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Interactions of Imbalance Settlement with Energy and Reserve Markets in Multi-Product European Balancing Markets

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Abstract

This paper provides a framework for analyzing the interaction of imbalance settlement with the clearing of real-time energy and reserve markets. We characterize the optimal strategies of price-taking flexibility providers that can participate in sequential capacity auctions for automatic and manual frequency restoration reserves, followed by an auction that is conducted by the system operator for activating balancing energy. We establish equilibria based on three market features: (i) reserve demand curves, (ii) the activation strategy implemented by the system operator, and (iii) the imbalance settlement scheme. The optimal activation strategy is derived and the effect of the imbalance pricing scheme on bidding incentives, cost efficiency, and reserve prices is discussed.

Keywords: Electricity Markets Design, Balancing Markets, Multi-Product Markets, Reserve

1. Introduction

Balancing the market is the last step in the sequence of electricity markets. It refers to the real-time dispatch of electricity assets in order to balance electricity generation and consumption. The efficiency of this process

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[1]The views expressed in this paper do not necessarily reflect the position of ACER.

is a critical element for the success of the energy transition, as it ensures the reliability of electricity power grid. In Europe, this process has evolved from a power system mechanism operated by vertically integrated monopolies to a sequence of geographically integrated *balancing markets* with the liberalization of the electricity sector and the creation of *transmission system operators* (TSOs) Meeus (2020). An emerging challenge when balancing the market is the increased short-term uncertainty that results from the energy transition Goodarzi et al. (2019). This increased uncertainty is caused by the fast-paced integration of renewable resources, the increased engagement of demand response, and the decentralization of energy supply.

The balancing process consists of system operators reacting to the real-time *imbalance* between electricity consumption and generation by activating upward or downward *balancing energy* from flexible power plants. The activation is decided through *balancing energy auctions.* System operators have a variety of balancing products at their disposal for covering imbalance and maintaining grid frequency. These products have different technical characteristics and are tailored to specific balancing issues. They are also traded on different time scales. This paper proposes a game-theoretical framework for understanding the interaction between different balancing products in a multi-product balancing system comprised of multiple distinct balancing energy auctions. Three issues are specifically investigated in this work: (i) the strategy of the system operator for activating balancing energy, (ii) the imbalance settlement scheme, and (iii) the demand curves for balancing capacity.

The activation strategy of the system operator refers to the trade-off that is faced by the system operator when allocating the imbalance between balancing products. Balancing the market is a continuous process and, broadly speaking, it involves the activation of fast balancing products with high reactivity over a short duration versus slow balancing products with low reactivity over longer duration. This tradeoff is not unique to European systems. Every electricity system has to deal with electricity imbalances to ensure the stability of the power grid. The fast balancing product studied in this paper is the *automatic frequency restoration reserve* (aFRR) product of the European market, which can react almost instantaneously to a dispatch order, whereas the slow balancing product analyzed in this work is the *manual frequency restoration reserve* (mFRR) product of the European market, and we assume that it cannot be adjusted in the very short term. The balancing process can be modelled as a sequence of short-term energy auctions for aFRR preceded by one longer-term energy auction for mFRR. The standard practice in Europe is a sequence of 225 four-second aFRR auctions preceded by a fifteen-minute mFRR auction. In terms of operation, the system operator sets a demand for mFRR balancing energy over the next fifteen minutes and the leftover imbalance, the original imbalance minus the mFRR balancing energy, is covered every four seconds in the aFRR activation auctions. The system operator activation strategy refers to the process for determining the demand for mFRR balancing energy. Reasoning about this process in terms of a two-stage decision process, the activation strategy can be interpreted as a first-stage decision whereas the demand for aFRR balancing energy is a recourse decision given the first-stage decision. The activation strategy of the system operator will be referred to as the *mFRR activation strategy* for the remainder of the paper.

The imbalance pricing scheme refers to the mechanism for pricing the demand side of balancing markets, which is represented by imbalances. These imbalances are generated by agents that are connected to the grid and deviate from their traded positions due to forecast errors in renewable energy supply or electricity demand, the sudden loss of assets due to failure, or the intentional deviations from scheduled for the purpose of arbitrage, to name a few reasons.[2] Imbalances are settled over fifteen-minute intervals, that are referred to as *imbalance settlement periods* (ISP), at the *imbalance price*. National TSOs have some freedom concerning the design of the imbalance price, however there exists a decision by the European Union Agency for the Cooperation of Energy Regulators, ACER, to harmonize and standardize imbalance settlement ACER (2020). This decision is referred to as the *imbalance settlement harmonization methodology*, ISHM, wherein article 9 states that the imbalance price should be based on the mFRR and aFRR prices. This paper reviews three imbalance pricing schemes considered in the power market design discourse: (i) the “mean mFRR and aFRR” price, (ii) the “max mFRR and mean aFRR” prices, and (iii) the “mFRR only” price. The first two are derived from the ISHM and constructed by respectively taking the mean of the mFRR and aFRR prices and the maximum of the mFRR and the mean aFRR price, and the “mFRR only” price accounts solely for the mFRR price.

[2] Agents generating imbalances are referred to as *balance responsible parties* (BRPs) in European balancing markets.

The balancing capacity demand curves refer to the administrative demand curves that are set by the TSOs in the procurement auctions for *balancing capacity*. Real-time balancing energy auctions are preceded by day-ahead tenders for balancing capacity, also referred to as *reserve* in US markets. These reserves are procured in order to ensure an adequate supply of balancing energy in real-time. The tendered quantity that is required in order to ensure the reliability of the system is determined through a *reserve dimensioning* process.

Each of these three elements (the TSO activation strategy, the imbalance pricing scheme, and the shape of reserve demand curves) affect the payoff that is associated with the agents' actions and their bidding behavior. The mFRR activation strategy impacts the expected aFRR and mFRR payoffs of the agents. The imbalance pricing scheme modifies the payoff of the self-scheduling option for flexible assets. Certain imbalance pricing schemes can incentivize flexible assets to opt out of the centralized aFRR and mFRR auctions and to self-activate their asset in order to be intentionally in imbalance and enjoy the imbalance price. The balancing capacity demand curve can generate non-zero capacity prices that reflect an opportunity cost for reserving assets, which interplays with different combinations of mFRR activation strategies and imbalance pricing schemes.

The investigation in this paper is related to the ongoing ratification of the ISHM in the Nordic countries (Norway, Sweden, Finland and Denmark) Stattnett et al. (2023). The discussion in these countries revolves around shifting from the "mFRR only" imbalance pricing scheme to the "mean mFRR and aFRR" or the "max mFRR and mean aFRR" approaches, in order to comply with the ISHM.

An additional motivation for this work is connected to the price incidents that occurred in Austria following the connection to the balancing platforms MARI and PICASSO. Austria has experienced aFRR prices above 7500 €/MWh for 0.17% of the PICASSO platform optimization runs over the last 6 months of 2022 ACER (2023). This corresponds to 7.5 hours of sustained extreme prices, which impact the Austrian imbalance cost. The study of the mFRR activation strategy was recommended by ACER as a mean for mitigating the aFRR price incidents.

Our analysis aims at contributing to the electricity market design literature. Analytical and game-theoretical techniques have been used in Fabra et al. (2006) to compare pay-as-bid and uniform pricing in energy auctions under conditions of market power. Similar frameworks are also used in Bushnell and Oren (1994) and Chao and Wilson (2002) in order to analyse coupled reserve and energy auctions. Bushnell and Oren (1994) analyze scoring rules with discriminatory pricing in the reserve auction and Chao and Wilson (2002) proves, through backward induction, that independent capacity and energy auctions do induce a truthful revelation of cost under uniform pricing and in the presence of price-taking agents. Multi-product capacity auctions are investigated in Kamat and Oren (2002) without an energy component. Similar methods have been applied in the analysis of European balancing markets. Ocker et al. (2018) focus on pricing rules for balancing markets and strategic interactions between agents. Ehrhart and Ocker (2021) include the day-ahead wholesale market in their analysis. Cartuyvels et al. (2023) examine the uncoordinated implementation of adders in integrated energy auctions and introduce an outside option to balancing energy auctions through imbalance settlement.

Non-analytical methods have also been used in order to address specific questions of market design in European balancing markets. Agent-based models have been used for investigating the effect of the imbalance pricing scheme van der Veen et al. (2012), market organization Poplavskaya et al. (2020), the introduction of free bids Poplavskaya et al. (2021), and the backpropagation of real-time balancing capacity prices to day-ahead markets Papavasiliou and Bertrand (2021). Petitet et al. (2019) use a simulation-based model in order to analyze the impact of the gate-closure time on operating cost.

Our paper extends the literature on balancing market design by proposing an analysis for the case of multiple reserve products that accounts for real-time balancing constraints and the intricate relationship between faster- and slower-moving reserves. The goal of this paper is to provide a quantitative framework for highlighting the incentives that are generated in a multi-product balancing market and the sensitivity of the market equilibrium to (i) the imbalance pricing scheme, (ii) the activation strategy for slow-moving reserve (mFRR in our case), and (iii) the balancing capacity demand curve.

The four main policy insights uncovered by our analysis are summarized as follows. (1) The "mFRR only" imbalance price allows for simple optimal strategies and prevents self-scheduling from participants. (2) Minimum balancing activation cost can be reached from the "mFRR only" and "mean mFRR and aFRR" imbalance pricing schemes under the optimal mFRR activation strategy. (3) If the optimal mFRR activation strategy is not available, the "mean mFRR and aFRR" imbalance pricing scheme incentivizes self-scheduling in a way that reduces the balancing cost compared to the no-reaction benchmark. (4) The "max mFRR and mean aFRR" imbalance price distorts price signals and induces an inefficient level of self-dispatching.

The remainder of the paper is structured as follows. Section 2 provides an overview on the current framework for balancing markets and their integration throughout Europe, and compares it to US-style real-time markets. Section 3 presents the multi-product balancing market model that is used in our analysis. Section 4 analyzes the impact of the mFRR activation strategy and the imbalance pricing scheme on the balancing energy equilibrium. The introduction of capacity demand curves is discussed in 5. Section 6 illustrates the results of our analytical models on an example and section 7 discusses them. Section 8 concludes.

2. Balancing Markets

Imbalances are covered by activating balancing products sequentially. The first line of defense in European system operation is *frequency containment reserve* (FCR). It is based on frequency control and its full activation time is thirty seconds. FCR is relieved by aFRR, which corresponds to a full activation time of five to seven and a half minutes. The demand for aFRR balancing energy is also set by an automatic controller. aFRR is relieved by mFRR, which has a full activation time of 15 minutes. mFRR is manually activated by TSOs. Finally, certain TSOs rely on *replacement reserve* (RR) to support or relieve mFRR. This paper focuses on the interaction between aFRR and mFRR, since FCR is a capacity product which is <u>not</u> activated through auctions[3] while RR is a regional product, as opposed to a universal balancing product.

One of the first steps in the integration of the European balancing market was to standardize the balancing products Meeus (2020), and this has been followed by several initiatives to allow for the cross-border trading of balancing products . Currently, (i) FCR capacity is jointly procured by the Austrian, Belgian, Czech, Danish, Dutch, French, German, Slovenian and Swiss TSOs, (ii) mFRR balancing energy activation is coordinated by the balancing platform MARI in Germany, Austria, and the Czech Republic, (iii) the activation of aFRR balancing energy is optimized by the balancing

[3]The balancing energy component of FCR is assumed to even out due to the symmetry of upward and downward activation.

platform PICASSO in Germany, Austria, the Czech Republic, Slovenia and Italy[4], and (iv) RR is exchanged through the TERRE initiative. To be more specific on aFRR and mFRR, TSOs connected to the MARI platform submit a demand for mFRR balancing energy over the next fifteen minutes[5], and the platform selects *balancing energy bids* at least cost in order to satisfy the demand given the network topology. Then, independent PICASSO runs are solved every four seconds based on the zonal imbalances that are yet to be covered. mFRR and aFRR balancing energy activated by the platforms is remunerated at the aFRR and mFRR platform prices respectively. The rules for setting the platform prices are framed by decision 2020/01 of ACER and the amendment to this initial decision in Decision 03/2022. This decision is often referred to as the platform *pricing methodologies*.

Two European idiosyncracies not included in this analysis are *dual-price* schemes for imbalance settlement and *direct mFRR activation*. The ISHM permits Member States to employ a dual-pricing structure. In this structure, the settlement of imbalances for a market participant is contingent upon the direction of its own imbalance in relation to the aggregate system position. This investigation considers a *single-price* scheme with a unique imbalance settlement price for all agents generating imbalances. Direct mFRR activation is the process that allows system operators to submit a demand for mFRR balancing energy during an imbalance settlement period. Only *scheduled mFRR activation* occurring at the beginning of an ISP is accounted for in our model.

The US counterparts to European balancing energy markets are the *real-time markets* with nuances concerning the treatment of automatically controlled balancing energy. The US counterpart of aFRR is *regulation reserve*. It is traded in an auxiliary market that does not affect the real-time energy price. This mechanism is similar to the "mFRR only" imbalance pricing scheme, analyzed further below, and is in stark contrast to current European practices for pricing imbalances, which according to some proposals should include an aFRR price component. Another divergence in the balancing process is the trading of balancing capacity. Balancing capacity is equivalently

[4]Italy has decided to suspend its participation in PICASSO starting on the 15th of March 2024.

[5]TSOs are allowed to submit a demand curve for mFRR balancing energy but price-elastic demand bids are ignored in this analysis, and we rather concentrate on the case of price-inelastic demand bids.

referred to as *reserve* in the US and is often co-optimized with energy in real time. The benefit of holding reserve is accounted for in the objective function of the pricing run and this generates real-time prices for reserve which reflect the stress that is experienced by the system. In contrast to the US, balancing capacity is not traded in real time in Europe. It is only traded in the day-ahead balancing capacity auctions. This lack of real-time trading hinders the formation of the balancing capacity prices as pointed out in Papavasiliou and Bertrand (2021); Papavasiliou (2020). The introduction of "adders" on the imbalance price by European TSOs, such as ELIA, the Belgian TSO, has been related to the value balancing capacity in real time Cartuyvels et al. (2023). In Cartuyvels et al. (2023), the authors outline the potential adverse distributive effects for a Member State of unilaterally introducing adders without a market for real-time balancing capacity in an integrated European balancing market.

3. Modelling Framework for Multi-Product Balancing

This section covers the modeling framework of our analysis. We begin by describing the sequence of auctions that constitute the balancing market, and how uncertainty is revealed throughout this process. We then characterize the behaviour of price-taking agents in this context.

3.1. Sequence of Auctions and Revelation of Uncertainty

Our model considers a two-product market, based on the current European practice that has been described previously. It consists of sequential capacity auctions for the procurement of aFRR and mFRR balancing capacity followed by distinct energy auctions for the activation of mFRR and aFRR (see figure 1). Two types of agents can participate in balancing markets in our model. Fast assets can offer both aFRR and mFRR, while slow assets can only offer mFRR.

The capacity auctions (also called *reserve auctions*) are assumed to be held consecutively[6]. The product of higher quality, aFRR, is auctioned first in our analysis. The fast assets that are not selected can compete in the mFRR capacity auction with the slow assets.

[6]The joint co-optimized procurement of aFRR and mFRR balancing capacity is out of scope, although this is a very active field of debate at present, for instance in the UK.

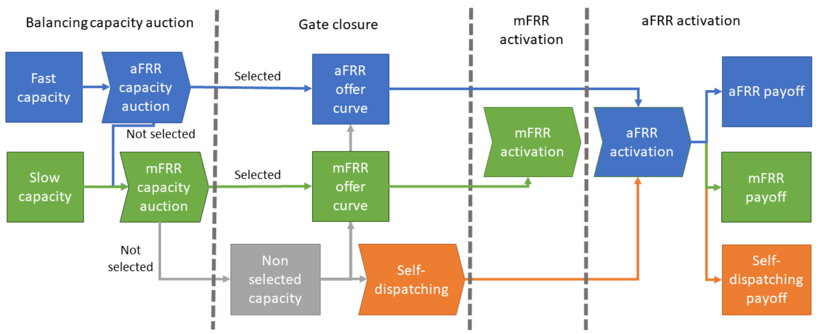


Figure 1: Sequence of auctions and revelation of uncertainty in the balancing markets. Colors depict the balancing product (blue for aFRR, green for mFRR and orange for self-dispatching).

The capacity that is cleared in either the aFRR or the mFRR capacity auction has to be offered to the corresponding energy auction. The capacity that is not cleared can either participate in one of the energy auctions through so-called free bids or it can participate in the balancing process, outside of the centralized markets that are held by the system operator. The latter is referred to as *self-dispatching* and its level is denoted as x^{SD} in our model. Self-dispatching is often referred to as *reactive-balancing*, *passive balancing* or *implicit balancing* in Europe.

The next step is the mFRR energy auction, in which the system operator dispatches/activates an amount x^{mFRR} of mFRR. This auction is followed by T aFRR energy auctions in which the system operator covers the leftover imbalance by dispatching/activating an amount x_t^{aFRR} of aFRR. This results in the following balancing constraint (1) for every sub-period $t = \{1, 2, \ldots, T\}$ given the system imbalance at sub-period t, x_t[7]:

$$x_t - x^{SD} - x^{mFRR} = x_t^{aFRR} \quad \forall t \in \{1, 2, \ldots, T\}. \tag{1}$$

The decisions of whether to self-dispatch or not, and the outcomes of the mFRR and aFRR energy auctions, depend on the information that is

[7]The balancing constraints in (1) are stylized representations of the system operators' constraints. In practice, system operators aim at minimizing the frequency deviation and the demand for aFRR balancing energy is based on the instantaneous *area control error*. Furthermore, the initial inelastic system imbalance is indistinguishable from agents' self-dispatching.

available to the agents at the time of decision-making. This is modeled in our setup by a gradual revelation of uncertainty as we get closer to real time:

- The capacity auction stage is in the day ahead. No uncertainty has been revealed yet.

- The second stage is the *gate-closure* time of the balancing energy auctions, 25 minutes before the beginning of the imbalance settlement period.[8] It is the final opportunity for participating in the aFRR and mFRR balancing energy auctions, or for opting out of the centralized auction and rather resorting to the possible alternative of self-dispatching[9]. The gate-closure forecast of the system imbalance is revealed at this time.

- The mFRR activation stage is at the beginning of the imbalance settlement period. It corresponds to the moment of activation of mFRR by the system operator. The demand for mFRR balancing energy depends on the forecast of the system operator regarding system imbalance and the mFRR activation strategy of the TSO.

- The aFRR activation stage lasts from the beginning of the imbalance settlement period to its end. The remaining uncertainty is revealed as the imbalance settlement period unfolds and the system reacts to the actual imbalances.

This process is illustrated with an example in figure 2. We represent the gradual revelation of uncertainty as the realization of random variables that affect the conditional distribution of imbalances in our model. The initial distribution of imbalances is quite wide, but it becomes narrower as we arrive closer to real time. At gate closure, a forecast y_1 is revealed, and this updates the conditional distribution of the system imbalance. This may impact the bidding behaviour of the agents. At the mFRR activation stage, an additional forecast y_2 is revealed. This also results in an update of the conditional distribution of the system imbalance and it determines the demand for mFRR balancing energy by the TSO. Finally, the real system

[8]Article 8.2 of the *Implementation Framework for the European Platform for the Exchange of Balancing Energy from Frequency Restoration Reserve with Automatic Activation.*

[9]The timing of self-dispatching is discussed in section 7.

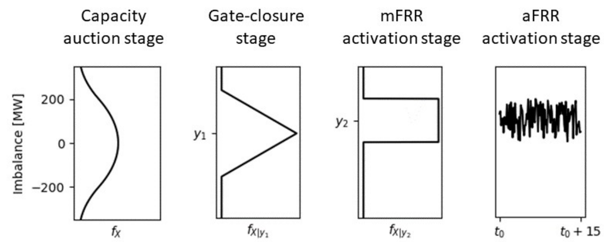


Figure 2: Example distributions of system imbalance at the capacity auction stage and as a function of the forecasts at the gate-closure stage (y_1) and the mFRR activation stage (y_2) and example realization of system imbalance at the aFRR activation stage.

imbalance at time t, x_t, is drawn from the system imbalance distribution conditional on the forecast of the TSO, $X|y_2$. The forecasts y_1 and y_2 are assumed to be drawn from the random variables Y_1 and $Y_2|y_1$.

There also exist two derivative random variables based on the mFRR activation strategy of the system operator. This activation strategy is represented by a function $G(\cdot)$ that maps the information that is available at the beginning of the imbalance settlement period to a level of mFRR activation. We can define the distribution of the demand for mFRR, X^{mFRR}, as

$$X^{mFRR} = G(Y_2). \tag{2}$$

The distribution of the demand for aFRR at subperiod t, X_t^{aFRR}, can be similarly defined. The special case for the demand for aFRR without self-dispatching by market participants is presented hereunder:

$$X_t^{aFRR} = X - X^{mFRR} = X - G(Y_2). \tag{3}$$

3.2. Price-Taking Bidding Behaviour

We now characterize the behaviour of rational fringe agents who react to exogenous demands and prices. This generalizes the optimal bidding strategy of a particular agent participating in this multi-stage game for any balancing market design. We specifically analyze the profit maximization strategy of an agent with upward balancing capacity and marginal cost θ who participates in the multi-stage multi-product reserve and energy game that is presented in figure 3. The specification θ has been dropped from the notation for brevity.

The optimal strategy of this game is obtained through backward induction by starting at the bottom of the tree at the aFRR and mFRR activation

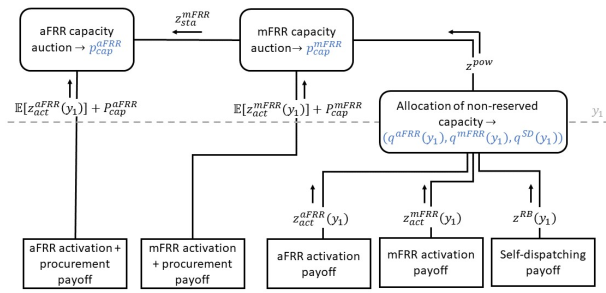


Figure 3: Multi-product reserve and energy balancing auction game.

payoffs and the self-dispatching payoff. The payoff of the non-reserved capacity is then back-propagated to the mFRR capacity auction and then to the aFRR capacity auction.

- The last stage of the game occurs at gate-closure, when the system imbalance forecast y_1 is revealed. At that point, the agent must allocate its generation capacity between the aFRR and mFRR balancing energy auctions as well as self-dispatching (see bottom right of figure 3). The action set of an agent conditional to the revealed uncertainty y_1 comprises of the aFRR price-quantity balancing energy bid, $(p^{aFRR}(y_1), q^{aFRR}(y_1))$, the mFRR price-quantity balancing energy bid, $(p^{mFRR}(y_1), q^{mFRR}(y_1))$, and the capacity allocated to self-dispatching $q^{SD}(y_1)$. This action set can be restricted to the capacity component of these bids, since bidding the true marginal cost as the price component of the balancing energy bid is a weakly dominant strategy. Agents are price-takers and they have no incentive to deviate from bidding their truthful cost as this would result in possible loss if they underbid or lost profit if they overbid. The optimal action of an agent given y_1 can then be found by maximizing (4):

$$\max_{q(y_1)} \quad q^{aFRR}(y_1) \cdot z_{act}^{aFRR}(y_1) + q^{mFRR}(y_1) \cdot z_{act}^{mFRR}(y_1) + q^{SD}(y_1) \cdot z^{SD}(y_1) \tag{4}$$

$$s.t. \quad q^{aFRR}(y_1) + q^{mFRR}(y_1) + q^{SD}(y_1) \leq Q \tag{5}$$

The objective function here consists of the sum of the product of the capacity bids, $q(y_1) = (q^{aFRR}(y_1), q^{mFRR}(y_1), q^{SD}(y_1))$ and the respective marginal payoff, $z_{act}^{aFRR}(y_1)$ for aFRR balancing energy, $z_{act}^{mFRR}(y_1)$ for mFRR balancing energy and $z^{SD}(y_1)$ for self-dispatching. Constraint (5) corresponds to the fact that the sum of the capacity bids is bounded by the capacity of the assets Q.

The marginal payoffs of the different options are dependent on the mFRR activation strategy and imbalance pricing scheme, but their exact characterization is not required in order to derive the agents' optimal strategies if we only look for pure strategies. Only the relative ranking of these marginal payoffs is required, as agents will offer their full capacity to the most profitable option. The expected payoff for the non-reserved capacity is then computed as the expectation over the realisations of y_1 (which is distributed according to F_{Y_1}, the cumulative distribution function of Y_1) of the maximum of these marginal payoffs. This expected payoff is denoted as z^{pow}:

$$z^{pow} = \int \max(z_{act}^{aFRR}(y_1); z_{act}^{mFRR}(y_1); z^{SD}(y_1))dF_{Y_1}(y_1) \tag{6}$$

- The expected non-reserved payoff is then backpropagated to the mFRR capacity auction (see top-right of figure 3). At this stage, agents must submit a price-quantity bid for the mFRR balancing capacity auction, $(p_{cap}^{mFRR}, q_{cap}^{mFRR})$, and a price bid in the balancing energy auction for the capacity selected in the capacity auction, $p_{cap,en}^{mFRR}$. Only pure strategies are considered, thus the quantity component of the balancing capacity bid is set at the maximum of the capacity. The tradeoff between (i) participating in the mFRR balancing capacity auction and then in the mFRR balancing energy auction and (ii) participating in the balancing market as non-reserved capacity corresponds to the price component of the balancing capacity bid. It determines whether a bid is selected in the capacity auction and is sufficient for characterizing pure strategies. As in the case of non-reserved capacity, bidding the marginal cost in the subsequent balancing energy auction is weakly dominant. The optimal bidding strategy at this stage can then be found by solving (7) given an exogenous mFRR capacity price P_{cap}^{mFRR}:

$$\max_{p_{cap}^{mFRR}} \begin{cases} P_{cap}^{mFRR} + \mathbb{E}_{Y_1}[z_{act}^{mFRR}(y_1)] & \text{if } p_{cap}^{mFRR} \leq P_{cap}^{mFRR} \\ z^{pow} & \text{if } p_{cap}^{mFRR} > P_{cap}^{mFRR} \end{cases} \tag{7}$$

Bidding the opportunity cost of participating in the mFRR balancing energy auction, $z^{pow} - \mathbb{E}_{Y_1}[z_{act}^{mFRR}(y_1)]$, is always optimal. If $P_{cap}^{mFRR} +$

$\mathbb{E}_{Y_1}[z_{act}^{mFRR}(y_1)] > z^{pow}$, then any price bid belonging to the interval $[0, P_{cap}^{mFRR}]$ is optimal. If $P_{cap}^{mFRR} + \mathbb{E}_{Y_1}[z_{act}^{mFRR}(y_1)] < z^{pow}$, then any price bid belonging to the interval $(P_{cap}^{mFRR}, +\infty)$ is optimal. This optimal strategy results in the expected payoff at the mFRR balancing capacity stage, z_{sta}^{mFRR}, characterized as the maximum between the mFRR procurement and activation payoff and the payoff from not being reserved:

$$z_{sta}^{mFRR} = \max(P_{cap}^{mFRR} + \mathbb{E}_{Y_1}[z_{act}^{mFRR}(y_1)]; z^{pow}) \tag{8}$$

- The expected payoff at the mFRR capacity auction stage is then back-propagated to the aFRR capacity auction (see top-left of figure 3). The action space at that stage is similar to that of the mFRR capacity auction stage with a price-quantity balancing capacity bid, $(p_{cap}^{aFRR}, q_{cap}^{aFRR})$, and a price bid for the subsequent aFRR balancing energy auction. The same argument as for the mFRR capacity auction can be used to restrict this action space to only the price component of the balancing capacity bid. The optimal bidding strategy is similar to the one in the mFRR capacity auction except that the tradeoff is between the aFRR procurement and activation payoff, $P_{cap}^{aFRR} + \mathbb{E}_{Y_1}[z_{act}^{aFRR}(y_1)]$, and the expected payoff at the mFRR capacity auction stage. This results in the following price offer in the aFRR balancing capacity auction:

$$p_{cap}^{aFRR} = z_{sta}^{mFRR} - \mathbb{E}_{Y_1}[z_{act}^{aFRR}(y_1)] \tag{9}$$

In summary, rational fringe agents participating in the multi-product reserve and energy balancing auction game (i) allocate their non-reserved capacity between the aFRR and mFRR balancing energy auction, as well as self-dispatching depending on the highest activation payoff (bottom right of figure 3), (ii) bid the difference between solely participating in the mFRR balancing energy auction and participating in the balancing market as non-reserved capacity in the mFRR capacity auction (top right of figure 3), and (iii) bid the difference between solely participating in the aFRR balancing energy auction and the payoff at the mFRR capacity auction stage in the aFRR capacity auction (top left of figure 3).

The activation payoffs are explicitly stated below, in order to clarify the impact of modifying the mFRR activation strategy on them.

mFRR Activation Payoff

The marginal mFRR activation payoff can be computed through the profit maximization problem (10) as a function of the price component of the mFRR balancing energy bid, p^{mFRR}, given the exogenous inverse supply curve for mFRR prices, P^{mFRR}, and the cumulative distribution function of the demand for mFRR balancing energy conditional on the gate-closure forecast, $F_{X^{mFRR}|y_1}$:

$$z_{act}^{mFRR}(\theta|y_1) = \max_{p^{mFRR}} \int_{P^{mFRR}(x) \geq p^{mFRR}} (P^{mFRR}(x) - \theta) dF_{X^{mFRR}|y_1}(x) \quad (10)$$

The objective function corresponds to a uniform price auction where an agent is selected as soon as the price of the auction exceeds the price component of its balancing energy bid. Its payoff is equal to the expectation of the mFRR price minus its marginal cost whenever the offer is selected. Accepted bids are assumed to be fully selected.

The weak dominance of bidding at marginal cost is due to the monotonicity of the mFRR prices. The marginal mFRR activation payoff can then be rewritten as the expectation over the maximum operator between the profit when being activated and zero:

$$\begin{aligned} z_{act}^{mFRR}(\theta|y_1) &= \int_{P^{mFRR}(x) \geq \theta} (P^{mFRR}(x) - \theta) dF_{X^{mFRR}|y_1}(x) & (11) \\ &= \mathbb{E}_{X^{mFRR}|y_1}[\max(P^{mFRR}(X^{mFRR}) - \theta; 0)]. & (12) \end{aligned}$$

The payoff of solely participating in the mFRR balancing energy auction is then computed as the expectation over the gate-closure forecast given the probability density function of the gate-closure forecast, F_{Y_1}:

$$z_{act}^{mFRR}(\theta) = \mathbb{E}_{Y_1}[z_{act}^{mFRR}(\theta|y_1)] = \int z_{act}^{mFRR}(\theta|y_1) dF_{Y_1}(y_1). \quad (13)$$

aFRR Activation Payoff

The aFRR activation payoff is similar to the mFRR activation payoff. except that it is composed of T aFRR energy auctions of duration $1/T$ each. The problem of maximizing profit from aFRR activation is described in problem (14) given the exogenous inverse supply curve for aFRR prices P^{aFRR} and the cumulative distribution function of the demand for aFRR balancing energy conditional on the gate-closure forecast, $F_{X^{aFRR}|y_1}$. The subperiod aFRR demands are assumed to be independent[10] and this allows us to express the total payoff over the imbalance settlement period as the sum of the payoff of each subperiod, and integrate independently over the distribution of each sub-period in order to compute the expected payoff of that sub-period:

$$z_{act}^{aFRR}(\theta|y_1) = \max_{p^{aFRR}} \sum_{t=1...T} \frac{1}{T} \int_{P^{aFRR}(x) \geq p^{aFRR}} (P^{aFRR}(x_t) - \theta) dF_{X^{aFRR}|y_1}(x_t) \tag{14}$$

The same reasoning as for mFRR applies concerning the weak dominance of bidding $p^{aFRR} = \theta$, i.e. the true (privately known) marginal cost. The aFRR activation payoff can then also be rewritten using the maximum operator:

$$z_{act}^{aFRR}(\theta|y_1) = \int_{P^{aFRR}(x) \geq \theta} (P^{aFRR}(x) - \theta) dF_{X^{aFRR}|y_1}(x) \tag{15}$$

$$= \mathbb{E}_{X^{aFRR}|y_1}[\max(P^{aFRR}(X^{aFRR}) - \theta; 0)] \tag{16}$$

Self-dispatching Payoff

The marginal self-dispatching payoff is not based on an energy auction but rather on the self-activation of an agent based on its expectation of the imbalance price. The payoff conditional on the revealed uncertainty at gate closure, y_1, can be found by maximizing the expected payoff of performing self-dispatching, ai, given the expected imbalance price conditional on the system imbalance forecast, $\mathbb{E}[P^{imb}|y_1]$. This is expressed in problem (17):

$$z^{SD}(\theta|y_1) = \max_{ai} (\mathbb{E}[P^{imb}|y_1] - \theta) \cdot ai \tag{17}$$

$$s.t. \quad 0 \leq ai \leq 1$$

The decision on how much capacity to allocate to self-dispatching is made at gate closure of the mFRR/aFRR balancing energy auctions, when y_1 is revealed. We directly see that $ai = 0$ if $\mathbb{E}[P^{imb}|y_1] \leq \theta$ and $ai = 1$ otherwise. This means that an agent should commit to self-dispatching its asset only if the expected imbalance price conditional on y_1 is greater than its marginal

[10] The independence assumption might not hold in practice as pointed out in Papavasiliou et al. (2018). The results of this paper are not dependent on this assumption but it makes the formulation more compact.

cost. This allows us to reformulate the self-dispatching payoff conditional on y_1 as

$$z^{SD}(\theta|y_1) = \max(\mathbb{E}[P^{imb}|y_1] - \theta; 0). \tag{18}$$

4. Equilibrium without Balancing Capacity Markets

This section discusses the equilibria that emerge for trading non-reserved capacity (bottom right of figure 3) depending on the mFRR activation strategy and imbalance pricing scheme. Two types of activation strategies are investigated: (i) the least-cost activation strategy that replicates the mFRR dispatch of a two-stage stochastic program, and (ii) every mFRR activation strategy activating less mFRR than the least-cost ideal. Three imbalance pricing schemes are investigated: (i) the "mFRR only" imbalance price inspired by the "law of one price", and (ii) the "mean mFRR and aFRR" and (iii) the "max mFRR and mean aFRR" imbalance price foreseen by the ISHM. This results in six possible cases that can be restricted to four equilibria: (1) the "mFRR only" imbalance price for both mFRR activation strategies, (2) the "mean mFRR and aFRR" imbalance price with the least-cost mFRR activation strategy, (3) the "mean mFRR and aFRR" imbalance price with an activation of mFRR which is less than that of the least-cost mFRR activation strategy, and (4) the "max mFRR and mean aFRR" imbalance price for both activation strategies. The equilibria are characterized by the allocation of capacity in the multi-product balancing energy game. Fast agents can allocate between aFRR and mFRR balancing energy auctions and self-dispatching and slow agents between the mFRR balancing energy auction and self-dispatching. The equilibria are summarised in table 1 and consist of (1) every slow agent participating in the mFRR balancing energy auction and every fast agent participating in the aFRR balancing energy auction for the "mFRR only" price, (2) same for the "mean mFRR and aFRR" imbalance pricing method with the least-cost activation strategy, (3) some slow agents self-dispatching and others participating in the mFRR balancing energy auction and every fast agent participating in the aFRR balancing energy auction for the "mean mFRR and aFRR" imbalance pricing method with less mFRR activation than that of the least-cost mFRR activation strategy, and (4) some fast and slow agents self-dispatching and others participating either in the aFRR or mFRR balancing energy auction for the "max mFRR and mean mFRR" imbalance pricing scheme.

Table 1: Summary of equilibria for non-reserved capacity.

<table>
<tr><th></th><th colspan="2">mFRR only</th><th colspan="2">mean mFRR and aFRR</th><th colspan="2">max mFRR and mean aFRR</th></tr>
<tr><td>Asset
Act. Strat.</td><td>Slow</td><td>Fast</td><td>Slow</td><td>Fast</td><td>Slow</td><td>Fast</td></tr>
<tr><td>Least-cost</td><td rowspan="2">mFRR</td><td rowspan="2">aFRR</td><td>mFRR</td><td>aFRR</td><td rowspan="2">self-disp. and mFRR</td><td rowspan="2">self-disp. and aFRR</td></tr>
<tr><td>Less mFRR</td><td>self-disp and mFRR</td><td>aFRR</td></tr>
</table>

The remainder of the section will cover the four cases. We assume a continuum of fast and slow agents represented by their inverse supply functions. We denote these inverse supply function as $O^F(\cdot)$ and $O^S(\cdot)$ respectively. Note that the equilibria depend on the forecast of the system imbalance at gate closure, y_1. The specification y_1 has been dropped for brevity.

4.1. "mFRR only"

The "mFRR only" imbalance pricing scheme is motivated by the "law of one price": homogeneous goods should trade at the same price Jevons (1871). This imbalance pricing scheme considers that imbalances and mFRR balancing energy are at least partially substitutable, based on the fact that they both represent balancing energy that is traded on a 15-minute timescale and should thus be priced similarly. This results in the imbalance price aligning to the mFRR price:

$$P^{imb}(x^{mFRR}) = P^{mFRR}(x^{mFRR}). \tag{19}$$

One property of this scheme is that no agents find it to their advantage to self-dispatch, since participating in the mFRR balancing auction is always more profitable.

Lemma 1. *The payoff from participating in the mFRR balancing auction is greater than or equal to the one from self-dispatching under the "'mFRR only" imbalance price.*

Proof. By Jensen's inequality,

$$\max(\mathbb{E}[P^{imb}] - \theta; 0) = \max(\mathbb{E}[P^{mFRR}] - \theta; 0) \leq \mathbb{E}[\max(P^{mFRR} - \theta; 0)]. \tag{20}$$

□

We can now analyze the impact of the mFRR activation strategy on the payoff of agents. We begin with the least-cost activation strategy and generalize the insight to strategies with less mFRR activation. The ideal least-cost activation strategy can be obtained by solving the two-stage stochastic multi-period dispatch of aFRR and mFRR. According to such an activation strategy, the system operator would activate mFRR and aFRR with the aim of minimizing the sum of the mFRR activation cost and the expected aFRR activation cost in equation (21) given the aFRR and mFRR aggregate cost curves, C^{mFRR} and C^{aFRR}, and the uncertain demand in sub-period t and scenario ω, $x_t(\omega)$, where the scenario ω belongs to the uncertainty set Ω. This problem is expressed as:

$$\min_{x^{mFRR}, x^{aFRR}} C^{mFRR}(x^{mFRR}) + \mathbb{E}_\Omega[\sum_{t=1\ldots T} \Delta_T C^{aFRR}(x_t^{aFRR}(\omega))] \tag{21}$$

$$s.t. \quad x_t(\omega) = x^{mFRR} + x_t^{aFRR}(\omega) \quad \forall \omega \in \Omega, \forall t \in \{1\ldots T\} \tag{22}$$

The uncertain sub-period demands $x_t(\omega)$ are assumed to be independent and corresponds to independent samples of a random variable, X, with cumulative density function F_X. Note that the aFRR and mFRR costs are obtained by integrating the aFRR and mFRR inverse supply curves, P^{aFRR} and P^{mFRR}:

$$C^i(x) = \int_0^x P^i(y)dy \quad i \in \{aFRR, mFRR\}. \tag{23}$$

The uncertain cost from activating balancing energy can be reformulated by substituting the balance constraint and by exploiting the independence of the sub-period demands:

$$\mathbb{E}_\Omega[\sum_{t=1\ldots T} \Delta_T C^{aFRR}(x_t^{aFRR}(\omega))] = \mathbb{E}_\Omega[\sum_{t=1\ldots T} \Delta_T C^{aFRR}(x_t(\omega) - x^{mFRR})] \tag{24a}$$

$$= \int \sum_{t=1\ldots T} \Delta_T \left(C^{aFRR}(x - x^{mFRR}) dF_X(x)\right) \tag{24b}$$

$$= \int C^{aFRR}(x - x^{mFRR}) dF_X(x) \tag{24c}$$

The first-order condition of the cost minimization problem shows us that the following condition holds at optimality:

$$P^{mFRR}(x^{mFRR}) = \int P^{aFRR}(x - x^{mFRR})dF_X(x) \tag{25}$$

Condition (25) indicates that the optimal activation of aFRR and mFRR should be such that the price of mFRR should be equal to the expectation of the price of aFRR. A consequence of this finding is the superiority of the expected aFRR activation payoff over the mFRR activation payoff, from the point of view of fast reserve providers, under this activation strategy.

Lemma 2. *The payoff from participating in the mFRR balancing energy auction is lower than or equal to the payoff of participating in the aFRR balancing energy auction if the mFRR activation strategy of the system operator follows the least-cost strategy.*

Proof. By Jensen's inequality,

$$\max(P^{mFRR} - \theta; 0) = \max(\mathbb{E}[P^{aFRR}] - \theta; 0) \leq \mathbb{E}[\max(P^{aFRR} - \theta; 0)]. \tag{26}$$

□

The difference between the aFRR and the mFRR payoffs can be considered as a flexibility premium. We are now ready to characterize the equilibrium under the "mFRR only" imbalance price.

Proposition 1. *Every agent offering their capacity to the best quality auction they can (fast agents to aFRR balancing energy and slow agents to mFRR balancing energy) is an equilibrium under the "mFRR only" imbalance price and with an mFRR activation strategy equal to the least-cost activation strategy.*

Proof. Lemmas 1 and 2 state that, for all agents, performing self-dispatching is less profitable than participating in the mFRR balancing energy auction and that the aFRR balancing energy is more profitable than the mFRR balancing energy auction. This leads to every slow agent offering their capacity to the mFRR balancing energy auction and every fast agent offering their capacity to the aFRR balancing energy auction. □

This results in aFRR prices following the inverse supply curve of fast agents and mFRR prices following the inverse supply curve of slow agents. The analysis for mFRR activation strategies with less mFRR activation than the least-cost mFRR activation is trivial, since lemma 1 is not affected by the mFRR activation strategy and lemma 2 remains valid for mFRR activation strategies that activate less mFRR capacity than that activated in the least-cost mFRR activation strategy.

4.2. "mean mFRR and aFRR" Imbalance Settlement with Least-Cost Activation Strategy

The "mean mFRR and aFRR" imbalance price is expressed in equation (27) as a function of the demands for mFRR and aFRR balancing energy, x^{mFRR} and x_t^{aFRR} for $t \in \{1 \dots T\}$:

$$P^{imb}(x^{aFRR}, x^{mFRR}) = \frac{\sum_{t=1\dots T} \frac{1}{T} P^{aFRR}(x_t^{aFRR}) + P^{mFRR}(x^{mFRR})}{2} \qquad (27)$$

Proposition 2. *Every agent offering their capacity to the best quality auction they can is an equilibrium under the "mean mFRR and aFRR" imbalance price and with an mFRR activation strategy that corresponds to the least-cost activation strategy.*

Proof. Under the least-cost mFRR activation strategy, the mFRR price is equal to the expected aFRR price, $P^{mFRR} = \mathbb{E}[P^{aFRR}]$, which establishes an expected imbalance price equal to the expected mFRR price (see equation (27)). Lemmas 1 and 2 can then both be applied to prove the desired result. □

4.3. "mean mFRR and aFRR" Imbalance Settlement with Less mFRR than Least Cost

If the mFRR activation strategy activates less than the amount of mFRR activated by the least-cost activation strategy, self-dispatching may become more profitable than participating in the mFRR balancing energy auction. This is due to the higher aFRR prices that lift the imbalance price. This results in the following proposition.

Proposition 3. *Every agent offering their capacity to the best quality auction they can is not always an equilibrium under the "mean mFRR and aFRR" imbalance price and with an mFRR activation strategy that activates less mFRR than the least-cost strategy.*

An example of truthful participation in the balancing energy auction not being an equilibrium, proving proposition 3, is provided in section 6.2. The equilibrium in this case results in self-dispatching from the slow agents and can be characterized as follows:

- Every fast agent participates in the aFRR balancing energy auction.
- Every slow agent with a marginal cost lower than $O^S(\alpha^S)$, i.e. the cheapest capacity up to a quantity α^S, self-dispatches.
- Every slow agent with a marginal cost higher than $O^S(\alpha^S)$ participates in the mFRR balancing energy auction.

A level α^S of self-dispatch from slow agents has two direct effects: it increases the mFRR prices by removing cheap assets from the mFRR merit order, and it decreases the aFRR prices by reducing the leftover imbalance that needs to be covered by aFRR. The updated mFRR price given a level α of self-dispatch from slow agents is described in equation (28). Generators with a marginal cost lower than $O^S(\alpha)$ opt out of the mFRR balancing energy auction and this translates the inverse supply function of slow assets to the left by α for upward balancing capacity.

$$P^{mFRR}(x|\alpha) = \begin{cases} O^S(x+\alpha) & \text{if } x \geq 0 \\ O^S(x) & \text{if } x < 0 \end{cases} \tag{28}$$

The effect of self-dispatching on the demand for mFRR and aFRR balancing energy is described in equations (29) and (30) for initial imbalances x_t, a system operator forecast y_2, and an mFRR activation strategy G:

$$x^{mFRR} = G(y_2), \tag{29}$$

$$x_t^{aFRR} = x_t - G(y_2) - \alpha \qquad \forall t \in \{1 \dots T\}. \tag{30}$$

The demand for mFRR balancing is not affected by the self-dispatching, as this demand is obtained from the system operator forecast and an mFRR activation strategy that is assumed to be static.

Increasing the self-dispatch level increases the marginal mFRR activation payoff, due to the increased mFRR prices, and decreases the marginal self-dispatching payoff, due to the decreased aFRR prices. An equilibrium can then be found by finding a level of self-dispatch α^S such that every agent below α^S on the inverse supply curve finds self-dispatching more profitable, every agent after α^S finds participating in the mFRR balancing energy auction more profitable, whereas the agent at α^S is indifferent between both options. This corresponds to solving the following identity where the marginal payoff for both mFRR activation and self-dispatch are dependent on the level of self-dispatching:

$$z_{act}^{mFRR}(O^S(\alpha^S)|\alpha^S) = z^{RB}(O^S(\alpha^S)|\alpha^S). \tag{31}$$

4.4. "max mFRR and mean aFRR" Imbalance Pricing

The "max mFRR and mean aFRR" imbalance price as a function of the demands for mFRR and aFRR balancing energy is characterized as follows:

$$P^{imb}(x^{aFRR}, x^{mFRR}) = \max\left(\sum_{t=1...T} \frac{1}{T} P^{aFRR}(x_t^{aFRR}); P^{mFRR}(x^{mFRR})\right) \tag{32}$$

Under this imbalance settlement scheme, the expected imbalance price is always greater than the expected aFRR price or the mFRR price. Both fast and slow agents may find it optimal to self-dispatch at equilibirum. As in the case of the "mean mFRR and aFRR" price, self-dispatch is performed by the cheapest agents.

Proposition 4. *Every agent offering their capacity to the best quality auction they can is not always an equilibrium under the "max mFRR and mean aFRR" imbalance price.*

The difference between proposition 3 and 4 is that even the least-cost activation strategy does not always sustain an equilibrium where every agent offers their capacity to the best quality auction. An example proving 4 is also given in section 6.2. The equilibrium in this case exhibits self-dispatching from both fast and slow agents, and can be characterized as follows:

- Every fast agent with a marginal cost lower than $O^F(\alpha^F)$, i.e. the cheapest fast capacity up to a quantity α^F, self-dispatches.
- Every fast agent with a marginal cost higher than $O^F(\alpha^F)$ participates in the aFRR balancing energy auction.
- Every slow agent with a marginal cost lower than $O^S(\alpha^S)$ self-dispatches.
- Every slow agent with a marginal cost higher than $O^S(\alpha^S)$ participates in the mFRR balancing energy auction.

Self-dispatching by fast assets is not an equilibrium outcome in the "mean mFRR and aFRR" case as the expected imbalance price is bounded within by the mean mFRR and aFRR prices and if the mean mFRR price becomes greater than the mean aFRR price, self-dispatching becomes strictly less profitable than offering mFRR. For the "max mFRR and mean aFRR" imbalance pricing design, if the mFRR price becomes greater than the aFRR price and dominates the formation of the imbalance price, self-dispatching is as profitable as offering mFRR and can be strictly greater than offering aFRR. This can result in self-dispatching from both fast and slow assets. The equilibrium can be obtained by finding the fast and slow agents that are indifferent between self-dispatching and offering aFRR or mFRR, depending on the level of fast and slow self-dispatching, α^F and α^S. These thresholds α^F and α^S are characterized by the following identity:

$$z_{act}^{mFRR}(O^S(\alpha^S)|\alpha^S) = z^{RB}(O^S(\alpha^S)|\alpha^S, \alpha^F), \tag{33}$$

$$z_{act}^{aFRR}(O^F(\alpha^F)|\alpha^S, \alpha^F) = z^{RB}(O^F(\alpha^F)|\alpha^S, \alpha^F). \tag{34}$$

5. Introduction of Full Capacity Demand Curves

The equilibria discussed in the previous section do not account for the aFRR and mFRR capacity auctions. This is a corner case of the complete multi-product reserve and energy games of figure 3 with zero aFRR and mFRR capacity demand curves. A second corner case of this game arises when the system operator uses *full capacity demand curves*, which procure the entirety of the balancing capacity for both fast and slow assets in the forward (day-ahead) market. This corresponds to inelastic reserve requirements for aFRR equal to the installed fast capacity and reserve requirements for mFRR equal to the installed slow capacity. This scenario results in the following equilibrium.

Proposition 5. *The equilibrium with full capacity demand curves is characterized by every fast agent being selected in the aFRR capacity auction and every slow agent being selected in the mFRR capacity auction.*[11]

[11]Technically, this equilibrium is generated from ϵ-*full capacity demand curves* that procures the entirety of the balancing capacity minus a small ϵ. The equilibrium requires a fringe agent to not be selected to ensure that the last agent that is selected in the balancing capacity auction bids its opportunity cost.

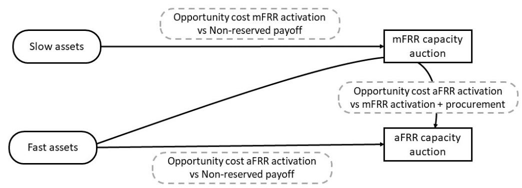


Figure 4: Opportunity cost in the capacity auctions.

Proof. At equilibrium, the mFRR capacity price is equal to the difference in payoff between participating in the mFRR balancing energy auction versus participating in the balancing market as non-reserved capacity for the agent with the highest such cost, and the aFRR capacity price is equal to the maximum between (a) the difference between participating in the aFRR balancing energy auction versus participating in the balancing market as non-reserved capacity for the agent with the highest such cost, and (b) the difference between participating in the aFRR balancing energy auction versus participating in the mFRR balancing energy and balancing capacity auctions for the agent with the highest such cost. These opportunity costs are presented in figure 4. No price-taking agent has an incentive to deviate from the equilibrium given these capacity prices. □

Full capacity demand curves result in mFRR and aFRR capacity prices that are equal to zero for the "mFRR only" imbalance pricing policy. Participating in the mFRR balancing energy auction is the optimal strategy for slow assets, therefore slow assets have no opportunity cost for doing so, which results in a zero mFRR capacity price. If there is no mFRR procurement payoff, participating in the aFRR balancing energy auction remains the optimal strategy for fast assets. They too have no opportunity cost, which results in a zero aFRR capacity price. More generally, every balancing energy market design, i.e. the combination of an mFRR activation strategy and an imbalance pricing scheme, that incentivizes agents to participate in the best quality auction they can will result in zero aFRR and mFRR balancing capacity prices. This includes the "mean mFRR and aFRR" imbalance price under the least-cost mFRR activation strategy.

Balancing energy market designs that do not incentivize agents to offer their capacity in the best quality balancing energy auction can generate non-zero balancing capacity prices. Slow assets require a compensation for participating in the mFRR balancing energy auction if it is not their optimal strategy. Even if participating in the aFRR balancing energy auction is still the optimal strategy for the fast non-reserved capacity, the mFRR procurement payoff pushes the aFRR capacity price up as this generates an opportunity cost between solely offering aFRR balancing energy and participating in the mFRR balancing capacity auction (see figure 4).

6. Results

The results that are presented in this section are based on an illustrative example. Let $N^F = 500$ MW and $N^S = 1000$ MW be the fast and slow capacity that is available for balancing. The marginal cost of upward balancing capacity is distributed linearly between 0 and 100 €/MWh. Let (i) Y_1, the forecast of imbalances at gate closure, be uniformly distributed between -200 and 200 MW, (ii) $Y_2|y_1$, the forecast of the system operator conditional on y_1, be uniformly distributed between $y_1 - 100$ and $y_1 + 100$, and (iii) $X|y_2$, the random variable from which the actual system imbalance is drawn given y_2, be uniformly distributed between $y_2 - 100$ and $y_2 + 100$. This revelation of uncertainty is illustrated in figure 2.

This section begins by describing the least-cost mFRR activation strategy in this setting. It continues with the equilibria for the different imbalance pricing schemes, illustrating proposition 3 and 4. The impact of the capacity demand curve is then described. The last point that we focus on concerns the activation cost under the different pricing schemes and mFRR activation strategies.

6.1. Least-Cost mFRR Activation Strategy

The least-cost mFRR activation strategy is obtained by expressing the demand for mFRR balancing energy, x^{mFRR}, as a function of the system operator forecast of the imbalance, y_2, as indicaed in equation (25). This strategy is obtained in a setting where every fast agent participates in the aFRR balancing energy auction and every slow agent participates in the mFRR balancing energy auction, which results in the aFRR and mFRR balancing energy prices following the fast and slow inverse supply curves respectively. Additionally, as no agent self-dispatches, the uncertain sub-period demands are drawn from the system imbalance distribution conditional on the system operator forecast, $X|y_2$. Applying equation (25) to our numerical settings results in the following identity:

$$O^S(x^{mFRR}) = \int O^F(x - x^{mFRR})dF_{X|y_2}(x) = \int_{y_2-100}^{y_2+100} O^F(x - x^{mFRR})\frac{1}{200}dx. \tag{35}$$

Given $O^S(x) = x/10$ and $O^F(x) = x/5$, the least cost activation strategy, $G^{LC}(y_2)$, can be expressed analytically as:

$$x^{mFRR} = \frac{2}{3}y_2 = G^{LC}(y_2). \tag{36}$$

6.2. Reactive Balancing

Figure 5 presents the capacity that is not offered to balancing energy auctions as a function of the gate-closure forecast, y_1, for the illustrative example under the "mean mFRR and aFRR" and "max mFRR and mean aFRR" imbalance prices. This figure presents the level of self-dispatching for an mFRR activation strategy that activates less mFRR than the least-cost activation strategy,

$$G(y_1) = 0.5 \cdot y_1 < G^{LC}(y_1), \tag{37}$$

and for the imbalance pricing schemes that generate self-dispatch, namely the "mean mFRR and aFRR" and the "max mFRR and mean aFRR" imbalance pricing schemes. The illustrative example is symmetric and exhibits a similar level of self-dispatching by downward flexible assets for negative y_1. . Higher gate-closure forecasts indicate a higher discrepancy between the aFRR and mFRR balancing energy prices and result in higher self-dispatching payoffs for imbalance pricing schemes that include an aFRR price component. There is a change of mode for the "max mFRR and aFRR" imbalance price when the gate-closure forecast reaches around 100 MW. Only slow agents self-dispatch before 100 MW, but, as soon as the forecast reaches 100 MW, fast agents start to self-dispatch and this increases the total capacity not offered to the balancing energy auctions. This coincides with the mFRR price dominating the mean aFRR price in the formation of the imbalance price.

Figure 6 illustrates the effect of reactive balancing on the expected payoffs of the agents as a function of their marginal cost for a given forecast y_1. Figure 6a shows that, under the mFRR activation strategy $G(y_1) = 0.5 \cdot y_1$ and the gate-closure forecast $y_1 = 140$, every fast asset offering aFRR and

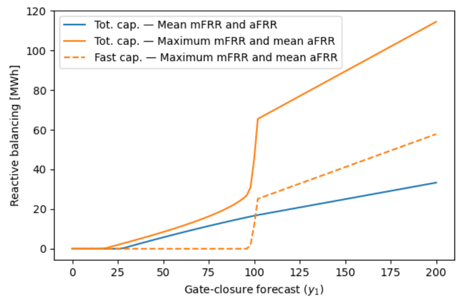


Figure 5: Fast and slow assets performing reactive balancing when the mFRR activation strategy is lower than that of the least-cost activation strategy ($G(y_1) = 0.5 \cdot y_1$).

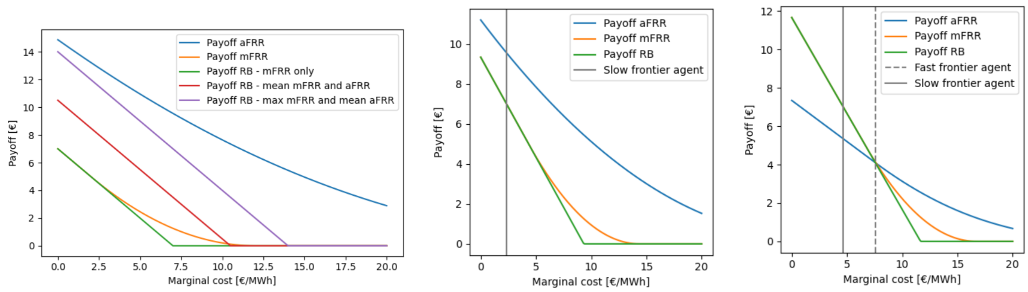
every slow asset offering mFRR cannot constitute an equilibrium for either the "mean mFRR and aFRR" or the "max mFRR and mean aFRR" imbalance prices. The mFRR balancing energy payoff is dominated by reactive balancing payoffs, and agents would rather perform reactive balancing than participate in the mFRR balancing energy auctions.

Figure 6b presents the payoffs under the equilibrium level of reactive balancing for the "mean mFRR and aFRR" imbalance price. Slow assets that perform reactive balancing reduce the aFRR prices and payoffs (by reducing the residual system imbalance that needs to be covered by aFRR) and increase the mFRR prices and payoffs (by removing cheap assets from the mFRR merit order). Slow assets perform reactive balancing up to the point where the expected payoff of performing reactive balancing and of participating in the mFRR balancing energy auction are equal. This characterizes a frontier agent such that all slow assets with lower marginal cost will perform reactive balancing and all assets with higher marginal cost will participate in the mFRR balancing energy auction.

Figure 6c provides a similar analysis for the "max mFRR and mean aFRR" imbalance price. The reactive balancing from slow assets results in an increased mFRR price dominating the mean aFRR price in the formation of the imbalance price. This leads to a large level of reactive balancing from fast assets.

6.3. Capacity Prices with Full Capacity Demand Curves

Self-dispatching can be restricted to the case of the "mean mFRR and aFRR" and the "max mFRR and mean aFRR" imbalance pricing schemes given sufficiently large capacity demand curves. The "mFRR only" imbal-



(a) No reaction.

(b) Optimal level of reactive balancing for "mean mFRR and aFRR".

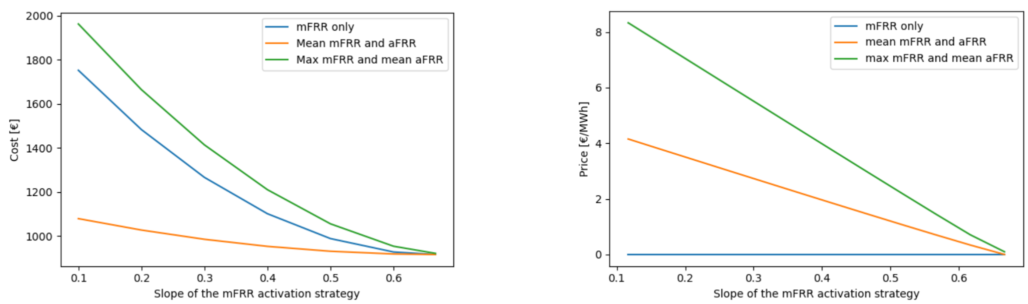
(c) Òptimal level of reactive balancing for "max mFRR and mean aFRR".

Figure 6: Activation payoffs for $y_1 = 140$ and $G(y_1) = 0.5 \cdot y_1$.

ance price does not incentivize self-dispatching and generates zero capacity prices. This behavior and the resulting mFRR balancing capacity prices are presented in figure 7b for the illustrative example. The horizontal axis in this figure represents the slope of a linear mFRR activation strategy up to the least-cost activation strategy at 2/3. The lower the slope, the less mFRR is activated by the TSO, and the higher the opportunity cost for participating in the mFRR auction. Note that the aFRR balancing capacity prices are equal to the mFRR balancing capacity prices as the dominating opportunity cost in figure 4 is the one related to participating in the mFRR capacity auction.

6.4. Activation cost

Figure 7a presents the activation that results from different mFRR activation strategies and imbalance pricing schemes. The minimum activation cost can be found under the least-cost mFRR activation strategy for the "mFRR only" and the "mean mFRR and aFRR" imbalance settlement schemes. The "mFRR only" imbalance price generates the benchmark activation cost where all agents offer their capacity to the best quality auction they can. The "mean mFRR and aFRR" can reduce the activation cost by correcting an inefficient mFRR activation strategy. It incentivizes slow agents to self-activate and it drives the equilibrium closer to the optimal least-cost dispatch. The self-activation resulting from "max mFRR and aFRR" imbalance pricing scheme is inefficient and over-compensates for the inaccurate mFRR activation strategy.



(a) Activation cost in the illustrative example as a function of the mFRR activation strategy.

(b) mFRR and aFRR capacity prices (which are equal) under full capacity demand curves for an mFRR activation strategy that is lower than or equal to that of the least-cost activation strategy.

Figure 7

7. Discussion

This section discusses the policy implications of the results that are illustrated in the previous section and the impact of some modelling assumptions.

7.1. Policy Implications

One finding of our analysis is that bidding the marginal cost in the balancing energy auctions is not always the optimal strategy for the "mean mFRR and aFRR" and the "max mFRR and mean aFRR" imbalance settlement schemes. The "mFRR only" imbalance settlement scheme has the advantage of providing a clear strategy for the agents: they should bid in the best-quality auction they can, and they have no opportunity cost in the capacity auctions. This strategy is *weakly dominant*, and independent of the other agents' strategies. An advantage of a clear optimal bidding strategy is that it fosters competition by reducing the barrier to entry. There is no need to rely on extended analytics in order to participate profitably in the market. The other imbalance pricing schemes can allow for pure strategy equilibria, but they can be harder to reach in practice. They require agents to perfectly forecast the behaviours of the other agents, since their strategy depends on the level of reactive balancing in the system.

Our analysis also goes against one argument in favor of reactive balancing. It is argued that reactive balancing decreases the capacity procurement cost, by reducing the *reserve requirement*. The reserve requirement drives the width of the capacity demand curve and it is computed based on the historical distribution of the system imbalance, which can be reduced through reactive balancing ELIA (2021). This reasoning ignores the potential capacity cost increase that is caused by a balancing setting that incentivizes reactive balancing and generates artificial opportunity cost that is driven by imbalance settlement pricing as opposed to the intrinsic economic value of reserve. The more reactive balancing is profitable, the higher the opportunity cost for agents to participate in the balancing energy auctions and the higher the prices in the capacity auctions. It is unclear which factor, the reduction in reserve requirements or the increase in capacity prices, exerts a greater impact on the procurement cost, and this should be evaluated in a system-specific manner, but the argument cited above that has been used in public discourse is incomplete.

7.2. Effect of Modelling Assumptions

The model assumes that assets commit to performing reactive balancing at gate closure. This is a realistic representation of some European countries where the system imbalance is revealed with a thirty-minute delay.[12] In other countries, such as Belgium and the Netherlands, the system imbalance is revealed in real time, at every minute. Our model underestimates the benefits of performing reactive balancing in this context, since the risk associated with performing reactive balancing decreases with the level of information that agents have on the system imbalance. Nevertheless, the insights of the "mFRR only" imbalance pricing scheme are independent of this assumption. Performing reactive balancing is always weakly dominated by participating in the mFRR balancing energy auction, regardless of the information that is available to the agent at the time of balancing. If assets can commit to performing reactive balancing at the beginning of the ISP, or adapt dynamically to the revelation of uncertainty as the ISP unfolds , a qualitative argument can be made for an increased level of reactive balancing for the "mean mFRR and aFRR" and the "max mFRR and mean aFRR" imbalance prices. A quantitative analysis would require the characterization of the equilibrium.

The model also assumes that the system operator demand for mFRR balancing energy is not endogenized. In particular, the activation strategy

[12]European regulation mandates system operators to publish the system imbalance with a delay of at most thirty minutes.

does not account for the induced self-dispatch. If the system operator is considered to be an agent that participates in the multi-stage game with the objective of minimizing activation cost, then only three cases based on the imbalance pricing scheme need to be inspected. There is no need anymore to consider different activation strategies, since the system operator would submit a demand for mFRR balancing energy that would minimize activation cost. The equilibrium with a responsive system operator and the "mFRR only" and the "mean mFRR and aFRR" prices would actually be identical to the one generated by an unresponsive system operator following the least-cost activation strategy. The case with the "max mFRR and mean aFRR" is not as straightforward, as there may exist an activation strategy such that the induced self-dispatching results in a lower activation cost than the least-cost activation strategy of an unresponsive system operator.

Finally, the *one-way substitutability* assumption of the fast assets can be discussed. It is common to assume that fast-moving assets can offer both mFRR and aFRR without restriction, however this ignores energy-constrained assets such as batteries or pump-hydro power plants. These assets can have difficulties in participating in the mFRR auction due to the longer activation time in the same direction. Without the one-way substitutability, our model could lead to price reversal in aFRR and mFRR capacity prices.

8. Conclusion

This paper proposes a framework for analyzing European multi-product balancing auctions. We analyze the impact of (i) the imbalance settlement scheme, (ii) the mFRR activation strategy, and (iii) the capacity demand curve on the balancing market equilibria. The reaction of rational fringe agents is endogenously accounted for by the model.

Four main insights can be derived from the model. (1) The "mFRR only" imbalance pricing scheme incentivizes agents to offer their capacity to the best-quality balancing energy auction they can. (2) The minimum balancing activation cost can be reached with the "mFRR only" and the "'mean mFRR and aFRR" imbalance prices under the least-cost mFRR activation strategy. (3) If the least-cost activation strategy is not applied, the "mean mFRR and aFRR" imbalance price incentivizes a level of reactive balancing compensating for the inefficient activation strategy and generate a lower balancing activation cost. (4) The "max mFRR and mean aFRR" imbalance price induces a level of reactive balancing that increases the balancing activation cost.

Future work aims at extending the framework to account for cross-border interaction through the European balancing platforms and to consider intermediate cases for capacity demand curves. Another line of research will focus on the characterization of elastic demand curve for mFRR balancing energy.

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