"Coherent and ballistic transport in InGaAs and Bi mesoscopic devices"

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ABSTRACT

In 'clean' confined conductors (the so-called mesoscopic systems), the electronic phase and momentum can be preserved over very long distances compared to the system dimensions. This gives rise to peculiar transport properties, bearing signatures of electron interferences, ballistic electron trajectories, electron-electron interactions, regular-chaotic electron dynamics and (in some cases) spin-orbit coupling. Examples of such effects are the Universal Conductance Fluctuations (UCFs) and the Weak Localization observed in the low-temperature magnetoconductance of many confined electronic systems. Of central importance, the electronic phase coherence time and the spin-orbit coupling time determine the amplitude of these quantum effects. In the first part of this thesis, we use UCFs to extract these characteristic timescales in open ballistic quantum dots (QDs) fabricated from InGaAs heterostructures. We observe an intrinsic saturation of the coherence time at low temperature in the In...
Chapter 5

Rectification in InGaAs nanojunctions

No amount of experimentation can ever prove me right; a single experiment can prove me wrong.

Albert Einstein

5.1 Introduction

The subject of this chapter is quite different from that of the two previous chapters, although it still deals with ballistic and coherent electron transport. The devices investigated here show electrical rectification effects, related to their particular shape and symmetry. Rectification is well-known in the field of fluid mechanics: obtaining such an effect from a geometrical asymmetry was proposed as early as in 1920, when Nikola Tesla patented the "valvular conduit". The device consists of a channel with asymmetric loops on its sides and presents a resistance which depends on the direction of the fluid flow. Eighty years later, a very similar effect was obtained for electrons (different electrical resistance for different current directions), using an asymmetric ballistic nano-channel patterned in a semiconductor heterostructure [107]. In the last few years, a wealth of ballistic rectification effects have also been predicted and evidenced in junctions of three [76, 137] and four [140, 108, 48, 53, 32, 61] nanochannels with different geometries and

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different degrees of asymmetry. Pioneering works have also shown that the very high frequency operation of these devices is feasible and very promising [141, 115]. In such applications, tuning the ballistic rectification effect in situ would be highly valuable.

Here, we examine nonlinear rectification effects in two asymmetric four-terminal ballistic devices (shown in Fig. 5.4 and 5.5). At first, we briefly recall the basic model of Fleischmann and Geisel [53, 32, 61], which inspired our devices' design, and we discuss recent experimental results for similar four-terminal devices [48] (section 5.2). Then, we detail the relevant parameters of our samples and the measurement methods (section 5.3). We discuss the results obtained in different measurement configurations (section 5.4). We show that the sign of the slope of the rectified voltage can be reversed as the current through the device is increased, and we compare this observation to predictions of the Fleischmann and Geisel model. In section 5.5, we provide evidence that the sign and amplitude of the rectified voltage can be tuned by illuminating the sample as well as by biasing in-plane gates. Moreover, we show that the channels’ conductances govern the sign and amplitude of the rectification effect. Finally, we demonstrate that the effect is ballistic and that sign reversal of the rectified voltage cannot be understood within the framework of existing models [53, 32, 61, 48, 123].

5.2 Principle of operation

5.2.1 Song’s experiment and model

The origin of the interest in four-terminal ballistic nanojunctions comes from the seminal experiment of Song and coworkers [146], based on the device shown in the inset of Fig. 5.1. It consists of a cross junction (with leads labeled $S$, $D$, $U$ and $L$) with a triangular scatterer inserted in the center, fabricated from a high-mobility AlGaAs/GaAs heterostructure. At low temperature, injecting a current between the source ($S$) and the drain ($D$) results in a negative voltage $V_{LV} = V_L - V_U$, whatever the current sign, as shown on Fig. 5.1(a). The effect was first explained using the following very simple model: electrons ejected from $S$ or $D$ into the junction are deflected toward the lower contact $L$ by the scatterer following trajectories such as indicated by the arrows in the inset of Fig. 5.1(a). Similarly, injecting the current from $U$ or $L$ gives rise to a nonlinear $V_{SD}$, as shown in Fig. 5.1(b), but with a much smaller amplitude. Song et al. explained the nonlinear effect in terms of a Landauer-Büttiker formalism [30]. The currents $I_i$ flowing in
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Figure 5.1: (a) $V_{LU}$ vs $I_{SD}$ and (b) $V_{SD}$ vs $I_{LU}$ in the device shown in the inset of (a) (AFM micrograph). Arrows represent typical electron paths. [From ref. [140]]

each of the four leads $i$ (potential $\mu_i$) can be expressed as:

$$I_i = \frac{2e}{h} \left[ (M_i - R_i)\mu_i - \sum_{j \neq i} T_{ij}\mu_j \right], \quad (5.1)$$

where $M_i$ is the number of quantum channels in lead $i$, $R_i$ is the reflection coefficient describing backscattering into lead $i$, and $T_{ij}$ is the transmission coefficient from lead $i$ to lead $j$. Song et al. [140] assumed that Eq. (5.1) still holds in a nonlinear ballistic transport regime, and introduced current-dependent transmission coefficients. In analogy to the linear transport regime, the resistance $R_{SD,LU} = V_{LU}/I_{SD}$ is then obtained from current balance equations, leading to the final expression [140]:

$$R_{SD,LU} = \frac{h}{e^2 D} [\mathcal{F}_{LS}(I)\mathcal{F}_{UD}(-I) - \mathcal{F}_{LD}(I)\mathcal{F}_{US}(I)], \quad (5.2)$$
where $D$ is a positive factor which does not depend on the current $I$. In a perfectly symmetric device in the linear regime, one expects $I_{LS} = I_{LD}$ and $I_{US} = I_{UD}$, equivalent to $R_{SD,LU} = 0$. When current rises, however, the electric field gradually changes the velocity distribution of electrons, so that part of the electrons will be accelerated and ejected from the narrow leads with a smaller angle with respect to the current direction (the collimation effect), and another portion will be decelerated and therefore decollimated. If an electric field is applied such that electrons coming out from lead $S$ are accelerated ($I_{SD} < 0$), $I_{LS}$ will increase due to collimation, while $I_{LD}$ will decrease as electrons coming from lead $D$ are decollimated. Therefore, $R_{SD,LU} > 0$, which corresponds to the positive slope of $V_L$ vs $I_{SD}$ for $I_{SD} < 0$ on Fig. 5.1(b). Using similar arguments, we obtain $R_{SD,LU} < 0$ for $I_{SD} > 0$, and hence the negative slope of $V_L$ vs $I_{SD}$ for $I_{SD} > 0$.

In this framework, nonlinear ballistic transport and geometrical asymmetry are the basic ‘physical ingredients’ entering into the explanation of the working principle of the devices discussed here. In the nonlinear ballistic transport regime, the electric field is large enough to change the momentum distribution of the electrons, but not large enough to destroy the ballistic nature of electron motion inside the device.

### 5.2.2 Fleischmann and Geisel proposal

An alternative interpretation to the rectification effect in four-terminal junctions was proposed by Fleischmann and Geisel (FG) [53], based on the same formalism as above.² In the FG model, the number of modes $M_i$ in the leads are energy-dependent, instead of the transmission coefficients. FG starts from an generalized expression of the incoming current ‘per unit energy $E$’ injected from lead $i$ at an energy $E$:

$$i^+_i(E, \mu_i, T) = \frac{2e}{h} M_i(E) f_i(E),$$

(5.3)

where $f_i(E)$ is the Fermi distribution in lead $i$ at temperature $T$. The outgoing current per unit energy is therefore $i^-_i(E, \{\mu_i\}, T) = \sum_j \mathcal{J}_{ij} i^+_j$. Total currents are then obtained by integration over the proper range in energy. In the following, we assume $T = 0$ so that Fermi distributions are step functions.

In order to understand the behaviour of a four-terminal rectifier, let’s first examine the scheme of Fig. 5.2(b) representing a voltage probe $P$ connected via two leads to the source $S$ and the drain $D$, with the potentials $\mu_S$, $\mu_P$.

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²The theoretical validity of this model was recently criticized, see ref. [32, 61]
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Figure 5.2: (a) Geometry of the experimental setup used in [140]. (b) Scheme of a voltage probe. (c) Optimized geometry for a 4-terminal rectifier. (d) Scheme of a four-terminal rectifier. [From ref. [53]]

and \( \mu_D \) characterizing the three reservoirs. FG considers two cases, and estimates \( \mu_P \) as a function of the given values of \( \mu_S \) and \( \mu_D \): (i) the leads are narrow (i.e. their width is similar to the Fermi wavelength \( \lambda_F \)) so that the number of channels remains constant in the considered energy range; (ii) the leads are wide (compared to \( \lambda_F \)) so that the number of leads increases as \( \sqrt{E} \). From the current balance equations, one can deduce the values of \( \mu_P \) [53]: in the situation (i), one has the simple result \( \mu_{P,i} = (\mu_S + \mu_D)/2 \); in the situation (ii) the result is \( \mu_{P,ii} = [(\mu_S^{3/2} + \mu_D^{3/2})/2]^{2/3} > (\mu_S + \mu_D)/2 \).

Fig. 5.2(d) shows a scheme of a four-terminal cross-junction fabricated from two three-terminal junctions similar to those discussed above. The top junction is in situation (i) (narrow leads, constant and quantized number of modes vs \( E \)), and the bottom junction is in situation (ii) (wide leads, continuously increasing number of modes with \( E \)). Based on the expressions of \( \mu_P \) given above, it is natural to expect a voltage drop between electrodes \( U \) and \( L \) (proportional to \( \mu_{P,i} - \mu_{P,ii} \)), which is identical upon reversal of the \( S-D \) current.

It is also worth noting that in the scheme proposed in Fig. 5.2(d), direct electron paths between \( U \) and \( L \) can be avoided, contrary to the device measured by Song et al. (next to the central scatterer). Such parasitic links lower the measured transverse voltage \( V_{LU} \). Therefore, the nonlinear effect
should be enhanced in the geometry shown in Fig. 5.2(c), proposed by FG. Most interestingly, based on their model, FG also predict that the sign of the slope of $V_{LU}$ vs $I_{SD}$ can be reversed at high current when an additional channel opens up in the narrow leads.

To summarize the FG model, as current flows between source and drain (S and D on Fig. 5.1(a)), electrons accumulate at the wider voltage probe provided that the narrow and wide channels of the device show quantized and ballistic transport, respectively. However, recent experiment on a device geometry similar to Fig. 5.2(c) revealed an accumulation of electrons at the opposite (narrower) voltage probe, i.e. $V_{LU} > 0$ [48]. The apparent discrepancy between the theory and the experiment was attributed to the fact that the narrow and wide channels in the experiment were in the ballistic and diffusive regimes, respectively, in opposition to the FG model assumptions.

5.2.3 Monte Carlo method

Very recently, nonlinear ballistic effects were analyzed in the framework of a semiclassical two-dimensional Monte Carlo (MC) method [115, 114]. The MC approach is very different from Landauer-Büttiker methods, based on an assumption of coherent transport in the junction (i.e., discrete number of quantum modes in the leads). Intuitively, this approach can be roughly summarized as follows: particles are injected into a device according to thermal and velocity distributions describing the source; then each of them travels a distance determined by a probability distribution depending on the total interaction cross section, to the site of a collision and scatter into another energy and/or direction according to the corresponding differential cross section (more details can be found in [114]. The simulations of four-terminal devices in [115, 114] are two-dimensional: the device is viewed as a single patterned layer of electrons; the effect of surface charges and charged impurities in the $\delta$-doping plane is taken into account by incorporating a ‘virtual’ doping in this layer (impurity scattering is switched off). A lateral surface charge is also included in the simulation. The concentrations of the background doping in the channel and of the surface charges are adjusted so that reasonable agreement with experimental data is obtained in very simple structures (wires, for example).

The result of the MC simulations of the $V_{LU}$ vs $V_{S}$ \(^3\) characteristic in Song's rectifier are reproduced in Fig. 5.3. All the simulation parameters have been adjusted on an InGaAs heterostructure very similar to our sub-

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\(^3\) $V_{BR}$ vs $V$ in the notations of Mateos [114].
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Figure 5.3: Result of the Monte Carlo simulations of the transverse voltage ($V_{BT}$ in the notations of [114]), as a function of the voltage $V = -V_S = V_D$ in the rectifier of Song, for different values of the surface charge $\sigma$. Parameters are adjusted for an InGaAs/InAlAs heterostructure. (From Ref. [114]).

strate. Clearly, the MC method reproduces the typical shape of the experimental curves of Song (Fig. 5.1(a)) at low $V$. In this model, space charges play a dominant role: a region of higher local electron concentration is found by simulations in the bottom part of the triangular scatterer (towards the I contact), which is the origin of the negative $V_{LT}$. This result shows that a quantum description of electron transport may not be essential for the explanation of the nonlinear ballistic effect. We point out that the advantages of the MC approach over previous models are to take the detailed geometry of the device into account, and to provide concentration maps and maps of the electrical potential inside the devices.

5.3 Experimental details

5.3.1 Samples

The four-terminal junctions studied here, shown on Fig. 5.4 and 5.5(b), are fabricated from two different $\delta$-doped InGaAs/InAlAs heterostructure (wafers A and C). Electron-beam lithography and wet etching were used to
pattern the junctions (see chapter 2), formed by three wide channels (‘source’ S, ‘drain’ D, and ‘lower’ L) and one narrow channel (‘upper’ U). A diamond-shaped etched antidot is shifted from the center of the junctions and defines two narrow branches joining channels S-U and D-U (width ~ 140 nm and 100 nm for C1 and A4, respectively) and two wider branches (width ~ 300 nm and 260 nm for C1 and A4, respectively) joining S-L and D-L. It therefore breaks the symmetry of the device with respect to the S-D axis. In sample C1, a control over the width of each branch of the device is provided by four in-plane gates patterned in the 2DEG, separated from the junction by 150 nm-wide etched trenches.

The measurements were performed between 4.2 K and 140 K in a 4He dewar. Temperatures above 4.2 K were obtained by maintaining the sample above the liquid 4He bath. In both samples, the low-temperature electron sheet density in the 2DEG is \( n_s \sim 1.7 \times 10^{16} \text{ m}^{-2} \) (see Appendix A for details on the measurement of \( n_s \) and \( \mu \)). Illuminating the sample with a red light emitting diode (LED)\(^4\) resulted in an increase of up to ~ 25% of \( n_s \) through the persistent photoconductivity effect. At 4.2 K, and without illumination, the electron mobility is \( \mu = 4 \text{ m}^2/\text{Vs} \) in sample C1 and 7 m\(^2/\text{Vs} \) in A4. This is equivalent to electron mean free paths \( l_\mu = 0.8 \mu\text{m} \) and 1.7 \( \mu\text{m} \) in C1 and A4, respectively, larger than the size of the devices.

5.3.2 Nonlinear measurements

We used two different setups to obtain the current-voltage characteristics presented in this chapter, namely the ac setup and the push-pull setup. Historically, our first measurements on rectifiers were performed using the ac setup on sample A4. Then, we brought several improvements to the setup as we wanted to extract additional informations to test the theories presented in section 5.2. In the next section we describe the original setup, its evolution to the final measurement scheme, and the justification for these evolutions.

ac setup

The ac setup, shown in Fig. 5.4(a), was only used for measurements on A4. On Fig. 5.4(a-b) (as well as on Fig. 5.5), the polarization circuit is colored in red. The probe signal, \( V_{D}^{\text{spec}} \), is the sum of a dc voltage which is varied, and a small alternative (ac) voltage of constant amplitude (frequency \( f = 7.33 \)

\(^4\)The sample is illuminated during its cooldown (between ~ 30 K and 60 K), for several seconds. The LED is mounted close to the sample, on the sample holder.
Figure 5.4: (a) ac setup used to measure the $V_{LU}$ vs $I_{SD}$ characteristics on sample A4 (polarization circuit is in red and detection circuit in blue; this code is the same for following figures). The white bar on the SEM micrograph represents 500 nm. (b) ac setup used to measure the $V_{DS}$ vs $I_{UL}$ characteristics on sample A4. (c) $dV_{LU}/dI_{SD}$ vs $I_{SD}$ in sample A4 at 4.2 K measured using the ac setup in (a). (d) $V_{LU}$ vs $I_{SD}$ obtained by integration of the curve in (c).
Hz). $V_{D}^{app}$ is applied on a large resistance ($R = 1\, \Omega$, much larger than the device resistance) in series with the sample, channel $S$ being connected to the ground. The resulting probe current is $I_{SD} = V_{D}^{app}/R$; the amplitude of its ac component is 20 nA, and the dc component $I_{SD}$ changes from -10 $\mu$A to 10 $\mu$A. The measurement wires are colored in blue on Fig. 5.4 (as well as on Fig. 5.5): we detect the ac signal at frequency $f$ between channels $L$ and $U$ using a lock-in technique. Actually, the lock-in measures the derivative of $V_{LU}$ with respect to $I_{SD}$: $\frac{dV_{LU}}{dI_{SD}}$. The transverse voltage $V_{LU} = V_{L} - V_{U}$ is therefore obtained by numerical integration of the lock-in signal.

As an example, the raw data, $\frac{dV_{LU}}{dI_{SD}}$ vs $I_{SD}$, and the integrated lock-in signal $V_{LU}$ vs $I_{SD}$ measured at 4.2 K in A4 are shown on Fig. 5.4(c-d). $V_{LU}$ vs $I_{SD}$ has a linear and a nonlinear contribution. The linear contribution comes both from an asymmetry of the polarization (one side of the device is grounded, and the potential of the other side changes as $I_{SD}$ changes from -10 $\mu$A to 10 $\mu$A) and from an asymmetry of the device (see section 5.3.3).

$V_{DS} = V_{D} - V_{S}$ is measured in the same way as a function of $I_{UL}$ using the setup shown in Fig. 5.4(b). In the following, this setup will be referred to as the ‘reverse configuration’, in opposition to the ‘direct configuration’ described above. Measurement performed using the ac setup on A4 are presented in section 5.4.

**Push-pull setup**

There are several important differences between the ac setup and the push-pull measurement setup, shown on Fig. 5.5(a-b). The main difference is the polarization symmetry. In the push-pull setup, polarization is realized by applying opposite dc voltages $+V$ and $-V$ on channels $D$ and $S$ with two large resistances ($R = 0.1\, \Omega$) in series. The advantage of this symmetric polarization scheme is that, in the case of a perfectly symmetric device, we no longer expect to observe the ‘parasitic’ linear contribution discussed above in $V_{LU}$ vs $I_{SD}$.

On the detection side, we measure the dc transverse voltage $V_{LU}$ using a nanovoltmeter. As two contacts are available for each channel, we also simultaneously measure the longitudinal voltage drop at the edges of the device $V_{SD}$. Therefore, $V_{SD}$ does not include any contribution of lead or contact resistance ($R_{CS}$ and $R_{CD}$ on Fig. 5.5(a)). The possibility to measure $V_{SD}$ was added in order to test the prediction of Fleischmann and Geisel [53],

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3We observe that the noise on the lock-in signal (Fig. 5.4(a)) increases as the current $I_{SD}$ increases. We do not have any explanation for this observation.
Figure 5.5: (a) scheme of the push-pull setup used to measure $V_{SD}$ and $V_{LU}$ as a function of $I_{SD}$. (b) scheme of the modified push-pull setup used for sample C$_1$. The white bar on the SEM micrograph of C$_1$ represents 2$\mu$m.
concerning the change of slope of the nonlinear voltage occurring as the number of quantum channels increases in the device branches. Measurements results obtained using this scheme are discussed in section 5.4.

Before the measurements on sample C1, some minor modifications were made to the push-pull setup (Fig. 5.5(b)). Instead of measuring only the difference $V_D - V_S$, we rather measured the source and drain voltages $V_S$ and $V_D$ separately, 2 $\mu$m away from the junction. This allowed us to correct the voltages applied on channels $S$ and $D$ ($-V + \Delta V_1$ and $+V + \Delta V_2$) so that we could obtain $V_S \sim -V_D$. Moreover, as four in-plane gates are patterned on sample C1, dc voltages ($V_{G_1-4}$) can be applied on each gate contact in order to independently tune the width of the channels.

Contrary to the ac method, the dc push-pull method does not require any post-treatment ($V_{LU}$ is directly measured), which is an additional advantage. Moreover, the precision of the two measurement schemes are comparable.

5.3.3 Channels' conductances

As we will show below, the knowledge of the conductance of each branch of the four terminal junction is essential in the understanding of its nonlinear properties. For each configuration of in-plane gate biases and illumination, the four-contacts conductances $G_{IJ}$ between channels I and J have been measured in sample C1 at 4.2 K using a lock-in technique. Keep in mind that, for adjacent channels I and J, $G_{IJ}$ is the conductance of the branch I-J in parallel with the conductance of branches I-K, K-M and M-J in series (where K and M are the other channels). $G_{SL}$, for example, is a relatively accurate measurement of the conductance of the branch S-L, since the conductance of branches S-U and U-D are small compared to that of the branch S-L. However, $G_{SU}$ certainly overestimates the conductance of branch S-U, because branches S-L and I-D have a higher conductance.

The conductances $G_{IJ}$ in A4, measured at 4.2 K, are shown in Table 5.1. The two rows correspond to two different cooldowns of the sample. We observe significant changes between the values of $G_{IJ}$ in the two rows, due to different illumination conditions during the two cooldowns. The symmetry of A4 with respect to the U-L axis can be inferred by noting that $G_{US} \neq G_{UD}$ and $G_{LS} \neq G_{LD}$ on the other side. This shows that the symmetry with respect to the U-L axis is slightly broken in A4.

In the case of sample C1, we could tune the channels' conductances using voltages on lateral gates and illumination of the sample during the cooldown. Many sets of $G_{ij}$ were obtained, and we won't reproduce all of them here. However, a graphic representation of the conductances of the upper and
Table 5.1: Four-contact conductances $G_{ij}$ (in units of $2e^2/h$) measured between channels $i$ and $j$ of sample $A_4$ (at 4.2 K). The first and second rows correspond to measurements performed in the ac and dc push-pull setup, respectively (measurements in the ac and dc setups were performed during different cooldowns of the sample).

<table>
<thead>
<tr>
<th>Measurement configuration</th>
<th>$G_{US}$</th>
<th>$G_{UD}$</th>
<th>$G_{LS}$</th>
<th>$G_{LD}$</th>
<th>$G_{UL}$</th>
<th>$G_{SD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ac</td>
<td>4.07</td>
<td>3.76</td>
<td>18.05</td>
<td>18.21</td>
<td>3.73</td>
<td>12.06</td>
</tr>
<tr>
<td>dc push-pull</td>
<td>6.14</td>
<td>5.37</td>
<td>18.16</td>
<td>19.24</td>
<td>6.32</td>
<td>11.72</td>
</tr>
</tbody>
</table>

lower part of the device is given in Fig. 5.11. As in $A_4$, differences of up to 20% are observed between $G_{LS}$ and $G_{LD}$ and up to 15% between $G_{US}$ and $G_{UD}$, indicating that $C_1$ is also slightly asymmetric with respect to the U-L axis.

5.4 Results

Fig. 5.6(a) shows $V_{LU}$ vs $I_{SD}$ in $A_4$, measured using the ac setup. Note that the voltage drop $V_{SD}$, calculated using measurements of $G_{SD}$ and $I_{SD}$, is shown as the top axis. At high temperature (130.5 K), $V_{LU}$ vs $I_{SD}$ is linear. As explained above, this linear contribution is purely ohmic, and originates from the fact that $V_S \neq -V_D$ in the ac setup, and from the broken symmetry of the junction with respect to the U-L axis.6

As the temperature decreases, a nonlinear contribution of growing amplitude superimposes on the linear background. In order to isolate the nonlinear contribution, we subtract a linear fit from the $V_{LU}$ vs $I_{SD}$ data, to obtain $\delta V_{LU}$ vs $I_{SD}$, shown on Fig. 5.6(b) at several temperatures. $\delta V_{LU}$ vs $I_{SD}$ is almost symmetric with respect to zero bias in the considered current range, except for very low currents around $I_{SD} = 0$. This result is qualitatively very similar to the first report on a ballistic rectifier [140]: in both cases the nonlinear part of $V_{LU}$ vs $I_{SD}$ is negative and has a bell-shape. In our sample, for a similar $I_{SD}$, the nonlinear effect is increased by a factor of $\sim 4 - 5$ with respect to Ref. [140] at 4.2 K. At first sight, this improve-

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6It was similarly observed at different levels in almost all previous experimental works on three- and four-terminals nanojunctions [76, 140, 48]. In the case of a perfectly symmetric device, one expects a vanishing $V_{LU}$ vs $I_{SD}$ in the diffusive limit.
Figure 5.6: (a) $V_{LU} \text{ vs } I_{SD}$ at 4.2 K (bold curve) and 130.5 K (light curve) in A4. (b) $\delta V_{LU} \text{ vs } I_{SD}$ at indicated temperatures. (c) $V_{UL} \text{ vs } I_{UL}$ at 4.2 K (bold curve) and 79 K (light curve). (d) $\delta V_{DS} \text{ vs } I_{UL}$ at indicated temperatures.
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![SEM micrograph of the device of Löfgren et. al.](image)

**Figure 5.7:** Left: SEM micrograph of the device of Löfgren et. al. Right: Output of the device shown at the left: $V_{LU}$ vs $V_{SD}$ [From ref. [108]]. Note that, here, $V_{LU}$ is averaged over positive and negative $V_{SD}$.

ment is consistent with the prediction of Fleischmann and Geisel [53], who attributed the enhancement to the suppression of the parasitic channel joining directly U and L. However, we recall that Ref. [140] concerns a larger device fabricated on a higher mobility substrate. Therefore, the comparison between the current-voltage characteristics in the two geometries is not straightforward, and the observed enhancement of the nonlinear effect may be coincidental.

A more rigorous verification of the prediction of FG would require measurements on a device similar to the one investigated in [140], but fabricated on our substrate and with dimensions very similar to the dimensions of $A_4$. This was almost realized in the recent work of Löfgren et al. [108]: a device with a geometry similar to [140], but with smaller dimensions and fabricated on an In$_{0.73}$Ga$_{0.25}$As/InP heterostructure is investigated (see Fig. 5.7; $\mu = 45$ m$^2$/Vs, $n_s = 4.5 \times 10^{13}$ m$^{-2}$ and $l_m = 5$ μm in this sample at low temperature). Comparing Fig. 5.7 and Fig. 5.6(b), we observe that the nonlinear signal in $A_4$ is significantly larger in our voltage range (between $\sim$ -11 mV and 11 mV), even if the mean free path is a factor of 2-3 smaller in our device. This is in agreement with the first prediction of FG, and therefore confirms the influence of the suppression of the parasitic electron
paths between the upper and lower channels.

In a similar way, we examine the reverse configuration $V_{DS}$ vs $I_{UL}$, shown on Fig. 5.6(c). Again, we observe a linear contribution to $V_{DS}$ vs $I_{UL}$, which is subtracted to isolate the nonlinear part $\delta V_{DS}$ vs $I_{LU}$, plotted on Fig. 5.6(d). Note that, although current scales are identical in Fig. 5.6(b) and (d), the voltage scales, shown on the top of the figure, are very different due to the difference between $G_{SD}$ and $G_{UL}$. $\delta V_{DS}$ vs $I_{LU}$ shows some differences with the results obtained in the direct configuration (Fig. 5.6(b)). At first, for a given applied current, the nonlinear contribution to the transverse voltage is a factor of ~2 smaller in the reverse configuration. The difference is even larger when these amplitudes are compared for a given longitudinal voltage drop. As an example, for $V_{DS} = V_{LU} = 10$ mV, the nonlinear contribution is more than five times smaller in the reverse configuration. This difference is a confirmation that the nonlinear effect is most likely related to the broken symmetry of the device, which is much more pronounced with respect to the S-D axis than to the U-L axis.

Interestingly, in contrast to $\delta V_{LU}$ vs $I_{SD}$ discussed above, $\delta V_{DS}$ vs $I_{LU}$ is asymmetric with respect to $I_{SD} = 0$. For positive currents, the sign of the slope of $\delta V_{DS}$ vs $I_{LU}$ changes when $I_{UL} \geq 5 \mu A$. Such a change of slope is reminiscent of the "oscillatory" behavior observed at a larger bias in the transverse voltage of the ballistic rectifier of Löfgren and coworkers (Fig. 5.7). In their work, a qualitative explanation is proposed for both the negative transverse voltage and the slope reversal, based on the following hypothesis: (1) as the current grows, electron collimation at the device opening is increased, resulting in a more efficient deflection of the electrons by the scattering center and (2) as the current increases, the number of lateral confinement modes at the openings increases, widening the angular distribution of electrons flowing from the opening to the scatterer. The second mechanisms changes the efficiency of the device to redirect electrons towards channel L, since less electrons hit the scatterer. In our particular device geometry, however, both effects are expected to be weaker. Indeed, the current is injected through larger channels (in which collimation effects should be smaller than in Löfgren's device) and it is much less likely that a change in the angular distribution of injected current would cause a change in the number of electrons hitting the scatterer. We can therefore rule out

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7The voltages scales are calculated from the current scales, using the measured values of $G_{SD}$ and $G_{UL}$, and assuming a linear current-voltage regime. This condition is not fulfilled at high current in our samples, as we show below. Therefore, the voltage scales are only indicative here (especially in Fig. 5.6(c-d)). In contrast, voltage drops are measured in the push-pull setup.
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this explanation in our case.

In order to get a further insight into this "oscillatory" behavior, we present $\delta V_{LU}$ vs $I_{SD}$ obtained using the push-pull measurement setup on Fig. 5.8(a), along with simultaneous measurements of the voltage drop $V_{SD}$ vs $I_{SD}$, Fig. 5.8(b) (both measured on A4). Fig. 5.8(a) also compares the transverse voltage measured using the push-pull setup (plain curve) and the ac setup (dotted curve). While both measurements are similar for positive $I_{SD}$, differences clearly appear for $I_{SD} < 0$. We recall that the curves have been measured for different cool downs of the sample, which results in slightly different values of the conductances of the branches during both measurements. As we will show below on the basis of data from sample C1, these changes of conductances are at the origin of the changes of the shape of the $\delta V_{LU}$ vs $I_{SD}$ characteristic.

Most importantly, we note that, in addition to the rectifying behavior already observed at low current, the sign of the slope of $\delta V_{LU}$ vs $I_{SD}$ successively reverses two times at higher negative current, reminiscent of the behavior observed on Fig. 5.6(d) for $\delta V_{DS}$ vs $I_{UL}$. These reversals are related to changes in the longitudinal conductance of the device. Indeed, although $V_{SD}$ seems almost linear as a function of $I_{SD}$ (Fig. 5.8(b)), we observe on Fig. 5.8(c) that the longitudinal conductance $G_{SD} (= \frac{\delta I_{SD}}{\delta V_{SD}})$ is not constant as $I_{SD}$ grows. $G_{SD}$ smoothly decreases for increasing negative currents, and shows two relatively abrupt jumps for $I_{SD} \approx -9\mu A$ and $I_{SD} \approx -12\mu A$ (indicated by the dashed lines on the graph); for positive currents, the differential conductance slightly increases. As indicated by the dashed lines on Fig. 5.8, the 'step-like' decreases of $G_{SD}$ coincide with the changes of slope in $\delta V_{LU}$ vs $I_{SD}$.

In the classical (ohmic) regime, increasing the bias voltage decreases the electron mean free path, and therefore also decreases the differential conductance $[80, 81]$. Nonlinear and asymmetric behavior of the differential conductance of narrow constrictions has also been reported in the quantum regime $[99, 142]$. In this case, the differential conductance increases or decreases with growing current depending on the energy subband separation, on the position of the Fermi level with respect to subband edges, and on the relative change of the electrochemical potential at the edges of the constriction $[99]$. In this framework, step-like changes of the differential resistance are associated to changes in the transverse subbands occupation. Our observation in Fig. 5.8 therefore seems to corroborate with the second prediction of Fleischmann and Geisel $[53]$, linking the change of the number of modes in the narrow branches of the device to the occurrence of a reversal of the
Figure 5.8: (a) $\delta V_{LU}$ vs $I_{SD}$ at 4.2 K, measured on $A_4$ using the push-pull setup (bold curve) and using the ac setup (dotted curve). (b) $V_{SD}$ vs $I_{SD}$ at 4.2K, measured using the push-pull setup (bold curve) and using the ac setup (dotted curve). (c) Differential longitudinal resistance $dV_{SD}/dI_{SD}$ vs $I_{SD}$ at 4.2K.
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slope of $\delta V_{LU}$ vs $I_{SD}$.

However, a more careful examination of our data reveals some contradictions with this simple picture. First, one can question whether the change of the number of lateral modes occurs in the narrow branches or in the wide branches. Between 0 and -14 $\mu A$, the conductance $G_{SD}$ is divided by a factor of $\sim 2$. As $G_{SD}$ is a measurement of the conductance of the narrow and wide branches in parallel and as the narrow branches have a much smaller conductance, a decrease of the conductance of the narrow branches could not cause a factor of $\sim 2$ decrease of $G_{SD}$. Secondly, Ref. [53] and [108] predict that the number of quantum modes should increase as $I_{SD}$ grows, which should increase the conductance, in opposition with our observation. Note that the model of Löfgren also invokes an increasing number of modes as $I_{SD}$ increases.

Figure 5.9: (a) $\delta V_{LU}$ vs $I_{SD}$ at 4.2 K in sample C$_1$. (b) Differential conductance $G_{SD}$ vs $I_{SD}$. The dashed curve is a fit to a quadratic law.

Measurements on C$_1$ also revealed a decrease of $G_{SD}$ as $I_{SD}$ grows. As shown on Fig. 5.9(b), $G_{SD}$ has a parabolic-like dependence on $I_{SD}$. This behavior is very similar to that observed by Horsey et al. in narrow constrictions [81]. Contrary to the measurements in A$_1$, $G_{SD}$ vs $I_{SD}$ does not show any abrupt jumps. On Fig. 5.9(a), a change is observed in the
slope of $\delta V_{LU}$ vs $I_{SD}$ for positive $I_{SD}$. However, it may originate from an improper subtraction of the linear component of $V_{LU}$ vs $I_{SD}$, so that the change is much less clear than on Fig. 5.8(a), and more data (for larger $I_{SD}$) would be needed to have a better view of the situation in this case.

Our observations therefore lead us to conclude that neither the model of Fleischmann and Geisler nor that of Löfgren fully explain the ‘oscillatory’ behavior of $\delta V_{LU}$ vs $I_{SD}$.

5.5 Tunable rectification

In this part, we mainly discuss data from C1, and we investigate the effect of side gates biases and of illumination on the rectifying properties. Fig. 5.10(b) shows $V_{LU}$ vs $V_{SD}$ measured using the push-pull method. These raw measurements are very similar to data on Fig. 5.6: at high temperature (130 K), $V_{LU}$ linearly depends on $V_{SD}$ (again, an ohmic effect, arising from the unintentional asymmetry of the device with respect to the U-L axis), and, as $T$ goes down, a nonlinear contribution superimposes on the linear background. Most importantly, changing the sample cooldown conditions tremendously affects this nonlinear contribution. Curves A and B on Fig. 5.10(b), both measured at 4.2 K, show opposite nonlinear contribution to $V_{LU}$ vs $V_{SD}$. Curve A was obtained after a brief illumination at 60 K by the LED, while curve B was obtained without illumination.

In order to shed light on the striking behavior shown in Fig. 5.10(b), we carefully investigate correlations between device parameters and the sign and amplitude of the nonlinear effect. In addition to illumination, biasing the in-plane gates also affects the nonlinear effect. This is clearly evidenced on Fig. 5.10(c), where $\delta V_{LU}$ vs $V_{SD}$ - obtained by subtracting the linear contribution from the $V_{LU}$ vs $V_{SD}$ data - is displayed for different values of in-plane gate voltages. The amplitude of the nonlinear signal $\delta V_{LU}$ changes between $\sim +0.5$ mV and $\sim -0.5$ mV at $V_{SD} = 70$ mV. As both illumination and side gate biasing influence the conductance of the channels, it is tempting to present the amplitude of the nonlinear effect as a function of the opening of the branches.

In order to quantify the opening of the upper part of the device, formed by the branches S-U and U-D, we define $G_U = (G_{US} + G_{UD})/2$. Similarly, the opening of the lower part of the device is proportional to $G_L = \ldots$ 

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8In Fig. 5.8(a), the changes in the slope do not depend on the subtraction of a linear component.
Figure 5.10: (a) Electron micrograph of device C\textsubscript{1} (dark regions have been etched). (b) \(V_{LU}\) vs \(V_{SD}\) at 130 K (bold curve) and at 4.2 K with (curve A) and without (curve B) high temperature illumination by a LED. (c) \(\delta V_{LU}\) vs \(V_{SD}\) at 4.2 K for different combinations of in-plane gate voltages and different cooldown conditions (curves 1 and 2 have been obtained after illumination). The curves have been offset for clarity.
Figure 5.11: Color plot of $\delta V_{LU}^{30 \text{ mV}}$ vs $G_U$ and $G_L$ at 4.2 K (see color scale at the right of the graph). Schematic representations of the device are drawn in the corners of the graph, with the sign of the accumulated charge at contacts U and L. The dash-dotted line corresponds to $G_U = G_L$. Inset: $\delta V_{LU}^{30 \text{ mV}}$ vs $G_L$ at 4.2 K. The dashed line is a linear fit to the data.
Fig. 5.11 gathers all our measurements of \( \delta V_{LU}^{30\,\text{mV}} = [\delta V_{LU}(V_{SD} = 30\,\text{mV}) + \delta V_{LU}(V_{SD} = -30\,\text{mV})]/2 \) for different combinations of \( G_U \) and \( G_L \), using a color map where positive values are shown in red and negative in blue.\(^9\) Clear trends are observed: positive \( \delta V_{LU}^{30\,\text{mV}} \) values concentrate at the large \( G_{LU} \) side of the graph and negative \( \delta V_{LU}^{30\,\text{mV}} \) values group at the small \( G_{LU} \) side. Furthermore, Fig. 5.11 (inset) shows that \( \delta V_{LU}^{30\,\text{mV}} \) vs \( G_L \) is linear, with a transition between positive and negative \( \delta V_{LU}^{30\,\text{mV}} \) around \( G_L = 8 \times 2 \, e^2/h \). The trend is similar in the case of \( \delta V_{LU}^{30\,\text{mV}} \) vs \( G_U \). In summary, widening all device branches leads to an accumulation of electrons at the narrow channel, and narrowing the branches results in accumulation of electrons at the wide one.

Importantly, we note on Fig. 5.11 that all our data points fall on the same side of the dash-dotted line (corresponding to \( G_U = G_L \)), so that \( G_U \) is always smaller than \( G_L \). Since branches L-S and L-D always remain wider than branches U-S and U-D, whatever the biases applied on in-plane gates, the observed sign reversal of \( \delta V_{LU} \) cannot originate from an asymmetry reversal of the device (in contrast to reference [48], where sign reversal is always observed as device asymmetry is reversed). Indeed, red data points on Fig. 5.11 correspond to negative charge accumulation at the narrower side of the device, similar to data in reference [48]. Conversely, blue data points correspond to accumulation of negative charges at the wider side of the device, similar to predictions of Fleischmann and Geisel [53, 32, 61]. We point out that our high conductance data show the same rectification sign as in ref. [48], where \( G_{LU} \) are always larger than in our sample.\(^1\) This work is therefore consistent with ref. [48], while unveiling new features in an uncovered range of \( G_{LU} \).

In a first analysis, the transition from negative to positive output voltage could be viewed as a transition between the two different transport mechanisms, described in references [48] and [53]. Within the framework of these models, the change from negative to positive \( \delta V_{LU} \) could in principle be explained, but only under specific conditions, i.e. widening the branches should cause a concomitant change of transport regime: from quantized to ballistic in the narrow branches, and from ballistic to diffusive in the wide ones. While unlikely, we now test this explanation in view of all available

\(^9\)Keep in mind that \( G_U \) overestimates the actual conductance of branches U-S and U-D (see section 5.3.3); comparatively, \( G_L \) is a much better estimate of the conductance of branches L-S and L-D.

\(^1\)The choice of \( V_{SD} = 30 \, \text{mV} \) is arbitrary. Choosing another value for \( V_{SD} \) would not change our conclusions.

\(^1\)as inferred from \( \mu, n \), and device size in reference [48].
The temperature dependence shown in Fig. 5.12(c) is clearly related to that of the electron mean free path, measured on an unpatterned part of the same wafer. This indicates that the nonlinear rectification is governed by ballistic effects, both for $\delta V_{LU} > 0$ and $\delta V_{LU} < 0$. This observation rules out the explanation given in the previous paragraph, since the $T^{-1}$ dependence expected for thermal broadening of the quantum steps [14] is not observed. On the other hand, our results show that, in addition to quantum transport, [53, 32, 61] ballistic effects can give rise to the accumulation of electrons in the wider branch of an asymmetric cross-junction device, and that the direction of accumulation can be reverted in situ. However, the understanding of the sign reversal and the tunability of the effect remains to be theoretically investigated.

It is also worth examining data from the other four-terminal rectifier ($A_4$), in light of the analysis performed above, on data from $C_1$. $G_L$ is significantly larger in $A_4$ ($\sim 18 - 19 \times 2 \text{ e}^2/\text{h}$) compared to $C_1$, while $G_U$ is almost in the same range in both samples ($\sim 4 - 7 \times 2 \text{ e}^2/\text{h}$ in $A_4$, see table 5.1). Data from $A_4$ should therefore appear beyond the range of the $G_L$ axis on Fig. 5.11, as blue dots (negative $\delta V_{LU}$). This observation raises the possibility of another transition from positive to negative $\delta V_{LU}$ as $G_L$ increases above $12 \times 2\text{ e}^2/\text{h}$. However, one could also argue that the geometry (channel length, shape of the scatterer) is different in both samples. This parameter is indeed likely to be very important in determining the device properties, as transport is in the ballistic regime. We therefore can not definitely conclude this issue, and suggest that Monte Carlo simulation [115, 114] may be a good way to predict the current-voltage characteristics of ballistic rectifiers, since they take into account the full device geometry, and not only the width of the channels.

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12 The current used in our experiment can be large enough to induce Joule heating of the electron system, so that it is logical to consider thermal voltages as a potential explanation to the observed nonlinear transverse voltage. However, an earlier work [123] showed that a necessary condition for the occurrence of such a thermal voltage in a four terminal configuration is a quantized regime of transport, at least in the narrower part of the device. As the temperature dependence of the effect is consistent with a ballistic regime of transport (and not with a quantized regime), we can exclude a thermal origin for the nonlinear effect.

13 Note that $\delta V_{2nV}$ cannot be calculated in $A_4$, since no data is available for $V_{SD} = 30 \text{ mV}$. We can just extrapolate up to $V_{SD} = 30 \text{ mV}$, assuming that the sign of $\delta V_{LU}$ does not change.
5.5. **TUNABLE RECTIFICATION**

Figure 5.12: Temperature dependence of $\delta V_{LU}$ vs $V_{SD}$ in the case of (a) positive $\delta V_{LU}$ and (b) negative $\delta V_{LU}$. (c) $i_\mu$ (bold curve, right axis) and $|\delta V_{LU}^{mV}|$ (left axis) as a function of the temperature for positive and negative $\delta V_{LU}$. 
5.6 Summary

In this chapter, we examine the nonlinear properties of asymmetric four-terminal junctions, in a geometry inspired by a recent proposal of Fleischmann and Geisel [53]. We observe a nonlinear transverse voltage as a function of the longitudinal applied voltage, very similar to previous experimental reports on ballistic four-terminal junctions. Importantly, we observe reversals of the slope of the transverse voltage at high longitudinal voltage, concomitant with a decrease of the channels' conductances. We show that this behaviour cannot be explained in the framework of available models, based either on an increasing number of modes in the channels, or on changes in the collimation effects at the channels edge when the longitudinal voltage increases.

Using a sample with in-plane gates patterned close to the channels, we demonstrated the tunability of the sign and amplitude of the nonlinear transverse voltage. The tunability was found to be governed by the conductances of the channels. Electrons accumulate in the wide (narrow) part of the device in the case of small (large) average conductance. Data suggest that both the upper and lower part of our device are in the ballistic (but not quantum) regime of transport, which differs from previously reported works. [53, 32, 61, 48, 123] In our case, the real geometry of the sample should be taken into account, and not only the width of the branches. We suggest that this might be accomplished by means of Monte Carlo simulations, which already proved successful with similar devices [115, 114].

Section 5.5 of this chapter, concerning the tunability of the transverse voltage, has been recently published in Ref. [67].
Chapter 6

Conclusions and perspectives

*Time brings truth to light*

Aristotle

The most significant contribution of this thesis concerns the temperature dependence of the electron phase coherence time $\tau_\phi$ in confined systems. We have observed a low-temperature saturation of $\tau_\phi$ in each of our InGaAs open quantum dots, over a wide range of temperature. We could rule out any 'extrinsic' origin, related to the substrate material or the measurement system. We found that it is the dwell time that limits the measured phase coherence time, *i.e.* $\tau_\phi^{sat} = \tau_d$ (where $\tau_\phi^{sat}$ is the saturated coherence time and $\tau_d$ is the dwell time). Electron interferences can only occur during the average time spent by electrons inside the quantum dot. The observed saturation is therefore directly related to the measurement method.

As a consequence, our work questions the experimental evidences for the existence of a low temperature intrinsic saturation of $\tau_\phi$ in confined systems. The conclusion of our work implies that the saturation of the 'true', or intrinsic, coherence time has not been observed yet in any experiment on quantum dots, since we are able to explain all experimental reports of saturation of the coherence time in these systems, and the only available experiment on samples with a virtually infinite dwell time (i.e. closed dots) did not reveal any saturation of the intrinsic $\tau_\phi$.

Furthermore, it might be necessary to reconsider previous experimental reports of saturation of the coherence time in many other confined systems, such as quantum wires. Indeed, a dwell time can also be defined in such systems, but has never been taken into account in the extraction of $\tau_\phi$, as
far as we know. Therefore, our conclusions for quantum dots are likely to lead to a similar study in quantum wires (in particular short wires, with a small dwell time), for which many report of low temperature saturation of $\tau_\phi$ have been published.

Our work also clears up the way for theoretical works. Previous experimental reports on quantum dots put constraints on a possible theory of the real saturation of $\tau_\phi$. As we could attribute previous observations in quantum dots to an intrinsic, but sample-specific origin, part of these constraints is lifted. Even though we do not bring the proof for the existence or nonexistence of an intrinsic saturation of $\tau_\phi$, our findings help to discriminate between experiments reporting a 'real' or 'artificial' sample specific saturation.

In addition, we provide new $\tau_\phi$ vs $T$ data for open quantum dots, in unexplored ranges of mean energy level spacings, number of modes, and materials parameters. In the temperature-dependent regime, we found that, in one sample, $\tau_\phi$ vs $T$ follows quantitatively the theoretical prediction for decoherence by electron-electron interactions in two-dimensional disordered systems. In the other samples, discrepancies were observed, that we attribute to the population of the second subband in the quantum well. However, theoretical work is needed to confirm the latter hypothesis.

In chapter 4, we study phase coherence and spin-orbit coupling in Bi films and in a single-crystal Bi cavity. We show that the cavity is quasi-ballistic and zero-dimensional for phase coherent processes. Our work therefore provides the first insight into the transport properties of this type of Bi systems. $\tau_\phi$ in films is significantly larger than in the cavity, while the spin-orbit scattering time $\tau_{so}$ is similar in both systems. The reason for this difference remains to be investigated. A large unexplained discrepancy is also found between our $\tau_\phi$ data and the prediction of the 2D Nyquist theory, although the temperature dependence ($\tau_\phi \propto T^{-1}$) is consistent with this model in the films and the dot.

Besides the questions over the origin of these discrepancies, our work on Bi samples raises a large number of interesting perspectives. In particular, we believe that the technique developed to obtain films with large crystallites can be further improved. Another substrate material could be used instead of SiO$_2$, with a lattice parameter closer to that of bismuth. This could be combined with a slower deposition process at higher temperature, and a thermal treatment to align the crystallites along the same direction in the plane of the film (which would suppress the uncertainty over the crystal orientation in the cavity). This would allow to produce monocrystalline devices on thinner films, e.g. in the vicinity of the semimetal-semiconductor
transition. The influence of the lateral size of the cavity on the weak antilocalization effect is another interesting problem to study. As explained in chapter 1, weak antilocalization can be suppressed in GaAs quantum dots with a lateral size smaller than $L_s$. One can naturally wonder if a similar suppression would occur in the same conditions in a Bi sample. In view of the $L_s$ values that we found, ballistic bimuth cavities with dimensions smaller than $L_s$ are possible to fabricate using the process that we developed. Such a small cavity, with a small electron dwell time would also allow to test our explanation of the saturation of $\tau_\phi$.

In the last part of the thesis, we discuss the nonlinear transverse voltage-longitudinal current characteristics of ballistic cross junctions. We show that the conductances measured between the channels of the junctions are crucial to the understanding of the nonlinear behaviour. Reversals of the slope of the transverse voltage are observed in some cases, coinciding with a decrease of the longitudinal conductance. Furthermore, the sign and the amplitude of the nonlinear transverse voltage can be tuned by changing the conductances (or the width) of the branches in the junction. Theories based on an energy-dependent Landauer-Büttiker formalism or on energy-dependent collimation effects can not fully explain these observations. We suggest that Monte Carlo (MC) methods are better suited to study the behavior of our structures. MC simulations of our cross structures have recently been undertaken in the group of Profs. Javier Mateos and Tomas Gonzalez in Salamanca University. We hope that the result of these simulations will help to explain our data.