively cuts all her links to reach the network $g' = g - \{jl | jl \in g\}$. In g', players having a link with player i in the network g successively add a link with player j to reach $g'' = g - \{jl | jl \in g\} + \{jl | l \in N_i(g)\}$. Then from g'', the players having a link with i successively delete this link and we reach the network $\tilde{g}$. In each network g_k in the sequence going from g to g' we have that $Y_j^{ce}(g_k, v) \leq Y_j^{ce}(g, v)$ since otherwise we would have $g_k \in F(g) \neq \varnothing$. Each network $\overline{g}$ in the path from $g' + jl$, where $l \in N_i(g)$, to $\tilde{g}$ is such that $Y_k(\overline{g}) = Y_m(\overline{g})$ for all $k, m \in N_i(g) \cup \{j\}$, since they belong to the same component. In addition, we have $Y_k(\overline{g}) < Y_k(g)$ for $k \in N_i(g)$ as otherwise, we would have $\overline{g} \in F(g)$. This contradicts anonymity of the value function since in the network $\tilde{g}$, the players from $N_i(g)$ are in a component identical to h but where player j has replaced player i. This establishes that $Y_i^{ce}(g, v) = Y_j^{ce}(g, v)$ for all $i, j \in N$ and for all $g \in E(v)$.

(i.b)To see that $Y_i^{ce}(g, v) = Y_i^{ce}(g', v)$ for all $g, g' \in E(v)$, suppose on the contrary that $Y_i^{ce}(g, v) \neq Y_i^{ce}(g', v)$, say $Y_i^{ce}(g, v) > Y_i^{ce}(g', v)$. Using the result (i.a), we then have $Y_j^{ce}(g, v) > Y_j{}^{ce}(g', v)$ for all $j \in N$, contradicting that g' is strongly efficient.

(ii) By contradiction, suppose that $Y_l^{ce}(g, v) < Y_l{}^{ce}(g', v)$ for some player l, and for some pair of networks $g \in E(v)$, $g' \notin E(v)$. Notice that $l \in N(g')$, since if it was not the case player l could successively delete links from the network g to reach a network g'' satisfying $Y_l^{ce}(g'', v) \geq Y_l{}^{ce}(g', v) > Y_l^{ce}(g, v)$. This would then contradict the fact that $F(g) = \varnothing$. Let $h' \subseteq g'$ such that $l \in N(h')$ be the component to which l belongs in the network g'. We have that $Y_i^{ce}(g, v) = Y_l{}^{ce}(g, v)$ for all $i \in N$ since $g \in E(v)$ (see part (i.a)). In addition, $Y_l^{ce}(g', v) = Y_i{}^{ce}(g', v)$ for all $i \in N(h')$ by definition of the componentwise allocation rule. Thus, $Y_i^{ce}(g, v) < Y_i{}^{ce}(g', v)$ for all $i \in N(h')$. These relations imply that $F(g) \neq \varnothing$, a contradiction. To see this, let us construct a path from the network g to the network $\tilde{g} = h' + g|_{N \setminus N(h')}$ as follows. From the network g, some player $k \in N(h')$ adds successively a link with all the other players from $N(h')$ to reach the network $g_1 = g + \{kj | j \in N(h')\}$. In g_1, let the players from $N(h')$ add the links that belong to h' but do not belong to g, leading to the formation of the network $g_2 = g_1 + \{ij | ij \in h' \setminus g\}$. Then let the players from $N(h')$ delete successively the links that belong to g_2 but do not belong to h' to reach $\tilde{g} = h' + g|_{N\setminus N(h')}$. Let $\overline{g}$ be a network in the path described from g to $\tilde{g}$. Let $S(\overline{g}) \subseteq N(h')$ be the set of players who have added or deleted a link in the path from the network g to the network $\overline{g}$. We have $Y_i^{ce}(\overline{g}, v) = Y_j^{ce}(\overline{g}, v)$ for all $i, j \in S(\overline{g})$ since any pair of players in $S(\overline{g})$ is in the same component in the network $\overline{g}$. Since $F(g) = \emptyset$, we have $Y_i^{ce}(\overline{g}, v) \leq Y_i^{ce}(g, v)$ for all $i \in S(\overline{g})$. This must be true for all network $\overline{g}$ in the path from g to $\tilde{g}$. However, we have supposed that $Y_l^{ce}(g, v) < Y_l^{ce}(g', v) = Y_l^{ce}(\tilde{g}, v)$ for all $l \in N(h')$, a contradiction. ■

Proof of Proposition 2: We only provide the proof of Part (iii). Proofs of Parts (i) and (ii) are available upon request.

(iii) Suppose that $c > \delta$. Let $\overline{c}(n) = \max\{c \in \mathbb{R} \mid \exists\, g \in \mathbb{G}$ such that $g \neq g^{\emptyset}$ and $Y_i(g) \geq 0 \forall i \in N\}$. For $c > \overline{c}(n)$ we have that for any $g \neq g^{\emptyset}$ there is some $i \in N$ such that $Y_i(g) < 0$; and for $c \leq \overline{c}(n)$ we have that there is some $g \neq g^{\emptyset}$ such that $Y_i(g) \geq 0$ for all $i \in N$. Thus, $\delta < \overline{c}(n) < \delta + ((n-2)/2)\delta^2$ if $n \geq 4$. Indeed, the payoff of a player in the circle g^C of n agents (a circle of n agents is a network where each agent is indirectly connected to the others and has two links) is $Y_i(g^C) = 2\delta + 2\delta^2 + \ldots + \delta^{n/2} - 2c$ if n is even and $Y_i(g) = 2\delta + 2\delta^2 + \ldots + 2\delta^{(n-1)/2} - 2c$ if n is odd. We have $Y_i(g^C) > 0$ for all $i \in N$ if $c = \delta$ as long as $n \geq 4$, implying that $\overline{c}(n) > \delta$. In addition, if $c = \delta + ((n-2)/2)\delta^2$, the star networks and the empty network are strongly efficient and generate a value of 0. In the star network, the hub player has a negative payoff. Inefficient networks generate a negative value, implying that some agent has a negative payoff. There is no network different than the empty network which ensures a nonnegative payoff to all agents when $c = \delta + ((n-2)/2)\delta^2$. Thus, $\overline{c}(n) < \delta + ((n-2)/2)\delta^2$.[17]

(iii.1) Suppose first that $c > \overline{c}(n)$. In order to show that a set consisting of the empty network (with a payoff of 0 for all players) is the unique pairwise farsightedly stable set of networks, we need to show that Corollary 1 in Herings *et al.* (2009) applies. That is, we need to show that $g^{\emptyset} \in F(g)$ for all $g \neq g^{\emptyset}$ and that $F(g^{\emptyset}) = \emptyset$. Since $c > c(\overline{n})$, in any other network g, there is some player with a negative payoff who prefers the empty network and hence, we have that $g \notin F(g^{\emptyset})$. Now, from g, let one of the players with a negative payoff delete one of her links. Since in any resulting network g' there is some player preferring the empty network, by letting one of such players delete one of her links at each step, we finally end up at the empty network $g^{\emptyset}$, and $g^{\emptyset} \in F(g)$. Thus, $g^{\emptyset} \in F(g)$ for all $g \neq g^{\emptyset}$ and Corollary 1 in Herings *et al.* (2009) applies.

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[17] When $n = 4$, the network that maximizes the allocation of the agent with the smaller payoff is the circle. We thus have $\overline{c}(4) = \delta + \delta^2/2$. In general, the network that maximizes the allocation of the agent with the smaller payoff is such that each agent is indirectly connected and has the same number of links.

(iii.2) If $\delta < c \leq \overline{c}(n)$, then $\min_i Y_i(g) \geq 0$ for all i for some $g \neq g^{\varnothing}$. We have that $g^{\varnothing} \notin F(g)$ and then $\{g^{\varnothing}\}$ is not a pairwise farsightedly stable set. ■

Proof of Proposition 3: First, we show that for every $g' \notin G_2$ there is $g \in G_2$ such that $g \in F(g')$. For every $g \in G_2$, each seller i receives $Y_i(g) = 1/2 - c_s > 0$, and each buyer j receives $Y_j(g) = 1/2 - c_b > 0$. Start with g' and build a sequence of networks as follows. At each step some agent l who receives a payoff smaller than $1/2 - c_l$ deletes a link looking forward to g until we reach a network g'' consisting only of linked pairs of agents and/or agents having no links. Then, agents successively add the missing links that belong to some $g \in G_2$ such that $g \supseteq g''$.

Second, for every $g \in G_2$ we have that $F(g) \cap G_2 = \varnothing$ since $Y_i(g) = 1/2 - c_s > 0$ and $Y_j(g) = 1/2 - c_b > 0$ for every $g \in G_2$. In addition, for each $g \in G_2$ we have $F(g) \cap (\mathbb{G} \setminus G_2) = \varnothing$ since the only networks $g' \notin G_2$ that some forward-looking agents may prefer to $g \in G_2$ are such that the agents deviating from g obtain a payoff of 1 in g' (ignoring the costs of maintaining links). To obtain 1 the deviating agents will have to form links along the sequence with agents that will obtain 0 in g' (ignoring the costs of maintaining links). But, before forming these additional links with the original deviating agents, these agents have a payoff of 1/2 (ignoring the costs of maintaining links), and thus, they have incentives to block the formation of any additional costly link. Hence, we have shown that $F(g') \cap G_2 \neq \varnothing$ for all $g' \notin G_2$ and $F(g) = \varnothing$ for all $g \in G_2$. Theorem 5 in Herings *et al.* (2009) states that the set G is the unique pairwise farsightedly stable set if and only if $G = \{g \in \mathbb{G} \mid F(g) = \varnothing\}$ and for every $g' \in \mathbb{G} \setminus G$, $F(g') \cap G \neq \varnothing$. Thus, G_2 is the unique pairwise farsightedly stable set. ■

Proof of Proposition 4: (i) Suppose $1/[l_b^*(l_b^* + 1)] > c_b$ or $1/[l_b^*(l_b^* + 1)] = c_b$ and $l_b^* > l_s^*$; and let $G_1 = \{g \in \mathbb{G}(\{s\}, B) \mid l(g) = \min\{l_s^*, l_b^*\}\}$. It is quite straightforward that (a) $g' \notin F(g)$ for all $g' \notin G_1$ and $g \in G_1$; (b) $g' \in F(g)$ for all $g, g' \in G_1$; (c) $g \in F(g')$ for all $g \in G_1$, $g' \notin G_1$. Then, it follows that $\{g\}$ with $g \in G_1$ are the unique pairwise farsightedly stable sets.

(ii) + (iii) Suppose $1/[l_b^*(l_b^* + 1)] = c_b$ and $2/[l_s^*(l_s^* + 1)] > c_s$ and $l_b^* \leq l_s^*$ or $1/[l_b^*(l_b^* + 1)] = c_b$ and $2/[l_s^*(l_s^* + 1)] = c_s$ and $l_b^* < l_s^*$. It is quite straightforward that (a) $g' \notin F(g)$ for all $g' \notin G_1$ and $g \in G_1$; (b) $g' \notin F(g)$ for all $g, g' \in G_1$; (c) $g \in F(g')$ for all $g \in G_1$, $g' \notin G_1$. (a) and (b) imply that $F(g) = \varnothing$ for all $g \in G_1$. (a) and (c) imply that G_1 is a pairwise farsightedly stable set while (b) implies that G_1 is the unique pairwise farsightedly stable set.

(iv) Suppose $1/[l_b^*(l_b^* + 1)] = c_b$, $2/[l_s^*(l_s^* + 1)] = c_s$, and $l_s^* = l_b^*$. Let $G_{-1} = \{g \in \mathbb{G}(\{s\}, B) \mid l(g) = l_s^* - 1\}$. We have $Y_s(g) = Y_s(g')$ for all $g, g' \in G_1 \cup G_{-1}$; $Y_i(g) = 0$ for all $g \in G_1$, $i \in B$; $Y_i(g) = 0$ for all $g \in G_{-1}$, $i \in B$ with $l_i(g) = 0$. It follows that (a) $g' \notin F(g)$ for all $g, g' \in G_1 \cup G_{-1}$; (b) for all

$g' \notin G_1 \cup G_{-1}$ there is $g \in F(g')$ such that $g \in G_1 \cup G_{-1}$; (c) $g' \notin F(g)$ for all $g' \notin G_1 \cup G_{-1}$ and $g \in G_1 \cup G_{-1}$. (a) and (b) imply that $G_1 \cup G_{-1}$ is a pairwise farsightedly stable set while (c) implies that $G_1 \cup G_{-1}$ is the unique pairwise farsightedly stable set. ■

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