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Friendship networks with farsighted agents

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Abstract

We reconsider de Marti and Zenou (2017) model of friendship network formation where individuals belong to two different communities and costs of forming links depend on community memberships. Many inefficient friendship networks such as segregation can arise when all individuals are myopic. Once there are myopic and farsighted individuals in both communities, we show that if there are enough farsighted individuals in the dominant community relatively to the number of individuals in the small community, then the friendship network where the smaller community ends up being assimilated into the dominant community is likely to emerge and is strongly and Pareto efficient. Moreover, this friendship network Pareto dominates the complete segregation network.

Key words: friendship networks; stable sets; myopia; farsightedness; assimilation; segregation.

JEL Classification: A14, C70, D20.

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1 Introduction

Social networks or friendship networks are important in obtaining information on goods and services, like product information or information about job opportunities. Individuals are often regrouped into communities based on their ethnicity, religion, income, education, etc. (see e.g. de Marti and Zenou, 2017; Patacchini and Zenou, 2016). Beside belonging to different communities, individuals often differ in their degree of farsightedness, i.e., their ability to forecast how others will react to the decisions they take. Indeed, recent experiments on network formation provide evidence in favour of a mixed population consisting of both myopic and (limited) farsighted individuals (see Kirchsteiger, Mantovani, Mauleon and Vannetelbosch, 2016; Teteryatnikova and Tremewan, 2020). The degree of farsightedness and the depth of reasoning are often correlated with other relevant attributes such as education, income, age, etc. (see Mauersberger and Nagel, 2018).

The aim of this paper is to provide a theoretical study of how different degrees of farsightedness affect the formation of friendship relationships when individuals can belong to various communities.[1] It is important to understand what happens when myopic individuals interact with farsighted individuals since, in general, some networks that are stable when all players are myopic could now be destabilized once individuals are mixed. In particular, we are interested in addressing the following set of questions. What are the friendship network structures that may endogenously arise once individuals belonging to two different communities can be either myopic or farsighted in forming links? When do we observe integration, segregation or (partial) assimilation? Does farsightedness help to bridge communities and to more integrated societies? Are farsighted individuals more likely to be linked to others who have different characteristics? How might the network structure change if the dominant community is relatively more farsighted than the other one? Do myopic individuals end up assimilated to the dominant community? Are individual incentives to link adequate from a social welfare point of view? Does it improve efficiency if some individuals become farsighted? And if yes, whom?

To answer these questions we reconsider de Marti and Zenou (2017) model of network formation where individuals belong to two different communities. Communities may be defined along social categories such as ethnicity, religion, education, income, etc. In contrast to de Marti and Zenou (2017) where all individuals were myopic, we now allow the possibility of having a mixed population composed of both myopic and farsighted individuals. Myopic or farsighted individuals decide with whom they want to form a link, according to a utility function that weights the costs and benefits of each connection. Farsighted individuals are able to anticipate that once they add or delete some links, other individuals could add or delete links afterwards. Benefits of a friendship connection

[1] Jackson (2008) and Goyal (2007) provide a comprehensive introduction to the theory of social and economic networks. Mauleon and Vannetelbosch (2016) give an overview of the solution concepts for solving network formation games. In Bramoullé, Galeotti and Rogers (2016), one can find the recent developments on the economics of networks.

decrease with distance in the network, while the cost of a link depends on the type of individuals involved. Two individuals from the same community face a low linking cost, while the cost of forming a friendship relationship between two individuals from different communities decreases with the rate of exposure of each of them to the other community.

We adopt the notion of myopic-farsighted stable set to determine the friendship networks that emerge when some individuals are myopic while others are farsighted.[2] A myopic-farsighted stable set is the set of networks satisfying internal and external stability with respect to the notion of myopic-farsighted improving path. That is, a set of networks is a myopic- farsighted stable set if there is no myopic-farsighted improving path between networks within the set and there is a myopic-farsighted improving path from any network outside the set to some network within the set. A myopic-farsighted improving path is simply a sequence of networks that can emerge when farsighted individuals form or delete links based on the improvement the end network offers relative to the current network while myopic individuals form or delete links based on the improvement the resulting network offers relative to the current network.

When all individuals are myopic, de Marti and Zenou (2017) show that many friendship networks can be stable. In the case of low intra-community costs, the complete integration is stable when inter-community costs are sufficiently low. For higher inter-community costs, the complete segregation becomes stable. They also point out that some asymmetric network configurations can be stable. For instance, the network in which both communities are fully intra-connected and where there is only one bridge link can be stabilized. However, a tension between efficiency and stability may occur since Pareto-dominated networks, like segregation, are stable.

What happens when the population is composed of both myopic and farsighted individuals and intra-community costs are low? We first show that, if farsighted individuals in the large community are relatively numerous and inter-community costs are large enough, then a friendship network where individuals of the small community are fully assimilated into the large community is likely to emerge in the long run and is efficient. The complete segregation is destabilized because farsighted individuals while they do not have immediate incentives to add or delete links, they anticipate that once they do so, other individuals will continue adding or deleting links leading to a friendship network where the small community is assimilated into the larger one. Precisely, farsighted individuals in the dominant community first push farsighted individuals in the small community in a situation where they are worst off compared to what they obtain when they are fully assimilated into the dominant community. Next farsighted individuals in the dominant community lure the myopic individuals in the small community with the prospect of forming a friendship network where the dominant community is fully assimilated into the

[2]Herings, Mauleon and Vannetelbosch (2020) were first to define the myopic-farsighted stable set for two-sided matching problems. This notion is extended to R&D network formation with pairwise deviations in Mauleon, Sempere-Monerris and Vannetelbosch (2020) and to general network formation problems in Luo, Mauleon and Vannetelbosch (2021).

smaller community. From such friendship network, farsighted individuals in the dominant community are able to induce a switch towards the opposite fully assimilated network, the friendship network where the small community is fully assimilated into the dominant community, where they achieve their best outcome. The less farsighted individuals in the small community are, the more likely this friendship network will arise. In the limit, when all individuals in the small community tend to be myopic, a singleton set consisting of the network where the smaller community is assimilated into the dominant one is the unique myopic-farsighted stable set.

In addition, we are able to provide the lower bound on the relative number of farsighted individuals in the dominant community relative to the number of individuals in the small community so that the friendship network where the smaller community ends up being assimilated into the dominant community is stable. Thus, turning myopic individuals into farsighted ones, especially in the dominant community, could be very helpful in avoiding (Pareto-) inefficient situations like segregation.

We also show that if all individuals in the small community are farsighted while there are both myopic and farsighted individuals in the dominant community, inter-community costs are large enough and the smaller community is not too small relatively to the other one, then a friendship network where individuals of the dominant community are fully assimilated into the small community could emerge in the long run and it Pareto dominates the complete segregation network.[3]

In the case of intermediate intra-community costs, we show that a mixed population of farsighted and myopic individuals again solve the tension between stability and efficiency. Many friendship networks are stable when all individuals are myopic, but once there are enough farsighted individuals, independently to which community they belong, then a star network with a myopic individual in the center is going to arise and is efficient.

We now turn to the related literature. There is an extensive literature using network models to explain the fact that individuals are more likely to be linked to individuals who have similar characteristics. Currarini, Jackson and Pin (2009) develop a dynamic random matching model with a population formed by groups of different sizes and show that segregation in social networks results from the decisions of the individuals involved and/or from the ways in which individuals meet and interact. In equilibrium, individuals' behavior is totally homogeneous within the same group of individuals. Bramoullé, Currarini, Jackson, Pin and Rogers (2012) develop a model of dynamic matching with both random meetings and network-based search. They show that majority and minority groups have different patterns of interactions and that relative homophily in the network is strongest when groups have equal size, and vanishes as groups have increasingly unequal sizes.[4]

[3]When inter-community costs are small, the complete integration network becomes again stable whatever the number of farsighted and myopic individuals within the population and it is efficient.

[4]Golub and Jackson (2012) study how the speed of learning and best-response processes depends on homophily. Pin and Rogers (2016) provide a survey on stochastic network formation and homophily. Mele

Despite strong empirical evidence, few models of network formation with differentiated communities have studied the impact of social networks on the long-run integration outcome of minorities. Jackson and Rogers (2005) extend the Jackson and Wolinsky (1996)'s connection model by including two communities and assuming that the cost of linking two individuals from different communities is exogenous and independent of the behavior of the two individuals involved in the link. Johnson and Gilles (2000) add a geographical dimension to Jackson and Wolinsky (1996)'s connection model assuming that the cost of a link is proportional to the geographical distance between two individuals. As already mentioned, de Marti and Zenou (2017) model is a variation of the connection model where the cost of a link is endogenous and depends on the neighbourhood structure of the two individuals involved in the link.

We go further the related literature by considering the impact of a mixed population along two dimensions (community membership and degree of farsightedness) on the stability of friendship networks. That is, we analyze how the presence of farsighted individuals can affect the long-run integration outcome and under which circumstances this can lead to either a segregated society or an integrated society or a society where one community is assimilated into the other one. By doing so, we are the first to stabilize in the long-run the efficient network structure where the smaller community ends up being assimilated into the larger community.[5]

Another strand of the literature studies the role of social networks in the assimilation of immigrants, a hot debate in the United States and in Europe. There is strong evidence showing that family, peers and communities affect assimilation decisions (see e.g. Bisin, Patacchini, Verdier and Zenou, 2016). In particular, there may be a conflict between an individual's assimilation choice and that of her peers and between an individual's assimilation choice and that of her family and community. Verdier and Zenou (2017) study the role of the immigrant network in the assimilation process of ethnic minorities. They show that, in an exogenous network, the more central minority individuals are located in the social network, the more they assimilate to the majority culture. By endogenizing the network structure, they show when the ethnic minority will integrate or not into the majority group.[6]

The paper is organized as follows. In Section 2 we present de Marti and Zenou (2017)

(2017) proposes a dynamic model of network formation that combines strategic and random networks features.

[5]Using data from the German Socio-Economic panel for the period 1996 to 2011, Facchini, Patacchini and Steinhardt (2015) find that first generation migrants who have a German friend are more similar to German natives than migrants who do not. In addition, the educational achievement is positively related to the likelihood of forming friendships with majority group members. Similarly, from data of the European Community Household Panel (1994-2001), de Palo, Faini and Venturini (2007) find that more educated migrants tend to socialize more intensively with the majority community.

[6]Verdier and Zenou (2018) study the population dynamics of cultural traits emphasizing different facets of the impact of forward looking cultural leaders in the process of cultural assimilation of minority communities.

model of friendship networks with two communities. In Section 3 we introduce the concept of myopic-farsighted stable sets. In Section 4 we provide a characterization of the myopic-farsighted stable sets when intra-community costs are low. In Section 5 we consider the case where intra-community costs are intermediate. In Section 6 we look at the tension between stability and efficiency, we discuss what could happen for intermediate inter-community costs and we conclude.

2 Friendship networks with two communities

We consider de Marti and Zenou (2017) model of friendship networks where individuals belong to two different communities.[7] Individuals benefit from direct and indirect connections to others, which can be interpreted as positive externalities. These benefits decay with distance between individuals and the cost of forming links may depend on community memberships. The novelty is that individuals can now be either farsighted or myopic when deciding about the friendship links they want to form. In de Marti and Zenou (2017) all individuals were supposed to be myopic.

The set of individuals is denoted by $N = N^M \cup N^F$, where N^M is the set of myopic individuals and N^F is the set of farsighted individuals. Let n be the total number of individuals and $n^M \geq 0$ ($n^F = n - n^M \geq 0$) be the number of myopic (farsighted) individuals. Moreover, the population is divided into two communities $N = N^B \cup N^G$, where N^B is the *blue* community and N^G is the *green* community. Each individual belongs to one of the two communities and the type of individual i is denoted as $\tau(i) \in \{N^B, N^G\}$. We have $n = n^B + n^G$, where n^B and n^G denote, respectively, the number of N^B individuals and the number of N^G individuals in the population. Let $n^{M,B}$ and $n^{F,B}$ be, respectively, the number of myopic and farsighted individuals in the blue community, with $n^B = n^{M,B} + n^{F,B}$. Let $n^{M,G}$ and $n^{F,G}$ be, respectively, the number of myopic and farsighted individuals in the green community, with $n^G = n^{M,G} + n^{F,G}$. Notice that $n^M = n^{M,B} + n^{M,G}$, $n^F = n^{F,B} + n^{F,G}$ and $n = n^M + n^F$. Without loss of generality, the green community is the largest one and there are at least two individuals in each community: $1 < n^B \leq n^G$.

A friendship network g is a list of which pairs of individuals are linked to each other and $ij \in g$ indicates that i and j are linked under g. The complete network on the set of individuals $S \subseteq N$ is denoted by g^S and is equal to the set of all subsets of S of size 2. It follows in particular that the empty network is denoted by $g^\emptyset$. The set of all possible networks on N is denoted by $\mathcal{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \text{there is } j \text{ such that } ij \in g\}$ be the set of individuals who have at least one link in the network g. Let $N_i(g) = \{j \in N \mid ij \in g\}$ be the set of neighbours (or friends) of individual i in

[7]See also Bjerre-Nielsen (2020) for a related model of network formation with multiple types.

g.[8] Let $n_i(g) = \#(N_i(g))$ be the number of neighbours (or friends) of individual i in g. A path in a network g between i and j is a sequence of individuals $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. A network g is connected if for all $i \in N$ and $j \in N \setminus \{i\}$, there exists a path in g connecting i and j. A non-empty sub-network $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. A star network is a network such that there exists some individual i (the center) who is linked to every other individual $j \neq i$ (the peripherals) and that contains no other links (i.e. g is such that $N_i(g) = N \setminus \{i\}$ and $N_j(g) = \{i\}$ for all $j \in N \setminus \{i\}$).

A network utility function (or payoff function) is a mapping $U_i : \mathcal{G} \to \mathbb{R}$ that assigns to each network g a utility $U_i(g)$ for each individual $i \in N$. A network $g \in \mathcal{G}$ is strongly efficient if $\sum_{i \in N} U_i(g) \geq \sum_{i \in N} U_i(g')$ for all $g' \in \mathcal{G}$. A network $g \in \mathcal{G}$ Pareto dominates a network $g' \in \mathcal{G}$ relative to U if $U_i(g) \geq U_i(g')$ for all $i \in N$, with strict inequality for at least one $i \in N$. A network $g \in \mathcal{G}$ is Pareto efficient relative to U if it is not Pareto dominated, and a network $g \in \mathcal{G}$ is Pareto dominant if it Pareto dominates any other network.

Preferences are given by

$$U_i(g) = \sum_{j \neq i} \delta^{t(i,j)} - \sum_{j \in N_i(g)} c_{ij}(g),$$

where $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j), $0 < \delta < 1$ is the benefit from a connection that decreases with the distance of the relationship, and $c_{ij}(g) > 0$ is the cost for individual i of maintaining a direct link with j. The cost of forming one link may vary as a function of the type of individuals connected by such link.

Definition 1 (de Marti and Zenou, 2017)**.** Given a network g, the rate of exposure of individual i to their own community $\tau(i)$ is

$$e_i^{\tau(i)}(g) = \begin{cases} n_i^{\tau(i)}(g)/(n_i(g) - 1) & \text{if } 0 < n_i^{\tau(i)}(g) < n_i(g) \\ 0 & \text{if } n_i^{\tau(i)}(g) = 0 \end{cases} \tag{1}$$

where $n_i^{\tau(i)}(g)$ is the number of i's same-type friends in network g while $n_i(g)$ is the total number of i's friends in network g.

Let c and C be strictly positive parameters, $c > 0$ and $C > 0$.[9] The cost for individual i of maintaining a link with j, $c_{ij}(g)$, depends on whether i and j belong or not to the same community:

$$c_{ij}(g) = \begin{cases} c & \text{if } \tau(i) = \tau(j) \\ c + e_i^{\tau(i)}(g) \cdot e_j^{\tau(j)}(g) \cdot C & \text{if } \tau(i) \neq \tau(j) \end{cases}.$$

[8] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, $\#$ will refer to the notion of cardinality.

[9] For $C = 0$ the model of de Marti and Zenou (2017) reverts to the connections model introduced by Jackson and Wolinsky (1996).

Such cost function assumes that it is less costly to interact with someone of the same type (intra-community cost) than with someone of a different type (inter-community cost). Notice that C is not present in the cost of a link between individuals of the same community. But, C becomes an additional cost when two individuals from different communities, having links with individuals of their own community, form a link between them. For instance, if a green individual has only green friends, then it will be more costly for her to interact with a blue individual that has mostly blue friends. However, the more similar the friendship composition of two individuals of different types, the easier it is for them to interact. If at least i or j has no friends of the same type (i.e., $e_i^{\tau(i)} = 0$ or $e_j^{\tau(j)} = 0$), then it is equally costly for them to interact with someone of the opposite type as with someone of the same type (i.e., the cost is c in both cases).[10] In Figure 1 we depict a friendship network among seven individuals and two communities ($N^G = \{1, 2, 3, 4\}$, $N^B = \{5, 6, 7\}$) with a bridge link between both communities. Green individuals are represented by solid circles while blue individuals are represented by circles. For instance, green individual 4's payoff is equal to $4\delta + 2\delta^2 - 4c - C$ since $e_4^{\tau(4)} = 3/(4-1) = 1$ and $e_7^{\tau(7)} = 2/(3-1) = 1$.

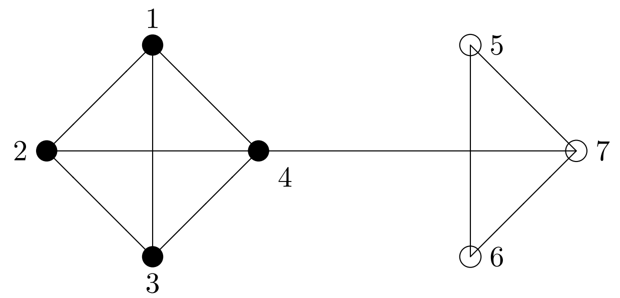

Figure 1: A bridge link between both communities. Greens are represented by solid circles while blues are represented by circles.

We now describe some prominent network configurations in the case of friendship networks with communities. Let $g_{\text{assi,green}}$ denote the network where all members of the blue community are fully assimilated to the dominant (or larger) green community. That is, each green individual is linked to all other (green and blue) individuals while each blue individual is only linked to all green individuals. Formally, $g_{\text{assi,green}} = g^{N^G} \cup \{ij \mid i \in N^G, j \in N^B\}$. In $g_{\text{assi,green}}$, a green individual obtains $(n-1)(\delta - c)$ as utility, while a blue obtains $(n^G)(\delta - c) + (n^B - 1)\delta^2$ as utility. In Figure 2 we depict $g_{\text{assi,green}}$ for

[10] In the definition of the rate of exposure (see the expression (1)), we subtract 1 in the denominator because, when computing the cost of a given bridge link between communities, this bridge link is not included in the computation of the cost. What is relevant for the cost is the rate of exposure according to the rest of the connections of each of the two individuals involved in the bridge link.

$N^G = \{1, 2, 3, 4\}$ and $N^B = \{5, 6\}$. Similarly, let $g_{\text{assi,blue}}$ denote the network where all members of the green community are fully assimilated to the smaller blue community. That is, each blue individual is linked to all other (green and blue) individuals while each green individual is only linked to all blue individuals. Formally, $g_{\text{assi,blue}} = g^{N^B} \cup \{ij \mid i \in N^B, j \in N^G\}$. In $g_{\text{assi,blue}}$, a blue individual obtains $(n-1)(\delta - c)$ as utility, while a green obtains $(n^B)(\delta - c) + (n^G - 1)\delta^2$ as utility. In Figure 3 we depict $g_{\text{assi,blue}}$ for $N^G = \{1, 2, 3, 4\}$ and $N^B = \{5, 6\}$. Let g_{int} denote the complete integration network where both communities are fully intra-connected and fully inter-connected: $g_{\text{int}} = g^N$ and is depicted in Figure 4. In g_{int}, a green individual and a blue individual obtain, respectively,

$$(n-1)(\delta - c) - n^B \frac{n^G - 1}{n-2} \frac{n^B - 1}{n-2} C \text{ and } (n-1)(\delta - c) - n^G \frac{n^B - 1}{n-2} \frac{n^G - 1}{n-2} C$$

as utility. Let g_{seg} denote the complete segregation network where both communities are fully intra-connected but isolated of each other: $g_{\text{seg}} = g^{N^G} \cup g^{N^B}$ and is depicted in Figure 5. In g_{seg}, a green individual obtains $(n^G - 1)(\delta - c)$ as utility, while a blue obtains $(n^B - 1)(\delta - c)$ as utility.

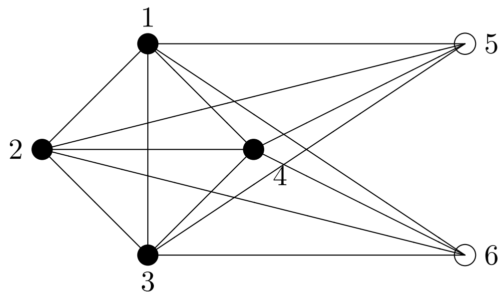

Figure 2: The blue community is fully assimilated within the green community.

de Marti and Zenou (2017) adopt the notion of pairwise stability, introduced by Jackson and Wolinsky (1996), to study the networks that will be formed at equilibrium. A network is pairwise stable if no individual benefits from deleting a link and no two individuals benefit from adding a link between them. Formally, a network $g \in \mathcal{G}$ is pairwise stable if (i) for all $ij \in g$, $U_i(g) \geq U_i(g - ij)$ and $U_j(g) \geq U_j(g - ij)$, (ii) for all $ij \notin g$, if $U_i(g) < U_i(g + ij)$ then $U_j(g) > U_j(g + ij)$. Pairwise stability presumes that individuals are myopic: they do not anticipate that other individuals may react to their changes. Denote $\Delta \equiv \delta - \delta^2 - c$. Many different network configurations can be pairwise stable depending on the exact intra- and inter-community costs. Beside the networks g_{int}, g_{seg}, $g_{\text{assi,green}}$ and $g_{\text{assi,blue}}$ that can be pairwise stable, some asymmetric network configurations can also be pairwise stable: (i) the network in which both communities are fully intra-connected and where there is only one bridge link (see Figure 1), (ii) the network

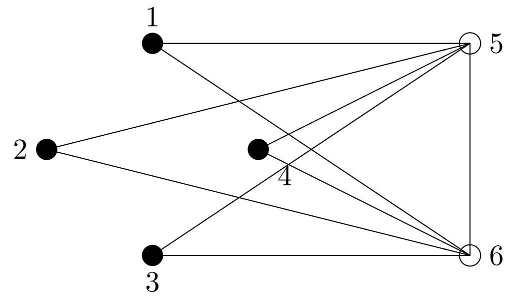

Figure 3: The green community is fully assimilated within the blue community.

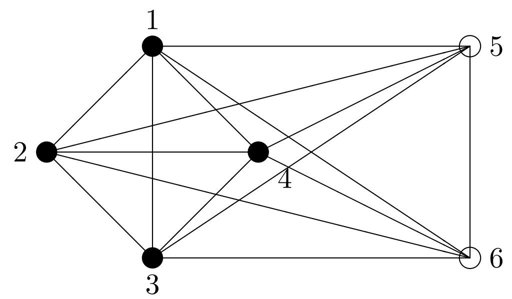

Figure 4: Both communities are fully integrated.

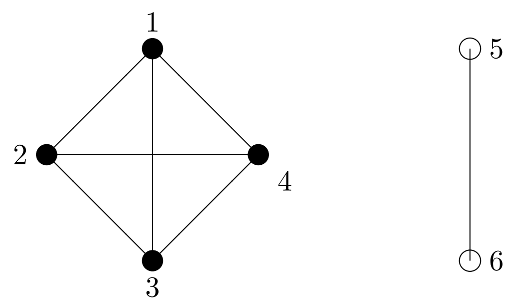

Figure 5: Both communities are segregated.

in which both communities are fully intra-connected, where each blue individual has one and only one bridge link, and where each green individual has at most one bridge link, (iii) the network in which both communities are fully intra-connected and with a unique blue individual connected with all green individuals, and (iv) the network in which both communities are fully intra-connected and in which one green individual is linked to all blue individuals.

We next allow the population to include not only myopic individuals but also farsighted ones. Farsighted individuals are able to anticipate that once they add or delete some links, other individuals could add or delete links afterwards.

3 Myopic-farsighted stable sets

We adopt the notion of myopic-farsighted stable set introduced by Herings, Mauleon and Vannetelbosch (2020) for two-sided matching problems and by Luo, Mauleon and Vannetelbosch (2021) for network formation games to determine the networks that are stable when some individuals are myopic while others are farsighted.[11] A set of networks G is said to be a myopic-farsighted stable set if it satisfies the following two types of stability. Internal stability: No network in G is dominated by any other network in G. External stability: Every network not in G is dominated by some network in G. A network g' is said to be dominated by a network g if there is a myopic-farsighted improving path from g' to g.

A myopic-farsighted improving path is a sequence of distinct networks that can emerge when farsighted individuals form or delete links based on the improvement the end network offers relative to the current network while myopic individuals form or delete links based on the improvement the resulting network offers relative to the current network. Since we only allow for pairwise deviations, each network in the sequence differs from the previous one in that either a new link is formed between two individuals or an existing link is deleted. If a link is deleted, then it must be that either a myopic individual prefers the resulting network to the current network or a farsighted individual prefers the end network to the current network. If a link is added between some myopic individual i and some farsighted individual j, then the myopic individual i must prefer the resulting network to the current network and the farsighted individual j must prefer the end network to the current network.[12]

[11]See Chwe (1994), Herings, Mauleon and Vannetelbosch (2009), Mauleon, Vannetelbosch and Vergote (2011), Ray and Vohra (2015, 2019), Roketskiy (2018) for definitions of the farsighted stable set when individuals are farsighted. Alternative notions of farsightedness for network formation are suggested by Dutta, Ghosal and Ray (2005), Dutta and Vohra (2017), Herings, Mauleon and Vannetelbosch (2004, 2019), Page, Wooders and Kamat (2005), Page and Wooders (2009) among others.

[12]Along a myopic-farsighted improving path, myopic players do not care whether other players are myopic or farsighted. They behave as if all players are myopic and they compare their resulting network's payoff to their current network's payoff for taking a decision. However, farsighted players know exactly who is farsighted and who is myopic and they compare their end network's payoff to their current network's

Definition 2. A myopic-farsighted improving path from a network g to a network g' is a finite sequence of distinct networks $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either

(i) $g_{k+1} = g_k - ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $i \in N^M$ or $U_j(g_K) > U_j(g_k)$ and $j \in N^F$; or

(ii) $g_{k+1} = g_k + ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $U_j(g_{k+1}) \geq U_j(g_k)$ if $i, j \in N^M$, or $U_i(g_K) > U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ if $i, j \in N^F$, or $U_i(g_{k+1}) \geq U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ (with one inequality holding strictly) if $i \in N^M, j \in N^F$.

If there exists a myopic-farsighted improving path from a network g to a network g', then we write $g \to g'$. The set of all networks that can be reached from a network $g \in \mathcal{G}$ by a myopic-farsighted improving path is denoted by $\phi(g)$, $\phi(g) = \{g' \in \mathcal{G} \mid g \to g'\}$. When all individuals are myopic, our notion of myopic-farsighted improving path reverts to Jackson and Watts (2002) notion of improving path. When all individuals are farsighted, our notion of myopic-farsighted improving path reverts to Jackson (2008) and Herings, Mauleon and Vannetelbosch (2009) notion of farsighted improving path. A set of networks G is a myopic-farsighted stable set if the following two conditions hold. Internal stability: for any two networks g and g' in the myopic-farsighted stable set G there is no myopic-farsighted improving path from g to g' (and vice versa). External stability: for every network g outside the myopic-farsighted stable set G there is a myopic-farsighted improving path leading to some network g' in the myopic-farsighted stable set G (i.e. there is $g' \in G$ such that $g \to g'$).

Definition 3. A set of networks $G \subseteq \mathcal{G}$ is a myopic-farsighted stable set if: **(IS)** for every $g, g' \in G$, it holds that $g' \notin \phi(g)$; and **(ES)** for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$.

When all individuals are farsighted, the myopic-farsighted stable set is simply the farsighted stable set as defined in Herings, Mauleon and Vannetelbosch (2009) or Ray and Vohra (2015). When all individuals are myopic, the myopic-farsighted stable set boils down to the pairwise CP vNM set as defined in Herings, Mauleon, and Vannetelbosch (2017) for two-sided matching problems.[13] Luo, Mauleon and Vannetelbosch (2021) characterize the myopic-farsighted stable set when all individuals are myopic (i.e. $N = N^M$): a set of networks is a myopic-farsighted stable set if and only if it consists of all pairwise stable networks and one network from each closed cycle.

4 Low intra-community costs

When all individuals are myopic each myopic-farsighted stable set contains all pairwise networks. Hence, many inefficient friendship networks can emerge in the long run when

payoff for taking a decision.

[13]The pairwise CP vNM set follows the approach by Page and Wooders (2009) who define the stable set with respect to path dominance, i.e. the transitive closure of ϕ.

both communities are composed of only myopic individuals. We now investigate what happens when the population is mixed in terms of their degree of farsightedness.

We next consider the following three cases: (1) all individuals in both communities are mixed; (2) all individuals in the large green community are farsighted; (3) all individuals in the small blue community are farsighted.

4.1 Farsighted and myopic agents in both communities

Suppose first that some individuals are farsighted while others are myopic in both communities. We show that if there are enough farsighted individuals in the dominant green community relatively to the size of the small blue community, then a friendship network where individuals of the small blue community are fully assimilated into the large green community is likely to emerge in the long run.

Let $\widehat{C}$ be the lower bound on the inter-community cost parameter C such that, if there are enough farsighted individuals in the green community, $\{g_{\text{assi,green}}\}$ is a myopic-farsighted stable set whatever the number of farsighted or myopic individuals within the blue community. Formally,

$$\widehat{C} = \Delta \frac{(n^{F,G} + n^{M,B} - 2)^2 (n^{F,G} + n^{M,B} - 3)}{n^{F,G}(n^{F,G} - 1)^2} \cdot \min\{1, n^{M,B}\}.$$

Precisely, it is the lower bound on C such that a myopic blue individual has incentives to cut a link with another myopic blue individual in the complete component between farsighted green individuals and myopic blue individuals, i.e. $g^{(N^F \cap N^G) \cup (N^M \cap N^B)}$.

Let $\underline{n}^G$ be given by

$$\underline{n}^G = (n^B - 1)\frac{(\delta - \delta^2 - c)}{(\delta - c)}.$$

It is the lower bound on the number of farsighted green individuals such that a farsighted blue individual prefers being fully assimilated into the green community than being unconnected from farsighted green individuals. That is, for $i \in N^{F,B}$, $U_i(g_{\text{seg}}) < U_i(g_{\text{assi,green}})$ if and only if $n^{F,G} > \underline{n}^G$. Notice that $\underline{n}^G < n^B \leq n^G$.

Lemma 1 shows that if the number of farsighted agents in the green community is large enough ($n^{F,G} > \underline{n}^G$) and inter-community costs are large enough ($C > \widehat{C}$), then there always exists a myopic-farsighted improving path emanating from any network $g \neq g_{\text{assi,green}}$ leading to $g_{\text{assi,green}}$ where blue individuals are fully assimilated into the large green community.

Lemma 1. *Assume low intra-community costs, $c < \delta - \delta^2$, and inter-community costs, $C > \widehat{C}$. If $n^{F,G} > \underline{n}^G$, then $\phi(g) \cap \{g_{assi,green}\} \neq \emptyset$ for all $g \neq g_{assi,green}$ where $g_{assi,green} = g^{N^G} \cup \{ij \mid i \in N^G, j \in N^B\}$.*

Proof. Take any network $g \neq g_{\text{assi,green}}$. We build in steps a myopic-farsighted improving path from g to $g_{\text{assi,green}}$.

Step 0: If g is such that blue individuals have links among themselves, i.e., $g \cap g^{N^B} \neq \emptyset$ then go to Step 1. Otherwise, starting from g, green individuals first build all the missing links between green individuals to reach $g' = g \cup g^{N^G}$. Since $c < \delta - \delta^2$ and $g' \cap g^{N^B} = \emptyset$, green individuals are not affected by C and so myopic or farsighted green individuals have incentives to add the links with the other green individuals. Farsighted green individuals look forward to $g_{\text{assi,green}}$, where they obtain their highest possible payoff given $c < \delta - \delta^2$, $U_i(g_{\text{assi,green}}) = (n-1)(\delta - c)$. From g' green individuals build all the missing links with blue individuals to finally reach $g'' = g' \cup \{ij \mid i \in N^G, j \in N^B\} = g_{\text{assi,green}}$. Since $c < \delta - \delta^2$ and $g'' \cap g^{N^B} = \emptyset$, blue and green individuals are not affected by C and so myopic or farsighted blue individuals have incentives to add the links with myopic or farsighted green individuals (and vice versa).

Step 1: Starting in g, farsighted green individuals delete successively all the links (if any) they have with green and blue individuals looking forward to $g_{\text{assi,green}}$, where they obtain their highest possible payoff given $c < \delta - \delta^2$ and $C > \widehat{C}$. We reach the network $g' = g \cap g^{N^B \cup (N^G \cap N^M)}$ where all the links involving farsighted green individuals in g have been deleted. Thus, $g' \subseteq g^{N^B \cup (N^G \cap N^M)}$ and go to Step 2.

Step 2: From $g' = g \cap g^{N^B \cup (N^G \cap N^M)}$, since $n^G \geq n^B$ and $n^{F,G} > \underline{n}^G$, blue individuals who are farsighted (if any) prefer $g_{\text{assi,green}}$ to g'. Indeed, if $n^{F,G} > \underline{n}^G$ or

$$n^{F,G} > (n^B - 1)\frac{(\delta - \delta^2 - c)}{(\delta - c)},$$

then farsighted blue individuals prefer the end network $g_{\text{assi,green}}$ where they get $(n^G)(\delta - c) + \delta^2(n^B - 1)$ to the current network where they obtain at most $(n^B - 1 + n^{M,G})(\delta - c)$. So, the $n^{F,B}$ farsighted blue individuals are ready to delete all their links looking forward to $g_{\text{assi,green}}$. We reach the network $g'' \subseteq g^{N^M}$. If $g'' = g^{\emptyset}$ is the empty network then go to Step 3. Otherwise, go to Step 4.

Step 3: From the empty network $g^{\emptyset}$ green individuals and blue individuals build all the links in $g^{N^G} \cup \{ij \mid i \in N^G, j \in N^B\}$ to finally reach the network $g_{\text{assi,green}}$. Since along the myopic-farsighted improving blue individuals have no links to other blue individuals, the payoffs of both green and blue individuals are not affected by C. So, each time a farsighted green or blue individual adds a link she prefers the end network $g_{\text{assi,green}}$ to the current network and each time a myopic green or blue individual adds a link she prefers the resulting network to the current network. Hence, $\phi(g) \cap \{g_{\text{assi,green}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,green}}$.

Step 4: From $g'' \subseteq g^{N^M}$, since $c < \delta - \delta^2$, myopic blue individuals have incentives to build all the links with the farsighted green individuals.[14] Farsighted green individuals

[14]Since farsighted green individuals are not linked to other green individuals, the cost for a myopic blue individual to link to such farsighted green individual is equal to c. In addition, the inter-community costs she may incur for her links to myopic green individuals decrease because she is becoming relatively less exposed to her own blue community.

are looking forward $g_{\text{assi,green}}$ and prefer the end network to the current one. We reach the network $g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\}$ where all possible links between myopic blue individuals and farsighted green individuals are formed. Thus, we reach the network where all farsighted green individuals are assimilated to the community of myopic blue individuals (but the community of myopic blue individuals is not necessarily fully intra-connected). Notice that farsighted blue individuals remain without links and farsighted green individuals prefer $g_{\text{assi,green}}$ to $g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\}$. Next go to Step 5.

Step 5: From $g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\}$, farsighted green individuals who look forward towards $g_{\text{assi,green}}$ build all the links between them to reach $g^{N^F \cap N^G} \cup g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\}$. Next go to Step 6.

Step 6: From $g^{N^F \cap N^G} \cup g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\}$, since $C > \widehat{C}$, myopic blue individuals have incentives to delete successively all the links they have with other myopic blue individuals to finally reach the network $g^{N^F \cap N^G} \cup g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\} \setminus \{ij \mid i,j \in N^M \cap N^B\}$. The condition $C > \widehat{C}$ guarantees that, along the myopic-farsighted improving path starting at $g_1 = g^{N^F \cap N^G} \cup g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\}$, followed by $g_{k+1} = g_k - ij$ with $ij \in g_k$ and $i,j \in N^M \cap N^B$ for $k \geq 1$, and ending at $g_K = g^{N^F \cap N^G} \cup g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\} \setminus \{ij \mid i,j \in N^M \cap N^B\}$, all myopic blue individuals have incentives to delete their links with other myopic blue individuals. Indeed, consider a sequence starting at $g_1 = g^{N^F \cap N^G} \cup g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\}$, followed by $g_{k+1} = g_k - ij$ with $i \in N^M \cap N^B, j \in N_i(g_k) \cap N^M \cap N^B$, for $k = 1, ...n^{M,B} - 1$. Along this sequence, a myopic blue individual i successively deletes all her links with the other myopic blue individuals and she has incentives to cut her kth link to some myopic blue individual if and only if

$$C > \Delta \frac{(n^{F,G} + n^{M,B} - 2)(n^{F,G} + n^{M,B} - 1 - k)(n^{F,G} + n^{M,B} - 2 - k)}{n^{F,G}(n^{F,G} - 1)^2}.$$

This condition is satisfied since $C > \widehat{C}$ and

$$\begin{aligned}\widehat{C} &= \Delta \frac{(n^{F,G} + n^{M,B} - 2)^2 (n^{F,G} + n^{M,B} - 3)}{n^{F,G}(n^{F,G} - 1)^2} \cdot \min\{1, n^{M,B}\} \\ &\geq \Delta \frac{(n^{F,G} + n^{M,B} - 2)(n^{F,G} + n^{M,B} - 1 - k)(n^{F,G} + n^{M,B} - 2 - k)}{n^{F,G}(n^{F,G} - 1)^2}.\end{aligned}$$

when $n^{M,B} \neq 0$. Next, from $g^{N^F \cap N^G} \cup g'' \cup \{ij \mid i \in N^F \cap N^G, j \in N^M \cap N^B\} \setminus \{ij \mid i,j \in N^M \cap N^B\}$, farsighted and myopic green individuals build all the missing links between them. Since myopic blue individuals are not linked to each other and farsighted blue individuals have no link, farsighted and myopic green individuals are not affected by C and so myopic or farsighted green individuals have incentives to add the links between them. Next, farsighted and myopic blue individuals build the missing links with all green individuals to reach the network $g_{\text{assi,green}}$ where all blue individuals are fully assimilated into the green community. Thus, $\phi(g) \cap \{g_{\text{assi,green}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,green}}$. □

In the proof of Lemma 1, we construct a myopic-farsighted improving path leading to $g_{\text{assi,green}}$ along which farsighted green individuals first delete all their links to push farsighted blue individuals in a situation where they are worst off compared to what they obtain when they are fully assimilated into the green community. Next, myopic blue individuals link to each other and to all farsighted green individuals. In fact, farsighted green individuals are luring myopic blue individuals with the prospect of a friendship network where green individuals are assimilated into the community of blue myopic individuals. Next, farsighted green individuals who are looking forward towards $g_{\text{assi,green}}$ build all the links between them to reach a network where farsighted green individuals and myopic blue individuals are integrated to each other. Since inter-community costs are large enough, $C > \widehat{C}$, myopic blue individuals have now incentives to delete all the links between them. Finally, farsighted blue individuals build the missing links with farsighted green individuals to form $g_{\text{assi,green}}$.

Proposition 1 shows that if the number of farsighted agents in the green community is large enough ($n^{F,G} > \underline{n}^G$) and inter-community costs are large enough ($C > \widehat{C}$), then the set $G = \{g_{\text{assi,green}}\}$ is a myopic-farsighted stable set.

Proposition 1. *Assume low intra-community costs, $c < \delta - \delta^2$, and inter-community costs, $C > \widehat{C}$. If $n^{F,G} > \underline{n}^G$, then the set $G = \{g_{assi,green}\}$, where $g_{assi,green} = g^{N^G} \cup \{ij \mid i \in N^G, j \in N^B\}$, is a myopic-farsighted stable set.*

Proof. The set $G = \{g_{\text{assi,green}}\}$ satisfies (**IS**) in Definition 3 since it is a singleton set. From Lemma 1 we have that $\phi(g) \cap \{g_{\text{assi,green}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,green}}$. Hence, $G = \{g_{\text{assi,green}}\}$ satisfies (**ES**). □

Remark that if there are more farsighted green individuals than myopic and farsighted blue individuals, then the condition $n^{F,G} > \underline{n}^G$ holds, and it becomes likely that the friendship network where blue individuals are assimilated into the dominant green community will emerge in the long run.

Suppose now that all blue individuals are myopic ($N^B \cap N^F = \emptyset$ and $n^{M,B} = n^B$). The next corollary shows that if there are enough farsighted individuals in the dominant group (green community) while all individuals in the other group (blue community) are myopic and $C > \widehat{C}$, then the friendship network where the blue individuals end up assimilated into the dominant green community will emerge for sure in the long run since $\{g_{\text{assi,green}}\}$ is the unique myopic-farsighted stable set.[15]

Corollary 1. *Assume low intra-community costs, $c < \delta - \delta^2$, and inter-community costs, $C > \widehat{C}$. If $n^{F,G} > \underline{n}^G$ and $N^M = N^B$, then the set $G = \{g_{assi,green}\}$, where $g_{assi,green} = g^{N^G} \cup \{ij \mid i \in N^G, j \in N^B\}$, is the unique myopic-farsighted stable set.*

[15]When $N^G = N^F$ and $N^B = N^M$, $\widehat{C}$ is equal to $\overline{C}_1$, where $\overline{C}_1$ is the lower bound on C such that a myopic blue individual has incentives to cut a link with another blue individual in the complete integrated network, and it is given by $\overline{C}_1 = \frac{(n-2)^2(n-3)}{n^G(n^G-1)^2}\Delta$.

Proof. From Proposition 1 we have that $G = \{g_{\text{assi,green}}\}$ both satisfies (**IS**) and (**ES**). Farsighted and myopic green individuals obtain their highest possible payoff in $g_{\text{assi,green}}$ and myopic blue individuals have no incentive to delete any link nor to add a new link since $C > \widehat{C}$ and $c < \delta - \delta^2$. Since $N^M = N^B$ it follows that $\phi(g_{\text{assi,green}}) = \emptyset$. So, since $\phi(g) \cap \{g_{\text{assi,green}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,green}}$ and $\phi(g_{\text{assi,green}}) = \emptyset$, the set $G = \{g_{\text{assi,green}}\}$ is the unique myopic-farsighted stable set (any other set would violate (**IS**) and/or (**ES**)). □

4.2 The larger community is farsighted

Suppose now that all individuals in the large green community are farsighted, $N^{F,G} = N^G$. First, notice that the condition in Lemma 1 and Proposition 1 on the number of farsighted individuals in the green community is trivially satisfied. Second, when all individuals in the small blue community are also farsighted (i.e. $N^{F,G} = N^G$ and $N^{F,B} = N^B$), the set $\{g_{\text{assi,green}}\}$ becomes a myopic-farsighted stable set for any inter-community costs $C > 0$.

Remark 1. *Assume low intra-community costs, $c < \delta - \delta^2$, and all individuals are farsighted, $N^F = N$. Then, $n^{M,B} = 0$, $\widehat{C} = 0$ and from Lemma 1 and Proposition 1 we have that the set $G = \{g_{\textit{assi,green}}\}$, where $g_{\textit{assi,green}} = g^{N^G} \cup \{ij \mid i \in N^G, j \in N^B\}$, is a myopic-farsighted stable set for any $C > 0$.*

4.3 The smaller community is farsighted

Suppose now that all individuals in the small blue community are farsighted. We first show that if inter-community costs are large enough and the blue community is not too small relatively to the green community, then a friendship network where individuals of the large green community are fully assimilated into the small blue community could emerge in the long run.

Let $\widetilde{C}$ be the lower bound on the inter-community cost parameter C such that, in the case of low intra-community costs, $g_{\text{assi,blue}}$ is a myopic-farsighted stable set whatever the number of farsighted or myopic individuals within the green community. Formally,

$$\widetilde{C} = \Delta \frac{(n^{F,B} + n^{M,G} - 2)^2 (n^{F,B} + n^{M,G} - 3)}{n^{F,B}(n^{F,B} - 1)^2} \cdot \min\{1, n^{M,G}\}.$$

It is the lower bound on C such that a myopic green individual has incentives to cut a link with another myopic green individual in the complete component between farsighted blue individuals and myopic green individuals, i.e. $g^{(N^F \cap N^B) \cup (N^M \cap N^G)}$.

Let $\underline{n}^B$ be given by

$$\underline{n}^B = (n^G - 1) \frac{(\delta - \delta^2 - c)}{(\delta - c)} \cdot \min\{1, n^{F,G}\}.$$

It is the lower bound on the size of the blue community such that a farsighted green individual prefers being fully assimilated into the blue community than being segregated from it. That is, for $i \in N^{F,G}$, $U_i(g_{\text{seg}}) < U_i(g_{\text{assi,blue}})$ if and only if $n^B > \underline{n}^B$.

Lemma 2 shows that if there are enough individuals in the blue community relatively to the green one ($n^B > \underline{n}^B$) and inter-community costs are large enough ($C > \widetilde{C}$), then there always exists a myopic-farsighted improving path emanating from any network $g \neq g_{\text{assi,blue}}$ leading to $g_{\text{assi,blue}}$ where green individuals are fully assimilated into the small blue community.

Lemma 2. *Assume low intra-community costs, $c < \delta - \delta^2$, and inter-community costs, $C > \widetilde{C}$. Assume all individuals in the blue community are farsighted, $N^B \subseteq N^F$. If $n^B > \underline{n}^B$, then $\phi(g) \cap \{g_{assi,blue}\} \neq \emptyset$ for all $g \neq g_{assi,blue}$ where $g_{assi,blue} = g^{N^B} \cup \{ij \mid i \in N^G, j \in N^B\}$.*

Proof. Take any network $g \neq g_{\text{assi,blue}}$. We build in steps a myopic-farsighted improving path from g to $g_{\text{assi,blue}}$. Notice that $N^B \subseteq N^F$ and so $n^B = n^{F,B}$.

Step 0: If g is such that green individuals have links among themselves, i.e., $g \cap g^{N^G} \neq \emptyset$ then go to Step 1. Otherwise, starting from g, blue individuals first build all the missing links between blue individuals to reach $g' = g \cup g^{N^B}$ looking forward to $g_{\text{assi,blue}}$, where they obtain their highest possible payoff given $c < \delta - \delta^2$, $U_i(g_{\text{assi,blue}}) = (n-1)(\delta - c)$. From g' blue individuals build all the missing links with green individuals to finally reach $g'' = g' \cup \{ij \mid i \in N^G, j \in N^B\} = g_{\text{assi,blue}}$. Since $c < \delta - \delta^2$ and $g'' \cap g^{N^G} = \emptyset$, green individuals are assimilated to the blue community in g'' and they are not affected by C and so myopic or farsighted green individuals have incentives to add the links with the blue individuals.

Step 1: Starting in g, blue individuals who are all farsighted ($N^B \subseteq N^F$) delete successively all the links (if any) they have with green and blue individuals looking forward to $g_{\text{assi,blue}}$, where they obtain their highest possible payoff given $c < \delta - \delta^2$ and $C > \widetilde{C}$, $U_i(g_{\text{assi,blue}}) = (n-1)(\delta - c)$. We reach the network $g' = g \cap g^{N^G}$ where all the links involving blue individuals in g have been deleted. Thus, $g' \subseteq g^{N^G}$ and go to Step 2.

Step 2: From $g' = g \cap g^{N^G}$, since $n^B > \underline{n}^B$, all green individuals who are farsighted (if any) prefer $g_{\text{assi,blue}}$ to g' and so farsighted green individuals (if any) are ready to delete all their links looking forward to $g_{\text{assi,blue}}$. We reach the network $g'' \subseteq g^{N^M \cap N^G}$. If $g'' = g^{\emptyset}$ is the empty network then go to Step 3. Otherwise, go to Step 4.

Step 3: From the empty network $g^{\emptyset}$ green individuals and blue individuals build all the links in $g^{N^B} \cup \{ij \mid i \in N^G, j \in N^B\}$ to finally reach the network $g_{\text{assi,blue}}$. Since along the myopic-farsighted improving green individuals have no links to other green individuals, the payoffs of both green and blue individuals are not affected by C. So, each time a farsighted green or blue individual adds a link she prefers the end network $g_{\text{assi,blue}}$ to the current network and each time a myopic green individual adds a link she prefers the resulting network to the current network. Hence, $\phi(g) \cap \{g_{\text{assi,blue}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,blue}}$ and $G = \{g_{\text{assi,blue}}\}$ satisfies (**ES**).

Step 4: From $g'' \subseteq g^{N^M \cap N^G}$, since $c < \delta - \delta^2$, myopic green individuals have incentives to build all the missing links with other myopic green individuals. We reach the network $g^{N^M \cap N^G}$. From $g^{N^M \cap N^G}$, since $c < \delta - \delta^2$, myopic green individuals have incentives to build all the links with the blue individuals. Blue individuals who are all farsighted and are looking forward $g_{\text{assi,blue}}$ prefer the end network to the current one. We reach the network $g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$ where all possible links between myopic green individuals and all blue individuals are formed. Thus, we reach the network where all the blue individuals are assimilated to the community of myopic green individuals and the community of myopic green individuals is fully intra-connected. Notice that all farsighted green individuals (if any) remain without links and all blue individuals prefer $g_{\text{assi,blue}}$ to $g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$. Next go to Step 5.
Step 5: From $g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$, blue individuals who are all farsighted and look forward towards $g_{\text{assi,blue}}$ build all the links between the blue individuals to reach $g^{N^B} \cup g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$. Notice that if $N^G = N^M$ then $g^{N^B} \cup g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$ is simply the complete network g^N. Next go to Step 6.
Step 6: From $g^{N^B} \cup g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$ since $C > \widetilde{C}$, myopic green individuals have incentives to delete successively all the links they have with other myopic green individuals to finally reach the network $g^{N^B} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$. The condition $C > \widetilde{C}$ guarantees that, along the myopic-farsighted improving path starting at $g_1 = g^{N^B} \cup g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$, followed by $g_{k+1} = g_k - ij$ with $ij \in g_k$ and $i, j \in N^M \cap N^G$ for $k \geq 1$, and ending at $g_K = g^{N^B} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$, all the myopic green individuals have incentives to delete their links with other myopic green individuals. Indeed, consider a sequence starting at $g_1 = g^{N^B} \cup g^{N^M \cap N^G} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$, followed by $g_{k+1} = g_k - ij$ with $i \in N^M \cap N^G, j \in N_i(g_k) \cap N^M \cap N^G$, for $k = 1, ...n^{M,G} - 1$. Along this sequence, a myopic green individual i successively deletes all her links with the other myopic green individuals and she has incentives to cut her kth link to some myopic green individual if and only if

$$C > \Delta \frac{(n^{F,B} + n^{M,G} - 2)(n^{F,B} + n^{M,G} - 1 - k)(n^{F,B} + n^{M,G} - 2 - k)}{n^{F,B}(n^{F,B} - 1)^2}.$$

This condition is satisfied since $C > \widetilde{C}$ and

$$\begin{aligned}\widetilde{C} &= \Delta \frac{(n^{F,B} + n^{M,G} - 2)^2(n^{F,B} + n^{M,G} - 3)}{n^{F,B}(n^{F,B} - 1)^2} \cdot \min\{1, n^{M,G}\} \\ &\geq \Delta \frac{(n^{F,B} + n^{M,G} - 2)(n^{F,B} + n^{M,G} - 1 - k)(n^{F,B} + n^{M,G} - 2 - k)}{n^{F,B}(n^{F,B} - 1)^2},\end{aligned}$$

when $n^{M,G} \neq 0$. Next, from $g^{N^B} \cup \{ij \mid i \in N^B, j \in N^M \cap N^G\}$, farsighted green individuals build the missing links $\{ij \mid i \in N^B, j \in N^F \cap N^G\}$ with all blue individuals to reach the network $g_{\text{assi,blue}}$ where all green individuals are fully assimilated into the blue community. Thus, $\phi(g) \cap \{g_{\text{assi,blue}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,blue}}$. □

Proposition 2. *Assume low intra-community costs, $c < \delta - \delta^2$, and inter-community costs, $C > \widetilde{C}$. Assume all individuals in the blue community are farsighted, $N^B \subseteq N^F$. If $n^B > \underline{n}^B$, then the set $G = \{g_{assi,blue}\}$, where $g_{assi,blue} = g^{N^B} \cup \{ij \mid i \in N^G, j \in N^B\}$, is a myopic-farsighted stable set.*

Proof. The set $G = \{g_{\text{assi,blue}}\}$ satisfies (**IS**) in Definition 3 since it is a singleton set. From Lemma 2 we have that $\phi(g) \cap \{g_{\text{assi,blue}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,blue}}$. Hence, $G = \{g_{\text{assi,blue}}\}$ satisfies (**ES**). □

Proposition 2 shows that the set $\{g_{\text{assi,blue}}\}$ is a myopic-farsighted stable set since from any other network there is a myopic-farsighted improving path leading to $g_{\text{assi,blue}}$. In the proof of Lemma 2 we construct such myopic-farsighted improving path. This myopic-farsighted improving path is similar to the one for Lemma 1 by switching blue individuals for green ones and vice versa. The major difference is that now the blue community has to be large enough relatively to the green community ($n^B > \underline{n}^B$) to ensure that farsighted green individuals are worst off once all blue individuals delete their links compared to what they obtain when they are fully assimilated into the blue community.

Suppose now that all green individuals are myopic ($N^G \cap N^F = \emptyset$ and $n^{M,G} = n^G$). Then, $\widetilde{C}$ is equal to $\overline{C}_2$, where $\overline{C}_2$ is the lower bound on C such that a myopic green individual has incentives to cut a link with another green individual in the complete integrated network, and it is given by

$$\overline{C}_2 = \frac{(n-2)^2(n-3)}{n^B(n^B-1)^2}\Delta.$$

Thus, if $C > \overline{C}_2$, each myopic green individual has an incentive to delete some link to another green individual in the complete integrated network g_{int}, i.e. g^N.

The next corollary shows that if the dominant group (green community) is myopic while the other group (blue community) is farsighted and $C > \overline{C}_2$, then the friendship network where the green individuals end up assimilated into the small blue community will emerge for sure in the long run since $\{g_{\text{assi,blue}}\}$ is the unique myopic-farsighted stable set.

Corollary 2. *Assume low intra-community costs, $c < \delta - \delta^2$, and inter-community costs, $C > \overline{C}_2$. Assume all individuals in the green community are myopic, $N^M = N^G$, and all individuals in the blue community are farsighted, $N^F = N^B$. Then, the set $G = \{g_{assi,blue}\}$, where $g_{assi,blue} = g^{N^B} \cup \{ij \mid i \in N^G, j \in N^B\}$, is the unique myopic-farsighted stable set.*

Proof. From Proposition 2 we have that $G = \{g_{\text{assi,blue}}\}$ both satisfies (**IS**) and (**ES**). Farsighted blue individuals obtain their highest possible payoff in $g_{\text{assi,blue}}$ and myopic green individuals have no incentive to delete any link nor to add a new link since $C > \overline{C}_2$ and $c < \delta - \delta^2$. Since $N^M = N^G$ it follows that $\phi(g_{\text{assi,blue}}) = \emptyset$. So, since $\phi(g) \cap \{g_{\text{assi,blue}}\} \neq \emptyset$ for all $g \neq g_{\text{assi,blue}}$ and $\phi(g_{\text{assi,blue}}) = \emptyset$, the set $G = \{g_{\text{assi,blue}}\}$ is the unique myopic-farsighted stable set (any other set would violate (**IS**) and/or (**ES**)).

□

Suppose now that both communities are farsighted ($N = N^F$) and the blue community is not too small relatively to the green one (i.e. $n^B > \underline{n}^B$). Then, the set $\{g_{\text{assi,blue}}\}$ becomes a myopic-farsighted stable set for any inter-community costs $C > 0$.

Remark 2. *Assume low intra-community costs, $c < \delta - \delta^2$, all individuals are farsighted, $N^F = N$, and $n^B > \underline{n}^B$. Then, $n^{M,B} = 0$, $\widetilde{C} = 0$ and from Lemma 2 and Proposition 2 we have that the set $G = \{g_{assi,blue}\}$, where $g_{assi,blue} = g^{N^B} \cup \{ij \mid i \in N^G, j \in N^B\}$, is a myopic-farsighted stable set for any $C > 0$.*

However, if both communities are fully farsighted and the blue community is relatively small (i.e. $n^B \leq \underline{n}^B$), then the set $\{g_{\text{assi,blue}}\}$ is never a myopic-farsighted stable set because $\phi(g_{\text{seg}}) \cap \{g_{\text{assi,blue}}\} = \emptyset$. Moreover, the set $\{g_{\text{seg}}\}$ is never a myopic-farsighted stable set because $\phi(g_{\text{assi,green}}) \cap \{g_{\text{seg}}\} = \emptyset$. Thus, the complete segregation network g_{seg} and the network $g_{\text{assi,blue}}$ in which all green individuals are fully assimilated into the smaller blue community are unlikely to emerge in the long run when the whole population is farsighted.

Remark 3. *Assume low intra-community costs, $c < \delta - \delta^2$, and inter-community costs, $C > 0$. Assume all individuals are farsighted, $N^F = N$.*

(i) *The set $G = \{g_{assi,blue}\}$ is never a myopic-farsighted stable set if $n^B \leq \underline{n}^B$.*

(ii) *The set $G = \{g_{seg}\}$ is never a myopic-farsighted stable set.*

5 Intermediate intra-community costs

We now consider situations where intra-community costs are intermediate, i.e. $\delta - \delta^2 < c < \delta$. So, it becomes more expensive to build links with individuals from the same community. We denote by g^{*i} the star network where individual i is the center of the star. We next show that, if the whole population is mixed, independently of the distribution of myopic and farsighted individuals in the two communities, then a star network encompassing all individuals from both communities with some myopic individual in the center is going to emerge in the long run.

Let $\gamma \in [0, 1]$ be given by

$$\gamma = \min \left[\frac{n^B - 1}{n^{F,G} + n^B - 2}, \frac{n^G - 1}{n^{F,B} + n^G - 2} \right] \text{ if } n^{F,G} \neq 0 \text{ and } n^{F,B} \neq 0$$

and $\gamma = 1$ otherwise. Notice that $(n^B - 1) \cdot (n^{F,G} + n^B - 2)^{-1}$ is the greatest rate of exposure of a farsighted blue individual in the center of a star component encompassing all farsighted green individuals and all myopic or farsighted blue individuals when adding a link to some myopic green individual. Similarly, $(n^G - 1) \cdot (n^{F,B} + n^G - 2)^{-1}$ is the greatest rate of exposure of a farsighted green individual in the center of a star component encompassing all farsighted blue individuals and all myopic or farsighted green individuals when adding a link to some myopic blue individual.[16] When $n^{F,G} \neq 0$ and $n^{F,B} \neq 0$, γ is simply the minimum of these two greatest rates of exposure.

Proposition 3. *Assume intermediate intra-community costs, $\delta - \delta^2 < c < \delta$, $N^F \neq \emptyset$ and $N^M \neq \emptyset$. If $c + \gamma C < (\delta - \delta^2)(1 + \delta(n^F - 1))$, then the set $G^* = \left\{g^{*i} \mid i \in N^M\right\}$ is the unique myopic-farsighted stable set.*

Proof. We first show that $G^* = \left\{g^{*i} \mid i \in N^M\right\}$ satisfies both internal stability (i.e. condition (**IS**) in Definition 3) and external stability (i.e. condition (**ES**) in Definition 3).
IS. Farsighted green and blue individuals are peripherals in all networks in G^* so that they always obtain the same payoff: $U_i(g) = \delta + (n-2)\delta^2 - c$ for all $i \in N^F$, $g \in G^*$. Myopic green and blue individuals who are peripherals have no incentive to delete their single link ($\delta + (n-2)\delta^2 - c > 0$) nor to add a new link to any other individual since $\delta - \delta^2 < c$. The center who is myopic has no incentive to delete one link since $c < \delta$. Hence, for every $g, g' \in G^*$, it holds that $g' \notin \phi(g)$.
ES. Take any network $g \notin G^*$. We build in steps a myopic-farsighted improving path from g to some $g^{*i} \in G^*$.
Step 1: Starting in g, farsighted green and blue individuals delete all their links successively looking forward to some $g^{*i} \in G^*$, where they obtain their highest possible payoff given $\delta - \delta^2 < c$. Notice that if g is a star network where the center is a farsighted green or blue individual, then the center starts by deleting all her links since only the center is better off in g^{*i} compared to g (and we go directly to Step 8). We reach a network g^1 where all farsighted green and blue individuals have no link and myopic individuals only keep the links to myopic individuals they had in g.
Step 2: From g^1, looking forward to $g^{*i} \in G^*$, farsighted green and blue individuals build a star network g^{*j^F} restricted to farsighted individuals with individual j^F being the center is such that either

$$j^F \in N^{F,B} \text{ if } \min\left[\frac{n^B - 1}{n^{F,G} + n^B - 2}, \frac{n^G - 1}{n^{F,B} + n^G - 2}\right] = \frac{n^B - 1}{n^{F,G} + n^B - 2} \text{ and } n^{F,G} \neq 0,\, n^{F,B} \neq 0,$$

or

$$j^F \in N^{F,G} \text{ if } \min\left[\frac{n^B - 1}{n^{F,G} + n^B - 2}, \frac{n^G - 1}{n^{F,B} + n^G - 2}\right] = \frac{n^G - 1}{n^{F,B} + n^G - 2} \text{ and } n^{F,G} \neq 0,\, n^{F,B} \neq 0,$$

or $j^F \in N^F$ if either $n^{F,G} = 0$ or $n^{F,B} = 0$. Notice that g^{*j^F} is such that $j \in N^F$, $N_j(g^{*j^F}) = N^F \setminus \left\{j^F\right\}$ and $N_k(g^{*j^F}) = \left\{j^F\right\}$ for all $k \in N^F \setminus \left\{j^F\right\}$), and we obtain $g^2 = g^1 \cup g^{*j^F}$ where all farsighted green and blue individuals are still disconnected from the myopic green and blue individuals.
Step 3: From g^2, looking forward to $g^{*i} \in G^*$, the farsighted individual j^F who is the center of g^{*j^F} adds a link to some myopic individual, say individual 1. Individual j^F is better off in g^{*i} compared to g^2, $\delta + (n-2)\delta^2 - c > (n^F - 1)(\delta - c)$, while individual 1 is better

[16] The greatest rate of exposure of a myopic green (blue) individual who is adding a link to a farsighted blue (green) individual in the center of the star component is equal to 1.

in g^2+j^F1 if $c+\left(e^{\tau(j^F)}_{j^F}(g^2+j^F1)\cdot e^{\tau(1)}_1(g^2+j^F1)\right)\cdot C<\delta+\delta^2(n^F-1)$ when $\tau(j^F)\neq\tau(1)$. Since $e^{\tau(j^F)}_{j^F}(g^2+j^F1)\leq\gamma$ and $e^{\tau(1)}_1(g^2+j^F1)\leq 1$, the sufficient condition becomes $c+(\gamma\cdot 1)\,C<\delta+\delta^2(n^F-1)$. This last inequality holds since $c+\gamma C<(\delta-\delta^2)(1+\delta(n^F-1))<\delta+\delta^2(n^F-1)$. Notice that the condition $c+\gamma C<(\delta-\delta^2)(1+\delta(n^F-1))$ also guarantees that individual 1 is better off when $\tau(j^F)=\tau(1)$.

Step 4: From g^2+j^F1, looking forward to $g^{*i}\in G^*$, the farsighted individual j^F adds a link successively to the myopic individuals who are neighbours of individual 1 (if any), say individual 2. Individual 2 who is myopic and linked to individual 1 has an incentive to add the link j^F2 if $\delta^2+(n^F-1)\delta^3<\delta-c-\left(e^{\tau(j^F)}_{j^F}(g^2+j^F1+j^F2)\cdot e^{\tau(2)}_2(g^2+j^F1+j^F2)\right)C+(n^F-1)\delta^2$ when $\tau(j^F)\neq\tau(2)$. Since $e^{\tau(j^F)}_{j^F}(g^2+j^F1+j^F2)\leq\gamma$ and $e^{\tau(2)}_2(g^2+j^F1+j^F2)\leq 1$, the sufficient condition becomes

$$c+\gamma C<\delta-\delta^2+(n^F-1)(\delta^2-\delta^3), \tag{2}$$

or

$$c+\gamma C<(\delta-\delta^2)(1+\delta(n^F-1)) \tag{3}$$

where n^F is the number of farsighted individuals in the whole population. We reach the network $g^2+j^F1+\{j^Fl\mid l\in N_1(g^2+j^F1)\cap N^M\}$, where individual j^F is (directly) linked to individual 1 and all her neighbours, and all other farsighted green and blue individuals.

Step 5: From $g^2+j1+\{j^Fl\mid l\in N_1(g^2+j^F1)\cap N^M\}$, the myopic individuals who are neighbours of individual 1 and have just added a link to the farsighted individual j^F delete their link successively with individual 1. They have incentives to do so since $\delta-\delta^2<c$ and we reach the network $g^2+j^F1+\{j^Fl\mid l\in N_1(g^2+j^F1)\cap N^M\}-\{1l\mid l\in N_1(g^2+j^F1)\cap N^M\}$.

Step 6: Next, looking forward to $g^{*i}\in G^*$, the farsighted individual j^F adds a link successively to the myopic individuals who are neighbours of some $l\in N_1(g^2+j^F1)\cap N^M$ and we proceed as in Step 4 and Step 5. We repeat this process until we reach a network g^3 where there is no myopic individual linked directly to the myopic neighbours of individual j^F (i.e. $N_k(g^3)\cap N^M=\emptyset$ for all $k\in N_{j^F}(g^3)\cap N^M$).

Step 7: From g^3, individual j^F adds a link to some myopic individual belonging to another component (if any) as in Step 3 and we proceed as in Step 4 to Step 6. We repeat this process until we end up with a star network g^{*j} with individual j^F (who is farsighted) in the center (i.e. $N_{j^F}(g^{*j^F})=N\setminus\{j^F\}$ and $N_k(g^{*j^F})=\{j^F\}$ for all $k\in N\setminus\{j^F\}$).

Step 8: From g^{*j^F}, looking forward to $g^{*i}\in G^*$, the farsighted individual j^F deletes all her links successively to reach the empty network $g^{\emptyset}$. From $g^{\emptyset}$, myopic and farsighted individuals have both incentives (since $\delta>c$) to add links successively to build the star network $g^{*i}\in G^*$ where some myopic individual $i\in N^M$ is the center.

Uniqueness. We now show that G^* is the unique myopic-farsighted stable set. Farsighted green and blue individuals who are peripherals in all networks in G^* obtain their highest possible payoff. Myopic green and blue individuals who are peripherals have no incentive to delete their single link nor to add a new link. The center who is myopic has no incentive to delete one link. Hence, $\phi(g) = \emptyset$ for every $g \in G^*$. Suppose that $G \neq G^*$ is another myopic-farsighted stable set. (1) G does not include G^*: $G \not\supseteq G^*$. External stability would be violated since $\phi(g) = \emptyset$ for every $g \in G^*$. (2) G includes G^*: $G \supsetneq G^*$. Internal stability would be violated since for every $g \in \mathcal{G} \setminus G^*$, it holds that $\phi(g) \cap G^* \neq \emptyset$. □

Once all individuals become farsighted (i.e. $N = N^F$), for $\delta - \delta^2 < c < \delta$ and for $C > 0$, every set consisting of a star network encompassing all individuals is a myopic-farsighted stable set

Proposition 4. *Assume intermediate intra-community costs, $\delta - \delta^2 < c < \delta$, and all individuals farsighted, $N = N^F$. If g is a star network then $\{g\}$ is a myopic-farsighted stable set.*

Proof. Since each set is a singleton set, internal stability (**IS**) is satisfied. (**ES**) Take any network $g \neq g^{*i}$, we need to show that $\phi(g) \ni g^{*i}$. **(i)** Suppose $g \neq g^{*j}$ $(j \neq i)$. From g, looking forward to g^{*i} (where they obtain their highest possible payoff), farsighted individuals $(\neq i)$ delete all their links successively to reach the empty network. From $g^{\emptyset}$, farsighted individuals have incentives (since $\delta > c$) to add links successively to build the star network g^{*i} with individual i in the center. **(ii)** Suppose $g = g^{*j}$ $(j \neq i)$. From g, looking forward to g^{*i}, the farsighted individual j deletes all her links successively to reach the empty network. From $g^{\emptyset}$, farsighted individuals have incentives (since $\delta > c$) to add links successively to build the star network g^{*i} with individual i in the center. □

While every set consisting of a star network is a myopic-farsighted stable set, there may be other myopic-farsighted stable sets.

6 Discussion

6.1 Low inter-community costs

Suppose now that both intra- and inter-community costs are low, i.e. $c + n^G C < \delta - \delta^2$. Then the complete integrated network is going to emerge whatever the composition of the population in terms of farsightedness.

Proposition 5. *Assume low intra-community costs and low inter-community costs, $c + n^G C < \delta - \delta^2$. The set $G = \{g_{int}\}$, where $g_{int} = g^N$, is a myopic-farsighted stable set.*

Proof. The set $G = \{g_{\text{int}}\}$, where $g_{\text{int}} = g^N$, satisfies (**IS**) in Definition 3 since it is a singleton set. We now show that it also satisfies (**ES**).
ES. Take any network $g \neq g_{\text{int}}$. Since $c + n^G C < \delta - \delta^2$, it follows that $U_i(g + ij) > U_i(g)$ and $U_j(g + ij) > U_j(g)$ as well as $U_i(g^N) \geq U_i(g + ij) > U_i(g)$ and $U_j(g^N) \geq$

$U_j(g+ij) > U_j(g)$. Hence, the sequence starting at $g_1 = g$, followed by $g_{k+1} = g_k + ij$ with $ij \in g^N \setminus g^k$, for $k = 1, 2...$, and ending at $g_K = g^N$, is a sequence along which $U_i(g_k + ij) > U_i(g_k)$, $U_j(g_k + ij) > U_j(g_k)$, $U_i(g^N) > U_i(g_k)$ and $U_j(g^N) > U_j(g_k)$. Thus, this sequence is a myopic-farsighted improving path from g to g^N whatever the composition of the population in terms of myopia and farsightedness (i.e. N^M and N^F), and $G = \{g_{\text{int}}\}$ satisfies (**ES**). □

6.2 Intermediate inter-community costs

Suppose now that inter-community costs are intermediate while intra-community costs are low. Then, it becomes unlikely that a singleton myopic-farsighted stable set exists. In fact, a myopic-farsighted stable set (if it exists) may include many different network architectures. For instance, any network where n^G green individuals are fully intra-connected, n^B blue individuals are fully intra-connected, and one green individual is linked to all blue individuals may belong to the myopic-farsighted stable sets for intermediate inter-community costs. Take $N^G = \{1, 2, 3, 4\}$, $N^B = \{5, 6, 7\}$, $N^F = \{4\}$ and $N^M = N \setminus N^F$. If $3\Delta/2 < C < \min\{25\Delta/9, 5\Delta/3 + 15(\delta^2(1-\delta))/3\}$ then the network $\widetilde{g} = g^{N^G} \cup g^{N^B} \cup \{4j \mid j \in N^B\}$ belongs to any myopic-farsighted stable set (if any) since $\phi(\widetilde{g}) = \emptyset$. The network $\widetilde{g}$ is depicted in Figure 6. Moreover, we have that $\phi(g_{\text{int}}) = \emptyset$ and so the complete integration network $g_{\text{int}} = g^N$ also belongs to any myopic-farsighted stable set (if any).[17]

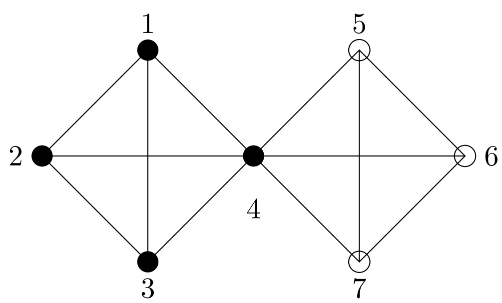

Figure 6: Both communities are fully intra-connected and one green individual is linked to all blue individuals.

Notice that green individual 4 who is farsighted obtains her highest possible payoff in the network $\widetilde{g}$. So, she will never engage a move towards some alternative network configuration. Since $C > 3\Delta/2$, any myopic green individual $i \neq 4$ has no incentive to

[17]Patacchini and Zenou (2016) look at friendship networks among US high-school students (Add Health data). They find that, for mixed schools, most of the white students have white friends while one part of the black students has mostly white friends and the other part has mostly black friends.

add a link with a blue individual. But, myopic green individuals prefer the complete integration network to the network $\widetilde{g}$. Hence, if they were farsighted, they would engage a move towards the complete integration network and destabilize the network $\widetilde{g}$.

6.3 Stability versus efficiency

6.3.1 Low intra-community costs

Suppose that intra-community costs are low, i.e. $c < \delta - \delta^2$. Since, $n^G \geq n^B$, the network $g_{\text{assi,green}}$ is always better than the network $g_{\text{assi,blue}}$ in terms of strong efficiency (i.e. sum of utilities of all individuals). Comparing the network $g_{\text{assi,green}}$ with the complete integrated network g_{int}, we have that the network $g_{\text{assi,green}}$ is better than the complete integrated network g_{int} in terms of strong efficiency if and only if

$$C > \frac{(n-2)^2}{2n^G(n^G-1)}\Delta = C^*.$$

In addition, the network $g_{\text{assi,green}}$ is always better than the complete segregated network g_{seg} in terms of strong efficiency. In terms of Pareto efficiency, the network $g_{\text{assi,green}}$ always Pareto dominates the complete segregated network g_{seg}, while it only Pareto dominates the complete integrated network g_{int} if

$$C \geq \frac{(n-2)^2}{n^G(n^G-1)}\Delta = C^{**},$$

where $C^{**} > C^*$. Finally, the network $g_{\text{assi,blue}}$ Pareto dominates the complete integrated network g_{int} if

$$C \geq \frac{(n-2)^2}{n^B(n^B-1)}\Delta = C^{***},$$

where $C^{***} \geq C^{**} > C^*$.

Remark 4. *Assume low intra-community costs, $c < \delta - \delta^2$. The complete segregated network g_{seg} is never strongly efficient and is Pareto dominated for any value of C. In terms of strong efficiency,*

- **(i)** *if $C < C^*$, the complete integrated network g_{int} is better than the complete segregated network g_{seg} and the networks with assimilation $g_{assi,green}$ or $g_{assi,blue}$;*
- **(ii)** *if $C > C^*$, the network $g_{assi,green}$ in which all blue individuals are fully assimilated into the dominant green community is better than the complete integrated network g_{int}, the complete segregated network g_{seg} and the network $g_{assi,blue}$ in which all green individuals are fully assimilated into the smaller blue community.*

In terms of Pareto efficiency, if $C \geq C^{**}$, the network $g_{assi,green}$ Pareto dominates the complete integrated network g_{int}, and if $C \geq C^{***} \geq C^{**}$, the network $g_{assi,blue}$ Pareto *dominates the complete integrated network g_{int}. The network $g_{assi,green}$ always Pareto dominates the complete segregated network g_{seg}, and if $n^B > \underline{n}^B$, the network $g_{assi,blue}$ Pareto dominates the complete segregated network g_{seg}.*

Whether a network is strongly efficient or not depends on C. The formation of a link between two individuals from two different communities (the same community) has a positive (negative) exposure effect for both individuals involved in the link because the decrease (increase) in the rate of exposure of each of these individuals to their own community reduces (increases) their inter-community costs that are proportional to C. When C is very small, the difference between inter- and intra-community costs is negligible and it is as if the entire population belong to a single community. Then, the complete integrated network is strongly efficient. When C is large enough, inter-community costs overcome benefits derived from connecting to the other community, and the network in which all blue individuals are fully assimilated into the dominant green community is strongly efficient.

Is there a tension between stability and efficiency when C is larger than $\widehat{C}$?[18] When the whole population is myopic, a conflict between stability and efficiency may occur since the complete segregated network is stable. However, once there are enough farsighted individuals in the dominant green community relatively to the size of the smaller community, $n^{F,G} > \underline{n}^G$, the tension vanishes. Indeed, the network $g_{\text{assi,green}}$ in which all blue individuals are fully assimilated into the dominant green community is likely to emerge in the long run since $\{g_{\text{assi,green}}\}$ is a myopic-farsighted stable set and $g_{\text{assi,green}}$ is not only better than $g_{\text{assi,blue}}$, g_{seg}, g_{int} in terms of strong efficiency but it also Pareto dominates g_{seg} and g_{int}. Thus, turning myopic players into farsighted players within the dominant community may improve efficiency by destabilizing inefficient and/or Pareto dominated networks.

Remark that when C is larger than $\widetilde{C}$ and all blue individuals are farsighted and sufficiently numerous ($n^B > \underline{n}^B$), then the network $g_{\text{assi,blue}}$ may emerge in the long run. Although the network $g_{\text{assi,green}}$ is better than the network $g_{\text{assi,blue}}$ in terms of strong efficiency, the network $g_{\text{assi,blue}}$ Pareto dominates the complete segregated network g_{seg}.

6.3.2 Intermediate intra-community costs

Remember that, for $C = 0$, de Marti and Zenou (2017) friendship model reverts to Jackson and Wolinsky (1996) connections model where a star network is strongly efficient for $\delta - \delta^2 < c < \delta$ (and $C = 0$). Hence, such star network is also strongly efficient for $\delta - \delta^2 < c < \delta$ and $C > 0$.

Remark 5. *Assume intermediate intra-community costs, $\delta - \delta^2 < c < \delta$. A star network is strongly efficient.*

[18] Notice that $C^* < C^{**} \leq \overline{C}_1 \leq \overline{C}_2$, $C^{**} \leq C^{***} \leq \overline{C}_2$, $C^{**} \leq \widehat{C}$ and $C^{***} \leq \widetilde{C}$.

When intra-community costs are intermediate and the population is formed by myopic and farsighted individuals, the set of star networks with a myopic individual at the center of the star is the unique myopic-farsighted stable set and each star network is strongly efficient. Thus, provided that the population is mixed, there is no tension between stability and efficiency.

6.4 Conclusion

We have reconsidered de Marti and Zenou (2017) model of friendship network formation where individuals are myopic and belong to two different communities (greens and blues). When all individuals are myopic many friendship networks, like fully integrated communities or segregated communities, can be pairwise stable and a tension between efficiency and stability may occur. We have added a second heterogeneity dimension: individuals can be either myopic or farsighted. We summarize our main results for low intra-community costs in Figure 7. When the population becomes mixed in terms of farsightedness and myopia, most inefficient friendship networks tend to be destabilized. Once there are enough farsighted individuals in the larger community and inter-community costs are large enough, the friendship network where the smaller community ends up being assimilated into the dominant community is likely to arise and is efficient. For instance, segregation is destabilized because farsighted individuals while they do not have immediate incentives to add or delete links, they anticipate that once they do so, other individuals will continue adding or deleting links leading to a friendship network where the small community is assimilated into the dominant one. When inter-community costs are small enough, the complete integration is stable whatever the number of farsighted and myopic individuals in both communities.[19]

For intermediate intra-community costs, a star network (encompassing both communities) with a myopic individual at the center of the star is going to arise and is strongly efficient provided that the population is mixed.

The degree of farsightedness of an individual is likely to be correlated with her level of education or grades at school. Hence, for future research it would be interesting to confront our theoretical predictions with empirical or experimental data. Segregation should mostly occur when both communities are low educated. When one community is high educated (i.e. a community with a large number of high educated individuals) while the other community is low educated (i.e. a community with a low number of high educated individuals), the individuals belonging to the less educated community are likely to end up assimilated into the high educated community, and even more likely if they are high

[19] When adding or deleting links, individuals know perfectly the links other individuals do have. Recently, Foerster, Mauleon and Vannetelbosch (2021) propose a solution concept for network formation games where individuals can form two types of links: public links observed by everyone and shadow links generally not observed by others. Then, it could happen that, some agents overestimate others' connections and hence under-connect (relative to stable networks under correct beliefs), while others underestimate connections and hence over-connect.

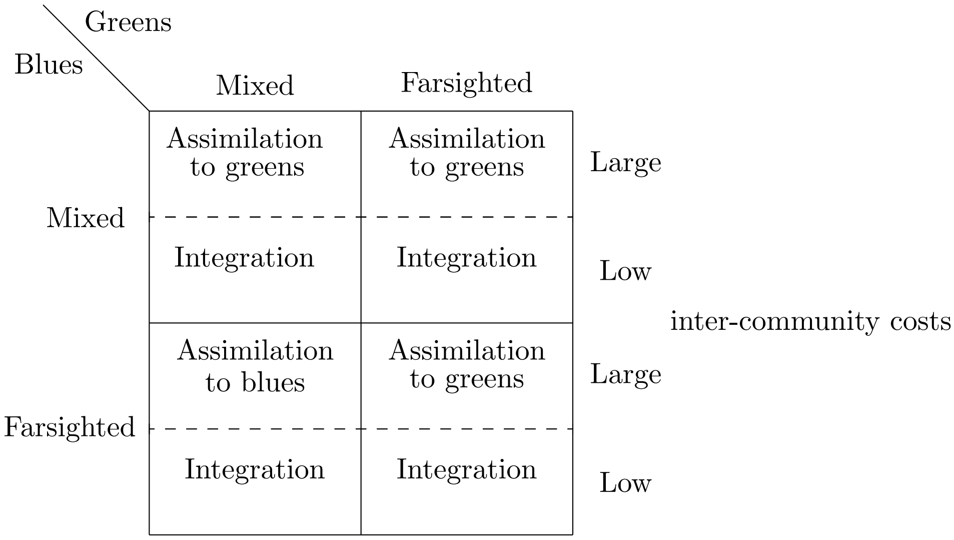

Figure 7: A summary of stable friendship networks with low intra-community costs.

educated individuals. Put it differently, it is more likely that the community with relatively less educated individuals will end up being (at least partially) assimilated into the other community, and this is even more true if there are more high educated in the dominant community and the dominant group is relatively larger than the other community. In addition, our results suggest that policies promoting mixing individuals and turning myopic individuals into farsighted ones (especially in the dominant community) could be helpful in avoiding (Pareto-) inefficient situations.

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