Strategic union delegation and strike activity*

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Abstract

We develop a model of wage determination with private information, in which the union has the option to delegate the wage bargaining to either surplus-maximizing delegates or to wage-maximizing delegates (such as senior union members). We show that the wage outcome in case of surplus-maximizing delegates is not necessarily smaller than the wage outcome in case of wage-maximizing delegates, even when the wage bargaining with private information is close to one with complete information. However, if it is commonly known that the union is stronger than the firm and the demand is sufficiently elastic, then delegating to wage-maximizing delegates increases for sure the wage at equilibrium. The maximum delay in reaching an agreement is greater whenever the union chooses wage-maximizing delegates instead of surplus-maximizing delegates and remains finite even when the period length shrinks to zero.

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Délégation syndicale stratégique et activité de grève. Nous étudions comment l'option pour les syndicats de déléguer la négociation salariale va influencer le salaire négocié et les incitants pour faire la grève. Nous développons un modèle de négociation salariale avec information incomplète dans lequel le syndicat a l'option de déléguer la négociation à un délégué qui maximise le surplus ou à un délégué qui maximise le salaire. Nous montrons que le salaire négocié par un délégué qui maximise le surplus n'est pas nécessairement inférieur au salaire négocié par un délégué qui maximise le salaire. Cependant, si le syndicat est plus fort que la firme et la demande est élastique, alors le fait de déléguer la négociation à un délégué qui maximise le salaire va augmenter le salaire d'équilibre. Finalement, nous montrons que de plus longues grèves sont observées lorsque le syndicat délègue la négociation à un délégué qui maximise uniquement le salaire.

1 Introduction

The purpose of this paper is to provide a theoretical study of how the option for unions to delegate the wage bargaining will affect the wage outcome and the incentives for strike activity. Up to now the literature has mainly focused on strategic delegation on behalf of shareholders. Fershtman and Judd (1987) have addressed the issue of strategic managerial delegation in the context of oligopolistic industries with Cournot competition (see also Sklivas, 1987). Regarding strategic union delegation, Jones (1989a) has shown that a divergence between the objectives of union leaders and union members will naturally arise in a democratic union as part of a rational bargaining strategy. Essentially, the reason is that in many bargaining situations, commitment can be valuable, and the union members can credibly commit to a bargaining stance, which they could not otherwise sustain, by delegating authority to a negotiator whose objectives make this stance an optimal one.

In case the union and the firm bargain over wages and employment levels, union delegation implies that inefficient bargains are reached (on a pseudo contract curve), which are preferred by the union to the efficient bargaining solution (on the true contract curve). In case the union and the firm only bargain over the wage (and the firm sets employment unilaterally), union delegation may mimic the monopoly union outcome where the union chooses its most preferred wage. More recently, Conlin and Furusawa (2000) have provided an explanation of why senior union members may represent the union in contract negotiations with a monopolist when parties also negotiate about the bargaining agenda. By strategically delegating contract negotiations to wage-maximizing individuals, the surplus-maximizing union may be better off than if surplus-maximizing individuals negotiate the contract.

But these previous studies have considered complete information frameworks so that delay in reaching an agreement (strikes or lockouts), which waste industry resources, cannot occur at equilibrium.[1] So, we go beyond the analysis offered in Jones (1989a) and Conlin and Furusawa (2000) by developing a model that enable us to investigate in presence of strategic union delegation how private information affects the wage level and the delay in reaching an agreement.[2]

Precisely, we develop a model of wage determination in which both the union and the firm have private information. In the first stage, the union chooses whether to use surplus-maximizing delegates or to use wage-maximizing delegates (such as senior union members or workers who are protected from being dismissed) who will negotiate the wage with the employer. So, only the union has the option to delegate bargaining authority. As pointed out by Jones (1989a), it is the differential ability to commit (here, the union can delegate bargaining authority to workers who cannot be laid off), and not differences in the desire to do so, that underlies one-sided delegation models. If one considers objectives for negotiators, which do not represent the true preferences of any worker, then it may be more appropriate to allow both parties to be able to delegate bargaining authority to a negotiator.[3] In the second stage, the wage bargaining occurs. To describe the wage bargaining process, we adopt Rubinstein's (1982) alternating-offer bargaining model with two-sided incomplete information, which allows the occurrence of strikes at equilibrium.[4] In the third and final stage, the firm chooses its output level to be produced.

As a benchmark we first consider the complete information situation and we show that, the weaker the union is, the more likely the union will choose to send wage-maximizing delegates. The choice of wage-maximizing delegates always increases the wage level and decreases the production output (and the employment level) as well as the consumer surplus. Once the negotiators have private information, the complete information results are not always valid. The wage outcome in case of surplus-maximizing delegates can be greater than the wage outcome in case of wage-maximizing delegates, even when the wage bargaining with private information is close to one with complete information. The less elastic the labor demand is, the more likely it could happen. However, if it is commonly known that the union is stronger than the firm and the labor demand is sufficiently elastic, then we recover the complete information result, namely that the wage outcome with surplus-maximizing delegates is always strictly smaller than the wage outcome with wage-maximizing delegates.

The maximum delay in reaching an agreement (or maximum strike activity) is greater whenever the union chooses wage-maximizing delegates instead of surplus-maximizing delegates. From conventional wage bargaining theory, firm's monopoly power increases the surplus to be divided between the union and the firm. Monopoly power (price-cost margin) is greater when the firm faces wage-maximizing delegates. As a result, delegating the wage bargain to wage-maximizing delegates raises the potential payoffs for the union and the firm but, in expanding the payoff set (or range of possible payoffs), also increases the scope for delay (longer strikes and lockouts may be needed for screening the private information). We show by means of an example that strategic union delegation can increase substantially the potential delay. Finally, we provide the necessary and sufficient condition such that, even in presence of private information, it is always optimal for the union to choose wage-maximizing delegates.

The paper is organized as follows. In Section 2 the model is presented. Section 3 describes the wage bargaining game and choice of delegates under complete information. Section 4 is devoted to the wage bargaining with private information. It offers some predictions regarding the strike duration. Finally, Section 5 concludes.

2 The basic model

Consider a market for a single homogenous product, where the demand is given by $P = a - b \cdot Q^c$, P is the market price, Q is the quantity produced, and $c > 0$. There is one firm producing the good. Let Π denote the profit level. The only variable input is labor. Technology exhibits constant returns to scale and is normalized in such a way that $Q = L$, where L is labor input, and the unit production cost of each firm is the wage W. Thus, the profit of each firm is given by

$$\Pi = (a - b\ Q^c)\ Q - W\ Q. \tag{1}$$

Without loss of generality, we let $b = 1$. The firm belongs to and is controlled by one risk-neutral owner whose objective is to maximize profits. In addition, the firm is unionized, and enters into a closed-shop agreement with its risk-neutral union. The union objective is to maximize the union surplus:

$$U = L\ (W - \overline{W}), \tag{2}$$

where $\overline{W}$ is the reservation wage. The wage rate is determined by negotiations between the firm and the union delegates. Preceding the negotiations, the union may affect the negotiation outcome by selecting delegates whose objective is either to maximize the union's surplus or to maximize the wage rate.

We develop a three-stage game. In stage one, the surplus-maximizing union chooses whether to use surplus-maximizing delegates or to use wage-maximizing delegates (such as senior union members) who will negotiate the wage with the employer. The objective of a wage-maximizing delegate is simply $V = W - \overline{W}$. In stage two, the wage bargaining occurs. Finally, in stage three the employer chooses the output level. The model is solved backwards.

In the last stage of the game, knowing that the wage level (W) has already been determined, the employer chooses

$$Q(W) = \left[\frac{a - W}{1 + c}\right]^{\frac{1}{c}} \tag{3}$$

to maximize its profits. In stage two, the negotiation takes place. We first consider the complete information bargaining as a benchmark.

3 The wage bargaining with complete information

First, we consider the case in which the union sends surplus-maximizing delegates whose interest is the same as the union's objective. The negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model. The firm and the union delegate make alternatively wage offers, with the firm making offers in odd-numbered periods and the union delegate making offers in even-numbered periods. The length of each period is Δ. The negotiation starts in period 0 and ends when one of the negotiators accepts an offer. No limit is placed on the time that may be expended in bargaining and perpetual disagreement is a possible outcome. The union is assumed to be on strike in every period until an agreement is reached. Both the firm and the union are assumed to be impatient. The firm and the union delegate have time preferences with constant discount rates $r_{\mathrm{f}} > 0$ and $r_{\mathrm{u}} > 0$, respectively.[5]

To capture the notion that the time it takes to come to terms is small relative to the length of the contract, we assume that the time between periods is very small. This allows a study of the limiting situations in which the bargaining procedure is essentially symmetric and the potential costs of delaying agreement by one period can be regarded as negligible. As the interval between offers and counteroffers is short and shrinks to zero, the alternating-offer model has a unique limiting subgame perfect equilibrium, which approximates the Nash bargaining solution to the bargaining problem (see Binmore, Rubinstein and Wolinsky, 1986). Thus the predicted wage is given by

$$W_{\mathrm{s}}^{\mathrm{SPE}} = \arg\max \left[U - U^0\right]^{\alpha} \cdot \left[\Pi - \Pi^0\right]^{1-\alpha} \tag{4}$$

where the lowerscript "s" means that wage bargaining is between the firm and surplus-maximizing union delegates, and where $U^0 = 0$ and $\Pi^0 = 0$ are, respectively, the disagreement payoffs of the union delegate and the firm. The parameter $\alpha \in (0,1)$ is the union bargaining power which is equal to $\frac{r_{\mathrm{f}}}{r_{\mathrm{u}}+r_{\mathrm{f}}}$. Simple computation gives us

$$W_{\mathrm{s}}^{\mathrm{SPE}} = \overline{W} + \alpha \frac{c}{1+c}(a - \overline{W}) = \overline{W} + \frac{r_{\mathrm{f}}}{(r_{\mathrm{u}} + r_{\mathrm{f}})} \frac{c}{(1+c)}(a - \overline{W}). \tag{5}$$

Obviously, the wage is increasing with the reservation wage $\overline{W}$, with the union bargaining power α, and with the parameter c.[6] Then, one can easily obtain the equilibrium employment level

$$L_{\mathrm{s}}^* = \left[\frac{1 + c - \alpha\, c}{(1+c)^2}\left(a - \overline{W}\right)\right]^{\frac{1}{c}}, \tag{6}$$

as well as the union's payoff and the firm's profit, which are denoted $U_{\mathrm{s}}^*(\alpha)$ and $\Pi_{\mathrm{s}}^*(\alpha)$, and are given by

$$U_{\mathrm{s}}^*(\alpha) = \left[\frac{\alpha\, c}{1+c}\right]\left[\frac{1 + (1-\alpha)\, c}{(1+c)^2}\right]^{\frac{1}{c}} \left[a - \overline{W}\right]^{\frac{1+c}{c}}, \tag{7}$$

$$\Pi_{\mathrm{s}}^*(\alpha) = c \cdot \left[\frac{1 + (1-\alpha)\, c}{(1+c)^2}\left(a - \overline{W}\right)\right]^{\frac{1+c}{c}}. \tag{8}$$

Second, we consider the case in which the union sends wage-maximizing delegates. Then, the predicted wage is given by

$$W_{\mathrm{w}}^{\mathrm{SPE}} = \arg\max \left[V - V^0\right]^{\alpha} \cdot \left[\Pi - \Pi^0\right]^{1-\alpha} \tag{9}$$

where the lowerscript "w" means that wage bargaining is between the firm and wage-maximizing union delegates, and where $V^0 = 0$ and $\Pi^0 = 0$ are, respectively, the disagreement payoffs of the union delegate and the firm. The parameter $\alpha \in (0,1)$ is still the union bargaining power which is equal to $\frac{r_{\mathrm{f}}}{r_{\mathrm{u}}+r_{\mathrm{f}}}$. Simple computation gives us

$$W_{\mathrm{w}}^{\mathrm{SPE}} = \overline{W} + \alpha \frac{c}{1 - \alpha + c}(a - \overline{W}) = \overline{W} + \frac{c\, r_{\mathrm{f}}}{(1+c)\, r_{\mathrm{u}} + c\, r_{\mathrm{f}}}(a - \overline{W}). \tag{10}$$

Again, the wage is increasing with the reservation wage $\overline{W}$, with the union bargaining power α, and with the parameter c. Then, one can easily obtain the equilibrium employment level

$$L_{\mathrm{w}}^* = \left[\frac{1-\alpha}{(1 - \alpha + c)}\left(a - \overline{W}\right)\right]^{\frac{1}{c}}, \tag{11}$$

as well as the union's payoff and the firm's profit, which are denoted $U^*_{\mathrm{w}}(\alpha)$ and $\Pi^*_{\mathrm{w}}(\alpha)$, and are given by

$$U^*_{\mathrm{w}}(\alpha) = \left[\frac{\alpha\, c}{1-\alpha+c}\right]\left[\frac{(1-\alpha)}{(1-\alpha+c)}\right]^{\frac{1}{c}}\left[a-\overline{W}\right]^{\frac{1+c}{c}}, \qquad (12)$$

$$\Pi^*_{\mathrm{w}}(\alpha) = c\cdot\left[\frac{(1-\alpha)}{(1-\alpha+c)}\left(a-\overline{W}\right)\right]^{\frac{1+c}{c}}. \qquad (13)$$

From (5), (8), (10) and (13) we obviously get that $W^{\mathrm{SPE}}_{\mathrm{w}} > W^{\mathrm{SPE}}_{\mathrm{s}}$ and $\Pi^*_{\mathrm{s}}(\alpha) > \Pi^*_{\mathrm{w}}(\alpha)$.

A natural question to ask at this point is whether union delegation reduces consumer surplus or social welfare. We denote by CS the consumer surplus. It is equal to

$$CS_{\mathrm{s}} = \frac{c}{1+c}\left[\frac{1+c-\alpha c}{(1+c)^2}(a-\overline{W})\right]^{\frac{1+c}{c}} \qquad (14)$$

for the case in which the union sends surplus-maximizing delegates, and it is equal to

$$CS_{\mathrm{w}} = \frac{c}{1+c}\left[\frac{1-\alpha}{1-\alpha+c}(a-\overline{W})\right]^{\frac{1+c}{c}} \qquad (15)$$

for the case in which the union sends wage-maximizing delegates. Comparing both expressions yields that the consumer surplus is always lower when the union sends wage-maximizing delegates rather than surplus-maximizing delegates.

In the first stage of the game, the union chooses whether to use surplus-maximizing delegates or wage-maximizing delegates to negotiate the wage with the employer. Comparing (7) with (12) we obtain the following proposition.

Proposition 1 *The union will send wage-maximizing delegates if and only if*

$$(1+c)^{c+2}(1-\alpha) \geq (1-\alpha+c)^{c+1}(1+c-\alpha c).$$

Proposition 1 tells us that: (i) for any given union bargaining power $\alpha \in (0,1)$, the more inelastic the product demand is (i.e. c is big), the more likely the union will choose to send wage-maximizing delegates; (ii) for any given degree of elasticity of the demand, the weaker the union is (i.e. α is small), the more likely the union will choose to send wage-maximizing delegates. In case the product demand is linear, $c = 1$, the union will choose to send wage-maximizing delegates if and only if $\alpha \leq \alpha^* \simeq .76$. So, if the union is relatively not too strong, then the union will delegate the negotiation task to wage-maximizing delegates.[7]

Proposition 1 gives us the optimal choice in terms of delegation under complete information subject to the restriction that the union can only choose between surplus-maximizing or wage-maximizing delegates. In case one allows for a broad class of objectives for union delegates or negotiators, Jones (1989a) has shown that appropriate selection of a negotiator can mimic the monopoly union outcome where the union selects its most preferred wage (on the labor demand curve). For instance, if the union delegates to a negotiator with objective $L \cdot \left(W - \overline{W}\right)^{\frac{1+c-\alpha c}{\alpha}}$ and gives appropriate incentives for the negotiator,[8] the outcome of the negotiation will be the monopoly union wage. But once such objective for the negotiator (an objective which does not represent the true preferences of any worker) is considered, it may be more appropriate to allow both parties, the union and the firm, to be able to delegate bargaining authority to a negotiator.

Both the asymmetric Nash bargaining solution and the Rubinstein's model predict efficient outcomes of the bargaining process (in particular agreement is reached immediately). This is not the case once we introduce incomplete information into the wage bargaining, in which the first rounds of negotiation are used for information transmission between the two negotiators.

4 The wage bargaining with private information

4.1 Perfect Bayesian equilibria

The main feature of the negotiation is that both negotiators have private information. Each negotiator does not know the impatience (or discount rate) of the other party. It is common knowledge that the firm's discount rate is included in the set $[r_{\mathrm{f}}^{\mathrm{P}}, r_{\mathrm{f}}^{\mathrm{I}}]$, where $0 < r_{\mathrm{f}}^{\mathrm{P}} \leq r_{\mathrm{f}}^{\mathrm{I}}$, and that the union's discount rate is included in the set $[r_{\mathrm{u}}^{\mathrm{P}}, r_{\mathrm{u}}^{\mathrm{I}}]$, where $0 < r_{\mathrm{u}}^{\mathrm{P}} \leq r_{\mathrm{u}}^{\mathrm{I}}$. The superscripts "I" and "P" identify the most impatient and most patient types, respectively. The types are independently drawn from the set $[r_i^{\mathrm{P}}, r_i^{\mathrm{I}}]$ according to the probability distribution p_i, for $i =$u,f. We allow for general distributions over discount rates. This uncertainty implies bounds on the union bargaining power which are denoted by $\underline{\alpha} = r_{\mathrm{f}}^{\mathrm{P}} \cdot \left[r_{\mathrm{u}}^{\mathrm{I}} + r_{\mathrm{f}}^{\mathrm{P}}\right]^{-1}$ and $\overline{\alpha} = r_{\mathrm{f}}^{\mathrm{I}} \cdot \left[r_{\mathrm{u}}^{\mathrm{P}} + r_{\mathrm{f}}^{\mathrm{I}}\right]^{-1}$.

Lemma 1 *Consider the wage bargaining with incomplete information in which the distributions p_f and p_u are common knowledge, and in which the period length shrinks to zero. For* *any perfect Bayesian equilibria (PBE), the payoff of the union belongs to* $[U^*(\underline{\alpha}), U^*(\overline{\alpha})]$ *and the payoff of the firm belongs to* $[\Pi^*(\overline{\alpha}), \Pi^*(\underline{\alpha})]$.

The proof of this lemma, as well as the other proofs, may be found in the appendix. In Lemma 1, $U^*(\underline{\alpha})$ and $\Pi^*(\underline{\alpha})$ denote, respectively, the SPE utility of the union and the SPE profit of the firm of the complete information game, when it is common knowledge that the union's bargaining power is $\alpha = \underline{\alpha}$. In case the union chooses to send surplus-maximizing delegates, then $U^*(\underline{\alpha})$ and $\Pi^*(\underline{\alpha})$ are given, respectively, by Expression (7) and Expression (8) with $\alpha = \underline{\alpha}$. In case the union chooses to send wage-maximizing delegates, then $U^*(\underline{\alpha})$ and $\Pi^*(\underline{\alpha})$ are given, respectively, by Expression (12) and Expression (13) with $\alpha = \underline{\alpha}$. Similarly for $\alpha = \overline{\alpha}$. Lemma 1 follows from Watson's (1998) analysis of Rubinstein's alternating-offer bargaining model with two-sided incomplete information.[9] Lemma 1 is not a direct corollary to Watson (1998) Theorem 1 because Watson's work focuses on linear preferences, but the analysis can be modified to handle the present case. Translating Watson (1998) Theorem 2 to our framework completes the characterization of the PBE payoffs.

Lemma 2 *Consider the wage bargaining with incomplete information in which the period length shrinks to zero. For any* $\widetilde{U} \in [U^*(\underline{\alpha}), U^*(\overline{\alpha})]$, $\widetilde{\Pi} \in [\Pi^*(\overline{\alpha}), \Pi^*(\underline{\alpha})]$, *there exists distributions* p_u *and* p_f, *and a PBE such that the PBE payoffs are* $\widetilde{U}$ *and* $\widetilde{\Pi}$.

In other words, whether or not all payoffs within the intervals given in Lemma 1 are possible depends on the distributions over types. As Watson (1998) stated, Lemma 1 and Lemma 2 establish that "each player will be no worse than he would be in equilibrium if it were common knowledge that he were his least patient type and the opponent were his most patient type. Furthermore, each player will be no better than he would be in equilibrium with the roles reversed". Since we allow for general distributions over types, multiplicity of PBE is not an exception (even when the game is almost with complete information; see Section 4.2). From Lemma 1 we obtain Proposition 2 and Proposition 3.

Proposition 2 *For any distribution over types, PBE wage bargaining outcomes in case of the union chooses to send surplus-maximizing delegates,* $W_s^*(\underline{\alpha}, \overline{\alpha})$, *satisfy the following inequalities*

$$\overline{W} + \frac{r_f^P}{(r_u^I + r_f^P)} \frac{c}{(1+c)}(a - \overline{W}) \leq W_s^*(\underline{\alpha}, \overline{\alpha}) \leq \overline{W} + \frac{r_f^I}{(r_u^P + r_f^I)} \frac{c}{(1+c)}(a - \overline{W}). \qquad (16)$$

Notice that each wage satisfying these bounds can be the outcome by choosing appropriately the distribution over types. The lower (upper) bound is the wage outcome of the complete information game, when it is common knowledge that the union's type is $r_{\mathrm{u}}^{\mathrm{I}}$ ($r_{\mathrm{u}}^{\mathrm{P}}$) and the firm's type is $r_{\mathrm{f}}^{\mathrm{P}}$ ($r_{\mathrm{f}}^{\mathrm{I}}$) (and the union bargaining power is $\underline{\alpha}$ ($\overline{\alpha}$)). For many distributions over types, there is a wide range of PBE: there are PBE in which the wage is close to the upper bound and there are PBE in which the wage is close to the lower bound (see Section 4.2 for some intuition). Expression (16) implies bounds on the firm's employment level, as well as on the firm's output, at equilibrium.

Proposition 3 *For any distribution over types, PBE wage bargaining outcomes in case of the union chooses to send wage-maximizing delegates, $W_w^*(\underline{\alpha},\overline{\alpha})$, satisfy the following inequalities*

$$\overline{W} + \frac{c\ r_f^P}{(1+c)\ r_u^I + c\ r_f^P}(a-\overline{W}) \leq W_w^*(\underline{\alpha},\overline{\alpha}) \leq \overline{W} + \frac{c\ r_f^I}{(1+c)\ r_u^P + c\ r_f^I}(a-\overline{W}). \qquad (17)$$

In complete information, the choice of wage-maximizing delegates always increases the wage level and decreases the production output (and the employment level) as well as the consumer surplus. But once the union and the firm have private information, these complete information results do not necessarily hold.

Corollary 1 *In case of union-firm bargaining with private information, the wage outcome with surplus-maximizing delegates is not necessarily smaller than the wage outcome with wage-maximizing delegates.*

Let

$$\overline{c} = \frac{r_{\mathrm{f}}^{\mathrm{P}} r_{\mathrm{f}}^{\mathrm{I}} - r_{\mathrm{u}}^{\mathrm{I}} r_{\mathrm{f}}^{\mathrm{I}} + r_{\mathrm{u}}^{\mathrm{P}} r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{u}}^{\mathrm{I}} r_{\mathrm{f}}^{\mathrm{I}} - r_{\mathrm{u}}^{\mathrm{P}} r_{\mathrm{f}}^{\mathrm{P}}} = \frac{\underline{\alpha}(1+\overline{\alpha}) - \overline{\alpha}}{\overline{\alpha} - \underline{\alpha}}. \qquad (18)$$

be the cutoff value on c which is obtained by equating the lower bound from (17) with the upper bound from (16). The larger the amount of private information $|\overline{\alpha} - \underline{\alpha}|$ is, the smaller the cutoff value $\overline{c}$ is. Whenever $c > \overline{c}$, the standard result of complete information may not hold: sending wage-maximizing delegates might decrease the wage level in equilibrium. Indeed, when the labor demand is inelastic, the objective of the surplus-maximizing delegate and the objective of the wage-maximizing delegate become close. Then, one can always find probability distributions over types such that there is a PBE wage with surplus-maximizing delegates which is higher than some PBE wage with wage-maximizing delegates. The necessary and sufficient condition such that the complete information result always holds is $c \leq \overline{c}$. The more elastic the labor demand is, the more likely the wage outcome in case of wage-maximizing delegates will be higher than the wage outcome in case of surplus-maximizing delegates even in presence of incomplete information.

There are industries in which unions have a dominant power. One example is the International Typographical Union (ITU) before the seventies. See Dertouzos and Pencavel (1981), Pencavel (1991).

Corollary 2 *Suppose it is commonly known that the union is stronger than the firm* ($\underline{\alpha} \geq \frac{2}{3}$). *For any distribution over types, if the product demand is sufficiently elastic* ($c \leq 1$)*, then* $W_w^*(\underline{\alpha}, \overline{\alpha}) > W_s^*(\underline{\alpha}, \overline{\alpha})$.

The intuition behind this corollary is the following one. Incomplete information in the model takes into account two main features. The first one is the amount of private information in possession of the players. By the amount of private information we mean the size of the set in which player's discount rate is contained and which is common knowledge between the players. The second one is the uncertainty about who is the more patient player, i.e. who is the stronger player. When it is common knowledge that the union is stronger than the firm, this second feature disappears, and information tends to play a less crucial role in the process of the negotiation between the firm and the union. The more elastic the product demand is, the smaller the mark-up of price over marginal cost is, and the less uncertainty there is in the wage bargain. Moreover, if the elasticity of product and labor demands is high, a wage increase will imply a significant drop in employment level and, hence, it will refrain surplus-maximizing delegates from demanding high wages. Therefore, the complete information result is recovered once it is common knowledge that the union is stronger than the firm and that the product demand is sufficiently elastic.

4.2 Wage bargaining with almost complete information

The previous analysis establishes bounds on the PBE payoffs, but it says nothing about the possible payoff vectors inside the bounds. It would be interesting to study the set of payoffs and the set of wages that are supported by perfect Bayesian equilibria in the wage bargaining game which is "close" to having complete information. Watson (1998) has also studied the PBE payoff set of Rubinstein's alternating-offer game under arbitrary sequences of distributions over the players' types which have the same (possibly wide) support,[10] yet which converge to a point mass distribution. That is, he has examined bargaining games in which with high probability a player's discount rate is close to a certain value, yet there is a slight chance that the player's discount rate is much higher or much lower. He has shown that the set of equilibrium payoffs does not converge to that of the complete information, despite that the game converges to one of complete information. More precisely, the set converges from above but not from below in the sense that a player cannot gain if there is a slight chance that he is very patient (has a low discount rate), yet he can suffer if there is a slight chance that he is impatient. In other words, a slight chance of being a patient type can't help a player, whereas a slight chance of being impatient can certainly hurt. The limiting set of equilibrium payoffs is defined by each player's greatest possible discount rate and the limiting discount rates; the players' lowest possible discount rates do not play a role. Watson's main result can be extended to our wage bargaining. It also furnishes intuition that is meaningful for general distributions.

Suppose that there is three possible types for both the union and the firm: r_i^{P}, r_i^*, r_i^{I} where $r_i^{\mathrm{P}} < r_i^* < r_i^{\mathrm{I}}$, for $i = $u,f. Suppose the distribution over these types $\left(r_i^{\mathrm{P}}, r_i^*, r_i^{\mathrm{I}}\right)$ is $(\beta, 1-2\beta, \beta)$ for both the union and the firm; β is the probability that player i's discount rate is r_i^{P}, $1-2\beta$ is the probability that player i's discount rate is r_i^*, and β is the probability that player i's discount rate is r_i^{I}. Then, we might wish to know how the set of PBE payoffs or the wage outcomes change as β converges to zero, where there is only a slight chance that player i is either of type r_i^{P} or type r_i^{I}. From Watson's (1998) Theorem 4 and Theorem 5, it follows that, as β converges to zero, PBE wage outcomes satisfy the bounds in Expression (16) and Expression (17) with $r_i^{\mathrm{P}} = r_i^*$. There are PBE in which the wage is close to the upper bound $W_{\cdot}^{\mathrm{SPE}}(r_{\mathrm{u}}^*, r_{\mathrm{f}}^{\mathrm{I}})$ and there are PBE in which the wage is close to the lower bound $W_{\cdot}^{\mathrm{SPE}}(r_{\mathrm{u}}^{\mathrm{I}}, r_{\mathrm{f}}^*)$. So, PBE wages do not converge to a single wage, despite that the distribution over types converges to a point mass distribution.[11]

Corollary 3 *There exists union-firm bargaining with almost complete information where the wage outcome with surplus-maximizing delegates is not necessarily lower than the wage outcome with wage-maximizing delegates.*

Suppose that the firm's type is known, while the union's type is private information to him: the union's discount rate is either $r_{\mathrm{u}}^{\mathrm{P}}$ or $r_{\mathrm{u}}^{\mathrm{I}}$ with $r_{\mathrm{u}}^{\mathrm{P}} < r_{\mathrm{u}}^{\mathrm{I}}$. Suppose that the distribution over types $(r_{\rm u}^{\rm P}, r_{\rm u}^{\rm I})$ is $(1-\varepsilon, \varepsilon)$ where ε is the probability of the impatient type $(r_{\rm u}^{\rm I})$; ε is small. The above analysis tells us that there is a wide range of PBE wages. It is possible that, in case of surplus-maximizing delegates, an outcome will be reached close to the upper bound wage,

$$\overline{W} + \frac{c\, r_{\rm f}}{(1+c)(r_{\rm u}^{\rm P} + r_{\rm f})}(a - \overline{W});$$

meanwhile, in case of wage-maximizing delegates, an outcome would be reached close to the lower bound wage,

$$\overline{W} + \frac{c\, r_{\rm f}}{(1+c)\; r_{\rm u}^{\rm P} + c\; r_{\rm f}}(a - \overline{W}).$$

Then, if the demand is not too elastic, i.e. $c > \frac{r_{\rm f}}{r_{\rm u}^{\rm I} - r_{\rm u}^{\rm P}} - 1$, the wage outcome in case of surplus-maximizing delegates is higher than the one that would be reached in case of wage-maximizing delegates.

4.3 Maximum delay in reaching an agreement

Inefficient outcomes are possible, even as the period length shrinks to zero. While the scope of possible inefficiency is clear from Lemma 1 and Lemma 2, what is not so obvious is the potential for delay. The wage bargaining game may involve delay (strikes or lock-outs), but not perpetual disagreement, at equilibrium (see Watson, 1998).[12] In fact, delay is positively related to the distance between the discount rates of the most and least patient types of the players. If the range of types is reduced, then this leads to a smaller range of possible payoffs and less delay. Delay can occur even when the game is close to one of complete information (as the type distributions converge to point mass distributions).

We propose to identify strike activity with the maximum delay time in reaching an agreement. Only in average this measure is a good proxy for actual strike duration.[13] It is not uncommon in the literature on bargaining to analyze the maximum number of delay time that may be expended before reaching an agreement.[14] In the appendix we compute the maximum delay in equilibrium which shows that an agreement is reached in finite time and that delay time equals zero as incomplete information vanishes (in that $r_i^{\rm P}$ and $r_i^{\rm I}$ converge).

In case the union chooses surplus-maximizing delegates, the maximum real delay time in reaching an agreement is given by

$$D_{\rm s} = \min\left\{D_{\rm s}^{\rm u}, D_{\rm s}^{\rm f}\right\} \tag{19}$$

where

$$D_{\mathrm{s}}^{\mathrm{u}} = -\frac{1}{r_{\mathrm{u}}^{\mathrm{P}}} \cdot \log \left[\left(\frac{r_{\mathrm{u}}^{\mathrm{P}} + r_{\mathrm{f}}^{\mathrm{I}}}{r_{\mathrm{u}}^{\mathrm{I}} + r_{\mathrm{f}}^{\mathrm{P}}} \right)^{\frac{1+c}{c}} \left(\frac{r_{\mathrm{u}}^{\mathrm{I}} + \frac{1}{1+c} r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{u}}^{\mathrm{P}} + \frac{1}{1+c} r_{\mathrm{f}}^{\mathrm{I}}} \right)^{\frac{1}{c}} \left(\frac{r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{f}}^{\mathrm{I}}} \right) \right] \tag{20}$$

is the maximum real time the union would spend negotiating, and

$$D_{\mathrm{s}}^{\mathrm{f}} = -\frac{1}{r_{\mathrm{f}}^{\mathrm{P}}} \cdot \frac{1+c}{c} \cdot \log \left[\left(\frac{r_{\mathrm{u}}^{\mathrm{P}} + \frac{1}{1+c} r_{\mathrm{f}}^{\mathrm{I}}}{r_{\mathrm{u}}^{\mathrm{I}} + \frac{1}{1+c} r_{\mathrm{f}}^{\mathrm{P}}} \right) \left(\frac{r_{\mathrm{u}}^{\mathrm{I}} + r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{u}}^{\mathrm{P}} + r_{\mathrm{f}}^{\mathrm{I}}} \right) \right] \tag{21}$$

is the maximum real time the firm would spend negotiating facing a surplus-maximizing union delegate. In fact, $D_{\mathrm{s}}^{\mathrm{u}}$ ($D_{\mathrm{s}}^{\mathrm{f}}$) is the maximum real time the union (firm) would spend negotiating in any equilibrium if the union (firm) was of its most patient type. So, $D_{\mathrm{s}}^{\mathrm{u}}$ ($D_{\mathrm{s}}^{\mathrm{f}}$) is the upper bound on the maximum time the union (firm) of type r_{u} (r_{f}) would spend negotiating; maximum time that is decreasing with type r_{u} (r_{f}). So, the more patient a player is the greater the delay that may be observed. Since $D_{\mathrm{s}}^{\mathrm{u}}$ and $D_{\mathrm{s}}^{\mathrm{f}}$ are positive, finite numbers, the maximum real delay in reaching an agreement in case of surplus-maximizing delegates is finite and converges to zero as r_i^{I} and r_i^{P} become close, for $i =$u,f.

In case the union chooses wage-maximizing delegates, the maximum real delay time in reaching an agreement is given by

$$D_{\mathrm{w}} = \min \left\{ D_{\mathrm{w}}^{\mathrm{u}}, D_{\mathrm{w}}^{\mathrm{f}} \right\} \tag{22}$$

where

$$D_{\mathrm{w}}^{\mathrm{u}} = -\frac{1}{r_{\mathrm{u}}^{\mathrm{P}}} \cdot \log \left[\left(\frac{r_{\mathrm{u}}^{\mathrm{P}} + \frac{c}{1+c} r_{\mathrm{f}}^{\mathrm{I}}}{r_{\mathrm{u}}^{\mathrm{I}} + \frac{c}{1+c} r_{\mathrm{f}}^{\mathrm{P}}} \right) \left(\frac{r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{f}}^{\mathrm{I}}} \right) \right] \tag{23}$$

is the maximum real time the union would spend negotiating, and

$$D_{\mathrm{w}}^{\mathrm{f}} = -\frac{1}{r_{\mathrm{f}}^{\mathrm{P}}} \cdot \frac{1+c}{c} \cdot \log \left[\left(\frac{r_{\mathrm{u}}^{\mathrm{I}} + \frac{c}{1+c} r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{u}}^{\mathrm{P}} + \frac{c}{1+c} r_{\mathrm{f}}^{\mathrm{I}}} \right) \left(\frac{r_{\mathrm{u}}^{\mathrm{P}}}{r_{\mathrm{u}}^{\mathrm{I}}} \right) \right] \tag{24}$$

is the maximum real time the firm would spend negotiating facing a wage-maximizing union delegate. Since $D_{\mathrm{w}}^{\mathrm{u}}$ and $D_{\mathrm{w}}^{\mathrm{f}}$ are positive, finite numbers, the maximum real delay in reaching an agreement in case of wage-maximizing delegates is finite and converges to zero as r_i^{I} and r_i^{P} become close, for $i =$u,f.

Comparing (20) with (23), (21) with (24), we have $D_{\mathrm{w}}^{\mathrm{u}} > D_{\mathrm{s}}^{\mathrm{u}}$ and $D_{\mathrm{w}}^{\mathrm{f}} > D_{\mathrm{s}}^{\mathrm{f}}$ for $r_i^{\mathrm{P}} < r_i^{\mathrm{I}}$, $i =$u,f. It follows that $D_{\mathrm{w}} > D_{\mathrm{s}}$.

Proposition 4 *The maximum delay in reaching an agreement is greater whenever the union chooses wage-maximizing delegates instead of surplus-maximizing delegates. That is, $D_w > D_s$ for $r_i^P < r_i^I$.*

From conventional wage bargaining theory, firm's monopoly power increases the surplus to be divided between the union and the firm. Monopoly power (price-cost margin) is greater when the firm faces wage-maximizing delegates.[15] As a result, delegating the wage bargain to wage-maximizing delegates raises the potential payoffs for the union and the firm but, in expanding the payoff set (or range of possible payoffs), also increases the scope for delay (longer strikes and lockouts may be needed for screening the private information).

We provide now an example about the maximum delay. In this example we let $r_{\mathrm{f}}^{\mathrm{P}} = r_{\mathrm{u}}^{\mathrm{P}} = r^{\mathrm{P}}$, $r_{\mathrm{f}}^{\mathrm{I}} = r_{\mathrm{u}}^{\mathrm{I}} = r^{\mathrm{I}}$ and $r^{\mathrm{P}} = 0.22 - r^{\mathrm{I}}$ with $r^{\mathrm{I}} \in [0.11, 0.21]$. Table 1 gives us the integer part of the maximum delay for different values of the parameter c.[16] We observe that (i) $D_{\mathrm{w}} > D_{\mathrm{s}}$ for $r_i^{\mathrm{P}} < r_i^{\mathrm{I}}$, (ii) D_{w} and D_{s} are increasing with the amount of private information $\left| r_i^{\mathrm{P}} - r_i^{\mathrm{I}} \right|$, (iii) D_{w} is decreasing with c and D_{s} is increasing with c (so, the difference between D_{w} and D_{s} is becoming larger the more elastic the demand is). Notice that real delay time in reaching an agreement is not negligible: many bargaining rounds may be needed at equilibrium before an agreement is reached. Results (i) and (ii) hold in general, while result (iii) does not. However, if the union has more private information than the firm has, it is likely that the difference between D_{w} and D_{s} is becoming larger the more elastic the demand is. Indeed, we have: $\partial D_{\mathrm{w}}^{\mathrm{u}} / \partial c < 0$; $\partial D_{\mathrm{s}}^{\mathrm{u}} / \partial c > 0$ if $r_{\mathrm{u}}^{\mathrm{I}} - r_{\mathrm{u}}^{\mathrm{P}} > \frac{1}{1+c} \left(r_{\mathrm{f}}^{\mathrm{I}} - r_{\mathrm{f}}^{\mathrm{P}} \right)$.

	$c = \frac{1}{2}$		$c = 1$		$c = 2$	
r^{I}	D_{s}	D_{w}	D_{s}	D_{w}	D_{s}	D_{w}
0.21	110	402	125	(366) 367	147	341
0.20	49	158	55	143	65	131
0.19	29	86	32	78	38	71
0.18	19	54	21	48	24	(43) 44
0.17	13	35	14	31	16	28
0.16	9	24	10	21	11	19
0.15	6	16	6	14	7	12
0.14	4	10	4	9	5	8
0.13	2	6	2	5	3	4
0.12	1	2	1	2	1	2
0.11	0	0	0	0	0	0

Table 1: Maximum delay in reaching an agreement

From Proposition 4 we know that if the union chooses to send wage-maximizing delegates then strike activity is going to increase. Before concluding, we briefly investigate whether and when it is optimal to delegate for the union. The necessary and sufficient condition such that it is always optimal for the union to choose wage-maximizing delegates is

$$U_{\mathrm{w}}^{*}(\underline{\alpha}) = \left[\frac{\underline{\alpha}\ c}{1-\underline{\alpha}+c}\right]^{c}\left[\frac{(1-\underline{\alpha})}{(1-\underline{\alpha}+c)}\right] \geq \left[\frac{\overline{\alpha}\ c}{1+c}\right]^{c}\left[\frac{1+(1-\overline{\alpha})\ c}{(1+c)^{2}}\right] = U_{\mathrm{s}}^{*}(\overline{\alpha}) \tag{25}$$

where $U_{\mathrm{w}}^{*}(\underline{\alpha})$ and $U_{\mathrm{s}}^{*}(\overline{\alpha})$ are given by Expressions (12) and (7), respectively. Take the case of a linear demand (c is equal to 1). Then, the above condition (25) becomes:

$$\frac{\underline{\alpha}\ (1-\underline{\alpha})}{(2-\underline{\alpha})^{2}} \geq \frac{\overline{\alpha}\ (2-\overline{\alpha})}{8} \tag{26}$$

For instance, if it is commonly known that the union is weaker than the firm (i.e. $\overline{\alpha} \leq \frac{1}{2}$) and the union is not too weak (i.e. $\underline{\alpha} \geq \frac{2}{5}$) then it is optimal for the union to send wage-maximizing delegates.

5 Conclusion

We have developed a model of wage determination with private information, in which the union has the option to delegate the wage bargaining to either surplus-maximizing delegates or to wage-maximizing delegates (such as senior union members). We have shown that the wage outcome in case of surplus-maximizing delegates is not necessarily smaller than the wage outcome in case of wage-maximizing delegates, even when the wage bargaining with private information is close to one with complete information. However, if it is commonly known that one party is stronger than the other and the labor demand is quite elastic, then delegating to wage-maximizing delegates increases for sure the wage at equilibrium. We have shown that strike activity is greater whenever the union chooses wage-maximizing delegates instead of surplus-maximizing delegates. We have also found that strategic delegation can increase substantially delays. From a policy perspective our analysis questions whether one should allow for strategic union delegation (for example, by means of laws protecting union delegates from being dismissed). From a research perspective our analysis questions theoretical results obtained under complete information as well as empirical studies of the trade union objectives.

Appendix A: Proofs

The proof of Lemma 1 is done in two main steps. First, we will show that all strategies that survive the iterative elimination of conditionally dominated strategies impose some bounds on the players payoffs (see Lemma A1). Second, we will show that each PBE conforms to Lemma A1. There are two players who must agree on a wage W from the set X, where X is the set of feasible agreements: $X \equiv \{W \in \mathbb{R} \,| 0 \leq W \leq a\}$. The players either reach an agreement in the set X, or fail to reach an agreement, in which case the disagreement event E occurs. The two players have well defined preferences over $X \cup E$. The negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model. The firm and the union delegate make alternatively wage offers, with the firm making offers in odd-numbered periods and the union delegate making offers in even-numbered periods. The negotiation starts in period 0 and ends when one of the negotiators accepts an offer. No limit is placed on the time that may be expended in bargaining and perpetual disagreement is a possible outcome. The union is assumed to be on strike in every period until an agreement is reached. Both the firm and the union are assumed to be impatient. The firm and the union (delegate) have time preferences with constant discount factors $\delta_{\text{f}} \in (0,1)$ and $\delta_{\text{u}} \in (0,1)$, respectively.

Let H_i be the set of histories after which player i has the move, for $i =$u,f. Let $H \equiv H_{\text{u}} \cup H_{\text{f}}$. Let $h = \emptyset$ be the history at the start of play. A pure strategy of player i is some function $s_i : H_i \rightarrow A$ which maps each possible history after which player i has the move into an action. In each period, the action space for the player who makes an offer is X, while the action space for the responder is {accept, reject}. Let S_i be the set of strategies for player i, $i =$u,f. Let $S \equiv S_{\text{u}} \times S_{\text{f}}$. Each strategy profile $s \in S$ induces an outcome which specifies agreement at some date or irrevocable disagreement. Payoffs in the wage bargaining are given as functions of the players' strategy profile according to the vN-M utility functions $Y_i : S \rightarrow \mathbb{R}$. For any strategy profile $s \in S$ which leads to an agreement W at period n, let $Y_{\text{f}}(s) = \delta_{\text{f}}^n \cdot \Pi(W, L(W))$ and $Y_{\text{u}}(s) = \delta_{\text{u}}^n \cdot U(W, L(W))$ be, respectively, the firm's payoff and the union's payoff. In case the union chooses a wage-maximizing delegate, the payoff of the delegate is simply $\delta_{\text{u}}^n \cdot V(W)$. For any strategy profile $s \in S$ which leads to perpetual disagreement, let $Y_i(s) = 0$. For $h \in H_i$, let $Y_i(s \mid h)$ be the payoff of player i in the game conditional on h describing the play and s describing the play thereafter. Define $Y_i^{\delta_i}$ as the payoff function of player i with discount factor δ_i.

The wage bargaining game with complete information about the players' discount factors has a unique subgame perfect equilibrium (SPE).

The SPE wage outcome in case of surplus-maximizing delegates is such that

$$\begin{cases} \Pi(W_{\mathrm{u}}, L(W_{\mathrm{u}})) = \delta_{\mathrm{f}} \cdot \Pi(W_{\mathrm{f}}, L(W_{\mathrm{f}})) \\ U(W_{\mathrm{f}}, L(W_{\mathrm{f}})) = \delta_{\mathrm{u}} \cdot U(W_{\mathrm{u}}, L(W_{\mathrm{u}})) \end{cases}$$

where W_{u} is the SPE wage outcome if the union delegate makes the first wage offer, and W_{f} is the SPE wage outcome if the firm makes the first offer. Since the union makes the first offer, the SPE wage is

$$W_{\mathrm{s}}^{\mathrm{SPE}}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}}) = \overline{W} + \frac{1 - (\delta_{\mathrm{f}})^{\frac{c}{1+c}}}{1 - \delta_{\mathrm{u}}\delta_{\mathrm{f}}}\left(a - \overline{W}\right),$$

from which we get the SPE profit and the SPE union's payoff:

$$U_{\mathrm{s}}^{*}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}}) = \left(1 - (\delta_{\mathrm{f}})^{\frac{c}{1+c}}\right)\left(\frac{(\delta_{\mathrm{f}})^{\frac{c}{1+c}} - \delta_{\mathrm{u}}\delta_{\mathrm{f}}}{1+c}\right)^{\frac{1}{c}}\left(\frac{a - \overline{W}}{1 - \delta_{\mathrm{u}}\delta_{\mathrm{f}}}\right)^{\frac{1+c}{c}}, \quad (27)$$

$$\Pi_{\mathrm{s}}^{*}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}}) = c \cdot \left(\frac{1}{1+c}\right)^{\frac{1+c}{c}}\left(\frac{(\delta_{\mathrm{f}})^{\frac{c}{1+c}} - \delta_{\mathrm{u}}\delta_{\mathrm{f}}}{1 - \delta_{\mathrm{u}}\delta_{\mathrm{f}}}\left(a - \overline{W}\right)\right)^{\frac{1+c}{c}}. \quad (28)$$

Similarly, the SPE wage outcome in case of wage-maximizing delegates is such that

$$\begin{cases} \Pi(W_{\mathrm{u}}, L(W_{\mathrm{u}})) = \delta_{\mathrm{f}} \cdot \Pi(W_{\mathrm{f}}, L(W_{\mathrm{f}})) \\ V(W_{\mathrm{f}}) = \delta_{\mathrm{u}} \cdot V(W_{\mathrm{u}}) \end{cases}$$

where W_{u} is the SPE wage outcome if the union delegate makes the first wage offer, and W_{f} is the SPE wage outcome if the firm makes the first offer. Since the union delegate makes the first offer, the SPE wage is

$$W_{\mathrm{w}}^{\mathrm{SPE}}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}}) = \overline{W} + \frac{1 - (\delta_{\mathrm{f}})^{\frac{c}{1+c}}}{1 - \delta_{\mathrm{u}}(\delta_{\mathrm{f}})^{\frac{c}{1+c}}}\left(a - \overline{W}\right),$$

from which we get the SPE profit and the SPE union's payoff:

$$V_{\mathrm{w}}^{*}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}}) = \frac{1 - (\delta_{\mathrm{f}})^{\frac{c}{1+c}}}{1 - \delta_{\mathrm{u}}(\delta_{\mathrm{f}})^{\frac{c}{1+c}}}\left(a - \overline{W}\right), \quad (29)$$

$$\Pi_{\mathrm{w}}^{*}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}}) = c \cdot \left(\frac{1}{1+c}\right)^{\frac{1+c}{c}}\left(\frac{(\delta_{\mathrm{f}})^{\frac{c}{1+c}}(1 - \delta_{\mathrm{u}})}{1 - \delta_{\mathrm{u}}(\delta_{\mathrm{f}})^{\frac{c}{1+c}}}\left(a - \overline{W}\right)\right)^{\frac{1+c}{c}}. \quad (30)$$

The players have private information. They are uncertain about each others' discount factors. Player i's discount factor is included in the set $\left[\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}\right]$, where $0 < \delta_i^{\mathrm{I}} \leq \delta_i^{\mathrm{P}} < 1$, for

i =u,f. The superscripts "I" and "P" identify the most impatient and most patient types, respectively. For solving the wage bargaining game with incomplete information about the players' discount factors, we use weaker solution concepts for games with incomplete information.

Definition 1 *Take a set of strategy profiles $T = T_u \times T_f \subset S$ such that T is a product set, and a type δ_i of player i. Strategy s_i is strictly conditionally dominated for type δ_i with respect to T if there exists some $s_i' \in S_i$ such that $Y_i^{\delta_i}(s_i', s_j \mid h) \geq Y_i^{\delta_i}(s_i, s_j \mid h)$ for all $s_j \in T_j$ and all $h \in H$, and there exists $h' \in H$ for which $Y_i^{\delta_i}(s_i', s_j \mid h') > Y_i^{\delta_i}(s_i, s_j \mid h')$ for all $s_j \in T_j$.*

A strategy s_i is strictly conditionally dominated if there exists a strategy s_i' such that s_i' performs at least as well as s_i after all histories and strictly better after at least one history, subject to $s_j \in T_j$.

Definition 2 *Take a set of strategy profiles $T = T_u \times T_f \subset S$ such that T is a product set, and a type δ_i of player i. Strategy s_i is weakly conditionally dominated for type δ_i with respect to T if there exists some $s_i' \in S_i$ such that $Y_i^{\delta_i}(s_i', s_j \mid h) \geq Y_i^{\delta_i}(s_i, s_j \mid h)$ for all $s_j \in T_j$ and all $h \in H$, and there exists $s_j' \in T_j$ and $h' \in H$ for which $Y_i^{\delta_i}(s_i', s_j' \mid h') > Y_i^{\delta_i}(s_i, s_j' \mid h')$.*

Definition 3 *A strategy s_i is strictly (weakly) conditionally dominated with respect to $T \subset S$ if and only if s_i is strictly (weakly) conditionally dominated for each type $\delta_i \in [\delta_i^I, \delta_i^P]$.*

Let $SD_i(T)$ be the set of strategies for player i that are not strictly conditionally dominated with respect to T, and let $SD(T) \equiv SD_{\mathrm{u}}(T) \times SD_{\mathrm{f}}(T)$. For each positive integer $k \geq 1$, let $SD^k \equiv SD(SD^{k-1})$ be defined recursively, starting from $SD^0 \equiv S$. $\left\{SD^k; k \geq 0\right\}$ is a weakly decreasing sequence, i.e. $\emptyset \neq SD^{k+1} \subseteq SD^k$, $k = 1, 2, ...\infty$. The limit set is given by $SD^\infty \equiv \lim_{k\to\infty} SD^k = \bigcap_{k=0}^\infty SD^k$. The set SD^∞ contains the strategy profiles that remain after the iterative elimination of strictly conditionally dominated strategies. Let $WD_i(T)$ be the set of strategies for player i that are not weakly conditionally dominated with respect to T, and let $WD(T) \equiv WD_{\mathrm{u}}(T) \times WD_{\mathrm{f}}(T)$. We define another iterative process for deleting dominated strategies. The process involves the recursive definition that starts from $(WD\left(SD^\infty\right))^0 \equiv S$ and goes on with $(WD\left(SD^\infty\right))^k \equiv$

$(WD(SD^{\infty}(WD(SD^{\infty}))^{k-1}))$ for each positive integer $k \geq 1$. $(WD(SD^{\infty}))^k$ can be written as the Cartesian product $(WD(SD^{\infty}))^k_{\mathrm{u}} \times (WD(SD^{\infty}))^k_{\mathrm{f}}$. $\left\{(WD(SD^{\infty}))^k; k \geq 0\right\}$ is a weakly decreasing sequence, i.e. $\emptyset \neq (WD(SD^{\infty}))^{k+1} \subseteq (WD(SD^{\infty}))^k$, $k = 1, 2, ..., \infty$. The limit set is given by $UD \equiv \lim_{k \to \infty} (WD(SD^{\infty}))^k = \bigcap_{k=0}^{\infty} (WD(SD^{\infty}))^k$. This limit set is the set of players' strategy profiles that are undominated by surviving the cautious iterative elimination of conditionally dominated strategies.

Lemma A1 characterizes the set UD. Let $Y^*_{\mathrm{u}}(\delta^{\mathrm{I}}_{\mathrm{f}}, \delta^{\mathrm{P}}_{\mathrm{u}})$ ($Y^*_{\mathrm{u}}(\delta^{\mathrm{P}}_{\mathrm{f}}, \delta^{\mathrm{I}}_{\mathrm{u}})$) be the union's unique SPE payoff in the game of complete information with discount factors $\delta^{\mathrm{P}}_{\mathrm{u}}$ ($\delta^{\mathrm{I}}_{\mathrm{u}}$) and $\delta^{\mathrm{I}}_{\mathrm{f}}$ ($\delta^{\mathrm{P}}_{\mathrm{f}}$), where the union makes the first wage offer. For instance, in case of surplus-maximizing delegates $Y^*_{\mathrm{u}}(\delta^{\mathrm{I}}_{\mathrm{f}}, \delta^{\mathrm{P}}_{\mathrm{u}})$ is given by Expression (27) with $\delta_{\mathrm{f}} = \delta^{\mathrm{I}}_{\mathrm{f}}$ and $\delta_{\mathrm{u}} = \delta^{\mathrm{P}}_{\mathrm{u}}$. Let $Y^*_{\mathrm{f}}(\delta^P_f, \delta^I_u)$ ($Y^*_{\mathrm{f}}(\delta^{\mathrm{I}}_{\mathrm{f}}, \delta^{\mathrm{P}}_{\mathrm{u}})$) be the firm's unique SPE payoff in the game of complete information with discount factors δ^I_u (δ^P_u) and δ^P_f (δ^I_f), where the firm makes the first wage offer.

Lemma A1 All $s \in UD$ are such that the union's payoff $Y_{\mathrm{u}}(s) \in \left[U^*(\delta^{\mathrm{I}}_{\mathrm{u}}, \delta^{\mathrm{P}}_{\mathrm{f}}), U^*(\delta^{\mathrm{P}}_{\mathrm{u}}, \delta^{\mathrm{I}}_{\mathrm{f}})\right]$ and the firm's payoff $Y_{\mathrm{f}}(s) \in \left[\Pi^*(\delta^{\mathrm{I}}_{\mathrm{f}}, \delta^{\mathrm{P}}_{\mathrm{u}}), \Pi^*(\delta^{\mathrm{P}}_{\mathrm{f}}, \delta^{\mathrm{I}}_{\mathrm{u}})\right]$, for $\delta_i \in [\delta^{\mathrm{I}}_i, \delta^{\mathrm{P}}_i]$, $i =$u,f.

Proof. First, we characterize the set SD^{∞}. The method used is similar to that employed by Shaked and Sutton (1984) and others (e.g. Vannetelbosch (1999)). This method looks for bounds on the behavior of the players and observe that a bound for one player implies a relative bound for the opponent. In turn, this implies a new bound on the behavior of the original player. For instance, if the union knows that the firm never expects more than $\Pi(W_0)$ in a period where the firm makes an offer, then the union can guarantee itself a payoff close to $U(W_1)$ by offering a wage W_1 such that $\delta^{\mathrm{P}}_{\mathrm{f}} \Pi(W_0) = \Pi(W_1)$; obviously, $W_1 > W_0$. Iterative application of this logic leads to the characterization. Watson (1994) did it for the case of linear preferences. Since the complete information game has a unique SPE, this logic leads to well defined bounds. Strategy profiles that survive the iterative elimination of strictly conditionally dominated strategies, $s \in SD^{\infty}$, are such that (i) the union never offers the firm a wage such that firm's payoff (undiscounted) exceeds $\delta^{\mathrm{P}}_{\mathrm{f}} Y^*_{\mathrm{f}}(\delta^{\mathrm{I}}_{\mathrm{u}}, \delta^{\mathrm{P}}_{\mathrm{f}})$, (ii) the firm never offers the union a wage such that union's payoff (undiscounted) exceeds $\delta^{\mathrm{P}}_{\mathrm{u}} Y^*_{\mathrm{u}}(\delta^{\mathrm{I}}_{\mathrm{f}}, \delta^{\mathrm{P}}_{\mathrm{u}})$, (iii) the union always accepts any wage which gives him a payoff (undiscounted) more than $\delta^{\mathrm{P}}_{\mathrm{u}} Y^*_{\mathrm{u}}(\delta^{\mathrm{P}}_{\mathrm{u}}, \delta^{\mathrm{I}}_{\mathrm{f}})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta^{\mathrm{I}}_{\mathrm{u}} Y^*_{\mathrm{u}}(\delta^{\mathrm{I}}_{\mathrm{u}}, \delta^{\mathrm{P}}_{\mathrm{f}})$, (iv) the firm always accepts any wage which gives him a payoff (undiscounted) more than $\delta^{\mathrm{P}}_{\mathrm{f}} Y^*_{\mathrm{f}}(\delta^{\mathrm{P}}_{\mathrm{f}}, \delta^{\mathrm{I}}_{\mathrm{u}})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta_{\rm f}^{\rm I} Y_{\rm f}^*(\delta_{\rm f}^{\rm I}, \delta_{\rm u}^{\rm P})$, (v) the union of type $\delta_{\rm u}$ always accepts any wage which gives him a payoff (undiscounted) more than $\delta_{\rm u} Y_{\rm u}^*(\delta_{\rm u}^{\rm P}, \delta_{\rm f}^{\rm I})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta_{\rm u} Y_{\rm u}^*(\delta_{\rm u}^{\rm I}, \delta_{\rm f}^{\rm P})$, (vi) the firm of type $\delta_{\rm f}$ always accepts any wage which gives him a payoff (undiscounted) more than $\delta_{\rm f} Y_{\rm f}^*(\delta_{\rm f}^{\rm P}, \delta_{\rm u}^{\rm I})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta_{\rm f} Y_{\rm f}^*(\delta_{\rm f}^{\rm I}, \delta_{\rm u}^{\rm P})$.

Second, we characterize the set UD. Notice that perpetual disagreement is even possible with strategies belonging to SD^∞. For example, the union (of type δ_u) may reject a wage giving him a payoff (undiscounted) of $\delta_{\rm u} Y_{\rm u}^*(\delta_{\rm u}^{\rm P}, \delta_{\rm f}^{\rm I})$ expecting that his counter-offer wage giving him a payoff (undiscounted) of $Y_{\rm u}^*(\delta_{\rm u}^{\rm P}, \delta_{\rm f}^{\rm I})$ will be accepted by the firm next period. A similar reasoning can be made for the firm. Therefore, bargainers may even play a strategy profile $s \in SD^\infty$ leading to an agreement reached with delay or to perpetual disagreement. But, one round of elimination of weakly conditionally dominated strategies will delete such strategies. Therefore, strategy profiles $s \in (WD\,(SD^\infty))^1$ are such that (i) the union always accepts any wage which gives him a payoff (undiscounted) greater or equal than $\delta_{\rm u}^{\rm P} Y_{\rm u}^*(\delta_{\rm u}^{\rm P}, \delta_{\rm f}^{\rm I})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta_{\rm u}^{\rm I} Y_{\rm u}^*(\delta_{\rm u}^{\rm I}, \delta_{\rm f}^{\rm P})$, (ii) the firm always accepts any wage which gives him a payoff (undiscounted) greater or equal than $\delta_{\rm f}^{\rm P} Y_{\rm f}^*(\delta_{\rm f}^{\rm P}, \delta_{\rm u}^{\rm I})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta_{\rm f}^{\rm I} Y_{\rm f}^*(\delta_{\rm f}^{\rm I}, \delta_{\rm u}^{\rm P})$, (iii) the union of type $\delta_{\rm u}$ always accepts any wage which gives him a payoff (undiscounted) greater or equal than $\delta_{\rm u} Y_{\rm u}^*(\delta_{\rm u}^{\rm P}, \delta_{\rm f}^{\rm I})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta_{\rm u} Y_{\rm u}^*(\delta_{\rm u}^{\rm I}, \delta_{\rm f}^{\rm P})$, (iv) the firm of type $\delta_{\rm f}$ always accepts any wage which gives him a payoff (undiscounted) greater or equal than $\delta_{\rm f} Y_{\rm f}^*(\delta_{\rm f}^{\rm P}, \delta_{\rm u}^{\rm I})$ and rejects all wages which give him a payoff (undiscounted) less than $\delta_{\rm f} Y_{\rm f}^*(\delta_{\rm f}^{\rm I}, \delta_{\rm u}^{\rm P})$, (v) the union never offers the firm a wage such that firm's payoff (undiscounted) exceeds $\delta_{\rm f}^{\rm P} Y_{\rm f}^*(\delta_{\rm u}^{\rm I}, \delta_{\rm f}^{\rm P})$, (vi) the firm never offers the union a wage such that union's payoff (undiscounted) exceeds $\delta_{\rm u}^{\rm P} Y_{\rm u}^*(\delta_{\rm f}^{\rm I}, \delta_{\rm u}^{\rm P})$. Then strategy profiles $s \in SD^1((WD\,(SD^\infty))^1)$ are such that (vii) the union will never offer a wage such that his payoff (undiscounted) is less than $Y_{\rm u}^*(\delta_u^I, \delta_f^P)$ knowing that the firm will accept (for sure next period) this wage giving him a payoff (undiscounted) $\delta_{\rm f}^{\rm P} Y_{\rm f}^*(\delta_{\rm f}^{\rm P}, \delta_{\rm u}^{\rm I})$, (viii) the firm will never offer a wage such that his payoff (undiscounted) is less than $Y_{\rm f}^*(\delta_{\rm u}^{\rm P}, \delta_{\rm f}^{\rm I})$ knowing that the union will accept (for sure next period) this wage giving him a payoff (undiscounted) $\delta_{\rm u}^{\rm P} Y_{\rm u}^*(\delta_{\rm f}^{\rm I}, \delta_{\rm u}^{\rm P})$, (ix) the six characterizations (i)-(vi) of $s \in (WD\,(SD^\infty))^1$

mentioned here-above.

Next rounds of the iterative procedure do not delete anymore strategies. Therefore, strategy profiles $s \in UD$ are such that $Y_{\mathrm{u}}(s) \in \left[Y_{\mathrm{u}}^{*}(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}}), Y_{\mathrm{u}}^{*}(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}})\right]$ and $Y_{\mathrm{f}}(s) \in \left[\delta_{\mathrm{f}}^{\mathrm{I}} Y_{\mathrm{f}}^{*}(\delta_{\mathrm{f}}^{\mathrm{I}}, \delta_{\mathrm{u}}^{\mathrm{P}}), \delta_{\mathrm{f}}^{\mathrm{P}} Y_{\mathrm{f}}^{*}(\delta_{\mathrm{f}}^{\mathrm{P}}, \delta_{\mathrm{u}}^{\mathrm{I}})\right]$, for $\delta_i \in [\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}]$, i =u,f. In case of surplus-maximizing delegates, $Y_{\mathrm{u}}^{*}(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}}) = U_{\mathrm{s}}^{*}(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}})$ and $Y_{\mathrm{u}}^{*}(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}}) = U_{\mathrm{s}}^{*}(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}})$ (where $U_{\mathrm{s}}^{*}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}})$ is given by Expression (27)), $\delta_{\mathrm{f}}^{\mathrm{I}} Y_{\mathrm{f}}^{*}(\delta_{\mathrm{f}}^{\mathrm{I}}, \delta_{\mathrm{u}}^{\mathrm{P}}) = \Pi_{\mathrm{s}}^{*}(\delta_{\mathrm{f}}^{\mathrm{I}}, \delta_{\mathrm{u}}^{\mathrm{P}})$ and $\delta_{\mathrm{f}}^{\mathrm{P}} Y_{\mathrm{f}}^{*}(\delta_{\mathrm{f}}^{\mathrm{P}}, \delta_{\mathrm{u}}^{\mathrm{I}}) = \Pi_{\mathrm{s}}^{*}(\delta_{\mathrm{f}}^{\mathrm{P}}, \delta_{\mathrm{u}}^{\mathrm{I}})$ (where $\Pi_{\mathrm{s}}^{*}(\delta_{\mathrm{f}}, \delta_{\mathrm{u}})$ is given by Expression (28)). Similarly, in case of wage-maximizing delegates. This completes the proof of Lemma A1.

To characterize the set UD we do not require the players to know the probability distribution of types in $\left[\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{u}}^{\mathrm{P}}\right] \times \left[\delta_{\mathrm{f}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}}\right]$. We only have to assume that it is common knowledge among the players that player i's type is included in the set $\left[\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}\right]$, $i = u, f$. Suppose now that players' types are independently drawn, with player i's discount factor drawn from the set $[\delta_i^{\mathrm{P}}, \delta_i^{\mathrm{I}}]$ according to the probability distribution p_i, for i =u,f. Let $p = (p_{\mathrm{u}}, p_{\mathrm{f}})$. Next we state some properties about the perfect Bayesian equilibria (PBE) of the wage bargaining game of incomplete information in which the distribution p is common knowledge between the players. A specification of strategies consists of a strategy $s_i^{\delta_i}$ for each type δ_i of player i. We can represent this specification as a mapping $s_i : [\delta_i^{\mathrm{P}}, \delta_i^{\mathrm{I}}] \times H_i \rightarrow A$. Let μ be a system of beliefs where $\mu(\cdot \mid h) = (\mu_{\mathrm{u}}(\cdot \mid h), \mu_{\mathrm{f}}(\cdot \mid h))$ are the beliefs of the players regarding each others' types, conditional on history h. A PBE is a (s, μ) that satisfies: (i) sequential rationality: for each $h \in H$, the continuation strategies are a Bayesian equilibrium in the continuation game given the beliefs $\mu(\cdot \mid h)$; (ii) correct initial beliefs: $\mu(\cdot \mid \emptyset) = p$; (iii) player i's belief about his opponent does not change as a result of player i's own actions; (iv) Bayes' rule is used to update beliefs whenever possible.

To show that each PBE conforms to Lemma A1, take a PBE strategy profile s. Suppose that for some k, $s_i^{\delta_i} \in SD_i^k$ for all $\delta_i \in [\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}]$. Then it must be that $s_j^{\delta_j} \in SD_j^{k+1}$ for all $\delta_j \in [\delta_j^{\mathrm{I}}, \delta_j^{\mathrm{P}}]$, $j \neq i$. (Otherwise there would exist an δ_j such that $s_j^{\delta_j}$ is dominated after some history h. This would contradict that player j of type δ_j maximizes her expected payoff in the continuation game following h, which contradicts that s is a PBE strategy profile.) Obviously $s_i^{\delta_i} \in SD_i^1$ for $\delta_i \in [\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}]$ and i =u,f. By induction, then, $s_i^{\delta_i} \in SD_i^k$ for all $\delta_i \in [\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}]$, all k, and i =u,f. This implies that $s_i^{\delta_i} \in SD_i^{\infty}$ for all $\delta_i \in [\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}]$,

i =u,f. Since conjectures are correct in a PBE, we have that $s_i^{\delta_i} \in (WD\,(SD^{\infty}))^1$ for all $\delta_i \in [\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}]$, i =u,f. So, we have that: (i) in case of surplus-maximizing delegates, for any PBE, the payoff of the union belongs to $[U_{\mathrm{s}}^*(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}}), U_{\mathrm{s}}^*(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}})]$ and the payoff of the firm belongs to $[\Pi_{\mathrm{s}}^*(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}}), \Pi_{\mathrm{s}}^*(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}})]$; (ii) in case of wage-maximizing delegates, for any PBE, the payoff of the union delegate belongs to $[V_{\mathrm{w}}^*(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}}), V_{\mathrm{w}}^*(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}})]$ and the payoff of the firm belongs to $[\Pi_{\mathrm{w}}^*(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}}), \Pi_{\mathrm{w}}^*(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}})]$.

It is customary to express the players' discount factors in terms of discount rates, r_{u} and r_{f}, and the length of the bargaining period, Δ, according to formula $\delta_i = \exp(-r_i\Delta)$, for i =u,f. With this interpretation, player i's type is identified with his discount rate r_i, where $r_i \in [r_i^{\mathrm{P}}, r_i^{\mathrm{I}}]$. We thus have that $\delta_i^{\mathrm{I}} = \exp(-r_i^{\mathrm{I}}\Delta)$ and $\delta_i^{\mathrm{P}} = \exp(-r_i^{\mathrm{P}}\Delta)$, for i =u,f. Taking the limit $\Delta \to 0$ on players' PBE payoffs completes the proof of Lemma 1. ■

Lemma 2 is a straightforward corollary of Theorem 2 in Watson (1998). From Lemma 1 and the above proof we obtain bounds on PBE wage bargaining outcomes. In case of the union chooses to send surplus-maximizing delegates, the bounds are

$$\left[\overline{W} + \frac{r_{\mathrm{f}}^{\mathrm{P}}}{(r_{\mathrm{u}}^{\mathrm{I}} + r_{\mathrm{f}}^{\mathrm{P}})}\frac{c}{(1+c)}(a - \overline{W}), \overline{W} + \frac{r_{\mathrm{f}}^{\mathrm{I}}}{(r_{\mathrm{u}}^{\mathrm{P}} + r_{\mathrm{f}}^{\mathrm{I}})}\frac{c}{(1+c)}(a - \overline{W})\right].$$

In case of the union chooses to send wage-maximizing delegates, the bounds are

$$\left[\overline{W} + \frac{c\ r_{\mathrm{f}}^{\mathrm{P}}}{(1+c)\ r_{\mathrm{u}}^{\mathrm{I}} + c\ r_{\mathrm{f}}^{\mathrm{P}}}(a - \overline{W}), \overline{W} + \frac{c\ r_{\mathrm{f}}^{\mathrm{I}}}{(1+c)\ r_{\mathrm{u}}^{\mathrm{P}} + c\ r_{\mathrm{f}}^{\mathrm{I}}}(a - \overline{W})\right].$$

Corollary 1 and Corollary 2 follows directly from Lemma 1, Proposition 1 and Proposition 2. Finally, Corollary 3 is a straightforward application of Theorem 4 and Theorem 5 in Watson (1998).

Appendix B: Maximum delay

In case the union chooses surplus-maximizing delegates, the maximum number of bargaining periods the union would spend negotiating, $I\,(m_{\mathrm{s}}^{\mathrm{u}})$, is given by

$$U_{\mathrm{s}}^*\left(\delta_{\mathrm{u}}^{\mathrm{I}}, \delta_{\mathrm{f}}^{\mathrm{P}}\right) = \left(\delta_{\mathrm{u}}^{\mathrm{P}}\right)^{m_{\mathrm{s}}^{\mathrm{u}}} \cdot U_{\mathrm{s}}^*\left(\delta_{\mathrm{u}}^{\mathrm{P}}, \delta_{\mathrm{f}}^{\mathrm{I}}\right),$$

(where $U_{\mathrm{s}}^*\,(\delta_{\mathrm{u}}, \delta_{\mathrm{f}})$ is given by Expression (27)) from which we obtain

$$m_{\mathrm{s}}^{\mathrm{u}} = \frac{1}{\log\left(\delta_{\mathrm{u}}^{\mathrm{P}}\right)} \cdot \log\left[\left(\frac{1 - \delta_{\mathrm{u}}^{\mathrm{P}}\delta_{\mathrm{f}}^{\mathrm{I}}}{1 - \delta_{\mathrm{u}}^{\mathrm{I}}\delta_{\mathrm{f}}^{\mathrm{P}}}\right)^{\frac{1+c}{c}} \left(\frac{1 - \left(\delta_{\mathrm{f}}^{\mathrm{P}}\right)^{\frac{c}{1+c}}}{1 - \left(\delta_{\mathrm{f}}^{\mathrm{I}}\right)^{\frac{c}{1+c}}}\right) \left(\frac{\left(\delta_{\mathrm{f}}^{\mathrm{P}}\right)^{\frac{c}{1+c}} - \delta_{\mathrm{u}}^{\mathrm{I}}\delta_{\mathrm{f}}^{\mathrm{P}}}{\left(\delta_{\mathrm{f}}^{\mathrm{I}}\right)^{\frac{c}{1+c}} - \delta_{\mathrm{u}}^{\mathrm{P}}\delta_{\mathrm{f}}^{\mathrm{I}}}\right)^{\frac{1}{c}}\right].$$

Notice that $I\left(m_{\mathrm{s}}^{\mathrm{u}}\right)$ is simply the integer part of $m_{\mathrm{s}}^{\mathrm{u}}$. It is customary to express the players' discount factors in terms of discount rates, r_{u} and r_{f}, and the length of the bargaining period, Δ, according to formula $\delta_i = \exp\left(-r_i\Delta\right)$, for i =u,f. With this interpretation, player i's type is identified with his discount rate r_i, where $r_i \in [r_i^{\mathrm{P}}, r_i^{\mathrm{I}}]$. We thus have that $\delta_i^{\mathrm{I}} = \exp(-r_i^{\mathrm{I}}\Delta)$ and $\delta_i^{\mathrm{P}} = \exp(-r_i^{\mathrm{P}}\Delta)$, for i =u,f. Note that $r_i^{\mathrm{I}} \geq r_i^{\mathrm{P}}$ since greater patience implies a lower discount rate. As Δ approaches zero, we have (using l'Hopital's rule): (i) $(1-\delta_{\mathrm{u}}^{\mathrm{P}}\delta_{\mathrm{f}}^{\mathrm{I}})/(1-\delta_{\mathrm{u}}^{\mathrm{I}}\delta_{\mathrm{f}}^{\mathrm{P}})$ converges to $(r_{\mathrm{u}}^{\mathrm{P}}+r_{\mathrm{f}}^{\mathrm{I}})/(r_{\mathrm{u}}^{\mathrm{I}}+r_{\mathrm{f}}^{\mathrm{P}})$, (ii) $(1-(\delta_{\mathrm{f}}^{\mathrm{P}})^{\frac{c}{1+c}})/(1-(\delta_{\mathrm{f}}^{\mathrm{I}})^{\frac{c}{1+c}})$ converges to $r_{\mathrm{f}}^{\mathrm{P}}/r_{\mathrm{f}}^{\mathrm{I}}$, (iii) $((\delta_{\mathrm{f}}^{\mathrm{P}})^{\frac{c}{1+c}} - \delta_{\mathrm{u}}^{\mathrm{I}}\delta_{\mathrm{f}}^{\mathrm{P}})/((\delta_{\mathrm{f}}^{\mathrm{I}})^{\frac{c}{1+c}} - \delta_{\mathrm{u}}^{\mathrm{P}}\delta_{\mathrm{f}}^{\mathrm{I}})$ converges to $(r_{\mathrm{u}}^{\mathrm{I}} + \frac{1}{1+c}r_{\mathrm{f}}^{\mathrm{P}})/(r_{\mathrm{u}}^{\mathrm{P}} + \frac{1}{1+c}r_{\mathrm{f}}^{\mathrm{I}})$, and (iv) $\Delta/\log(\delta_{\mathrm{u}}^{\mathrm{P}})$ converges to $(-1/r_{\mathrm{u}}^{\mathrm{P}})$. These facts imply that

$$D_{\mathrm{s}}^{\mathrm{u}} = \lim_{\Delta\to 0}\left(m_{\mathrm{s}}^{\mathrm{u}}\cdot\Delta\right) = -\frac{1}{r_{\mathrm{u}}^{\mathrm{P}}}\cdot\log\left[\left(\frac{r_{\mathrm{u}}^{\mathrm{P}}+r_{\mathrm{f}}^{\mathrm{I}}}{r_{\mathrm{u}}^{\mathrm{I}}+r_{\mathrm{f}}^{\mathrm{P}}}\right)^{\frac{1+c}{c}}\left(\frac{r_{\mathrm{u}}^{\mathrm{I}}+\frac{1}{1+c}r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{u}}^{\mathrm{P}}+\frac{1}{1+c}r_{\mathrm{f}}^{\mathrm{I}}}\right)^{\frac{1}{c}}\left(\frac{r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{f}}^{\mathrm{I}}}\right)\right],$$

which is a positive, finite number. Notice that $D_{\mathrm{s}}^{\mathrm{u}}$ converges to zero as r_i^{P} and r_i^{I} become close, for i =u,f. Similarly, the maximum number of bargaining periods the firm would spend negotiating, $I(m_{\mathrm{s}}^{\mathrm{f}})$, is given by

$$\Pi_{\mathrm{s}}^{*}\left(\delta_{\mathrm{u}}^{\mathrm{P}},\delta_{\mathrm{f}}^{\mathrm{I}}\right) = \left(\delta_{\mathrm{f}}^{\mathrm{P}}\right)^{m_{\mathrm{s}}^{\mathrm{f}}}\cdot\Pi_{\mathrm{s}}^{*}\left(\delta_{\mathrm{u}}^{\mathrm{I}},\delta_{\mathrm{f}}^{\mathrm{P}}\right),$$

(where $\Pi_{\mathrm{s}}^{*}\left(\delta_{\mathrm{u}},\delta_{\mathrm{f}}\right)$ is given by Expression (28)) from which we obtain

$$m_{\mathrm{s}}^{\mathrm{f}} = \frac{1}{\log\left(\delta_{\mathrm{f}}^{\mathrm{P}}\right)}\cdot\log\left[\left(\frac{1-\delta_{\mathrm{u}}^{\mathrm{I}}\delta_{\mathrm{f}}^{\mathrm{P}}}{1-\delta_{\mathrm{u}}^{\mathrm{P}}\delta_{\mathrm{f}}^{\mathrm{I}}}\right)^{\frac{1+c}{c}}\left(\frac{\left(\delta_{\mathrm{f}}^{\mathrm{I}}\right)^{\frac{c}{1+c}}-\delta_{\mathrm{u}}^{\mathrm{P}}\delta_{\mathrm{f}}^{\mathrm{I}}}{\left(\delta_{\mathrm{f}}^{\mathrm{P}}\right)^{\frac{c}{1+c}}-\delta_{\mathrm{u}}^{\mathrm{I}}\delta_{\mathrm{f}}^{\mathrm{P}}}\right)^{\frac{1+c}{c}}\right],$$

and as Δ approaches zero,

$$D_{\mathrm{s}}^{\mathrm{f}} = \lim_{\Delta\to 0}\left(m_{\mathrm{s}}^{\mathrm{f}}\cdot\Delta\right) = -\frac{1}{r_{\mathrm{f}}^{\mathrm{P}}}\cdot\frac{1+c}{c}\cdot\log\left[\left(\frac{r_{\mathrm{u}}^{\mathrm{P}}+\frac{1}{1+c}r_{\mathrm{f}}^{\mathrm{I}}}{r_{\mathrm{u}}^{\mathrm{I}}+\frac{1}{1+c}r_{\mathrm{f}}^{\mathrm{P}}}\right)\left(\frac{r_{\mathrm{u}}^{\mathrm{I}}+r_{\mathrm{f}}^{\mathrm{P}}}{r_{\mathrm{u}}^{\mathrm{P}}+r_{\mathrm{f}}^{\mathrm{I}}}\right)\right].$$

Then, the maximum real delay time before reaching an agreement is given by

$$D_{\mathrm{s}} = \min\left\{D_{\mathrm{s}}^{\mathrm{u}}, D_{\mathrm{s}}^{\mathrm{f}}\right\}.$$

In case the union chooses wage-maximizing delegates, the maximum number of bargaining periods the union would spend negotiating, $I\left(m_{\mathrm{w}}^{\mathrm{u}}\right)$, is given by

$$V_{\mathrm{w}}^{*}\left(\delta_{\mathrm{u}}^{\mathrm{I}},\delta_{\mathrm{f}}^{\mathrm{P}}\right) = \left(\delta_{\mathrm{u}}^{\mathrm{P}}\right)^{m_{\mathrm{w}}^{\mathrm{u}}}\cdot V_{\mathrm{w}}^{*}\left(\delta_{\mathrm{u}}^{\mathrm{P}},\delta_{\mathrm{f}}^{\mathrm{I}}\right),$$

(where $V^*_{\mathrm{w}}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}})$ is given by Expression (29)) from which we obtain

$$m^{\mathrm{u}}_{\mathrm{w}} = \frac{1}{\log\left(\delta^{\mathrm{P}}_{\mathrm{u}}\right)} \cdot \log\left[\left(\frac{1-\left(\delta^{\mathrm{P}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}}{1-\left(\delta^{\mathrm{I}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}}\right)\left(\frac{1-\delta^{\mathrm{P}}_{\mathrm{u}}\left(\delta^{\mathrm{I}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}}{1-\delta^{\mathrm{I}}_{\mathrm{u}}\left(\delta^{\mathrm{P}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}}\right)\right],$$

and as Δ approaches zero

$$D^{\mathrm{u}}_{\mathrm{w}} = \lim_{\Delta\to 0}\left(m^{\mathrm{u}}_{\mathrm{w}}\cdot\Delta\right) = -\frac{1}{r^{\mathrm{P}}_{\mathrm{u}}}\cdot\log\left[\left(\frac{r^{\mathrm{P}}_{\mathrm{u}}+\frac{c}{1+c}r^{\mathrm{I}}_{\mathrm{f}}}{r^{\mathrm{I}}_{\mathrm{u}}+\frac{c}{1+c}r^{\mathrm{P}}_{\mathrm{f}}}\right)\left(\frac{r^{\mathrm{P}}_{\mathrm{f}}}{r^{\mathrm{I}}_{\mathrm{f}}}\right)\right],$$

which is a positive, finite number. Notice that $D^{\mathrm{u}}_{\mathrm{w}}$ converges to zero as r^{P}_{i} and r^{I}_{i} become close, for i =u,f. Similarly, the maximum number of bargaining periods the firm would spend negotiating, $I(m^{\mathrm{f}}_{\mathrm{w}})$, is given by

$$\Pi^*_{\mathrm{w}}\left(\delta^{\mathrm{P}}_{\mathrm{u}}, \delta^{\mathrm{I}}_{\mathrm{f}}\right) = \left(\delta^{\mathrm{P}}_{\mathrm{f}}\right)^{m^{\mathrm{f}}_{\mathrm{w}}}\cdot\Pi^*_{\mathrm{w}}\left(\delta^{\mathrm{I}}_{\mathrm{u}}, \delta^{\mathrm{P}}_{\mathrm{f}}\right),$$

(where $\Pi^*_{\mathrm{w}}(\delta_{\mathrm{u}}, \delta_{\mathrm{f}})$ is given by Expression (30)) from which we obtain

$$m^{\mathrm{f}}_{\mathrm{w}} = \frac{1}{\log\left(\delta^{\mathrm{P}}_{\mathrm{f}}\right)}\cdot\log\left[\left(\frac{1-\delta^{\mathrm{I}}_{\mathrm{u}}\left(\delta^{\mathrm{P}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}}{1-\delta^{\mathrm{P}}_{\mathrm{u}}\left(\delta^{\mathrm{I}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}}\right)^{\frac{1+c}{c}}\left(\frac{\left(\delta^{\mathrm{I}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}\left(1-\delta^{\mathrm{P}}_{\mathrm{u}}\right)}{\left(\delta^{\mathrm{P}}_{\mathrm{f}}\right)^{\frac{c}{1+c}}\left(1-\delta^{\mathrm{I}}_{\mathrm{u}}\right)}\right)^{\frac{1+c}{c}}\right],$$

and as Δ approaches zero,

$$D^{\mathrm{f}}_{\mathrm{w}} = \lim_{\Delta\to 0}\left(m^{\mathrm{f}}_{\mathrm{w}}\cdot\Delta\right) = -\frac{1}{r^{\mathrm{P}}_{\mathrm{f}}}\cdot\frac{1+c}{c}\cdot\log\left[\left(\frac{r^{\mathrm{I}}_{\mathrm{u}}+\frac{c}{1+c}r^{\mathrm{P}}_{\mathrm{f}}}{r^{\mathrm{P}}_{\mathrm{u}}+\frac{c}{1+c}r^{\mathrm{I}}_{\mathrm{f}}}\right)\left(\frac{r^{\mathrm{P}}_{\mathrm{u}}}{r^{\mathrm{I}}_{\mathrm{u}}}\right)\right].$$

Then, the maximum real delay time before reaching an agreement is given by

$$D_{\mathrm{w}} = \min\left\{D^{\mathrm{u}}_{\mathrm{w}}, D^{\mathrm{f}}_{\mathrm{w}}\right\}.$$

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Notes

[1]Strikes data seem to have a significant impact on the wage-employment relationship for collective negotiations (see e.g. Kennan and Wilson, 1989).

[2]See Kennan and Wilson (1989, 1993) for surveys of bargaining models with private information and their relation to strike data. See Kennan (1986) for a survey of the empirical results on strike activity.

[3]Jones (1989b) has extended Jones (1989a) analysis of one-sided delegation to symmetric, two-sided delegation case. The negotiators have the same structure of preferences as the concerned parties, but with potentially different taste parameters. A large choice of negotiators is available to each party. Although one party may gain at the expense of the other by the process of delegation relative to self-representation, it is often the case that two-sided delegation leads to utility losses for both parties.

[4]The union and the firm only bargain over wages (then, the firm chooses the employment level). This assumption allows us to model the wage bargaining process by means

of Rubinstein's (1982) alternating-offer bargaining model, and to use Watson's (1998) analysis of Rubinstein's model in case of two-sided incomplete information.

[5]Two versions of Rubinstein alternating-offer bargaining model capture different motives that induce parties to reach an agreement rather than to insist indefinitely on incompatible demands. In a first version the parties' incentive to agree lies in the fact that they are impatient : player i is indifferent between receiving $x \cdot \exp(-r_i\Delta)$ today and x tomorrow, where $r_i > 0$ is player i's discount rate. In a second version the parties are not impatient but they face a risk that if agreement is delayed then the opportunity they hope to exploit jointly may be lost : player i believes that at the end of each bargaining period there is a positive probability $1 - \exp(-r_i\Delta)$ that the process will break down, $r_i > 0$. So, r_i can be interpreted either as player i's discount rate or as his estimate about the probability of a breakdown of the negotiations.

[6]The larger c is the less elastic (or the more inelastic) the demand is. So, the larger c is the more inclined the firm is to accept high wages since the firm can easily pass wages on prices without loosing too many consumers.

[7]As pointed out by Jones (1989a), one has to be cautious with respect to the interpretation of econometric estimates of trade unions objectives done in the past since these estimates did not distinguish between the objective of the trade union and the objective of the union delegate who actually negotiated. See Pencavel (1991) for a survey of the empirical results on trade union objectives.

[8]Jones (1989a) has designed a monetary payment schedule which provides appropriate incentives at all levels of fees and hence allows to set the fee to zero.

[9]Watson (1998) characterized the set of PBE payoffs which may arise in Rubinstein's alternating-offer bargaining game and constructed bounds (which are met) on the agreements that may be made. The bounds and the PBE payoffs set are determined by the range of incomplete information and are easy to compute because they correspond to the SPE payoffs of two bargaining games of complete information. These two games are defined by matching one player's most impatient type with the opponent's most patient type.

[10]If r_i^{P} and r_i^{I} converge (for $i =$u,f) then the PBE payoffs of the incomplete information game converge to the unique SPE payoff vector of some complete information game.

[11]This lopsided convergence follows from the construction of PBE strategies, where players will punish one another if they depart from their equilibrium strategies. An effective form of punishment in the bargaining game is that when a player takes some deviant action, beliefs about him are updated *optimistically* -putting probability one on his weakest type. The existence of a very impatient type (a type near r_i^{I} as compared to r_i^*) allows the threat of such a revision of beliefs, however small is the probability of the impatient type. The existence of a very patient type has little effect, since it would not be used in punishing a player.

[12]Watson (1998) has constructed equilibria with delay in which the types of each player behave identically (no information is revealed in equilibrium), players use pure strategies, and players make non-serious offers until some appointed date.

[13]In the literature on strikes, three different measures of strike activity are usually proposed: the strike incidence, the strike duration, and the number of work days lost due to work stoppages. See Cheung and Davidson (1991), Kennan and Wilson (1989). Since we allow for general distributions over types and we may have a multiplicity of PBE, we are unable to compute measures of strike activity as the ones just mentioned.

[14]Cramton (1992) has constructed a sequential equilibrium in a bargaining model with two-sided uncertainty, where types are revealed after a maximum of two rounds, but where delay is directly related to the types of the players. There is also a continuum of other sequential equilibria where types are not fully revealed. Unlike in Rubinstein's model in which the time between offers is fixed, in Cramton (1992) each player can delay making offers. This ability to delay offers enables each player to commit to not revising or rescinding an offer until a counter-offer is made. Cai (2003) has also analyzed the maximum number of delay periods but in a multilateral bargaining. His model has a finite number of Markov perfect equilibria, some of which exhibiting wasteful delays. The maximum number of delay periods that can be supported in Markov perfect equilibria increases in the order of the square of the number of players. These results are robust to a relaxing of the Markov requirements and to more general surplus functions. Ausubel, Cramton and Deneckere

(2002) provide a recent survey of bargaining models with incomplete information.

[15]In complete information, wages are higher in case of wage-maximizing delegates, but prices are much higher too. Indeed, surplus-maximizing delegates care about employment (hence, output) which pushes prices down. Moreover, high wages tend to push prices up.

[16]We can interpret r_i as the annual discount rate and the numbers in Table 1 as the maximum number of days needed to reach an agreement. Indeed, the integer part of the maximum delays for $\Delta = 1/365$ are exactly the numbers of Table 1, except when there are brackets (then it is the number in brackets). The data in Table 1 seem consistent with U.S. strike durations reported in Cramton and Tracy (1994).