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Myopic-farsighted absorbing networks

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Abstract

We propose the notion of myopic-farsighted absorbing set to determine the networks that emerge in the long run when some players are myopic while others are farsighted. A set of networks is a myopic-farsighted absorbing set if (No External Deviation) there is no myopic-farsighted improving path from networks within the set to some networks outside the set, (External Stability) there is a myopic-farsighted improving path from any network outside the set to some network within the set, and (Minimality) there is no proper subset satisfying No External Deviation and External Stability. Contrary to the notion of myopic-farsighted stable set, we show that a myopic-farsighted absorbing set always exists. We partially characterize the myopic-farsighted absorbing sets and we provide sufficient conditions for the equivalence between a myopic-farsighted absorbing set and a myopic-farsighted stable set. We also introduce and fully characterize the notion of proper myopic-farsighted absorbing set that refines the concept of myopic-farsighted absorbing set by selecting the more absorbing networks. Finally, we consider a threshold game that illustrates the role of the relative number of farsighted and myopic players for reaching efficiency.

Keywords Networks · Absorbing sets · Myopia · Farsightedness

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1 Introduction

In many situations, networks are neither fixed nor randomly determined, but rather emerge through the decisions taken by agents. For instance, in R&D networks, trade networks, buyers-sellers networks or criminal networks, agents decide about the links they want to form or to maintain with other agents, and mutual consent is usually required for forming or maintaining a link. A central question is predicting the networks that agents will form.[1] Jackson and Wolinsky (1996) propose the notion of pairwise stability to predict the networks that one might expect to emerge in the long run. A network is pairwise stable if no agent benefits from deleting a link and no two agents benefit from adding a link between them. Pairwise stability presumes that agents are myopic: they do not anticipate that other agents may react to their changes.[2] Farsighted agents might not add a link that appears valuable to them given the current network, as that might in turn lead to the formation of other links and ultimately lower their payoffs. Notions of farsightedness for network formation are proposed by Dutta et al. (2005), Herings et al. (2009, 2019) , Page and Wooders (2009).

Until recently, the literature assumes that either all agents are myopic or all agents are farsighted. However, in many real-life situations it happens that myopic agents do interact with farsighted ones. For instance, between sellers and buyers, one would expect the sellers to be more farsighted than the buyers. Sellers have much more market experience and therefore may be better positioned to contemplate the chain of reactions that follows a deviation. In addition, recent experiments provide evidence in favour of a mixed population consisting of both myopic and farsighted agents. See Kirchsteiger et al. (2016) and Teteryatnikova and Tremewan (2020).

In the present paper, we propose the notion of myopic-farsighted absorbing set to determine the networks that emerge in the long run when some agents are myopic while others are farsighted.

> A set of networks is said to be *myopic-farsighted absorbing* if it satisfies the following three conditions.
>
> NO EXTERNAL DEVIATION. There is no myopic-farsighted improving path from networks within the set to some networks outside the set;
>
> EXTERNAL STABILITY. There is a myopic-farsighted improving path from any network outside the set to some network within the set; and
>
> MINIMALITY. There is no proper subset satisfying NO EXTERNAL DEVIATION and EXTERNAL STABILITY.

[1] Mauleon and Vannetelbosch (2016) provide a comprehensive overview of the solution concepts for solving network formation games.

[2] Jackson and Watts (2002) study a dynamic, but myopic, network formation process in which agents form and sever links based on the improvement that the resulting network offers them relative to the current network. See also Tercieux and Vannetelbosch (2006).

A myopic-farsighted improving path is a sequence of networks that can emerge when farsighted agents form or delete links based on the improvement the end network offers relative to the current network while myopic agents form or delete links based on the improvement the resulting network offers relative to the current network. If a link is deleted, then it must be that either a myopic agent prefers the resulting network to the current network or a farsighted agent prefers the end network to the current network. If a link is added between some myopic agent i and some farsighted agent j, then the myopic agent i must prefer the resulting network to the current network and the farsighted agent j must prefer the end network to the current network.

We show that a myopic-farsighted absorbing set always exists and we partially characterize the myopic-farsighted absorbing sets. Since myopic-farsighted absorbing sets could be quite inclusive, we introduce and fully characterize the notion of proper myopic-farsighted absorbing set that refines the concept of myopic-farsighted absorbing set by selecting the more absorbing networks. The proper myopic-farsighted absorbing set is unique and coincides with the set of all basins of attraction, where the basins of attraction consist of all networks from which there are no myopic-farsighted deviations together with the networks belonging to all closed cycles.

We next consider a threshold game that illustrates the role of the relative number of farsighted and myopic agents for reaching efficiency. In the threshold game, the worth of link creation turns non-negative after some threshold in the connectedness of the network is reached, both for the agents and on aggregate, but the individual benefits are negative below this threshold. If network externalities take this form, myopic agents can be stuck in insufficiently dense networks. Farsightedness may take care of this problem and achieve efficiency. In the presence of both myopic and farsighted agents, their ability to pass the threshold will depend on the number of farsighted agents. Only if there are enough farsighted agents that, by linking among them, could pass the threshold, the myopic agents would also start forming links achieving the efficient network. In addition, we introduce conditions on the allocation rule under which the efficient complete network constitutes the unique myopic-farsighted absorbing set.

Luo et al. (2021) propose the notion of myopic-farsighted stable set.

A set of networks is said to be *myopic-farsighted stable* if the following two conditions hold.

INTERNAL STABILITY. For any two networks in the myopic-farsighted stable set there is no myopic-farsighted improving path from one network to the other one;
EXTERNAL STABILITY. For every network outside the myopic-farsighted stable set there is a myopic-farsighted improving path leading to some network in the myopic-farsighted stable set.

One drawback of such concept is that a myopic-farsighted stable set does not always exist. Contrary to the notion of myopic-farsighted stable set, a myopic-farsighted absorbing set not only requires that from any network outside the set there is a myopic-farsighted deviation leading to some network in the set, but also that, once in the set, there is no myopic-farsighted deviation leading to some network outside the set. We provide sufficient conditions for the equivalence between a myopic-farsighted absorbing set and a myopic-farsighted stable set. In addition, we show that myopic-farsighted stable sets and myopic-farsighted absorbing sets have a non-empty intersection

The paper is organized as follows. In Sect. 2, we introduce networks. In Sect. 3, we define the myopic-farsighted absorbing sets and we partially characterize them. In Sect. 4, we define the proper myopic-farsighted absorbing set and we provide its characterization. In Sect. 5, we relate the myopic-farsighted absorbing set to the myopic-farsighted stable set. In Sect. 6, we consider a threshold game to illustrate the notion of myopic-farsighted absorbing set. In Sect. 7, we study the relationship between efficiency and myopic-farsighted absorbing sets. In Sect. 8, we conclude.

2 Networks

The population of players (or agents) consists of both myopic and farsighted players. The set of players $N = \{1, 2, ..., n\}$, where n is the total number of players, is partitioned: $N = M \cup F$, where M is the set of myopic players and F is the set of farsighted players. Let $m \geq 0$ ($n - m \geq 0$) be the number of myopic (farsighted) players. A network g is a list of which pairs of players are linked to each other and $ij \in g$ indicates that i and j are linked under g. The complete network on the set of players $S \subseteq N$ is denoted by g^S and is equal to the set of all subsets of S of size 2. It follows in particular that the empty network is denoted by $g^{\emptyset}$. The set of all possible networks on N is denoted by $\mathcal{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \text{there is } j \text{ such that } ij \in g\}$ be the set of players who have at least one link in the network g. Let $N_i(g) = \{j \in N \mid ij \in g\}$ be the set of neighbors of player i in g. A path in a network g between i and j is a sequence of players $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. A network g is connected if for all $i \in N$ and $j \in N \setminus \{i\}$, there exists a path in g connecting i and j. A non-empty network $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. The set of components of g is denoted by $H(g)$. The partition of N induced by g is denoted by $\Pi(g)$, where $S \in \Pi(g)$ if and only if either there exists $h \in H(g)$ such that $S = N(h)$ or there exists $i \notin N(g)$ such that $S = \{i\}$.[3]

[3] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, $\#$ will refer to the notion of cardinality.

A network utility function (or payoff function) is a mapping $u : \mathcal{G} \to \mathbb{R}^N$ that assigns to each network g a utility $u_i(g)$ for each player $i \in N$. A network $g \in \mathcal{G}$ is strongly efficient relative to u if it maximizes $\sum_{i \in N} u_i(g)$; i.e., if $\sum_{i \in N} u_i(g) \geq \sum_{i \in N} u_i(g')$ for all $g' \in \mathcal{G}$. A network $g \in \mathcal{G}$ Pareto dominates a network $g' \in \mathcal{G}$ relative to u if $u_i(g) \geq u_i(g')$ for all $i \in N$, with strict inequality for at least one $i \in N$. A network $g \in \mathcal{G}$ is Pareto efficient relative to u if it is not Pareto dominated, and a network $g \in \mathcal{G}$ is Pareto dominant if it Pareto dominates any other network. To determine which networks can be formed in the long run, Jackson and Wolinsky (1996) propose a myopic notion of stability: a network g is pairwise stable with respect to u if and only if (i) for all $ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, and (ii) for all $ij \notin g$, if $u_i(g) < u_i(g + ij)$ then $u_j(g) > u_j(g + ij)$. Let P_1 be the set of pairwise stable networks.

3 Myopic-farsighted absorbing sets of networks

A myopic-farsighted improving path is a sequence of networks that can emerge when farsighted players form or delete links based on the improvement the end network offers relative to the current network while myopic players form or delete links based on the improvement the resulting network offers relative to the current network. If a link is deleted, then it must be that either a myopic player prefers the resulting network to the current network or a farsighted player prefers the end network to the current network. If a link is added between some myopic player i and some farsighted player j, then the myopic player i must prefer the resulting network to the current network and the farsighted player j must prefer the end network to the current network.

Definition 1 A *myopic-farsighted improving path* from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either

(i) $g_{k+1} = g_k - ij$ for some ij such that $u_i(g_{k+1}) > u_i(g_k)$ and $i \in M$ or $u_j(g_K) > u_j(g_k)$ and $j \in F$; or

(ii) $g_{k+1} = g_k + ij$ for some ij such that $u_i(g_{k+1}) > u_i(g_k)$ and $u_j(g_{k+1}) \geq u_j(g_k)$ if $i, j \in M$, or $u_i(g_K) > u_i(g_k)$ and $u_j(g_K) \geq u_j(g_k)$ if $i, j \in F$, or $u_i(g_{k+1}) \geq u_i(g_k)$ and $u_j(g_K) \geq u_j(g_k)$ with one inequality holding strictly if $i \in M, j \in F$.

If there exists a myopic-farsighted improving path from a network g to a network g', then we write $g \to g'$. The set of all networks that can be reached from a network $g \in \mathcal{G}$ by a myopic-farsighted improving path is denoted by $\phi(g)$, $\phi(g) = \{g' \in \mathcal{G} \mid g \to g'\}$. Along a myopic-farsighted improving path, myopic players do not care whether other players are myopic or farsighted, while farsighted players know exactly who is farsighted and who is myopic. When all players are myopic, our notion of myopic-farsighted improving path reverts to Jackson and Watts (2002) notion of improving path; while when all players are farsighted, it reverts to Jackson (2008) or Herings et al. (2009) notion of farsighted improving path. For $N = F$, Jackson (2008) defines a network to be farsightedly pairwise stable if there is no farsighted improving path emanating from it: $g \in \mathcal{G}$ is pairwise farsightedly stable if $\phi(g) = \emptyset$. This concept refines the set of pairwise stable networks, and so often fails to exist. Let P_∞ be the set of farsightedly pairwise stable networks.

To determine the networks that emerge in the long run when the population of players is composed of both myopic and farsighted players, we propose the notion of myopic-farsighted absorbing set. It is based on the following three main requirements: No External Deviation (**NED**), External Stability (**ES**) and Minimality (**MIN**).

Definition 2 A set of networks $G \subseteq \mathcal{G}$ is a *myopic-farsighted absorbing set* if: (**NED**) for every $g \in G$, it holds that $\phi(g) \cap (\mathcal{G} \setminus G) = \emptyset$; (**ES**) for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$; and (**MIN**) $\nexists\ G' \subsetneq G$ such that G' satisfies conditions **NED** and **ES**.

That is, a set of networks G is said to be a myopic-farsighted absorbing set if the following three conditions hold.

NED. From any network $g \in G$ there is no myopic-farsighted improving path to some network $g' \notin G$ (i.e. for every $g \in G$, it holds that $\phi(g) \subseteq G$);

ES. For every network $g' \notin G$ there is a myopic-farsighted improving path leading to some network $g \in G$ (i.e. $g' \to g$); and

MIN. There is no proper subset of G satisfying **NED** and **ES**.

Let $\mathcal{A}(F)$ be the collection of myopic-farsighted absorbing sets of the network formation game when F is the set of farsighted players and $M = N \setminus F$ is the set of myopic players. Notice that, for $F = N$ $(F = \emptyset)$, $\mathcal{A}(N)$ $(\mathcal{A}(\emptyset))$ is simply the collection of farsighted (myopic) absorbing sets.

Example 1 Consider now Jackson and Wolinsky (1996) co-author model with three players. Each player is a researcher who spends time writing papers. If two players are connected, then they are working on a paper together. The amount of time researcher i spends on a given project is inversely related to the number of projects, $\#N_i(g)$, that player i is involved in. Formally, player i's payoff is given by

$$u_i(g) = \sum_{j:ij \in g} \left(\frac{1}{\#N_i(g)} + \frac{1}{\#N_j(g)} + \frac{1}{\#N_i(g)\#N_j(g)} \right)$$

for $\#N_i(g) > 0$. For $\#N_i(g) = 0$ we assume that $u_i(g) = 0$. The utilities the players obtain from different network configurations are given in Figure 1.

Suppose that all players are farsighted ($N = F$): we have $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_4, g_5\}$, $\phi(g_2) = \{g_4, g_6\}$, $\phi(g_3) = \{g_5, g_6\}$, $\phi(g_4) = \phi(g_5) = \phi(g_6) = \{g_7\}$ and $\phi(g_7) = \emptyset$. It is easily verified that there are three myopic-farsighted absorbing sets: $\mathcal{A}(\{1,2,3\}) = \{\{g_4, g_5, g_7\}, \{g_4, g_6, g_7\}, \{g_5, g_6, g_7\}\}$.

Suppose now that players 1 and 2 are farsighted while player 3 is myopic ($F = \{1, 2\}$): we have $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$, $\phi(g_1) = \{g_4, g_5, g_7\}$,

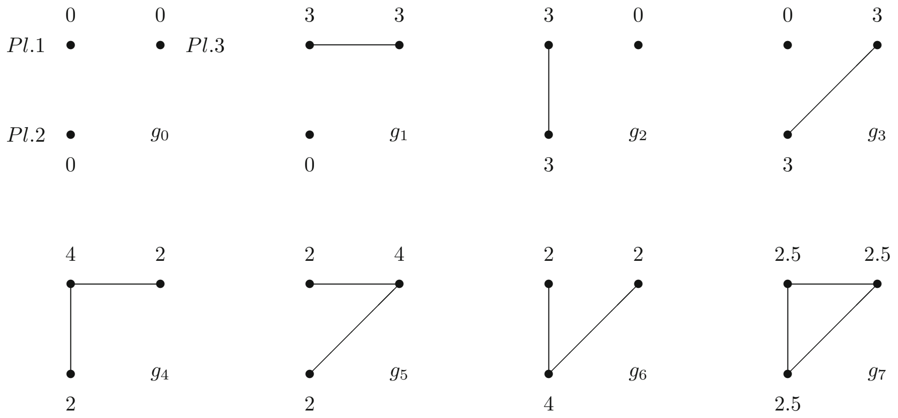

Fig. 1 The co-author model with three players

$\phi(g_2) = \{g_4, g_6\}$, $\phi(g_3) = \{g_5, g_6, g_7\}$, $\phi(g_4) = \phi(g_5) = \phi(g_6) = \{g_7\}$ and $\phi(g_7) = \emptyset$. It follows that there are two myopic-farsighted absorbing sets: $\mathcal{A}(\{1,2\}) = \{\{g_4, g_7\}, \{g_6, g_7\}\}$.

Suppose now that player 1 is farsighted while players 2 and 3 are myopic ($F = \{1\}$): we have $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$, $\phi(g_1) = \{g_4, g_5, g_7\}$, $\phi(g_2) = \{g_4, g_6, g_7\}$, $\phi(g_3) = \{g_5, g_6, g_7\}$, $\phi(g_4) = \phi(g_5) = \phi(g_6) = \{g_7\}$ and $\phi(g_7) = \emptyset$. It follows that there is a single myopic-farsighted absorbing set: $\mathcal{A}(\{1\}) = \{\{g_7\}\}$.

Theorem 1 *A myopic-farsighted absorbing set of networks exists.*

Proof Notice that $\mathcal{G}$ satisfies conditions **NED** and **ES**. Let us proceed by contradiction. Assume that there does not exist any set of networks $G \subseteq \mathcal{G}$ that is a farsighted absorbing set. This means that for any $G^0 \subseteq \mathcal{G}$ that satisfies conditions **NED** and **ES** in Definition 2, we can find a proper subset $G^1 \subsetneq G^0$ that satisfies conditions **NED** and **ES**. Iterating this reasoning we can build an infinite decreasing sequence $\{G^k\}_{k \geq 0}$ of subsets of $\mathcal{G}$ satisfying conditions **NED** and **ES**. But since $\mathcal{G}$ has finite cardinality, this is not possible. □

Proposition 1 *If $G \subseteq \mathcal{G}$ and $G' \subseteq \mathcal{G}$ are myopic-farsighted absorbing sets, then $G \cap G' \neq \emptyset$, $G \nsubseteq G'$ and $G \nsupseteq G'$.*

Proof Suppose that $G \subseteq \mathcal{G}$ and $G' \subseteq \mathcal{G}$ are both myopic-farsighted absorbing sets. (i) If $G \subsetneq G'$, then **MIN** is violated. (ii) If $G \cap G' = \emptyset$, then **NED** implies that both sets G and G' violate **ES**. □

Proposition 2 *Let $G \subseteq \mathcal{G}$ be a myopic-farsighted absorbing set. If $\phi(g) = \emptyset$ then $g \in G$.*

Proof Take any g such that $\phi(g) = \emptyset$. Then, g should belong to the myopic-farsighted absorbing set G. Otherwise, G would violate **ES**. $\square$

Let $\phi^2(g) = \phi(\phi(g)) = \{g'' \in \mathcal{G} \mid \exists g' \in \phi(g) \text{ such that } g'' \in \phi(g')\}$ be the set of networks that can be reached by a composition of two myopic-farsighted improving paths from g. We extend this definition and, for $r \in \mathbb{N}$, we define $\phi^r(g)$ as those networks that can be reached from g by means of r compositions of myopic-farsighted improving paths. The transitive closure of ϕ is denoted by ϕ^∞ and defined as $\phi^\infty(g) = \bigcup_{r\in\mathbb{N}} \phi^r(g)$. Since the set $\mathcal{G}$ is finite, it holds that, for some $r' \in \mathbb{N}$, for every $g \in \mathcal{G}$, $\phi^\infty(g) = \bigcup_{r=1}^{r'} \phi^r(g)$. We now extend Jackson and Watts (2002) notions of cycle and closed cycle to a mixed population of myopic and farsighted players. A set of networks C forms a cycle if for any $g \in C$ and $g' \in C$ there exists a sequence of myopic-farsighted improving paths connecting g to g', i.e., $g' \in \phi^\infty(g)$. A cycle C is a closed cycle if no network in C lies on a myopic-farsighted improving path leading to a network that is not in C, i.e., $\bigcup_{g\in C} \phi^\infty(g) = C$.

Proposition 3 *Let $G \subseteq \mathcal{G}$ be a myopic-farsighted absorbing set and $C^1, ..., C^r$ $(r \geq 1)$ be the closed cycles. We have that $\left(\bigcup_{k=1}^{r} C^k\right) \subseteq G$.*

Proof Take the closed cycles $C^1, ..., C^r$ $(r \geq 1)$ and any myopic-farsighted absorbing set G. (i) If $\left(\bigcup_{k=1}^{r} C^k\right) \cap G = \emptyset$, then G would violate **ES** since for every $g \in \left(\bigcup_{k=1}^{r} C^k\right)$ we have $\phi(g) \subseteq \left(\bigcup_{k=1}^{r} C^k\right)$. (ii) If $\left(\bigcup_{k=1}^{r} C^k\right) \cap G \neq \left(\bigcup_{k=1}^{r} C^k\right)$, then G would violate **NED** and/or **ES**. $\square$

Example 1 (Continued). Consider again Jackson and Wolinsky (1996) co-author model with three players. When $F = \{1,2,3\}$ we have that $\phi(g_7) = \emptyset$. Hence, from Proposition 2 we have that g_7 belongs to all myopic-farsighted absorbing sets. Indeed, we have $\mathcal{A}(\{1,2,3\}) = \{\{g_4, g_5, g_7\}, \{g_4, g_6, g_7\}, \{g_5, g_6, g_7\}\}$. Moreover, we have that $G \cap G' \neq \emptyset$, $G \nsubseteq G'$ and $G \nsupseteq G'$ for all $G, G' \in \mathcal{A}(\{1,2,3\})$ as shown, in general, in Proposition 1. Similarly, for the cases where $F = \{1,2\}$ and $F = \{1\}$.

If we look more deeply at the networks belonging to each myopic-farsighted absorbing set, we notice that, once we reach an absorbing set, players will leave some networks for sure and never go back to them along any sequence of myopic-farsighted improving paths. For instance, take $\{g_4, g_5, g_7\} \in \mathcal{A}(\{1,2,3\})$. From g_4 or g_5 players will deviate for sure to end up in g_7. In other words, there might exist networks that are more absorbing than others in a myopic-farsighted absorbing set. We next provide a refinement of the notion of myopic-farsighted absorbing set that captures this property.

4 Proper myopic-farsighted absorbing sets

We now introduce the notion of proper myopic-farsighted absorbing set that refines the notion of myopic-farsighted absorbing set. Given a myopic-farsighted absorbing set, a proper myopic-farsighted absorbing set is defined by an iterative process. At each step of the process, we delete, among the remaining networks belonging to the myopic-farsighted absorbing set, the networks that both are defeated by some remaining network and do not defeat any other remaining network. A network g is defeated by some other network g' if there is a myopic-farsighted improving path from g to g'. In other words, the proper myopic-farsighted absorbing set is formed by the networks that absorb the rest of networks in the myopic-farsighted absorbing set.

Definition 3 Let $G^0 \subseteq \mathcal{G}$ be a myopic-farsighted absorbing set. For $k \geq 1$, G^k is inductively defined as follows: (i) $G^k \subseteq G^{k-1}$, (ii) for every $g \in G^k$, $\phi(g) \subseteq G^k$, and (iii) for every $g \in G^{k-1} \setminus G^k$, $\phi(g) \cap G^k \neq \emptyset$. The set $G^\infty = \lim_{k\to\infty} G^k$ is a *proper myopic-farsighted absorbing set*.

To characterize the proper farsighted absorbing set, we first introduce the notion of basin of attraction.[4]

Definition 4 A set of networks $G \subseteq \mathcal{G}$ is a *basin of attraction* if (**NED**) for every $g \in G$, it holds that $\phi(g) \cap (\mathcal{G} \setminus G) = \emptyset$ and (**MIN***): $\nexists G' \subsetneq G$ such that G' satisfies **NED**.

That is, a set of networks G is a basin of attraction if the following two conditions hold.

NED. From any network $g \in G$ there is no myopic-farsighted improving path to some network $g' \notin G$ (i.e., for any network $g \in G$ it holds that $\phi(g) \subseteq G$);

MIN*. There is no proper subset of G satisfying **NED**.

As shown in the next proposition, in any network formation game, there is a disjoint collection of basins of attraction, say $\{B^1, B^2, .., B^s\}$, where for each $k = 1, ..., s$ ($s \geq 1$), $B^k \subseteq \mathcal{G}$ is either a singleton set $\{g\}$ with $\phi(g) = \emptyset$ or a closed cycle C^l. Hence, the number of basins of attraction is simply given by $s = \#\{g \in \mathcal{G} \mid \phi(g) = \emptyset\} + \#\{C^1, ..., C^r\}$.

Proposition 4 *A set of networks $B \subseteq \mathcal{G}$ is a basin of attraction if and only if either $B = \{g\}$ with $\phi(g) = \emptyset$ or B is a closed cycle.*

Proof ($\Rightarrow$) Take any set of networks $G \subseteq \mathcal{G}$. If $G = \{g\}$ with $\phi(g) = \emptyset$ then **NED** and **MIN*** are satisfied. If G is a closed cycle C^k then **NED** and **MIN*** are satisfied. Thus, both conditions in Definition 4 are satisfied and hence $G = B$ is a basin of attraction.

($\Leftarrow$) We need to show that any set of networks $G \neq B^k$, $B^k = \{g\}$ with $\phi(g) = \emptyset$ or $B^k = C^l$, would violate either **NED** or **MIN***. Four cases have to be considered.

(i) Any set G such that $G \supsetneq \{g\}$ with $\phi(g) = \emptyset$ violates **MIN***.

(ii) Any set G such that $G \supsetneq C^l$ violates **MIN***.

(iii) Any set G such that $G \cap B^k = \emptyset$ violates **NED** or **MIN***. Since $G \cap B^k = \emptyset$, G is such that, for all $g \in G$, we have $\phi(g) \neq \emptyset$ and $g \notin C^l$. Suppose G satisfies **NED** and **MIN***. By **NED**, it holds that $\bigcup_{g\in G} \phi(g) \subseteq G$. By **MIN***, it holds that

[4] Page and Wooders (2009) define a basin of attraction as a set of networks to which the network formation process might tend and from which there is no escape.

$\bigcup_{g\in G}\phi(g) = G$, with G being a closed cycle. A contradiction.

(iv) Any set G such that $G\cap B^k \subsetneq B^k$ violates **NED**. □

We now show that there exists a unique proper myopic-farsighted absorbing set that contains all the basins of attractions; i.e., all networks $g\in\mathcal{G}$ such that $\phi(g)=\emptyset$ together with all closed cycles.

Proposition 5 *Let $B^1,\dots,B^s$ be the basins of attraction. The set of networks $\bigcup_{k=1}^{s}B^k = \{g\in\mathcal{G}\mid\phi(g)=\emptyset\}\cup\left(\bigcup_{k=1}^{r}C^k\right)$ is the unique proper myopic-farsighted absorbing set.*

Proof Take any farsighted absorbing set $G^0\subseteq\mathcal{G}$. We have $\bigcup_{k=1}^{s}B^k = \{g\in\mathcal{G}\mid\phi(g)=\emptyset\}\cup\left(\bigcup_{k=1}^{r}C^k\right)\subseteq G^0$. We proceed inductively to show that $G^\infty = \bigcup_{k=1}^{s}B^k$.

(Step 1) If $\bigcup_{k=1}^{s}B^k = G^0$ we have that $\bigcup_{k=1}^{s}B^k = G^0 = G^1 = G^\infty$. Otherwise, $\bigcup_{k=1}^{s}B^k\subsetneq G^0$ and there exists $g\in G^0$ such that (i) $g\notin\bigcup_{k=1}^{s}B^k$ and (ii) $g\notin\phi(g')$ for every $g'\in G^0$. All $g\in\bigcup_{k=1}^{s}B^k$ belong to G^1; otherwise, conditions (ii) and/or (iii) in Definition 3 would be violated. The set $G^1 = G^0\setminus\{g\in G^0\mid g\notin\bigcup_{k=1}^{s}B^k$ and $g\notin\phi(g')$ for every $g'\in G^0\}$ satisfies (i), (ii) and (iii) in Definition 3.

(Step 2) If $\bigcup_{k=1}^{s}B^k = G^1$ we have that $\bigcup_{k=1}^{s}B^k = G^1 = G^2 = G^\infty$. Otherwise, $\bigcup_{k=1}^{s}B^k\subsetneq G^1$ and there exists $g\in G^1$ such that (i) $g\notin\bigcup_{k=1}^{s}B^k$ and (ii) $g\notin\phi(g')$ for every $g'\in G^1$. All $g\in\bigcup_{k=1}^{s}B^k$ belong to G^2; otherwise, conditions (ii) and/or (iii) in Definition 3 would be violated. The set $G^2 = G^1\setminus\{g\in G^1\mid g\notin\bigcup_{k=1}^{s}B^k$ and $g\notin\phi(g')$ for every $g'\in G^1\}$ satisfies (i), (ii) and (iii) in Definition 3.

(Step l) If $\bigcup_{k=1}^{s}B^k = G^{l-1}$ we have that $\bigcup_{k=1}^{s}B^k = G^{l-1} = G^l = G^\infty$. Otherwise, $\bigcup_{k=1}^{s}B^k\subsetneq G^{l-1}$ and there exists $g\in G^{l-1}$ such that (i) $g\notin\bigcup_{k=1}^{s}B^k$ and (ii) $g\notin\phi(g')$ for every $g'\in G^{l-1}$. All $g\in\bigcup_{k=1}^{s}B^k$ belong to G^l; otherwise, conditions (ii) and/or (iii) in Definition 3 would be violated. The set $G^l = G^{l-1}\setminus\{g\in G^{l-1}\mid g\notin\bigcup_{k=1}^{s}B^k$ and $g\notin\phi(g')$ for every $g'\in G^{l-1}\}$ satisfies (i), (ii) and (iii) in Definition 3.

Since $\mathcal{G}$ is finite, there is $\bar{l}$ such that $G^{\bar{l}} = G^{\bar{l}+1} = G^{\bar{l}+2} = G^\infty = \bigcup_{k=1}^{s}B^k$. □

Example 1 (Continued). Consider again Jackson and Wolinsky (1996) co-author model with three players. When $F=\{1,2,3\}$ we have that $\mathcal{A}(\{1,2,3\}) = \{\{g_4,g_5,g_7\},\ \{g_4,g_6,g_7\},\ \{g_5,g_6,g_7\}\}$, but the proper myopic-farsighted absorbing set $\{g_7\}$ singles out the complete network g_7. Notice that the network g_7 is the intersection of the three myopic-farsighted absorbing sets.

Without loss of generality, let $A^1,\dots,A^t$ and $B^1,\dots,B^s$ be, respectively, the myopic-farsighted absorbing sets and the basins of attraction in the network formation game. Since we have $\left(\{g\in\mathcal{G}\mid\phi(g)=\emptyset\}\cup\left(\bigcup_{k=1}^{r}C^k\right)\right)\subseteq A^k$, for every $k=1,\dots,t$, it follows that $\bigcap_{k=1}^{t}A^k\supseteq\bigcup_{k=1}^{s}B^k$. The example of Figure 2 illustrates the fact that the unique proper myopic-farsighted absorbing set $\bigcup_{k=1}^{s}B^k$ can be a strict subset of $\bigcap_{k=1}^{t}A^k$. For $N=F$, we have $\phi(g_0)=\{g_1,g_2,g_3,g_4,g_5,g_6,g_7\}$, $\phi(g_1)=\{g_4\}$, $\phi(g_2)=\{g_4\}$, $\phi(g_3)=\{g_1,g_2,g_5,g_6,g_7\}$, $\phi(g_4)=\phi(g_5)=\phi(g_6)=\{g_7\}$ and

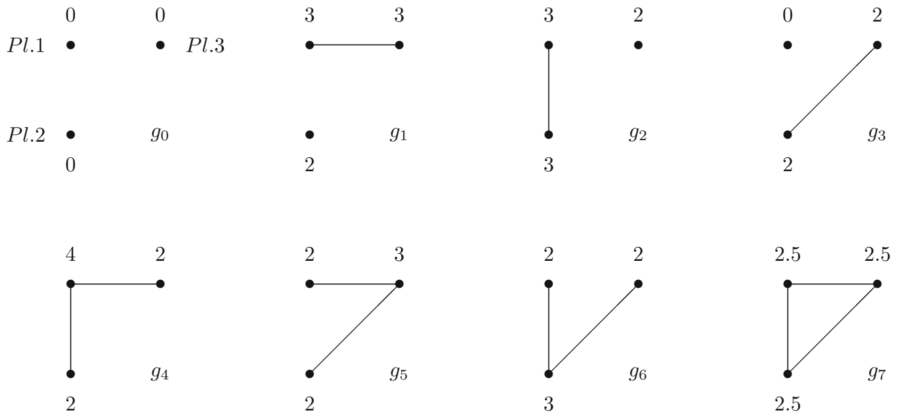

Fig. 2 A network formation game among three players

$\phi(g_7) = \emptyset$. It is easily verified that there is a unique myopic-farsighted absorbing set, $\{g_4, g_7\}$, while $\{g_7\}$ is the unique proper myopic-farsighted absorbing set.

Suppose that the utility function u is such that, for any given network, all players get the same payoff: $u_i(g) = u_j(g)$ for all $i,j \in N$. With the egalitarian utility function u, each player's payoff depends on the network but not on the specific role she plays within the network. The next proposition shows that the set of networks such that there are no myopic-farsighted improving paths emanating from them is the unique (proper) myopic-farsighted absorbing set under the egalitarian utility function.

Proposition 6 *Take any u such that $u_i(g) = u_j(g)$ for all $i,j \in N$. The set $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ is the unique myopic-farsighted absorbing set and the unique proper myopic-farsighted absorbing set.*

Proof (i) First, we show that for every $g' \in \mathcal{G} \setminus \{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$, it holds that $\phi(g') \cap \{g \in \mathcal{G} \mid \phi(g) = \emptyset\} \neq \emptyset$. Given the egalitarian utility function u, we have that, for $0 \leq m \leq n$, there are no closed cycles: for all $g \in \mathcal{G}$ we have $g \notin \phi^{\infty}(g)$. In addition, given the egalitarian utility function u, we have that $u_i(g_k) < u_i(g_K)$ for all $i \in N$, $k = 1, ..., K-1$, along any myopic-farsighted improving path $(g_1, ..., g_K)$. It follows that if $g' \in \phi(g)$ and $g'' \in \phi(g')$ then $g'' \in \phi(g)$. Hence, $\phi(g) = \phi^{\infty}(g)$. Thus, for any g' such that $\phi(g') \neq \emptyset$ we have $\phi(g') \cap \{g \in \mathcal{G} \mid \phi(g) = \emptyset\} \neq \emptyset$, and the set $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ satisfies **ES**. Second, since $\phi(g) = \emptyset$ for all $g \in \{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$, the set $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ satisfies **NED**. Third, $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ satisfies **MIN** since any set $G \subsetneq \{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ violates **ES**. From Proposition 2 we have that $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ is included in any myopic-farsighted absorbing set. Since $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ is a myopic-farsighted absorbing set, any set $G \supsetneq \{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ would violate **MIN**. Hence, $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ is the unique myopic-farsighted absorbing set.

(ii) Since there are no closed cycles, it follows from Proposition 5 that $\{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ is the proper myopic-farsighted absorbing set. □

5 Relationship with myopic-farsighted stable Sets

Luo et al. (2021) propose the notion of myopic-farsighted stable set to determine the networks that are stable when some players are myopic while others are farsighted.[5] A set of networks G is a myopic-farsighted stable set if G satisfies Internal Stability (**IS**) and External Stability (**ES**).

Definition 5 A set of networks $G \subseteq \mathcal{G}$ is a *myopic-farsighted stable set* if: **(IS)** for every $g, g' \in G$ $(g \neq g')$, it holds that $g' \notin \phi(g)$; and (**ES**) for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$.

When all players are farsighted ($N = F$), the myopic-farsighted stable set is simply the vNM farsighted stable set as defined in Herings et al. (2009) or Ray and Vohra (2015).[6] When all players are myopic ($N = M$), the myopic-farsighted stable set boils down to the pairwise CP vNM set as defined in Herings et al. (2017) for two-sided matching problems.[7]

We provide conditions for the equivalence between the unique myopic-farsighted absorbing set and the unique myopic-farsighted stable set.

Proposition 7 *If $G \subseteq \mathcal{G}$ is such that* (**i**) *for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$, and* (**ii**) *for every $g \in G$, it holds that $\phi(g) = \emptyset$, then G is both the unique myopic-farsighted stable set and the unique myopic-farsighted absorbing set.*

Proof If $G \subseteq \mathcal{G}$ is such that (i) for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$, and (ii) for every $g \in G$, it holds that $\phi(g) = \emptyset$, then G satisfies **ES**, **IS**, **NED** and **MIN**. Hence, G is both a myopic-farsighted stable set and a myopic-farsighted absorbing set.

Suppose that $G' \neq G$ is a myopic-farsighted stable set. Since for every $g \in G$, it holds $\phi(g) = \emptyset$, then $G \subseteq G'$. Otherwise, G' violates **ES**. But, if $G \subsetneq G'$ then G' violates **IS**. Hence, G is the unique myopic-farsighted stable set.

Suppose that $G' \neq G$ is a myopic-farsighted absorbing set. Since for every $g \in G$, it holds that $\phi(g) = \emptyset$, then $G \subseteq G'$. Otherwise, G' violates **NED**. But, if $G \subsetneq G'$ then G' violates **MIN**. Hence, G is the unique myopic-farsighted absorbing set. □

Example 2 [Non-existence of a myopic-farsighted stable set[8]] Consider the situation where three players can form links and where the payoffs are given in Figure 3. When all players are farsighted ($F = N$), we have $\phi(g_0) = \phi(g_7) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_2, g_3\}$, $\phi(g_2) = \{g_3, g_4, g_5\}$, $\phi(g_3) = \{g_4, g_5\}$, $\phi(g_4) = \{g_1, g_5, g_6\}$, $\phi(g_5) = \{g_1, g_6\}$, $\phi(g_6) = \{g_1, g_2, g_3\}$.

[5] Herings et al. (2020) define first the myopic-farsighted stable set for two-sided matching problems, and Mauleon et al. (2018) extend it to R&D network formation with pairwise deviations.

[6] Alternative notions of farsightedness are suggested by Chwe (1994), Diamantoudi and Xue (2003), Dutta and Vohra (2017), Herings et al. (2004), Herings et al. (2019), Mauleon and Vannetelbosch (2004), Page et al. (2005), Ray and Vohra (2019), Xue (1998) among others.

[7] The pairwise CP vNM set follows the approach by Page and Wooders (2009) who define the stable set with respect to path dominance, i.e., the transitive closure of ϕ.

[8] Luo et al. (2021) provide conditions on the utility function that guarantee the existence and uniqueness of a myopic-farsighted stable set.

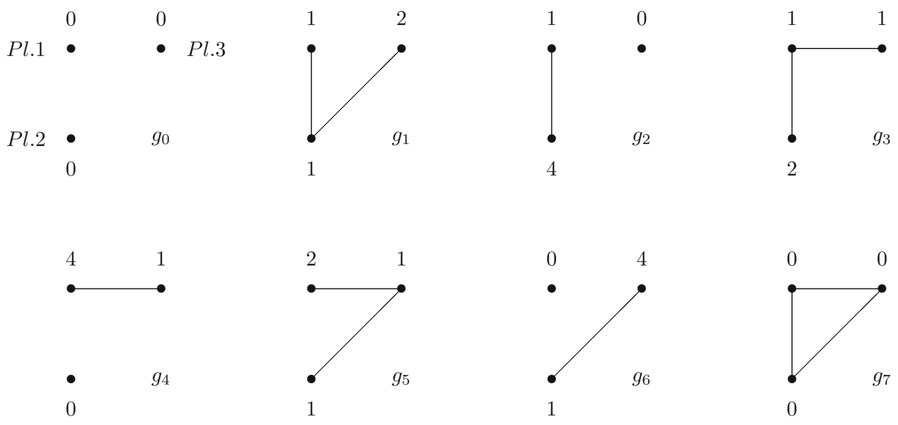

Fig. 3 Non-existence of myopic-farsighted stable sets

There is no myopic-farsighted stable set.

Suppose on the contrary that G is a myopic-farsighted stable set. Suppose $g_1 \in G$. **IS** implies that no other network can belong to G. Since $\phi(g_2) \cap \{g_1\} = \emptyset$ it follows that **ES** is violated, a contradiction. As a consequence, $g_1 \notin G$. A symmetric argument leads to the result that $g_3 \notin G$ and $g_5 \notin G$. Suppose now that $g_2 \in G$. **IS** implies that no other network can belong to G. Since $\phi(g_5) \cap \{g_2\} = \emptyset$ it follows that **ES** is violated, a contradiction. By symmetry it follows that $g_4 \notin G$ and $g_6 \notin G$. Suppose $g_0 \in G$. **IS** implies that no other network can belong to G. Since $\phi(g_1) \cap \{g_0\} = \emptyset$, it follows that **ES** is violated, a contradiction. By a similar argument, we can show that $g_7 \notin G$. The only remaining possibility is $G = \emptyset$. This clearly violates **ES**.

However, there is a unique myopic-farsighted absorbing set: $\{g_1, g_2, g_3, g_4, g_5, g_6\}$. Any subset would violate **NED**.

When a myopic-farsighted stable set does exist, it has a non-empty intersection with a myopic-farsighted absorbing set.

Proposition 8 *Suppose that G is a myopic-farsighted stable set. If G' is a myopic-farsighted absorbing set, then $G \cap G' \neq \emptyset$.*

Proof Suppose that G is a myopic-farsighted stable set and G' is a myopic-farsighted absorbing set such that $G \cap G' = \emptyset$. Since G is a myopic-farsighted stable set, **ES** implies that for every $g \in G'$, it holds that $\phi(g) \cap G \neq \emptyset$. But since G' is a myopic-farsighted absorbing set, **NED** implies that for every $g \in G'$, it holds that $\phi(g) \subseteq G'$ contradicting $G \cap G' = \emptyset$. □

6 The threshold game

We now consider the threshold game to illustrate the notion of myopic-farsighted absorbing sets and to point out the role of the relative number of farsighted and myopic players for reaching efficiency. In the threshold game, every player can have a link with another player at a cost of c $(0 < c < 1)$. Every player receives a benefit of $\#N_i(g)$ if there are at least $\bar{l}$ links in the network, but benefits are zero if there are less than $\bar{l}$ links. Thus, player i's payoff is given by

$$u_i(g) = \begin{cases} (1-c)\#N_i(g) & \text{if } \#g \geq \bar{l} \\ -c\#N_i(g) & \text{if } \#g < \bar{l} \end{cases},$$

where $\#g$ denotes the number of links in the network g and $1 \leq \bar{l} \leq n(n-1)/2$. In Figure 4 we have depicted the 3-player case with $\bar{l} = 2$.

Suppose that player 1 is farsighted while players 2 and 3 are myopic: we have $\phi(g_0) = \emptyset$, $\phi(g_1) = \{g_0, g_4, g_5, g_7\}$, $\phi(g_2) = \{g_0, g_4, g_6, g_7\}$, $\phi(g_3) = \{g_0, g_5, g_6, g_7\}$, $\phi(g_4) = \phi(g_5) = \phi(g_6) = \{g_7\}$ and $\phi(g_7) = \emptyset$. It is easily verified that there is a unique myopic-farsighted absorbing set: $\{g_0, g_7\}$.

Suppose now that players 1 and 2 are farsighted while player 3 is myopic: we have $\phi(g_0) = \{g_2, g_4, g_6, g_7\}$, $\phi(g_1) = \{g_0, g_4, g_5, g_6, g_7\}$, $\phi(g_2) = \{g_0, g_4, g_5, g_6, g_7\}$, $\phi(g_3) = \{g_0, g_4, g_5, g_6, g_7\}$, $\phi(g_4) = \phi(g_5) = \phi(g_6) = \{g_7\}$ and $\phi(g_7) = \emptyset$. It follows that there is a unique myopic-farsighted absorbing set: $\{g_7\}$.

Proposition 9 *The unique myopic-farsighted absorbing set in the threshold game is*

(i) $\{g^N\}$ if $(n-m)(n-m-1)/2 \geq \bar{l} - 1$, and
(ii) $\{g^\emptyset, g^N\}$ if $(n-m)(n-m-1)/2 < \bar{l} - 1$.

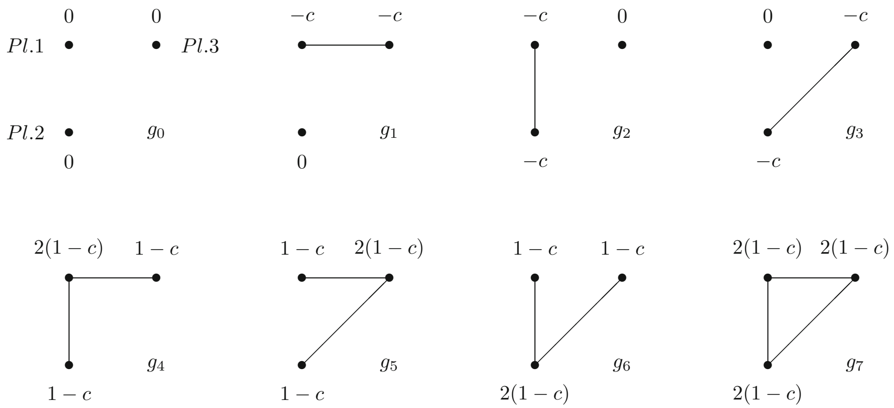

Fig. 4 The threshold game with three players and $\bar{l} = 2$

Proof We first show that g^N belongs to any absorbing set. Notice that the complete network g^N is the unique Pareto efficient network (and Pareto dominates any other network): $u_i(g^N) = (1-c)(n-1) \geq u_i(g)$ for all $g \neq g^N$ and $u_i(g^N) > u_i(g)$ for all g such that $\#N_i(g) < n-1$. Hence, there is no myopic-farsighted improving path emanating from g^N: $\phi(g^N) = \emptyset$. Hence, g^N belongs to any absorbing set. Otherwise, **ES** would be violated.

(i) If the number $(n-m)$ of farsighted players is such that $(n-m)(n-m-1)/2 \geq \bar{l}-1$, then from any $g \neq g^N$ there is a myopic-farsighted improving path to the complete network g^N, i.e. $g^N \in \phi(g)$ for any $g \neq g^N$. Take any $g \neq g^N$. From g, looking forward towards g^N, farsighted players build links with other farsighted players to reach network g' with $g^F \subseteq g'$. Notice that g' is such that $\#g' \geq \bar{l}-1$. Hence, from g' both myopic and farsighted players have now incentives to successively add links to reach g^N. Since $g^N \in \phi(g)$ for any $g \neq g^N$ and $\phi(g^N) = \emptyset$, the set $\{g^N\}$ satisfies **ES**, **NED** and **MIN**, and is the unique myopic-farsighted absorbing set.

(ii) If the number $(n-m)$ of farsighted players is such that $(n-m)(n-m-1)/2 < \bar{l}-1$, then there is no myopic-farsighted improving path emanating from the empty network $g^{\emptyset}$. Myopic players have no incentive to build a link and farsighted players are not numerous enough to form a network g with $\#g \geq \bar{l}-1$ so that myopic players would now have incentives to add links. Hence, $\phi(g^{\emptyset}) = \emptyset$ and $g^{\emptyset}$ belongs to any absorbing set. For any g such that $\#g < \bar{l}-1$, myopic players have incentives to cut links and farsighted players who look forward to $g^{\emptyset}$ have also incentives to delete links. Thus, for any g such that $\#g < \bar{l}-1$, $g^{\emptyset} \in \phi(g)$. For any g such that $\#g \geq \bar{l}-1$, myopic players have incentives to add links and farsighted players who look forward to g^N have also incentives to add links. Thus, for any g such that $\#g \geq \bar{l}-1$, $g^N \in \phi(g)$. Since $\phi(g^N) = \phi(g^{\emptyset}) = \emptyset$, $g^{\emptyset} \in \phi(g)$ for any $g \neq g^{\emptyset}$ such that $\#g < \bar{l}-1$ and $g^N \in \phi(g)$ for any $g \neq g^N$ such that $\#g \geq \bar{l}-1$, the set $\{g^{\emptyset}, g^N\}$ satisfies **ES**, **NED** and **MIN**, and is the unique myopic-farsighted absorbing set. □

Let $m^*(n,\bar{l})$ be such that $(n-m^*)(n-m^*-1)/2 = \bar{l}-1$. If $m \leq m^*$ then $\{g^N\}$ is the unique myopic-farsighted absorbing set, but if $m > m^*$ then $\{g^{\emptyset}, g^N\}$ is the unique myopic-farsighted absorbing set. Thus, if $m > m^*$, then turning $m - m^*$ myopic players into farsighted ones would guarantee the emergence of the efficient outcome.

7 Efficiency

Herings et al. (2019) define the property of increasing returns to link creation for network utility functions. A network utility function u displays no externalities across components (**NEC**) if for every $g \in \mathcal{G}$, for every $h \in H(g)$, we have $u_i(g) = u_i(h)$ for all $i \in N(h)$ and $u_i(g) = 0$ for all $i \in N \setminus N(g)$. In particular, it holds that $u_i(g^{\emptyset}) = 0$ for all $i \in N$. If a network utility function u satisfies **NEC**, then it is sufficient to specify it for connected networks. Let $\overline{\mathcal{G}} = \{g \in \mathcal{G} \,|\, \#H(g) = 1\}$ be the set of connected networks and let $\overline{\mathcal{G}}^{+} = \{h \in \overline{\mathcal{G}} | \sum_{i \in N} u_i(h) \geq 0\}$ be the set of connected networks with non-negative aggregate payoffs. A network utility function u is said to satisfy increasing returns to link creation (**IRL**) if:

(i) u satisfies **NEC**.

(ii) If $h \in \overline{\mathcal{G}}^{+}$ and $h \subseteq h' \in \overline{\mathcal{G}}$, then $h' \in \overline{\mathcal{G}}^{+}$.

(iii) If $h \in \overline{\mathcal{G}}^{+}$ and $ij \in h$, then $u_i(h - ij) \leq u_i(h)$ and $u_j(h - ij) \leq u_j(h)$ with at least one inequality holding strictly.

(iv) There exists $h' \in \overline{\mathcal{G}}^{+}$ such that for all $h \in \overline{\mathcal{G}} \cup \{g^{\emptyset}\}$ with $h \subsetneq h'$, for all $i \in N(h')$, we have $u_i(h) < u_i(h')$.

Condition (iv) of **IRL** implies that there is a connected network h' for which the utility of all players having at least one link is greater than the utility they could obtain in any network $h \subsetneq h'$. If we take $h = g^{\emptyset}$, then it follows that $u_i(h') > 0$ for all $i \in N(h')$. Let $\overline{m} = n - \#N(h')$. Condition (ii) of **IRL** implies that the aggregate utilities in any connected network containing h' are non-negative. Hence, $\sum_{i \in N} u_i(g^N) \geq 0$. Condition (iii) of **IRL** implies that utilities increase when making additional links in connected networks containing h'.

Given a permutation of players π (a bijection from N to N) and any $g \in \mathcal{G}$, let $g^{\pi} = \{\pi(i)\pi(j) \mid ij \in g\}$ be the network that shares the same architecture as g but with the specific players permuted. A network utility function u satisfies equal treatment of equals (**ETE**) if for any $g \in \mathcal{G}$, $i \in N$ and permutation π, we have $u_{\pi(i)}(g^{\pi}) = u_i(g)$.

Lemma 1 *Let the network utility function u satisfy* **IRL** *and* **ETE**, *and be such that* $u_i(g) = u_j(g)$ *for all* $i, j \in S \in \Pi(g)$. *For all* $m \leq \overline{m}$, *we have that* $g^N \in \phi(g)$ *for every* $g \neq g^N$.

Proof Since u satisfy **IRL** and $u_i(g) = u_j(g)$ for all $i, j \in S \in \Pi(g)$, we have that g^N strictly Pareto dominates any $g \neq g^N$. From condition (iv) of **IRL**, there is a network $h' \in \overline{\mathcal{G}}^{+}$ such that for all $h \in \overline{\mathcal{G}}$ with $h \subsetneq h'$ it holds that $u_i(h') > u_i(h)$ for all $i \in N(h')$. In particular, we have that $u_i(h') > u_i(g^{\emptyset})$ for all $i \in N(h')$. Let h' be such a network with $\widetilde{l}$ links. Since u satisfies **ETE**, we have $u_{\pi(i)}(h'^{\pi}) = u_i(h')$ for any $i \in N$ and π, and so h'^{π} satisfies condition (iv) of **IRL** for any π. Remember that $\overline{m} = n - \#N(h')$.

(a) First, consider the empty network $g^{\emptyset}$. If $m \leq \overline{m}$, then $\#N(h')$ farsighted players have incentives to form sequentially the missing links in $g^{\emptyset}$ to form h' (or some h'^{π}) foreseeing the Pareto dominating network g^N. From h' myopic players have incentives (by **IRL**) to form sequentially the missing links in h' as well as farsighted players have incentives to form sequentially the missing links in h' looking forward to g^N.

(b) Second, consider any network $\widetilde{g} \neq \emptyset$ such that $h'^{\pi} \nsubseteq \widetilde{g}$ for any π. If $m \leq \overline{m}$, then $\#N(h')$ farsighted players have incentives to form sequentially the missing links in $\widetilde{g}$ to form $\widetilde{g}'$ such that $\widetilde{g}' \supseteq h'^{\pi}$ for some π foreseeing the Pareto dominating network g^N. From $\widetilde{g}'$ myopic players have incentives (by **IRL**) to form sequentially the missing links in $\widetilde{g}'$ as well as farsighted players have incentives to form sequentially the missing links in $\widetilde{g}'$ looking forward to g^N.

(c) Third, consider any network $\widetilde{g} \neq \emptyset$ such that $h'^{\pi} \subseteq \widetilde{g}$ for some π. From $\widetilde{g}$ myopic players have incentives (by **IRL**) to form sequentially the missing links in $\widetilde{g}$ as well as farsighted players have incentives to form sequentially the missing links in $\widetilde{g}$ looking forward to g^N.

Hence, if $m \leq \overline{m}$, then $g^N \in \phi(g)$ for every $g \neq g^N$. □

Next proposition shows that under **IRL** and **ETE**, the (strongly and Pareto) efficient complete network constitutes the unique myopic-farsighted absorbing set.

Proposition 10 *Let the network utility function u satisfy* **IRL** *and* **ETE** *and be such that $u_i(g) = u_j(g)$ for all $i,j \in S \in \Pi(g)$. For all $m \leq \overline{m}$, $\{g^N\}$ is the unique myopic-farsighted absorbing set.*

Proof Take any u satisfying **IRL** and **ETE**. (i) From Lemma 1 we have that for all $m \leq \overline{m}$, $\{g^N\}$ satisfies **ES** since $g^N \in \phi(g)$ for every $g \neq g^N$. (ii) Since $u_i(g) = u_j(g)$ for all $i,j \in S \in \Pi(g)$, g^N (strictly) Pareto dominates any $g \neq g^N$. Hence, $\phi(g^N) = \emptyset$ and $\{g^N\}$ satisfies **NED**. (iii) Since $\phi(g^N) = \emptyset$, it follows from Proposition 2 that g^N belongs to any myopic-farsighted absorbing set. By **MIN**, $\{g^N\}$ is the unique myopic-farsighted absorbing set. □

8 Conclusion

We have proposed the notion of myopic-farsighted absorbing set to predict the networks that emerge when some players are myopic while others are farsighted. A set of networks is a myopic-farsighted absorbing set if (No External Deviation) there is no myopic-farsighted deviation from networks within the set to some networks outside the set, (External Stability) there is a myopic-farsighted deviation from any network outside the set to some network within the set, and (Minimality) there is no proper subset satisfying No External Deviation and External Stability. Contrary to the notion of myopic-farsighted stable set, a myopic-farsighted absorbing set always exists. Since myopic-farsighted absorbing sets could be quite inclusive, we have proposed the notion of proper myopic-farsighted absorbing set that refines the concept of myopic-farsighted absorbing set by selecting the more absorbing networks. There is a unique proper myopic-farsighted absorbing set and it coincides with the set of all basins of attraction. Finally, we have introduced a threshold game that illustrates the role of the relative number of farsighted and myopic players for reaching efficiency.

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