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Single Pixel Hyperspectral Imaging using Fourier Transform Interferometry

Amirafshar Moshtaghpour*, Jose M. Bioucas Dias†, and Laurent Jacques*
* ISPGroup, ICTEAM/ELEN, UCLouvain, Louvain-la-Neuve, Belgium.
† Instituto de Telecomunicações, Instituto Superior Técnico, Universidade de Lisboa, Portugal.

***Abstract*—Single-Pixel (SP) imaging is now a reality in many applications, *e.g.,* biomedical ultrathin endoscope and fluorescent spectroscopy. In this context, many schemes exist to improve the light throughput of these device, *e.g.,* using structured illumination driven by compressive sensing theory. In this work, we consider the combination of SP imaging with the Fourier Transform Interferometry (SP-FTI) to reach high-resolution HyperSpectral (HS) imaging, as desirable in, *e.g.,* fluorescent spectroscopy. While this association is not new, we here focus on optimizing the spatial illumination, structured as Hadamard patterns, during the optical path progression. We follow a variable density sampling strategy for space-time coding of the light illumination, and show theoretically and numerically (not displayed in this abstract) that this scheme allows for reduced number of measurements and light-exposure of the observed object compared to the conventional compressive SP-FTI.**

Fourier Transform Interferometry (FTI), shown in Fig. 1 (right), works on the principle of a Michelson interferometer. A coherent wide-band beam entering the FTI device is first divided into two beams by a Beam-Splitter (BS). Those beams are then reflected back either by a fixed mirror or by the moving mirror, controlling the Optical Path Difference (OPD) of the two beams, and interfere after being recombined by the BS. The resulting beam is later recorded by an external imaging sensor. Physical optics shows that the outgoing beam from the FTI, as a function of OPD $\xi \in \mathbb{R}$, is the Fourier transform of the entering beam, as a function of wavenumber $\nu \in \mathbb{R}$.

We have proposed two compressive sensing-FTI methods in [1], *i.e.,* Coded Illumination-FTI (CI-FTI) and Structured Illumination-FTI (SI-FTI), that can efficiently reduce the light exposure on the observed object, which is desirable in fluorescence spectroscopy. The former operates by temporal coding of the global light source. In SI-FTI, spatial modulation of the illumination (*e.g.*, using a spatial light modulator), allows for different OPD coding per spatial locations, but with no spatial mixing during the acquisition. Single Pixel-Fourier Transform Interferometry (SP-FTI), introduced in [2], is another variant of compressive FTI, where the structured illumination modulates the spatial content of the scene before being integrated into a single beam (as apposed to SI-FTI).

To study our SP-FTI, suppose that $\boldsymbol{X} \in \mathbb{R}^{N_\nu \times N_\mathrm{p}}$ is the discretization of the HS volume $\boldsymbol{X}_c$ (see Fig. 1) over $N_\nu = N_\xi$ wavenumber samples and $\bar{N}_\mathrm{p}$ pixels in each x- and y-axis such that $N_\mathrm{p} := \bar{N}_\mathrm{p}^2$. We assume that the light source provides constant illumination during N_ξ time slots (or OPD samples). As shown in Fig. 1 (top-left), at each OPD sample $\ell \in [\![N_\xi]\!]$, a Coded Aperture (CA) is programmed $M_\ell \ll N_\mathrm{p}$ times according to the Hadamard patterns, as opposed to [2], where M_ℓ is constant for all OPDs. Every programmed CA gives a spatially coded HS light beam, which is later integrated into a single beam, *e.g.*, by means of an optical collimator. Following the previous discussion, the FTI measurement is then the Fourier transform of the coded and integrated HS light beam. Finally, the whole SP-FTI acquisition results in $M := \sum_{\ell=1}^{N_\xi} M_\ell$ measurements. Note that in [2], a fixed set of Hadamard patterns are initially programmed, that are repeated at every OPD sample.

In a simplified setting, the SP-FTI mixing model reads

$$\boldsymbol{y} = \boldsymbol{P}_\Omega \boldsymbol{\Phi} \boldsymbol{x} + \boldsymbol{n} \in \mathbb{R}^M, \quad \boldsymbol{\Phi} := \boldsymbol{H} \otimes \boldsymbol{F}, \quad \boldsymbol{x} := \mathrm{vec}(\boldsymbol{X}), \qquad (1)$$

where $\boldsymbol{F} \in \mathbb{C}^{N_\xi \times N_\xi}$ and $\boldsymbol{H} \in \mathbb{R}^{N_\mathrm{p} \times N_\mathrm{p}}$ is the DFT and Hadamard transform, receptively. The operator $\boldsymbol{P}_\Omega \in \{0,1\}^{M \times N} (N = N_\xi N_\mathrm{p})$ extracts $M = |\Omega|$ rows of a matrix indexed in $\Omega = \{\Omega_i\}_{i=1}^M \subset [\![N]\!]$ and $\boldsymbol{n} = \{n_i\} \in \mathbb{R}^M$ models an additive noise. In (1), the Fourier matrix is imposed by the FTI system, while the Hadamard matrix is an aspect of our sensing design. In this work, we address *the optimal design of* Ω, in a sense defined below.

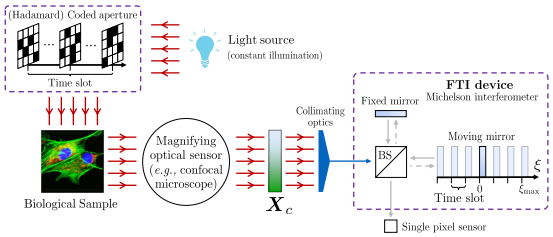

Fig. 1: In SP-FTI, a continuous HS volume $\boldsymbol{X}_c$ (*e.g.*, observed from a confocal microscope) is spatially and temporally modulated from space-time coding of the light source.

Analogous to the other variants of compressive FTI systems in [1], the proposed SP-FTI leverages the Variable Density Sampling (VDS) scheme in [3] for an optimum CA programing (or subsampling set Ω), *i.e.,* an optimum probability mass function (pmf) $p(l) := \mathbb{P}[\beta = l]$ of a random variable (r.v.) $\beta \in [\![N]\!]$ such that $\Omega_i \sim_{\text{i.i.d.}} \beta$ for $i \in [\![M]\!]$.

A stable and robust HS recovery is then reached by solving

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{u} \in \mathbb{R}^N} \|\boldsymbol{\Psi}^\top \boldsymbol{u}\|_1 \text{ s.t. } \|\boldsymbol{D}(\boldsymbol{y} - \boldsymbol{P}_\Omega \boldsymbol{\Phi} \boldsymbol{u})\| \leq \varepsilon \sqrt{M}, \qquad (2)$$

where $\boldsymbol{\Psi} := \boldsymbol{\Psi}_\mathrm{p} \otimes \boldsymbol{\Psi}_\nu$, with spectral and spatial (analysis) sparsity bases $\boldsymbol{\Psi}_\nu \in \mathbb{R}^{N_\nu \times N_\nu}$ and $\boldsymbol{\Psi}_\mathrm{p} \in \mathbb{R}^{N_\mathrm{p} \times N_\mathrm{p}}$, respectively, and $\boldsymbol{D} \in \mathbb{R}^{M \times M}$ is a diagonal matrix such that $d_{ii} = 1/p(\Omega_i)^{1/2}$. To ensure a controlled recovery guarantee of (2), the VDS in [3] optimizes the selection of indices in Ω w.r.t $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ according to the pmf that is proportional to the *local coherence* of $\boldsymbol{\Phi}\boldsymbol{\Psi}$.

In our study of SP-FTI, we set $\boldsymbol{\Psi}_\nu$ and $\boldsymbol{\Psi}_\mathrm{p}$ to the 1-D discrete Haar wavelet and 2-D isotropic Haar wavelet basis, respectively. Careful estimation of the local coherence parameter yields the optimum 3-D pmf $p(l) \propto \min\{1, |l^\xi - N_\xi/2|^{-1} \cdot |\max\{l^x, l^y\}|^{-1}\}$ where $l^\xi \in [\![N_\xi]\!]$ is the OPD index, $l^x, l^y \in [\![\bar{N}_\mathrm{p}]\!]$ are the spatial Hadamard frequencies, such that $l := N_\mathrm{p}(l^\xi - 1) + \bar{N}_\mathrm{p}(l^y - 1) + l^x$. Moreover, in order to recover K-sparse signals we must have $M \gtrsim K \log^3(K) \log^3(N)$.

In a nutshell, for biological specimens where *(i)* the spatial content is supported by a minority of spatial wavelet coefficients and *(ii)* the spectral signatures of its elements are represented by few wavelet coefficients, we have $K \ll N$; our compressive SP-FTI scheme then requires a small number of measurements, and thus reduced light exposure for successful HS image reconstructions.

References

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