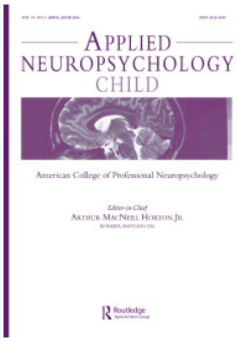

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CASE REPORT

Enhancing mathematics learning through finger-counting: A study investigating tactile strategies in 2 visually impaired cases

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ABSTRACT
Finger-counting plays a crucial role in grounding and establishing mathematics, one of the most abstract domains of human cognition. While the combination of visual and proprioceptive information enables the coordination of finger movements, it was recently suggested that the emergence of finger-counting primarily relies on visual cues. In this study, we aimed to directly test this assumption by examining whether explicit finger-counting training (through tactile stimulation) may assist visually impaired children in overcoming their difficulties in learning mathematics. Two visually impaired participants (2 boys of 8.5 and 7.5 years) were therefore trained to use their fingers to calculate. Their pre- and post-training performance were compared to two control groups of sighted children who underwent either the same finger counting training (8 boys, 10 girls, Mage = 5.9 years; 10 kindergarteners and eight 1st graders) or another control vocabulary training (10 boys, 8 girls, Mage = 5.9 years; 11 kindergarteners and seven 1st graders). Results demonstrated that sighted children's arithmetic performance improved much more after the finger training than after the vocabulary training. Importantly, the positive impact of the finger training was also observed in both visually impaired participants (for addition and subtraction in one child; only for addition in the other child). These results are discussed in relation to the sensory compensation hypothesis and emphasize the importance of early and appropriate instruction of finger-based representations in both sighted and visually impaired children.

KEYWORDS
Arithmetic abilities; finger-counting; finger training; visual impairment

Introduction

In their model of early number development, Krajewski and Schneider (2009) assume that the acquisition of early numerical competencies follows three consecutive levels. On level I, children discriminate quantities (Starkey & Cooper, 1980; Wynn, 1992) and learn to recite the exact number word sequence. At this stage, children become skilled at counting but do not yet map number words and quantities. On level II, children start to link discrete quantities with the counting procedure and therefore acquire the so-called cardinality principle. Finally, on level III, children gather experience with the relations between quantities. They become able to compose, decompose and quantify the difference between numbers.

Importantly, accumulating evidence in recent years suggested that finger-based representations might provide a preliminary access to these foundational mathematical constructs (Gelman & Gallistel, 1978; Krenger & Thevenot, 2024; Roesch & Moeller, 2015; Neveu et al., 2023; Poletti et al., 2022a). At the first developmental stage, when children learn the sequence of number words, they indeed often start to use the finger-counting system of their own culture (Wiese, 2003a, 2003b). In Belgium, for example, the typical finger-counting procedure involves raising the fingers of one hand, from the thumb to the little finger, to count from 1 to 5, and raising the fingers of the other hand in the exact same order to count from 6 to 10. By associating each raised finger with a specific number word, finger-counting supports the segmentation of the counting sequence (Bender & Beller, 2011) and thus help children to understand the one-to-one correspondence and stable order principles (Brissiaud, 2003; Fayol & Seron, 2005). As the number of fingers raised directly represents the number of counted elements, finger-counting moreover prompts the understanding of the cardinality principle (level II; Neveu et al., 2023). Children indeed name typical finger-counting configurations faster/better than non-canonical finger representations (Di Luca et al., 2006; Noël, 2005; Marlair et al., 2021; Sixtus et al., 2017, 2020; Soylu et al., 2019). This fast access to number semantics can definitively help children to broaden the set of numbers whose magnitudes can be accurately represented. Finally, on level III, children not only use finger-counting to keep track of the numbers while calculating (Fuson, 1988) but also use this procedure to represent the combined quantities (e.g., 2 fingers + 1 finger raised = 3;

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Siegler & Shrager, 1984). Finger-counting therefore also supports the development of basic arithmetic (Artemenko et al., 2022; Barrocas et al., 2020; Baroody, 1987; Fayol et al., 1998; Krenger & Thevenot, 2024; Zhang et al., 2020).

To effectively use finger-counting, children need to recognize each finger as separate entities and assign them different numerical labels. The success in using one's fingers therefore relies on good finger gnosis ability and fine motor skills (for a review see Barrocas et al., 2020; Fischer et al., 2022). While this can be achieved visually as well as through tactile and proprioceptive sensations, it has been suggested that the emergence of finger-counting mostly relies on imitation (Fuson, 1988) and on the visual recognition of canonical hand shape patterns (Crollen et al., 2011). As the inability to see correctly is a major impediment to imitation, it is not surprising to find that individuals presenting visual impairment use the finger-counting strategy less frequently and/or less typically than their sighted peers (Crollen et al., 2011, 2014). However, until now, we do not know whether vision is mandatory for the establishment of finger-counting or whether visually impaired children just need systematic instruction to take advantage of this procedure.

The current study

Given that training fine motor skills (Asakawa et al., 2019) and finger gnosis (Gracia-Bafalluy & Noël, 2008; but see Fischer, 2010 and Jay & Betenson, 2017) can overcome numerical difficulties in sighted children, the present paper aims to examine whether an explicit teaching of finger-counting (by touch) may also improve the arithmetic abilities of visually impaired children presenting mathematics learning difficulties (MLD). A tactile finger training was therefore proposed to 2 visually impaired children presenting MLD and to 18 younger sighted children (the finger group), who were selected based on their mathematical abilities to match those of the visually impaired children. Another group of 18 sighted children underwent a vocabulary control training. As the 2 visually impaired children presented the same mathematical level as the sighted children, it was possible to compare the effects of the finger training in both populations. According to the sensory compensation hypothesis, the lack of vision can be compensated by one of the remaining senses (Bell et al., 2019; Braun, 2016). We therefore hypothesize that our finger training will induce similar positive impacts on the arithmetic performances of both populations.

Methods

Design

Based on the models of early number development mentioned above (Krajewski & Schneider, 2009; Roesch & Moeller, 2015), our finger-counting training included 3 different steps. First, children were trained to identify and dissociate each finger (i.e., finger gnosis training). Second, children learned to associate each finger with a specific number word, to count and to represent a quantity (i.e., finger-counting training). Finally, they learned how to solve basic arithmetic operations (simple addition and subtraction) with their fingers ($3+5$ was for example solved by raising 3 fingers on one hand and 5 fingers on the other hand). The efficiency of the training was assessed by asking children to perform addition, subtraction, and multiplication operations both before and after the training sessions. Performances of the sighted children (the finger group) were compared to those of the two visually impaired cases (VI1 and VI2) and to those of another group of sighted control children to whom stories were read (the vocabulary group).

Participants

VI1 was a boy of 8.5 years old and VI2 a boy of 7.5 years old. These two VI children were referred to us by their teachers as presenting mathematics difficulties but no other associated troubles. Both were native French speakers living in Belgium, right-handed and in the 2nd grade of primary school. They both suffered from congenital visual impairment. VI1 had oculocutaneous albinism with visual acuity of 1/20 in both the left and right eyes and 1/10 in binocular vision. VI2 had nystagmus with visual acuity of 4/10 in the left eye, 3/10 in the right eye and 5/10 in binocular vision with correction.

The finger sighted group was composed of 18 children (8 boys and 10 girls, 2 left-handed, 2 bilinguals; $M_{AGE} \pm SD =$ 5.9 years ± 0.6) from kindergarten ($N = 10$) and 1st grade of primary school ($N = 8$). The vocabulary sighted group also included 18 children (10 boys and 8 girls, 1 left-handed, 2 bilinguals; $M_{AGE} \pm SD =$ 5.9 years ± 0.6) from kindergarten ($N = 11$) and 1st grade of primary school ($N = 7$). To take part in the study, sighted children had to meet the following inclusion criteria: have a corrected-to-normal vision, be a native French speaker living in Belgium and be able to count up and down between 1 and 10. These counting skills were evaluated by asking the children to count orally as far as possible (they were stopped at 31 if they went that far) and count backwards from 15 (Tedi-math battery, Van Nieuwenhoven et al., 2001). Children with a neuro-cognitive trouble were excluded from the study.

Parents gave a written consent for the participation of their child and the procedures were approved by the local research ethics committee.

Procedure

The finger and vocabulary groups included eighteen children each. This sample size was determined based on a previous study using a similar training method (Honoré & Noël, 2016) and power analyses for case-control comparisons (McIntosh & Rittmo, 2021). Children from both groups were selected to be matched in terms of arithmetic abilities, and to match the mathematical level of the two VI cases. These abilities were evaluated with an oral version of the TempoTest Automatiseren (TTA, De Vos, 1992), requiring children to perform a maximum of arithmetic operations (addition, subtraction, multiplication) in a limited time (2 min for each operation). An independent samples t-test confirmed that there was no difference between the finger ($M \pm SD = 5.61 \pm 2.77$) and the vocabulary groups ($M \pm SD = 5.67 \pm 2.77$), $t(34) = -0.06$, $p = .952$.

The TTA scores of both VI cases were compared to those of the control children with the Crawford modified t-test (program Singlims_ES.exe; Crawford & Howell, 1998; Crawford & Garthwaite, 2002). Results showed that VI1's addition performance before training did not differ from that of the control group (VI1's score = 7, Mean score of the control group = 5.64, SD = 2.73, $t = 0.49$, $p = .63$), nor did VI2's performance (VI2's score = 3, $t = -0.95$, $p = .35$). The same results were obtained for subtraction (VI1's and VI2's score = 6, Mean of the control group = 3.69, SD = 2.29, $t = 0.99$, $p = .33$) and multiplication (VI1's and VI2's score = 0, Mean of the control group = 0.08, SD = 0.28, $t = -0.29$, $p = .77$).

Children were also matched in terms of verbal IQ, evaluated with the subtest Information from the fifth Edition of the Wechsler Intelligence Scale for Children (WISC-V, Wechsler & Dannay-Penhouët, 2016) ($M_{FINGER} \pm SD = 7.39 \pm 2.55$; $M_{VOCABULARY} \pm SD = 8.44 \pm 2.61$; $t(34) = -1.23$, $p = .229$). Analyses performed on the Information subtest scores revealed that VI1's (IQ score = 6) and VI2's (IQ score = 7) verbal IQ were not different from the one of the controls (Mean score of the control group = 7.92, SD = 2.60, both $p_s > .40$).

Training protocol

Children attended 8 individual training sessions of 30 min per week. Due to the school holiday schedule, however, half of the children did the first two sessions in the same week, four days apart.

Finger training

The finger training was composed of three different phases (phase 1: finger gnosis, phase 2: finger-counting and phase 3: finger calculation). During the first two sessions, finger gnosis was trained with a tactile adaptation of the protocol used by Gracia-Bafalluy and Noël (2008). This includes finger identification, association and dexterity. At the beginning of the session, children learned to associate each finger with a specific texture: thumb with velvet, index with sandpaper, middle finger with soft hair, ring finger with silk and little finger with metal chain. They had to place their fingers on textured rectangles organized as if they were piano keys (Figure 1a). Then, children were asked to touch every texture one by one starting with the thumb of the dominant hand to the little finger of the non-dominant hand and then with both hands at the same time. Afterwards, three games were proposed to the kids: the road game, the texture-to-finger game and the finger-to-texture game.

The road game (Figure 1b) consisted of five textured pathways that children had to follow with the corresponding finger (e.g., follow the sandpaper pathway with the index finger, the silk pathway with the ring finger, etc). Children started with the thumb of the dominant hand, then the index finger and so on to the little finger. Then, they had to follow the different pathways with the non-dominant hand (again starting from the thumb). In the texture-to-finger game (Figure 1c), textured rectangles were presented to the children one by one, in a random order. They had to touch it with one hand and show on the other hand the finger that had been associated with the presented texture. The child could choose with which hand (s)he wanted to start (e.g., touch the texture with the right index and show the corresponding finger on the left hand) and then did the same exercise with the other hand (e.g., touch the texture with the left index and show the corresponding finger on the right hand). For the finger-to-texture game (Figure 1d), children had to find back the texture associated with each finger. Fingers (of the dominant hand first and then the non-dominant hand) were touched one by one by the experimenter in a random order. Children had to indicate to which texture it corresponded by selecting the correct texture among all the textures randomly organized. Finally, children were shown specific finger configurations on a wood hand and were asked, after exploring it by touch, to reproduce the configuration with their own hands. Five configurations were presented in the following order: thumb raised, index raised, middle finger raised, ring finger raised and little finger raised (Figure 1e).

The sessions 3 and 4 consisted in a finger-counting training during which children were taught to associate each finger with a specific number (to represent a quantity and to count). Different tasks were used, inspired from Crollen et al. (2011), Jay and Betenson (2017) and Frey et al. (unpublished, cited by Schild et al., 2020). In the first two tasks of session 3, children had to place their hands palm down on the table. Two spacers were used, one before the thumb of the dominant hand, to indicate the start of the counting, and one after the last finger of the counting sequence (e.g., after the middle finger of the dominant hand to count to three, Figure 2a). Children were thus asked to count the number of fingers between the two spacers. The second spacer was moved successively to follow the counting sequence: first, after the thumb of the dominant hand, second, after the index finger of the dominant hand, third, after the middle finger of the dominant hand, and so on until the ten fingers were all included between the two spacers. During this first task, children were encouraged to move the fingers between the spacers to increase their proprioceptive sensation. In a second task, children were asked to tell the number of fingers included between the two spacers, directly, without counting them (Figure 2b). The second spacer was randomly placed after each finger (i.e., not following the counting sequence) in order to consolidate the counting configurations from 1 to 10. The third task of session 3 required children to count from 1 to 10 by raising their fingers, one by one, starting from the thumb of the dominant hand to the little finger of the non-dominant hand (Figure 2c). Children then had to show numbers with their fingers (e.g., can you show me "seven" with your

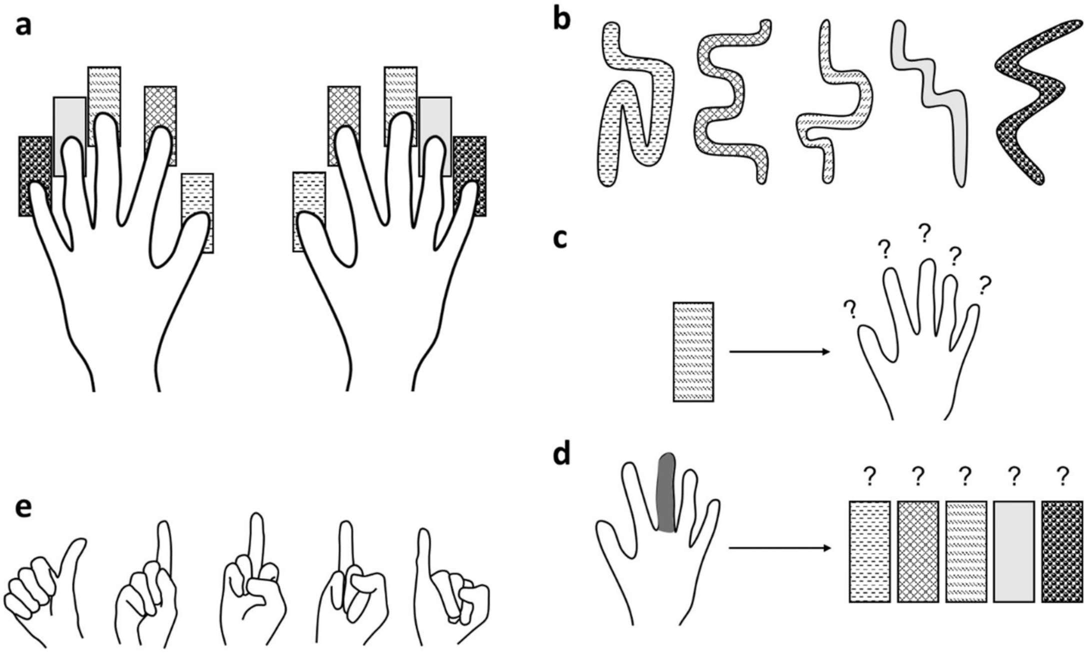

Figure 1. Illustration of the finger gnosis training (sessions 1–2). (a) finger-texture associations, (b) the road game, (c) the texture-to-finger game, (d) the finger-to-texture game and (e) finger configurations to reproduce.

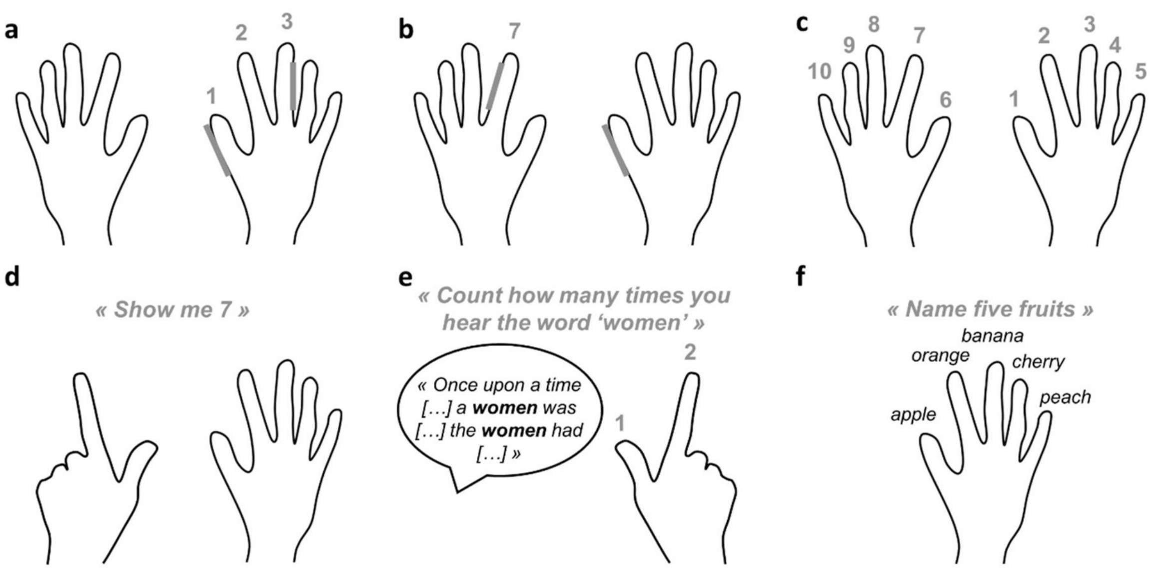

Figure 2. Illustration of the finger counting training (sessions 3–4). Children were asked to (a) count the fingers between two spacers (one by one), (b) identify the number of fingers between two spacers (all at the same time), (c) count from 1 to 10 with fingers, (d) show a specific number with fingers, (e) use fingers to count the occurrence of a target word in a story and (f) use fingers to give a specific number of exemplars from a category.

fingers? Figure 2d) and were trained to raise their fingers simultaneously without counting.

Session 4 started with the same exercise as in the second task of session 3. Children were asked to tell the number of fingers included between two spacers without counting them (Figure 2b). Then, they were required to count forwards or backwards between two specific numbers (e.g., count forward from five to ten or count backwards from seven to two). To do so, they were instructed to represent the first number on their fingers and then to raise or to remove fingers one by one until they reached the second number. Finally, at the end of session 4, children were taught how to use fingers to keep track of a counting sequence. First, they were presented with a short story and were required to count, with their fingers, the number of times they heard a specific target word (e.g., woman, Figure 2e). Children were also asked questions about the content of the story to ensure that they were paying attention to the story itself. Second, children were asked to name a specific number of exemplars from different categories (e.g., can you give me five names of fruits?, nine boy's names?, Figure 2f). Children were again required to use their fingers to keep track of the number of items uttered.

The last four sessions of the training (sessions 5–8) were dedicated to the resolution of arithmetic operations with the fingers (the finger calculation phase). The sessions always started with the task "can you show me *N* with your fingers?" to ensure that the children were still able to represent the numbers 1–10. Then, twenty addition and twenty subtraction operations were proposed to the children. For the first ten operations, children could be helped by the experimenter and some materials (e.g., spacers), but had to perform the last 10 operations by themselves.

Over the course of the training sessions, children were taught finger strategies of increasing complexity (Baroody, 1987, 2006; Barrouillet, 2006; Carpenter & Moser, 1984; Siegler, 1987; Siegler & Shrager, 1984). To solve addition, session 5 started with the most basic strategy called "counting all": to calculate a sum (e.g., $2+4$), children represented each operand on their hands (e.g., raised 2 fingers on one hand and 4 fingers on the other hand) and then counted all the fingers raised (Figure 3a). In sessions 6 and 7, children learned the "counting min" strategy: to solve $2+7$, children were required to represent the largest of the two operands with their fingers (i.e., 7) and then to count until they had reached the second operand on their fingers (Figure 3b). Children were first presented addition operations whose sums were always equal to 10 (session 6) and then different (but smaller) than 10 (session 7). Finally, in session 8, children were trained to solve addition whose sum was between 10 and 20. To do so, they had to mentally represent the largest operand (e.g., 12) and then to count on their fingers until they had reached the second operand (Figure 3c).

To solve subtraction (Barrouillet, 2006; Carpenter & Moser, 1984), children started with the "separating from" strategy which consists in representing the first operand on the fingers (e.g., 9) and then removing the second operand (e.g., 2, Figure 4a). During the first session (session 5), the fingers corresponding to the second operand could be removed one by one, then, they had to be removed simultaneously (sessions 6 and 7). In session 6, children were presented subtraction whose first operand was always 10 (e.g., 10 – *N*). In session 7, they were presented subtraction whose first operand was different (but smaller) than 10 (e.g., 9 – *N*). Finally, children learned to solve subtraction whose first operand was bigger than 10 with the "counting down from" strategy: they had to mentally represent the first operand (e.g., 12) and then had to count backwards while raising the number of fingers corresponding to the second operand of the subtraction (Figure 4b).

Vocabulary training

For the control vocabulary training (Honoré & Noël, 2016; Crollen et al., 2020), short stories (adapted to the children's age) were read to the children in each session and the meaning of five difficult words was explained at the end of the story (see Table 1). To add some variability in the training and encourage the child to stay focused during the word explanation, different strategies were proposed: one of the words was simply defined with a dictionary, two other words were explained with pictures and finally, the last two words were defined and the children were then required to use them in a sentence.

Pre- and post-training measures

Different tasks were submitted to the children before and after the eight training sessions, in order to evaluate the efficiency of the training. The pretest was done maximum two weeks before the first training session and the post-test

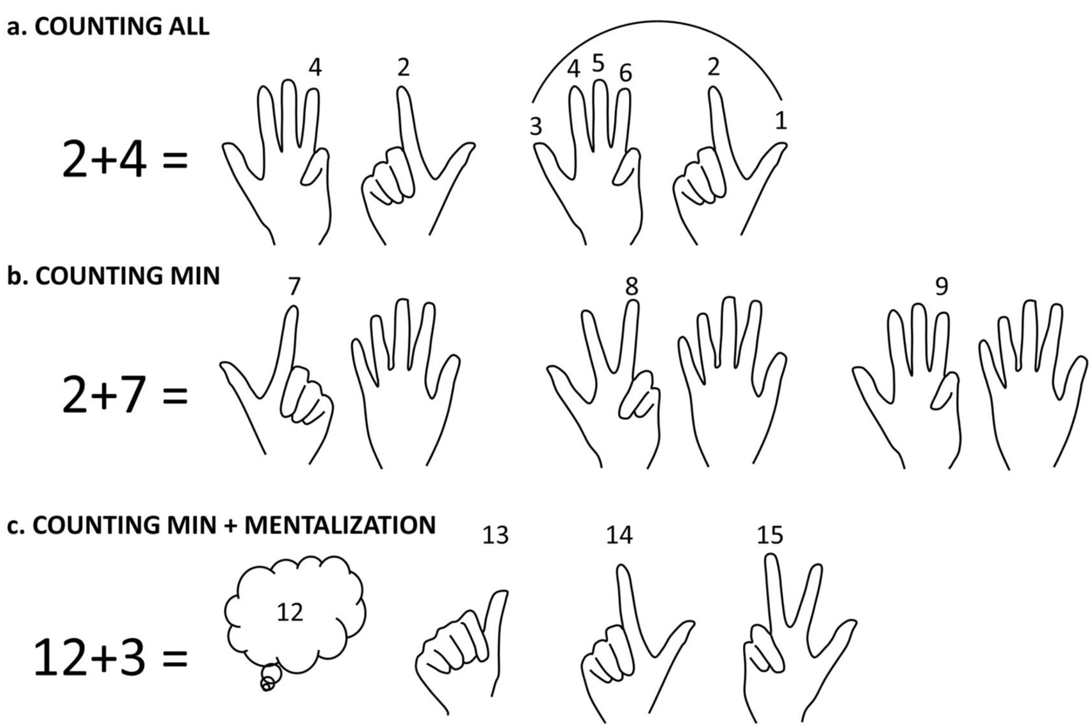

Figure 3. Illustration of the finger calculation training for additions. Children were taught (a) the "counting all" strategy, (b) the "counting min" strategy and (c) the "counting min" strategy with mentalization.

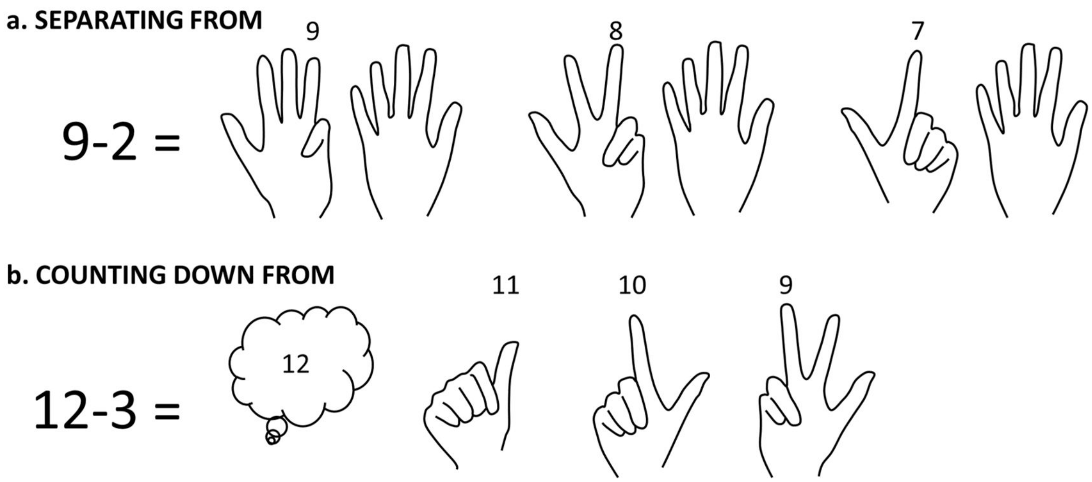

Figure 4. Illustration of the finger calculation training for additions. Children were taught (a) the "separating from" strategy and (b) the "counting down from" strategy.

Table 1. Vocabulary control training.

Training	Story	Words
Session 1	"La sorcière qui voulait être laide" (*The witch who wanted to be ugly*) Renaud and Villeminot (2009)	• "gracieux" (*graceful*) • "joufflu" (*chubby*) • "croupie" (*rancid*) • "infesté" (*infested*) • "pachyderme" (*pachyderm*)
Session 2	"Nino dino, t'es plus mon copain !" (*Dino Nino, you're no longer my friend*) Mim (2019)	• "protester" (*protest*) • "mastodonte" (*mastodon*) • "mastiquer" (*chew*) • "glousser" (*chuckle*) • "ronchonner" (*moan*)
Session 3	"Le Pays sans fleurs" (*The land without flowers*) Conte d'Océanie, No author (n.d.)	• "jadis" (*formerly*) • "berge" (*riverbank*) • "croître" (*expand*) • "étendue" (*stretch*) • "s'efforcer" (*strive*)
Session 4	"Martine à la montagne" (*Martine goes to the mountains*) Delahaye and Marlier (1993)	• "se tourmenter" (*fuss*) • "piolet" (*ice axe*) • "marchepied" (*step stool*) • "téléphérique" (*cable car*) • "élan" (*momentum*)
Session 5	"Les aventures de Pinocchio" (*Pinocchio*) Collodi (1997)	• "repentant" (*repentant*) • "braire" (*bray*) • "naïf" (*naive*) • "pantin" (*puppet*) • "surgir" (*spring up*)
Session 6	"Le vilain petit canard" (*The ugly duckling*) Morand and Pons (2006)	• "se ruer" (*stampede*) • "marécage" (*chubby*) • "couver" (*brood*) • "jonc" (*bulrush*) • "barboter" (*dabble*)
Session 7	"Le petit chaperon rouge" (*Little red riding hood*) Schlossberg et al. (2006)	• "lisière" (*fringe*) • "chevillette" (*peg*) • "ricaner" (*giggle*) • "bobine" (*coil*) • "chaumière" (*cottage*)
Session 8	"Cendrillon" (*Cinderella*) Disney Adaptation (1995)	• "émoi" (*commotion*) • "ébloui" (*dazzled*) • "orpheline" (*orphan*) • "triomphe" (*triumph*) • "enchantement" (*enchantment*)

maximum seven days following the last training session. Pre- and post-training measures first included a test of finger gnosis. In this test, children were required to identify the fingers touched by the experimenter. The stimulated hand was hidden from the child's view (Noël, 2005). To give their responses, children had a wooden hand at their disposal and had to show the finger touched by the experimenter on this wooden hand. One point was given for each correct identification, with a maximum score of 60. Then, children's use of finger-counting was assessed with the PAJI task (Crollen et al., 2011). Children were exposed to 10 series of two phonetically dissimilar syllables (/pa/and/ji/) and instructed to count the numbers of/pa/and/ji/sounds in a sequence (e.g., "/pa/,/ji/,/ji/,/pa/,/pa/,/ji/" = 3x/pa/and

3x/ji/). One point was attributed for every correct response of/pa/and/ji/syllables count (maximum score of 20). The number of times children spontaneously used their fingers to count the number of syllables was also calculated (maximum score of 10). Finally, children's mathematics abilities were evaluated with the TempoTest Automatiseren (De Vos, 1992), requiring them to perform a maximum of arithmetic operations (addition, subtraction, multiplication) in a limited time (2 min for each operation). The arithmetic problems were presented orally to both sighted and visually impaired children, who were instructed to respond as quickly as possible. One point was given for each correct response.

Statistical analyses

Raw scores were analyzed for each measure (i.e., finger gnosis, finger-counting and arithmetic). Results of the finger and vocabulary groups (see Table 2) were first compared and analyzed with a 2 (*session*: pre- *vs*. post-training) × 2 (*group*: finger *vs*. vocabulary) mixed model ANOVA. Then, results of both VI cases (see Table 2) were compared to the performance of the sighted children with a Crawford modified t-test (Crawford & Howell, 1998). To do so, a gain score was computed by subtracting children's performance in the pre-training session from children's performance in the post-training session.

Results

Finger gnosis

In the finger gnosis task, results of the ANOVA did not show any effect of the session, $F(1,34) = 2.83$, $p = .102$, of the training, $F(1,34) = 0.80$, $p = .378$, and no interaction between those two factors, $F(1,34) = 0.49$, $p = .489$. No significant gain was highlighted in the finger group ($M_{GAIN} \pm SD = 1.06 \pm 4.78$), nor in the vocabulary group ($M_{GAIN} \pm SD = 2.56 \pm 7.75$). The gain obtained by VI1 was not different from that of the finger group (VI1's gain $= -4$, Crawford $t = -1.03$, $p = .317$), nor was VI2's gain (VI2's gain $= 4$, Crawford $t = 0.60$, $p = .557$) (Figure 5).

Finger-counting

Analyses on the accuracy scores of the PAJI task revealed a main effect of the session (post > pre), $F(1,34) = 57.00$, $p < .001$, $\eta_p^2 = 0.626$, a main effect of the training (finger > vocabulary), $F(1,34) = 18.33$, $p = < .001$, $\eta_p^2 = 0.350$, and most importantly, a significant interaction between those two factors, $F(1,34) = 18.57$, $p = < .001$, $\eta_p^2 = 0.353$ (Figure 6a). While children's performance in the pre-training session was similar in both groups (Group difference = 0.11, $p = .887$), the finger group outperformed the vocabulary group in the post-training session (Group difference = 7.06, $p < .001$, Table 2). This means that the gain obtained by the finger group ($M_{GAIN} \pm SD = 9.56 \pm 5.66$) was significantly greater than the one obtained by the vocabulary group ($M_{GAIN} \pm SD = 2.61 \pm 3.84$), $t(1,34) = 4.31$, $p < .001$. Furthermore, VI1's gain was not different from that of the finger group (VI1's gain = 9, Crawford $t = -0.10$, $p = .924$), nor was VI2's gain (VI2's gain = 7, Crawford $t = -0.44$, $p = .665$). Children from the finger group and both VI cases were therefore more accurate at counting the/pa//ji/syllables in the post-training session than in the pre-training session (Figure 6a).

In terms of finger use, a main effect of the session was found (post > pre), $F(1,34) = 32.90$, $p < .001$, $\eta_p^2 = 0.492$, no main effect of the training, $F(1,34) = 0.93$, $p = .342$, but a significant interaction between those two factors, $F(1,34) = 11.95$, $p = .001$, $\eta_p^2 = 0.260$ (Figure 6b). While both groups showed an equivalent use of fingers in the pre-training session (Group difference = 1.61, $p = .572$), children from the finger group, but not those of the vocabulary group, significantly increased their use of finger in the post-training session (Session difference for finger group = 6.94, $p < .001$; Session difference for vocabulary group = 1.72, $p = .232$, Table 2). The gain obtained by VI1 and VI2 was not different from that of the finger group (VI1's and VI2's gain = 10, Mean of the control group = 6.94, SD = 4.39, Crawford $t = 0.68$, $p = .507$). Like

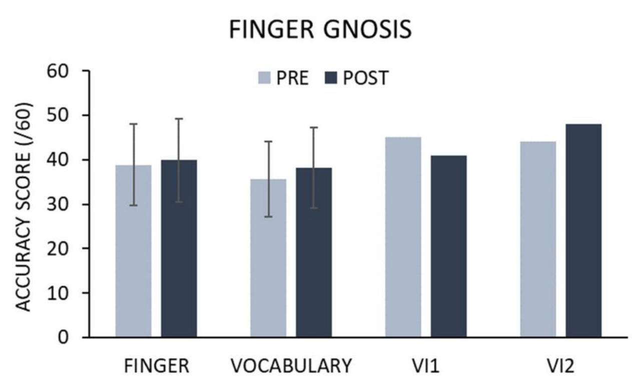

Figure 5. Finger gnosis results. Results in the pre- and post-training sessions for the finger group, the vocabulary group, VI1 and VI2. Bars represent standard error of the mean.

Table 2. Pre- and post-training raw scores for the finger group, vocabulary group, VI1 and VI2.

	Pre-training				Post-training			
	Finger *M(SD)*	Vocabulary *M(SD)*	VI1	VI2	Finger *M(SD)*	Vocabulary *M(SD)*	VI1	VI2
Finger Gnosis	38.83 (10.46)	35.61 (7.75)	45	44	39.89 (10.96)	38.17 (5.23)	41	48
Paji								
Accuracy	2.11 (2.56)	2 (2.09)	1	0	11.67 (4.30)	4.61 (4.38)	10	7
Use of fingers	3.06 (4.39)	4.67 (4.54)	0	0	10 (0.00)	6.9 (4.42)	10	10
Arithmetic								
Addition	5.61 (2.77)	5.67 (4.06)	7	3	10.56 (3.97)	7.39 (3.52)	12	10
Subtraction	3.33 (2.09)	4.06 (2.48)	6	6	7.5 (2.01)	4.83 (2.28)	10	5
Multiplication	0.06 (0.24)	0.11 (0.32)	0	0	0.56 (1.29)	0.06 (0.24)	2	1

children in the finger group, both VI1 and VI2 were indeed using their fingers much more in the post-training session than in the pre-training session (100% of the time in the post-training session *versus* 0% of the time in the pre-training session, see Table 2, Figure 6b).

Arithmetic

Regarding the addition operations, results showed a main effect of the session (post > pre), $F(1,34) = 137.99$, $p < .001$, $\eta_p^2 = 0.802$, no main effect of the training, $F(1,34) = 2.15$, $p = .152$, but a significant interaction between those two factors, $F(1,34) = 32.24$, $p = < .001$, $\eta_p^2 = 0.487$ (Figure 7a). Before the training, children's performance in solving addition operations was not different between both groups (Group difference = 0.06, $p = .952$). In contrast, after the training, the finger group performed significantly better than the vocabulary group (Group difference = 3.17, $p = .032$, Table 2). Therefore, the finger group improved its performance much more than the vocabulary group (Finger group:

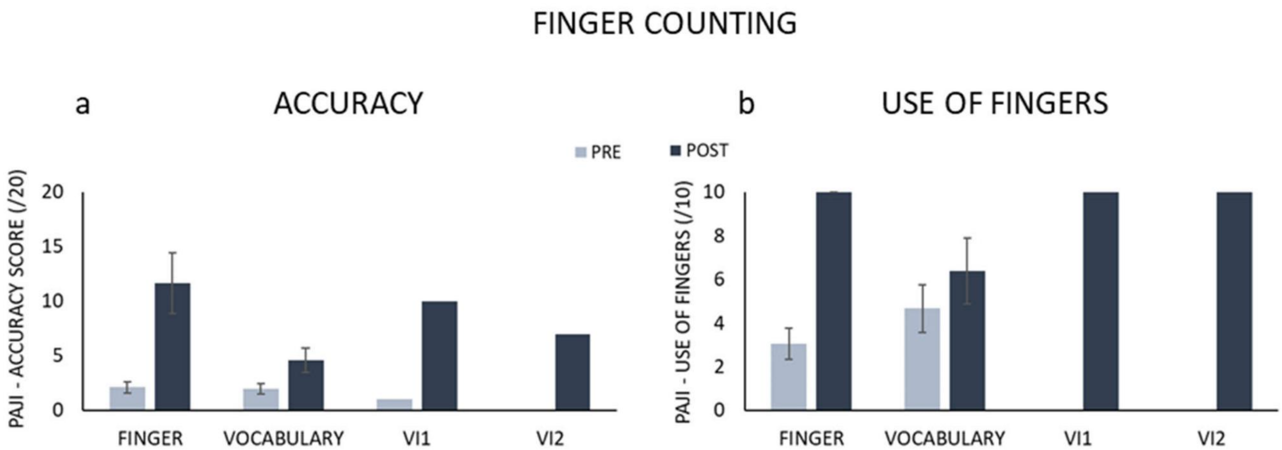

Figure 6. (a) PAJI accuracy scores and (b) PAJI use of fingers in the pre- and post-training sessions for the finger group, vocabulary group, VI1 and VI2. Bars represent standard error of the mean.

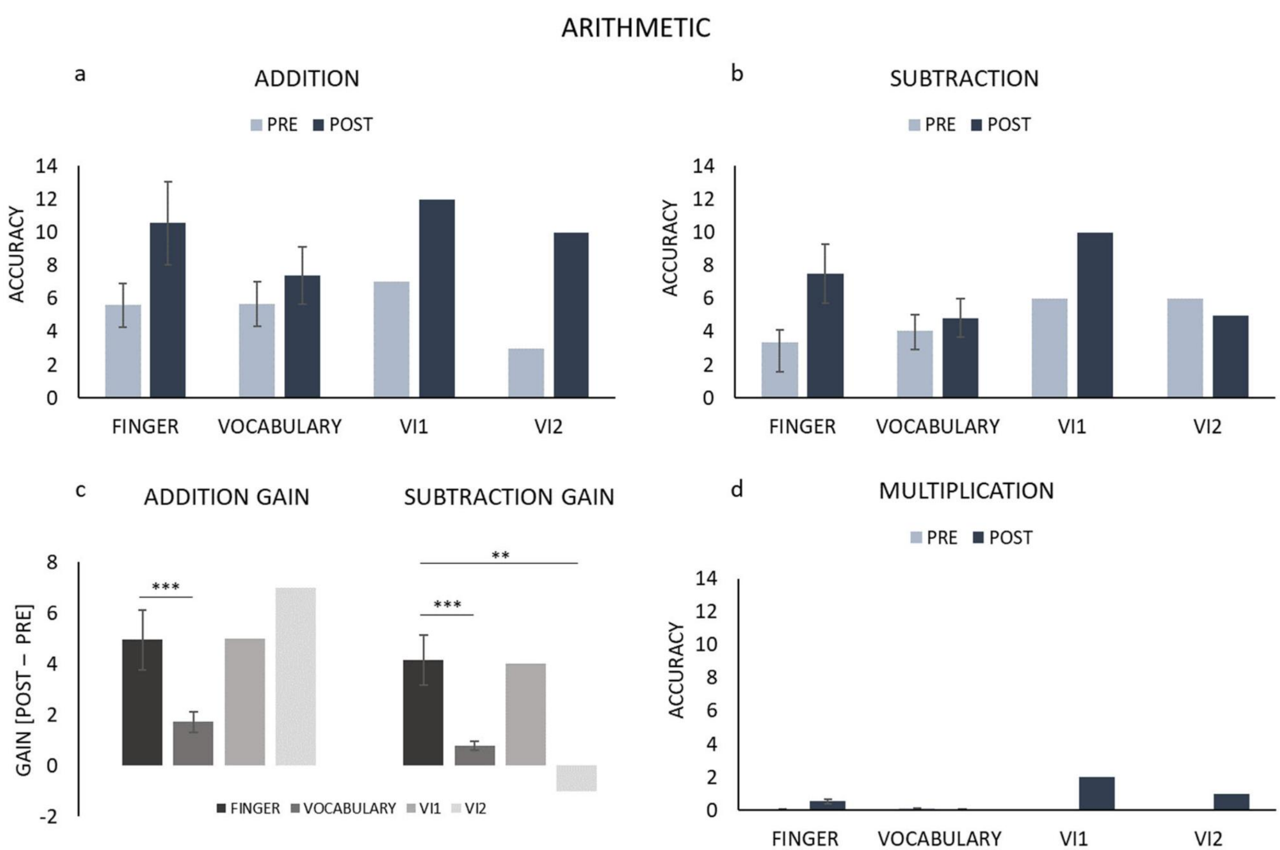

Figure 7. Children's performance for (a) Addition operations and (b) Subtraction operations. (c) Arithmetic gain for addition and subtraction, calculated as the difference between the pre- and post-training scores. (d) Children's multiplication resolution performances in pre- and post-training. Bars represent standard error of the means. Asterisks indicate a significant difference with $p < .001$ (***) or $p < .01$ (**).

$M_{GAIN} \pm SD = 4.94 \pm 1.63$; Vocabulary group: $M_{GAIN} \pm SD = 1.72 \pm 1.78$), $t(1,34) = 5.68$, $p < .001$ (Figure 7c). Moreover, VI1's and VI2's gain in addition was similar to that of the finger group (VI1's gain = 5, Crawford $t = 0.04$, $p = .972$; VI2's gain = 7, Crawford $t = 1.23$, $p = .235$). Both VI cases and the sighted children from the finger group thus benefited from the training to improve their addition resolution performance.

When it comes to subtraction, a main effect of the session was found (post > pre), $F(1,34) = 79.92$, $p < .001$, $\eta_p^2 = 0.702$, no main effect of the training, $F(1,34) = 2.00$, $p = .166$, but a significant interaction between those two factors was highlighted, $F(1,34) = 37.54$, $p = < .001$, $\eta_p^2 = 0.525$ (Figure 7b). While the finger and vocabulary groups started with a similar subtraction resolution performance in the pre-training session (Group difference = 0.72, $p = .704$), only children from the finger group significantly increased their performance in the post-training session (Session difference for the finger group = 4.17, $p < .001$; Session difference for the vocabulary group = 0.78, $p = .110$, Table 2). VI1's subtraction gain was not different from that of the finger group (VI1's gain = 4, Mean of the control group = 4.17, SD = 1.65, Crawford $t = -0.10$, $p = .921$), but VI2's gain was significantly different from the gain obtained by the finger group (VI2's gain = −1, Crawford $t = -3.05$, $p = .007$, Figure 7c). VI2's gain in subtraction was indeed lower than 99.64% (95% CI = [97.64–100.00]) of the sighted children's gain.

The scores for multiplication were very low for all children, both before and after training (Figure 7d). In fact, over 85% of the children were unable to solve any multiplication operations in both the pre- and post-tests. These results were expected, as children in Belgium typically start learning multiplication during the second year of primary school. Consequently, we decided not to conduct further analysis on these specific results.

Discussion

The aim of the present study was to examine whether an explicit teaching of finger-counting by touch may help visually impaired children to overcome their mathematics learning difficulties. Sighted and visually impaired children were therefore trained to use their fingers to calculate. The efficiency of the training was evaluated within and across populations. Our data first demonstrated that the training had no effect on finger gnosis abilities. Fischer et al., 2022 This finding aligns with prior research which suggests that when general cognitive abilities are considered, the correlation between finger gnosis and arithmetic performance decreases (Long et al., 2016; Wasner et al., 2016). However, it is crucial to acknowledge that some children already demonstrated high scores before training. Hence, it is advisable to thoroughly evaluate these competencies before implementing finger counting techniques. It could even be more important in the visually impaired population as these children need to be sensorily aware of which fingers they are moving when counting. As visual control cannot efficiently guide these children, finger sense or finger gnosia could be the most important skill required in performing this task. Moreover, in case of visual impairment, spatial proximity between the fingers often lead to interference in fingers' discrimination. This interference can even be more pronounced in Braille readers using several fingers to read (while VI2 was not learning braille, we don't have information regarding VI1). These individuals were indeed shown to present a disordered or fused cortical representation of their reading fingers (Sterr et al., 1998). This induces misperception of which fingers have been touched and can in turn be maladaptive for finger-counting learning. To further explore this possibility, future studies should compare the efficiency of finger-counting training in children exposed to Braille *versus* those not exposed.

In line with this, Crollen et al. (2011) reported that blind children reading Braille, when using their fingers, often use their both hands to represent even numbers in a symmetric way (e.g., raising the thumb, index and middle fingers of both hands to represent 6). This strategy induces the activation of two body maps (a finger schema as well as a hand schema; Haggard et al., 2006) and is therefore probably used to enhance children's proprioceptive discrimination acuity. Even if this strategy seems adaptative, the present paper did not adopt it but rather relied on the teaching of typical Belgian finger-counting habit. Three reasons supported this decision. First, it has already been shown that children benefit most of the finger-counting procedure when the procedure sticks to a stable motor sequence (De La Cruz et al., 2014; see Kamawar et al., 2010 for similar results with object counting). Second, it was suggested that explicit teaching of finger counting could help less gifted children to overcome their arithmetic difficulties (Dupont-Boime & Thevenot, 2018; Poletti et al., 2022b). Third, judgements concerning the amount of tactually stimulated fingers were found to be faster and more accurate when the stimulated fingers were close to each other than when they consisted of non-neighboring fingers (Cohen et al., 2014). The present study accordingly highlighted that systematically teaching finger-number associations can improve basic numerical abilities of both sighted and visually impaired children. Our 2 VI cases indeed used the Belgian typical finger-counting strategy much more in the post-training session than in the pre-training session. They did so efficiently (accuracy increase) and similarly to the sighted children of the finger group.

Importantly, learning the typical Belgian finger-counting not only supports counting abilities but also sustains the execution of basic arithmetic operations in both populations. Sighted children of the finger group indeed improved their addition and subtraction resolution performances (much more than the children in the vocabulary group). This observation supports neurocognitive studies suggesting that finger-counting may be a building block for arithmetic learning in sighted individuals (Dupont-Boime & Thevenot, 2018; Krenger & Thevenot, 2024; Moeller et al., 2011). The systematic use of canonical finger configurations to calculate indeed gives these configurations a specific status in long-term memory: they are automatically produced and recognized (Marlair et al., 2021). This will in turn allow the child to progressively move from an externalized finger-based representation to an internalized calculation procedure. Our results demonstrated that this conclusion can be extended to the visually impaired population. However, while VI1 improved his addition and subtraction resolution performances, VI2 only improves his resolution of addition operations. This observation does nevertheless not question the idea that finger-counting can be learned by touch, with or without residual vision. Finger-counting can therefore be considered as a multisensory (proprioceptive and visual) tool conveying ordinal and cardinal information about numbers (Moeller et al., 2011; Sixtus et al., 2023). While it has already been demonstrated that finger-counting can be used without proprioceptive information (Poeck, 1964), the present paper suggests that finger-counting can also be used without visual information or with impaired visual information.

Visually impaired children not only lack the ability to observe others performing finger-counting but also miss the visual experience of their own hands. Consequently, it is not surprising that they do not exhibit finger-counting behavior inspired or patterned by observing others. However, given their presumably unimpaired experience of their own hands, including internal or embodied sensation, proprioception, and motor control, it is surprising to note that they rarely display idiosyncratic finger-counting (finger-counting behavior not inspired by others). In contrast to Piaget's claim in his genetic epistemology theory, this suggests that fingers are not essential for activating and accessing the mental representation of numbers (Piaget, 1950, 1970, 1972). Instead, they serve as a convenient external tool that informs and influences numerical development, similar to tallies, abacuses, and notations (Newen et al., 2018; Overmann, 2023; Radman, 2013; Zahidi, 2021).

To sum up, our results support a new line of research demonstrating that vision is not as necessary as previously assumed for the development of numerical skills (for a review see Crollen & Collignon, 2020). Previous studies on early visual deprivation have indeed shown that blind individuals can compensate their lack of vision and perform efficiently in various numerical tasks, such as counting (Crollen et al., 2014) and arithmetic (Dormal et al., 2016). These findings led some researchers to suggest that blind and sighted children might develop an understanding of numbers through different pathways (Crollen & Collignon, 2020). While using fingers to keep track of the counted elements or to learn calculation is quite common in sighted children, it is much less present in blind children (Crollen et al., 2011). However, because the lack of vision enables the brain to access new pathways in the quest to overcome the visual limitation (Kanjlia et al., 2016; Crollen et al., 2019), the lack of finger-counting doesn't have any negative impact on the blind's numerical performance. Visually impaired children, in contrast, often experience difficulties in following the process of mathematics learning (Bell & Silverman, 2019; Cryer et al., 2013). As they get older, the gap between them and their sighted peers progressively increases (Giesen et al., 2012), with 75% of visually impaired children being one grade behind and 20% being four or more grades behind (Gulley et al., 2017). Like blind children, these children do not use the finger-counting strategy spontaneously but are, at the same time, not sufficiently relying on their remaining senses to develop efficient compensatory strategies alone. Visually impaired children typically need a variety of opportunities to explore the world by touch (Gast et al., 1992; Wright & Wright, 1998). Some accommodations should therefore be done to give these children a safe and full access to the benefits of finger-counting. Once these accommodations are afforded to the children, they will be able to use finger-counting as an appropriate physical calculation resource.

Limitations of the study

First, our finger-counting training might have been too short to enhance the subtraction performance of our VI2 case. In Belgium, arithmetic operations are always learned in the following order: addition first, subtraction second and then multiplication and division. As VI2 was one year younger than VI1, he may have had less knowledge of the subtraction procedure and may therefore have needed more practice sessions to improve his subtraction performance. Second, our finger-counting training involved 3 different steps (i.e., finger gnosis training, finger-counting training, simple arithmetic training). Given the absence of significant improvement in finger gnosis, it is reasonable to dismiss it as a contributing factor to the positive effect we observed. However, identifying the specific role of the two other phases remains challenging. Future studies should address this limitation by evaluating children's performances after the completion of each step. Finally, as the current study involved only two visually impaired cases, it is important not to hastily extrapolate the findings to larger populations. Testing children with diverse profiles (e.g., including various visual and associated troubles), would be beneficial to better delineate the necessary components of effective finger-counting training.

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Authors contributions

Conceptualization (CM, AG, MV, VC); Data curation (CM, VC); Formal analysis (CM); Funding acquisition (CM); Investigation (CM, AG, MV); Methodology (CM, AG, MV, VC); Project administration (CM, VC); Resources (CM, VC); Software (CM, VC); Supervision (CM, VC); Visualization (CM, VC); Roles/Writing - original draft (CM, VC); and Writing - review & editing (CM, VC).

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Data availability statement

The data supporting the findings of the current study are available within the supplementary material.

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