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Old age or dependence. Which social insurance?*

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Abstract

We consider a society where individuals differ according to their productivity and their risk of mortality and dependency. We show that according to the most reasonable estimates of correlations among these three characteristics, if one had to choose between a public pension system and a long-term care social insurance, the latter should be chosen by a utilitarian social planner. With a Rawlsian planner, the balance between the two schemes does depend on the comparison between the ratio of the survival probability to the dependence risk of the poor with its population average.

JEL: H2, H5.

Keywords: long term care, pension, mortality risk, optimal taxation, liquidity constraints.

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1 Introduction

Due to the ageing process, the rise in long-term care needs constitutes a major challenge of the coming decades. Long-term care (LTC) concerns individuals who are no longer able to carry out basic daily activities such as eating, washing, dressing, etc. Nowadays, the number of persons in need of LTC is substantial and rapidly increasing. This raises the question of the provision of care. As stressed by Norton (2000), about two thirds of LTC is generally provided by informal care givers (mainly the family, i.e. spouses, daughters and step daughters). However, the involvement of the family seems to reach a ceiling. Furthermore, although markets for private LTC insurance exist, these remain thin in most countries. According to Brown and Finkelstein (2007), only about 9 to 10 % of the population at risk of facing future LTC costs has purchased a private LTC insurance in the U.S. In the light of the expected decline in informal care, and of the difficulties faced by the market for private LTC insurance, one would hope that the public sector plays a more important role in the provision and funding of LTC. Nowadays, in most advanced economies, the State is involved either in the provision or in the funding of LTC services, but to an extent that varies strongly across countries. However, the involvement of the public sector in LTC is not as comprehensive and generous as it is for the funding of general health services or that of pensions. The LTC "pillar" of the Welfare State remains quite thin in comparison with other pillars of the social insurance system.

In this paper, we raise the question of the relative desirability of LTC insurance and public pensions. To proceed we look at the hypothetical situation in which the government has to choose between a public pension system and a LTC insurance. We show that priority should be given to the LTC scheme. This finding is a bit paradoxical given that in the real world the opposite result seems to prevail.

To make our point, we use a simple model of a two-period economy with three states of nature: the first period (people work and save), the second period (people retire) with a healthy state, and the second period with dependency (Cremer et al. (2010), Cremer and Pestieau (2014), Leroux et al. (2019)). Society comprises a number of individuals who differ in their productivity and their probability of survival and dependence. We have strong evidence concerning the correlation between those probabilities. According to the recent waves of Survey of Health, Ageing and Retirement in Europe (SHARE), we indeed observe a positive correlation between income and longevity, and a negative correlation between income and dependence, conditional or not upon survival.

To provide an intuition of our results, assume that there are no liquidity constraints, namely, individuals can purchase negative amounts of either annuities or LTC insurance. Keeping a balanced government budget, low-income people will always prefer to have LTC benefits than pensions, since their ratio of dependency risk to their probability of survival is higher than its average of the rest of society. In other words, a redistributive LTC scheme brings more utility to the poor than a redistributive social security. One dollar devoted to LTC public benefits them more than a dollar spent on public pensions. This is true even when there are liquidity constraints, as long as individuals, particularly those with low income, have a positive saving and a positive LTC insurance. Even when this possibility does not hold, we show that the superiority of LTC social insurance over public pensions is maintained under some plausible conditions.

When the objective of the government is Rawlsian, then the desirability of public LTC scheme depends on the comparison between the ratio of the survival probability to the dependence risk of the poor with its population average. However, the superiority of LTC benefits over social security is more limited than in the utilitarian case.

The rest of the paper is organized as follows. Section 2 presents the basic model. Sections 3 deals with the utilitarian case. Section 4 is devoted to numerical simulation. Sections 5 deals with the Rawlsian case. Finally, Section 6 concludes.

2 The Model

Consider a two-period model, where individuals work and save in the first period and retire in the second. In the second period people face different risks of mortality and dependence. There are I types of individuals. The proportion of type i ($i = 0, 1, ..., I$) individuals is denoted by n_i, with the total number of individuals born in the first period being normalized to unity: $\sum_{i=0}^{I} n_i = 1$. Each individual of type i is characterized by three characteristics: (i) w_i (labor productivity in the first period), (ii) π_i (the probability to be alive in the second period), and (iii) p_i (the probability of becoming dependent in the second period). From the Survey of Health, Ageing and Retirement in Europe (SHARE), we know that at least for the European countries covered the following relations hold:

- longevity (π_i) increases with income.
- conditional upon survival, the probability of dependency (p_i) decreases with income.
- the probability of dependency ($\pi_i p_i$) decreases with income.

The last correlation may appear surprising, given that the probability of survival increases with income. It just means that this relation is dominated by the negative link between income and dependence risk. Consistent with these facts, we posit that $cov(w_i, \pi_i) > 0$, $cov(w_i, p_i) < 0$ and $cov(w_i, \pi_i p_i) < 0$.

Let c_i denote individual i's first period consumption, $\ell_i \in [0, \bar{\ell}]$, labor supply, d_i, the second period consumption if (s)he is healthy, and m_i, LTC expenditures in the case of dependency. An individual's expected lifetime utility is given by

$$U_i = u\left(c_i - v(\ell_i)\right) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i H(m_i).$$

In the following we denote $x_i \equiv c_i - v(\ell_i)$. We assume $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$, $H' > 0$, $H'' < 0$, $v'(0) = 0$ and $v'(\bar{\ell}) > \max_{i=1,\dots,I} w_i$. We also assume $H(y) < u(y)$ and $H'(y) > u'(y)$ for all $y > 0$, to reflect costly needs for dependency. As shown by Ameriks et al. (2020), the desires to self-insure against the risk against dependency explains a substantial fraction of the wealth holding of elderly people.

Private saving is invested in a perfect annuity market with a zero interest rate. From saving s_i, type i has a return s_i/π_i. There is also a private insurance market against dependency. From the insurance purchase P_i, type i receives $P_i/(\pi_i p_i)$, where the return is inversely proportional to the individual's risk of dependency.

The government's policy comprises three intruments (i) linear income tax ($\tau \geq 0$), (ii) flat-rate pension ($r \geq 0$), and (iii) uniform long-term care benefit ($q \geq 0$). Individuals choose labor supply (ℓ_i), annitized savings (s_i), and private insurance (P_i) while taking the government's scheme as given:

$$\begin{aligned} U_i &= u\left((1-\tau)w_i\ell_i - s_i - P_i - v(\ell_i)\right) \\ &\quad +\pi_i(1-p_i)u(s_i/\pi_i + r) + \pi_i p_i H(s_i/\pi_i + P_i/(\pi_i p_i) + q + r). \end{aligned} \tag{1}$$

In Section 3, we assume that $x_i = c_i - v(\ell_i) > 0$ holds for all i at the optimum of (1). Namely, individuals would not transfer all his/her first-period incomes to the second period through P_i and s_i. Here we assume that w_i's are sufficiently high, and/or the expected cost for dependency is sufficiently low. The case where there is an individual with $w_i = 0$ will be discussed in Section 5.

The FOCs with respect to ℓ_i, s_i and P_i are:

$$u'(x_i)\left((1-\tau)w_i - v'(\ell_i)\right) = 0, \tag{2}$$

$$-u'(x_i) + (1-p_i)u'(d_i) + p_i H'(m_i) \leq 0, \tag{3}$$

$$-u'(x_i) + H'(m_i) \leq 0. \tag{4}$$

Let the solution values be ℓ_i^*, s_i^* and P_i^* respectively. The first condition is written with an equal sign, implying for an interior solution for labor. The two other solutions are not necessarily interior, implying that some individuals may be constrained to have a non-negative level of saving or of LTC insurance. Formally: $s_i^* \geq 0; P_i^* \geq 0$. In the case of interior solutions, we have

$$u'(c_i) = u'(d_i) = H'(m_i).$$

Concerning those solutions, we distinguish two cases in which they are interior:

- Given the parameters of the model, all the solutions are interior. This will be the case when both q and r are small, or alternatively when the tax rate is low for some reason (political decision or tax distortions).

- Liquidity constraints are assumed away, implying that individuals can have negative saving or insurance premium.

We now turn to the optimal level of public benefits chosen by a government that is utilitarian or Rawlsian.

3 Utilitarian Case

The problem of the utilitarian government is to maximize the following Lagrangian:

$$\begin{aligned}
\pounds &= \sum n_i\{u\left((1-\tau)w_i\ell_i^* - s_i^* - P_i^* - v(\ell_i^*)\right) \\
&\quad +\pi_i(1-p_i)u(s_i^*/\pi_i + r) + \pi_i p_i H(s_i^*/\pi_i + P_i^*/(\pi_i p_i) + q + r)\} \\
&\quad +\mu \sum n_i(\tau w_i \ell_i^* - \pi_i r - \pi_i p_i q), \qquad (5)
\end{aligned}$$

For simplicity, the stars with respect to x_i, d_i, m_i and ℓ_i are dropped in the remainder of the paper. The FOCs on q and r are as follows:

$$\frac{\partial \pounds}{\partial q} = \sum n_i \pi_i p_i H'(m_i) - \mu \overline{\pi p}, \qquad (6)$$

$$\frac{\partial \pounds}{\partial r} = \sum n_i \pi_i \{(1-p_i)u'(d_i) + p_i H'(m_i)\} - \mu \overline{\pi}, \qquad (7)$$

where the bar denotes the population average of the respective parameter.

We adopt the viewpoint of tax reform wherein we consider that the tax is given, not necessarily optimal, and we look at the welfare incidence of increasing q at the expense of r while keeping a balanced budget. This is given by:

$$\begin{aligned}
\frac{\partial \pounds^c}{\partial q} &\equiv \frac{\partial \pounds}{\partial q} + \frac{\partial \pounds}{\partial r}\frac{\partial r}{\partial q}\Big|_{d\tau=0} \\
&= \left(1 - \frac{\overline{\pi p}}{\overline{\pi}}\right) \sum n_i \pi p_i H'(m_i) - \frac{\overline{\pi p}}{\overline{\pi}} \sum n_i \pi_i (1-p_i) u'(d_i),
\end{aligned}$$

or

$$\frac{\partial \pounds^c}{\partial q} = \left(1 - \frac{\overline{\pi p}}{\overline{\pi}}\right) \overline{\pi p} \left(cov\left(H'(m_i), \frac{\pi_i p_i}{\overline{\pi p}}\right) - cov\left(u'(d_i), \frac{\pi_i(1-p_i)}{\overline{\pi(1-p)}}\right) + \varDelta \right), \qquad (8)$$

where $\varDelta \equiv \sum n_i\{H'(m_i) - u'(d_i)\}$.

Regarding the second-period consumption (m_i and d_i), the following property holds (regardless of the liquidity constraints). The proof is given in the Appendix:

Lemma 1 *$\partial m_i/\partial w_i \geq 0$ and $\partial d_i/\partial w_i \geq 0$ with strict inequality when $s_i^* > 0$.*

We have to ascertain that the wage gap is wider than the probability gap. To show why this is important, assume for a moment that the wage dispersion is negligible whereas the variance of the dependence risk is very high. Then it is not impossible that low-income individuals save more than high-income ones. Assuming as it is reasonable that such situation is not possible, we can be sure that $cov(H'(m_i), \pi_i p_i/\overline{\pi p}) > 0$ and $cov(u'(d_i), \pi_i(1 - p_i/\overline{\pi(1-p)})) < 0$. The LTC social insurance realizes targeted expenditures but the public pension favors the productive individuals who also live longer. Note that we assume that the tax distortions are independent of the type of insurance. Therefore, as long as $\Delta = 0$ ($H'(m_i) - u'(d_i) = 0$ for all i), it is always desirable to increase q at the expense of r; in other words, $\partial \pounds^c/\partial q > 0$. $\Delta = 0$ holds if both s_i^* and P_i^* are positive for all individuals. For low values of both q and r (and τ), this condition is fulfilled.

To get $\dfrac{\partial \pounds^c}{\partial q} \leq 0$, we have to assume that Δ is negative and large enough. For an illustrative purpose, suppose that $H(y) = u(y - L)$ where $L > 0$ stands for the resources needed to compensate for the dependency.

Lemma 2 *Suppose that $H(y) = u(y - L)$. If $q \leq L$, then $\Delta \geq 0$.*

Only if $q > L$ (the government fully compensates resources the need for the dependency), we have $H'(m_i) = u'(s_i^*/\pi_i + q + r - L) < u'(s_i^*/\pi_i + r)$,[1] so that $\Delta < 0$. However, note that $q > L$ is a necessary but not sufficient condition for $\dfrac{\partial \pounds^c}{\partial q} \leq 0$. This leads us to our first theorem.

Theorem 1 *It is always desirable to have a balanced budget increase in LTC benefits at the expense of social security benefits, as long as the liquidity constraint is not binding for any individual. In the case when part of the population is subject to a liquidity constraint, this dominance of LTC over social security still holds as long as LTC benefits are not too high relative to pension benefits.*

[1]When $q > L$, $-u'(x_i) + H'(m_i) < -u'(x_i) + (1 - p_i)u'(d_i) + p_i H'(m_i)$. From (3) and (4), we conclude that $P_i^* = 0$ for all i.

Combining (6) and the revenue-side optimization, we obtain:

$$\begin{aligned}\frac{\partial \pounds^c}{\partial \tau} &= \frac{\partial \pounds}{\partial \tau} + \frac{\partial \pounds}{\partial q}\frac{\partial q}{\partial \tau}\Big|_{dr=0} \\ &= \sum n_i u_i'(x_i)(-w_i\ell_i) + \sum n_i \pi_i p_i H'(m_i)\frac{\overline{y}+\tau\partial\overline{y}/\partial\tau}{\overline{\pi p}} \qquad (9) \\ &= -cov(u'(x_i), y_i) + cov(H'(m_i), \frac{\pi_i p_i}{\overline{\pi p}})\overline{y} - \mathit{\Gamma}\overline{y} + \mu\tau\frac{\partial \overline{y}}{\partial \tau},\end{aligned}$$

where $y_i \equiv w_i\ell_i$ and $\mathit{\Gamma} \equiv \sum n_i\{u'(x_i) - H'(m_i)\} \geq 0$. This derives the following optimal tax formula:

$$\tau^* = \frac{-cov\left(u'(x_i), \dfrac{y_i}{\overline{y}}\right) + cov\left(H'(m_i), \dfrac{\pi_i p_i}{\overline{\pi p}}\right) - \mathit{\Gamma}}{-\mu\partial\overline{y}/\partial\tau \cdot 1/\overline{y}} > 0. \qquad (10)$$

The denominator is the conventional efficiency term. It is positive. The first term of the numerator is the traditional equity term $-cov(u'(x_i), y_i) > 0$. These two terms correspond to the conventional optimal tax formula (e.g., Sheshinski (1972) and Hellwig (1986)). This redistributive impact of the conventional first term of the numerator is reinforced by the second term, which is positive and reflects the redistributive impact of the LTC benefit. Note that if instead of using the tax proceeds for LTC they were used for pensions, the second term of the numerator would be negative, reflecting the fact that pensions tend to benefit the high-income individuals. The last term of the numerator represents the cost of the binding liquidity constraints (4).

Let (τ^*, q^*, r^*) be the social optimum. With respect to q^*, it is characterized by $\partial\pounds^c/\partial q = 0$ or $\partial\pounds^c/\partial q|_{r=0} > 0$. From (8) and Lemma 2, we conclude the following:

Theorem 2 *Suppose that $H(y) = u(y - L)$. If $q^* \leq L$, then $r^* = 0$.*

Theorem 2 is easily extended to the case where the private saving and private insurance accrue loading costs, with the loading costs of P_i being higher than those of s_i. With sufficiently high loading costs, some individuals may prefer not to buy insurance, in which case the role of the public LTC insurance is strengthened. See the Appendix for the proof of Lemma 2.

Evaluated at $q \leq L$ and $r = 0$, taking account of the government budget balance, the total effect of the tax increase for the increase of q is given by (9). Whether there exists the optimum at $q^* \leq L$ depends on the sign of (9) at $q \leq L$. The qualitative features are as follows. Other things being equal, (9) is lower (and the optimal LTC social insurance q^* is lower) when the distribution of income and risk of dependency are more equal, or when the tax distortions are high.[2]

Between q and r, the priority is given to q until $q = L$. $r^* > 0$ might happen only when $q^* > L$ (individuals are overly insured under dependency), which we do not observe in reality.

4 Numerical example

To illustrate the above results, we now resort to a numerical example that will make possible to see the incidence of parameters changes on the final outcome. The society comprises two types of individuals. We use the following specification:

$$u(x) = ax - 0.5x^2; v(\ell) = \frac{\gamma/k}{1+\gamma}\ell^{\frac{1+\gamma}{\gamma}}.$$

The initial values of the parameters that make our benchmark scenario are:

$$a = 400,\ n_1 = n_2 = 0.5,\ w_1 = 5,\ w_2 = 8,\ \pi_1 = 1,\ \pi_2 = 1,$$

$$p_1 = 0.5,\ p_2 = 0.2,\ L = 50, \gamma = 1,\ k = 12.32.$$

In this benchmark scenario, if we restrict $r = 0$, then the utilitarian social welfare is maximized at $q = 92.5$. Given these parameters, social welfare is maximized at $r^* = 7.05$ and $q^* = 85.80 \approx 1.72L$ for a tax rate $\tau^* = 0.073$. This result, as well as other results following changes in parameters, are presented in Table 1. Note that in all the scenarios studied, individuals do not purchase any LTC insurance at the social optimum.

The values of k in the first row (the benchmark case) and the second row were chosen so that $\tau = 0.033$ at $q = L$ and $r = 0$. We now interpret this table by looking at the effect of parameters change. An increase in labor elasticity implies a lower tax and consequently lower LTC benefits and zero pension. Decreasing π_1 from

[2]When $q \leq L$ and $r = 0$, we can show that $\Gamma = 0$. Dividing (9) by $\overline{y}$, it is increasing in $-cov(u'(x_i), y_i/\overline{y})$ and decreasing in $\partial\overline{y}/\partial\tau \cdot 1/\overline{y}$. Evaluated at the left side of the Laffer curve ($\overline{y} + \tau\partial\overline{y}/\partial\tau > 0$), (9) is also increasing in $cov(H'(m_i), \pi_i p_i/\overline{\pi p})$.

	r^*	q^*	τ^*	s_1^*	y_1^*	s_2^*	y_2^*
benchmark	7.05	85.80	0.073	53.70	285.53	162.30	730.95
$\gamma = 2 > 1$ (k=1.33)	0	73.99	0.051	25.50	199.12	126.59	815.59
$\pi_1 = 0.8 < 1$	3.92	85.28	0.056	51.37	290.67	170.07	744.10
$w_1 = 4 < 5$	14.45	91.07	0.105	21.98	176.41	146.55	705.65
$n_1 = 0.35 < 0.5$	11.55	85.54	0.065	52.67	288.01	163.04	737.32
$p_1 = 0.65 > 0.5$	0	94.53	0.080	50.76	283.48	162.53	725.70
$p_2 = 0.35 > 0.2$	10.02	65.96	0.075	56.88	284.96	160.87	729.51

Table 1: Numerical Examples

1 to 0.8 leads to lower pensions and to a lower tax rate. Having a lower wage (from 5 to 4) for type 1 individuals has the result of a quite a higher tax rate with increased pensions and LTC benefits. If the relative number of type 1 individuals decreases from 0.5 to 0.35 (n_2 increases from 0.5 to 0.65), then the tax rate goes down and the pension level goes up. Finally we look at changes in the risk of dependence. If p_1 increases from 0.5 to 0.65, then the expenditure is devoted to the LTC benefits, and the pensions vanish. On the other hand, if p_2 goes from 0.2 to 0.35, pensions increase whereas LTC benefits decrease, even though the number of dependent people increased. This decrease of q^* is due to the decreased covariance between $\pi_i p_i$ and w_i.

5 Rawlsian Case

Suppose that there is an individual 0 whose wage $w_0 = 0$. For this individual, we have $s_0^* = 0$ and $P_0^* = 0$. Suppose that the government's social objective is to maximize the second-period utility of individual 0:

$$\pounds = \pi_0(1-p_0)u(r) + \pi_0 p_0 H(q+r) + \mu \sum n_i(\tau w_i \ell_i^* - \pi_i r - \pi_i p_i q), \quad (11)$$

Since individual 0 does not pay the payroll tax, the optimal tax rate under this social objective is the peak of the Laffer curve: $\tau^* = \dfrac{\overline{y}}{-\partial\overline{y}/\partial\tau}$. The issue here is how to allocate the tax revenue between

q and r. The FOCs with respect to q and r are:

$$\frac{\partial \pounds}{\partial q} = \pi_0 p_0 H'(r+q) - \mu \overline{\pi p}, \tag{12}$$

$$\frac{\partial \pounds}{\partial r} = \pi_0 \{(1-p_0)u'(r) + p_0 H'(r+q)\} - \mu \bar{\pi}. \tag{13}$$

From these FOCs, we have:

$$\frac{\partial \pounds^c}{\partial q} = \left(1 - \frac{\overline{\pi p}}{\bar{\pi}}\right) \pi_0 p_0 H'(r+q) - \frac{\overline{\pi p}}{\bar{\pi}} \pi_0 (1-p_0) u'(r) \tag{14}$$

In other words, a compensated increase of LTC benefits, q, is desirable if and only if:

$$H'(r+q) > u'(r)\varPhi, \tag{15}$$

where $\varPhi = \frac{1-p_0}{p_0} \frac{\overline{\pi p}/\bar{\pi}}{1-\overline{\pi p}/\bar{\pi}} < 1$.

The inequality $\varPhi < 1$ can also be expressed as:

$$\frac{\pi_0}{p_0 \pi_0} < \frac{\bar{\pi}}{\overline{p\pi}}.$$

This inequality compares the ratio of the survival probability to the dependence risk of the poor with the same ratio for the whole society. The survival probability π_0 corresponds to the benefit r whereas the probability $\pi_0 p_0$ corresponds to the benefit $r+q$. Assume that p_0 increases, with all the other probabilities being constant. Then $\varPhi$ decreases, which implies a decrease of the social marginal utility of $d = r$ relative to $m = q + r$. Note that in the case of $\pi_i = \overline{\pi}$ for all i, $\varPhi < 1$ since $p_0 > \overline{p}$.

Clearly in the Rawlsian case, the superiority of LTC benefits over social security is more limited than in the utilitarian case.

Theorem 3 *In the Rawlsian case, a balanced budget increase in LTC benefits is desirable as long as these are not too high relative to pension benefits and if the dependence probability of the poorest is higher than that of the average population.*

6 Conclusion

This paper has studied the design of both a social LTC insurance and a public pension system. Both benefits were uniform as well as the payroll tax rate. Under the realistic assumption of a positive correlation between income and the survival probability and of a negative correlation of the dependency probability and income for the skilled and a lower probability of turning dependent, we show that a utilitarian government should give priority to the LTC scheme relative to the pension program. When the government adopts a Rawlsian criterion, both programs are needed and the relative advantage of one over the other will depend on the comparison between the ratio of the survival probability to the dependence risk of the poor with its population average. In this paper, we use linear instruments. In a companion paper (Nishimura and Pesitieau (2016)) we instead use non-linear instruments but with just two types of individuals. In the optimal non-linear scheme, our stylized facts determine the features of the optimal tax policies on the saving and the LTC schemes.

It is fair to recognize that one of the reasons why LTC has a negligible role in most social insurance schemes is that LTC is mainly supplied by the families, which is not the case for the old age support. Introducing family solidarity would clearly modify our results but only partially, if we take into account the possibility of solidarity default. In this paper, we have made a number of assumptions to keep the presentation simple. We have assumed zero interest rate, quasi-linear preferences, no time preference, a pure Beveridgian social security. Relaxing any of the assumptions would not change our basic results.

Appendix

Proof of Lemma 1: Let $f(\tau, w_i) \equiv (1-\tau)w_i\ell_i^* - v(\ell_i^*)$. From the Envelope Theorem, $\partial f/\partial w_i = (1-\tau)\ell_i^* > 0$. When $s_i^* > 0$ and $P_i^* > 0$, (3) and (4) imply $u'(f(\tau, w_i) - s_i^* - P_i^*) = H'(m_i) = u'(s_i^*/\pi_i + r)$, so $x_i = \dfrac{s_i^*}{\pi_i} + r = \dfrac{f(\tau, w_i) - P_i^* - r}{1+\pi_i} + r$. Differentiating $-u'\left(\dfrac{f(\tau, w_i) - P_i^* - r}{1+\pi_i} + r\right) + H'\left(\dfrac{f(\tau, w_i) - P_i^* - r}{1+\pi_i} + \dfrac{P_i^*}{\pi_i p_i} + q + r\right) = 0$, we obtain:

$$\frac{\partial P_i^*}{\partial w_i} = \frac{(u''(x_i) - H''(m_i))/(1+\pi_i) \cdot \partial f/\partial w_i}{(u''(x_i) - H''(m_i))/(1+\pi_i) + H''(m_i)/(\pi_i p_i)}. \tag{16}$$

The denominator of (16) is negative due to the second-order condition with respect to P_i^*. Then $\frac{\partial x_i}{\partial w_i} = \frac{1}{1+\pi_i}\left(\frac{\partial f}{\partial w_i} - \frac{\partial P_i^*}{\partial w_i}\right) = \frac{H''(m_i)/(\pi_i p_i)\partial f/\partial w_i}{u''(x_i) - H''(m_i) + H''(m_i)(1+\pi_i)/(\pi_i p_i)} > 0$. Since $H''(m_i)\frac{\partial m_i}{\partial w_i} = u''(d_i)\frac{\partial d_i}{\partial w_i} = u''(x_i)\frac{\partial x_i}{\partial w_i}$, we have $\frac{\partial m_i}{\partial w_i} > 0$ and $\frac{\partial d_i}{\partial w_i} > 0$.

When the individual optimum faces the liquidity constraint for P_i^*, then (3) and (4) are characterized by $-u'(f(\tau, w_i) - s_i^*) + (1 - p_i)u'(s_i^*/\pi_i + r) + p_i H'(s_i^*/\pi_i + q + r) \leq 0$ and $P_i^* = 0$. When $s_i^* > 0$, differentiating the former equation, we obtain $\frac{\partial s_i^*}{\partial w_i} = \frac{u''(x_i)}{u''(x_i) + (1-p_i)u''(d_i)/\pi_i + p_i H''(m_i)/\pi_i}\frac{\partial f}{\partial w_i} > 0$. When $s_i^* = 0$ and $P_i^* = 0$, $\partial m_i/\partial w_i = 0$.

When $s_i^* = 0$ and $P_i^* > 0$, then differentiating $-u'\left(f(\tau, w_i) - P_i^*\right) + H'\left(\frac{P_i^*}{\pi_i p_i} + q + r\right) = 0$, $\frac{\partial x_i}{\partial w_i} = \frac{\partial f}{\partial w_i} - \frac{\partial P_i^*}{\partial w_i} = \frac{H''(m_i)/(\pi_i p_i)}{u''(x_i) + H''(m_i)/(\pi_i p_i)}\frac{\partial f}{\partial w_i} > 0$. Since $H''(m_i)\frac{\partial m_i}{\partial w_i} = u''(x_i)\frac{\partial x_i}{\partial w_i}$, we have $\frac{\partial m_i}{\partial w_i} > 0$. *Q.E.D.*

Proof of Lemma 2 (with the loading costs): Suppose that the private saving and the private LTC insurance accrue the loading costs. With $f(\tau, w_i) \equiv (1-\tau)w_i\ell_i^* - v(\ell_i^*)$, the FOCs of individual optimization (3) and (4) are modified to:

$$-\frac{u'(f(\tau, w_i) - s_i - P_i)}{\lambda^s} + (1-p_i)u'\left(\frac{\lambda^s s_i}{\pi_i} + r\right) + p_i H'\left(\frac{\lambda^s s_i}{\pi_i} + \frac{\lambda^P P_i}{\pi_i p_i} + q + r\right) \leq 0, \tag{3'}$$

$$-\frac{u'(f(\tau, w_i) - s_i - P_i)}{\lambda^P} + H'\left(\frac{\lambda^s s_i}{\pi_i} + \frac{\lambda^P P_i}{\pi_i p_i} + q + r\right) \leq 0, \tag{4'}$$

where λ^s and λ^P represent the loading factors of the private savings and the private LTC, respectively. It is reasonable to assume $\lambda^P \leq \lambda^s \leq 1$, i.e., the loading costs of the private LTC are greater than those of the private savings. When $\lambda^s < 1$ and $\lambda^P < 1$, some individuals may prefer not to have private savings or private insurances.

Suppose that $q \leq L$. If (4') holds in equality, then (3') implies that $-\dfrac{\lambda^P}{\lambda^s}H'(m_i) + (1-p_i)u'(d_i) + p_iH'(m_i) = (1-p_i)(u'(d_i) - H'(m_i)) + \dfrac{\lambda^s - \lambda^P}{\lambda^s}H'(m_i) \leq 0$ for all i. When $q \leq L$ and $P_i^* = 0$, $u'(d_i) = u'(\lambda^s s_i^*/\pi_i + r) \leq H'(\lambda^s s_i^*/\pi_i + r + q) = H'(m_i)$ for all i. In both cases, we have $u'(d_i) \leq H'(m_i)$ for all i when $q \leq L$. We therefore have $\Delta \geq 0$. *Q.E.D.*

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