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Fabbro, Vincent ; Jeannin, N. ; Djafri, Kahina ; Lemorton, Joël ; Vanhoenacker-Janvier, Danielle

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Scintillation modelling in troposphere using Multiple Phase Screen

V. Fabbro\textsuperscript{a},*, N. Jeannin\textsuperscript{a}, K. Djafri\textsuperscript{b}, J. Lemorton\textsuperscript{a} and D. Vanhoenacker-Janvier\textsuperscript{b}

\textsuperscript{a}ONERA, Toulouse, France
\textsuperscript{b}UCL, ICTEAM, Bâtiment Maxwell, Louvain-la-Neuve, Belgium

Abstract. Microwaves propagation modelling in clear air troposphere (i.e. without rain) is investigated. Large scale variations of refractivity are computed from mesoscale meteorological modelling. Small scale variations are deduced from large scale considering that the inertial regime of Kolmogorov spectrum is established. The propagation effects are estimated applying launching ray to take into account large scale refractivity effects and resolution of Parabolic Wave Equation with Multiple Phase Screen technique for small scale. The proposed approach has been evaluated versus earth satellite measurements of log-amplitude scintillation measured at Louvain-la-Neuve.

Keywords: Tropospheric scintillation, propagation effect, Parabolic Wave Equation, Multiple Phase Screen

1. Introduction

In the troposphere, variations of refractive index at small and large scales (i.e. from hundreds of metres to several kilometres for the large scale, from few centimetres to hundreds of meters for the small scale, along the propagation path) induce refraction and scintillation effect on radio-wave propagation. They have an impact on different space-borne remote sensing systems, Earth Observation systems or satellite telecommunication systems such as: Radio-occultation (Fig. 1 path a), SAR systems (Fig. 1 path b), Earth – satellite link (Fig. 1 path c), UAV link or propagation at very low angle (Fig. 1 path d). Large scale index variations are impacting electromagnetic systems for long propagation path (typically of more than about ten kilometres at low elevation). This variation is assumed to be the stable part of the atmosphere with respect to time and space and can be characterised by average pressure, temperature and humidity values. Values of these parameters can be computed by meteorological mesoscale models by using forecasting or computational analysis of past situations. Those models compute the state of the atmosphere through the modelling of the mechanical and thermodynamical processes, with boundary conditions given by observations or outputs from coarser scale meteorological models. Non hydrostatic models like WRF (Weather Research and Forecasting) used in this study enables a computation in 3D at resolution approaching 1 km in the horizontal plane and some tens of meters in the first layers of the atmosphere. From these inputs, large scale effects of propagation can then be computed. On the other hand, the turbulent part of troposphere (i.e. turbulence or small scale refractivity variations) induces fast fluctuation of log-amplitude (ratio, expressed in dB, of the instantaneous amplitude of the observed signal to the mean amplitude) and phase of the signal, also called scintillation, due to electromagnetic wave propagation through the turbulent atmosphere. Signal scintillation increases with frequency and becomes important from X band and above. Asymptotic and semi-empirical models have been proposed to describe expected statistics of log amplitude and are valid in weak scattering regime. These approaches allow modelling the second moment of the log-amplitude and phase of the signal. As it is difficult to characterize the turbulent structure constant $C_n^2$ (in m$^{-2/3}$) determining the turbulence strength along the propagation path [3], semi-empirical...
models have been derived. Numerous models can be found in the open literature, such as: ITU-R model of recommendation 618-10 [8], Karasawa model [4], Ortigies models [6], Otung model [7], Marzano models [5] or van de Kamp models [15]. These models are based on regression of the log-amplitude Probability Density Function (PDF) and variance on earth satellite link measured data. All these models are based on the Rytov-Tatarskii-Ishimaru [3, 11, 14] work and have been fitted on data essentially for elevations around $30^\circ$.

In this paper, a new approach is investigated composed of three main parts: first, the modelling of stable propagation effects applying launching ray (based on Bouguer’s rule) for the large scale refraction effects, second PWE-MPS (Parabolic Wave Equation - Multiple Phase Screen) method for scintillation effect and third NWF model (Numerical Weather Forecasting) for medium characterisation. Section 2 begins by describing briefly the propagation modelling in stable troposphere. The main equations of PWE-MPS technique are then reminded, and a test case of comparison to asymptotic formulas is lastly presented. In Section 3, the medium characterisation using NWF model is described. An example of comparison between modelled pressure, temperature and humidity profiles with radiosondes measurements is proposed. The approach developed to derive $C_n^2$ the structure constant of turbulence is briefly discussed. Finally, the ability of the proposed approach to reproduce existing experimental measurements carried out in Louvain-la-Neuve measuring OLYMPUS beacon signal is tested and the results validate the proposed approach. Encouraging results for the relevance of the proposed approach are found considering those data.

2. Modelling of tropospheric effects

2.1. Stable troposphere

Propagation of electromagnetic wave in the stable medium is modelled by an iterative ray launching based on Bouguer’s approach [1]. The atmosphere is implicitly considered as stratified in altitude and the iterative process is applied versus a constant step range along the path. This approach enables to get the geometric path of the ray: considering atmosphere variables (pressure, temperature, humidity) known in a 3D volume, the refractive index can be computed and the iterative launching ray applied to model the electromagnetic wave path. The geometry is referenced to the centre of earth (cf. Fig. 2).

From values of pressure, temperature and humidity, gaseous attenuation is computed along the propagation path applying the International Telecommunication Union (ITU) recommendation 676-9 [9].

2.2. Turbulent troposphere

Turbulent troposphere can be statistically characterised by the refractive index autocorrelation function or by its Fourier transform (spectrum) with respect
to space or time. Considering the von Karman-Kolmogorov spectrum [3], the turbulence strength is described by the structure constant \( C_n^2 \) and the outer and inner scales of turbulence are \( L_w \) and \( l_k \) respectively. \( L_w \) varies from \( \sim 1 \) m to \( \sim 550 \) m when \( l_k \) is a few millimetres.

In the proposed approach, the scintillation of propagated wave is computed using Parabolic Wave Equation (PWE) resolution introducing Multiple Phase Screen (MPS). The PWE-MPS is a resolution of the scalar wave Helmholtz equation only considering forward propagation and taking into account turbulence effect via phase screen generations. For more details, the reader should refer to [10] and [2].

As large scale refraction effect is modelled by launching ray, \( n_0(x, y, z) \) is assumed to be equal to 1. The 3D PWE method is based on scalar Helmholtz equation and can be written:

\[
\frac{\partial}{\partial x} \left( E(x, y, z) \right) = jk_0 \left[ 1 + \frac{1}{k_0^2} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] + n_0(x, y, z) E(x, y, z) \tag{1}
\]

where \( x \) is the propagation axis, \( y \) and \( z \) are transversal dimensions, \( E \) the scalar field component (electric or magnetic) and \( k_0 \) the wave number in free space. The PWE is solved iteratively with respect to the forward propagation path applying the Split-Step Fourier approach (SSF) [3]. This method can be summarized by the equation:

\[
E(t_0 + \delta x, y, z) = \int_{-\infty}^{+\infty} dk_x dk_y e^{j(2\pi f \delta t - k_x x)} E(t_0, y, z) e^{-j\phi(x, y, z)} \tag{2}
\]

where along \( x \) the range step is \( \delta x \), \( y \) and \( z \) are the transverse coordinates, \( k_x \) and \( k_y \) are the spatial wave number dual of \( y \) and \( z \) in Fourier space. For one step forward, the turbulent propagation index \( n_1 \) is integrated over the propagation step by generation of a phase screen \( \phi(y, z) \) [10]:

\[
\phi(y, z) = \int \left[ \frac{\xi}{n_1(\xi, y, z)} \right] d\xi. \tag{3}
\]

To generate the phase screens, the Kolmogorov-von Karman spectrum is used, defined by [18]:

\[
S^{Dc}_n(k_x, k_y, k_z) = 0.033 C_n^2 \left( k_x^2 + k_y^2 + k_z^2 \right)^{11/6} \tag{4}
\]

and induces the phase screen spectrum [2]:

\[
S^{Dc}(k_x, k_y) = 2\pi \xi S^{Dc}_n(k_x = 0, k_y, k_z) \tag{5}
\]

where \( K_{ref} = 2\pi/L_{ref} \) with \( L_{ref} \) is the outer scale of the turbulence, and \( C_n^2 \) the turbulent structure constant. In this paper, the equations are given for a 3D Problem but a 2D resolution is also possible. 2D and 3D PWE-MPS method allow generating electromagnetic field realisations in terms of amplitude and phase. 2D or 3D fields can be computed but 2D resolution induces a slight underestimation of log-amplitude variance in Fresnel regime [2, 10]. The antenna pattern can be considered in the modelling, and variations of \( C_n^2 \) or \( L_{ref} \) along the propagation path can be taken into account. The PWE-MPS is theoretically valid for weak and strong scattering.

Examples of 3D PWE-MPS realisations are presented in term of power density spectrum for log-amplitude and phase in Figs. 3 and 4 respectively. The configuration corresponds to a Fresnel turbulent regime, with a frequency of 30 GHz, a path length in turbulence \( R = 3 \) km with \( C_n^2 = 10^{-12} \) m\(^{-2/3} \) and \( L_{ref} = 100 \) m, and the wind velocity component transverse to the propagation direction \( u_w = 10 \) m/s. Taylor’s hypothesis (i.e. the medium is moved frozen at the mean wind velocity) is assumed. Under the weak fluctuation assumption, Rytov, Tatarskii and Ishimaru [3, 11, 14] have derived asymptotic formulations of the temporal Power Spectral Density (PSD) of the log-amplitude and phase \( W_p^0 \) and infinity \( \{ W_p^\infty_n \} \). Their analytical expressions in Fresnel regime are given in Equation 7.
with \( <\chi^2>_{3D\text{Fresnel}} \) the log-amplitude variance in \( N_p^2 \), \( f \) the temporal frequency in Hz, \( f_F \) the Fresnel frequency given by \( f_F = v_s/\sqrt{2\pi\lambda R} \) in Hz and the corner frequency \( f_c \) in Fresnel regime is given by \( f_c = 1.43 f_F \) in Hz. For the Fresnel turbulence regime, the log amplitude variance in \( N_p^2 \) is estimated by the asymptotic formulation [14]:

\[
<\chi^2>_{3D\text{Fresnel}} = 0.307C_p^2\lambda^{5/6}R^{11/6} \tag{7}
\]

With \( R \) the turbulence layer thickness. As shown in Figs. 3 and 4, log-amplitude and phase generation derived from 3D PWE-MPS scheme match fairly well the asymptotic formulae.

3. Tropospheric medium characterisation

3.1. Refractivity modelling

The non-hydrostatic numerical weather prediction model WRF-ARW [12] has been used to describe large scale refractivity variations in the troposphere. CFSR (Climate Forecast System Reanalysis) reanalysis data have been used for the initialization and the boundary conditions of the model. The grid nesting is chosen such as each sub grid can have a spatial and temporal resolution of 1/3 or 1/4 with respect to the parent grid. A run of WRF initialised by concurrent CFSR data has been executed on different days over the area of Louvain-la-Neuve, illustrated in Fig. 5.

The profiles of temperature, pressure and humidity deduced from radiosondes launched near Louvain-la-Neuve have been compared to the profiles that are extracted from the simulation. In Figs. 6 to 8, examples of temperature, pressure and humidity profiles computed and measured the 02/07/90 at 11H00 are reported. The horizontal displacement of the radiosonde balloon has been taken into account in the processing of mesoscale model outputs. This comparison with concurrent data demonstrates a capability of the model to reproduce observed fluctuations, even for humidity. These outputs allow computing matrix of refractivity covering all the area of Louvain-la-Neuve that can be used for refraction and tropospheric attenuation modelling.
3.2. Turbulence modelling

Equation (5) has to be parametrised in term of the turbulent structure constant \( C_n^2 \) and outer scale \( L_0 \). First, \( L_0 \) is assumed greater than the Fresnel turbulent diameter \( \sqrt{\lambda R} \) (with \( \lambda \) the wavelength and \( R \) the propagation path in turbulence). This hypothesis induces a Fresnel turbulence regime (or diffraction regime). Second the Kolmogorov spectrum is assumed valid over some kilometres in the horizontal plane. Then \( C_n^2 \) is computed in 2D considering horizontal plane at different altitudes, this 2D approach has been chosen because of problems induced by the sampling variations in altitude of NWF outputs. This limitation has to be investigated in further work. From these hypotheses, the local variance of the refractivity \( \sigma_n^2 \) is computed from mesoscale NWP models outputs, and the turbulent structure constant is estimated by:

\[
C_n^2 \approx \sigma_n^2 \left( \frac{L_0}{0.033} \right)^{5/3} \int_{K_{\min}}^{K_{\max}} K^{-5/3} dK
\]  

(8)

Where \( K \) is the 2D spatial wave number in the horizontal plane, \( K_{\max} = \frac{2\pi}{l} \) and \( K_{\min} = \frac{2\pi}{L} \), \( l \) is the resolution of the NWP data, \( L \) is the size of the sliding window. An example of \( C_n^2 \) computation applying this approach is represented on Fig. 9. The turbulent structure constant is computed at an altitude of 2100 m in the Pyrenees region. The logarithm of the turbulent structure constant is represented in color for an area of 120 km × 120 km. The outputs of WRF are processed considering a sliding widow of size \( L \) equal to 5 km, and a resolution \( l \) of 1 km.

4. Comparison with scintillation measurements

The proposed approach has been used to reproduce the configuration from Earth-satellite link measurements collected during the OLYMPUS experiment [16, 17]. The standard deviation of the signal at 30 GHz is computed every 10 minutes and compared to the measured one. Figures 9 and 10 provide illustrations of measured and modelled standard deviation of the log-amplitude signal versus UTC time. Each figure
Fig. 9. Example of turbulence structure constant computation (represented in color and log) obtained from WRF modelling applied in the Pyrenees region.

Fig. 10. Example of winter day scintillation measurements and modelling at 30 GHz, the 06/02/1990 in Louvain-la-Neuve. This corresponds to a different condition: Fig. 10 represents results in a winter period with low scintillations, and Fig. 11 in summer with a rather high level of scintillation.

The comparison with measurements demonstrates the capability of the model to reproduce the scintillation pattern. The results presented are very encouraging but further validations are necessary, considering longer periods.

5. Conclusion

A novel approach has been developed to model coherent propagation effects in clear sky. The characterisation of the medium (stable and turbulent component) is performed using the non-hydrostatic numerical prediction model WRF. From grids of pressure, temperature and humidity, refractivity and turbulent structure constant are computed. The propagation of electromagnetic wave is computed by launching ray in the stable part of the medium, and by PWE-MPS method for the turbulent part. The developed approach can be applied to different remote sensing and telecommunications configurations. The proposed approach has been validated theoretically against analytical approaches and exhibits good trends when it is compared to data. Further complementary validations have to be performed, in earth satellite configurations but also for other remote sensing configurations such as radio occultation.

Acknowledgments

The presented work has been carried out in the framework of contract ESTEC22797/09/NL/LvH “Advanced Modelling of Coherent Propagation Effects on Active Microwave Remote Sensing.” The authors would like to thank European Space Agency for its support.

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