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1 Introduction

The recent developments in digital technologies (e-commerce, social media and networks, mobile computing, sensor technologies) have not only driven individuals to leave an increasingly long digital trace behind them, but have also made available the tools to assemble, harness and analyse large and complex datasets (so-called ‘Big data’). As a consequence, firms are now able to target advertising, product offerings and prices to their customers with an unprecedented precision.

When it comes to prices, firm’s enhanced ability to price discriminate implies a reduction in consumer surplus. Yet, the same technological developments have also enabled individuals to protect their privacy (e.g., by erasing their digital trace or by concealing their actions online). Although one would expect that such countermeasures would restore (at least part of) the lost consumer surplus, we show in this note that the opposite may actually happen. Adding insult to injury, the use of privacy-protecting technologies may decrease consumer surplus even further.

We establish this point in a monopoly setting where the firm has access to a ‘tracking’ technology that allows it to identify the willingness to pay of its consumers with some probability; the firm then charges personalized prices to the consumers it identifies and a common regular price to the consumers it does not identify. Consumers have the possibility to acquire a ‘hiding’ technology that makes the firm’s tracking technology inoperative. Our main result is to show that consumer surplus is larger when this hiding technology is not available. In fact, when the technology is available, the firm has an incentive to limit or eliminate its use by raising the regular price of its product. As a result, what some consumers gain by protecting their privacy is more than offset by what the other consumers lose by paying a higher price.

Compared to the existing literature on privacy (see Acquisti et al., 2015, for a comprehensive and recent survey), the simple setting adopted in this note leaves aside a number of important features: price competition (as, e.g., in Taylor and Wagman, 2014, or in Montes et al., 2015), repeat purchases (as, e.g., in Conitzer et al., 2012), or data intermediaries (as, e.g., in Bergemann and Bonatti, 2015). However, this setting is novel in that it considers a tracking technology whose degree of precision can range between no and full identification of the consumers (in contrast with the existing lit-
erature that only considers the two extreme cases). It also considers costs of protecting privacy that can either be constant or heterogeneous across consumers. We examine the former case in Sections 3 and 4, and the latter case in Section 5; before that, we present the model in Section 2.

2 The model

A monopolist produces some product at a constant marginal cost, which is set to zero for simplicity. A unit mass of consumers have a unit demand for the monopolist’s product. A consumer’s valuation for the product, noted $r$, is supposed to be drawn from the uniform distribution on $[0,1]$.

The monopolist can have access to a ‘tracking technology’ that allows it to identify the valuation of a consumer with probability $\lambda$ (with $0 \leq \lambda \leq 1$). The parameter $\lambda$ can be interpreted as the precision of the tracking technology. In terms of pricing, this means that with probability $\lambda$, the monopolist knows the valuation of consumer $r$ and charges this consumer a personalized price $p(r) = r$ (which captures the consumer’s entire surplus), whereas with probability $(1 - \lambda)$, the monopolist does not know the consumer’s valuation and charges then a ‘regular’ price $p$. Arbitrage is supposed to be impossible or prohibitively costly.

Consumers have access to some ‘hiding technology’ that allows them to prevent the monopolist from discovering their valuation. The technology is assumed to have the following simple form: by paying a cost $c$, any consumer can make sure that the monopolist cannot identify her valuation, whatever the precision of its detection technology.

We analyse the following three-stage game. First, the monopolist decides whether or not to use the tracking technology. Second, the monopolist sets its prices (i.e., the regular price $p$ and, possibly, a schedule of personalized prices $p(r)$), while consumers decide whether or not to acquire the hiding technology. Third, consumers observe the price that the monopolist charges them and decide whether or not to buy the product. The equilibrium con-

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1 An exception is Johnson (2013), who allows for gradations in information quality in his model of targeted advertising and advertising avoidance.

2 Alternatively, we can assume that each valuation $r$ is shared by a unit mass of consumers and that the technology allows the monopolist to identify a fraction $\lambda$ of those consumers.

3 As shown in Section 5, our results also hold if we assume heterogeneous hiding costs.
cept is subgame perfection.

We consider two benchmarks. First, if the monopolist does not use the tracking technology, it charges the regular price to all consumers. It is easily seen that the optimal price is $p_0 = 1/2$, which yields a profit of $\pi_0 = 1/4$ and a consumer surplus of $CS_0 = 1/8$.

Second, if no hiding technology were available, the monopolist would charge $p_0 = 1/2$ to unidentified consumers and their valuation $r$ to identified consumers. Hence the monopolist’s profit would be equal to:

$$\pi_n(\lambda) = \lambda \int_0^1 r dr + (1 - \lambda) p_0 (1 - p_0) = \frac{1}{4} (1 + \lambda).$$

The consumer surplus is computed as

$$CS_n(\lambda) = \lambda \times 0 + (1 - \lambda) \int_{p_0}^1 (r - p_0) dr = \frac{1}{8} (1 - \lambda).$$

We observe that when consumers have no way to hide their identity, the monopolist’s profit increases and the consumer surplus decreases when the precision of the tracking technology (i.e. $\lambda$) increases.\(^4\)

3 Equilibrium

Suppose that the monopolist uses the tracking technology. At stage 2, consumers anticipate that they will pay a price $p$ if they are not identified or a personalized price equal to their valuation if they are. Hence, any consumer $r$ with $r \geq p$ will have a surplus of $(1 - \lambda) (r - p)$ if she does not acquire the hiding technology and a surplus of $r - p - c$ if she does. Comparing the two options, we find that it is worth acquiring the hiding technology if and only if $c \leq \lambda (r - p)$, i.e., if the cost of hiding one’s valuation ($c$) is inferior to the benefit of hiding it (i.e., to keep the surplus $r - p$ when the tracking technology would discover one’s valuation if it is not hidden).\(^5\) The latter

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\(^4\) Social welfare, equal to $(3 + \lambda)/8$, increases with $\lambda$ because improved price discrimination reduces the deadweight loss. Belleflamme and Peitz (2015, Section 8.2) obtain the same results by assuming an alternative tracking technology that allows the monopolist to partition the $[0, 1]$ interval into $n$ subintervals of equal length and to identify to which subinterval the valuation of a consumer belongs. With this technology, the monopolist can practice third-degree price discrimination and achieve a profit of $\pi(n) = 1/2 - (2n - 1)/(4n^2)$, which clearly increases with $n$. Setting $\lambda = (n-1)^2/n^2$ makes the two models equivalent in terms of profits.

\(^5\) Consumers with $r < p$ do not find it profitable to hide: if they do not hide, their surplus is zero, whereas if they hide, their surplus is $-c$ (as they do not buy the good).
inequality can be rewritten as \( r \geq r_c(p) \equiv p + c/\lambda \). Noting that \( r_c(p) < 1 \) as long as \( p < 1 - c/\lambda \), we see that the monopolist faces the following alternative: either it sets any \( p \geq 1 - c/\lambda \) so that no consumer finds it profitable to acquire the hiding technology, or it sets \( p < 1 - c/\lambda \) and accepts that some consumers hide their valuation.

Consider the first option. As no consumer hides her identity, the analysis of the previous section applies. Optimally, the monopolist would set \( p_0 = 1/2 \) for the consumers that it cannot identify. This price satisfies the constraint as long as \( p_0 \geq 1 - c/\lambda \) or \( c \geq \lambda/2 \). In that case, the hiding technology is so expensive that the monopolist can ignore it and keep its pricing decisions unchanged. The monopolist achieves then the profit \( \pi_n(\lambda) = (1 + \lambda)/4 \). In contrast, if \( c < \lambda/2 \), then the monopolist needs to set \( p_1 = 1 - c/\lambda \) to unidentified consumers to discourage consumers from hiding. The monopolist’s profit is computed as

\[
\pi_1(c, \lambda) = \frac{1}{2\lambda} \left( -2 (1 - \lambda) c^2 + 2\lambda (1 - \lambda) c + \lambda^3 \right).
\]

Consider now the second option, where some consumers decide to hide. The monopolist’s maximization problem becomes

\[
\max_p \lambda \int_0^{r_c(p)} r \, dr + (1 - \lambda) \int_p^{r_c(p)} p \, dr + \int_{r_c(p)}^1 p \, dr \quad \text{s.t.} \quad r_c(p) \leq 1.
\]

Using the definition of \( r_c(p) \) and developing, we can rewrite the problem as

\[
\max_p \lambda \left( p + \frac{c}{\lambda} \right)^2 + p (1 - p - c) \quad \text{s.t.} \quad p \leq 1 - \frac{c}{\lambda}.
\]

From the first-order condition, we find the unconstrained optimum as \( p_2 = 1/(2 - \lambda) \).\(^6\) We observe that for \( \lambda > 0 \), the monopolist charges a larger regular price when some consumers find it profitable to hide their valuation: \( p_2 > p_0 = 1/2 \). Two reasons motivate this behavior: on the one hand, raising \( p \) discourages hiding (as \( r_c(p) \) increases with \( p \)); on the other hand, hiding consumers are high-valuation consumers and it is profitable for the monopolist to charge them a higher price. The unconstrained optimum satisfies the constraint as long as \( p_2 \leq 1 - c/\lambda \), which is equivalent to \( c \leq \)

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\(^6\)It appears that \( p \) and \( c \) play independent roles in the profit function; as a result \( p_2 \) does not depend on \( c \). This is an artifact of the linearity of the model.
λ(1−λ)/(2−λ) ≡ ˜c(λ). Otherwise, the monopolist chooses the corner solution \( p = 1 - c/λ \) and we are back in the case examined previously. When \( c \leq ˜c(\lambda) \), the monopolist’s profit is computed as

\[
\pi_2(\lambda, c) = \lambda \frac{1}{2} \left( p_2 + \frac{c}{\lambda} \right)^2 + \lambda \frac{1}{2} \left( 1 - p_2 - c \right) = \frac{\lambda + (2 - \lambda)c^2}{2\lambda(2 - \lambda)}.
\]

**In sum**, the monopolist always charges a price equal to their valuation to the consumers that it can identify. As for the regular price charged to unidentified consumers, three cases are possible according to the cost of the hiding technology. First, if \( c \geq \lambda/2 \), no consumer acquires the technology and the monopolist sets \( p_0 = 1/2 \). Second, if \( \tilde{c}(\lambda) \leq c < \lambda/2 \), no consumer acquires the technology and the monopolist sets \( p_1 = 1 - c/\lambda \). Finally, if \( c < \tilde{c}(\lambda) \), consumers with a high valuation acquire the hiding technology and the monopolist sets \( p_2 = 1/(2-\lambda) \). As illustrated in Figure 1 (and formally shown in Appendix 7.2), the equilibrium profit increases with the cost of the hiding technology (\( c \)) and increases with the precision of the tracking technology (\( \lambda \)).

We still need to check whether using the tracking technology brings a larger profit than not using it. We show in Appendix 7.1 that it is indeed the case: \( \pi_1(\lambda, c) > \pi_0 \) and \( \pi_2(\lambda, c) > \pi_0 \) in the range of parameters where these profit levels are relevant.\(^7\)

## 4 How does hiding affect consumers?

We saw in Section 2 that, absent any hiding technology, improved tracking (i.e., larger \( \lambda \)) allows the monopolist to increase its profit by grabbing a larger share of the consumer surplus (and also by reducing the deadweight loss). We now want to evaluate the extent to which the availability of the hiding technology challenges this result. A priori, we expect a move in the opposite direction: the possibility to hide one’s valuation should reduce the monopolist’s profit and increase the consumer surplus. The first point has already been established above as the equilibrium profit was shown to increase with \( c \), meaning that a cheaper hiding technology results in lower profits for the monopolist. As for the second point, however, our initial guess

\(^7\)We assume that the tracking technology is available for free. Posing a positive fixed cost would not add any useful insight to the analysis.
actually proves wrong: as stated in the following proposition, consumers prefer, on aggregate, that none of them acquires the hiding technology!

**Proposition 1** Consumers are collectively better off when the hiding technology is not available (or, equivalently, when its cost is large enough for no consumer to find it profitable to acquire it).

**Proof.** See Appendix 7.3. ■

The result is illustrated in Figure 1 where consumer surplus is measured against the cost of the hiding technology. We observe that the consumer surplus first decreases with $c$, then increases with $c$ and finally is independent of $c$. In the last segment, the consumer surplus is larger than when the hiding technology is available for free (i.e., $c = 0$).

We have already sketched the intuition for this result in Section 3. When the cost of the hiding technology is relatively low, the monopolist optimally sets a larger regular price than in the case where hiding is not a profitable option for consumers ($p_2 > p_0$). With probability $(1 - \lambda)$, all consumers who did not acquire the hiding technology pay this higher regular price and obtain therefore a lower consumer surplus. In contrast, by a simple revealed preference argument, those consumers who acquired the hiding technology obtain a larger surplus. However, what they gain does not compensate what the other consumers lose, which explains the result of Proposition 1.\(^8\)

In a nutshell, the consumers who hide their valuation are better off but at the expense of those who do not: hiding consumers exert a negative externality on the other consumers. Only an association representing all consumers would be able to internalize this externality. This association’s best conduct would be to prevent individual consumers from acquiring the hiding technology, thereby securing a consumer surplus of $CS_n(\lambda) = (1 - \lambda)/8$. The association could even improve consumer surplus further by acquiring

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\(^8\)Montes et al. (2015) reach a similar result with a similar intuition but in a different framework; they assume that the monopolist serves two groups of consumers, one it can fully identify and the other it cannot; in this setting, the consumer surplus is shown to be U-shaped in the cost of hiding. Conitzer et al. (2012) also conclude that making hiding less costly may, in certain cases, lower the consumer surplus. The intuition is, however, totally different. They derive this result in a setting with repeat purchase where identifying the consumers allows the monopolist to practice behavior-based price discrimination (BBPD). As is well known, a monopolist would like to commit not to use BBPD; the possibility for consumers to hide their past purchase at a low cost gives the monopolist the required commitment power, which increases its profit at the expense of consumers.
itself μ units of the hiding technology (with 0 ≤ μ ≤ 1) and distributing them randomly to the consumers. This would reduce the precision of the tracking technology from λ to λ (1 − μ), generating an increase in consumer surplus equal to $CS_n (\lambda (1 - \mu)) - CS_n (\lambda) = \lambda \mu / 8$. However, for such tactic to be profitable, the increase in surplus must be larger than μc, which is the total cost of the hiding technology for the association. This is so as long as $\lambda \mu / 8 \geq \mu c$ or $c \leq \lambda / 8$. As this condition is independent of μ, the association’s optimum would then be to set μ = 1, i.e., to distribute the hiding technology to each and every consumer, thereby annihilating the monopolist’s price discrimination abilities. Yet, if $c > \lambda / 8$, the consumer association cannot do better than preventing the individual use of the hiding technology.

![Figure 1: Equilibrium profit and consumer surplus](image)

### 5 Heterogeneous cost of hiding

We have assumed so far that all consumers face the same cost of hiding their valuation. This assumption suits well the idea of some technology that can be installed off the shelf. In other cases, it may be more reasonable to assume that consumers differ in their (opportunity) cost of hiding. To account for this, we assume here that each consumer is characterized not only by her valuation $r$ for the product, but also by her cost $c$ of hiding this valuation, with $c$ being drawn from a uniform distribution on $[0,1]$, independently from $r$.\footnote{This assumption is made for simplicity. It seems reasonable if $c$ is seen to reflect a consumer’s ability to, e.g., erase cookies or install some software on her computer. Alter-}
As before, a consumer would find it profitable to hide if \( r - p - c \geq (1 - \lambda) (r - p) \), which is equivalent to \( r \geq p + c/\lambda \). As the monopolist will not set a price above 1, there is always a mass of consumers who decide to hide. This mass is computed as \( H(p, \lambda) = \lambda (1 - p)^2/2 \). The monopolist’s profit can then be expressed as (see Figure 2):

\[
\pi(p, \lambda) = (1 - \lambda) \left( \int_0^{\lambda(1-p)} \int_p^{p+c/\lambda} dr \, dc + \int_{\lambda(1-p)}^{1} \int_p^1 dr \, dc \right) p
+ \lambda \left( \int_0^{\lambda(1-p)} \int_0^{p+c/\lambda} r \, dr \, dc + \int_{\lambda(1-p)}^{1} \int_0^1 r \, dr \, dc \right) + H(p, \lambda) p
= \frac{1}{6} \left( 2\lambda^2 p^3 + 6 \left( 1 - \lambda + \lambda^2 \right) p (1 - p) + \lambda (3 - 2\lambda) \right).
\]

The first-order condition for profit maximization is a second-order polynomial in \( p \), which has two positive roots. The second-order condition designates the smaller root as the profit-maximizing price (for \( 0 < \lambda < 1 \)):

\[
p^* = \frac{1}{\lambda^2} \left( 1 - \lambda + \lambda^2 - \sqrt{(1 - \lambda) (1 - \lambda + \lambda^2)} \right).
\]

It is easily shown that \( p^* \geq 1/2 \): as in the previous setting, the monopolist reacts to hiding by raising its regular price above the price that it would set were hiding not possible. This entails that hiding has the same negative impact on consumer surplus as in the previous setting, as we now check.

The consumer surplus is computed as (see Figure 2):

\[
CS(p, \lambda) = \int_0^{\lambda(1-p)} \int_p^{p+c/\lambda} (r - p - c) \, dr \, dc
+ (1 - \lambda) \left( \int_0^{\lambda(1-p)} \int_p^{p+c/\lambda} (r - p) \, dr \, dc + \int_{\lambda(1-p)}^{1} \int_p^1 (r - p) \, dr \, dc \right)
= \frac{1}{6} (1 - p)^2 \left( 3 (1 - \lambda) + \lambda^2 (1 - p) \right).
\]

Evaluating consumer surplus at \( p^* \) and defining \( K \equiv \sqrt{(1 - \lambda) (1 - \lambda + \lambda^2)} \), we have:

\[
CS(p^*, \lambda) = \frac{1}{6} \left( \frac{1}{\lambda^2} (K - (1 - \lambda)) \right)^2 (2 (1 - \lambda) + K).
\]

A simple plot of the two expressions reveals that \( CS_b(\lambda) = (1 - \lambda)/8 > CS(p^*, \lambda) \) for all \( 0 < \lambda < 1 \). We can therefore conclude that the result natively, \( r \) and \( c \) should be correlated if, for instance, consumers with a larger willingness to pay have higher incentives to hide their identity and are therefore willing to acquire more expensive hiding technologies.
of Proposition 1 still holds under the assumption of heterogeneous hiding costs.\textsuperscript{10}

\begin{center}
\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Distribution of consumer valuations and hiding costs}
\end{figure}
\end{center}

6 Conclusion

In this note, we have shown that when a monopolist has some probability to identify the consumers’ valuation and, thereby, charge them personalized prices, the possibility for consumers to hide their valuation has the effect of reducing consumer surplus (even when hiding can be done at no cost). The reason is that the monopolist raises the regular price that it charges to unidentified consumers, which harms consumers who choose not to hide their valuation. Hiding generates thus a hidden cost as consumers who hide exert a negative externality on consumers who do not. In future research, we aim at extending our analysis to a duopoly situation; in particular, we want to allow sellers to choose the precision of the tracking technology (parametrized by $\lambda$ in our setting), a decision that existing studies (e.g., Montes et al., 2015) are ill-equipped to analyze.

\textsuperscript{10}The result holds even if we ignore hiding costs. For a given $p$, total hiding costs are equal to $T(p) \equiv \lambda^2 (1 - p)^3 / 6$. A few lines of computations establish that $CS(p^*, \lambda) - T(p^*) \leq CS_b(\lambda)$ for all $0 < \lambda < 1$. It is thus the increase in the regular price that fully explains the reduction in consumer surplus.
7 Appendix

7.1 Profitability of the tracking technology

We compute

\[
\pi_2 (\lambda, c) - \pi_0 = \frac{\lambda + (2 - \lambda) c^2}{2\lambda (2 - \lambda)} - \frac{1}{4} = \frac{\lambda^2 + 2c^2 (2 - \lambda)}{4\lambda (2 - \lambda)},
\]
\[
\pi_1 (\lambda, c) - \pi_0 = \frac{-4 (1 - \lambda) c^2 + 4\lambda (1 - \lambda) c + \lambda^2 (2\lambda - 1)}{4\lambda^2}.
\]

As \(0 \leq \lambda \leq 1\), the first expression is clearly positive. As for the second expression, the second-degree polynomial in \(c\) on the numerator has the following two roots:

\[
c_- = \lambda \left(1 - \lambda\right) - \sqrt{\frac{\lambda (1 - \lambda)}{2}} \quad \text{and} \quad c^+ = \lambda \left(1 - \lambda\right) + \sqrt{\frac{\lambda (1 - \lambda)}{2}}.
\]

Recall that profit \(\pi_1\) applies for \(\bar{c} (\lambda) \leq c < \lambda/2\). It is readily shown that (i) \(c_- < \bar{c} (\lambda)\), (ii) \(c^+ > \lambda/2\), and (iii) \(c_- > 0\) if and only if \(\lambda < 1/2\). It follows that the polynomial is positive for all admissible values of \(c\), implying that \(\pi_1 (\lambda, c) > \pi_0\) on the relevant range of parameter values.

7.2 Comparative statics of equilibrium profit

\[
\pi (\lambda, c) = \begin{cases} 
\pi_n (\lambda, c) = \frac{1}{4} (1 + \lambda) & \text{if } c \geq \lambda/2, \\
\pi_1 (\lambda, c) = -\frac{2(1-\lambda)c^2 + 2\lambda(1-\lambda)c + \lambda^3}{2\lambda^2} & \text{if } \bar{c} (\lambda) \leq c < \lambda/2, \\
\pi_2 (\lambda, c) = \frac{(2-\lambda)c^2 + \lambda}{2\lambda(2-\lambda)} & \text{if } c < \bar{c} (\lambda).
\end{cases}
\]

It is immediate that \(\pi_n (\lambda, c)\) increases with \(\lambda\) and is independent of \(c\). As for the second segment, we compute

\[
\frac{d}{d\lambda} \pi_1 (\lambda, c) = \frac{(1-\lambda)(\lambda - 2c)}{\lambda^2} \quad \text{and} \quad \frac{d}{d\lambda} \pi_1 (\lambda, c) = \frac{2(2-\lambda)c^2 - 2\lambda c + \lambda^3}{2\lambda^2}.
\]

The first expression is clearly positive as \(c < \lambda/2\) in this case; as for the second, a few lines of computations show that it is positive as well. Finally, in the third segment, we compute

\[
\frac{d}{dc} \pi_2 (\lambda, c) = \frac{c}{\lambda} \quad \text{and} \quad \frac{d}{dc} \pi_2 (\lambda, c) = \frac{(2c + \lambda(1-c))(\lambda - (2-\lambda)c)}{2\lambda^2 (2-\lambda)^2}.
\]

Both expressions are clearly positive (for the second, we have indeed that \(c < \bar{c} (\lambda)\) implies that \(\lambda - (2 - \lambda) c > 0\)).
7.3 Proof of Proposition 1

For $c \geq \lambda/2$, we have already computed $CS_n(\lambda) = (1 - \lambda)/8$, which clearly decreases with $\lambda$ and is independent of $c$. For $\tilde{c}(\lambda) \leq c < \lambda/2$, we compute

$$CS_1(\lambda, c) = \lambda \times 0 + (1 - \lambda) \int_{1 - c/\lambda}^{1} (r - (1 - c/\lambda)) \, dr = (1 - \lambda) \frac{c^2}{2\lambda^2}.$$ 

It is easily seen that the latter expression increases with $c$ and decreases with $\lambda$. Quick computations also reveal that $c < \lambda/2$ implies that $CS_1(\lambda, c) \leq CS_n(\lambda)$. As $c$ decreases, some consumers are tempted to hide their identity, which drives the monopolist to raise its uniform price so as to discourage hiding. As a result, fewer unidentified consumers decide to buy the good and those who do pay a higher price than in the case where hiding is not an option. Finally, for $c < \tilde{c}(\lambda)$, we compute

$$CS_2(\lambda, c) = (1 - \lambda) \int_{p_a}^{p_a + \frac{c}{\lambda}} (r - p_a) \, dr + \int_{p_a + \frac{c}{\lambda}}^{p_a + \frac{c}{\lambda} + \frac{\lambda}{2}} (r - p_a - c) \, dr = \frac{(2 - \lambda)^2 c^2 - 2\lambda(1 - \lambda)(2 - \lambda) c + \lambda(1 - \lambda)^2}{2\lambda(2 - \lambda)^2}.$$ 

A few lines of computations establish that $CS_2(\lambda, c)$ decreases with $\lambda$ and increases with $c$. To complete the proof, we show that consumers as a whole are worse off when $c = 0$ than when $c$ is large enough for hiding to be blockaded:

$$CS_n(\lambda) - CS_2(\lambda, 0) = -\frac{1 - \lambda}{8} - \frac{\lambda(1 - \lambda)^2}{2\lambda(2 - \lambda)^2} = \lambda^2 \frac{1 - \lambda}{8(2 - \lambda)^2} > 0.$$

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