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Does Democracy Inevitably Lead to Aggressive Redistribution? A Family Perspective

Simon Fan Yu Pang Pierre Pestieau*

Abstract

This paper explains why democracies marked by inequalities may not experience aggressive redistribution through the lens of parent-child interactions. Parental concerns about the negative impacts of high taxation on their children's motivation to study and pursue high-paying careers deter the poor majority from harboring an inclination to expropriate the rich. We construct an overlapping generations model in which workers vote on the redistributive policy under majority rule, while considering the incentive costs that the policy imposes on their children. We analyze the stationary Markov perfect equilibrium where the likelihood that a moderate income tax can be credibly enforced increases with the degree of parental altruism. In an extended model where career prospects are jointly determined by study efforts and received educational resources, we provide an analytical and numerical characterization of the conditions under which full redistribution does not materialize in the steady state under both private and public school systems.

JEL Classifications: D72, H31, I24

Keywords: credible tax policy; parental altruism; Markov perfect equilibrium; education; majority voting

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1 Introduction

Redistributive preferences theory posits that income inequality can correct itself in a democracy because the poor, who often hold the political power, tend to raise taxes for the rich and share the proceeds among themselves (See a survey by Boadway and Keen 2000). Much empirical research indicates that this theory is not representative of typical behavior in democratic societies (Kelly and Enns 2010, Kuziemko, Norton, Saez, and Stantcheva 2015). The question of why the poor develop reservations toward large-scale welfare benefits has sparked considerable discussions, but rigorous theoretical work seems limited.[1] In a seminal contribution, Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2003) examine the decay and survival of a welfare state in an overlapping generations model with income inequality. They show that the low-income old agents may vote for low redistribution, as it incentivizes the young to invest in human capital. Such increased investments, in turn, bolster total tax collection and thus intergenerational transfers to the low-income voters.

This paper explains the inequality-redistribution puzzle from the parent-child interactions perspective. By developing a dynamic model *à la* Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2003) that considers incentive costs of significant redistribution on human capital formation of the future generation, we address three distinctive aspects. First, our model emphasizes parental altruism toward children, as opposed to self-interest that aims at the acquisition of pecuniary gains, as a main intergenerational link. Many parents care about their children's future educational attainment and career outcome (Cremer, De Donder, and Pestieau 2010). They will be concerned that a socio-political environment where high earners are heavily taxed to finance downward redistribution may demotivate their children to strive for high-paying careers. If the poor are aware that the immediate material gains they receive from massive redistribution are outweighed by the potential long-term losses their children would suffer, they hesitate to soak their rich peers. For example, Okun (2015) attributes the American blue-collar workers' protest against a proposal for confiscatory estate taxes in 1972 to their aspiration that they "*wanted some big prizes maintained in the game*" and "*did not want the yacht clubs closed forever to their children and grandchildren.*" (p. 47)

[1]For instance, Bonica, McCarty, Poole, and Rosenthal (2013) provide several perspectives, including immigration, growing acceptance of free-market capitalism, wealth-biased lobbying, and distorted political process. De Donder and Pestieau (2015) postulate that the poor majority may not support a high estate tax when exposed to a strong political and media environment where any such wealth transfer encroaches on individual autonomy.

Second, our framework examines the credibility of redistributive policies under majority rule.[2] While the analysis of time inconsistency problems of fiscal policies has deep roots in public economics, going back at least to Boadway, Marceau, and Marchand (1996) and Boadway and Keen (1998), it is seldom applied in an intergenerational context with parental altruism. When tax rules are in place, poor voters – if they constitute the majority – may demand confiscatory taxation to equalize wealth. But the exercise of discretion to fulfill such demands inadvertently discourages their children from making investments in human capital that concerns them, which erodes the policy credibility. We establish a credibility constraint that exhibits a growing likelihood as the level of parental altruism intensifies in the stationary Markov perfect equilibrium.

Third, this paper is generalized to include parents' educational expenditure and investigate its interactions with children's study efforts in determining their career development. In particular, the substantial allocation of funds toward educational expenses has the power to reshape redistributive preferences, potentially leading to a shift where the majority aligns with high earners. Our analysis builds on the literature that explores the welfare implications of educational systems under majority rule. Epple and Romano (1996) characterize an ends-against-the-middle equilibrium in a majority voting with dual educational systems. De la Croix and Doepke (2009) argue that the presence of a large private education sector benefits public schools if the society is politically dominated by the rich and crowds out public education spending otherwise. In this paper, we compare the respective impacts of private and public school systems on the political economy.

Some previous research analyzing the impact of high income taxation on the young generation concentrates on its disincentive effect on the labor market participation of young workers. Kremer (2002) demonstrates that the young have more elastic labor supply than the old with respect to taxes in a static model. Weinzierl (2011) predicts that lower taxes on high-paid young workers reduce the efficiency costs that arise from the distortion of their efforts in a dynamic Mirrleesian paradigm. Lindbeck and Nyberg (2006) show that generous redistributive programs weaken parents' motives to impart work norms to their children. Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2003) postulate that such a disincentive effect is more profound than the literature suggests: apart from the adverse impact on the young's labor supply and productivity, high taxes discourage schoolchildren

[2]Macroeconomic policies can be conducted by rules or discretion. A policymaker who has repeated interactions with private agents has an incentive to abide by the rules accounting for the potential loss of reputation (or credibility). See, for example, Kydland Prescott (1977) and Barro and Gordon (1983).

from devoting academic effort. This view is also adopted in our paper.

There is extensive economic literature investigating the return to schooling and pre-employment training (See the review by Stantcheva 2020). Having a sense of motivation propels schoolchildren to remain focused and engage themselves in learning activities. Bishop (1996) posits that the primary cause of low effort by some American students is a lack of anticipated future rewards. Chadi, De Pinto, and Schultze (2019) show that dim job prospects undermine student effort in Germany. Hard work, in turn, is the mechanism promoting educational attainment and prospective incomes. Hall and Farkas (2011) find that American students with constructive attitudinal/behavioral traits receive higher future earnings on average. Best and Kleven (2013) estimate that a 10% increase in the young-age effort raises the old-age pay by 2% – 4%. Metcalfe, Burgess, and Proud (2019) find that students in England perform worse in exams during the periods of the FIFA World Cup and interpret this result as reflecting a reduction in student effort. All these empirical analyses provide evidence in support of our main arguments.

This paper builds an overlapping generations model in which individuals who live through two periods ("childhood" and "adulthood") first invest in human capital at school and then progress to their working lives. Subject to the labor market uncertainty, workers may earn a high or low wage. Childhood human capital investments are costly but increase the probability of being a high earner in adulthood. Redistributive policies, as collectively set by workers through majority voting, will affect how children judge the future financial rewards of the efforts they put into studying.

In Section 2, we start with the case in which parents vote myopically, ignoring their children's career prospects. The poor majority will support full redistribution from high earners to themselves to maximize their consumption without incurring a cost. Children know that effort will not pay off under such a confiscatory tax policy and therefore stay inactive in human capital acquisition. In the long run, nobody will be a high earner, and the economy stagnates. A less damaging prediction may arise when parents are altruistic toward their children. We characterize a stationary Markov perfect equilibrium in which an existing, moderate tax rate is preferable to a discretionary confiscatory tax under majority rule when parents have strong concern for their children's career prospects. In other words, parental altruism plays a prominent role of limiting the poor's demand for full redistribution, which facilitates the tax policy to be credibly imposed. Given a credible tax rate, each generation exert the same effort, and low redistribution tends to encourage that effort.

Section 3 presents an extended model in which children's career prospects are jointly shaped by the efforts they supply and their received educational resources. Rather than assume the median voter is a low earner, we let the median wage be endogenously determined. We focus our analysis on the steady state, wherein the fraction of high earners is time-invariant, in the stationary Markov perfect equilibrium. We examine both private and public school systems by resorting to theoretical analysis and numerical solutions. Under private education, each forward-looking child makes a life plan, including her current study efforts in response to the voting outcome and parental inputs as well as her future allocation of income between consumption and educational investments. Under public education, the government collects income taxes not only for welfare transfers but also for equal provision of educational resources, while individuals choose their study efforts in childhood and vote on redistributive and educational policies in adulthood.

Our numerical examples in Section 4 provide some additional results. First of all, a tax policy is credible under the private school system if (i) the rich constitute the majority, or (ii) the poor constitute the majority and the tax rate is sufficiently high. Second, a credible tax should be neither too low nor too high under the public school system. A low tax does not suffice to finance public education, while a high tax tends to dampen children's human capital accumulation. Meanwhile, a generous fund for public education is less likely to be credible for the poor majority because (i) it reduces the resource allocation to transfers that low earners desire, and (ii) it creates a sizeable cohort of high earners, thereby increasing the appeal for low earners to demand full redistribution. Third, the children's efforts and parental/shcool investments are strategic complements in general circumstances. Intense parental altruism drives substantial study efforts and educational investments so that the entire population strives to become high earners. Comparing the optimal credible policies between the two educational systems show that public education is socially preferable to private education only if parents are not very altruistic toward their children.

2 The Basic Model

Consider an overlapping generations economy populated by individuals who live for childhood and adulthood. Each generation has a unit mass. In period t, members of generation t are born and devote a study effort s_t in pursuit of future career success. In period $t+1$, each member supplies one unit of labor inelastically.[3] Her wage rate, which depends on career development, is given by

$$w_t^i = \begin{cases} \omega > 1 & \text{if } i = H \\ 1 & \text{if } i = L \end{cases}, \tag{1}$$

where H and L denote high and low earners, respectively. The probability of being a high earner is governed by one's study effort s_t, namely $p_t = p(s_t) \in [0, 1]$, where $p' > 0$, $p'' < 0$, and $p(0) = 0$. Given that the population size is one, p_t also represents the fraction of high earners in generation t.

In each period, the fiscal policy of the economy is decided by all workers through majority rule. On behalf of the median voter of generation t, the government implements an income tax at the rate of $T_t \in [0, 1]$ in period $t + 1$. Tax revenues are then returned equally to workers so that they each receive a non-negative lump-sum transfer payment of

$$F_t = T_t(1 - p_t + \omega p_t). \tag{2}$$

Each i-type member of generation t, where $i \in \{H, L\}$, spends all her disposable income y_t^i on consumption c_t^i without leaving any bequests. In period $t + 1$, she treats (T_t, F_t) as exogenous and receives a disposable income (and hence consumption) at the level of

$$c_t^i = y_t^i = w_t^i(1 - T_t) + F_t. \tag{3}$$

Her lifetime utility is composed of two separable terms, as follows:

$$u_t^i = -\frac{s_t^2}{2} + d_t^i \qquad \text{where } \; d_t^i := \ln c_t^i + \alpha p_{t+1}. \tag{4}$$

The first term reflects the disutility caused by childhood study effort, and the second term captures the adulthood utility, which in turn has two sources – the private consumption c_t^i and the concern about the quality of the child (as measured by the child's probability of earning a high wage, p_{t+1}).[4]

[3] To highlight our argument that children's study efforts are responsive to the income tax, we assume inelastic labor supply. We aim to show that low-income workers choose not to support full redistribution even in the absence of the taxation-driven distortion of the labor market.

[4] We assume away the impact of family background on the quality of the child and will relax this assumption in the next section.

The parameter $\alpha \geqslant 0$ measures the degree of such parental concern.

The timeline of our model is as follows. The government moves first to announce an income tax rate $T \in [0,1]$. In period $t+1$, the median voter of generation t decides whether or not to accept $T_t = T$. If she accepts it, then all children, who observe that the voting result is consistent with the preannouncement, expect T to persist into the future. If she deviates to $\widehat{T} \neq T$, the government loses its intergenerational reputation, which makes all children believe that complete appropriation will happen in their adulthood, i.e., $T_{t+1} = 1$. In either scenario, children optimally choose their study efforts based on tax expectations.

In this section, we assume that it is easier to fail than to succeed even if a maximal study effort is spent, namely $p(\max\{s\}) < \frac{1}{2}$. Since there are more low earners than high earners, the median voter must be a low-income worker. We will remove this assumption in the next section to let the median wage to be endogenously determined.

Our analysis proceeds with two cases. The first case assumes that parents are not altruistic in the sense that they do not care about their children's life outcomes ($\alpha = 0$). We will demonstrate that the government levies a confiscatory tax rate in equilibrium, causing persistent economic stagnation. In the other case where parents are concerned for their children's future career achievement (i.e., $\alpha > 0$), the poor majority may not support such radical expropriation of wealth.

2.1 No Concern about Children's Career Development

We start with the benchmark case where parents are not altruistic toward their children ($\alpha = 0$). Inserting (3) into (4) and then using (2) shows that the median voter (or any low-wage worker) of generation t obtains the following utility in adulthood:

$$d_t^L = \ln(1 - T_t + F_t) = \ln[1 + (\omega - 1)p_t T_t]. \tag{5}$$

Given ω and p_t, the rational median voter will choose the maximal tax rate (i.e., $T_t = 1$) regardless of the announcement. After observing $T_t = 1$, the children generation believe that the same thing will occur in the future ($T_{t+1} = 1$). By (3), they expect to live on transfers only ($c_{t+1} = F_{t+1}$) and thus obtain a lifetime utility of

$$u_{t+1} = -\frac{s_{t+1}^2}{2} + \ln F_{t+1}.$$

It is straightforward to show that the equilibrium individual choice of study effort is $s_{t+1} = 0$. The next proposition summarizes this outcome.

Proposition 1 *If parents care little about children* $(\alpha = 0)$ *, then* $T = 1$ *and* $s = 0$ *in equilibrium.*

Proposition 1 addresses the question of why some democratic governments prioritize redistribution over mobility. Myopic voters, who favor immediate gains of equalization over future gains of children's achievement, will vote for a unity income tax, under which wages are all expropriated for redistribution. Foreseeing that everyone will obtain an equal disposable income and hence the same consumption, each child infers that becoming a high earner in the future is not beneficial to his quality of life. The perception that studying entails sheer costs and brings no pecuniary reward makes children lose motivation to deliver effort at school ($s = 0$), which will, in turn, limit their career prospects ($p(0) = 0$). In equilibrium, nobody succeeds in career so that the economy is caught in a low-consumption trap ($c = 1$). Proposition 1 implicates that the poor's vote for complete equalization ruins the economy.

2.2 Concern about Children's Career Development

This section aims to show that $T < 1$ may constitute an equilibrium when parents care about the quality of their children ($\alpha > 0$). We identify the circumstances under which the median voter is willing to accept $T < 1$ and investigate the optimal credible fiscal policy. Following the literature on the infinitely-repeated intergenerational interactions, we focus on the stationary Markov perfect equilibrium, as defined below:[5]

Definition 1 *The sequence* $\{x_t\}_0^\infty$ *is a stationary Markov perfect equilibrium if (i) for any* t*,* x_t *is the optimal strategy for a member of generation* t *who believes that future generations will play the strategy* $\{x_k\}_{t+1}^\infty$*, (ii) an individual's strategies are functions of the payoff-relevant state variable(s) only, and (iii) individuals facing the same state variable(s) play the same strategy,* $x_t = x$ *for all* t*.*

In Definition 1, part (i) requires that the equilibrium is a Nash equilibrium of every subgame. Part (ii) suggests that individuals play a Markov strategy. An equilibrium satisfying both (i) and

[5] Maskin and Tirole (2001) note that a stationary Markov perfect equilibrium commands computational simplicity, practical virtues, and philosophical considerations.

(ii) is a Markov perfect equilibrium. Part (iii) indicates that any subgame with the same state will be played in the same way. An equilibrium satisfying all three conditions is a *stationary* Markov perfect equilibrium.

The median voter of generation t chooses her preferred redistributive policy by assessing the consequences of accepting or deviating from the preannounced tax rate $T < 1$ in period $t + 1$. If she accepts it, then children will anticipate the continuation of the tax policy in the next period (i.e., $T_{t+1} = T$). With the probability $p(s_{t+1})$ of becoming a high earner in the future, a child expects his lifetime utility at the level of

$$u_{t+1} = -\frac{s_{t+1}^2}{2} + p(s_{t+1}) \ln[\omega(1 - T_{t+1}) + F_{t+1}] + [1 - p(s_{t+1})] \ln(1 - T_{t+1} + F_{t+1}) + \alpha p(s_{t+2}).$$

Clearly, a greater study effort exacerbates disutility in childhood but enhances earning prospects. Facing this tradeoff, each child optimally chooses his study effort based on the expected future tax policy. His Markov strategy depends only on the state variables affecting his utility, i.e., s_{t+1} is a function of (T_{t+1}, F_{t+1}). It follows that s_{t+2} is not a function of s_{t+1}.

The optimal study effort is in accordance with the following first order condition:

$$\frac{\partial u_{t+1}}{\partial s_{t+1}} = -s_{t+1} + p'(s_{t+1}) \ln \frac{\omega(1 - T_{t+1}) + F_{t+1}}{1 - T_{t+1} + F_{t+1}} = 0.$$

By substituting (2) into the above equation, we derive that the study effort in the stationary Markov perfect equilibrium (i.e., $s_{t+1} = s$ for all t) satisfies

$$\frac{s}{p'(s)} = \ln \left[1 + \frac{(\omega - 1)(1 - T)}{1 + (\omega - 1)p(s)T} \right], \tag{6}$$

Based on (6), we conduct comparative static analysis in the following proposition:

Proposition 2 $s'(\omega) > 0$ *and* $s'(T) < 0$.

Proof. See Appendix. ■

Proposition 2 implies that an individual's concern toward future career may affect her incentive to apply current effort. First, the motive for children to study respond positively to the pretax wage ratio, ω. As income inequality between high-paying and low-paying jobs intensifies, children will aspire to study hard in order to increase their chances of securing a high-paying job in the future. Second, children's efforts depend on their expectation about the future tax policy. When the government plans to increase taxes, children will be less motivated to make efforts that would lead to high-paying jobs, as the after-tax returns to these efforts would be lower.

Since individual efforts are time-independent in the stationary Markov perfect equilibrium, i.e., $s_t = s(T)$ where $T < 1$, we have $p_t = p[s(T)]$ for all t. Therefore, the median voter of generation t obtains the following utility in period $t+1$:

$$d_t^L = \ln(1 - T_t + F_t) + \alpha p(s_{t+1}) = \ln\left\{1 + (\omega - 1)p[s(T)]T\right\} + \alpha p[s(T)]. \tag{7}$$

The median voter's alternative option is to vote against the preannounced tax rate T. Her deviation to $T_t = \widehat{T} \neq T$ triggers a change in belief: children then believe that the tax rate is 100% in the next period ($T_{t+1} = 1$). As any income is expected to be taken away for redistributive purposes, the motivation for children to study is removed ($s_{t+1} = 0$). Foreseeing that all children, including her own child, will fail at career ($p_{t+1} = 0$), the median voter will optimally deviate to $\widehat{T} = 1$, which result in complete wealth equalization. Her utility in period $t+1$ amounts to

$$\widehat{d_t^L} = \ln\left[1 - \widehat{T} + \widehat{T}(1 - p_t + \omega p_t)\right] + \alpha p(0) = \ln\left\{1 + (\omega - 1)p[s(T)]\right\}. \tag{8}$$

Comparing (7) with (8) demonstrates that the median voter's decision-making involves a trade-off between her material benefit and her child's upward mobility: her deviant behavior increases her consumption level by $(\omega-1)(1-T)p[s(T)]$ but leads her child to have little chance to succeed in the future. A deviation will be deterred if and only if d_t^L is no less than $\widehat{d_t^L}$. In other words, if universal suffrage and majority rule are adopted in a democracy marked by economic inequalities, poor individuals, who entertain the hope that their children will achieve successful careers through hard work, may reject aggressive redistribution out of concern that it will demotivate their children. The next proposition follows directly from (7) and (8):

Proposition 3 $T < 1$ *is credible in the stationary Markov perfect equilibrium if and only if*

$$\alpha + \frac{1}{p[s(T)]} \ln\left\{T + \frac{1 - T}{1 + (\omega - 1)p[s(T)]}\right\} > 0. \tag{9}$$

Proof. See Appendix. ■

Proposition 3 suggests that the tax rate $T < 1$ that an economy starts with can be sustained over time. When condition (9) is valid, the median voter lacks the incentive to renege: she supports the preexisting policy that is more moderate in terms of redistribution rather than that aiming to completely equalize wealth. The likelihood that condition (9) holds increases with α. As a low-income parent, the median voter knows that confiscatory taxation (i.e., a deviation to $T = 1$) produces a negative side effect on her child's educational attainment and upward mobility. To the extent that the median voter strongly cares about the quality of her child, she may forgo the benefits of wealth equalization and accept the announced tax rate. As Fleurbaey (1995) and Hassler, Rodríguez Mora, and Zeira (2007) note, the aversion of low-income groups to inequality is inversely related to how assured the prospects of success are for their children. In addition, a larger ω results in a larger s (Proposition 2), making condition (9) less likely to be satisfied. Interpreted literally, a larger pretax wage ratio stimulates the poor to demand the social welfare transfers from the rich.

In contrast to Proposition 1 that predicts $c = 1$ when $\alpha = 0$, Proposition 3 shows that parental altruism ($\alpha > 0$) may help to get rid of a persistent state of low consumption. The median voter's deviation to $\widehat{T} = 1$ not only makes low earners worse off ($\widehat{d_t^L} < d_t^L$) but also causes high earners to suffer doubly: in addition to a lowered consumption level, they will find that their children lose will to study, which accelerates downward mobility. Rather, the acceptance of T creates a Pareto improvement under condition (9). This result echoes the prospect of upward mobility hypothesis, which argues that the tolerance of low-income groups toward current inequality is linked to their belief in the upward mobility of future selves (Piketty 1995, Bénabou and Ok 2001).

Note that having a credible tax rate in a stationary Markov perfect equilibrium may not achieve social optimum. We proceed to investigate the *optimal* credible policy from the utilitarian planner perspective. Define social welfare in period t as the expected lifetime utility of generation t:

$$W_t = -\frac{s_t^2}{2} + p_t \ln[\omega(1 - T_t) + F_t] + (1 - p_t) \ln(1 - T_t + F_t) + \alpha p_{t+1}.$$

In the stationary Markov perfect equilibrium, social welfare in each period can be written as

$$W = -\frac{s^2}{2} + p \ln[\omega - (\omega - 1)(1 - p)T] + (1 - p) \ln[1 + (\omega - 1)pT] + \alpha p. \tag{10}$$

The first order condition of W in (10) derives the interior solution to the optimal income tax rate T^*, as presented in the following proposition:

Proposition 4 *T^* satisfies (9) and*

$$p(1-T)\left(T-\frac{1-p}{p's'}\right)=\left[\alpha\left(\frac{1}{\omega-1}+pT\right)+T\right]\left[\frac{\omega}{\omega-1}-(1-p)T\right]. \tag{11}$$

Proof. See Appendix. ■

Proposition 4 characterizes the income tax rate that maximizes the social welfare in (10). There are a number of Nash majority voting equilibria from which the utilitarian planner can select the welfare-maximizing option. If the credible income tax that the economy starts with is suboptimal ($T \neq T^*$), the median voter cannot help to restore the social optimum by simply voting for T^*. This is because, under the predefined belief system, any deviation of the median voter away from T would make her offspring believe that the future income tax rate will be 100%, thereby resulting in a "bad" Nash equilibrium of persistent low-consumption (i.e., $s = 0$ and $c = 1$). To avoid this bad equilibrium, the median voter rationally maintains the default T. Nonetheless, if T^* is initially imposed, it can be sustained in a Nash equilibrium by the same logic, as long as condition (9) is satisfied. In other words, social optimum cannot be reached unless the optimal credible tax rate is announced in the initial stage.

3 An Extension: Educational Spending Affects Career Development

To highlight our essential idea, the simple analytical framework presented in Section 2 assumes that an individual's life outcome is determined by her childhood effort only. In reality, the prospect of earning capacities depends not only on one's own effort but also on family and school inputs. It is well established in the theoretical literature that altruistic parents invest in education to improve the quality of their children (e.g., Becker 1981, De la Croix and Michel 2002). Also, some empirical studies have validated the hypothesis that educational resources provided by parents and schools are beneficial to children's future achievements.[6] In this section, we aim to show how the private

[6] Breen (2010) shows that educational expansion promotes intergenerational social mobility in Germany, Sweden, and the UK. Chetty, Hendren, and Katz (2016) find that moving into a low-poverty neighborhood before teenage years increases an individual's likelihood of college attendance and future earnings in the US.

and public school systems shape individual behaviors in the stationary Markov perfect equilibrium. Moreover, to make the model more general, we relax the assumption that the median voter is a low earner but allow the median wage to be endogenously determined.

3.1 Private Education

We first consider that parents provide their children with private education. Under the private school system, individuals choose their study efforts in childhood and educational investments in adulthood given the median voter's decision on the tax rate. In period $t+1$, each i-type member of generation t allocates her disposable income, y_t^i, between private consumption, c_t^i, and educational spending on her child, e_t^i. Her budget constraint thus amounts to

$$c_t^i = y_t^i - e_t^i = w_t^i(1 - T_t) + F_t - e_t^i. \tag{12}$$

Each member of generation $t+1$ with an i-type parent makes a study effort of s_{it+1} and receives an educational investment of e_t^i from his parent. Her probability of succeeding in the labor market, p_{it+1}, is given in the following Cobb-Douglas form:

$$p_{it+1} = (s_{it+1})^{\beta}(e_t^i)^{\delta}. \tag{13}$$

Note that we use the superscript i to denote one's own type and the subscript i to denote his family background. Parameters $\beta, \delta \in (0, 1)$ capture that study efforts and educational investments exhibit diminishing returns.[7] In period $t+1$, the workforce is composed of a proportion p_t of high earners and a proportion $1 - p_t$ of low earners. In period $t+2$, the children of high and low earners will become high earners with a probability of p_{Ht+1} and p_{Lt+1}, respectively. Given a unit mass of each generation, the fraction of high earners in generation $t+1$ can be written as

$$p_{t+1} = p_t p_{Ht+1} + (1 - p_t)p_{Lt+1}. \tag{14}$$

The interactions between the government and individuals proceed as follows. The government

[7] Our formulation in (13) has an empirical counterpart. For instance, Attanasio, Boneva, and Rauh (2022) find that, in England, parents perceive the returns to time and material investments in children's education to be diminishing and there is no difference in such returns by parents' socioeconomic background.

announces a tax rate $T \in [0, 1)$. In period $t+1$, the median voter decides whether or not to accept the redistributive policy. If she accepts it, children believe that the same tax rate will be in place when they grow up, i.e., $T_{t+1} = T$. Otherwise, they believe $T_{t+1} = 1$. In either case, each forward-looking rational child maximizes his expected lifetime utility by choosing his current study effort and future educational spending on his (unborn) child.

Suppose that the median voter accepts T in period $t+1$. Children anticipate $T_{t+1} = T$ as they enter the workforce. A j-type worker, where $j \in \{H, L\}$, consumes at the level of c^j_{t+1} in period $t+2$, and his child will become a high earner with a probability of p_{jt+2}. As such, his utility in adulthood can be expressed by

$$d^j_{t+1} = \ln c^j_{t+1} + \alpha p_{jt+2} = \ln(y^j_{t+1} - e^j_{t+1}) + \alpha (s_{jt+2})^\beta (e^j_{t+1})^\delta. \tag{15}$$

By (3), (13), and (15), a j-type worker with an i-type parent expects his adulthood utility to be a weighted sum $p_{it+1} d^H_{t+1} + (1 - p_{it+1}) d^L_{t+1}$ and hence his lifetime utility to be

$$\begin{aligned} u_{it+1} &= -\frac{(s_{it+1})^2}{2} + p_{it+1} \left[\ln(y^H_{t+1} - e^H_{t+1}) + \alpha (s_{Ht+2})^\beta (e^H_{t+1})^\delta\right] \\ &\quad + (1 - p_{it+1}) \left[\ln(y^L_{t+1} - e^L_{t+1}) + \alpha (s_{Lt+2})^\beta (e^L_{t+1})^\delta\right] \\ &= -\frac{(s_{it+1})^2}{2} + (s_{it+1})^\beta (e^i_t)^\delta \Big\{ \ln\left[\omega(1 - T_{t+1}) + F_{t+1} - e^H_{t+1}\right] + \alpha (s_{Ht+2})^\beta (e^H_{t+1})^\delta \Big\} \\ &\quad + \left[1 - (s_{it+1})^\beta (e^i_t)^\delta\right] \left[\ln(1 - T_{t+1} + F_{t+1} - e^L_{t+1}) + \alpha (s_{Lt+2})^\beta (e^L_{t+1})^\delta\right]. \end{aligned}$$

A child, who has an i-type parent and expects himself to become a j-type worker in the future, optimally chooses his study effort s_{it+1} and investment in child's education e^j_{t+1}, taking his parent's educational investment e^i_t and fiscal policies (T_{t+1}, F_{t+1}) as given. As all individuals play Markov strategies, s_{jt+2} is not affected by (s_{it+1}, e^j_{t+1}) where $i, j \in \{H, L\}$. The first order conditions of u_{it+1} with respect to s_{it+1} and e^j_{t+1} obtain

$$\frac{\partial u_{it+1}}{\partial s_{it+1}} = -s_{it+1} + \beta(s_{it+1})^{\beta-1}(e_t^i)^\delta \Bigg\{ \ln \frac{\omega(1-T_{t+1}) + F_{t+1} - e_{t+1}^H}{1 - T_{t+1} + F_{t+1} - e_{t+1}^L} + \alpha\Big[(s_{Ht+2})^\beta (e_{t+1}^H)^\delta - (s_{Lt+2})^\beta (e_{t+1}^L)^\delta\Big]\Bigg\} = 0,$$

$$\frac{\partial u_{it+1}}{\partial e_{t+1}^H} = (s_{it+1})^\beta (e_t^i)^\delta \left[-\frac{1}{\omega(1-T_{t+1}) + F_{t+1} - e_{t+1}^H} + \alpha\delta(s_{Ht+2})^\beta (e_{t+1}^H)^{\delta-1}\right] = 0,$$

$$\frac{\partial u_{it+1}}{\partial e_{t+1}^L} = \left[1 - (s_{it+1})^\beta (e_t^i)^\delta\right] \left[-\frac{1}{1 - T_{t+1} + F_{t+1} - e_{t+1}^L} + \alpha\delta(s_{Lt+2})^\beta (e_{t+1}^L)^{\delta-1}\right] = 0.$$

Substituting (2) and $T_{t+1} = T$ into the above three equations solves (s_{it+1}, e_{t+1}^i) as follows:

$$\frac{(s_{it+1})^{2-\beta}}{\beta(e_t^i)^\delta} = \ln \frac{\omega - (\omega-1)(1-p_{t+1})T - e_{t+1}^H}{1 + (\omega-1)p_{t+1}T - e_{t+1}^L} + \alpha\big[(s_{Ht+2})^\beta (e_{t+1}^H)^\delta - (s_{Lt+2})^\beta (e_{t+1}^L)^\delta\big], \quad (16)$$

$$\frac{(s_{Lt+2})^\beta}{(e_{t+1}^L)^{1-\delta}}\big[1 + (\omega-1)p_{t+1}T - e_{t+1}^L\big] = \frac{(s_{Ht+2})^\beta}{(e_{t+1}^H)^{1-\delta}}\left[\omega - (\omega-1)(1-p_{t+1})T - e_{t+1}^H\right] = \frac{1}{\alpha\delta}. \quad (17)$$

Next, the median voter of generation t chooses whether or not to accept T. If $p_t < \frac{1}{2}$, then the median voter is a low earner. Given p_t and s_{Lt+1}, her acceptance of T results in a utility of

$$d_t^L = \ln(y_t^L - e_t^L) + \alpha p_{Lt+1} = \ln\left[1 + (\omega-1)p_t T - e_t^L\right] + \alpha(s_{Lt+1})^\beta (e_t^L)^\delta. \quad (18)$$

However, her deviation to $\widehat{T} \neq T$ will lead generation $t+1$ to believe $T_{t+1} = 1$ so that everyone lives on transfers F_{t+1}. As study effort does not pay off in life outcome, no child chooses to study ($s_{t+1} = 0$). Foreseeing that all children will become low earners in the future ($p_{it+1} = 0$), parents choose not to waste money in children's education ($e_t^i = 0$). Therefore, the median voter optimally deviates to $\widehat{T} = 1$ and obtains the following utility in period $t+1$:

$$\widehat{d_t^L} = \ln\left[1 + (\omega-1)p_t\right]. \quad (19)$$

The deviation will be constrained if and only if d_t^L in (18) is no less than $\widehat{d_t^L}$ in (19).

If $p_t \geqslant \frac{1}{2}$, the median voter is a high earner. For simplicity, we assume that the high-income median voter will never propose a tax cut by virtue of moral obligation. Her deviation to $\widehat{T} > T$ will lead the children generation to believe $T_{t+1} = 1$, which in turn induces parents not to invest in their children's education ($e_t^i = 0$). As a result, all children become low earners in the future ($p_{it+1} = 0$). Expecting negative consequences on both her disposable income and her child's career outcome, the high-income median voter find a deviation from the initial tax rate T unattractive.

In the stationary Markov perfect equilibrium, $s_{it} = s_i$ and $e_t^i = e^i$ for $i \in \{H, L\}$ and all t. It follows from the dynamic path of p_t for all t presented in equation (13) that $p_{it} = p_i$. To further simplify analysis, we restrict our attention to the steady state as for the proportion of high earners in this dynamic system, which is defined as follows:

Definition 2 *The economy reaches the steady state when the proportion of high earners remains constant across generations (i.e., $p_t = p$ for all t).*

By Definition 2 and (13), we characterize the steady state by rewriting (14) as

$$p = \frac{p_L}{1 + p_L - p_H} = \frac{(s_L)^\beta (e^L)^\delta}{1 + (s_L)^\beta (e^L)^\delta - (s_H)^\beta (e^H)^\delta}. \tag{20}$$

The next proposition summarizes the conditions for policy credibility and the equilibrium outcome in the steady state.

Proposition 5 *Under the private school system, in the steady state,*
(i) when $(s_L)^\beta (e^L)^\delta + (s_H)^\beta (e^H)^\delta \geqslant 1$, the tax rate $T < 1$ is always credible;
(ii) when $(s_L)^\beta (e^L)^\delta + (s_H)^\beta (e^H)^\delta < 1$, the tax rate $T < 1$ is credible if and only if

$$\alpha (s_L)^\beta (e^L)^\delta \geqslant \ln \frac{\alpha\delta (s_L)^\beta (e^L)^{\delta-1} [1 + \omega (s_L)^\beta (e^L)^\delta - (s_H)^\beta (e^H)^\delta]}{1 + (s_L)^\beta (e^L)^\delta - (s_H)^\beta (e^H)^\delta}; \tag{21}$$

(iii) given a credible tax policy, (s_H, s_L, e^H, e^L) are solved by

$$\frac{(s_H)^{2-\beta}}{\beta (e^H)^\delta} = \frac{(s_L)^{2-\beta}}{\beta (e^L)^\delta} = \left(\frac{2-\beta}{\delta} - 2\right) \ln \frac{s_H}{s_L} + \alpha \Big[(s_H)^\beta (e^H)^\delta - (s_L)^\beta (e^L)^\delta \Big], \tag{22}$$

$$\begin{aligned} &(s_H)^\beta (e^H)^{\delta-1} \left[\omega - e^H - \frac{(\omega - 1) T [1 - (s_H)^\beta (e^H)^\delta]}{1 + (s_L)^\beta (e^L)^\delta - (s_H)^\beta (e^H)^\delta} \right] \\ &= (s_L)^\beta (e^L)^{\delta-1} \left[1 - e^L + \frac{(\omega - 1) T (s_L)^\beta (e^L)^\delta}{1 + (s_L)^\beta (e^L)^\delta - (s_H)^\beta (e^H)^\delta} \right] = \frac{1}{\alpha\delta}. \end{aligned} \tag{23}$$

Proof. See Appendix. ■

Proposition 5(i) and (ii) discuss the conditions under which the taxation policy is credible in the steady state under the private school system. Specifically, when $(s_L)^\beta (e^L)^\delta + (s_H)^\beta (e^H)^\delta \geqslant 1$, the median voter is a high earner, $T < 1$ is unconditionally credible; when $(s_L)^\beta (e^L)^\delta + (s_H)^\beta (e^H)^\delta < 1$, the median voter is a low earner, and condition (21) ensures that the potential for her *ex post* deviation is eliminated. Proposition 5(iii) presents an implicit characterization of stationary Markov perfect equilibrium in the steady state. Equations (22) and (23) show that the simultaneous endogenous determination of four variables (s_H, s_L, e^H, e^L) is governed by five parameters $(T; \alpha, \beta, \delta, \omega)$. We will rely on a numerical exercise in Section 4 to further examine the solutions.

Finally, we analyze the social optimum by determining the welfare-maximizing choice among all the potential credible tax rates under the private school system. In the steady state, high earners' children make an effort s_H and become a high earner in the future with a probability of p_H. Their expected lifetime utility can be expressed by $-\frac{(s_H)^2}{2} + p_H(\ln c^H + \alpha p_H) + (1-p_H)(\ln c^L + \alpha p_L)$. Meanwhile, children born in poor families make an effort s_L and become a high earner in the future with a probability of p_L. Their expected lifetime utility can be expressed by $-\frac{(s_L)^2}{2} + p_L(\ln c^H + \alpha p_H) + (1-p_L)(\ln c^L + \alpha p_L)$.

As in Section 2.2, we think of the social planner's objective as reflecting a representative individual's preferences. Given a fraction p of high earners and a fraction $1-p$ of low earners in the economy, the steady-state social welfare can be written as the following weighted sum:

$$\begin{aligned} W = {} & p\left[-\frac{(s_H)^2}{2} + p_H(\ln c^H + \alpha p_H) + (1-p_H)(\ln c^L + \alpha p_L)\right] \\ & + (1-p)\left[-\frac{(s_L)^2}{2} + p_L(\ln c^H + \alpha p_H) + (1-p_L)(\ln c^L + \alpha p_L)\right] \\ = {} & -\frac{p(s_H)^2 + (1-p)(s_L)^2}{2} + p\ln\left[\omega - (\omega-1)(1-p)T - e^H\right] \\ & + (1-p)\ln\left[1 + (\omega-1)pT - e^L\right] + \alpha p. \end{aligned} \tag{24}$$

The optimal tax rate, $T^* \equiv \arg\max\{W\}$, is a function of $(\alpha, \beta, \delta, \omega)$. To circumvent the difficulty of analytically deriving T^* subject to the credibility constraints in Proposition 5, we will find out a numerical solution in Section 4.

3.2 Public Education

We now consider that the government takes the responsibility of financing education and assume away any private investment in education. Suppose that children receive the same education at public schools so that their career prospects are unaffected by their family backgrounds. With an equal chance of career success facing all children, the governing equation can be written in a manner analogous to (13):

$$p_{t+1} = (s_{t+1})^{\beta}(E_t)^{\delta}, \tag{25}$$

where E_t denotes the public educational spending per child funded by generation t.

The interactions between the government and individuals are the same as in Section 3.1 except for two things. First, the government initially announces both the income tax rate and the public educational expenditure. In period $t+1$, the government allocates tax revenues between income redistribution among generation t and public education for generation $t+1$. Given that population sizes of both generations are one, the balanced government budget implies

$$F_t + E_t = T_t(\omega p_t + 1 - p_t). \tag{26}$$

Second, individuals choose their study efforts but not educational expenditure on their children. It follows that $c_t^i = y_t^i$ for $i \in \{H, L\}$.

The median voter of generation t decides whether to accept (T, E) in period $t+1$. If she accepts the policy, then generation $t+1$ believe $T_{t+1} = T$ and $E_{t+1} = E$ in the next period. Each of them chooses a study effort to maximize her expected lifetime utility as expressed by

$$\begin{aligned} u_{t+1} &= -\frac{s_{t+1}^2}{2} + p_{t+1} \ln c_{t+1}^H + (1 - p_{t+1}) \ln c_{t+1}^L + \alpha p_{t+2} \\ &= -\frac{s_{t+1}^2}{2} + (s_{t+1})^{\beta}(E_t)^{\delta} \ln[\omega(1 - T_{t+1}) + F_{t+1}] \\ &\quad + \left[1 - (s_{t+1})^{\beta}(E_t)^{\delta}\right] \ln(1 - T_{t+1} + F_{t+1}) + \alpha(s_{t+2})^{\beta}(E_{t+1})^{\delta}. \end{aligned}$$

As individuals choose Markov strategies, s_{t+2} is unaffected by s_{t+1}. Taking the first order condition of u_{t+1} with respect to s_{t+1} and rearranging yields

$$\frac{s_{t+1}}{\beta(s_{t+1})^{\beta-1}(E_t)^{\delta}} = \ln\left[1 + \frac{(\omega - 1)(1 - T_{t+1})}{1 + (\omega - 1)p_{t+1}T_{t+1} - E_{t+1}}\right].$$

In the stationary Markov perfect equilibrium, each generation choose the same study effort under the same circumstance, i.e., $s_{t+1} = s(T, E)$ for all t. It is easy to show that the economy is always in the steady state, i.e., $p_t = p = s^\beta E^\delta$. By (25) and (26), rewrite the above equation as

$$\frac{s^{2-\beta}}{\beta E^\delta} = \ln\left[1 + \frac{(\omega-1)(1-T)}{1+(\omega-1)s^\beta E^\delta T - E}\right], \tag{27}$$

which implicitly solves s as a function of (T, E).

If $p < \frac{1}{2}$, then the median voter is a low earner. Her acceptance of (T, E) is expected to lead her current-period utility to be

$$d_t^L = \ln[1 + (\omega-1)p_t T_t - E_t] + \alpha p_{t+1} = \ln\left[1 - E + (\omega-1)s^\beta E^\delta T\right] + \alpha s^\beta E^\delta. \tag{28}$$

However, her deviation to $\widehat{T} \neq T$ or $\widehat{E} \neq E$ will lead the children generation to believe $T_{t+1} = 1$. Because study efforts are in vain, each child optimally chooses $s_{t+1} = 0$. Foreseeing that her child will be unsuccessful in career ($p_{t+1} = 0$), the low-paid median voter optimally deviates to $\widehat{T} = 1$ and $\widehat{E} = 0$, which brings her a utility of

$$\widehat{d_t^L} = \ln\left[1 + (\omega-1)s^\beta E^\delta\right]. \tag{29}$$

By (28) and (29), a sufficient and necessary condition for policy to be credible is $d_t^L \geqslant \widehat{d_t^L}$.

If $p \geqslant \frac{1}{2}$, then the median voter is a high earner. Clearly, a high paid median voter will never deviate from (T, E) because a deviation not only reduces her disposable income (and thus private consumption) but also hinders her child's career success. Put it another way, full redistribution will never happen when $p \geqslant \frac{1}{2}$.

Based on the above analysis, we establish the next proposition:

Proposition 6 *Under the public school system,*
(i) when $s^\beta E^\delta \geqslant \frac{1}{2}$, (T, E) *are always credible;*
(ii) when $s^\beta E^\delta < \frac{1}{2}$, (T, E) *are credible if and only if*

$$\alpha s^\beta E^\delta \geqslant \ln \frac{1 + (\omega-1)s^\beta E^\delta}{1 - E + (\omega-1)s^\beta E^\delta T}. \tag{30}$$

(iii) Given a credible policy, s *is determined by equation (27), which always decreases with* T *and increases with* E *if* $\delta(\omega-1)s^\beta E^{\delta-1}T < 1$.

Proof. See Appendix. ■

Proposition 6(i) and (ii) discuss the credibility of government policies under the public school system. In the case of $p = s^\beta E^\delta \geqslant \frac{1}{2}$, the rich constitute the majority of the population, and the policy is always credible. In the case of $p = s^\beta E^\delta < \frac{1}{2}$, the poor hold the political power, and the policy credibility is hence contingent on condition (30). Proposition 6 (iii) characterizes the study effort in the stationary Markov perfect equilibrium. The intuition behind the negative relationship between s and T is the same as analyzed in Proposition 2. The impact of educational expenditure on study effort is less straightforward. We present a sufficient condition under which s is positively related to E. Intuitively, student efforts and school inputs may be strategic complements. Hopland and Nyhus (2016) provide supportive evidence from Norwegian public schools that student efforts increase with the quality of teaching materials.

Finally, we analyze the social optimum under the public school system. Each generation consists of a fraction p of high earners, who obtain a utility of $-\frac{s^2}{2} + \ln[\omega - (\omega - 1)(1-p)T - E] + \alpha p$ and a fraction $1-p$ of low earners, who obtain a utility of $-\frac{s^2}{2} + \ln[1 + (\omega - 1)pT - E] + \alpha p$. By the utilitarian approach, the social welfare can be written as the following weighted sum:

$$W = \alpha s^\beta E^\delta - \frac{s^2}{2} + s^\beta E^\delta \ln[\omega - (\omega-1)(1 - s^\beta E^\delta)T - E] + (1 - s^\beta E^\delta)\ln[1 + (\omega - 1)s^\beta E^\delta T - E].$$

The first order conditions of W with respect to T and E yield the optimal policies (T^*, E^*), which are summarized in the next proposition:

Proposition 7 *Under the public school system, the social optimum under a credible policy is determined by*

$$p(1-T)\left[T - \frac{(1-p)s}{\beta p(\partial s/\partial T)}\right] = \left[\alpha\left(\frac{1-E}{\omega-1} + pT\right) + T\right]\left[\frac{\omega - E}{\omega - 1} - (1-p)T\right], \tag{31}$$

$$\left(\frac{\alpha\delta}{E} + \frac{\delta s^2}{\beta p E} + \frac{\alpha\beta}{s}\frac{\partial s}{\partial E}\right)\left(\frac{1-E}{\omega-1} + pT\right) = \left[\frac{p(1-T)}{\frac{\omega-E}{\omega-1} - (1-p)T} - 1\right]\left[\frac{\delta T}{E} - \frac{1}{(\omega-1)p} + \frac{\beta T}{s}\frac{\partial s}{\partial E}\right]. \tag{32}$$

Proof. See Appendix. ■

Equations (31) and (32) pin down the interior solutions to (T^*, E^*), where $\frac{\partial s}{\partial T}$ and $\frac{\partial s}{\partial E}$ can be derived from equation (27) (see appendix). In particular, equation (31) is similar to equation (11) in Proposition 4. Nevertheless, if $p < \frac{1}{2}$ and condition (30) is binding, then the social planner needs to solve a constrained optimization problem, namely the maximization of the objective function W

subject to the strict equality of (30). We will have a further discussion on the numerical solutions to (T^{*}, E^{*}) in the next section.

4 A Numerical Example

To better visualize the steady-state outcomes under different educational systems, we numerically solve our model. Specifically, we undertake quantitative exercises to examine: (a) individual study effort and private educational spending in response to the changing tax rates, (b) individual study effort in public schools in response to different policies, and (c) the optimal policies under either private or public educational system. Our simulations are performed based on the following parameter configuration:

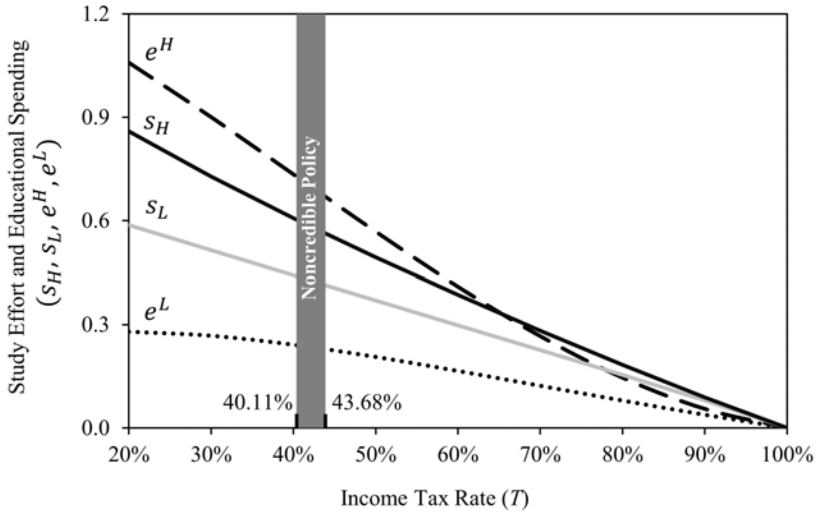
$$\alpha = 1.5 \qquad \beta = 0.6 \qquad \delta = 0.4 \qquad \omega = 3. \tag{33}$$

Figure 1 illustrates the effect of income tax rates on steady-state individual behaviors under the private school system given configuration (33). The income tax rate measured on the horizontal axis can be divided into three intervals.[8] When the tax rate is low ($T \in [20\%, 40.11\%]$), children have a strong incentive to study hard and are hence likely to succeed so that the median voter will be a high earner in the steady state. In contrast, the median voter will be a low earner when the tax rate is set high ($T > 40.11\%$). In particular, condition (21) fails to hold for $T \in (40.11\%, 43.68\%)$, as shown by the shaded area. In this case, the low-paid median voter will find such a tax rate not acceptable and vote for full redistribution. However, as the tax rate gets even higher ($T \geqslant 43.68\%$), the low-paid voter would rather preserve the status quo, and the tax policy is thus credible.

Moreover, given that tax rates are credible, the curves (s_H, s_L) in Figure 1 have a negative slope, implicating that a lower tax stimulates children to study harder. The curves (e^H, e^L) also follow downtrend: all parents – both high-income and low-income – cut back on educational expenses when the tax rate increases, although high-income parents tend to invest more for a given tax rate. Children's efforts and parental inputs are strategic complements, which is consistent with earlier empirical evidence (e.g., De Fraja, Oliveira, and Zanchi 2010). One immediate inference is that, all else being equal, a smaller T results in larger p_L and p_H, which translates into a greater fraction

[8] $T < 15.4\%$ would lead to an infeasible outcome $p > 1$ and hence is beyond discussion.

Figure 1: Income Tax, Private Educational Spending, and Study Effort in the Steady State

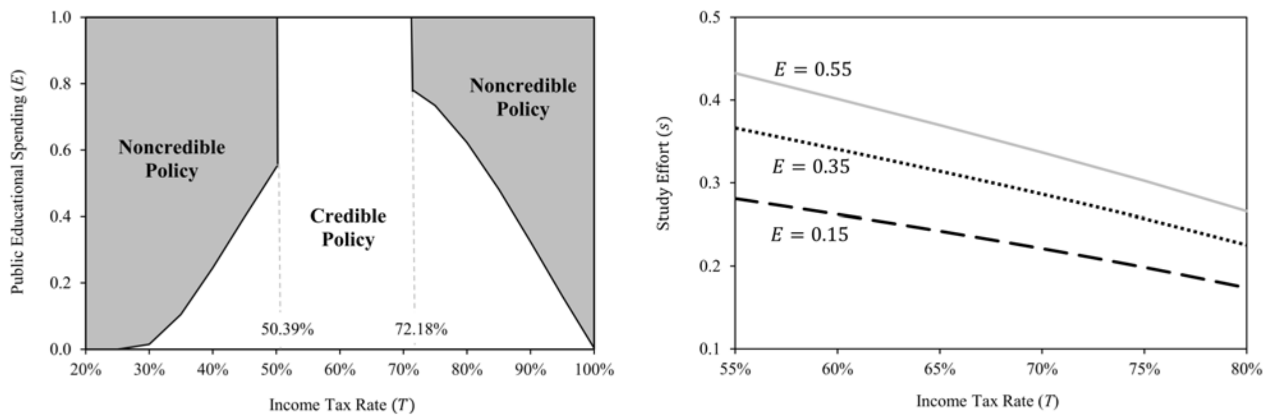


of high earners (larger p) and a higher level of expected steady-state income (larger $p(\omega - 1) + 1$).

Figure 2 demonstrates how government policy influences individual behaviors under the public school system. In the plot on the left, we present different possible combinations of (T, E). There are three disjoint areas divided by two bold curves, wherein the shaded area represents violation of condition (30). A tax rate below 25% is not credible: poor voters would turn against a tax rule that generates inadequate funds of public education and transfers. For a tax rate between 25% and 50.39%, low earners comprise a majority of the population ($p < \frac{1}{2}$). The curve separating the credible and noncredible policies displays an upward trend: given the government budget constraint, increasing educational funds means decreasing transfer payments, which will make the policy less acceptable to poor voters. For a tax rate between 50.39% and 72.18%, high earners have a higher weight in the voting ($p \geqslant \frac{1}{2}$), and the policy is credible for any level of public educational spending. For a tax rate above 72.18%, the median voter turns to be a low earner ($p < \frac{1}{2}$). In this case, a larger E may undermine the policy credibility: increasing educational funds, which effectively enlarges the relative size of high earners, tends to enhance the demand of poor voters for large-scale redistribution because it increases both the total proceeds contributed and their share thereof.

It is also noteworthy that given the level of the public educational spending, a credible income tax rate should be neither too low nor too high. With a too low tax, educational expenditures cannot be fully financed. With a too high tax, children will lack motivation to study and hence have a slim chance of succeeding in the labor market in the future.

Figure 2: Income Tax, Public Educational Spending, and Study Effort



The plot on the right charts the study efforts under different credible policies. The three curves represent three levels of public educational expenditure per child, namely $E \in \{0.15, 0.35, 0.55\}$. Given educational expenditure, a higher income tax tends to hinder study effort. At a given tax rate, increasing public educational spending induces children to exert more study effort. These results are in line with the prediction of Proposition 6: a low income tax combined with a large educational fund serves as a policy signal promoting hard work by children under general circumstances.

Finally, we examine some numerical solutions to the fiscal policies that maximize the steady-state social welfare under private and public school systems. Table 1 reports the simulated optimal credible policies and the consequent steady-state outcomes based on three configurations. Results in the first column ("Benchmark") are obtained based on (33). Under private education, the optimal policy is $T^* = 0.1540$ and $F^* = 0.4620$, which enables the entire population to earn high income ($p^* = 1$) and bear zero net income tax ($\omega T^* - F^* = 0$). All (rich) parents make an investment of $e^{H*} = 1.1249$ in their children's education, and all children devote a study effort of $s_H^* = 0.9244$. Compared with private school system, public school system is associated with a higher income tax rate ($0.4557 > 0.1540$). As everyone is a high earner ($p^* = 1$), all tax revenues are used to support public education ($E^* = 1.3672$ and $F^* = 0$). As the private school system (laissez-faire) achieves Pareto efficiency, the public school system financed by distortionary tax policies tends to make the society worse off ($1.6608 < 1.7009$).

The middle column of Table 1 differs from the first column only in that the degree of parental altruism, α, falls from 1.5 to 0.8. The reduced parental concern for children's career development makes the poor majority ($p^* = 0.0604$) more inclined to raise taxes on the rich (i.e., T^* increases from 0.1540 to 0.7431, and F^* increases from 0.4620 to 0.8329). As a consequence, the rich bear a net tax at the rate of 51.59%, while the poor get a transfer payment accounting for 8.22% of their earnings under the private school system. In addition, parents cut back on their private educational investments, thereby leading children to repond by reducing study efforts; the rich invest more in children's education than their low-income peers ($0.0477 > 0.0199$), and children with rich parents study harder than those with poor parents ($0.1525 > 0.1188$). Public school system facilitates the allocation of educational resources toward children born in low-income families, which may motivate them to study harder; however, the increased tax rate ($0.8150 > 0.7431$) tends to generate an opposing effect. Overall, children choose to put less effort ($s^* = 0.1144$) into learning under the public school system. Also, compared with private education, public education leads to a greater share of high earners ($0.0741 > 0.0604$) and a higher level of social welfare ($0.1535 > 0.1312$).

The last column reports the steady-state results under configuration (33) except that high earners' pretax wage, ω, falls from 3 to 2. It is natural to conceive that the optimal credible tax rates under both the private and public school systems are lower than they are in the benchmark case. A

Table 1. Socially Optimal Policies and the Steady-State Outcomes

		Benchmark	$\alpha = 0.8$	$\omega = 2$
Private School System	T^*	0.1540	0.7431	0.1425
	F^*	0.4620	0.8329	0.1818
	e^{H*}	1.1249	0.0477	0.3855
	e^{L*}	N.A.	0.0199	0.1206
	s_H^*	0.9244	0.1525	0.4537
	s_L^*	N.A.	0.1188	0.3256
	p^*	1	0.0604	0.2757
	W^*	1.7009	0.1312	0.3992
Public School System	T^*	0.4558	0.8150	0.3490
	F^*	0	0.8972	0.1694
	E^*	1.3672	0.0387	0.2892
	s^*	0.8118	0.1144	0.3318
	p^*	1	0.0741	0.3140
	W^*	1.6608	0.1535	0.4015

decrease in after-tax income induces high earners to invest less in their children's education under the private school system ($0.3855 < 0.6724$). Children then become less motivated at school for two reasons. First, the reduction in private educational funding lowers the marginal payoff of study effort. Second, a narrower pretax wage ratio reduces the return to success in the labor market. In the steady state, the economy contains a smaller proportion of high earners in comparison with that in the benchmark case, and in particular, the poor constitute the majority ($p^* = 0.2752$). Likewise, a smaller ω translates into smaller values of (T^*, E^*, s^*, p^*) under the public school system. We also show that, in this case, public education has an advantage over private education in promoting human capital development (higher p^*) and social welfare (higher W^*).

5 Conclusion

In democracies, the majority of the population holds the power to make important government decisions, including national redistributive policies. As inequality persists and becomes a pressing issue, a politico-economic concern then arises: Will low-income voters support policies that result in massive welfare transfers from high earners to themselves? Surprisingly, no democratic society has witnessed this outcome, prompting the question of why the poor hesitate to advocate for wealth redistribution from the rich (Pestieau and Lefebvre 2018). This paper proposes that one plausible explanation lies in the parental perspective, specifically how parents perceive the adverse effect of such policies on their children's motivation to learn and career prospects. Parents may worry that extensive welfare measures could undermine their children's incentive for personal growth and hinder their future professional development. By considering this familial viewpoint, we can gain valuable insights into the complex relationship between democracy, redistributive policies, and the concerns of individual families.

We build an overlapping generations model with a collection of agents who live for childhood and adulthood. Adults work for a salary and vote on the redistributive policy through majority rule, while children choose effort in accumulating human capital based on the voting result. All agents are identical in childhood, and their life outcome uncertainties are resolved in adulthood, turning a fraction of them into high earners and the remainder into low earners. If the median voter is a low earner, she may follow the preexisting tax rule or exercise her discretion to adopt a confiscatory tax rate, and her decision-making involves a tradeoff between her material gain and her child's upward mobility. We characterize a stationary Markov perfect equilibrium, in which tax rules are credibly enforced over time. Each generation make the same study effort, which decreases with the tax rate and increases with the pretax wage ratio. Moreover, intense parental concerns for children's future career success help to enhance the likelihood of policy credibility.

We then consider that study effort and educational resources combine to determine human capital acquisition. Under the private school system, the rational forward-looking individuals choose their childhood study effort and adulthood income allocation between consumption and their children's education, taking into account the positive effect of the costly effort on their future earnings. Under the public school system, children choose study effort based on the extent of redistribution and the level of public educational expenses. We examine the circumstances in which government policies are credible under both systems and characterize the steady state in the stationary Markov perfect equilibrium given a credible policy. Our simulation exercises demonstrate that children's efforts and parental/school inputs may be strategic complements in the steady state. Comparing the optimal credible policies under private education vis-à-via public education shows that (i) the optimal income tax rate decreases with the pretax wage ratio and increases with the degree of parental altruism, and (ii) the introduction of the tax-financed public education causes distortions if parents exhibit profound altruism toward their children, and helps to improve efficiency otherwise.

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Appendix: Mathematical Proofs

Proof of Proposition 2

Rewrite (6) as $f = \frac{s}{p'} - \ln\left[1 + \frac{(\omega-1)(1-T)}{1+(\omega-1)pT}\right] = 0$. It is straightforward to derive that $\frac{\partial f}{\partial \omega} < 0$ and $\frac{\partial f}{\partial T} > 0$. Because $p' > 0$ and $p'' \leqslant 0$, differentiating f with respect to s obtains

$$\begin{aligned}\frac{\partial f}{\partial s} &= \frac{p' - sp''}{(p')^2} - \frac{1}{1 + \frac{(\omega-1)(1-T)}{1+(\omega-1)pT}} \frac{[(\omega-1)(1-T)][(\omega-1)Tp']}{[1+(\omega-1)pT]^2} \\ &= \frac{p' + s(-p'')}{(p')^2} + \frac{(\omega-1)^2 T(1-T)p'}{[1+(\omega-1)pT][\omega-(\omega-1)(1-p)T]} > 0. \end{aligned} \tag{A1}$$

Therefore, the effects of ω and T on s satisfy $\dfrac{\partial s}{\partial \omega} = -\dfrac{\partial f/\partial \omega}{\partial f/\partial s} > 0$ and $\dfrac{\partial s}{\partial T} = -\dfrac{\partial f/\partial T}{\partial f/\partial s} < 0$.

Proof of Proposition 3

By (7) and (8), the median voter finds it beneficial to maintain T if and only if

$$\begin{aligned} d_t^L - \widehat{d_t^L} &= \ln\{1 + (\omega-1)p[s(T)]T\} + \alpha p[s(T)] - \ln\{1+(\omega-1)p[s(T)]\} \\ &= \alpha p[s(T)] + \ln\frac{T + (\omega-1)p[s(T)]T + (1-T)}{1+(\omega-1)p[s(T)]} > 0, \end{aligned} \tag{A2}$$

which can be rearranged as condition (9).

Proof of Proposition 4

Taking the first order condition of W in (10) with respect to T and then using (6) yields

$$\begin{aligned} \frac{\partial W}{\partial T} &= -ss' + p's'\ln[\omega-(\omega-1)(1-p)T] + p\frac{-(\omega-1)(1-p-Tp's')}{\omega-(\omega-1)(1-p)T} \\ &\quad - p's'\ln[1+(\omega-1)pT] + (1-p)\frac{(\omega-1)(p+Tp's')}{1+(\omega-1)pT} + \alpha p's' = 0 \\ \Rightarrow\ & \ln\frac{\omega-(\omega-1)(1-p)T}{1+(\omega-1)pT} - \frac{s}{p'} + (\omega-1)T\left[\frac{1-p}{1+(\omega-1)Tp} + \frac{p}{\omega-(\omega-1)(1-p)T}\right] \\ &+ \frac{(\omega-1)p(1-p)}{p's'}\left[\frac{1}{1+(\omega-1)Tp} - \frac{1}{\omega-(\omega-1)(1-p)T}\right] + \alpha = 0 \end{aligned}$$

$$\Rightarrow \quad \frac{(\omega-1)T\Big[1+(\omega-1)(2pT-p-T+1)+\frac{p(1-p)(\omega-1)(1-T)}{p's'T}\Big]}{[1+(\omega-1)pT][\omega-(\omega-1)(1-p)T]} = -\alpha$$
$$\Rightarrow \ p+T-2pT-\frac{\omega}{\omega-1}-\frac{p(1-p)(1-T)}{p's'T} = \frac{\alpha[1+(\omega-1)pT][\omega-(\omega-1)(1-p)T]}{(\omega-1)^2T}$$
$$\Rightarrow \ p(1-T)-\frac{p(1-T)(1-p)}{p's'T} = \left[\frac{\alpha}{T}\left(\frac{1}{\omega-1}+pT\right)+1\right]\left[\frac{\omega}{\omega-1}-(1-p)T\right], \quad \text{(A3)}$$

which can be rearranged as (11).

Proof of Proposition 5

(i) By (20), $p = \frac{(s_L)^\beta(e^L)^\delta}{1+(s_L)^\beta(e^L)^\delta-(s_H)^\beta(e^H)^\delta} \geqslant \frac{1}{2}$ if and only if $(s_L)^\beta(e^L)^\delta+(s_H)^\beta(e^H)^\delta \geqslant 1$. In this case, the median voter is a high earner, who will never deviate from T in the steady state.

(ii) By (20), $p = \frac{(s_L)^\beta(e^L)^\delta}{1+(s_L)^\beta(e^L)^\delta-(s_H)^\beta(e^H)^\delta} < \frac{1}{2}$ if and only if $(s_L)^\beta(e^L)^\delta+(s_H)^\beta(e^H)^\delta < 1$. In this case, the median voter is a low earner, who will not deviate from T in the steady state if and only if $d^L \geqslant \widehat{d^L}$, which is equivalent to

$$\ln\left[1-e^L+\frac{(\omega-1)T(s_L)^\beta(e^L)^\delta}{1+(s_L)^\beta(e^L)^\delta-(s_H)^\beta(e^H)^\delta}\right]+\alpha(s_L)^\beta(e^L)^\delta \geqslant \ln\left[1+\frac{(\omega-1)(s_L)^\beta(e^L)^\delta}{1+(s_L)^\beta(e^L)^\delta-(s_H)^\beta(e^H)^\delta}\right]$$
$$\Rightarrow \quad \alpha(s_L)^\beta(e^L)^\delta \geqslant \ln\left[\alpha\delta(s_L)^\beta(e^L)^{\delta-1}\right]+\ln\frac{1+\omega(s_L)^\beta(e^L)^\delta-(s_H)^\beta(e^H)^\delta}{1+(s_L)^\beta(e^L)^\delta-(s_H)^\beta(e^H)^\delta}, \quad \text{(A4)}$$

which can be rearranged as condition (21).

(iii) Given $e_t^i = e^i$ and $s_{it} = s_i$ where $i \in \{H, L\}$, we substitute (17) into (16) to obtain

$$\frac{(s_i)^{2-\beta}}{\beta(e^i)^\delta} = \ln\frac{(s_L)^\beta(e^L)^{\delta-1}}{(s_H)^\beta(e^H)^{\delta-1}}+\alpha\Big[(s_H)^\beta(e^H)^\delta-(s_L)^\beta(e^L)^\delta\Big], \quad \text{(A5)}$$

which can be simplified as (22). In the steady state, rewrite (17) as

$$(s_H)^\beta(e^H)^{\delta-1}\left[\omega-e^H-(\omega-1)(1-p)T\right] = (s_L)^\beta(e^L)^{\delta-1}\left[1-e^L+(\omega-1)pT\right] = \frac{1}{\alpha\delta}. \quad \text{(A6)}$$

Inserting (20) into (A6) yields (23).

Proof of Proposition 6

(i) When $p = s^\beta E^\delta \geqslant \frac{1}{2}$, the median voter is a high earner, and hence the policy is always credible.

(ii) When $p = s^\beta E^\delta < \frac{1}{2}$, the median voter is a low earner, and the policy is credible if and only if

$$d^L = \ln\left[1 - E + (\omega - 1)s^\beta E^\delta T\right] + \alpha s^\beta E^\delta \geqslant \ln[1 + (\omega - 1)s^\beta E^\delta] = \widehat{d}^L, \tag{A7}$$

which can be rearranged as (30).

(iii) Rewrite (27) as $g = \dfrac{s^{2-\beta}}{\beta E^\delta} - \ln\left[1 + \dfrac{(\omega - 1)(1 - T)}{1 - E + (\omega - 1)s^\beta E^\delta T}\right]$. It is straightforward to show that $\dfrac{\partial g}{\partial T} > 0$. Differentiating g with respect s and E obtains

$$\begin{aligned}
\frac{\partial g}{\partial s} &= (2-\beta)\frac{s^{1-\beta}}{\beta E^\delta} - \frac{1}{1 + \frac{(\omega-1)(1-T)}{1-E+(\omega-1)s^\beta E^\delta T}} \frac{[(\omega - 1)(1 - T)][(\omega - 1)(\beta s^{\beta-1})E^\delta T]}{-[1 - E + (\omega - 1)s^\beta E^\delta T]^2} \\
&= \frac{2-\beta}{\beta s^{\beta-1}E^\delta} + \frac{\beta(\omega - 1)^2(1 - T)Ts^{\beta-1}E^\delta}{[1 - E + (\omega - 1)s^\beta E^\delta T][\omega - E - (\omega - 1)(1 - s^\beta E^\delta)T]} > 0, \qquad \text{(A8)} \\
\frac{\partial g}{\partial E} &= \frac{s^{2-\beta}}{\beta}(-\delta E^{-\delta-1}) - \frac{1}{1 + \frac{(\omega-1)(1-T)}{1-E+(\omega-1)s^\beta E^\delta T}} \frac{(\omega - 1)(1 - T)[-1 + (\omega - 1)s^\beta T\delta E^{\delta-1}]}{-[1 - E + (\omega - 1)s^\beta E^\delta T]^2} \\
&= -\frac{\delta s^{2-\beta}}{\beta E^{\delta+1}} - \frac{(\omega - 1)(1 - T)[1 - \delta(\omega - 1)s^\beta E^{\delta-1}T]}{[1 - E + (\omega - 1)s^\beta E^\delta T][\omega - E - (\omega - 1)(1 - s^\beta E^\delta)T]}, \qquad \text{(A9)}
\end{aligned}$$

which is negative if $1 - \delta(\omega - 1)s^\beta E^{\delta-1}T > 0$. It follows that $\frac{\partial s}{\partial T} = -\frac{\partial g/\partial T}{\partial g/\partial s} < 0$ always holds and $\frac{\partial s}{\partial E} = -\frac{\partial g/\partial E}{\partial g/\partial s} > 0$ if $\delta(\omega - 1)s^\beta E^{\delta-1}T < 1$.

Proof of Proposition 7

The first order condition of W in (31) with respect to T yields

$$\begin{aligned}
\frac{\partial W}{\partial T} = \Bigg\{ &- s + \alpha\beta s^{\beta-1}E^\delta + \beta s^{\beta-1}E^\delta \ln\frac{\omega - (\omega - 1)(1 - s^\beta E^\delta)T - E}{1 + (\omega - 1)s^\beta E^\delta T - E} \\
&+ \frac{s^\beta E^\delta(\omega - 1)T(\beta s^{\beta-1}E^\delta)}{\omega - (\omega - 1)(1 - s^\beta E^\delta)T - E} + \frac{(1 - s^\beta E^\delta)(\omega - 1)T(\beta s^{\beta-1}E^\delta)}{1 + (\omega - 1)s^\beta E^\delta T - E}\Bigg\}\frac{\partial s}{\partial T} \\
&- \frac{s^\beta E^\delta(\omega - 1)(1 - s^\beta E^\delta)}{\omega - (\omega - 1)(1 - s^\beta E^\delta)T - E} + \frac{(1 - s^\beta E^\delta)(\omega - 1)s^\beta E^\delta}{1 + (\omega - 1)s^\beta E^\delta T - E} = 0 \\
\Rightarrow \quad &\frac{\beta p}{s}\left[\alpha + \frac{p(\omega - 1)T}{\omega - (\omega - 1)(1 - p)T - E} + \frac{(1 - p)(\omega - 1)T}{1 + (\omega - 1)pT - E}\right]\frac{\partial s}{\partial T} \\
&- (\omega - 1)(1 - p)p\left[\frac{1}{\omega - (\omega - 1)(1 - p)T - E} - \frac{1}{1 + (\omega - 1)pT - E}\right] = 0
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow\quad & \frac{\alpha}{\omega-1}+T\left[\frac{p}{\omega-E-(\omega-1)(1-p)T}+\frac{1-p}{1-E+(\omega-1)pT}\right]\\
&=\frac{s(1-p)}{\beta}\left(\frac{\partial s}{\partial T}\right)^{-1}\left[\frac{1}{\omega-E-(\omega-1)(1-p)T}-\frac{1}{1-E+(\omega-1)pT}\right]\\
\Rightarrow\quad & \frac{\alpha}{(\omega-1)^2}+\frac{T[(\omega-E)/(\omega-1)+2pT-p-T]}{[\omega-E-(\omega-1)(1-p)T][1-E+(\omega-1)pT]}\\
&=\left(\frac{\partial s}{\partial T}\right)^{-1}\frac{-(1-T)(1-p)s}{\beta[\omega-E-(\omega-1)(1-p)T][1-E+(\omega-1)pT]}\\
\Rightarrow\quad & \alpha\left[\frac{\omega-E}{\omega-1}-(1-p)T\right]\left(\frac{1-E}{\omega-1}+pT\right)+\left[\frac{\omega-E}{\omega-1}-(1-p)T\right]T-p(1-T)T\\
&=-\frac{(1-T)(1-p)s}{\beta}\left(\frac{\partial s}{\partial T}\right)^{-1}\\
\Rightarrow\quad & \frac{(1-T)(1-p)s}{\beta(-\partial s/\partial T)}+pT(1-T)=\left[\alpha\left(\frac{1-E}{\omega-1}+pT\right)+T\right]\left[\frac{\omega-E}{\omega-1}-(1-p)T\right], \qquad \text{(A10)}
\end{aligned}
$$

which can be rearranged as (31). The first order condition of W in (31) with respect to E yields

$$
\begin{aligned}
\frac{\partial W}{\partial E}&=\alpha\delta s^{\beta}E^{\delta-1}+\delta s^{\beta}E^{\delta-1}\ln\frac{\omega-(\omega-1)(1-p)T-E}{1+(\omega-1)pT-E}+s^{\beta}E^{\delta}\frac{(\omega-1)T(\delta s^{\beta}E^{\delta-1})-1}{\omega-(\omega-1)(1-p)T-E}\\
&+(1-s^{\beta}E^{\delta})\frac{(\omega-1)T(\delta s^{\beta}E^{\delta-1})-1}{1+(\omega-1)pT-E}+\Bigg[-s+\beta s^{\beta-1}E^{\delta}\ln\frac{\omega-(\omega-1)(1-p)T-E}{1+(\omega-1)pT-E}\\
&+\frac{s^{\beta}E^{\delta}(\omega-1)T(\beta s^{\beta-1}E^{\delta})}{\omega-(\omega-1)(1-p)T-E}+\frac{(1-s^{\beta}E^{\delta})(\omega-1)T(\beta s^{\beta-1}E^{\delta})}{1+(\omega-1)pT-E}+\alpha\beta s^{\beta-1}E^{\delta}\Bigg]\frac{\partial s}{\partial E}=0\\
\Rightarrow\quad & \frac{\alpha\delta p}{E}+\frac{\delta s^2}{\beta E}+\left[\frac{\delta p(\omega-1)T}{E}-1\right]\left[\frac{p}{\omega-E-(\omega-1)(1-p)T}+\frac{1-p}{1-E+(\omega-1)pT}\right]\\
&+\frac{\beta p}{s}\left[\alpha+\frac{p(\omega-1)T}{\omega-E-(\omega-1)(1-p)T}+\frac{(1-p)(\omega-1)T}{1-E+(\omega-1)pT}\right]\frac{\partial s}{\partial E}=0\\
\Rightarrow\quad & \left(\delta pT-\frac{E}{\omega-1}+\frac{\beta pET}{s}\frac{\partial s}{\partial E}\right)\frac{(\omega-1)[\omega-E-(\omega-1)(p+T-2pT)]}{[\omega-E-(\omega-1)(1-p)T][1-E+(\omega-1)pT]}\\
&=-\left(\alpha\delta p+\frac{\delta s^2}{\beta}+\frac{\alpha\beta pE}{s}\frac{\partial s}{\partial E}\right)\\
\Rightarrow\quad & -\left(\alpha\delta p+\frac{\delta s^2}{\beta}+\frac{\alpha\beta pE}{s}\frac{\partial s}{\partial E}\right)\left[\frac{\omega-E}{\omega-1}-(1-p)T\right]\left(\frac{1-E}{\omega-1}+pT\right)\\
&=\left[\frac{\omega-E}{\omega-1}-(1-p)T-p(1-T)\right]\left(\delta pT-\frac{E}{\omega-1}+\frac{\beta pET}{s}\frac{\partial s}{\partial E}\right), \qquad \text{(A11)}
\end{aligned}
$$

which can be rearranged as (32).