Cointegration analysis among European crude oil, natural gas and coal prices between 1980 and 2015.
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I. Introduction

The beginning of cointegration theory lies in the fact that a lot of financial and macroeconomic series are non-stationary. But when common methods are applied, a problem arises and is called “spurious regression”. Spurious regression is a situation in which variables that are related appear to be statistically significant when the variables are unrelated.

Non-stationarity is a property of many time series and arises when a variable doesn’t have a clear tendency to return to a constant value or linear trend. This property consequently leads to invalidate classical inference methods.

Cointegration theory allows the study of such non-stationary time series which are stationary when a linear combination is applied. Then it allows specifying stable long-run relationships while analysing the short-run dynamic of variables under consideration. Two cointegration methods will be used in this thesis. The first one is the Engle-Granger (1987) method which has the characteristic to be very simple to implement but has the main limit to allow the analysis of only two variables at a time. In order to counter this limit, the multivariate Johansen’s (1991) approach will be used as the second cointegration method.

In this thesis, we will apply cointegration theory among European price series of crude oil, natural gas and coal in a relative large sample of 35 years from 1980 to 2015. Such empirical research on cointegration among energy fossil fuels have already been led by Serletis and Herbet (1999), Villar and Joutz (2006), Mohammadi (2009), Bencivenga et al. (2010) and Westgaard et al. (2011). Nevertheless, none of them took as large a sample as in this thesis. Moreover, we also take into account data from the ten last years which are characterised by a high volatility, especially with the worldwide financial and economic crisis in 2008 and 2009.

Based on monthly price series of Brent crude oil, natural gas and coal from European countries, and using the software R, several hypotheses are tested in this thesis:

- Hypothesis n°1: There is (are) cointegration relationship(s) using the Engle-Granger (1987) method between prices of crude oil/natural gas, natural gas/coal, and coal/crude oil.
- Hypothesis n°2: There is (are) cointegration relationship(s) using Johansen’s (1991) approach.
- Hypothesis n°3: Considering a structural break, there is (are) cointegration relationship(s) using Johansen’s (1991) approach.
After this introduction, this thesis is divided into two mains parts: a theoretical part and a practical part. Each part is also divided into three distinct chapters.

The theoretical part is composed of three chapters. The first chapter is dedicated to the literature review which contains a non-exhaustive list of research that has already been made in the field of cointegration analysis.

The second chapter of the theoretical part gives a summary of the European energy markets in order to have an overview of the consumption, production, imports and exports of crude oil, natural gas and coal in the 28 European countries.

The third and last chapter of this theoretical part concerns the methodology used in the practical part. This part explains in a theoretical way all the concepts and tools in order to analyse the cointegration relationship among our variables.

The following part is the practical part which is also divided into three chapters. The fourth chapter is devoted to the data used in this thesis. A summary of the principal statistical analysis but also a basic analysis of the crude oil, natural gas and coal graphs are made.

The fifth chapter is then dedicated to the analysis of our data using the software R. The methodology applied is the same as explained in the theoretical part with the cointegration method of Engle-Granger (1987) and Johansen (1991).

The sixth and last chapter goes through all the principle results obtained from the analysis with their interpretations.

Finally, the conclusion gives an end to this thesis which is followed by the bibliography and the appendices.
II. Theoretical Part

Chapter 1: Literature review

1.1. Cointegration theory

The beginning of the cointegration theory lies in the fact that numerous financial and macroeconomic series have the characteristic of being non-stationary. By applying common regression methods to this kind of series, we face a problem which was put forward by Granger and Newbold (1974) and is called “spurious regressions”. This type of regressions are characterised by significant coefficients when those coefficients are actually wrongly specified in the estimated model.

Cointegration theory was introduced for the first time by Granger (1981, 1986) who based his work on Sargan (1964) and Hendry (1978) for using the difference between two series to explain changes in them. This theory has the specificity to counter this “spurious regression” phenomenon but also allows studying non-stationary series which are actually stationary by applying a linear combination on them. Moreover, it also allows both the specifications of long-term relationships and the short-term dynamic analysis. The author enunciated a theorem in order to bring to light the link between cointegration and error correction models (ECMs). This theorem establishes that cointegrated series of order 1 can always be represented by an ECM.

A large literature is devoted to the error correction models. Those models were especially introduced by Hendry\(^1\) and allow adjustments estimations in order to achieve a long-term equilibrium. Those dynamic models are characterised by variables evolutions of both short and long-term. The link between cointegration and ECM is explained in Granger (1981, 1983), Granger and Weiss (1983) and Engle and Granger (1987).

Engle and Granger (1987) developed more specifically the concept of cointegration in their article with estimation procedures, tests and empirical examples. Their main contribution consists in the “Engle and Granger approach” in estimating ECMs. This approach is a two-step estimation method and its advantage lies in its simplicity. However, this technique is only valid on cointegrated series of order 1, meaning that there is only one cointegration relationship among the series. Another contribution of this article is the cointegration tests. Engle and Granger (1987) propose a total of seven tests in order to analyse

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the presence of cointegration relationships in series. After comparison, the most powerful seems to be the Dickey-Fuller test (DF) and Augmented Dickey-Fuller test (ADF).

The limit of the Engle and Granger approach is the fact that it only allows the determination of one cointegration relationship. In order to face this problem, Johansen (1988) introduced a multivariate approach of cointegration based on the maximum likelihood process. This method is divided into two steps. First, the estimators of the maximum likelihood are settled. Secondly, the number of cointegration vectors is determined with two statistics proposed by Johansen which are the trace test and the maximum eigenvalue test. The Johansen’s approach is also developed in Johansen and Juselius (1990) and Johansen (1991).

In a more humorous and popular way, Murray (1994) tries to explain the concept of cointegration by taking the metaphor of a drunken woman walking and her dog in order to explain the variation of two co-integrated time series.

More recently, Hendry and Juselius (2000, 2001) summarized all the cointegration theories by first explaining the condition of non-stationarity and then by explaining the case of the multivariate nature of cointegration analysis.

There are also a lot of monographs on the cointegration theory like “Cointegration, Error Correction, and the Econometric Analysis of Non-stationary Data” from Banarjee et al. (1993), “Unit roots, cointegration and structural change” from Maddala and Kim (1998) and “Econométrie des series temporelles, macroéconomiques et financières” from Lardic and Mignon (2002) to name but a few.

1.2. Non-stationarity and unit root tests

The non-stationarity characteristic is also described in a large literature. Indeed, the cointegration framework comes from the fact that some series are non-stationary and cannot be analysed with traditional tools. Box and Jenkins (1970) presented a procedure in order to detect when series are non-stationary. In their framework, non-stationarity can be identified by the graphical representation of the series, a correlogram or by the spectral density function.

Nelson and Plosser (1982) stated the fact that non-stationarity is analysed from two distinct processes, the “trend stationary” (deterministic characteristic) and the “difference stationary” (stochastic characteristic).

One of the biggest issues of macroeconomic theory is the differentiation of “cycle” (short term variations) and “trend” (long term variations). Beveridge and Nelson (1981) solved this problem by considering that stationary series are the summation of a transitory component and a permanent component.
The most common methods to detect non stationarity in series are the graphical analysis and the Bartlett (1937) test which consists in the correlogram study. However, more rigorous tests were needed and Fuller (1976) and Dickey and Fuller (1979, 1981) were the first to present unit root tests\(^2\). Dickey-Fuller tests are parametric and have the advantage to be very simple but suffer from several limitations like residuals autocorrelation. Those limitations led to other unit root tests like Phillips and Perron (1988) which are non-parametric tests\(^3\), Perron (1989), Zivot and Andrews (1992) with a structural break variable, Perron and Vogelsang (1992), Kwiatkowski, Phillips, Schmidt and Shin (1992) Schmidt and Phillips (1992) and finally Elliott, Rothenberg and Stock (1996) for the most famous stationarity tests.

1.3. **Empirical studies**

1.3.1. **Energy**

A lot of empirical studies about energy prices based their research on a cointegration framework. Three main themes were actually treated with the first one being the cointegration relations among energy sources prices (crude oil, gas and coal). The second theme concerns the “crack spread” and is defined as “differences between wholesale petroleum product prices and crude oil prices”\(^4\). The crack spread is commonly used in the oil industry or futures trading and literature about it frequently used the cointegration framework in order to analyse short-term and long-term relationships among those price series. The third theme focuses on the effects of political decisions on energy prices. The most treated political decision concerns the deregulation of the energy markets that has been implemented over the last decades in the U.S.A., Europe and China.

Alexander (1999) deals with the first theme with her article by taking the comparison of energy markets with correlation and cointegration. Based on daily futures prices of WTI crude oil, NYMEX sweet crude oil and natural gas, she begins with a correlation analysis but rapidly comes to the conclusion that those common methods have limitations and is confronted to “spurious regressions” issues. In the second part of her work, she focuses on cointegration analysis and identifies that the first difference with correlation analysis is that “cointegration refers not to comovements in returns, but comovements in asset prices”\(^5\) and comes to the conclusion that this type of framework fits better with energy prices series.

\(^3\) See also Phillips (1987).
\(^5\) Alexander (1999).
Serletis and Herbert (1999) carried out a study among the North American energy prices based on Henry Hub and Transco Zone 6\textsuperscript{6} natural gas prices but also on fuel oil and power prices in the same zone. Using the Engle and Granger (1987) cointegration methods, their main findings were that those energy price series shared a similar trend during the sample period. On the same subject and with the same framework, Bachmeier and Griffin (2006) worked on the degree of market integration on the U.S. continent in and between three of the main sources of energy: crude oil, coal and natural gas. In accordance to their results, it seems that oil markets are much more integrated than coal markets. Moreover, oil and natural gas exhibit a cointegration relationship in the long-run.

More recently and still in the U.S zone, Villar and Joutz (2006) have presented a report for the U.S. Energy Information Administration dealing with the relationship between crude oil and natural gas prices. With the same approach as Alexander (1999), they first did a correlation analysis resulting in spurious regressions and then a cointegration analysis but in a multivariate approach using the Johansen’s (1991) procedure\textsuperscript{7}. They analysed the relations among Henry Hub prices and WTI crude oil between 1989 and 2005 and found out the existence of a cointegrating relationship between those price series. On the same topic, Brown and Yücel (2007) made a report for the Federal Reserve Bank of Dallas. Their findings were the same as Villar and Joutz (2006) about the cointegration relationship between Henry Hub natural gas and WTI crude oil, but they found out causality from crude oil to natural gas. Moreover, they included in their model (Johansen’s procedure) variables in order to take account of weather, seasonality, storage and production disruptions.

Always on the same topic, Mohammadi (2009) also analysed the cointegrating relationships among electricity, coal, natural gas and crude oil prices in the U.S., but by taking a larger sample of 47 years from 1960 to 2007. Surprisingly, his results showed that energy sources’ prices do not lead the electricity prices in the U.S. and a long-run relationship is only significant between electricity and coal prices. Moreover, it seems that there is no such thing as “a unified energy market” with electricity, coal, natural gas and crude oil pointing the weak long-term effect of policies on electricity prices.

Bencivenga et al. (2010) investigated the level of integration between gas, oil and electricity markets on the European markets. Based on the prices of the Ice Brent crude oil, the NBP natural gas (U.K.) and the European Energy Exchange\textsuperscript{8} index and using the

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\textsuperscript{6} “Transco Zone 6 is an important segment of the Transco pipeline extending from Northern Virginia to New York City, serving the eastern seaboard” (Serletis and Herbert, 1999).

\textsuperscript{7} See Johansen (1988) and Hendry and Juselius (2000).

\textsuperscript{8} European Energy Exchange is the leading energy exchange in Central Europe. It develops, operates and connects secure, liquid and transparent markets for energy and related products. On the EEX spot and derivatives
Johansen’s procedure, the authors discovered cointegration relationships between oil, gas and electricity prices. More precisely, it seems that the energy market is led by one common trend which can be interpreted as “a simple source of risk (the oil market), which affects the dynamics of the two other commodities (gas and electricity)” (Bencivenga et al, 2010). The same results appeared for Moutinho et al. (2011) who focused their attention on the Spanish energy market using this time the OMEL electricity market data. They discovered that Brent crude oil prices tend to lead gas and fuel prices movements.

Still in Europe, Westgaard et al. (2011) also analysed the cointegration dynamics between ICE gas oil and Brent crude oil prices but the particularity of their study is that they focused their research on futures contracts of multiple lengths (1, 2, 3, 6 and 12 month contracts). The results tend to show that cointegration relationships are more significant for 1 and 2 month futures contracts and even more so between 1994 and 2009. In fact, those relationships do not seem so obvious between 2002 and 2009, resulting from the high volatility of the period. Joëts and Mignon (2012) decided to analyse the same issue but using forward prices of 35 different maturities. This time, the existence of a cointegration relationship is proved in all forward energy prices. The study also put forward the asymmetric phenomena among the error correction model.

### 1.3.2. Crack spread

The second main theme concerning the crack spread has been analysed by many authors with the frequent objective to settle trading and hedging methods targeting to oil industries. Gjolberg and Johnsen (1999) based their work on the co-movements between the spot prices of crude oil and refined products (gasoline, naphta, jet fuel, gas oil, light fuel oil and heavy fuel oil). Their results showed a cointegration relationship among all refined products. Girma and Paulson (1999) study was intended to crack spread traders “willing to find some valuable information on risk management in the oil industry” and concluded with the same results concerning a cointegration relationship.

Haigh and Holt (2002) focused their work on hedging methodologies of energy futures prices and crack spreads. They based their analysis on a MGARCH model which has the particularity to have frequent portfolio updating but seems to be a more expensive process. They concluded that cointegration can be a very powerful tool intended for hedgers.

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*OMIE manages the wholesale electricity market (referred to as cash or “spot”) on the Iberian Peninsula. Like any other, the electricity market caters for the trading of electricity between agents (producers, consumers, retailers, etc.) at a price that is known, transparent and accessible. ([www.omel.es](http://www.omel.es)).*
same subject, Asche et al. (2003) analysed the crack spread between the Brent crude oil (U.K.) and four major oil products: gas oil, heavy fuel oil, naphtha and kerosene. Cointegration relationships arise from the analysis and crude oil seems to be the driving factor in the price generating process for refined products. Nevertheless, heavy fuel oil tends to be apart without showing long-run relationships with crude oil.

Lanza et al. (2005) enlarged their working horizon by comparing the crack spread between European and American energies markets. Using a multivariate cointegration framework, Brent crude oil and WTI crude oil prices were compared to products prices (unleaded gasoline, gasoil and fuel oil). It showed up that crude oil prices and products prices are cointegrated but also the fact that the long-run relationships seem to be specific to each area with a stronger significance in the E.U. than in America.

Christian et al. (2006) dealt with the crack spread between the WTI crude oil prices and the unleaded gasoline prices using a cointegration framework but based on the non-linear Enders and Grangers (1998) model which allowed asymmetric adjustment in the estimated model. The results clearly proved the non-linearity between WTI crude oil prices and gasoline prices.

Murat and Tokat (2009) were interested in the forecasting characteristics of the crack spread futures contracts in order to predict the oil price movements. Despite some structural breaks in their model, they manage to demonstrate that futures contracts on crack spread can be as good predictors as futures contracts on crude oil.

1.3.3. **Political impacts.**

The third main theme concerns the political impact and is mostly focused on the numerous deregulation processes engaged in Europe and in the U.S.A. Serletis and Rangel-Ruiz (2004) analysed the impact of the major energy market deregulation decisions in the U.S. (U.S. Natural Gas Policy Art in 1978 and the Natural Gas Decontrol Act in 1989) based on the Henry Hub gas prices and WTI crude oil prices. The results demonstrated that a cointegration relationship was present but mostly the fact that the deregulation and political decisions have weakened the relationship between U.S. crude oil and natural gas prices.

On the same topic, but located on the European energy markets, Jamasb and Pollitt (2005) discussed the impact and the effectiveness of the beginning of the process of such liberalization in the European Union. A lot of differences emerged in comparison to the U.S. zone especially because of cross border and physical interconnections issues present on the European markets. Eight distinct regional markets divide this market and it seems to slow down and complicate the liberalization and integration process on the continent. Asche et al.
(2006) dealt with the same subject but with an interest in the decoupling effect on the gas market in the U.K. followed by the opening of the Interconnector\(^\text{10}\). Panagiotidis and Rutledge (2007) went deeper into the subject by analysing the impact on the Brent crude oil and the natural gas markets in the U.K.

Ma and Oxley (2010) concentrated their work on the China markets in order to identify the existence of a cointegrated energy market including the effects of four major energy reforms which occurred in 1997, 1999, 2002 and 2004. It results that regional markets have emerged in China in response to those numerous political decisions.

More recently, Rosa (2014) analysed the impact of U.S. monetary policy on energy prices on three different levels of surprise. Based on intraday data, the results demonstrate that those monetary policies have an important impact on the volatility of futures prices, on the transaction volume but also negative responses from energy prices.

1.3.4. Other themes

Other themes are treated using the cointegration framework like Bunn and Fezzi (2007) in their work in order to analyse the impact of the E.U. Emission Trading Scheme on electricity and gas prices. Ellen and Zwinkels (2010) also use cointegration analysis to investigate the impact of agents’ behaviours on oil price dynamics. But cointegration methods are not only used on the energy prices theme as Yunus (2013) demonstrated in his study. The author analysed the contagion phenomena using an international framework based on ten equity markets among North America, Europe, Latin America and Asia.

\(^\text{10}\) The Interconnector is a natural gas pipeline between the United Kingdom and continental Europe. It crosses the North Sea between Bacton Gas Terminal in England and Zeebrugge in Belgium. The construction of the pipeline was completed in 1998. It provides bi-directional transport capability to facilitate energy trading in both markets. (http://www.interconnector.com/)
Chapter 2: European Energy Overview

2.1. General overview

European energy markets have many energy sources in order to satisfy the needs of the 28 members of the European Union (EU-28). In 2013, the gross inland consumption (GIC)\(^\text{11}\) of energy in the EU-28 was approximately 1,667 billion tonnes of oil equivalent (TOE)\(^\text{12}\). Based on the GIC of energy in the EU-28, the three main energy sources are crude oil, natural gas and coal followed by nuclear energy, renewable energy and other sources like waste burning.

![Figure 1: Evolution of the cumulative European GIC of energy between 1990 and 2013.](image)

We can see on Figure 1 that crude oil, natural gas and coal always had (and still have) an important place in European energy sources. Nevertheless, it seems that coal consumption has a declining tendency while natural gas and renewable energies tend to be a good alternative. Those tendencies can be seen as an effect of political decisions of the EU aiming to switch from fossil fuels to renewable energy sources. The GIC of energy stayed stable between 2003 and 2008 but decreased by about 5.8% in 2009, which was the

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\(^{11}\) Gross inland energy consumption, sometimes abbreviated as gross inland consumption, is the total energy demand of a country or region. It represents the quantity of energy necessary to satisfy inland consumption of the geographical entity under consideration. (http://ec.europa.eu/)

\(^{12}\) Tonne(s) of oil equivalent, abbreviated as TOE, is a normalized unit of energy. By convention it is equivalent to the approximate amount of energy that can be extracted from one tonne of crude oil. It is a standardized unit, assigned a net calorific value of 41,868 kilojoules/kg and may be used to compare the energy from different sources. (http://ec.europa.eu/)
consequence of a lower level of economic activity resulting from the financial and economic crisis.

In 2013, oil, gas and coal accounted respectively for 33%, 23% and 17% of the total European consumption (Appendix 1). When combined, it represents approximately 73% of the total gross inland energy consumption.

The top 3 consumer countries are Germany, France and Italy (Appendix 2 - 3). Those three countries consume together almost half of the total EU-28 consumption. This consumption can be divided into different sectors. The biggest consumers are the transport sector which accounts for 32% of the total final energy consumption\(^\text{13}\), the residential sector for 27% and the industry sector for 25% which represents, all combined, 82% of the entire consumers’ consumption (Appendix 4 - 5).

The energy production of EU-28 is the most concentrated on nuclear energy (Appendix 6 - 7 - 8). In 2013, it represented about 29% of the total primary production\(^\text{14}\) with 226.3 million TOE. Nevertheless, we can see in Figure 2 that the primary production is not sufficient to cover the entire gross inland consumption. Indeed, in 2013, the energy primary production only covered 47.4% of the total gross inland consumption. The surplus principally comes from importations.

\(^{13}\) Final energy consumption is the total energy consumed by end users, such as households, industry and agriculture. It is the energy which reaches the final consumer's door and excludes that which is used by the energy sector itself. (http://ec.europa.eu/)

\(^{14}\) Primary production of energy is any extraction of energy products in a useable form from natural sources. This occurs either when natural sources are exploited (for example, in coal mines, crude oil fields, hydro power plants) or in the fabrication of biofuels. (http://ec.europa.eu/)
The most imported energy sources are crude oil, followed by natural gas and coal (Figure 3). EU-28 is very dependent on energy imports with an energy dependency of 53.2% in 2013. The origins of those imports are principally Russia followed by South Africa, the United States, Australia and Columbia (Appendix 9). Since 2004, energy net imports of EU-28 have been higher than its primary production. In other words, more than half of its GIC was covered by net imports.

Figure 3: Evolution of the cumulative European energy imports between 1990 and 2013.

The European Union also exports energy sources mainly oil and natural gas but the comparison between imports and exports clearly shows an imbalance in the relation (Figure 4) (Appendix 10 - 11).

Figure 4: Comparative of the European energy imports and exports between 1990 and 2013.
2.2. Crude oil

Crude oil is the most consumed fossil fuel in EU-28 with 33% of the total GIC consumption in 2013 which represented 556.7 million TOE. The biggest consumers of crude oil in the EU-28 are Germany, France and the United Kingdom (Figure 5).

This consumption is supported by the primary production and mostly imports. The biggest crude oil producers are Norway with 87.4 million TOE, the United Kingdom with 41.9 million TOE and Denmark with 8.7 million TOE (Figure 6).

The rest of crude oil consumption is based on crude oil imports coming from Russia which accounted for about 32% of the total EU-28 imports (Figure 7). The European Union also relies on Saudi Arabia and Nigeria crude oil imports in order to satisfy its crude oil demand.
2.3. Natural Gas

The second source of fossil fuel in EU-28 is natural gas. Its consumption represented in 2013 about 23% of the total EU-28 GIC with 386.9 million TOE. The biggest consumers in EU-28 are Germany, United Kingdom and Italy (Figure 8).

Concerning the production of natural gas, Norway is still the leader based on the primary production with 95.6 million TOE, followed by the Netherlands and the United Kingdom (Figure 9).

Natural gas imports are also very important with about 340.6 million TOE in 2013. The origins of those importations are Russia, Algeria and Qatar (Figure 10).
2.4. Coal

The third most used fossil fuel in EU-28 is coal. Even if its consumption has had a decreasing tendency over the last decades, its consumption still corresponds to more than 15% of the total GIC in 2013. The biggest coal consumer in EU-28 is, like for crude oil and natural gas, Germany with 81.6 million TOE followed by Poland and the U.K. (Figure 11).

The EU-28 satisfies its coal demand with coal production and imports. The three main producers of coal are Poland with 56.8 million TOE followed by Germany and the Czech Republic (Figure 12).

The European Union also relies on imports in order to satisfy its demand. The three main coal importers are Russia, Colombia and the United States which, when combined represented about 65% of the entire coal imports in 2013 (Figure 13).
Chapter 3: Methodology

This chapter sets up the methodology used in the following analysis in Chapter 5. The methodology is divided into two main parts. The first main part is dedicated to the unit roots analysis in which we will investigate the presence of non-stationary processes of our price series using the Augmented Dickey-Fuller (1981) test. The second main part is devoted to cointegration analysis with two different approaches being the Engle and Granger approach (1987) and the Johansen’s procedure (1991).

But before going deeper into unit root tests and cointegration analysis, we will now recall some basic definitions and properties of stationarity but also non-stationarity processes.

3.1. Stationarity

There exist two stationarity cases that can be presented as following:

3.1.1. Stationarity stricto sensu

A process $X_t$ is stationary stricto sensu if $\forall \ t_1, t_2, ..., t_n$ with $t_i \in T, i = 1, ..., n$ and if $\forall \tau \in T$ with $t_{i+\tau} \in T$, so the joint probability distribution of $\{X_{t_1}, ..., X_{t_n}\}$ is the same as the one of $\{X_{t_{1+\tau}}, ..., X_{t_{n+\tau}}\}$.

Nevertheless, this definition is too restrictive to be used in empirical research. In this thesis, we will actually use the definition when $j$ is equal to 2. This is a well-known case and it is called “weak stationarity”.

3.1.2. Stationarity of order $j$

A process $X_t$ is stationary of order $j$ if $\forall \ t_1, t_2, ..., t_n$ with $t_i \in T, i = 1, ..., n$ and if $\forall \tau \in T$ with $t_{i+\tau} \in T$, every joint moment of order $j$ of $\{X_{t_1}, ..., X_{t_n}\}$ exists and is identical to the joint moments of $\{X_{t_{1+\tau}}, ..., X_{t_{n+\tau}}\}$ which can be written as:

$$E \left[(X_{t_1})^{j_1} ... (X_{t_n})^{j_n}\right] = E \left[(X_{t_{1+\tau}})^{j_1} ... (X_{t_{n+\tau}})^{j_n}\right]$$

with $j_1, ..., j_n \geq 0$ and $j_1 + \cdots + j_n \leq j$.

This definition has the characteristic of being less restrictive and allows empirical research. In this thesis, we will actually use the definition when $j$ is equal to 2. This is a well-known case and it is called “weak stationarity”. 
3.1.3. Weak stationarity

The process $X_t$ is stationary of order 2 (or weak stationary) if:

- $E(X_t^2) < \infty \forall t \in Z$, \hspace{2cm} (3.1)
- $E(X_t) = m \forall t \in Z$, \hspace{2cm} (3.2)
- $\text{Cov}(X_t, X_{t+h}) = \gamma_h, \forall t, h \in Z$ \hspace{2cm} (3.3)

where $\gamma$ is the auto covariance function of the process.

Those three conditions exhibit the fact that $X_t$ is stationary of order 2 if its mean, its variance (finite) and its covariance do not depend on time. If a series satisfies those three conditions, it is stationary.
3.2. **Non-stationarity**

A series is said to be non-stationary if it does not satisfy at least one of the three conditions of stationarity (3.1; 3.2; 3.3). The reason we actually use cointegration analysis is because the time series we want to analyse have the characteristic to be non-stationary (they have a unit root process). Those kinds of series have some specific properties that are established here.

Nelson and Plosser (1982) defined two different types of non-stationary processes which are TS (*Trend Stationary*) and DS (*Difference Stationary*). It is important to identify the type of non-stationarity because a misspecification can lead to a wrong model estimation and strong residuals autocorrelation. The TS and DS processes have different behaviours. When a shock occurs, the TS process will come back to its pre-shock level while the DS process does not come back and evolves on a stochastic path.

### 3.2.1. *Trend stationary*

The trend stationary process (TS) represents processes characterised by a deterministic non-stationarity. A TS process can be written as:

\[
X_t = \gamma + t\beta + \varepsilon_t
\]  

(3.4)

where \(\gamma + t\beta\) is a deterministic function of time and \(\varepsilon_t\) is a white noise of moments \((0, \sigma^2\varepsilon)\) and is supposed to be stationary.

The TS process properties are:

- \(E[X_t] = \gamma + t\beta\)  

(3.5)

- \(V[X_t] = \sigma^2\varepsilon\)  

(3.6)

- \(\text{Cov}[X_t, X_s] = 0, \forall t \neq s\)  

(3.7)

The mean of a TS process has a deterministic trend but its variance is fixed in time. Those properties exhibit the main characteristic of a TS process as being deterministic. In other words, the effects of shocks on \(X_t\) are transitory.
Figure 14 exhibits a simulation of series composed of 100 observations and a TS process with a shock occurring at observation 50. We can clearly see the deterministic trend because the shock has a temporary effect.

Nevertheless, it is possible to make a TS process stationary by applying a deterministic tendency regression on the series.

3.2.2. Difference stationary

The difference stationary (DS) process represents processes characterised by a stochastic non-stationarity. A DS process can be written as an AR(1) process:

$$X_t = \rho X_{t-1} + \beta + \varepsilon_t$$  \hspace{1cm} (3.8)

where $\varepsilon_t$ is a stationary process. By recurrence, we can write:

$$X_t = \rho^\tau X_{t-\tau} + \beta \sum_{j=0}^{\tau-1} \rho^j + \sum_{j=0}^{\tau-1} \rho^j \varepsilon_{t-j}$$  \hspace{1cm} (3.9)

If we set $|\rho| = 1$ (there is a unit root process meaning non-stationarity) and $\tau = t$:

$$X_t = X_0 + t\beta + \sum_{j=1}^{t} \varepsilon_j$$  \hspace{1cm} (3.10)

where $X_0$ is the first term of $X_t$ series.

An important difference in comparison to TS process arises around the error term of the process. In fact, we can see that the error term of the DS process ($\sum_{j=1}^{t} \varepsilon_j$) corresponds to an accumulation of random shocks. This shows us that a shock at a specific date has permanent consequences.
Figure 15 shows a simulation of series composed of 100 observations and a DS process with a shock occurring at observation 50. We can clearly see the stochastic trend because the shock has a permanent effect.

The DS process properties are:
- \( E[X_t] = X_0 + t\beta \) \hspace{1cm} (3.11)
- \( V[X_t] = \sigma^2 \) \hspace{1cm} (3.12)
- \( \text{Cov}[X_t, X_s] = \min(t, s)\sigma^2, \forall t \neq s \) \hspace{1cm} (3.13)

We can see here that the mean and the variance do depend on time unlike a TS process.

A DS process can be transformed into a stationary process by differentiating the series:
\[
(1 - L)^d X_t = \beta + \epsilon_t
\]
(3.14)

where \( \epsilon_t \) is a stationary process. \( d \) is the order of differentiation (or integration). If \( d = 1 \), the process is said to be of order 1. More generally, we can see the definition of Granger (1980): “\( X_t \) is integrated of order \( d \) (\( X_t \sim I(d) \)) if it is necessary to differentiate \( X_t \) \( d \) times in order to make it stationary. In other words, \( X_t \sim I(d) \) if and only if \( (1 - L)^d X_t \sim I(0) \).”
Two cases:
- \( d = 0 \Rightarrow X_t \sim I(0) \):
  
  The series is stationary and it fluctuates around its mean. The variance of \( X_t \) is finite and a shock has a transitory effect.

- \( d = 1 \Rightarrow X_t \sim I(1) \):
  
  The series is non-stationary in level but stationary in difference. The variance of \( X_t \) tends to infinity when \( t \) tends to infinity and a shock has a permanent effect (the series has an infinite memory of shocks).

It is often difficult to identify if a time series has a TS process or a DS process but the distinction is fundamental for the economic analysis. In this thesis, we will exclusively focus on time series exhibiting DS processes.
3.3. **Unit root tests**

In order to test the non-stationarity characteristic of our series, numerous tests called “unit root tests” are present in the literature. Traditional methods like the graphical analysis of the time series or the correlogram study are not sufficient to clearly specify the non-stationarity of a series.

In this thesis, we will base our analysis of the presence of unit root mostly on the Augmented Dickey-Fuller (ADF) test from Dickey and Fuller (1981) which is the most used and known test. The ADF test is a parametric test and is based on the estimation of an autoregressive process. Nevertheless, in order to make our analysis robust, we will also test the stationarity of our series with three other tests being the Phillips-Perron (1988) unit root test, the Zivot-Andrews (1992) unit root test and finally the Kwiatkowski, Phillips, Schmidt and Shin (1992) or KPSS unit root test.

Before going deeper into the unit root test, it is important to explain what a unit root process is. We call a unit root process any sequence with one or more characteristic roots that are equal to one. A simple example of model with a unit root process is the AR(1) model:

\[ X_t = \rho X_{t-1} + \varepsilon_t \]  

where \( \varepsilon_t \) is a white noise of moments \((0, \sigma^2_\varepsilon)\).

The important term in equation (3.15) is the constant \( \rho \). Two cases can arises: if \( \rho = 1 \) then equation (3.15) is a model with a random walk without drift, meaning a non-stationary process. We are in the presence of a unit root problem. The series is said to be non-stationary. The second case is when \( \rho < 1 \). In this case, equation (3.15) is said to be stationary.

The principle behind the ADF unit root test for non-stationarity is to simply find out if the estimated \( \rho \) from equation (3.15) is statistically equal to 1 or not. The two following sections explain the Simple Dickey-Fuller test and then the Augmented Dickey-Fuller test.
3.3.1. Simple Dickey-Fuller test.

Simple Dickey and Fuller (1979) test is based on three basic models.

- Model 1: no constant, no deterministic trend:
  \[ X_t = \rho X_{t-1} + \varepsilon_t \tag{3.16} \]

- Model 2: constant without a deterministic trend:
  \[ X_t = \alpha + \rho X_{t-1} + \varepsilon_t \tag{3.17} \]

- Model 3: constant with a deterministic trend:
  \[ X_t = \alpha + \beta t + \rho X_{t-1} + \varepsilon_t \tag{3.18} \]

(\(\varepsilon_t\) is supposed to be a white noise of moments \((0, \sigma^2_{\varepsilon})\) in the three models).

If \(\rho = 1\), it means that a root of the lagged polynomial is equal to 1. Then, we can say that there is a unit root in the process. In other words, \(X_t\) is a non-stationary process and has a stochastic characteristic (DS process).

We actually test the null hypothesis of the existence of a unit root (\(X_t\) is integrated of order 1, so non-stationary) against the alternative hypothesis of the non-existence of a unit root (\(X_t\) is integrated of order 0, so stationary).

Equation (3.15) can be manipulated by subtracting \(X_{t-1}\) from both sides:

- Model 1:
  \[ \Delta X_t = \phi X_{t-1} + \varepsilon_t \tag{3.19} \]

- Model 2:
  \[ \Delta X_t = \alpha + \phi X_{t-1} + \varepsilon_t \tag{3.20} \]

- Model 3:
  \[ \Delta X_t = \alpha + \beta t + \phi X_{t-1} + \varepsilon_t \tag{3.21} \]

with for every model, \(\phi = \rho - 1\) and \(\varepsilon_t\) is a white noise of moments \((0, \sigma^2_{\varepsilon})\).

Equations (3.19), (3.20) and (3.21) are estimated in practice. We test the null hypothesis \(\phi = 0\) (non-stationarity) against the alternative hypothesis \(\phi < 0\) (stationarity) by referring to the table of Fuller (1976) and Dickey and Fuller (1979, 1981).

- If the t-statistic of the \(\phi\) value is lower than the critical value, then the null hypothesis of non-stationarity is rejected (so the series is stationary).
- If the t-statistic of the \(\phi\) value is higher than the critical value, then the null hypothesis of non-stationarity is not rejected (so the series is non-stationary).
Nevertheless, the Simple Dickey-Fuller test has the main disadvantage of having a problem of residual autocorrelation. In order to solve this problem, Dickey and Fuller (1981) found the necessity to model a process with “additional correlation” in the stochastic component. They proposed an update version of their test called the Augmented Dickey-Fuller test (ADF) where the lags of the first difference are included in the regression equation.

3.3.2. Augmented Dickey-Fuller test.

The Augmented Dickey-Fuller also distinguishes three models:

- Model 1: with no constant and no trend:

\[
\Delta X_t = \phi X_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta X_{t-j} + \epsilon_t
\]

(3.22)

- Model 2: with constant and no trend:

\[
\Delta X_t = \alpha + \phi X_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta X_{t-j} + \epsilon_t
\]

(3.23)

- Model 3: with constant and trend:

\[
\Delta X_t = \alpha + \beta t + \phi X_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta X_{t-j} + \epsilon_t
\]

(3.24)

The additional differenced variable allows correcting the residuals autocorrelation problem and then authorizing the application of the Dickey-Fuller tests.

We here test the null and alternative hypothesis:

- \( H_0 : \phi = 0 \).
- \( H_1 : \phi < 0 \).

The test statistic is as follows:

\[
F_t = \frac{\hat{\phi}}{SE(\hat{\phi})}
\]

(3.25)

where \( SE(\hat{\phi}) \) is the standard error of \( \hat{\phi} \).

Then we have to compare the calculated test statistic in (3.10) with the critical value from Dickey-Fuller (table 1). If \( F_t \) is lower than the critical value, the null hypothesis of unit root is rejected meaning that the variables of the series does not contain a unit root and the series is then non-stationary.
Table 1: Dickey-Fuller critical values.

<table>
<thead>
<tr>
<th>T</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
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<td>-1.61</td>
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<td>-2.58</td>
<td>-1.95</td>
<td>-1.62</td>
</tr>
<tr>
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<td>-2.58</td>
<td>-1.95</td>
<td>-1.62</td>
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<tr>
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<tr>
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<td></td>
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<td>-2.89</td>
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<tr>
<td>∞</td>
<td>-3.96</td>
<td>-3.41</td>
<td>-3.12</td>
</tr>
</tbody>
</table>

3.3.3. Phillips-Perron test

Phillips and Perron (1988) introduced a non-parametric correction of the Simple Dickey-Fuller unit root test in order to correct the autocorrelation and the heteroscedasticity of residuals. The null hypothesis is similar to the ADF unit root test and is the presence of a unit root, meaning that the variable is non-stationary.

Like the ADF, the Phillips-Perron unit root test is based on three different models:

- Model 1: with no constant and no trend:

\[ \Delta X_t = \phi X_{t-1} + \varepsilon_t \quad (3.26) \]

Two statistical tests:

\[ Z(\hat{\phi}) = T \cdot \hat{\phi} - \frac{0.5 \cdot T^2 (\hat{\sigma}_\pi^2 - \hat{\sigma}^2)}{\sum_{t=2}^T X_{t-1}^2} \quad (3.27) \]

\[ Z(t_\phi) = \left( \frac{\hat{\sigma}}{\hat{\sigma}_\pi} \right) t_\phi - \frac{0.5 \cdot T (\hat{\sigma}_\pi^2 - \hat{\sigma}^2)}{\left( \hat{\sigma}_\pi \sum_{t=2}^T X_{t-1}^2 \right)^{1/2}} \quad (3.28) \]
with: $\hat{s}^2 = \frac{1}{T}\sum_{t=2}^{T} \hat{\varepsilon}_t^2$ ; $\hat{\sigma}_\pi^2 = \hat{s}^2 + \frac{2}{T} \sum_{t=1}^{T} w_{st} ; w_{st} = 1 - \frac{s}{t+1}$ is the weight of autocovariance.

- Model 2: with constant and not trend:

$$\Delta X_t = \alpha + \phi X_{t-1} + \varepsilon_t \quad (3.29)$$

Two statistical tests:

$$Z(\hat{\phi}) = T \hat{\phi} - \frac{0.5 T^2 (\hat{\sigma}_\pi^2 - \hat{s}^2)}{\sum_{t=2}^{T} (X_{t-1} - \hat{X}_{-1})^2} \quad (3.30)$$

$$Z(t_{\hat{\phi}}) = \left(\frac{\hat{s}}{\hat{\sigma}_\pi}\right) t_{\hat{\phi}} - \frac{0.5 T (\hat{\sigma}_\pi^2 - \hat{s}^2)}{(\hat{\sigma}_\pi^2 \sum_{t=2}^{T} (X_{t-1} - \hat{X}_{-1})^2)^{1/2}} \quad (3.31)$$

with: $\hat{X}_{t-1} = \frac{1}{T-1} \sum_{t=2}^{T} X_{t-1}$ ; and the other variable defined as before.

- Model 3: with constant and trend:

$$\Delta X_t = \alpha + \beta t + \phi X_{t-1} + \varepsilon_t \quad (3.32)$$

Two statistical tests:

$$Z(\hat{\phi}) = T \hat{\phi} - \frac{T^6 (\hat{\sigma}_\pi^2 - \hat{s}^2)}{24 D_{xx}} \quad (3.33)$$

$$Z(t_{\hat{\phi}}) = \left(\frac{\hat{s}}{\hat{\sigma}_\pi}\right) t_{\hat{\phi}} - \frac{0.5 T (\hat{\sigma}_\pi^2 - \hat{s}^2)}{4 (3 \hat{\sigma}_\pi^2 D_{xx})^{1/2}} \quad (3.34)$$

With $D_{xx}$ being a value defined by Bresson and Pirotte (1995, p.428) and the other values defined as before.
3.3.4. **Zivot-Andrews test**

The Zivot and Andrews (1992) test has for null hypothesis that the series $X_t$ is non-stationary without any structural break. In consequence, the alternative hypothesis of the unit root test corresponds to the fact that $X_t$ is a stationary process with a trend and a structural break at an unknown date.

The Zivot-Andrews unit root test is also based on three different models:

- **Model A**: one-time shift in the series in level:
  \[
  X_t = \mu^A + \theta^A DU_t(\lambda) + \beta^A t + \rho^A X_{t-1} + \sum_{i=1}^{p} c_i \Delta X_{t-1} + \varepsilon_t
  \]
  \[H_0 = \rho^A = 1; \beta^A = 0; \theta^A = 0\]  

- **Model B**: a change in the rate of growth:
  \[
  X_t = \mu^B + \beta^B t + \phi^B DT_t^*(\lambda) + \rho^B X_{t-1} + \sum_{i=1}^{p} c_i \Delta X_{t-1} + \varepsilon_t
  \]
  \[H_0 = \rho^B = 1; \beta^B = 0; \theta^B = 0\]

- **Model C**: a combination of A and B:
  \[
  X_t = \mu^C + \theta^C DU_t(\lambda) + \beta^C t + \phi^C DT_t^*(\lambda) + \rho^C X_{t-1} + \sum_{i=1}^{p} c_i \Delta X_{t-1} + \varepsilon_t
  \]
  \[H_0 = \rho^C = 1; \beta^C = 0; \theta^C = 0\]

with: $DU_t(\lambda) = \begin{cases} 1 & \text{if } t > T\lambda \\ 0 & \text{otherwise} \end{cases}$ and $DT_t^*(\lambda) = \begin{cases} t - T\lambda & \text{if } t > T\lambda \\ 0 & \text{otherwise} \end{cases}$

The test statistic is as follows:

\[t_\rho[\lambda_{i\inf}^t] = \inf_{\rho} t_{\rho,i}(\lambda) ; i = A, B, C\]

3.3.5. **Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test**

The Kwiatkowski et al. (1992) unit root test has the specificity, unlike the other unit root tests, to test for the null hypothesis of the absence of a unit root (the variable is stationary). The alternative hypothesis is by consequence the presence of a unit root (the variable is non-stationary).

Kwiatkowski et al. (1992) decompose the series $X_t$ into a sum of a deterministic trend, a random walk and a stationary error term $\varepsilon_t$. Under the null hypothesis, the variance of the random walk is equal to zero.
\[ X_t = \alpha t + r_t + \varepsilon_t \]  

(3.38)

with \( r_t \) is a random walk, \( r_t = r_{t-1} + \mu_t \) and \( \mu_t \) is a white noise of moments \( (0, \sigma_{\mu}^2) \).

The test is then based on a Lagrange Multiplier (LM) in order to test the null hypothesis of stationarity, i.e. \( \sigma_{\mu}^2 = 0 \). Because \( \varepsilon_t \) is stationary, under the null hypothesis, \( X_t \) is a stationary process around a trend. If \( \alpha = 0 \), then, under the null hypothesis, \( X_t \) is stationary around a level and not around a trend.

If we say that \( e_t, t = 1, \ldots, T \), being the residuals from the regression \( X_t \) on a constant and a deterministic trend, the LM statistic is:

\[ \text{LM} = \frac{\sum_{t=1}^{T} S_t^2}{\hat{\sigma}_r^2} \]  

(3.39)

with \( S_t \) is an estimator of the variance of the residuals \( e_t \).

We then have two statistic tests:

- Model 1: stationarity around a level:
  \[ \eta_{\mu} = \frac{\sum_{t=1}^{T} S_t^2}{T^2 \hat{\sigma}_r^4} \]  

(3.40)

- Model 2: stationarity around a deterministic trend:
  \[ \eta_{\tau} = \frac{\sum_{t=1}^{T} S_t^2}{T^2 \hat{\sigma}_r^4} \]  

(3.41)

Kwiatkowski et al. (1992) computed the critical values for both tests \( \eta_{\mu} \) and \( \eta_{\tau} \). The decisions rules are:

- If the values of \( \eta_{\mu} \) and \( \eta_{\tau} \) are lower than the critical values, we do not reject the null hypothesis.

- If the values of \( \eta_{\mu} \) and \( \eta_{\tau} \) are higher than the critical values, we reject the null hypothesis.
3.4. Cointegration tests

The reason for using cointegration theory comes from the fact that a lot of macroeconomic and financial time series are non-stationary. Several authors tried to use regular methods (like correlation analysis) with this kind of time series but all of them faced a problem of “spurious regressions” put forward by Granger and Newbold (1974).

Cointegration theory was introduced by Granger (1981) and allows studying non-stationary time series but which can be transformed by a linear combination into stationary time series. Since, it has known many developments and the link between cointegration and error correction models was explained by Granger (1981, 1983), Granger and Weiss (1983) and Engle and Granger (1987).

Before we focus on cointegration, Lardic and Mignon (2002) highlight four main properties of integrated series:

- If \( X_t \sim I(d) \) then \( a + bX_t \sim I(d) \) where \( a \) and \( b \) are constants with \( b \neq 0 \).
- If \( X_t \sim I(0) \) and \( Y_t \sim I(0) \) then \( aX_t + bY_t \sim I(0) \) where \( a \) and \( b \) are constants.
- If \( X_t \sim I(0) \) and \( Y_t \sim I(1) \) then \( aX_t + bY_t \sim I(1) \) where \( a \) and \( b \) are constants.
- If \( X_t \sim I(d_1) \) and \( Y_t \sim I(d_2) \) then \( aX_t + bY_t \sim I(\max(d_1, d_2)) \) where \( a \) and \( b \) are constants.

3.4.1. Definition and properties

We can define cointegration as follows:

If \( X_t \) and \( Y_t \) are both \( I(d) \), then the linear combination \( z_t : \)

\[
z_t = X_t - aY_t
\]

is also \( I(d) \).

However, it is possible that \( z_t \) is not \( I(d) \) but \( I(d - b) \) where \( b \) is a positive integer. In this case, \( X_t \) and \( Y_t \) are said to be cointegrated. \( a \) is the cointegration coefficient and the vector \([1, -a] \) is the cointegration vector.

The most studied case is when \( d = b = 1 \). It means that two non-stationary series \( I(1) \) are cointegrated if a stationary linear combination \( I(0) \) of those two series exists.

Lardic and Mignon (2002) give an intuitive explanation of cointegration:

"In the short term, \( X_t \) and \( Y_t \) can have both divergent evolutions (both are non-stationary), but they evolve together in the long term. Then, a stable relationship on the long-run exists between \( X_t \) and \( Y_t \). This relationship is called cointegration relationship or long-term relationship. It is given by \( X_t = aY_t \) (assuming \( z_t = 0 \). In the long-run, similar movements of
and compensate in order to have a stationary series. Then \( z_t \) measures the magnitude of the disequilibrium between \( X_t \) and \( Y_t \) and is called the equilibrium error.

Based on the fact that integrated variables of order 1 \( I(1) \) could have a cointegration relationship, several methods allow the investigation of such relationships. If each variable of a group is integrated of the same order, and if there exists at least one linear combination of these variables that is stationary, then we can conclude that the variables are cointegrated. The characteristic of such variables is that they will never move far apart and a long-run relationship will attract those variables. Testing for cointegration relationships implies testing the existence of such long-run relationship. In this thesis, we will use two different methods being firstly the Engle and Granger’s approach (1987) and secondly, the Johansen’s procedure (1991).

3.4.2. Error correction model.

Granger (1981) enounced a theorem in order to link cointegration to error correction models. This theorem establishes that, in the case of variables cointegrated of order (1,1), such series can be represented by an error correction model. We take this theorem for granted but the demonstration can be found in Granger (1981) and Engle and Granger (1987).

Error correction models (ECMs) allow the modelling of adjustments leading to a long-term equilibrium situation. Those are dynamic models with both short-term and long-term evolutions of variables.

Assuming \( X_t \) and \( Y_t \) are two cointegrated variables \( CI(1,1) \). The ECM can be written as:

\[
\begin{align*}
\Delta X_t &= \gamma_1 z_{t-1} + \sum_{i} \beta_i \Delta X_{t-i} + \sum_{j} \delta_j \Delta Y_{t-j} + d_1(L) \varepsilon_{X_t} \\
\Delta Y_t &= \gamma_2 z_{t-1} + \sum_{i} \beta'_i \Delta X_{t-i} + \sum_{j} \delta'_j \Delta Y_{t-j} + d_2(L) \varepsilon_{Y_t}
\end{align*}
\]  

where

- \( \varepsilon_{X_t} \) and \( \varepsilon_{Y_t} \) are white noises,
- \( z_t = X_t - aY_t \) is the residual of the cointegration relationship between \( X_t \) and \( Y_t \),
- \( d_1 \) and \( d_2 \) are (L) finite polynomial.
The error correction model describes an adjustment process. It includes two types of variables:

- Variables in first difference (stationary) which represents short-term movements.
- Variables in level \( z_t \) here which are a stationary linear combination of non-stationary variables and assure the long-term movements.

### 3.4.3. Granger-Causality

Before testing for cointegration with the Engle-Granger method, it could be useful to test among the variables of interest if there exists a link between them, and more specifically the sense of causality of those relations.

In order to investigate the sense of causality prior to the regression analysis, a well-known test can be used and is named the Granger Causality test. The principle of the Granger causality test between two variables \( X_t \) and \( Y_t \) is to evaluate if the past values of \( X_t \) and \( Y_t \) are useful to predict \( Y_t \). The null hypothesis of the test being that the past values of \( X_t \) do not help to predict the value of \( Y_t \).

The test is divided into several steps. First, we regress \( Y_t \) on \( p \) past values of \( Y_t \) and \( p \) past values \( X_t \).

The non-constrained model (NC) is:

\[
Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \varepsilon_t
\]  

(3.45)

The constrained model (C) is:

\[
Y_t = \alpha + \beta_1 Y_{t-1} + \varepsilon_t
\]  

(3.46)

We then use an F-test computed as:

\[
F^* = \frac{(SSR_C - SSR_{NC})/c}{SSR_{NC}/(n - k - 1)}
\]  

(3.47)

with:

- SSR: the sum of squared residuals.
- \( c \): the number of restrictions.
- \( K \): the number of variables.
The result of the $F$-test is then compared to the critical values from the Fisher law with $(1,n-2)$ degrees of freedom. If the $F$-test is superior to the critical value, the null hypothesis (causality from $X_t$ to $Y_t$) is then rejected.

It is important to add that the Granger causality test seems to be sensitive to the choice of the number of lags $p$. Therefore, the test should be experimented with multiple lags in order to have a robust conclusion.

3.4.4. **Engle-Granger method**

The reason we use cointegration analysis comes from the fact that the regression of non-stationary series on other series could lead to spurious regression. Nevertheless, if each variable of the time series is found to be integrated of order one $I(1)$, it contains a unit root. Then, in this case, the regression can still be meaningful (there are not spurious regressions) if the variables are cointegrated.

In order to test variables for cointegration, we estimate the regression via the ordinary least square method and then the residuals from the same regression are tested for the presence of a unit root. If the residuals do not have a unit root (i.e. the residuals are stationary), it means that the residuals are $I(0)$ and that the variables from the time series are said to be cointegrated and have a long-run (equilibrium) relationship. The Engle-Granger method is based on the latter principle and is divided into two distinct steps. The main advantage of the Engle-Granger method is its simplicity. Nevertheless, it is important to add that this technic is only available for series which are integrated of order one $I(1)$.

i. **Cointegration test**

The first step consists in determining the order of integration of each variable. In fact, cointegration requires that two variables have to be integrated of the same order. In order to do that, Engle-Granger (1987) advise to use the Augmented Dickey-Fuller (ADF) unit root test (see parts 3.3.2. of the same chapter). The test is applied as follows:

$$
\Delta \hat{y}_t = \alpha + \beta t + \gamma \hat{y}_{t-1} + \sum_{i=1}^{\rho} \delta_i \Delta \hat{y}_{t-i} + \epsilon_{it}
$$

(3.48)

where $\alpha$ is a constant, $\beta$ the coefficient of the trend, $\gamma$ the coefficient of $y_{t-1}$, $\rho$ is the lag order of the autoregressive process, $\Delta \hat{y}_t = \hat{y}_t - \hat{y}_{t-1}$ is the first difference of $\hat{y}_t$, $\hat{y}_{t-1}$ is the first lagged value of $\hat{y}_t$, $\epsilon_{it}$ is the change in the lagged value and $\epsilon_{it}$ is considered as a white noise.
When the hypothesis of the presence of a unit root can be rejected (meaning that the differenced series are stationary) and that all variables are integrated of the same order, we estimate the long-run relationship by using a linear regression estimated by the OLS method:

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$  \hspace{1cm} (3.49)

where $\beta_0$ is the intercept, $\beta_1$ represents the slope and $\varepsilon_t$ is the error term.

The estimation of the parameters is calculated from:

$$\hat{\beta}_1 = \frac{\sum(x_t - \bar{x}_t)(y_t - \bar{y}_t)}{\sum(x_t - \bar{x}_t)^2}$$  \hspace{1cm} (3.50)

$$\hat{\beta}_0 = \bar{y}_t - \hat{\beta}_1 \bar{x}_t$$  \hspace{1cm} (3.51)

where $\bar{x}_t$ and $\bar{y}_t$ are the mean of $x_t$ and $y_t$, respectively.

The estimated regression line then has the form:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$  \hspace{1cm} (3.52)

If the two variables exhibit a cointegration relationship, then the last equation gives “super-consistent” estimator (Enders, 2004), meaning that a strong relationship exists between those variables.

The method to determine if the variables have a cointegration relationship is to test the residuals of the estimated linear regression using ADF unit root tests. The residuals $\varepsilon_t$ are a series of estimated values which represents the deviation from the long-run relationship:

$$\varepsilon_t = y_t - \bar{y}_t$$  \hspace{1cm} (3.53)

where $\bar{y}_t$ are estimated values from $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$. Testing for the presence of unit roots in the residuals helps to determine if those deviations are stationary or not. If it is the case, then we can conclude that the series are cointegrated. But if the residuals are not stationary, the series are not cointegrated. The ADF is then performed on those residuals:

$$\Delta \hat{\varepsilon}_t = \alpha_1 \hat{\varepsilon}_{t-1} + \varepsilon_t$$  \hspace{1cm} (3.54)

where $\Delta \hat{\varepsilon}_t$ are the estimated first difference of the residuals, $\hat{\varepsilon}_{t-1}$ are the estimated lagged residuals, $\alpha_1$ is the slope (which is the parameter of interest) and $\varepsilon_t$ is the error term.

The statistic test on $\alpha_1$ in order to determine the stationarity of those residuals is based on the following null and alternative hypothesis:

$H_0: \alpha_1 = 0$

$H_1: \alpha_1 < 0$
The test statistic is as follows:

\[ F_{t} = \frac{\hat{a}_1}{SE(\hat{a}_1)} \]  

(3.55)

where \( SE(\hat{a}_1) \) represents the standard error of \( \hat{a}_1 \), the estimation of \( a_1 \).

When we have calculated the test statistic, we compare the values with the critical values of Engle and Yoo (1987) or the critical values from McKonnon (1991) (Appendix 12 - 13 - 14 - 15). In fact, we cannot compare the test statistic with the critical values from Dickey-Fuller because the cointegration test is based on estimated values of the residuals and not their true values.

If \( F_{t} \) is higher than the critical values, \( H_0 \) is not rejected meaning that the residuals are stationary. The variables are then cointegrated.

\textit{ii. Error correction model}

In the second step, an error correction model (ECM) is estimated. This ECM is obtained from the following equation:

\[ y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} + \alpha_1 y_{t-1} + \varepsilon_t \]  

(3.56)

By subtracting \( y_{t-1} \) on both sides we obtain:

\[ \Delta y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} - (1 - \alpha_1) y_{t-1} + \varepsilon_t \]  

(3.57)

By subtracting \( y_0 x_{t-1} \) on both sides again, the equation becomes:

\[ \Delta y_t - y_0 x_{t-1} = \alpha_0 + \gamma_0 x_t - y_0 x_{t-1} + \gamma_1 x_{t-1} - (1 - \alpha_1) y_{t-1} + \varepsilon_t \]  

(3.58)

\[ \Delta y_t = \alpha_0 + \gamma_0 \Delta x_t + (\gamma_0 + \gamma_1) x_{t-1} - (1 - \alpha_1) y_{t-1} + \varepsilon_t \]  

(3.59)

\[ \Delta y_t = \gamma_0 \Delta x_t - (1 - \alpha_1) \left[ y_{t-1} - \frac{\alpha_0}{1 - \alpha_1} - \frac{(\gamma_0 + \gamma_1)}{(1 - \alpha_1)} x_{t-1} \right] + \varepsilon_t \]  

(3.60)

Setting the values \( \beta_0 \) and \( \beta_1 \) as:

\[ \beta_0 = \frac{\alpha_0}{1 - \alpha_1} \]  

(3.61)

\[ \beta_1 = \frac{(\gamma_0 + \gamma_1)}{(1 - \alpha_1)} \]  

(3.62)

The equation can be written as:

\[ \Delta y_t = \gamma_0 \Delta x_t - (1 - \alpha_1) [y_{t-1} - \beta_0 - \beta_1 x_{t-1}] + \varepsilon_t \]  

(3.63)

The last equation is the ECM with the term \(-(1 - \alpha_1)\) giving the speed of adjustment and \( \varepsilon_{t-1} = y_{t-1} - \beta_0 - \beta_1 x_{t-1} \) being the error correction mechanism which
evaluates the distance of the system from the equilibrium. The value of \( \varepsilon_{t-1} \) should be negative which is a sign that the system converges to the equilibrium.

The coefficient \(- (1 - \alpha_1)\) is an indication of the speed of adjustment to the equilibrium. It can also give some indications depending on its sign and its value:

- Tending to \(-1\) (small values): economic agents are responsible for the removing of the disequilibrium in each period.
- Tending to 0 (large values): the adjustment is slow.
- Tending to -2 (very small values): overshoot of the economic equilibrium.
- Positive values: the system does not stay on the long-run equilibrium.

The Engle-Granger cointegration method is mostly used for its simplicity but it suffers from many limits. The two most important are that this technic can only be applied to series which are integrated of order one \( I(1) \) and it only allows the analysis of one cointegration relationship (by pair of variables).

3.4.5. **Johansen’s procedure**

In order to bypass the limits of the Engle and Granger approach, Johansen (1988) proposed a multivariate cointegration approach based on the estimation of the maximum likelihood. This approach is further developed in Johansen and Juselius (1990) and Johansen (1991).

This method takes as a start the vector autoregressive (VAR) model of order \( p \) as follows:

\[
X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \cdots + \Pi_p X_{t-p} + u_t
\]  

(3.64)

where:

- \( X_t : n \times 1 \) vector of variables integrated of order one \( I(1) \),
- \( u_t : n \times 1 \) vector of innovations,
- \( \Pi_1, \Pi_2, \ldots, \Pi_p : m \times m \) coefficients matrices.

We focus on the following null hypothesis: there exist \( r \) cointegration relations between the \( n \) variables. In other words, under the null hypothesis, \( X_t \) is cointegrated of rank \( r \).

Subtracting \( X_{t-1} \) on both sides of the equation gives:

\[
\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \cdots + \Gamma_{p-1} \Delta X_{t-p+1} - \Pi X_{t-p} + u_t
\]  

(3.65)
where:
- \( \Gamma_1 = \Pi_1 - I \),
- \( \Gamma_2 = \Pi_2 - \Gamma_1 \),
- \( \Gamma_3 = \Pi_3 - \Gamma_2 \),
- \( \Pi = I - \Pi_1 - \Pi_2 - \cdots - \Pi_p \). The matrix \( \Pi \) exhibits how the system is cointegrated and is called the impact matrix.

We now take the first equation of the system:

\[
\Delta X_{1t} = \gamma_{11}' \Delta X_{t-1} + \gamma_{12}' \Delta X_{t-2} + \cdots + \gamma_{1p-1}' \Delta X_{t-p+1} - \Pi_1' X_{t-p} + u_{1t} \tag{3.66}
\]

where:
- \( \gamma_{ij}' \) is the first row of \( \Gamma_p, j = 1, 2, \ldots, p - 1 \).
- \( \Pi_1' \) is the first row of \( \Pi \).

On the last equation, \( \Delta X_{1t}, \Delta X_{t-j} (j = 1, 2, \ldots, p - 1) \) and \( u_{1t} \) are all stationary (i.e. \( I(0) \)) and then, in order to have a meaningful equation, \( \Pi_1' X_{t-p} \) have to be stationary, \( I(0) \).

If \( X_t \) has variables which are cointegrated, then all the rows of \( \Pi \) must be cointegrated. In fact, the number of cointegrating vectors depends on the rank of matrix \( \Pi \) (Harris, 1995).

The matrix \( \Pi \) is an \( m \times m \) matrix and has for rank \( m \). It means that \( m \) is the linearly independent rows or columns which can be used as a basis for \( m \)-dimensional vector space. Then all \( m \times 1 \) vectors can be generated as linear combinations of its row and any of these linear combinations leads to stationarity, meaning that \( X_{t-p} \) has stationary components if the rank of \( \Pi \) is \( r < m \).

We can write \( \Pi X_{t-p} = \beta \alpha' \), with \( \beta \) and \( \alpha' \) being \( m \times r \) matrices:

- \( \alpha' \) is a \( (r \times m) \) matrix with the \( r \) cointegration vectors \( (r \) is then the cointegration rank).
- \( \beta \) is a \( (m \times r) \) matrix with the weights of the associated cointegration vectors.

\( \Pi X_{t-p} = \beta \alpha' X_{t-p} \) and all linear combinations of \( \alpha' X_{t-p} \) are stationary.

Johansen’s procedure estimates the VAR with \( \Pi = \beta \alpha' \) using the maximum likelihood estimator, for various values of \( r \) number of cointegrating vectors and assuming that \( u_t \) is independently and identically distributed and follows a normal distribution of moments \( (0, \sigma^2) \). The estimation is as follows:
\[ \Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \cdots + \Gamma_{p-1} \Delta X_{t-p+1} - \beta \alpha' X_{t-p} + u_t \] (3.67)

In order to estimate the different matrices, Johans en uses the maximum likelihood method. Under the normality hypothesis of \( \varepsilon_t \), the log-likelihood equation can be written as:

\[
\log L (\beta, \alpha, \Pi_1, \ldots, \Pi_{p-1}, \Omega) = - \frac{NT}{2} \log(2\pi) - \frac{T}{2} \log(\det(\Omega)) - \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t' \Omega^{-1} \varepsilon_t
\] (3.68)

where \( T \) is the number of observations, \( N \) is the number of variables in \( X \) and \( \det(\Omega) \) is the determinant of the variance-covariance matrix of \( \varepsilon_t \).\(^{15}\)

The next step consists in the detection of the number of cointegrating vectors. In order to do that, Johansen proposed two likelihood ratios:

i. The trace test

The trace test is based on the null hypothesis of \( r \leq q \), i.e. there are not more than \( r \) cointegration vectors. This test is the same as testing the null hypothesis \( Rk(\Pi) = r \).

The trace test statistic is composed as:

\[
TR = -T \sum_{i=r+1}^{N} \ln(1 - \hat{\lambda}_i)
\] (3.69)

where \( T \) is the sample size and \( \hat{\lambda}_i \) is the \( i^{th} \) largest canonical correlation.

The critical values for the \( TR \) statistic are from Osterwald-Lenum (1992) (Appendix 16). We reject the null hypothesis of \( r \) cointegration relationships when the \( TR \) statistic is higher than its critical value.

Three cases exist:
- \( Rk(\Pi) = 0 \), i.e. \( r = 0 \): there is no cointegration relationships.
- \( Rk(\Pi) = r \), with \( 0 < r < N \): there are \( r \) cointegration relationships.
- \( Rk(\Pi) = N \), i.e. \( r = N \): there is no cointegration relationship.

ii. The maximum eigenvalue

The maximum eigenvalue test is based on the null hypothesis of \( r \) cointegrating vectors against the alternative hypothesis of \( (r + 1) \) cointegrating vectors. The test statistic is given by:

\(^{15}\) The development of the log-likelihood estimation can be found in Lardic and Mignon (2002).
Nevertheless, the trace test is more used than the eigenvalue test.

It is important to say that we supposed here:
- No constant in the ECM model.
- No constant and no deterministic trend in the cointegration relationships.

But the critical values change if a constant or a trend is present. Four different cases can occur:
- (1): no constant in the ECM and the cointegration relationships.
- (2): constant in the ECM and in the cointegration relationships.
- (3): constant in the cointegration relationships but not in the ECM.
- (4): constant in the ECM and constant+trend in the cointegration relationships.

3.4.6. Structural shift

In this last section, we will analyse if the introduction of a structural break in the models has an impact on the presence of cointegration relationships. In order to do that, we base our analysis on the work of Lütkepohl, Saikkonen and Trenkler (2004) which is based on the Johansen’s (1991) procedure.

We base our analysis on the following model:

\[
y_t = \mu_0 + \mu_1 t + \delta d_{t \tau} + x_t
\]  

with \( y_t \) being a \( K \times 1 \) vector process with a constant \( \mu_0 \), a linear trend \( \mu_1 t \) and level shift part \( \delta d_{t \tau} \). \( d_{t \tau} \) is a dummy variable with a value of one if \( t \geq \tau \) and zero if not.

It is important to add that this method does not indicate the moment of the structural break so the analysis is made assuming a structural break at an unknown date.

The test statistic is as follows:

\[
LR(r) = T \sum_{j=r+1}^{N} \ln(1 - \hat{\lambda}_j)
\]  

The values of \( LR(r) \) are then compared to the critical values found in Lütkepohl, Saikkonen and Trenkler (2004).
III. Practical Part

Chapter 4: Data

This section covers the data used in our following analysis for the crude oil, natural gas and coal price series. An explanation of their origins, their main characteristics and some descriptive statistics are briefly exposed.

All the three price series used for crude oil, natural gas and coal have a monthly frequency\(^{16}\) and the time period chosen is comprised between January 1980 and January 2015 with a total of 424 observations for each price series. The choice of such a long period is motivated by the willingness to determine long-run relationships using the cointegration framework. All the data were collected on Macrobond.

4.1. Origin of data

The price series used for crude oil is the Brent crude oil monthly average of period prices from Hamburg Institute of International Economics\(^{17}\). The unit used is in USD/Barrel\(^{18}\).

![Figure 16: Brent Crude Oil Prices (1980-2015).](image)

We can see in Figure 16 the evolution of Brent crude oil prices from 1980 to 2015. The evolution is characterized by many shocks, especially the one around 2008-2009 corresponding to the financial crisis.

\(^{16}\) Monthly series were used because they have sufficient texture to capture short-run movements over time, without adding unnecessary complexity to the analysis.

\(^{17}\) http://www.hwwi.org/

\(^{18}\) 1 barrel = 159 liters.
The price series used for natural gas is the European natural gas monthly average of period prices from the World Bank\textsuperscript{19}. The unit used is in USD/MMBtu\textsuperscript{20}.

Figure 17: Natural Gas Prices (1980-2015).

Figure 17 exhibits the evolution of natural gas prices between 1980 and 2015. We can observe that the natural gas prices are relatively stable from 1980 to 1998 and then some fluctuations begin to be more present. This is consistent with the construction of the North Sea Interconnector which is the natural gas pipeline between the United Kingdom (Bacton Gas Terminal in England) and Continental Europe (Zeebrugge in Belgium).

This Interconnector has an important economic impact because it has the capacity to export 20 billion cubic metres of natural gas per year from the U.K. and it is still in expansion\textsuperscript{21}.

In the same way as Brent crude oil, we can see the important impact of the financial and economic crisis that occurs in 2008-2009.

The price series used for coal is the Steam Coal\textsuperscript{22} monthly average of period prices from Hamburg Institute of International Economics. The unit used is in USD/Metric Ton.

\textsuperscript{19} http://www.worldbank.org/
\textsuperscript{20} The British thermal unit (BTU or Btu) is a traditional unit of energy equal to about 1055 joules. It is the amount of energy needed to cool or heat one pound of water by one degree Fahrenheit. One MMBtu is equal to one million Btu.
\textsuperscript{21} http://www.interconnector.com/
\textsuperscript{22} Steam coal - also known as thermal coal - is mainly used in power generation. (http://www.worldcoal.org/)
In Figure 18 we observe that the same shock applies to coal prices during the financial and economic crisis in 2008-2009.

**Figure 18**: Coal Prices (1980-2015).

**Figure 19**: Brent Crude oil, Natural Gas and Coal Prices (1980 - 2015).
4.2. *Logarithm prices*

When we look at Figure 19, it is not easy to see if any link is present among those three price series. In order to make easier the comparison and the analysis of those series, a logarithm transformation is applied to the original prices.

The logarithm transformation is used to remove the scale effects in the variables and reduce the possible effect of heteroskedasticity. Logarithm transformation also has the beneficial property of allowing the analysis to proceed on variables in the same units.

Figure 20 illustrates those logarithmic prices for Brent crude oil, natural gas and coal.

Even if the graphical inspection of stationarity is not sufficient, we can say that the three price series might be non-stationary.

Moreover, we can see that the three price series might have a common evolution structure maybe leading to cointegration relationships.
4.3. **Logarithm returns**

Figure 21 exhibits the Brent crude oil, natural gas and coal returns in natural logarithm.

The log-returns are calculated using the common formula:

\[ r = \log(P_t) - \log(P_{t-1}) \]

where \( P_t \) corresponds to the price at time \( t \) of Brent crude oil, natural gas or coal. So the log-return series are equivalent to the 1st differentiation of the log-price series.

When looking at the three returns series, we can see that Brent crude oil is much more volatile than natural gas or coal returns.

Moreover, it seems that the differentiation seems to solve the problem of non-stationarity.

This is a characteristic of integrated series but the determination of stationarity will be deeper analysed in Chapter 5 with more pertinent stationarity tests.
Concerning the probability distributions of the three series, we can see in Figure 22 that the probability distributions of price series are not normally distributed but appear bimodal and skewed relative to the normal distribution.

Regarding this time the probability distribution of returns series, we can see that the shapes of probability distributions get closer to normal distribution but we are actually pretty far from a truly normal distribution.
4.5. Autocorrelation

Regarding the autocorrelation functions (ACFs) for the price series and return series through 15 lags, it seems that the differencing transformation appears to have resolved the non-stationarity in the series. In fact, we can see that the autocorrelations are much smaller for returns (right column) than for prices (left column).

![Figure 23: Brent Crude Oil, Natural Gas and Coal Prices and Returns Autocorrelation.](image)

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<th>Brent Crude Oil Returns (log) Autocorrelation</th>
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<td>2058.4</td>
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<td>2448.4</td>
<td>0.000</td>
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<tr>
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<td>0.739</td>
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<td>0.000</td>
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<td>0.000</td>
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</tr>
<tr>
<td>9.000</td>
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<td>3568.7</td>
<td>0.000</td>
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<td>10.000</td>
<td>0.647</td>
<td>3926.7</td>
<td>0.000</td>
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<td></td>
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<tr>
<td>11.000</td>
<td>0.616</td>
<td>4277.1</td>
<td>0.000</td>
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<td></td>
</tr>
<tr>
<td>12.000</td>
<td>0.586</td>
<td>4640.0</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.000</td>
<td>0.556</td>
<td>4953.3</td>
<td>0.000</td>
<td></td>
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</tr>
<tr>
<td>14.000</td>
<td>0.525</td>
<td>5283.3</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.000</td>
<td>0.495</td>
<td>5603.8</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
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<td>32.780</td>
<td>0.000</td>
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<td></td>
</tr>
<tr>
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<td>0.111</td>
<td>5255.7</td>
<td>0.022</td>
<td></td>
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</tr>
<tr>
<td>3.000</td>
<td>0.074</td>
<td>17.650</td>
<td>0.000</td>
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<td></td>
</tr>
<tr>
<td>4.000</td>
<td>0.096</td>
<td>58.005</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.053</td>
<td>61.901</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.056</td>
<td>63.216</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.000</td>
<td>0.004</td>
<td>64.071</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.000</td>
<td>-0.010</td>
<td>64.150</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.000</td>
<td>-0.056</td>
<td>65.442</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.000</td>
<td>-0.007</td>
<td>65.467</td>
<td>0.000</td>
<td></td>
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</tr>
<tr>
<td>11.000</td>
<td>0.015</td>
<td>66.897</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.000</td>
<td>-0.100</td>
<td>71.840</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coal Prices (log) Autocorrelation</th>
<th>Coal Returns (log) Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date: 06/02/15  Time: 22:55</td>
<td>Date: 06/02/15  Time: 22:56</td>
</tr>
<tr>
<td>Included observations: 424</td>
<td>Included observations: 423</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.995</td>
<td>422.86</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.000</td>
<td>0.977</td>
<td>827.29</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.000</td>
<td>0.947</td>
<td>1606.05</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.000</td>
<td>0.926</td>
<td>2829.8</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.000</td>
<td>0.892</td>
<td>4840.6</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.000</td>
<td>0.868</td>
<td>5633.8</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.277</td>
<td>32.780</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.000</td>
<td>0.111</td>
<td>5255.7</td>
<td>0.022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.000</td>
<td>0.074</td>
<td>17.650</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.000</td>
<td>0.096</td>
<td>58.005</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.000</td>
<td>0.053</td>
<td>61.901</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.000</td>
<td>0.056</td>
<td>63.216</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.000</td>
<td>0.004</td>
<td>64.071</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.000</td>
<td>-0.010</td>
<td>64.150</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.000</td>
<td>-0.056</td>
<td>65.442</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.000</td>
<td>-0.007</td>
<td>65.467</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.000</td>
<td>0.015</td>
<td>66.897</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.000</td>
<td>-0.100</td>
<td>71.840</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.6. **Normality test**

In order to test the normality of our series, we use the Shapiro-Wilk normality test. We already did a quick inspection of the probability distribution of the series, which showed us that series tend to not be normally distributed.

The normality test of the data has the null and the alternative hypothesis as follows:

- $H_0$: The data are normally distributed.
- $H_1$: The data are not normally distributed.

Using the R function `shapiro.test()` on each variable (Appendix 17).

Table 2 provides a summary of the results:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shapiro-Wilk</th>
<th>$p$-value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$lnoil$</td>
<td>0.9174</td>
<td>2.098e-14</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>$lngas$</td>
<td>0.9016</td>
<td>7.763e-16</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>$lncoal$</td>
<td>0.9229</td>
<td>1.009e-15</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

Regarding the results, the $p$-values seem to be very small which mean that we can reject the null hypothesis of normality at a 1% level of significance for the three series. We can affirm that the time series of all the variables are not normally distributed.
Chapter 5: Analysis

5.1. Introduction

In this chapter, we will investigate the existence of cointegration relationships between prices of Brent crude oil, natural gas and coal on the European continent applying the Engle-Granger method (1988a) and Johansen’s procedure (1991) using the software \textit{R}.

As mentioned in Chapter 4, three time series are used, that is, Brent crude oil, natural gas and coal prices. Data have a monthly frequency starting in January 1980 until December 2014 (35 years). All the variables are transformed into natural logarithms to reduce variances and allow an analysis in a same scale. Three variables will be used:

i. $lnoil$ : Brent crude oil prices in natural logarithm.

ii. $lngas$ : Natural gas prices in natural logarithm.

iii. $lncoal$ : Coal prices in natural logarithm.

This chapter is divided into several parts. First of all, we begin with an analysis of the stationarity of our three series using four different unit root tests. Next, we will use the Augmented Dickey-Fuller unit root test in order to determine the order of integration of our variables. After determining that the series have the right order of integration, the next step will consist in cointegration analysis using the Engle-Granger approach and Johansen’s procedure. In both methods, a study for the presence of cointegration will be done followed by the estimations of their error correction models.

5.2. Stationarity tests

In this first step, we will determine if our price series are stationary or not. As specified in Chapter 4, cointegration analysis is applied on non-stationary series. Testing for a unit root in each of the series implies testing for non-stationarity in those same series. In order to do that, four tests are applied in this thesis. The first one is the well-known and most used test for stationarity Augmented Dickey Fuller (ADF) unit root test. We will then complete the process by testing the stationarity using the Zivot-Andrews (ZA) test, the Phillips-Perron (PP) test and finally the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test.
5.2.1. Lag Length Selection, Trend and Intercept

Before applying those tests, a preliminary step is required. In fact, all of those tests require the specification of a lag length and the decision to include or not an intercept and/or a trend in the estimated models for the stationarity tests.

Firstly, the choice of the lag length depends on the unit root test used. For the ADF test, we base our choice on the Akaike Information Criterion (AIC)\(^{23}\) and on the Bayesian Information Criterion (BIC)\(^{24}\). Those criteria are automatically implemented in the \textit{ur.df} function in \texttt{R}. For the ZA test, we use the Schwert (1989) formula, for the PP test, we take the lag length with the minimum value of the Akaike Information Criterion value and for the KPSS, we take the same specifications as those used in the ADF test.

Secondly, the decision to implement an intercept and/or a trend in the estimated models is based on an elimination procedure. We begin by implementing an intercept and a trend. If the trend is significant, we keep the estimated model. If the trend is not significant, we estimate the model with an intercept only. If the intercept is significant, we keep the estimated model with the intercept only but if the intercept is not significant, then we have to estimate the model without intercept and trend.

5.2.2. Augmented Dickey-Fuller test

The first unit root test applied is the Augmented Dickey-Fuller (1981) unit root test with a lag length of one (based on the AIC and BIC), a constant and a trend.

We will test the following null and the alternative hypothesis:

- \( H_0 \): the variable is non-stationary.
- \( H_1 \): the variable is stationary.

The \texttt{R} function \textit{ur.df} is applied on each variable (Appendix 18).

We are using the following ADF model (with a constant and a trend):

\[
Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \alpha_1 \sum_{i=1}^{m} \Delta Y_{t-i} + \varepsilon_t
\]  

\textit{(5.1)}

\(^{23}\) The Akaike information criterion (Akaike, 1973) is a relative measure of the quality of a statistical model. The AIC equation is: \( AIC = 2k - 2\ln(L) \) (\( k \) = number of parameters; \( L \) = maximum likelihood of the model).

\(^{24}\) The Bayesian information criterion (Schwarz, 1978) is also an information criterion based on the AIC. The difference with the AIC is that the BIC also depends on the number of observations. The BIC equation is: \( BIC = -2\ln(L) + \ln(N)k \) (\( N \) = number of observations).
Based on the coefficients from the test, we get the following regressions:

\[ lnoilt = 0.05103 + 0.00009163tt - 0.02002lnoilt_{t-1} + 0.3067\Delta lnoilt_{t-1} \]  
\[ lngast = 0.0045 + 0.00005493tt - 0.009754lngast_{t-1} + 0.1132\Delta lngast_{t-1} \]  
\[ lncoal_t = 0.04071 - 0.010443lncoal_{t-1} + 0.2922\Delta lncoal_{t-1} \]

where:
- \( lnoilt \), \( lngast \), \( lncoal_t \): the price function series,
- \( tt \): the trend,
- \( lnoilt_{t-1} \), \(.lngast_{t-1} \), \( lncoal_{t-1} \): the lagged values of the series,
- \( \Delta lnoilt_{t-1} \), \( \Delta lngast_{t-1} \), \( \Delta lncoal_{t-1} \): the first difference lagged values of the series.

You can see that an intercept and a trend have been introduced in the models for \( lnoil \) and \( lngas \) because of their significance but only an intercept has been introduced for the variable \( lncoal \) (the trend is not significant).

The test statistic is calculated using the formula:

\[ F_t = \frac{\delta}{SE(\delta)} \]  

\[ lnoil F_t = \frac{-0.02002}{0.0076} = -2.6342 \]  
\[ lngas F_t = \frac{-0.009754}{0.005659} = -1.7238 \]  
\[ lncoal F_t = \frac{-0.010443}{0.005993} = -1.7425 \]

Using the critical values extracted from \( R \):

<table>
<thead>
<tr>
<th>Trend + Intercept</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>-3.98</td>
<td>-3.42</td>
<td>-3.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intercept</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>-3.44</td>
<td>-2.87</td>
<td>-2.57</td>
</tr>
</tbody>
</table>

We can compare those critical values with the values of the ADF test. We can see that all the calculated statistical tests fall into the non-rejection region, which is to the right of the \( \tau \) critical values.
Table 4: Summary of the ADF unit root test.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test statistic</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnoil</td>
<td>-2.6342</td>
<td>No reject the null hypothesis</td>
</tr>
<tr>
<td>lngas</td>
<td>-1.7238</td>
<td>No reject the null hypothesis</td>
</tr>
<tr>
<td>lncoal</td>
<td>-1.7425</td>
<td>No reject the null hypothesis</td>
</tr>
</tbody>
</table>

The results are that we cannot reject the null hypothesis for the presence of unit roots at a 10% level of significance meaning that all variables are non-stationary, each of them contains at least one unit root, which means that the variables are probably integrated of order one $I(1)$.

5.2.3. Phillips-Perron Test

The second unit root test is the Phillips-Perron (1988) unit root. The first step is to determine the truncated parameter. Using the Schwert (1989) formula $l_4 = \text{int} \left[ 4 \left( \frac{T}{100} \right)^{1/4} \right]$ or the Newey and West (1987) formula $l = \text{int} \left[ 4 \left( \frac{T}{100} \right)^{2/9} \right]$ ($T$ is the number of observations, which is 420 in our case), we find a value of $l = 4$.

We test the following null and the alternative hypothesis:

- $H_0$: the variable is non-stationary.
- $H_1$: the variable is stationary.

The Phillips-Perron test can be computed in $R$ using the function `ur.pp()` (Appendix 19).

We also decide to implement an intercept and a trend in the Phillips-Perron unit root test. We can see that both of them are significant in the three series. Table 5 computes the critical values of the test which are the same as the ADF test.

Table 5: PP unit root test critical values.

<table>
<thead>
<tr>
<th>Trend + Intercept</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Values</td>
<td>-3.98</td>
<td>-3.42</td>
<td>-3.13</td>
</tr>
</tbody>
</table>

Table 6: Summary of the PP unit root test.

<table>
<thead>
<tr>
<th>Variable</th>
<th>PP test statistic</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnoil</td>
<td>-2.5278</td>
<td>No reject the null hypothesis</td>
</tr>
<tr>
<td>lngas</td>
<td>-1.8576</td>
<td>No reject the null hypothesis</td>
</tr>
<tr>
<td>lncoal</td>
<td>-1.8576</td>
<td>No reject the null hypothesis</td>
</tr>
</tbody>
</table>
In Table 6, you can see that we do not reject the null hypothesis of stationarity at a level of 10% meaning that the three series are non-stationary.

5.2.4. Zivot-Andrews Test

The Zivot-Andrews (1992) has the particularity to test for the presence of a unit root with an unknown structural break in the variable. We test the following null and the alternative hypothesis:

- \( H_0 \): the variable is non-stationary with an unknown structural break.
- \( H_1 \): the variable is stationary with an unknown structural break.

Like in the ADF or the PP unit root tests, we have to specify the length of lag. In order to do that, we take the length of lags that minimize the Akaike Information Criterion and on the Bayesian Information Criterion.

This test is applied in R using the function \( ur.za() \) (Appendix 20).

We choose to introduce a trend and an intercept in the \( \text{lnoil} \) and \( \text{lngas} \) unit root test because both are significant. An intercept only was introduced in the \( \text{lngcoal} \) unit root test because the trend was not significant.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Potential Breakpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{lnoil} )</td>
<td>October 2008</td>
</tr>
<tr>
<td>( \text{lngas} )</td>
<td>January 2009</td>
</tr>
<tr>
<td>( \text{lngcoal} )</td>
<td>August 2008</td>
</tr>
</tbody>
</table>

We can see in Table 7 the breakpoints identified by the Zivot-Andrews unit root test for each variable. The dates coincide to the financial and economic crisis in 2008 and 2009 where a huge crash occurs on the three fossil fuel markets. Table 8 gives the critical values given from R and Table 5 summarizes the results of the Zivot-Andrews unit root test.

<table>
<thead>
<tr>
<th>Critical Values</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Intercept + Trend} )</td>
<td>-5.57</td>
<td>-5.08</td>
<td>-4.82</td>
</tr>
<tr>
<td>( \text{Intercept} )</td>
<td>-5.34</td>
<td>-4.8</td>
<td>-4.58</td>
</tr>
</tbody>
</table>
Table 9: Summary of the ZA unit root test.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ZA test statistic</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnoil</td>
<td>-4.4623</td>
<td>No reject the null hypothesis</td>
</tr>
<tr>
<td>lngas</td>
<td>-4.1829</td>
<td>No reject the null hypothesis</td>
</tr>
<tr>
<td>lncoal</td>
<td>-4.4177</td>
<td>No reject the null hypothesis</td>
</tr>
</tbody>
</table>

When we compare the critical values with the ZA test statistics for the three variables (Table 9), we can see that, despite the presence of a structural break in the series, the three variables remain non-stationary.

5.2.5. Kwiatkowski–Phillips–Schmidt–Shin Test

The last unit root test is the Kwiatkowski–Phillips–Schmidt–Shin (1992) (KPSS) and it has the specificity of testing the null and alternative hypothesis:
- \( H_0 \): the variable is stationary.
- \( H_1 \): the variable is non-stationary.

The first step is to determine if we introduce an intercept and/or a trend in the models and secondly determine the lag length. We can actually base our analysis on the ADF unit root test we made previously and choose the same specifications for the KPSS unit root test (Lardic and Mignon, 2002).

The KPSS unit root test can be implemented in R using the function `ur.kpss()` (Appendix 21).

Table 10 and 11 give the critical values and the results from the KPSS test.

Table 10: KPSS unit root test critical values.

<table>
<thead>
<tr>
<th>Critical Values</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept + Trend</td>
<td>-0.216</td>
<td>-0.146</td>
<td>-0.119</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.739</td>
<td>-0.463</td>
<td>-0.347</td>
</tr>
</tbody>
</table>

Table 11: Summary of the KPSS unit root test.

<table>
<thead>
<tr>
<th>Variable</th>
<th>KPSS test statistic</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnoil</td>
<td>4.2222</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>lngas</td>
<td>4.2524</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>lncoal</td>
<td>9.0657</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>
The results are that all the test statistics are higher than the critical values leading to the rejection of the null hypothesis of stationarity. The conclusion is that the three variables are non-stationary.

5.2.6. Stationarity tests summary

<table>
<thead>
<tr>
<th>Stationarity tests</th>
<th>lnoil, lngas, lncoal</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>Non-stationary</td>
</tr>
<tr>
<td>PP</td>
<td>Non-stationary</td>
</tr>
<tr>
<td>Z-A</td>
<td>Non-stationary</td>
</tr>
<tr>
<td>KPSS</td>
<td>Non-stationary</td>
</tr>
</tbody>
</table>

We can see in Table 12 that, testing with four different unit root tests, all variables are stationary.
5.3. **Cointegration tests**

5.3.1. **Order of integration**

After applying the unit root test of the variables in level, we now conduct the same test but using the variables in their 1st differenced in order to check if all the variables are integrated of the same order $I(1)^{25}$. In order to come to such a conclusion, series in 1st difference have to be stationary, which implies the rejection of the null hypothesis of ADF unit root test.

The determination of including a trend and/or an intercept has been made by elimination like in the stationarity tests. The conclusion is that neither intercept nor trends are significant. The choice of the lag length is based on the *Akaike Information Criterion*.

The stationarity test on the 1st difference variables is implemented in *R* (Appendix 22)

As it was expected, we can see in Table 14 that all ADF test statistics are lower than the critical values. This leads to the rejection of the null hypothesis of the non-stationarity of the variable. We can then accept the alternative hypothesis of stationarity.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test statistic</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dlnoil$</td>
<td>-12.5757</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>$dlngas$</td>
<td>-11.51</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>$dlncal$</td>
<td>-11.6396</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

Based on the fact that the three variables are non-stationary on level but stationary on their 1st difference, we can conclude that each variable $lnoil$, $lngas$ and $lncoal$ is integrated of order one $I(1)$. This condition being completed, we can now focus on the link shared by those three price series using cointegration framework like the Engle-Granger method (1987) and Johansen’s approach (1991).

---

25 Cointegration analysis requires all series to be integrated of the same order.
5.3.2. *Engle-Granger method*

The first method we will use in order to investigate cointegration relationships between our variables is the Engle-Granger methodology (1987). The particularity (but also the limit) of this method is that it allows testing only two variables at a time. This method is divided into many different steps. The first one consists in analysing which pair of variables exhibits a causality in the Granger sense. We then test each possible pair of variable and we keep the pairs for which a Granger causality has been proved. The second step is to test all those pairs using the Engle-Granger methodology. In the third and final step, we will estimate the error correction models (ECMs) of pairs which have a cointegration relationship.

*i. Granger Causality*

In this first step, the Granger Causality test requires to determine a lag length. Because the results from Granger causality tests are sensitive to this choice, we decide to test with five different lags and then keep the linear relations with the most robust results.

The Granger-Causality test is based on the following null and alternative hypothesis with $Y_t$ and $X_t$ being two stationary series:

$$Y_t \sim X_t$$

- $H_0$: $X_t$ does not Granger cause $Y_t$.
- $H_1$: $X_t$ does Granger cause $Y_t$.

The test is computed in R using the function `grangerest()` (Appendix 23). Table 15 summarizes the results from the Granger causality tests with lags 1, 2, 3, 4 and 5.
Based on those results, we can see that three cases exhibit Granger causality:

- \( \text{dlnoil} \) does Granger cause \( \text{dlngas} \).
- \( \text{dlnoil} \) does Granger cause \( \text{dlngas} \).
- \( \text{dlnoil} \) does Granger cause \( \text{dlngas} \).

We then decide to test those three pairs of variables for cointegration using the Engle-Granger method.

### ii. Cointegration test

The first step of this test consists in estimating linear regressions with our interested variables in \( R \) (Appendix 24).

The results shows that the all the variables are highly significant. The cointegration equations can be written as follows:

\[
\text{lnoil} = -1.37139 + 1.27876 \times \text{lncal}
\]  
(5.6)

\[
\text{lngas} = -1.24474 + 0.77842 \times \text{lnoil}
\]  
(5.7)

\[
\text{lngas} = -2.50343 + 1.04584 \times \text{lncal}
\]  
(5.8)

We can see that the \( p \)-values of the independent variables are very small, meaning that the regression coefficients are statistically significant at the 0.1\% level of significance.
The $R$-squared values are very high, which can be explained by the fact that the variations of the dependent variables are explained by changes in the independent variables.

After this first step, we would like to know if the variables are cointegrated. In order to do that, we test if the residuals from the regressions are stationary (ADF unit root test).

In order to do that, we begin by extracting the residuals from the estimated linear regressions and we test them with the ADF unit root test (Appendix 25).

When applying the ADF test to residuals (Appendix 26), we actually estimate the following relation:

$$\Delta \hat{\epsilon}_t = a_1 \hat{\epsilon}_{t-1} + \epsilon_t \tag{5.9}$$

where:
- $\epsilon_t$ : residual from the long-run relationships,
- $a_1$ : estimated regression coefficient of the lagged residuals.

The ADF test on residuals is based on the null and alternative hypotheses:
- $H_0$ : $a_1 = 0$.
- $H_1$ : $a_1 < 0$.

If the null hypothesis can be rejected, we conclude that the residuals do not contain a unit root (i.e. the residuals are stationary), meaning that the variables are cointegrated.

Because the values of the errors are estimated, we cannot base our analysis on Dickey-Fuller critical values provided by $R$ but on Engle and Yoo (1987) critical values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF test statistic</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>error.oc.eq</td>
<td>-3.4745</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>error.go.eq</td>
<td>-6.1407</td>
<td>Reject the null hypothesis</td>
</tr>
<tr>
<td>error.gc.eq</td>
<td>-3.0334</td>
<td>Reject the null hypothesis</td>
</tr>
</tbody>
</table>

Based on the cointegration test results, we can reject the null hypothesis for the presence of a unit root (the residuals are stationary). Given the fact that $lnoil$, $lngas$ and $lncoal$ are all integrated of order one $I(1)$, we can conclude that there exist three cointegration
relationships; one between $lnoil$ and $lncoal$, one between $lngas$ and $lnoil$ and one between $lngas$ and $lncoal$.

iii. Error correction model

After concluding that the variables are cointegrated, we can now specify the error correction models (ECMs). The ECMs will be:

$$\Delta lnoil = \beta_0 + \beta_1 \Delta lncoal_t + \alpha^* \varepsilon_{t-1} + \hat{u}_t$$

(5.10)

$$\Delta lngas = \beta_0 + \beta_1 \Delta inoil_t + \alpha^* \varepsilon_{t-1} + \hat{u}_t$$

(5.11)

$$\Delta lngas = \beta_0 + \beta_1 \Delta incoal_t + \alpha^* \varepsilon_{t-1} + \hat{u}_t$$

(5.12)

where:

- $\alpha^* = -(1 - \bar{\alpha})$,

The ECMs can be generated using R (Appendix 27).

Using R outputs, we can summarize the estimated ECMs:

$$\Delta lnoil = 0.0005864 + 0.3180181 \Delta lncoal_t - 0.0196971 \varepsilon_{t-1}$$

(5.13)

$$\Delta lngas = 0.001976 - 0.029021 \Delta inoil_t - 0.101288 \varepsilon_{t-1}$$

(5.14)

$$\Delta lngas = 0.001956 + 0.02713 \Delta incoal_t - 0.018055 \varepsilon_{t-1}$$

(5.15)

Regarding the three ECMs, we can directly see that the coefficients of interest $\alpha^*$ (long-run coefficient) have a correct negative sign and all coefficients in the three models are significant. This first observation tells us that there exist long-run relationships between each pairs of variables. In the long term, disequilibrium between the pairs of variables compensates in order to have a similar evolution between pairs of series. Moreover, it seems that the values of $\alpha^*$ is relatively small, meaning that the speed of adjustment is slow.

Regarding the coefficients of the variables in 1st difference which represents the short-term coefficients in the ECMs, we can see that only the coefficient of the 1st difference in coal is significant. The other coefficients are also significant, meaning that we can base an interpretation of the short-term relationships on those ECMs. Moreover, it seems that the growth of Brent crude oil prices depends positively of the growth of coal prices. The growth of natural gas prices depends negatively on the growth of Brent crude oil prices but positively on the growth of coal prices.
5.3.3. Johansen’s approach

In this second section, we will use Johansen’s (1991) multivariate procedure in order to determine the presence of cointegration relationships. In order to do that, Johansen proposes two likelihood ratio tests which are the trace test and the maximum eigenvalue test.

The particularity of Johansen’s approach is that it allows a multivariate analysis of cointegration when the Engle and Granger method only allows an univariate analysis.

The first step consists in the determination of the length of lag we will insert in the VAR model. For this purpose, we then estimate a VAR model with \textit{lnoil}, \textit{lngas} and \textit{lncoal} for dependent variable and then we test this model with multiple information criteria (Appendix 29).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
&AIC&HQ&SC&FPE\
\hline
4&2&2&4
\hline
\end{tabular}
\caption{Lag length criterions.}
\end{table}

Based on those results, we decide to include two lags in the VAR models we will use for the trace test and the maximum eigenvalue test.

\textit{i. Trace test}

The trace test is based on the following null and alternative hypothesis:

- \(H_0\): There are \(r\) cointegrating vectors.
- \(H_1\): There are \(n\) cointegrating vectors.

This test can be implemented in \textit{R} using \textit{ca.jo()} (Appendix 30).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
&\(H_0\)&\(H_1\)&t-statistic&10\%&5\%&1\%&Result\
\hline
r \leq 2&r > 2&1.01&7.52&9.24&12.97&Fail to reject \(H_0\)
\hline
r \leq 1&r > 1&12.27&17.85&19.96&24.60&Fail to reject \(H_0\)
\hline
r = 0&r > 0&100.21&32.00&34.91&41.07&Reject \(H_0\)
\hline
\end{tabular}
\caption{Trace test.}
\end{table}

\textit{ii. The maximum eigenvalue test}

The second test is the maximum eigenvalue test and it tests the null and alternative hypothesis:

- \(H_0\): There are \(r\) cointegrating vectors.
- \(H_1\): There are \((r+1)\) cointegrating vectors.
The $R$ function to implement the maximum eigenvalue test is also `cajo()` (Appendix 31).

### Table 20: Maximum eigenvalue test.

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$H_1$</th>
<th>t-statistic</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 2$</td>
<td>$r = 3$</td>
<td>1.01</td>
<td>7.52</td>
<td>9.24</td>
<td>12.97</td>
<td>Fail to reject $H_0$</td>
</tr>
<tr>
<td>$r = 1$</td>
<td>$r = 2$</td>
<td>11.26</td>
<td>13.75</td>
<td>15.67</td>
<td>20.20</td>
<td>Fail to reject $H_0$</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>87.94</td>
<td>19.77</td>
<td>22.00</td>
<td>26.81</td>
<td>Reject $H_0$</td>
</tr>
</tbody>
</table>

Table 19 and Table 20 exhibit the results of the trace test and the maximum eigenvalue test from the Johansen cointegration approach. Firstly, we can see that we can reject the null hypothesis of no cointegration ($r = 0$) in both tests at a 1% level of significance. This means that cointegration relationships exist among the three variables $\text{lnoil}$, $\text{lngas}$ and $\text{lncoal}$. Secondly, we can see that we fail to reject the null hypothesis ($r \leq 1$) and ($r \leq 2$) in both tests. With this last result, this implies that there is no more than one cointegration equation in the model.

#### iii. Vector error correction model

### Table 21: VECM results.

<table>
<thead>
<tr>
<th></th>
<th>$\text{lnoil.d}$</th>
<th>$\text{lngas.d}$</th>
<th>$\text{lncoal.d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{ect1}$</td>
<td>-0.055668</td>
<td>-0.099688</td>
<td>-0.009785</td>
</tr>
<tr>
<td></td>
<td>(-2.9215110)</td>
<td>(9.28470785)</td>
<td>(0.7225607)</td>
</tr>
<tr>
<td>$\text{lnoil.d1}$</td>
<td>0.276818</td>
<td>-0.001941</td>
<td>0.063533</td>
</tr>
<tr>
<td></td>
<td>(5.8058452)</td>
<td>(-3.07224174)</td>
<td>(1.8749977)</td>
</tr>
<tr>
<td>$\text{lngas.d1}$</td>
<td>0.068040</td>
<td>-0.068597</td>
<td>0.058959</td>
</tr>
<tr>
<td></td>
<td>(0.7892165)</td>
<td>(-2.41209991)</td>
<td>(0.9623084)</td>
</tr>
<tr>
<td>$\text{lncoal.d1}$</td>
<td>0.169489</td>
<td>0.039345</td>
<td>0.262528</td>
</tr>
<tr>
<td></td>
<td>(2.4976759)</td>
<td>(2.02897804)</td>
<td>(5.4437496)</td>
</tr>
</tbody>
</table>

In Table 21, we can see the estimation of the vector error correction model which contains the coefficients of the different VECM terms and their t-statistics. The results from the VECM estimation (Appendix 32) show us that the error correction term ($\text{ect1}$) have the correct negative sign and are significantly different from zero in the relation relative to the growth of $\text{lnoil}$ and $\text{lngas}$ meaning that there is a long-run equilibrium in those relations. Nevertheless, it seems that the error correction term of the relations relative to the growth of $\text{lncoal}$ is not significant, meaning that the growth of $\text{lncoal}$ is not characterised by a long-run
equilibrium. Based on the values of those coefficients, we can also add that the speed of the correction of disequilibrium is relatively slow.

Regarding the other short-term coefficients \( (\text{lnoil.dl1, lngas.dl1, lncoal.dl1}) \) in the three relations we can see that the growth of \( \text{lnoil} \) depends on its own lagged values and the lagged values of the growth of \( \text{lncoal} \). Concerning the relation relative to the growth of \( \text{lngas} \), we can see that it depends negatively on the lagged values of the growth of \( \text{lnoil} \) and its own lagged values and positively on the lagged values of \( \text{lncoal} \). Finally, in the last relation, we can see that the growth of \( \text{lncoal} \) depends positively on the lagged values of the growth of \( \text{lnoil} \) and on its own lagged values. When compared to the results of the Granger causality tests, we can see that the VECM results are robust with the causalities found in the Granger causality tests.

5.3.4. Structural shift

i. Trace test

In this last step, we now test for the cointegration rank of the VAR process but with a level shift at an unknown time. In order to do this, we use the function \textit{cajolst} in R (Appendix 33) which implements the procedure of Luetkepohl et al. (2004).

<table>
<thead>
<tr>
<th>( H_0 )</th>
<th>( H_1 )</th>
<th>t-statistic</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r \leq 2 )</td>
<td>( r &gt; 2 )</td>
<td>2.84</td>
<td>3.00</td>
<td>4.12</td>
<td>6.89</td>
<td>Fail to reject ( H_0 )</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>( r &gt; 1 )</td>
<td>19.65</td>
<td>10.45</td>
<td>12.28</td>
<td>16.42</td>
<td>Reject ( H_0 )</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>( r &gt; 0 )</td>
<td>96.28</td>
<td>21.80</td>
<td>24.28</td>
<td>29.47</td>
<td>Reject ( H_0 )</td>
</tr>
</tbody>
</table>

We can see in Table 22 that we reject the null hypothesis at a 1% level of significance in each case meaning that there are at most two cointegration equations. Based on this last result, it seems that the presence of a structural break has an impact on the determination of cointegration relationships.
ii. Vector error correction model

Table 23: VECM results.

<table>
<thead>
<tr>
<th></th>
<th>lnoil.d</th>
<th>lngas.d</th>
<th>lncoal.d</th>
</tr>
</thead>
<tbody>
<tr>
<td>ect1</td>
<td>-0.057387</td>
<td>-0.098268</td>
<td>-0.018409</td>
</tr>
<tr>
<td></td>
<td>(-2.9479862)</td>
<td>(8.9612192)</td>
<td>(1.3463160)</td>
</tr>
<tr>
<td>ect2</td>
<td>-0.058610</td>
<td>-0.112796</td>
<td>-0.004206</td>
</tr>
<tr>
<td></td>
<td>(2.6434676)</td>
<td>(-9.0311070)</td>
<td>(0.2700541)</td>
</tr>
<tr>
<td>lnoil.d1</td>
<td>0.275549</td>
<td>-0.002990</td>
<td>0.069905</td>
</tr>
<tr>
<td></td>
<td>(5.7631392)</td>
<td>(-3.1110182)</td>
<td>(2.0814296)</td>
</tr>
<tr>
<td>lngas.d1</td>
<td>0.066484</td>
<td>-0.069883</td>
<td>0.066767</td>
</tr>
<tr>
<td></td>
<td>(0.7697784)</td>
<td>(-2.4363620)</td>
<td>(1.1005365)</td>
</tr>
<tr>
<td>lncoal.d1</td>
<td>0.173342</td>
<td>0.042530</td>
<td>0.243189</td>
</tr>
<tr>
<td></td>
<td>(2.5311600)</td>
<td>(2.1024218)</td>
<td>(5.0553554)</td>
</tr>
</tbody>
</table>

The VECM of the model including a structural break at an unknown date (Appendix 34) shows us that all error correction terms (ect1, ect2) also have the correct negative sign for the three relations relative to the growth of lnoil, lngas and lncoal, meaning that there is a long-run relationship and there is a correction of disequilibrium in the models. Nevertheless, we can see that, like in the previous VECM, the error correction terms of the relation relative to the growth of lncoal are not significant, which means that the growth of lncoal is not characterised by a long-run equilibrium. Regarding the short-run coefficients, we can see that the results are similar to the previous VECM without structural shift. The relation relative to the growth of lnoil depends positively on its own lagged values, and the lagged values of the growth of lngas. The relation relative to the growth of lngas depends negatively on the lagged values of the growth of lncoal but negatively on the growth of lngas and its own lagged values. And finally, the growth of lncoal depends positively on the growth of lnoil and its own lagged values.
Chapter 6: Results

In Chapter 5, we first began with a stationarity analysis of our prices series. Using multiple unit root tests like the Augmented Dickey-Fuller test (1981), the Phillips-Perron test (1988), the Zivot-Andrew test (1992) and finally the Kwiatkowski–Phillips–Schmidt–Shin test (1992), we found that all price series, converted into natural logarithm (Brent crude oil, natural gas and coal), have a unit root, meaning that they are not-stationary.

After that, we applied the Augmented Dickey-Fuller unit root test on the same price series but when taking the first difference of them. For the three price series in first difference, we rejected the null hypothesis of the presence of a unit root, meaning that the first difference series are stationary. Knowing that the price series in level are non-stationary but the price series in first difference are stationary, we can then say that the three price series are integrated of the same order one, $I(1)$. With this condition reached, we then conducted cointegration tests using Engle-Granger (1987) method and Johansen’s (1991) approach.

Before applying the Engle-Granger approach for cointegration analysis, we first tested our variables using the Granger-causality test in order to determine the causality sense of hypothetical relationships between our variables. Using the first difference of our variable, the results showed three causalities: coal prices Granger caused oil prices, oil prices Granger caused natural gas prices and coal prices Granger caused natural gas prices.

We then tested, by pairs, those three relations using the Engle-Granger (1987) approach. After estimating the linear regressions of those relations, a stationarity test of their residuals led to the rejection of the null hypothesis of non-stationarity resulting in the conclusion of the existence of three cointegration relationships.

After establishing cointegration relationships between those pairs of variables, we then estimated an error correction model. The main result of this estimation is that the error correction terms ($ect1$) had the right negative signs in the three ECMs indicating the existence of an error correction mechanism: in the long-run, disequilibrium between pairs of variables compensate in a way that the pairs of variables have a similar evolution. Moreover, it seems that the growth of $lnoil$ and $lngas$ depends positively on the growth of $lncoal$ when the growth of $lngas$ depends negatively on the growth of $lnoil$. This last result can be interpreted by the substitution effect between gas and oil: when crude oil prices rise resulting from an increase in crude oil demand, this could lead to an increase in natural gas production as an alternative to oil, which leads to a decrease in natural gas prices (Villar and Joutz, 2006).
Then, a second cointegration analysis is made using Johansen’s approach (1988) which has the characteristic to be a multivariate approach. Using the trace test and the eigenvalue test on an estimated VAR model with two lags, we found the existence of cointegration relationships between $\ln\text{oil}$, $\ln\text{gas}$ and $\ln\text{coal}$ and also that the model contained one cointegration vector. This being settled, we then estimated a vector error correction model which gave us the following results: all relations have the correct negative signs meaning that there is an error correction mechanism which tends to attract the three variables on the same path. Nevertheless, these long-run relationships seem to be only significant for the relations relative to the growth of $\ln\text{oil}$ and $\ln\text{gas}$. Regarding the growth of $\ln\text{oil}$, it seems to be positively dependent on its own lagged values and on the growth of $\ln\text{coal}$. The growth of $\ln\text{gas}$ is negatively dependent on $\ln\text{oil}$ and its own lagged values but positively dependent on $\ln\text{coal}$. And the growth of $\ln\text{coal}$ is positively dependent on the growth of $\ln\text{oil}$ and its own lagged values.

The third and last cointegration analysis is based on the same idea as Johansen’s approach but by integrating the possibility of a presence of a structural break at an unknown date (Luetkepohl et al., 2004). The first step consists in the trace test of the estimated VAR model, which gave us the same results as Johansen’s procedure of the existence of cointegration relationships between $\ln\text{oil}$, $\ln\text{gas}$ and $\ln\text{coal}$ but with two cointegration vectors. The result that there is one more cointegration vector exhibits the fact that the presence of a structural break has an impact on cointegration relationships.

A vector error correction model is then estimated from the VAR model. The results showed that the two error correction terms ($\text{ect1}$ and $\text{etc2}$) have the right negative sign and are significantly different from zero for the relations relative to the growth of $\ln\text{oil}$ and $\ln\text{gas}$ like in Johansen’s procedure which give us the fact that the growth of $\ln\text{coal}$ does not seem to have an error correction mechanism in the long-run. Regarding the dependency of the growth of $\ln\text{oil}$, $\ln\text{gas}$ and $\ln\text{coal}$, the results seem to be the same as Johansen’s procedure and then stay robust when adding a structural break component.
IV. Conclusion

Economic and financial price series exhibit, most of the time, the characteristic to be non-stationary. The problem with those kinds of series which have this property is that classical analytical methods cannot be used on them, at the risk of having as a result "spurious regression".

In order to get around this difficulty, we applied the cointegration theory on our variables in order to detect some hypothetical relationships among prices of crude oil, natural gas and coal. Two main methods were used with the first one being the Engle-Granger (1987) method which allows us to analyse the cointegration relationships between pairs of variables and the second one being the multivariate Johansen’s (1991) approach which allows us to determine the number of cointegration relationships from VAR models.

The results from the Engle-Granger (1987) method (and from the Granger causality tests) show us that there are three cointegration relationships among our variables seeing that the prices of coal have an impact on the prices of crude oil and natural gas and the prices of crude oil have an impact on the prices of natural gas.

After estimating their respective error correction model for each relation, it has been interpreted that all of them exhibit a long-run relationship, meaning that an error correction mechanism is present in order to compensate some potential disequilibrium. This mechanism helps to keep the evolution of the three price series on the same path in the long-run. Moreover, it seems that the growth of coal prices has a positive impact on the growth of crude oil and natural gas prices when the growth of crude oil prices has a negative impact on the growth of natural gas prices. This first step accomplished, we can say that our hypothesis n°1 is verified for those three relations.

The second method used for cointegration analysis is Johansen’s (1991) approach and it allows the analysis of multiple variables at the same time and not only by pairs, as it was only possible with the Engle-Granger (1987) method. Based on the estimation of a VAR model with our three variables, the results of this second method show us the existence of one cointegration vector among the three variables which allows us the estimation of the VECM.

The results from the estimated VECM show us that, like in the Engle-Granger (1987) method, there is an error correction mechanism but only in the relations relative to the crude
oil prices and the natural gas prices. The relation relative to the coal prices does not have a significant error term in its relation. Concerning the short-term indicators, the results coincide with the Engle-Granger (1987) method. In fact, the growth of crude oil prices is positively dependent on the past values of the growth of coal prices and its own past growth in price values. The growth of natural gas prices are negatively linked to the past values of the growth of crude oil and coal but also to its own past growth in price values. And finally, the growth of coal prices is positively dependent on the past values of the growth of crude oil prices and on its own growth in price values. Based on those results, we can then assert that our hypothesis n°2 is verified and there is in fact a cointegration relationship among our variables.

The third and final analysis used the same Johansen’s (1991) approach but with the difference that, in the estimated model, a structural break at an unknown date has been introduced. The method is based on Luetkepohl et al (2004) and allows the analysis of cointegration relationships when a structural break in our data is taken into account. The reason for this analysis is to determine if strong variations in our data have an impact on the presence of cointegration relationships we found earlier. The strong variations refer to the high volatility period resulting from the economic and financial crisis in 2008 and 2009 when the prices of our three variables experienced a huge drop.

The results from this third cointegration analysis show us the existence of an additional cointegration vector in the estimated model compared to the previous analysis which gives us two cointegration vectors. The estimation of the VECM gives us the same interpretation as Johansen’s approach with an error correction mechanism in the relation relative to the crude oil and natural gas prices but not for the coal prices.

With this last result, we can then validate our hypothesis n°3 and say that, despite the presence of a structural break in the model, there exist two cointegration relationships among our variables which have a long-run relationship.

The fact that the presence of a structural break has a positive impact on the presence of cointegration relationships shows us that there are strong relations between crude oil, natural gas and coal prices, even if a high volatility period occurs in the prices series. In fact, such highly volatile periods, like the one from the ten last years, could have an important impact on the models’ estimations but it seems that the cointegration relationships among our variables do not react the same way and there is no decoupling effect.
The results of this thesis could be particularly useful for the energy industries especially for hedgers who are looking to diminish risk during a crisis period. In fact, during high volatility periods, it is relatively difficult to reduce the risk linked to energy sources. The fact that the three prices seem to be cointegrated could be helpful to better anticipate the behaviour of energy prices during economical or financial crisis periods.

Nevertheless, this thesis has its own limits. Because of the accessibility and the availability of appropriate data, we based our study on data that we estimated the best for their use. This limit leads us to appropriate recommendations we can give in order to improve this thesis or go deeper into the analysis. First of all, it could be interesting to use other data which, maybe, give a better representation of fossil fuels in Europe but this kind of data can be difficult to gather, especially over a long period. The second recommendation is to use other types of data. In this thesis, we use essentially average monthly spot prices and it could be interesting to see if our results hold if using futures prices. Moreover, the frequency could also have an impact on results just as the size of the history chosen. The third and last recommendation concerns the methodology. It could be useful to use another cointegration method like the Philips-Ouliaris approach in order to compare the results. Finally, in order to better estimate the impact of structural breaks on cointegration relationships, it could be interesting to use a more precise estimation method.

To conclude this thesis, it is important to say that cointegration analysis is a relative new-born theory that allows us the analysis of economic and financial series that could not have been done before, especially because of the non-stationarity characteristic of such series. Nevertheless, cointegration theories could still have a lot to offer in terms of research possibilities and discoveries in economic and financial fields. But it maybe needs some further research, but as a great professor once said:

“If we knew what it was we were doing, it would not be called research, would it?”

(Albert Einstein)
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