IMPROVING RECYCLING: HOW FAR SHOULD WE GO?

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Improving recycling: How far should we go?*

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Abstract

We analyze the strategic interaction between competing firms that source their inputs from either primary or recycled material. Because the manufacturers' primary production today serves as input for the recyclers' production tomorrow, manufacturers can limit the recyclers' scale of operation by reducing their output. Improving the recycling process generates then two opposite effects: it reduces primary production tomorrow by exposing manufacturers to stronger competition from recyclers, but it also lowers the manufacturers' incentives to reduce their primary production today. Making the recycling process too efficient might then be counterproductive for the environment. This intuition equally applies to remanufacturing.

Keywords: Recycling, remanufacturing, circular economy, strategic entry accommodation
JEL-Classification: L13, L72, O13, Q58

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1 Introduction

Research question. Authorities around the world have committed to scale up the collection of scraps for recycling. The expectation is that, by increasing the inputs for the recycling sector, they can reduce the volume of primary production and hence, lower the economy's impacts on the environment.[1] The European Parliament, for instance, signed in 2019 a legislation procedure requiring the Member States to achieve a 90% collection target for plastic bottles by 2029.

While recycling can reduce the negative impact of the economy on the environment, one may ask *to which extent an improvement in the collection of scraps for recycling can reduce the volume of primary production.* To answer this question properly, we need to analyse the strategic interaction between competing firms that source their inputs from either primary or recycled material. What makes this competition peculiar is that the firms producing from primary material (which we call the '*manufacturers*') also supply inputs for the firms producing from recycled material (which we call the '*recyclers*'). In consequence, the manufacturers can control the scale at which the recyclers can operate; in particular, they may want to reduce their current production to limit the competition that recyclers will exert in the future. It is thus crucial to take this possibility into account when evaluating the impacts of improving the recycling process.[2]

Main result. Our analysis establishes that *improvements in the recycling process do not necessarily contribute to reduce the extraction of primary material.* The intuition behind this result is the following. Improving the recycling process generates two opposite effects. On the one hand, it reduces primary production once recyclers enter the market because the competition they exert on manufacturers gets stronger as recycling is improved. On the other hand, the better the recycling process, the lower the incentives for manufacturers to reduce their primary production *before* recyclers enter. The benefit for manufacturers of limiting the recyclers' future entry must indeed be measured against the cost of forgoing current profits. An improvement of the recycling process worsens the benefit/cost ratio of this strategy because it forces manufacturers to accept a larger decrease of its current primary production to reach a given reduction of the recyclers' scale of entry.

To establish this result, we consider a model with three periods. In period 0, an authority determines the efficiency of the recycling process (e.g., the rate of scrap collection). In period 1, manufacturers extract primary material and use it to produce some final products. In period 2, recyclers enter the market; they produce the final product using recycled material while manufacturers continue to produce from primary material.

[1]Recycling of aluminum products, for example, requires as little as 5% of the energy and emits as little as 5% of green-house gas compared to production of primary aluminum (International Aluminium Institute, 2009). Recycling can also reduce the negative impact of waste on the environment (arguably better than landfill and incineration).

[2]As we discuss in Section 3.1, the same intuition applies to remanufacturing, which is "a specific type of recycling in which used durable goods are repaired to a like-new condition" (Bernard, 2011, p. 337).

Periods are linked as follows: the available recycled material in period 2 is a fraction of what manufacturers produced in period 1, and this fraction is determined by the authority's choice in period 0. In Section 2, we analyse a simplified version of this model with one manufacturer, one recycler and linear demand and costs. In Section 3, we show that our results hold in more general settings, with general demand and cost functions or with an arbitrary number of manufacturers and recyclers.

Related literature. Economists have studied the "recycling problem" since the notorious Alcoa case (Walter, 1951). In 1945 Alcoa, the producer of primary aluminum, was found in a monopolistic position by virtue of its control over 90% of primary aluminum output, limiting the competitiveness of the recycling industry, which captured roughly 20% of the total aluminum market. Judge Learned Hand concluded that Alcoa constituted an illegal monopoly, in violation of the Sherman Antitrust Act: Alcoa was found to control strategically the recycling sector's supply by manipulating the primary aluminum production. Questioning the correctness of this judgment, a strand of literature in industrial organization started in the mid-1970s to analyze theoretically and empirically the so-called "recycling problem."

The literature started with Gaskins (1974). Using an optimal control model to simulate the Alcoa scenario, the paper proposes that the manufacturer dominates the market when demand grows at steady state and confirms Hand's judgment. Swan (1977) then criticizes Gaskin's model for the sensitivity of its results to the rate of demand growth. In an overlapping-generations setting, he predicts that the long-run price is close to the monopoly price in the absence of recycling, proving that Alcoa strategically controls the supply of primary aluminum to maintain its monopoly position. In another paper, Suslow (1986) estimates that it is the degree of substitutability between primary and recycled aluminum on the demand side that determines the Alcoa's market power. Finally, Grant (1999) presents a more general model of the "recycling problem" and proposes that Alcoa's market power is due to the recyclers' inability to recycle the aluminum scraps economically, and that the existence of recycling is welfare-reducing relative to a monopoly in all aluminum production. However, these studies, among others[3], only focus on the impact of a competitive recycling sector on the market power of the manufacturer by integrating the collection decision in the recycling entities. In reality, while reprocessing entities are mostly private, the collection system relies heavily on the government's effort to scale it up. Playing a significant role in organizing the curb-side collection and subsidizing the collection entities, governments' commitments influence the collection rates beyond the market-based mechanism. Therefore, the impact of an exogenous variation of the collection rate is worth further analyses.

A second related strand of the literature focuses on the performance of the collection system and echoes our recommendation that recycling should not be pushed too far.

[3]See, for example, Hoel (1984), Hollander and Lasserre (1988), Gaudet and Van Long (1999), Gaudet and Van Long (2003), Eichner (2005) and Honma and Chang (2010).

Kinnaman, Shinkuma, and Yamamoto (2014) use data in Japan to estimate the average social cost of waste management as a function of the recycling rate. Defining the social cost as the sum of all municipal costs and revenues, costs to recycling households, external disposal costs and external benefits of recycling, the authors suggest that the recycling rate that minimizes the average social costs in Japan should only be 10% and concluded that *"the 20% recycling rate in Japan is higher than the socially optimal rate"* and that *"the current recycling rates in the United States (35%) and the EU27 (34%) may also be too high."* Dijkgraaf and Gradus (2014) estimate the cost function resulting from different policies in waste recycling in the Netherlands and find that it seems nearly impossible for the Netherlands to reach the EU-goal of 70% recycling rate because of the high cost of the recycling system. These studies, however, largely ignore interaction in the industry between the primary and secondary producers.[4]

Our work is also closely related to the literature on "remanufacturing"(see the discussion in Section 3.1). This strand of literature analyses the impacts of remanufacturing on primary manufacturers' profitability. For instance, Atasu, Sarvary, and Van Wassenhove (2008) investigate the conditions for the benefits from remanufacturing to outweigh the losses from cannibalization when manufacturers conduct remanufacturing themselves. They show that remanufacturing is more beneficial under competition than under monopoly. In another paper, Ferguson and Toktay (2009) analyze the competition between a manufacturer and a remanufacturing firm. They discuss the conditions for the manufacturer to choose to remanufacture its products or not, and compare two entry-deterrent strategies: remanufacturing and preemptive collection.

Three papers are closer to our work. First, Örsdemir, Kemahlioğlu-Ziya, and Parlaktürk (2014) study the competition between a manufacturer and a remanufacturer, incorporating the constraint that the remanufactured product quantity cannot exceed the quantity of the original product. Their model shares some features with ours but also differs in important aspects: competition takes place on a single period and public policy is not considered. Second, in a two-period model with linear demand, Mitra and Webster (2008) discuss the impact of the government's subsidies on remanufacturing by conducting numerical simulations with some given collection rates. The closest analysis to ours is Ba and Mahenc (2019). They study the impact of recycling on a monopolistic extractor of exhaustible resources, proving that recycling can speed up or slow down primary resource extraction relative to the Hotelling rule, depending on the objective of the extractor (for-profit or social welfare improvement). However, similar to the other strand of literature, these studies are not concerned with the impact of changes in the collection rate of end-of-life products on the manufacturer's strategy.

[4]In this stream of the literature, we can also cite Hamilton, Sproul, Sunding, and Zilberman (2013), Kinnaman (2013), Fullerton and Kinnaman (1995), Kinnaman and Fullerton (2000), Callan and Thomas (2001), Kinnaman (2006), Bohm, Folz, Kinnaman, and Podolsky (2010), Kinnaman (2010), and Hamilton et al. (2013).

2 A baseline model

We consider here a simplified setting with one manufacturer, one recycler and simple specifications for demand, costs and recycling technology. In the next section, we discuss our assumptions and show the robustness of our results by extending the model in several directions. We consider the market for some homogenous good (think, e.g., of aluminum cans). The manufacturer produces the good from primary material (that it extracts itself or acquires on some, not modelled, upstream market), whereas the recycler does so by reprocessing a fraction of the manufacturer' end-of-life products. The collection of end-of-life products is organized by some authorities, which we refer to as the *government*.

Because the government aims at setting the 'rules of the game' with the objective to reduce the environmental impact of the extraction of primary material, and because the manufacturer's initial production conditions the recycler's production capacity, we assume the following timeline. *In period 0*, the government sets a commitment for the collection rate $\tau \in [0, 1]$, with $\tau = 0$ corresponding to the total absence of collection, and $\tau = 1$ corresponding to the complete collection of all the scraps. *In period 1*, the manufacturer learns this information and chooses the quantity of production for this period, q_1; by the end of the period, all products in use wear out and the government collects the committed proportion τ of the scraps (the rest of the scraps is dumped). *In period 2*, the recycler enters the market and uses the scraps collected as input to compete with the manufacturer *à la Cournot*; we denote by r the quantity produced by the recycler and by q_2 the quantity produced by the manufacturer.

The other ingredients of the baseline model are as follows.

Demand. The inverse demand for the good is $p_1 = 1 - q_1$ in period 1 and $p_2 = 1 - q_2 - r$ in period 2. That is, we assume that consumers perceive the manufacturer's and the recycler's products as homogenous, and that the recycler's entry does not contribute to increase total demand (i.e., the maximum price that consumers are willing to pay is the same in both periods and is normalized to 1).

Production costs. We assume that both firms have a constant marginal cost of production and no fixed cost. Without loss of generality, we normalize the manufacturer's production cost to zero. Meanwhile, the recycler bears the cost of buying, sorting, and reprocessing old scraps to produce recycled products; the total cost to produce a quantity r of recycled products is equal to cr, with $c \geq 0$, meaning that recycling is at least as expensive as primary production. Prior to the recycler's entry, the manufacturer does not know precisely the value of c; it expects c to be drawn from a uniform distribution over the interval $[0, 1/2]$. As we assume no entry cost, $c \leq 1/2$ guarantees that the recycler enters the market.[5]

[5] For $c > 1/2$, the recycler stays out because entry is not profitable even when the manufacturer produces the monopoly quantity. Using the terminology of Bain (1956), we say that entry is 'blockaded' in this case. In our setting, the manufacturer cannot 'deter' entry and must 'accommodate' it when $c < 1/2$.

Recycling technology. We assume for simplicity a 1:1 recycling technology that allows the recycler to produce one unit of output with one unit of scrap as input (which explains the recycler's marginal cost of production). As for the collection of scraps, we put its organization in a black box. That is, we abstract away all the mechanisms that need to be put in place to implement a given collection rate.[6] We just assume, realistically, that the government has the capacity to modify this rate and, thereby, the 'rules of the game' by which the two firms will play.

We solve the game for its subgame-perfect equilibrium, assuming that the manufacturer does not discount its future profit when choosing its quantity in period 1. Before doing so, we briefly outline the benchmark case with no possibility of recycling. This is so, in our setting, when the collection rate τ is equal to zero: with no scrap collected, the recycler cannot enter the market. In this case, the manufacturer would simply behave as an unconstrained monopolist in both periods: it would choose q_1 and q_2 to maximize $\pi = (1-q_1)q_1 + (1-q_2)q_2$, which yields $q_1 = q_2 = q^m = 1/2$. Over the two periods, the manufacturer would then produce a total quantity of primary products $2q^m = 1$ and earn a total profit of $\Pi(1/2, 1/2) = 1/2$.

We now turn to the situations in which $\tau > 0$: scraps are collected and the recycler can enter the market in period 2. Solving the game backwards, we first analyse the Cournot competition in period 2; we then move to the manufacturer's choice in period 1 before considering the government's problem in period 0.

2.1 Competition between manufacturer and recycler

In period 2, the two firms simultaneously choose their quantity. The manufacturer chooses q_2 to maximize $\pi_2 = (1 - q_2 - r)\, q_2$; we derive the manufacturer's best-response function from the first-order condition:

$$q_2\,(r) = (1 - r)\,/2. \tag{1}$$

The recycler chooses r to maximize $\pi_r = (1 - q_2 - r)r - cr$ under the constraint $r \le \tau q_1$ (as it cannot produce more than the amount of scrap collected, i.e., τq_1). Solving the constrained maximization program, we find that the recycler's best-response function is kinked:

$$r^*\,(q_2) = \begin{cases} \frac{1}{2}\,(1 - c - q_2) & \text{if} \quad \frac{1}{2}\,(1 - c - q_2) \le \tau q_1, \\ \tau q_1 & \text{otherwise}\,. \end{cases} \tag{2}$$

Crossing the two best-response functions, we can identify two possible Cournot-Nash equilibria in period 2, depending on the amount of scraps collected (τq_1) and the recycler's unit cost (c): an *'unconstrained equilibrium'* in which scraps are in large supply and/or the recycler is not efficient enough to reprocess them all, and a *'constrained equilibrium'*

[6] The collection system can be organized into centralized or decentralized industries, with different policies to encourage consumers and firms to participate in scrap collection. To compare the merits of different organizations, see, e.g., Beatty, Berck, and Shimshack (2007), Viscusi, Huber, and Bell (2012), Hamilton et al. (2013), Kinnaman (2013), or Kinnaman et al. (2014).

in which scraps are in short supply and/or the recycler is efficient enough to be bounded by the input availability. These two equilibria are characterized as follows:

- Quantities at the *unconstrained equilibrium* are found by solving the system of equations made of (1) and the top branch of (2):

  $$q_2^u = \tfrac{1}{3}(1+c) \text{ and } r^u = \tfrac{1}{3}(1-2c),$$

  (with $r^u \geq 0$ as we assume $c \leq 1/2$). The equilibrium profits are then computed as

  $$\pi_2^u = \tfrac{1}{9}(1+c)^2 \text{ and } \pi_r^u = \tfrac{1}{9}(1-2c)^2.$$

- In the *constrained equilibrium*, the recycler's quantity is bounded by the input constraint and the manufacturer reacts according to (1), so that

  $$q_2^c = \tfrac{1}{2}(1-\tau q_1) \text{ and } r^c = \tau q_1,$$

  leading to equilibrium profits of

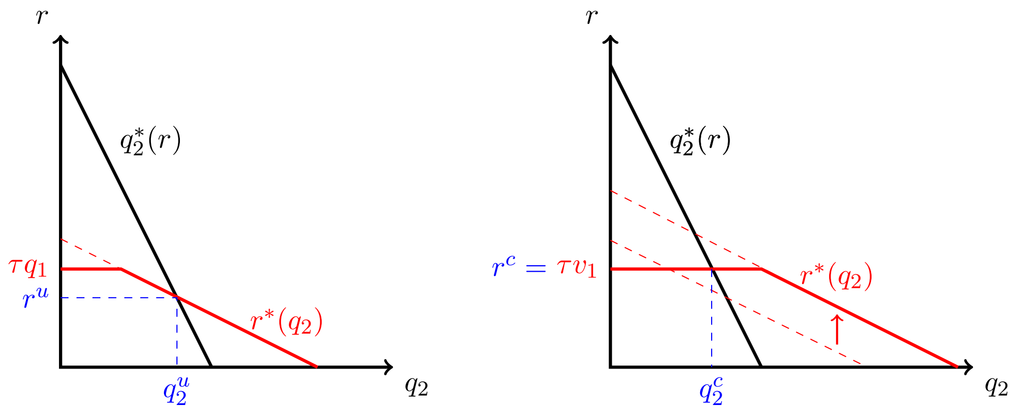
  $$\pi_2^c(q_1) = \tfrac{1}{4}(1-\tau q_1)^2 \text{ and } \pi_r^c(q_1) = \tfrac{1}{2}\tau q_1(1-2c-\tau q_1).$$

Figure 1 depicts the two possible equilibria in period 2. Due to the input constraint, the recycler's reaction function is kinked at $r = \tau q_1$. If the recycler is not efficient enough, it cannot economically recycle all the scraps collected. In this case, we obtain the unconstrained equilibrium (q_2^u, r^u) in which $r^u < \tau q_1$ (Figure 1a). Because the input constraint is not binding, the firms' outputs are independent of the collection rate τ and the quantity of primary products in the first period q_1, but they depend on the marginal recycling cost c (a higher marginal cost c leads to lower recycling r^u and higher primary production q_2^u in period 2). In contrast, if the recycler is efficient enough, its best-response function is shifted upward, as in Figure 1b. Here, the quantity of recycled product is constrained by the initial primary production q_1. The market then reaches the equilibrium (q_2^c, r^c) in which the recycler reprocesses all the scraps collected ($r^u = \tau q_1$), whereas the manufacturer produces a larger quantity q_2^c than in the unconstrained equilibrium. In this case, the manufacturer can control the scale of the recycler through its initial production q_1 (a lower q_1 leads to a lower r^c and a larger q_2^c).

We observe that, given a quantity of scraps collected τq_1, the unconstrained equilibrium occurs if $r^u < \tau q_1$ and the constrained equilibrium occurs otherwise. Since r^u decreases with the recycler's marginal cost c, we obtain the following lemma.

Lemma 1. *(1) For a given quantity of scraps collected τq_1, the unconstrained equilibrium ($r^u < \tau q_1$) obtains if the recycler's marginal cost c is above $\tilde{c} \equiv (1-3\tau q_1)/2$ and the constrained equilibrium ($r^u = \tau q_1$) obtains otherwise. (2) As the threshold $\tilde{c}$ decreases with τ and q_1, only the unconstrained equilibrium can occur if $\tau q_1 \geq 1/3$.*

Proof. (1) The threshold $\tilde{c}$ is the value of c that solves $r^u = \frac{1-2c}{3} = \tau q_1$; for $c > \tilde{c}$, we have $r^u > \tau q_1$. (2) Given that $c \geq 0$, only the unconstrained equilibrium can occur if $\tilde{c} \leq 0$, which is equivalent to $\tau q_1 \geq \frac{1}{3}$; for instance, if $q_1 = q_1^m = \frac{1}{2}$, then the condition becomes $\tau \geq \frac{2}{3}$. □



(a) Unconstrained equilibrium (b) Constrained equilibrium

Figure 1: Cournot-Nash equilibrium in period 2

2.2 Manufacturer's 'limit entry' strategy

We now analyse whether and how the manufacturer wants to follow a 'limit entry' strategy, whereby the firm reduces its production in period 1 so as to limit the quantity of input that the recycler will be able to use in period 2. We are also interested in evaluating how the level of the collection rate affects the manufacturer's decision.

In period 1, the manufacturer chooses the quantity q_1 to maximize its expected profits over the two periods. There are two possible courses of action. First, we know from Lemma 1 that if the manufacturer sets a sufficiently large quantity – namely, $q_1 \geq 1/(3\tau)$ – it can make sure that the equilibrium in period 2 will be unconstrained irrespective of the marginal cost drawn by the recycler. In that case, the manufacturer's profit in period 2 is $\pi_2^u = (1+c)^2/9$, which is independent of q_1. Hence, the manufacturer's optimal quantity is period 1 is $q_1^m = 1/2$. This quantity satisfies the constraint as long as $q_1^m \geq 1/(3\tau)$, or $\tau \geq 2/3$.

Alternatively, the manufacturer can choose $q_1 < 1/(3\tau)$. Then, the equilibrium prevailing in period 2 depends on the cost drawn by the recycler: the manufacturer obtains the profit π_2^c in the constrained equilibrium if $c < \tilde{c}$, and obtains the profit π_2^u in the unconstrained equilibrium if $c > \tilde{c}$. Consequently, if $q_1 < 1/(3\tau)$, the manufacturer's expected profit function can be written as

$$\Pi^e = \pi_1(q_1) + \hat{\pi}_2^c(q_1) + \hat{\pi}_2^u(q_1), \tag{3}$$

where $\pi_1(q_1) = q_1(1-q_1)$ and, given the uniform distribution of c over the interval $[0, 1/2]$,

$$\hat{\pi}_2^c(q_1) = \int_0^{\tilde{c}} 2\pi_2^c(q_1)\,dc = \int_0^{\frac{1}{2}(1-3\tau q_1)} \tfrac{1}{2}(1-\tau q_1)^2\,dc = \tfrac{1}{4}(1-\tau q_1)^2(1-3\tau q_1), \tag{4}$$

$$\hat{\pi}_2^u(q_1) = \int_{\tilde{c}}^{\frac{1}{2}} 2\pi_2^u dc = \int_{\frac{1}{2}(1-3\tau q_1)}^{\frac{1}{2}} \tfrac{2}{9}(1+c)^2\,dc = \tfrac{1}{4}\tau q_1\left(\tau^2 q_1^2 - 3\tau q_1 + 3\right). \tag{5}$$

Because $\tilde{c}$ decreases with q_1, the manufacturer faces a tradeoff when increasing q_1. From (5), we observe that $\hat{\pi}_2^u(q_1)$ increases with q_1 as the probability that the unconstrained equilibrium occurs increases with q_1 while the manufacturer's profit remains constant. In contrast, we observe from (4) that $\hat{\pi}_2^c(q_1)$ decreases with q_1 for two reasons: not only the constrained equilibrium becomes less likely but also the manufacturer gets a smaller profit (as $q_2^*(\tau q_1)$ decreases with q_1 because of strategic substitutability).

Clearly, the level of the collection rate τ affects the balance between these two conflicting forces. As we now show, it does so in a non-monotonic way. Denote by $q_1^*(\tau)$ the quantity that maximizes expression (3) for a given τ. Note first that if τ is close to zero, the unconstrained equilibrium is very unlikely (as $\tilde{c}$ is close to $1/2$) and the manufacturer is hardly affected by the small scale of the recycler's operation in the constrained equilibrium. It follows that the manufacturer choice of quantity $q_1^*(\tau)$ tends to $q_1^m = 1/2$ as τ tends to zero. Note also that we have just established that the manufacturer chooses $q_1^m = 1/2$ as well for $\tau \geq 2/3$. To understand how $q_1^*(\tau)$ evolves with τ for $0 < \tau < 2/3$, we use the implicit function theorem to write

$$\tfrac{dq_1^*(\tau)}{d\tau} = -\tfrac{\partial^2 \Pi^e(q_1^*)}{\partial q_1 \partial \tau} \Big/ \tfrac{\partial^2 \Pi^e(q_1^*)}{\partial q_1^2}.$$

Because $q_1^*(\tau)$ maximizes the firm's expected profit, $\partial^2 \Pi^e(q_1^*)/\partial q_1^2 < 0$ by the second-order condition. So $dq_1^*(\tau)/d\tau$ takes the sign of

$$\frac{\partial^2 \Pi^e(q_1^*)}{\partial q_1 \partial \tau} = \underbrace{\frac{\partial^2 \pi_1(q_1^*)}{\partial q_1 \partial \tau}}_{=0} + \underbrace{\frac{\partial^2 \hat{\pi}_2^c(q_1^*)}{\partial q_1 \partial \tau}}_{(i)} + \underbrace{\frac{\partial^2 \hat{\pi}_2^u(q_1^*)}{\partial q_1 \partial \tau}}_{(ii)}.$$

As τ only influences the first-period profit via its impact on q_1, the first term is equal to zero. The variation of q_1 with respect to τ depends then on two factors: (i) the marginal impact of τ on the expected loss following an increase in q_1^* under the constrained equilibrium and (ii) the marginal impact of τ on the expected gain following an increases in q_1^* under the unconstrained equilibrium. Therefore, the profit-maximizing quantity in period 1, q_1^*, decreases with τ if the first impact dominates the second, and increases with τ otherwise. As noted above, the former case certainly occurs when τ is close to zero (as the second impact vanishes), while the latter case certainly occurs when τ is close to $2/3$ (as the first impact vanishes). We expect thus $q_1^*(\tau)$ to be a U-shaped function of τ.

We now confirm our intuition by computing the exact value of $q_1^*(\tau)$. The first-order condition for profit-maximization is:

$$\frac{\partial \Pi^e}{\partial q_1} = \frac{1}{2}\left(-3\tau^3 q_1^2 - 4\left(1-\tau^2\right) q_1 + 2 - \tau\right) = 0. \quad (6)$$

At $\tau = 0$, it is equivalent to $1 - 2q_1 = 0$, which confirms that $q_1^*(0) = 1/2 = q_1^m$. For $\tau > 0$, the solution to Equation (6) is[7]

$$q_1^*(\tau) = \frac{\sqrt{4 - 8\tau^2 + 6\tau^3 + \tau^4} - 2\left(1-\tau^2\right)}{3\tau^3}. \quad (7)$$

[7]It can easily be checked that the other root is negative and that the second-order condition is satisfied.

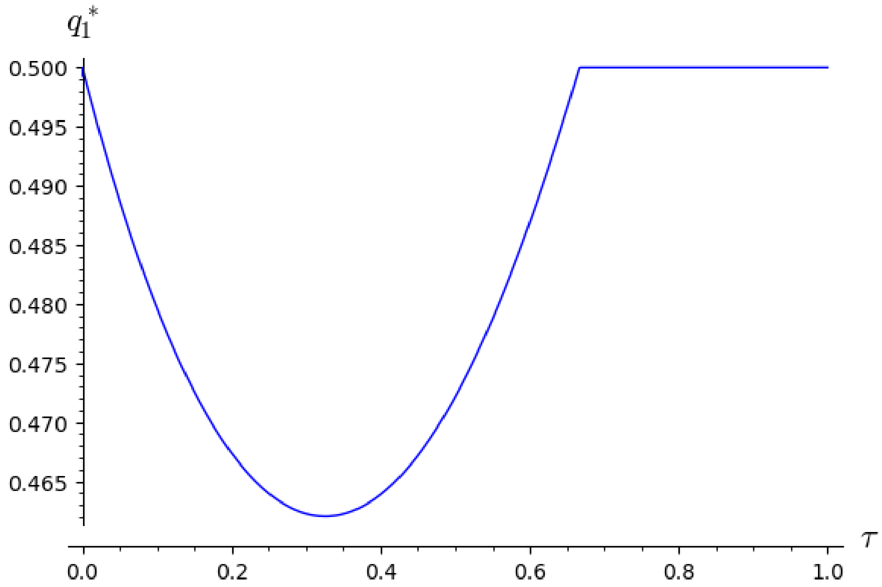


Figure 2: variation of first-period production with respect to the collection rate

We check that $q_1^*(\tau) < 1/(3\tau)$ if and only if $\tau < 2/3$. We also observe that for $\tau < 2/3$, $q_1^*(\tau) < 1/2$ (while $q_1^*(2/3) = 1/2$). As represented in Figure 2, $q_1^*(\tau)$ is a U-shaped function of τ: $q_1^* = 1/2$ at the two extreme values of the interval ($\tau = 0$ and $\tau = 2/3$), it decreases with τ for $0 < \tau < \tilde{\tau} \approx 0.325$ and increases with τ for $\tilde{\tau} < \tau < 2/3$. For $\tau \geq 2/3$, $q_1^*(\tau) = 1/2$: as explained above, the manufacturer can no longer constrain the recycler's input once the collection rate becomes too large; it therefore maintains the monopolistic production q_1^m to maximize its profits in the first period.

The next proposition records our results.

Proposition 1. *(1) If the collection rate τ is lower than $2/3$, then the manufacturer contracts its period 1 production, $q_1^*(\tau) < q_1^m$, to limit the recycler's scale of operation in period 2; the contraction is the largest for $\tau = \tilde{\tau} \approx 0.326$. (2) If the collection rate τ is larger than $2/3$, then the manufacturer maintains the monopolistic production level in period 1, $q_1^*(\tau) = q_1^m$.*

Intuitively, when the collection rate is small, the manufacturer can constraint the recycler's scales by reducing slightly its production in period 1. Hence, in this case, the manufacturer finds it profitable to sacrifice part of its period 1 profits to increase its expected period 2 profits. As long as the collection rate remains smaller than $\tilde{\tau}$, the manufacturer reduces further its initial production. Yet, once the collection rate becomes larger than $\tilde{\tau}$, the manufacturer continues to apply the limit entry strategy, but it does so by reducing its initial production by smaller amounts; in fact, as the recycler can access a larger share of the initial production, the manufacturer must forgo more profits in period 1 to reach a given increase in expected profits in period 2. Eventually, the limit entry strategy becomes unprofitable (and even unfeasible) when the collection rate gets larger than 2/3; the manufacturer is then no longer willing to contract its initial production and prefers to produce the monopoly output in period 1, as though the recycler's was not to enter in period 2.

2.3 Government's choice of collection rate

Finally, we examine the choice of the collection rate τ by the government.[8] We first assume that the government's sole objective is to minimize the extraction of primary material. We then examine how the optimal collection rate according to this purely environmental objective compares with the collection rate that would maximize consumer surplus. Throughout the analysis, we take the simplified view that increasing the collection rate is costless.

2.3.1 How to minimize primary production?

Total primary production is the addition of the quantities produced by the manufacturer in periods 1 and 2. As for period 1 production, we found above that $q_1^*(\tau)$ is equal to expression (7) for $\tau < 2/3$ and to $q_1^m = 1/2$ for $\tau \geq 2/3$. The manufacturer's production in period 2 also depends on whether the collection rate τ is below or above the threshold of 2/3. If $\tau < 2/3$, the manufacturer produces $q_1^*(\tau) < q_1^m$ to limit the recycler's entry. If the recycler's cost is such that $c < \tilde{c}$, then the manufacturer produces $q_2^c = (1/2)[1 - \tau q_1^*(\tau)]$; otherwise, if the recycler's cost is such that $c > \tilde{c}$, then the manufacturer produces $q_2^u = (1+c)/3$. In contrast, if $\tau \geq 2/3$, the manufacturer knows that it cannot limit the recycler's entry; then, irrespective of the recycler's cost c, the manufacturer's period 2 production is $q_2^u = (1+c)/3$. Hence, the expected quantity of primary production over the two periods can be written as

$$q_2^*(\tau) = \begin{cases} q_1^* + 2\int_0^{\frac{1}{2}(1-3\tau q_1^*)} \dfrac{1-\tau q_1^*}{2} dc + 2\int_{\frac{1}{2}(1-3\tau q_1^*)}^{\frac{1}{2}} \dfrac{1+c}{3} dc & \text{if} \quad \tau < \dfrac{2}{3} \\ \dfrac{1}{2} + 2\int_0^{\frac{1}{2}} \dfrac{1+c}{3} dc & \text{otherwise.} \end{cases}$$

It is worth noting that period 2 primary production is the average of q_2^c and q_2^u weighted by the probability that each equilibrium occurs. While q_2^u does not depend on τ, q_2^c decreases with τ. Moreover, the threshold $\tilde{c}(q_1^*)$ increases with τ. In other words, increasing the collection rate reduces primary production under the constrained equilibrium and, at the same time, increases the probability of occurrence of the unconstrained equilibrium, in which the manufacturer produces a smaller quantity of primary production. Under

[8] As our objective is to demonstrate the counter-intuitive impacts of modifying the collection rate, we limit our analysis to this policy instrument, abstracting away other instruments – such as taxes on primary products or subsidies on recycled products – that the government could use to limit the extraction of primary resources. For a survey on the economics of environmental policy instruments, see, e.g., Sterner and Robinson (2018).

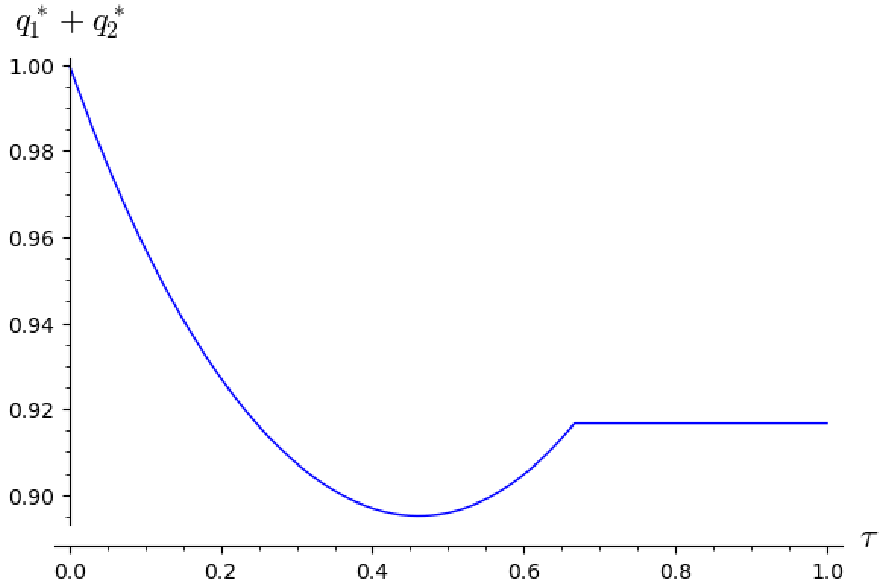


Figure 3: variation of the expected primary production with respect to τ

these two effects, *the expected primary production in period 2 decreases when τ increases.* We recall, however, that q_1^* does not vary monotonically with τ: it first decreases with τ, then increases with τ and finally reaches a plateau. We observe that the same pattern applies to the total primary production, $q^*(\tau) = q_1^*(\tau) + q_2^*(\tau)$. As represented in Figure 3, $q^*(\tau)$ decreases with τ for $0 < \tau < 0.46$, then increases with τ for $0.46 < \tau < 2/3$ and finally stays constant for $2/3 \leq \tau \leq 1$. We therefore conclude the following.

Proposition 2. *A government that aims at minimizing total primary production (and that can modify the collection rate at zero cost) chooses to set the collection rate at an intermediate level that achieves the best balance between the incentives given to the manufacturer to reduce its production in period 1 (so as to limit the recycler's entry) and the competition exerted by the recycler to limit the manufacturer's production in period 2.*

2.3.2 Do consumers support environmental measures?

In our baseline model with homogeneous products, the consumer surplus increases with the level of total production (primary *and* recycled). For a given pair of quantities (q_1, q_2) produced by the manufacturer in periods 1 and 2, and a given quantity r produced by the recycler in period 2, the consumer surplus is indeed computed as $CS = \frac{1}{2}[q_1^2 + (q_2 + r)^2]$. A priori, recycling has ambiguous impacts on the consumer surplus: on the one hand, the recycler's entry in period 2 benefits consumers (because in a homogeneous product market, duopolists produce together a larger equilibrium quantity than a monopolist does); on the other hand, the prospect of the recycler's entry induces the manufacturer to (weakly) decrease its production in period 1. As we state in the next lemma, it turns

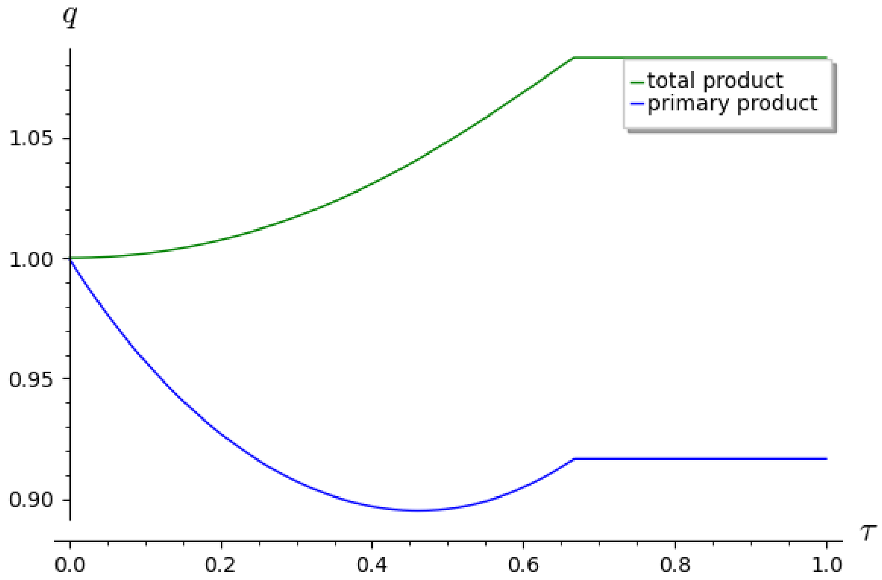


Figure 4: evolution of primary and total production with respect to τ

out that the former effect always outweighs the latter, and even more so as the collection rate increases.

Lemma 2. *Consumer surplus weakly increases with the collection rate.*

We thus see from Lemma 2 that consumers would vote for pushing the improvement of the recycling process to at least $\tau = 2/3$, that is, past the level that would be optimal for the environment. Protecting the environment may thus reduce the consumers' well-being in the short-run, as summarized in the next proposition and illustrated in Figure 4 (which contrasts the evolutions of primary and total production with respect to τ).

Proposition 3. *Increasing the collection rate from a low level benefits both consumers and the environment. However, increasing the collection rate over a certain threshold creates a tradeoff between consumer surplus and environment: while consumers have access to more products, the quantity of primary production also increases with the collection rate.*

The intuition behind this proposition should be clear by now. While it is clear that consumers will enjoy more surplus when there are more products in the economy, this also implies a tradeoff between consumer surplus and the environment. However, when the collection rate is initially low, the increasing substitution of primary by recycled products can reduce the primary production and increase total production at the same time. Hence, any improvement of the collection rate below $\tilde{\tau}$ is a win-win situation for both consumers and the environment. However, if $\tau > \tilde{\tau}$, improving the collection rate leads to an increase in the primary production. In this case, increasing the collection rate will benefit consumers but make the economy deviate from the best scenario for the environment (not to mention that it is costly to improve the collection system, particularly when the collection rate is already high).

2.3.3 Do firms support environmental measures?

Unsurprisingly, the firms' preferences regarding the level of the collection rate are completely at odds with one another: the manufacturer's profit is the largest for $\tau = 0$, while the recycler's profit reaches its maximum level at values of τ equal or larger than $2/3$. So, *if the government aims at minimizing total primary production, the chosen collection rate is too high from the manufacturer's viewpoint and too small from the recycler's viewpoint* (the recycler sides thus with the consumers). The chosen collection rate is also too high if we take the point of view of total industry profits, as they decrease with τ; this follows from the fact that, in our model, the manufacturer earns profit over one more period than the recycler.

3 Generalisation

In this section, we show that the results obtained in our baseline model continue to hold in more general settings. First, we broaden the scope of the baseline model by reformulating some assumptions, so as to adapt the setting to a wider set of industries and address the case of remanufacturing. Second, in the original setting with one manufacturer and one entrant, we generalize the demand function for the final product. Finally, we revert to a linear demand function but we extend the model to an arbitrary number of symmetric firms in each group.

3.1 Broader interpretation of the baseline model

We discuss here some of the assumptions of the baseline model and indicate how they can be modified and/or reinterpreted to extend the scope of the model.

Remanufacturing. We mentioned in the introduction that the baseline model can also be used to address the case of remanufacturing. Although remanufacturing differs from recycling from a technological perspective, it shares with recycling the same procedure of collection and reprocessing of end-of-life products. Furthermore, remanufacturers compete with manufacturers of primary products in the same way as recyclers do. Thus, the basic mechanisms of our model are also present in the case of remanufacturing. As an illustration, Örsdemir et al. (2014) explain that manufacturers have an incentive to reduce the competitive threat exerted by remanufacturers "through limiting quantity, specifically by creating scarcity of cores available for remanufacturing." They give the example of Lexmark, which made cores ineligible for remanufacturing.[9]

To adapt the baseline model to the case of remanufacturing, we can simply interpret the parameter τ as the degree of product repairability instead of the rate of scrap collection. A larger value of τ means then that the primary product is easier to repair, which

[9]See `https://archive.grrn.org/lexmark/background.html`, last accessed June 2, 2021.

increases the remanufacturing possibilities. Policies such as the 'repairability scores' for electronic devices in France aims to foster this process.[10]

Relative cost of recycling. We assumed that recycling (or remanufacturing) is at least as expensive as primary production. It could be objected that this assumption is limitative, as in some industries (e.g., the aluminum industry), more energy is needed for virgin production than for recycling. We claim, however, that our model still applies to such industries if we also take into account that consumers may perceive the recycler's product as of lower quality than the manufacturer's product. We would then write the demand for the recycler's product as $p_2^r = 1 - d - q_2 - r$, where $0 < d < 1$ measures the difference in the consumers' willingness to pay between the manufacturer's and the recycler's product. This formulation would fit, for instance, the case of recycled plastic in the food industry (the demand may be lower because recycled plastic does not meet certain safety requirements). Relabelling the recycler's marginal cost as c_r, we can define $c \equiv c_r + d$. In this linear model, c can be seen as the 'true' unit cost: The recycler's profit is indeed equal to $\pi_r = (1 - d - q_2 - r)r - c_r r = (1 - q_2 - r)r - cr$. It is then perfectly possible to have $c_r < 0$ (the recycler has a lower marginal cost than the manufacturer, which we normalized to zero), while $c > 0$ (the manufacturer has a competitive advantage over the recycler).

One-for-one recycling technology. We made the simplifying assumption that one unit of scrap can be transformed into one unit of recycled product. Alternatively, we could assume a linear technology that transforms one unit of scrap into μ units of output with $0 < \mu \leq 1$. Then, given a quantity q_1 of primary production and a collection rate τ', the maximum production for the recycler would be equal to $\mu\tau' q_1$. Letting $\tau \equiv \mu\tau'$ brings us back to the original formulation.

No discounting and homogeneous products. We assumed that the manufacturer does not discount its future profit and that recycled products are perfect substitutes for primary products. Note first that the latter assumption is generally correct in the case of metal recycling (the quality of aluminum, copper, iron after the recycling process is the same as the virgin metal). Second, we want to stress that these two assumptions are meant to put the manufacturer in the worst-case scenario as far as entry is concerned. If the manufacturer had a stronger preference for the present or if products were (horizontally or vertically) differentiated, entry would be less of a threat. As a result, the optimal collection rate from an environmental point of view would be larger (as the effect of the collection rate on the manufacturer's incentives to reduce the scale of entry would be weaker). However, as long as products are substitutes and the manufacturer takes second-period profits into account in its first-period decision, pushing the collection rate

[10] See Staub, C. (2020). *France will assign devices a repair rating.* E-ScrapNews (October 22, 2020); `https://tinyurl.com/32w9tn7w`, last accessed June 14, 2021.

to 100 percent would not be the optimal way to reduce primary production.

Separation of primary and recycling market. We did not allow the manufacturer to be active in the recycling market as well, contrary to what is observed in some industries (for instance, Rio Tinto produces aluminum from both bauxite and recycled scraps). In our setting, even if the manufacturer could enter the recycling market, it would decide against it. This is so because we assume constant marginal costs and higher production costs from recycling. To consider properly this possibility, we would thus need to modify our model substantially, which we leaves for future research.

3.2 General demand function

We now take a general demand function, $P = P(Q)$, that is strictly decreasing and twice differentiable in $\mathbb{R}^+$, and that satisfies the following assumptions: (i) $P(Q) = 0$ for a finite Q; (ii) $P''(Q)Q + P'(Q) < 0$ for all $Q > 0$; and (iii) $P(Q) > 0$. Under these assumptions, the best-response functions are downward sloping with the slope belonging to the interval $(-1, 0]$. These are the sufficient conditions to assure the existence of a unique and locally stable Cournot equilibrium in period 2.[11] These conditions also assure that products are substitutes so that per-firm outputs decrease with the number of firms in the symmetric equilibrium.[12]

In period 2, the manufacturer chooses q_2 to maximize its profit $\pi_2 = P(q_2+r)q_2$, while the recycler chooses r to maximize its profit $\pi_r = P(q_2+r)r - cr$ under the constraint that $r \leq \tau q_1$. Letting $r^*(q_2) = \text{argmax}_r\, \pi_r$, we can write the recycler's best-response function as

$$r^* = \begin{cases} r^*(q_2) & \text{if} \quad r^*(q_2) \leq \tau q_1, \\ \tau q_1 & \text{otherwise}. \end{cases}$$

Hence, the unconstrained and constrained equilibria, $\big(r^u(c), q_2^u(c)\big)$ and $\big(r^c(\tau q_1), q_2^c(\tau q_1)\big)$, are respectively given by

$$\begin{cases} P(Q) - q_2^u P'(Q) = 0 \\ P(Q) - r^u P'(Q) - c = 0 \end{cases} \quad \text{where } Q = r^u + q_2^u,$$

and

$$\begin{cases} P(Q) - q_2 P'(Q) = 0 \\ r^c = \tau q_1 \end{cases} \quad \text{where } Q = \tau q_1 + q_2^c.$$

Using the fact that q_2^c satisfies the first-order condition for profit maximization (i.e., $\frac{d\pi_2}{dq_2} = P'(Q)q_2^c + P(Q) = 0$, with $Q = \tau q_1 + q_2^c$), along with the implicit function theorem,

[11] See (Novshek, 1985) for the existence, (Kolstad and Mathiesen, 1987) on the uniqueness and (Dastidar, 2000) on the local stability of Cournot equilibrium. See Vives (2001) for the discussion of these conditions.

[12] In fact, Amir and Lambson (2000) prove that the necessary condition can be weaker: $P(Q)$ is log-concave, i.e. $P''(Q)P(Q) - P'^2 < 0$. However, for the existence of a unique Cournot equilibrium, it requires strictly increasing, convex cost functions. Therefore, we use the assumption of declining marginal revenue to cover the case with zero production cost.

we can compute the derivative of q_2^c with respect to q_1 as

$$\frac{dq_2^c}{dq_1} = -\frac{\frac{\partial \pi_2}{\partial^2 q_2 \partial q_1}}{\frac{\partial^2 \pi_2}{\partial q_2^2}} = -\tau \frac{P'' q_2^c + P'}{P'' q_2^c + 2P'}.$$

Because $P''q_2^c+P' < 0$ and $P''q_2^c+2P' < 0$ under our assumptions, we obtain that $\frac{dq_2^c}{dq_1} < 0$; that is, q_2^c decreases with q_1.

From there, we can show that the results of Lemma 1 still hold with the general demand $P(Q)$. The quantities of recycled and primary product in the unconstrained equilibrium $r^u(c)$ and $q_2^u(c)$ are indeed such that

$$\begin{cases} P' q_2^u(c) + P = 0, \\ P + P' r^u(c) - c = 0. \end{cases}$$

Because $P' < 0$ by assumption, $r^u(c)$ decreases in c, equals zero at $c = \bar{c}$, and reaches its maximum at $c = 0$, where the two firms are symmetric.[13] We then obtain $r^u(c) < r^u(0) = q_2^u(0)$ for all $c \in [0, \bar{c})$. As per-firm outputs decrease with the number of firms under our assumptions, we also have that $q_2^u(0) < q^m$.[14] Therefore, there exists a threshold $\bar{\tau} < 1$ such that $r^u(0) = \bar{\tau}$ and hence, $r^u(c) < \bar{\tau} q^m$ for all $c \in [0, \bar{c})$. (In the linear case, we had $\bar{\tau} = 2/3$.) Moreover, for any $\tau < \bar{\tau}$ and $q_1 < q^m$, there must exist a value $\tilde{c} \in (0, \bar{c})$ such that $r^u(\tilde{c}) = \tau q_1$. Because $r^u(c)$ decreases in c, we have $\tilde{c}$ decreasing with q_1 and τ, such that

$$\begin{cases} r^u(c) \leq \tau q_1 & \text{if} \quad c \geq \tilde{c}(q_1) \\ r^u(c) > \tau q_1 & \text{otherwise.} \end{cases}$$

We can then proceed (see the details in Appendix A) by solving the manufacturer's maximization problem in period 1. In particular, we establish that $q_1^* = q^m$ for $\tau \geq \bar{\tau}$ and $q_1^* < q^m$ for $\tau < \bar{\tau}$, with q_1^* decreasing with τ when τ is close to zero, and increasing in τ when τ is close to $\bar{\tau}$; we also show that the expected quantity produced by the manufacturer in period 2 decreases with τ. This allows us to state that the results of Proposition 1 continue to hold in the general case. In particular, we show that the total primary production is a U-shaped function of τ, which implies that *an environmentally-oriented government does not want to improve the collection process beyond a certain point.*

3.3 Competition in the manufacturing and recycling sectors

In this second extension, we revert to the linear formulation but we consider an arbitrary number of symmetric firms in each sector; that is, we assume m identical manufacturers and n identical recyclers. We adjust our notation as follows. First, we let q_{i1} and q_{i2} denote the quantities produced by manufacturer $i = 1 \ldots m$ in period 1 and 2 respectively; Q_1 and

[13] By definition, $\bar{c}$ is such that $\pi_r^u(r^u(\bar{c}), q_2^u(\bar{c})) = 0$; in a Cournot competition without degeneration following our assumption, this is equivalent to $r^u(\bar{c}) = 0$.

[14] See (Amir and Lambson, 2000)

Q_2 are the corresponding total quantities, summing over all m manufacturers. Second, we note r_j is the quantity produced by recycler $j = 1 \ldots n$ in period 2; R is the total quantity produced by the n recyclers, with $R \leq \tau Q_1$. Finally, the inverse demand is written as $P = 1 - Q_1$ in period 1 and $P = 1 - Q_2 - R$ in period 2.

As in the baseline model, we assume that all firms produce at a constant marginal cost; this cost is normalized to zero for manufacturers and is equal to c for all recyclers, with c drawn from a uniform distributed over $[0, 1/(m+1)]$. We solve the game by backward induction for its subgame-perfect equilibrium.

3.3.1 Period 2

Suppose that $n \geq 1$ recyclers have entered the market. The maximization problem of recycler j is $\max_{r_j} (1 - c - r_j - r_{-j} - Q_2) r_j$ subject to $r_j + r_{-j} \leq \tau Q_1$, where r_{-j} denotes the total quantity produced by the other recyclers. We derive recycler j's reaction function as follows. From the first-order condition, we find the quantity that recycler j would choose if it were unconstrained: $r_j = \frac{1}{2}(1 - c - r_{-j} - Q_2)$. This quantity is valid as long as the constraint is satisfied, i.e.,

$$\tfrac{1}{2}(1 - c - r_{-j} - Q_2) + r_{-j} \leq \tau Q_1 \Leftrightarrow r_{-j} - Q_2 \leq 2\tau Q_1 - (1 - c).$$

In sum, we have

$$r_j(r_{-j}, Q_2) = \begin{cases} \frac{1}{2}(a - c - r_{-j} - Q_2) & \text{if } r_{-j} - Q_2 \leq 2\tau Q_1 - (a - c) \\ \tau Q_1 - r_{-j} & \text{otherwise.} \end{cases}$$

Defining

$$Q_1^{\lim} \equiv \frac{1}{\tau} \frac{n(1 - (m+1)c)}{m + n + 1},$$

we can establish the following result (the proof is relegated to Appendix B).

Lemma 3. *The Nash equilibrium at the second stage of the game is characterized as follows. (1) For $Q_1 < Q_1^{\lim}$, all recyclers are constrained and equilibrium profits for manufacturers and recyclers are respectively given by*

$$\pi_2^c = \frac{(1 - \tau Q_1)^2}{(m+1)^2} \text{ and } \pi_r^c = \frac{1 - (m+1)c - \tau Q_1}{n(m+1)} \tau Q_1.$$

(2) For $Q_1 \geq Q_1^{\lim}$, no recycler is constrained and equilibrium profits are equal to

$$\pi_2^u = \frac{(1 + nc)^2}{(m + n + 1)^2} \text{ and } \pi_r^u = \frac{(1 - (m+1)c)^2}{(m + n + 1)^2}.$$

The condition for the unconstrained equilibrium ($Q_1 \geq Q_1^{\lim}$) can be rewritten as

$$c \geq \frac{n - \tau(m + n + 1) Q_1}{n(m+1)} \equiv \tilde{c}(m, n).$$

We note that $\tilde{c}(m, n)$ decreases with m and increases with n, implying that, other things being equal, the unconstrained equilibrium is more likely if there are more manufacturers or fewer recyclers. We also note that the unconstrained equilibrium is the only possible equilibrium if Q_1 is sufficiently large, that is

$$\tilde{c}(m,n) \leq 0 \Leftrightarrow Q_1 \geq \frac{n}{\tau(m+n+1)}. \tag{8}$$

3.3.2 Period 1

Our objective here is not to solve the game fully but to characterize the two symmetric equilibria that correspond to what we found in the baseline model, namely a 'limit strategy' subgame-perfect equilibrium in which manufacturers reduce their total quantity below the oligopoly level, and an 'unconstrained equilibrium' in which they do not. We start with the latter. As we just established, the equilibrium in period 2 is unconstrained, irrespective of the retailers' marginal cost if the total quantity produced by the manufacturers is above some threshold. In this case, the manufacturers' profits in the two periods are independent of one another. It follows that the equilibrium in period 1 is the classic Cournot-Nash equilibrium. In the present setting, each firm produces a quantity $q_1 = 1/(m+1)$. Then, condition (8) is satisfied as long as

$$Q_1 = \frac{m}{m+1} \geq \frac{n}{\tau(m+n+1)} \Leftrightarrow \tau \geq \frac{n(m+1)}{m(m+n+1)} \equiv \tilde{\tau}(m,n).$$

We note that $\tilde{\tau}(1,1) = 2/3$ (as we found in the baseline model), $\tilde{\tau}(m,n)$ decreases with m, increases with n, and $\tilde{\tau}(m,n) < 1$ if and only if $n < m(m+1)$. We can thus conclude that *the symmetric 'unconstrained equilibrium' occurs if the collection rate is above some threshold and becomes more likely as the number of manufacturers increases and the number of recyclers decreases.*

We now characterize the symmetric 'limit equilibrium'. If $Q_1 < Q_1^{\lim}$, a manufacturer's profit in period 2 is π_2^c if $c < \tilde{c}(m,n)$, and π_2^u if $c > \tilde{c}$. Letting $q_{-i1} = Q_1 - q_{i1}$, we can write manufacturer i's expected profit function as

$$\Pi^e(Q_1) = \pi_1(Q_1) + \hat{\pi}_2^c(Q_1) + \hat{\pi}_2^u(Q_1),$$

where $\pi_1(Q_1) = (1 - q_{i1} - q_{-i1})\,q_{i1}$ and, given the uniform distribution of c over the interval $[0, 1/(m+1)]$,

$$\begin{aligned}
\hat{\pi}_2^c(Q_1) &= \int_0^{\frac{n-\tau(m+n+1)Q_1}{n(m+1)}} \tfrac{(1-\tau Q_1)^2}{m+1} dc = \tfrac{1}{n(m+1)^2}(1-\tau Q_1)^2 (n - \tau(m+n+1)Q_1), \\
\hat{\pi}_2^u(Q_1) &= \int_{\frac{n-\tau(m+n+1)Q_1}{n(m+1)}}^{\frac{1}{m+1}} \tfrac{(m+1)(1+nc)^2}{(m+n+1)^2} dc = \tfrac{m+n+1}{3n(m+1)^2} \tau Q_1 \left(\tau^2 Q_1^2 - 3\tau Q_1 + 3\right).
\end{aligned}$$

The first-order condition for profit-maximization evaluated at $q_{i1} = q_1$ for all i can be written as

$$-2m^2\tau^3(m+n+1)\,q_1^2 + \left(2m(m+2n+1)\tau^2 - n(m+1)^3\right) q_1 + n\left((m+1)^2 - 2\tau\right) = 0.$$

Solving for q_1, we find

$$q_1^*(\tau, m, n) = \frac{(m+1)\sqrt{4m^2\tau^4+8m^2n(m+n+1)\tau^3-4mn(m+1)(m+2n+1)\tau^2+n^2(m+1)^4}}{4m^2(m+n+1)\tau^3} - \frac{n(m+1)^3-2m(m+2n+1)\tau^2}{4m^2(m+n+1)\tau^3},$$

and we check that the total quantity $mq_1^*(\tau, m, n)$ is inferior to $n/(\tau(m+n+1))$ if and only if $\tau < \tilde{\tau}(m, n)$.

We can now compute the value of τ that minimizes $q_1^*(\tau, m, n)$ and assess how this value, which we denote $\tilde{\tau}(m, n)$, changes with the numbers of manufacturer and recyclers in the market. Given the complexity of the expressions, we only consider cases with one or two firms in each group; we find:

$$\tilde{\tau}(2,1) = 0.178 < \tilde{\tau}(2,2) = 0.272 < \tilde{\tau}(1,1) = 0.326 < \tilde{\tau}(1,2) = 0.474,$$

which suggests that $\tilde{\tau}(m, n)$ decreases with m and increases with n. Further computations show that the same conclusion seems to apply to the total quantity of primary production, $Q_1(m, n) + Q_2(m, n)$, as illustrated in Figure 5. This suggests that a government aiming at limiting primary production should choose a larger collection rate if more recyclers can enter the market and a lower collection rate if more manufacturers are present on the market (other things being equal). Now, if the government can also regulate the number of firms, Figure 5 suggests that it should limit the number of manufacturers while increasing the collection rate and facilitating the entry of more recyclers.

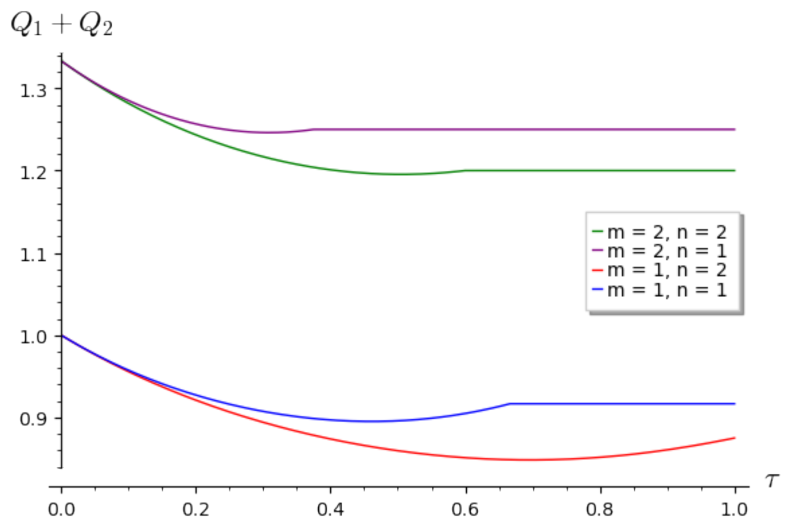


Figure 5: Evolution of total primary production

4 Conclusion

We show in this paper that increasing the rate of scrap collection for recycling (or the degree for product repairability to foster remanufacturing) does not reduce monotonically the quantity of primary production. This is due to the strategic reaction of manufacturers in period 1 when anticipating the entry by recyclers (or remanufacturers) in the next period. In fact, increasing the collection rate from a low level reduces the quantity of primary production as manufacturers constrain the recyclers' scale of operation to soften competition in period 2. However, if the initial collection rate is higher than a certain threshold, increasing the rate will lead to an increase in the quantity of primary production as constraining the recyclers' entry becomes too expensive for manufacturers (as they need to reduce further their production – and thus their profit – in period 1). Consequently, it may be counterproductive from an environmental point of view to make the recycling process too efficient because, above some level, manufacturers would prefer to increase their extraction of primary material. Our model also shows that the collection rate that minimizes the extraction of primary material would be considered as too low by the consumers and the recyclers, and too large by the manufacturers.

Although we show that our results continue to hold in richer settings, future research should aim at extending our model in other directions. For instance, it would be interesting to allow manufacturers of primary products to be active on the segment of recycled (or remanufactured) products as well. It would also be useful to study the interaction between the improvement of recycling with other policy instruments, such as tax and subsidies. Finally, a more dynamic setting would be needed to consider the depletion of the primary resource.

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Appendix

A Proof of Proposition 1 with a general demand function

Manufacturer's maximization problem in period 1. We compute the total differentiation of the manufacturer's expected profit function with respect to q_1:

$$\frac{d\Pi^e(q_1)}{dq_1} = \frac{d\pi_1(q_1)}{dq_1} + \frac{d\hat{\pi}_2^c(q_1)}{dq_1} + \frac{d\hat{\pi}_2^u(q_1)}{dq_1}.$$

The impact of q_1 on the expected profit in the second period is computed as

$$\begin{aligned}\frac{d\hat{\pi}_2^c(q_1)}{dq_1} + \frac{d\hat{\pi}_2^u(q_1)}{dq_1} =& \frac{1}{\bar{c}}\frac{d}{dq_1}\int_0^{\tilde{c}(q_1)} \pi_2^c(q_1)dc + \frac{1}{\bar{c}}\frac{d}{dq_1}\int_{\tilde{c}(q_1)}^{\bar{c}} \pi_2^u(c)dc \\ =& \frac{1}{\bar{c}}\left[\underbrace{\frac{d\tilde{c}(\tau q_1)}{\partial q_1}\left(\pi_2^c(q_1) - \pi_2^u(\tilde{c})\right)}_{(a)} + \underbrace{\tilde{c}\frac{d\pi_2^c(q_1)}{dq_1}}_{(b)}\right].\end{aligned}$$

Because $r^u(\tilde{c}) = r^c = \tau q_1$, we obtain $\pi_2^c(q_1) = \pi_2^u(\tilde{c})$, which makes $(a) = 0$. Therefore, we obtain

$$\frac{\partial \hat{\pi}_2^c(q_1)}{\partial v q_1} + \frac{\partial \hat{\pi}_2^u(q_1)}{\partial q_1} = \frac{1}{\bar{c}}\left[\tilde{c}(q_1)\frac{\partial \pi_2^c(q_1)}{\partial q_1}\right]. \tag{9}$$

Given that the profit of the manufacturer under the constrained equilibrium is $\pi_2^c(q_1) = P(\tau q_1 + q_2^*)q_2^*$, with $q_2^* = q_2^*(\tau q_1)$ the best response of the manufacturer when the recycler produces τq_1, we have

$$\begin{aligned}\frac{\partial \pi_2^c(q_1)}{\partial q_1} &= \left(\tau + \frac{\partial q_2^*}{\partial q_1}\right)P' q_2^* + P\frac{\partial q_2^*}{\partial q_1}\\ &= \tau P' q_2^* + \frac{\partial q_2^*}{\partial q_1}(P' q_2^* + P).\end{aligned} \tag{10}$$

Because q_2^* maximizes π_2^c, according to the first-order condition of the profit-maximization problem, $P' q_2^* = -P$. Replacing in (10), we obtain

$$\frac{\partial \pi_2^c(q_1)}{\partial q_1} = -P\tau < 0. \tag{11}$$

The first derivative of the manufacturer's profit function can then be written as

$$\frac{d\Pi^e}{dq_1} = P'(q_1)q_1 + P(q_1) - \frac{\tilde{c}}{\bar{c}}P(q_2 + r)\tau. \tag{12}$$

Since $\frac{\tilde{c}}{\bar{c}}P(.)\tau$ is positive and $P'(q_1)q_1 + P(q_1)$ is negative by assumption, there can be a value q_1^* so that $\frac{d\pi}{dq_1} = 0$. Furthermore, $P'(q_1)q_1 + P(q_1)$ decreases with all $q_1 > 0$ as $\pi_1(q_1)$ is concave and $\frac{\tilde{c}}{\bar{c}}P\tau$ increases with all $q_1 > 0$ as both P and $\tilde{c}$ decrease with q_1. Therefore, q_1^* is unique if it exists.

Profit-maximizing primary production in period 1. When q_1 converges to 0, $\frac{d\Pi}{dq_1}$ converges to $P'(0)0 + P(0)$, which is strictly positive under our assumptions (i). Furthermore, at $q_1 = q^m$, the manufacturer's period 1 profit is separately maximized, i.e., $P'(q_m)q_m + p(q_m) = 0$ by the first-order condition. Equation (12) then becomes

$$\frac{d\Pi}{dq_1} = -\frac{\tilde{c}}{\bar{c}}P\tau. \tag{13}$$

Since $\frac{\tilde{c}}{\bar{c}}P\tau > 0$, $\frac{d\Pi}{dq_1}$ is strictly negative at $q_1 = q^m$ (ii). Together, (i) and (ii) impose that, if $\tau < \bar{\tau}$, the profit function increases with q_1, reaches a unique maximum then decreases with q_1 when q_1 tends to q^m. Therefore, for $\tau < \bar{\tau}$, choosing $q_1^* \in (0, q^m)$ is the optimal choice for the manufacturer.

At $\tau = \bar{\tau}$, because $\tilde{c}(\bar{\tau}q^m) = 0$, Equation (12) is equivalent to

$$P'(q_1)q_1 + P(q_1) = 0.$$

Thus, the firm's profit is maximized at $q_1^* = q^m$ if $\tau = \bar{\tau}$. Because q_1^* is unique, this is also the global optimal choice of the manufacturer.

For $\tau > \bar{\tau}$, $\tilde{c}(\tau q^m)$ is negative, imposing that the constrained equilibrium does not obtains. The manufacturer's profit is independent of τ. The firm then chooses $q_1^* = q^m$.

Impact of the collection rate on primary production in period 1. To formalize the impact of $\tau \in (0, \bar{\tau})$ on q_1^*, we use the implicit function theorem to write

$$\frac{dq_1^*(\tau)}{d\tau} = -\frac{\frac{\partial^2 \pi(q_1^*)}{\partial q_1 \partial \tau}}{\frac{\partial^2 \pi(q_1^*)}{\partial q_1^2}}.$$

Because q_1^* maximizes the firm's expected profit, $\frac{\partial^2 \pi(q_1^*)}{\partial q_1^2} < 0$ by the second-order condition. So $\frac{dq_1^*(\tau)}{d\tau}$ takes the sign of

$$\frac{\partial^2 \pi(q_1^*)}{\partial q_1 \partial \tau} = \frac{\partial^2 \pi_1(q_1^*)}{\partial q_1 \partial \tau} + \underbrace{\frac{\partial^2 \hat{\pi}_2^c(q_1)}{\partial q_1 \partial \tau}}_{(i)} + \underbrace{\frac{\partial^2 \hat{\pi}_2^u(q_1)}{\partial q_1 \partial \tau}}_{(ii)}.$$

From (9), we obtain

$$\begin{aligned}\frac{\partial^2 \pi(q_1^*)}{\partial q_1 \partial \tau} &= \frac{\partial^2 \hat{\pi}_2^c(q_1)}{\partial q_1 \partial \tau} + \frac{\partial^2 \hat{\pi}_2^u(q_1)}{\partial q_1 \partial \tau} = \frac{1}{\bar{c}} \frac{\partial}{\partial \tau}\left[\tilde{c}(q_1^*) \frac{\pi_2^c(q_1^*)}{\partial q_1}\right] \\ &= \tilde{c}(q_1^*) \frac{\partial^2 \pi_2^c(q_1^*)}{\partial q_1 \partial \tau} + \frac{\partial \tilde{c}(q_1^*)}{\partial \tau} \frac{\partial \pi_2^c(q_1^*)}{\partial q_1} = -\frac{1}{\bar{c}} \frac{\partial}{\partial \tau}[\tilde{c} \tau P] \\ &= -\frac{1}{\tilde{c}}\left[P \tilde{c} + \tau\left(\frac{\partial \tilde{c}}{\partial \tau} P + \frac{\partial P}{\partial \tau} \tilde{c}\right)\right].\end{aligned}$$

Because both $\tilde{c}$ and P decrease with τ, the sign of $\frac{\partial^2 \pi(q_1^*)}{\partial q_1 \partial \tau}$ is ambiguous.

Consider τ close to 0, we have

$$\lim_{\tau \to 0} \frac{\partial}{\partial \tau}\left[\tilde{c} \frac{\pi_2^c}{\partial q_1}\right] = [-P\tilde{c}]_{\tau=0} < 0.$$

Furthermore, when $\tau \to 0$, $q_1^* \to \operatorname{argmax}_{q_1}(\pi_1 + \pi_2^c)$ but $\pi_2^c(\tau q_1, q_2^*(\tau q_1)) \to \pi_2^c(0, q_2^*(0))$, which is independent of q_1, thus $q_1^* \to \operatorname{argmax}_{q_1} \pi_1 = q^m$.

Consider $\tau = \bar{\tau}$, we have $q_1 = q^m$, leading to $\tilde{c} = 0$. In this case,

$$\lim_{\tau \to \bar{\tau}} \frac{\partial}{\partial \tau}\left[\tilde{c} \frac{\pi_2^c}{\partial q_1}\right] = -\bar{\tau} P \frac{\partial \tilde{c}}{\partial \tau} > 0.$$

As $\frac{\partial}{\partial \tau}\left[\tilde{c} \frac{\pi_2^c}{\partial q_1}\right]$ is negative for τ close to zero, whereas $q_1^* \to q^m > 0$ when $\tau \to \bar{\tau}$, we can deduce that the initial production of the manufacturer decreases from the monopolistic value when τ is small and increases with τ when τ is close to $\bar{\tau}$ to reach the monopolistic level $q_1^* = q^m$ again at this threshold.

Recall that the threshold $\tilde{c}$ is determined at $r^u(\tilde{c}) = \tau q_1$, from the first-order condition of profit maximization of the recycler, we obtain

$$\tilde{c} = P'(Q) \tau q_1 + P(Q), \tag{14}$$

with $Q = \tau q_1 + q_2^c(\tau q_1)$.

To study the variation of $\tilde{c}$ with respect to $\tau \in (0, \bar{\tau})$, we take the total differentiation of equation (14) with respect to τ and rearrange the terms to obtain

$$\begin{aligned}\frac{d\tilde{c}}{d\tau} &= \left(q_1 + \frac{\partial q_2}{\partial \tau}\right) P' + P'' \tau q_1 \left(q_1 + \frac{\partial q_2}{\partial \tau}\right) + P' q_1 \\ &= \left(q_1 + \frac{\partial q_2}{\partial \tau}\right)(P'' \tau q_1 + P') + P' q_1 \\ &= P' q_1 \left(\frac{P'' \tau q_1 + P'}{P'' q_2 + 2P'} + 1\right) < 0.\end{aligned}$$

Hence, given a level of q_1, we have

$$\begin{aligned}\frac{d\tilde{c}}{d\tau} &= q_1 P' + \tau P' \frac{dq_1}{d\tau} + \left(q_1 + \tau \frac{dq_1}{\tau} + \frac{dq_2}{d\tau}\right)(P'' \tau q_1 + P') \\ &= P'\left(\tau \frac{dq_1}{d\tau} + q_1\right)\left(\frac{P'' \tau q_1 + P'}{P'' q_2 + 2P'} + 1\right).\end{aligned}$$

For $\frac{d\tilde{c}}{d\tau} \geq 0$, we should have $\frac{dq_1}{d\tau} \leq -\frac{q_1}{\tau}$. Intuitively, this inequality means that, at an initial level of τ, in order to compensate for the marginal effect of an increase in τ on the reduction of $\tilde{c}$, the manufacturer has to reduce its initial production by more than q_1/τ, which is larger than q_1. That means that the manufacturer has to reduce more than what it is producing. Since this is impossible, we can conclude that $\tilde{c}$ always decreases with τ.

Impact of the collection rate on primary production in period 2. In period 2, the manufacturer's profits in the constrained equilibrium is given by $\pi_2^c = P(\tau q_1 + q_2)q_2$. Applying the implicit function theorem on the first-order condition $\frac{\partial \pi_2^c}{\partial q_2} = P'q_2 + P = 0$, we obtain the variation of q_2 with respect to τ due to the substitution effect in period 2 as

$$\frac{\partial q_2}{\partial \tau} = -\frac{\partial \pi_2}{\partial^2 q_2 \partial q_1} / \frac{\partial^2 \pi_2}{\partial q_2^2} = -\tau q_1 \frac{P'' q_2 + P'}{P'' q_2 + 2P'}.$$

Hence, the total variation of q_2 with respect to τ, taking into account both the substitution effect in period 2 and the strategic effect in period 1 is given by

$$\begin{aligned}
\frac{dq_2}{d\tau} =& \frac{\partial q_2}{\partial \tau} + \frac{\partial q_2}{\partial q_1}\frac{dq_1}{d\tau} \\
=& -\tau q_1 \frac{P'' q_2 + P'}{P'' q_2 + 2P'} - \tau \frac{P'' q_2 + P'}{P'' q_2 + 2P'} \frac{dq_1}{d\tau} \\
=& -\frac{P'' q_2 + P'}{P'' q_2 + 2P'} \left(\frac{dq_1}{d\tau} + q_1 \right) \tau.
\end{aligned}$$

Similar to the logic above, since $\frac{dq_1}{d\tau} > q_1$, we have $\frac{dq_2}{d\tau} < 0$; that is, the quantity of primary product in the constrained equilibrium decreases in τ.

We know that $\tilde{c}$ decreases in τ; that is, the probability of occurrence of the constrained equilibrium is smaller. Moreover, given the same recycling cost c, we have $q_2^c > q_2^u$. Hence, the expected quantity of primary product in period 2 decreases τ. Now given that q_1^* decreases with τ when τ is small, the total quantity of primary production decreases when τ is small. However, since q_2^u is independent of τ, the higher the weight of q_2^u in the computation of expected primary production in period 2, the smaller the variation of $\hat{q}_2$ with respect to τ. Hence, when τ is large, the negative impact of increasing τ on the total primary production via $\hat{q}_2$ converge to zero while the positive impact via q_1^* is larger. Therefore, the total primary production decreases, then increases with τ, that is, it also has an interior minimum with respect to τ.

B Proof of Lemma 3

We first show that *there cannot be an equilibrium where some but not all recyclers are constrained.* Suppose, by contradiction, that we have an equilibrium with a strict subset of recyclers being in the constrained part of their reaction function. Suppose that recyclers 1 to k are in the unconstrained part, while recyclers k to n are in their constrained part (with $1 < k < n$). Define

$$R_u \equiv \sum_{s=1}^{k} r_s \text{ and } R_c \equiv \sum_{t=k+1}^{n} r_t.$$

We can sum up the first-order conditions of the unconstrained recyclers:

$$\begin{aligned}
\sum_{s=1}^{k} r_s \quad &= \quad \sum_{s=1}^{k} \tfrac{1}{2}\left(1 - c - r_{-s} - Q_2\right) \Leftrightarrow R_u = \tfrac{1}{2}k\left(1 - c - Q_2 - R_c\right) - \tfrac{1}{2}\left(k-1\right)R_u \\
&\Leftrightarrow \quad \left(k+1\right)R_u + kR_c = k\left(1 - c - Q_2\right).
\end{aligned}$$

Proceeding in the same way for the constrained recyclers, we have

$$\sum_{t=k+1}^{n} r_t = \sum_{t=k+1}^{n} (\tau Q_1 - r_{-t}) \Leftrightarrow R_c = (n-k)(\tau Q_1 - R_u) - (n-k-1) R_c$$
$$\Leftrightarrow R_c + R_u = \tau Q_1$$

Solving the system of the last two equations gives:

$$R_u = k(1-c-Q_2-\tau Q_1),$$
$$R_c = (k+1)\tau Q_1 - k(1-c-Q_2).$$

We need to check if the condition for being in the unconstrained part of the reaction function is satisfied for all recyclers $s \in \{1, k\}$. If it is, then we have:

$$\sum_{s=1}^{k} (r_{-s} - Q_2) \leq k(2\tau Q_1 - (1-c)) \Leftrightarrow k\tau Q_1 - R_u \leq k(2\tau Q_1 - (1-c))$$
$$\Leftrightarrow \tau Q_1 - (1-c-Q_2-\tau Q_1) \leq 2\tau Q_1 - (1-c)$$
$$\Leftrightarrow 2\tau Q_1 - (1-c) - Q_2 \leq 2\tau Q_1 - (1-c) \iff Q_2 \leq 0,$$

a contradiction.

It follows that the only two equilibria are such that either none or all recyclers are constrained. Take first the case such that *no recycler is constrained.* Summing up the reaction functions of the n recyclers, we have $R_u(Q_2) = n(1-c-Q_2)/(n+1)$. As for manufacturer i, its problem is $\max_{q_{i2}} (a - q_{i2} - q_{-i2} - R_u) q_{i2}$, where q_{-i2} is the sum of the quantities produced by the other manufacturers. The first-order condition yields $q_{i2}(q_{-i2}, R_u) = (1 - q_{-i2} - R_u)/2$. Summing up the m previous expressions, we have

$$Q_2 = \tfrac{1}{2}(ma - (m-1)Q_2 - mR_u) \Leftrightarrow Q_2(R_u) = \tfrac{m}{m+1}(1-R_u).$$

A the Nash equilibrium, we have $Q_2^* = Q_2(R_u^*)$ and $R_u^* = R_u(Q_2^*)$; solving and assuming $c < 1/(m+1)$, we obtain

$$Q_2^* = m\frac{1+nc}{m+n+1}, R_u^* = n\frac{1-(m+1)c}{m+n+1}.$$

At the symmetric equilibrium, each manufacturer produces Q_2/m and each recycler produces R_u/n. This equilibrium is valid as long as all recyclers are indeed in the unconstrianed part of their reaction function, which supposes

$$\sum_{j=1}^{n} (r_{-j} - Q_2^*) \leq n(2\tau Q_1 - (1-c)) \Leftrightarrow (n-1)R_u^* - nQ_2^* \leq n(2\tau Q_1 - (1-c))$$

$$\Leftrightarrow Q_1 \geq \frac{1}{\tau}\frac{n}{m+n+1}(1-(m+1)c) \equiv Q_1^{\lim}.$$

Consider now the case in which *all recyclers are constrained.* Summing up the reaction functions of the n recyclers, we have $R_c = \tau Q_1$.As $Q_2(R_c) = m(1-R_c)/(m+1)$, we find that $Q_2^{**} = m(1-\tau Q_1)/(m+1)$ and $R_c^{**} = \tau Q_1$. At the symmetric equilibrium, each manufacturer produces Q_2/m and each recycler produces R_c/n. This equilibrium is valid as long as all recyclers are indeed in the constrained part of their reaction function, which supposes that $(n-1)R_c^{**} - nQ_2^{**} > n(2\tau Q_1 - (1-c))$. A few lines of computations establish that the latter condition is equivalent to $Q_1 < Q_1^{\lim}$.

We can now compute the equilibrium profits in both cases. In the unconstrained case, we have

$$\begin{aligned}
\pi_2^u &= \left(1 - Q_2^* - R_u^*\right) \frac{1}{m} Q_2^* \\
&= \left(1 - m\frac{1+nc}{m+n+1} - n\frac{1-(m+1)c}{m+n+1}\right) \frac{1}{m} m \frac{1+nc}{m+n+1} \\
&= \frac{(1+nc)^2}{(m+n+1)^2}, \\
\pi_r^u &= \left(1 - c - Q_2^* - R_u^*\right) \frac{1}{n} R_u^* \\
&= \left(1 - c - m\frac{1+nc}{m+n+1} - n\frac{1-(m+1)c}{m+n+1}\right) \frac{1-(m+1)c}{m+n+1} \\
&= \frac{(1-(m+1)c)^2}{(m+n+1)^2}.
\end{aligned}$$

In the constrained case, we have

$$\begin{aligned}
\pi_2^c &= \left(1 - Q_2^{**} - R_u^{**}\right) \frac{1}{m} Q_2^{**} \\
&= \left(1 - \frac{m(1-\tau Q_1)}{m+1} - \tau Q_1\right) \frac{1-\tau Q_1}{m+1} \\
&= \frac{(1-\tau Q_1)^2}{(m+1)^2}, \\
\pi_r^c &= \left(1 - Q_2^{**} - R_u^{**}\right) \frac{1}{n} R_u^{**} \\
&= \left(1 - c - \frac{m(1-\tau Q_1)}{m+1} - \tau Q_1\right) \frac{1}{n} \tau Q_1 \\
&= \frac{1-(m+1)c-\tau Q_1}{n(m+1)} \tau Q_1.
\end{aligned}$$